Kagome spin liquid, Symmetry protected topological phase and Deconfined criticality

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PHYSICS
VE RI TAS
HARVARD

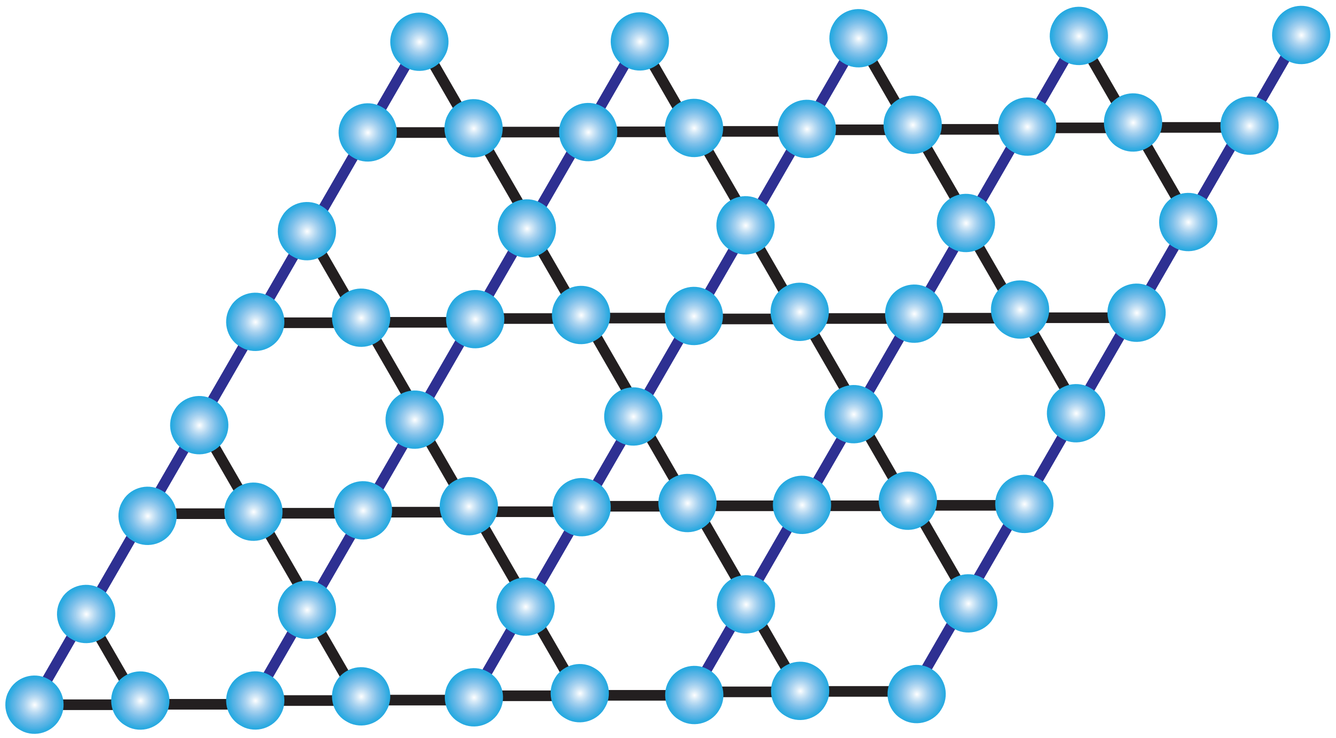
YCH, Zaletel, Oshikawa, Pollmann, to appear
YCH, Fuji, and Bhattacharjee, arXiv:1512.05381 (2015).

GORDON AND BETTY MOORE FOUNDATION

Other related work:

YCH, Bhattacharjee, Pollmann, and Moessner, PRL 115, 267209 (2015).
YCH, Bhattacharjee, Moessner, and Pollmann, PRL 115, 116803 (2015).
YCH and Chen, PRL 114, 037201 (2015).
YCH, Sheng and Chen, PRL 112, 137202 (2014).

Spin liquids on kagome lattice



Kagome Heisenberg model

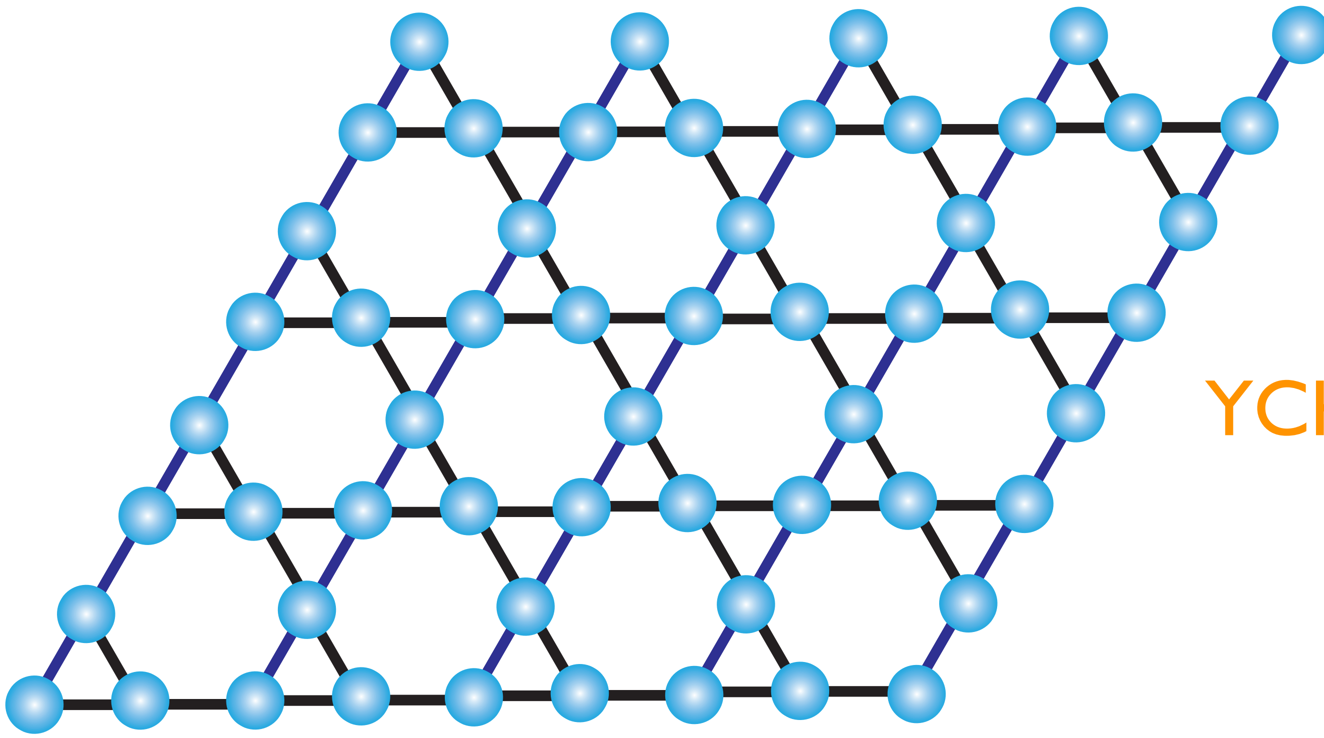
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j , \quad J > 0$$

What is the ground state?

Every possible candidate has been proposed?!

Read & Sachdev (1991); Marston & Zeng (1991); Chalker, Holdsworth, Shender (1992); Yang, Warman & Girvin (1993); Hastings (2000); Wang & Vishwanath (2006); Ran, Hermele, Lee & Wen (2007); Singh & Huse (2007); Jiang, Sheng, Weng (2008); Evenbly & Vidal (2010); Yan, Huse & White (2011); Lauchli, Sudan, Sorensen (2011); Iqbal, Becca & Poilblanc (2011); Depenbrock, McCulloch & Schollwock (2012); Jiang, Wang & Balents (2012); Xie, et. al., Xiang (2014); YCH, Sheng, & Chen (2014)….

Spin liquids on kagome lattice



Kagome spin liquid

Kagome Heisenberg model

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j, \quad J > 0$$

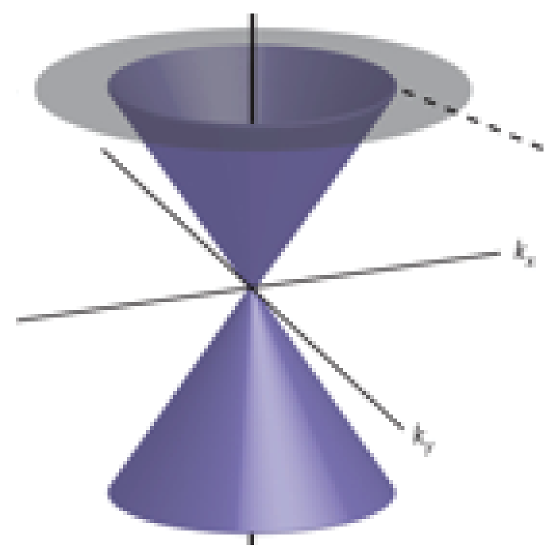
What is the ground state?

Yan, Huse, and White

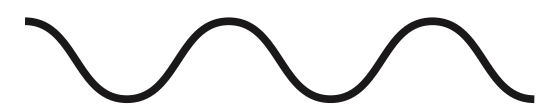
"Dirac spin liquid"
(conformal/critical phase)

YCH, Zaletel, Oshikawa, Pollmann (to appear)

QED3



a_μ



$$\mathcal{L} = \sum \bar{\psi}_i [i\gamma^\mu (\partial_\mu - i a_\mu)] \psi_i$$

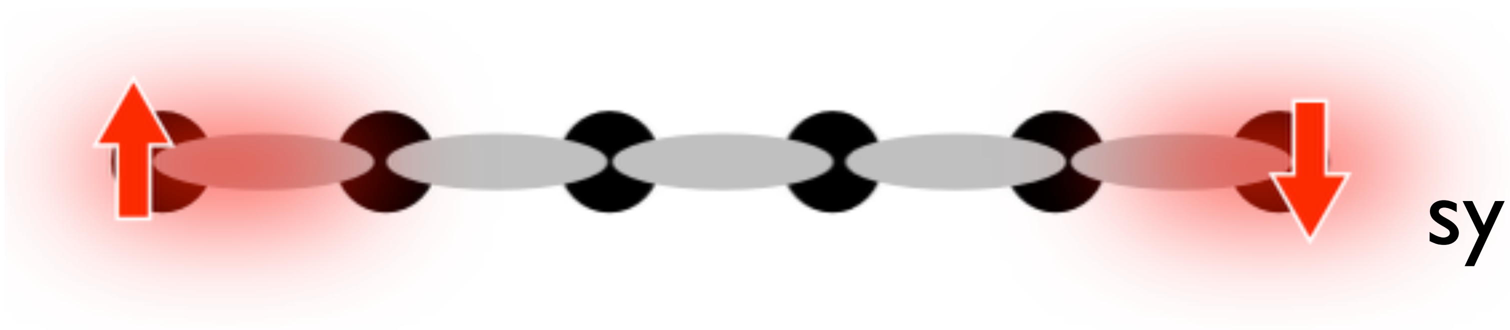
Hastings; Ran, Hermele, Lee & Wen

Symmetry protected topological phase

interacting system

Chen, Gu, Liu, Wen

1D bosonic SPT: Haldane's spin-1 chain

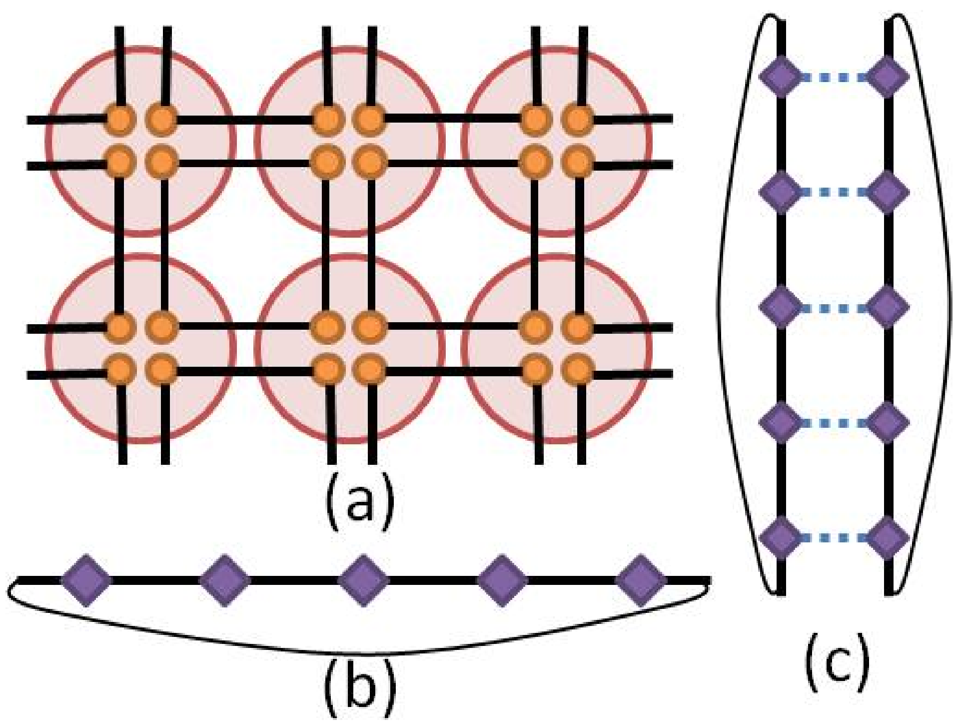


projective symmetry group

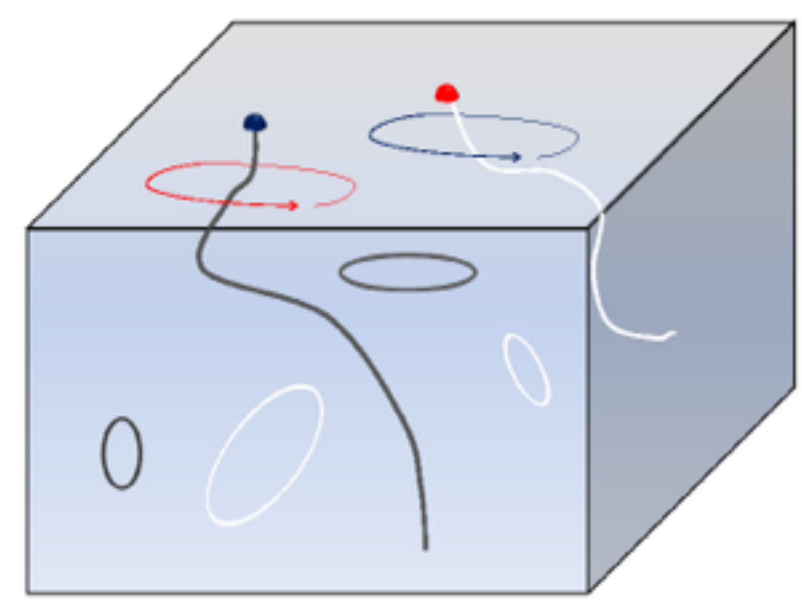
Pollmann, Turner, Berg, Oshikawa

beyond 1D: cohomology group

Chen, Gu, Liu, Wen



Chen, Liu, Wen

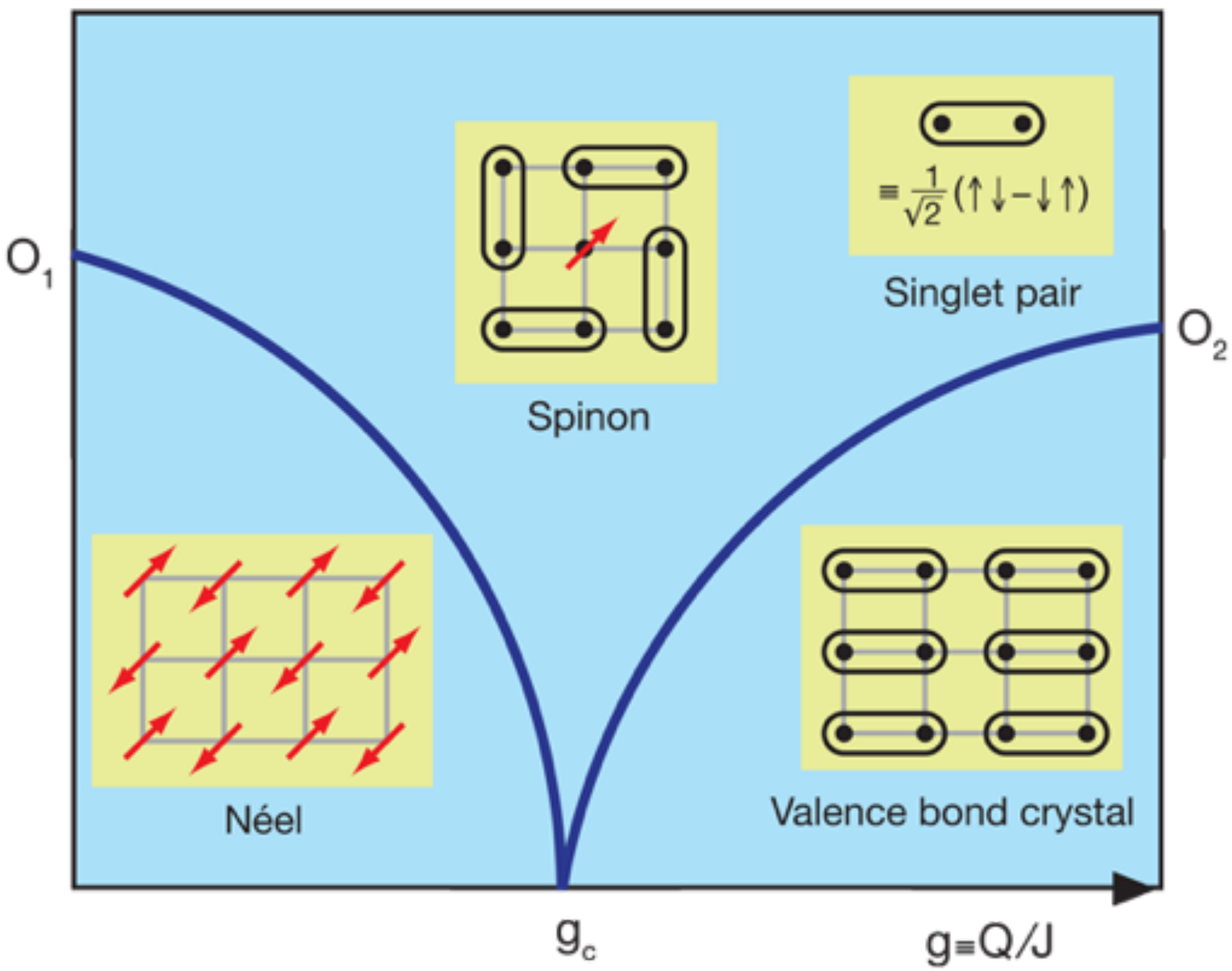


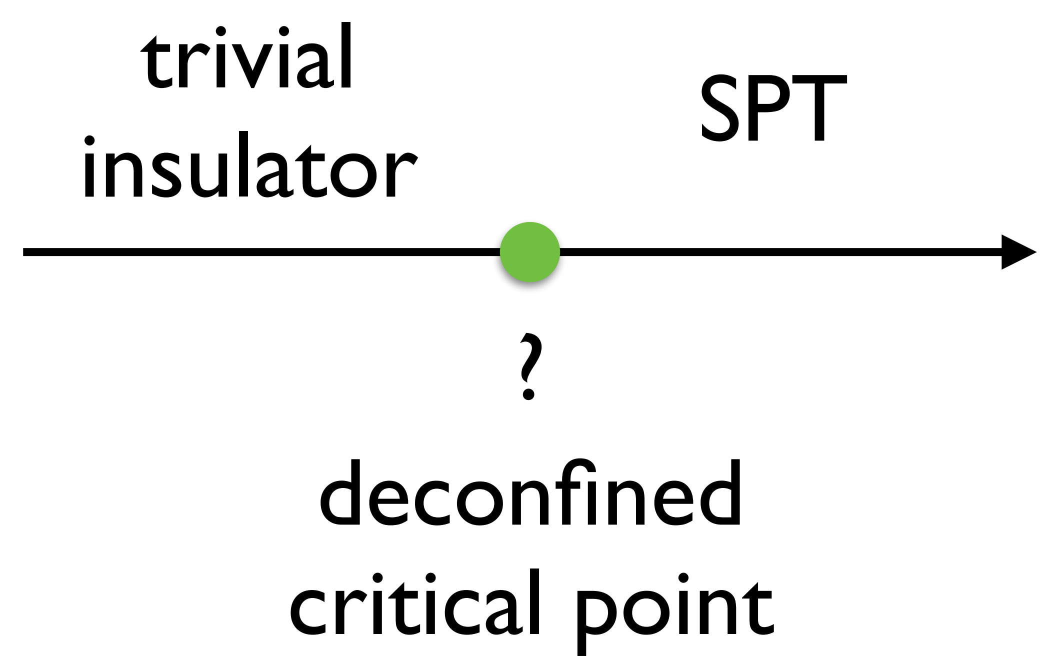
Vishwanath, Senthil

Deconfined criticality

Senthil, Vishwanath, Balents, Sachdev, Fisher

Neel to VBS



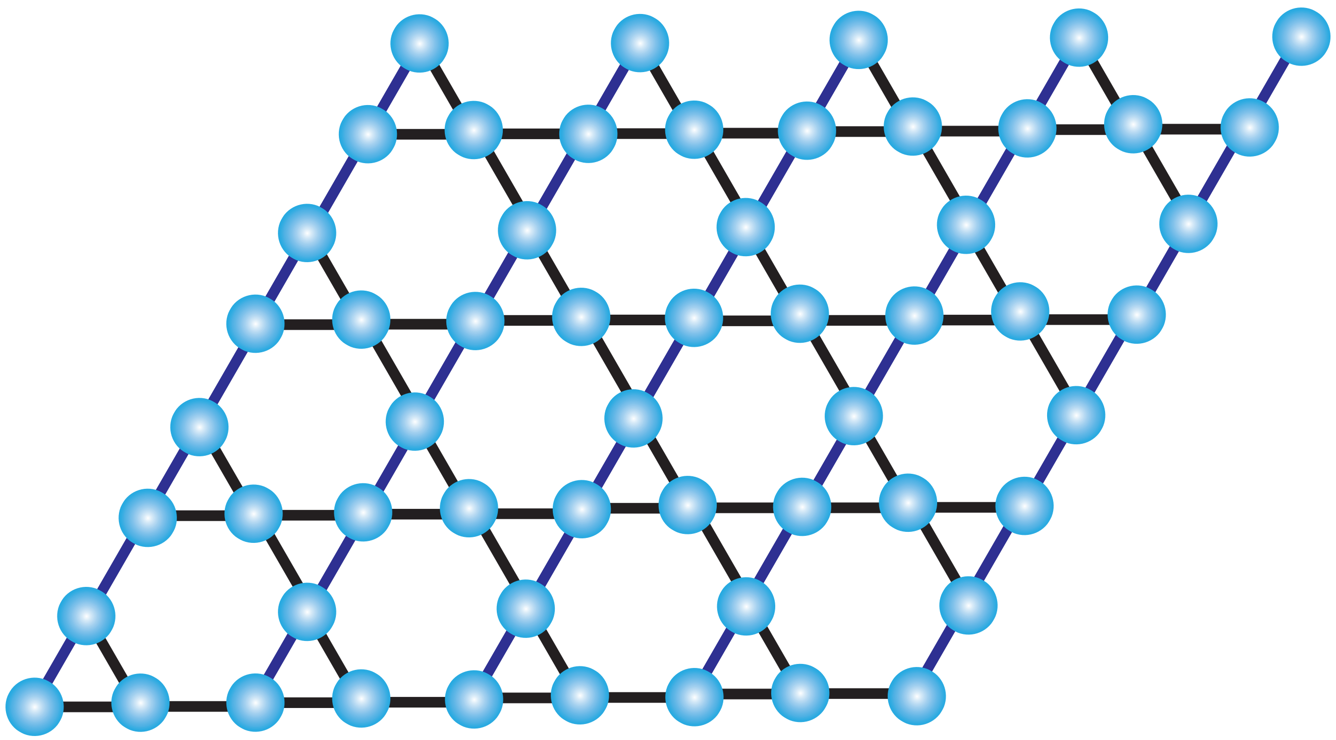


Numerics (e.g. J-Q model): Sandvik

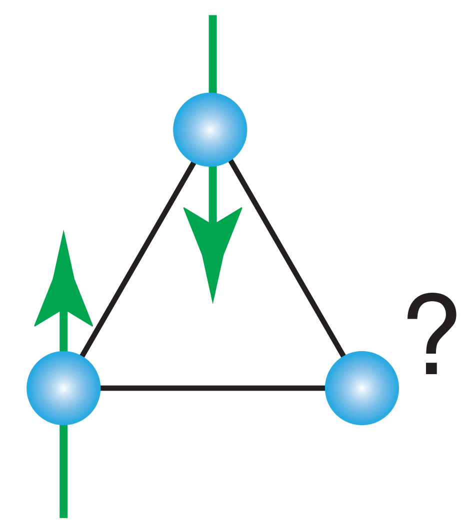
Outline

1. Introduction
2. Numerics of the kagome spin liquid: signatures of Dirac cone
3. Theory of the kagome spin liquid
   - An unbiased lattice gauge mapping
   - Lattice gauge model: symmetry protected topological phase, deconfined criticality

Spin liquid: a state without magnetic order



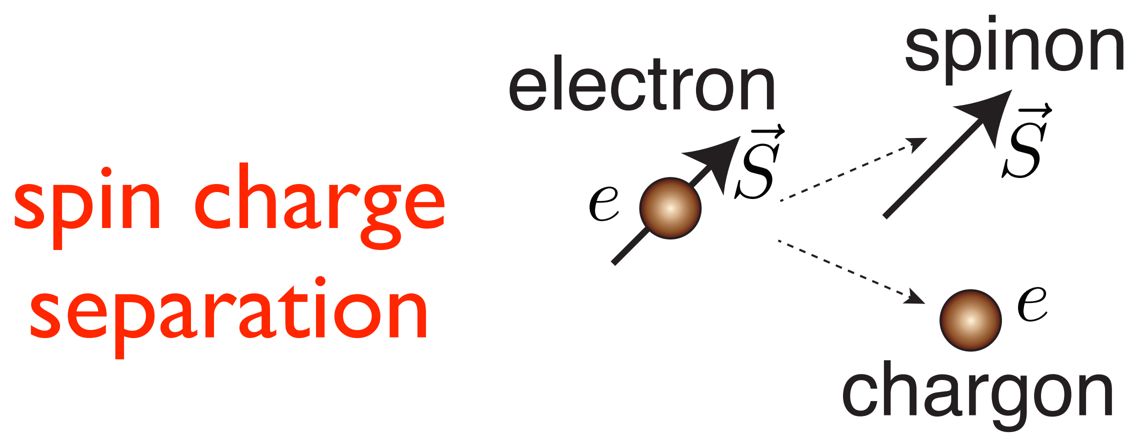
Frustration



corner sharing: large quantum fluctuation

Spin liquid: more than absence of order

- Fractionalization in 2D/3D



- Emergent gauge field: U(1), Z2……
- Fractional quasiparticles (anyon)
- Parent state of a superconductor
- …

Examples of spin liquid

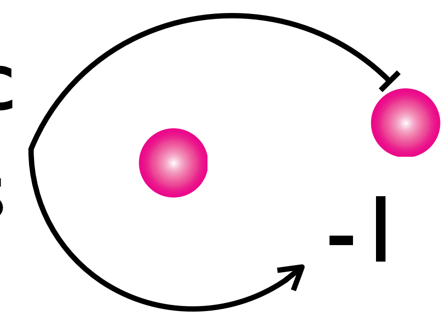
Gapped spin liquid with topological order

- Chiral spin liquid

Kalmeyer & Laughlin 1987 PRL

spinon 1/2 spin

Semionic statistics

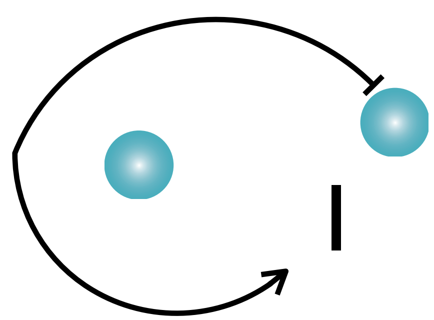


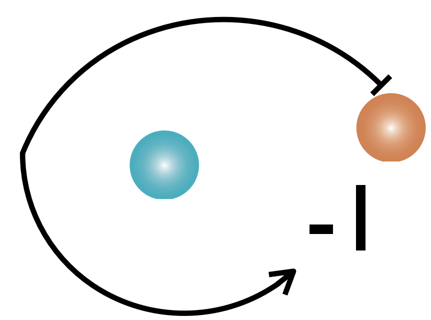
- Z2 spin liquid

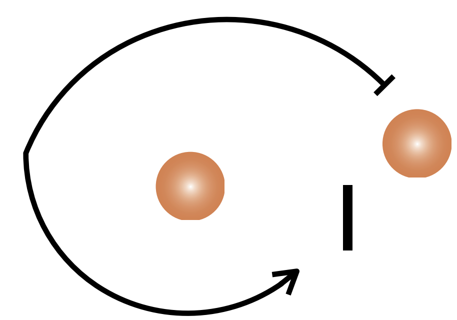
Read & Sachdev PRL 1991; Moessner & Sondhi PRL 2001…

spinon

vison





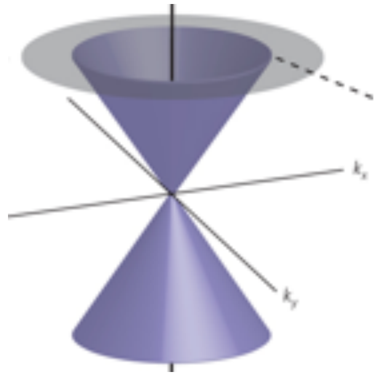


Examples of spin liquid

Gapless spin liquid

Critical matter

- Dirac spin liquid



Strongly interacting gauge theory: QED3

$$\mathcal{L} = \sum \bar{\psi}_i [i\gamma^\mu (\partial_\mu - i a_\mu)] \psi_i$$

Hastings PRB 2000; Ran, Hermele, Lee & Wen PRL 2007

Numerical tools: DMRG

DMRG: unbiased*, large system size for 2D (compared with ED)

DMRG's success in topological order

Topological degenerate GS…
Modular matrix (anyonic statistics) e.g. Cincio, Vidal
Entanglement spectrum (edge CFT)

Successful examples for DMRG:

Fractional quantum Hall state
Zaletel, Mong, Pollmann

Chiral spin liquid
YCH, et al.; Gong et al; Bauer et al.

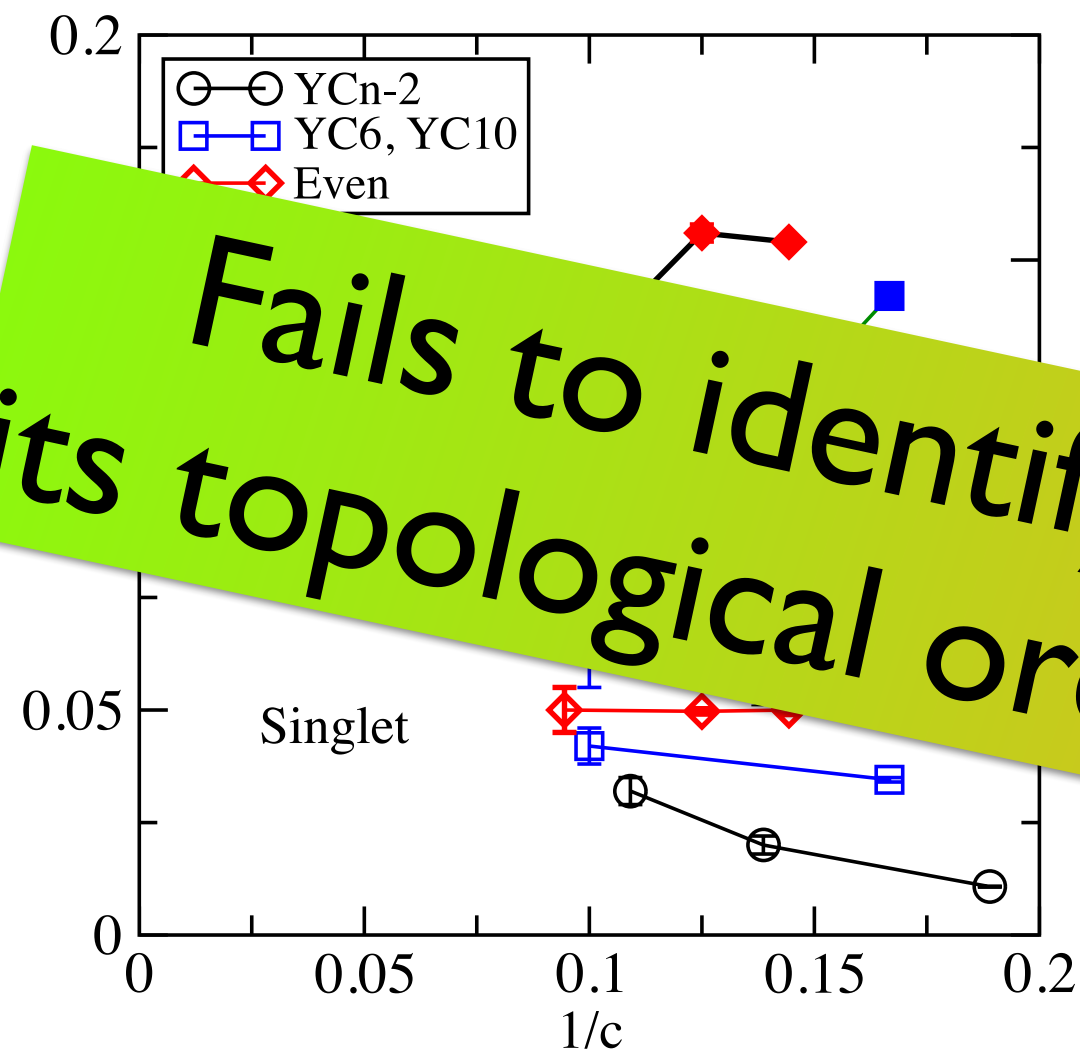
Z2 spin liquid
YCH, Sheng, Chen

Balents, Fisher, Girvin (2002)

Numerics on the kagome spin liquid

DMRG $_{\text{unbiased*}}$

Identify it as a Spin liquid



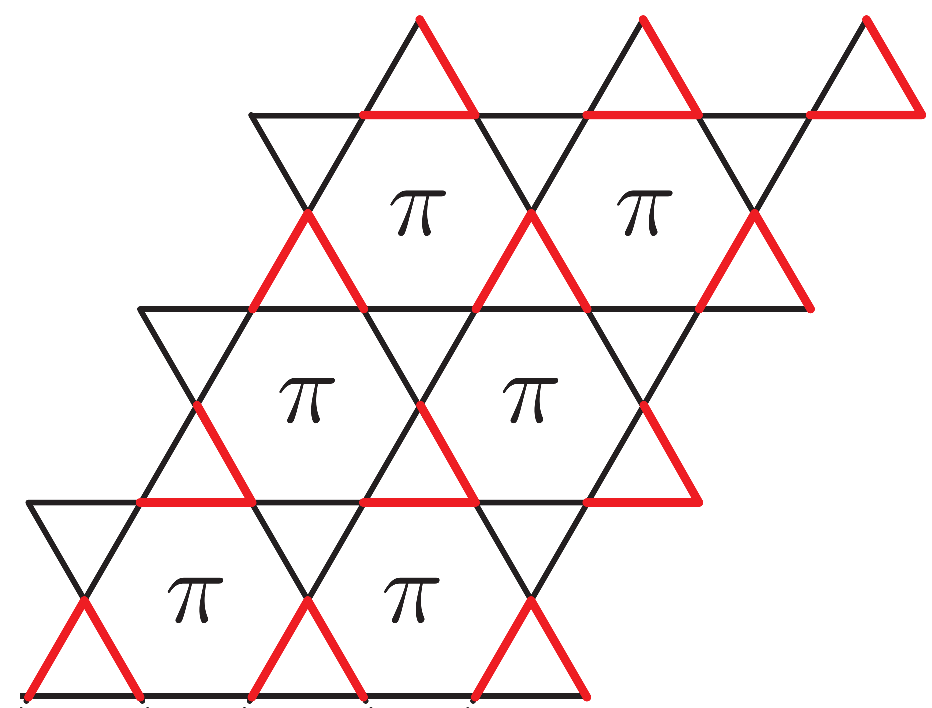
Fails to identify its topological order

Yan, Huse & White;
Depenbrock, McCulloch & Schollwock;
Jiang, Wang & Balents

VMC $_{\text{biased}}$

π-flux state, U(1) Dirac?

$$\vec{S}_i = c_i^\dagger \vec{\sigma}_i c_i \quad \langle c_i^\dagger c_j \rangle = \chi_{ij}$$

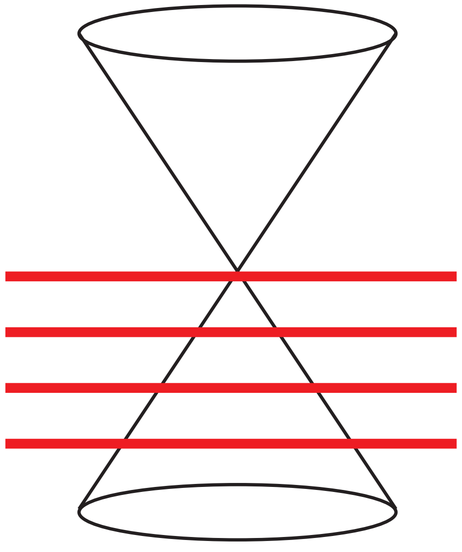


Ran, Hermele, Lee & Wen
Iqbal, Becca & Poilblanc

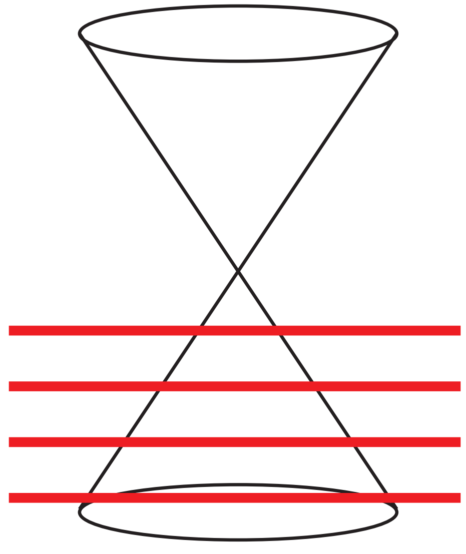
Dirac like spectrum, caution!

DMRG simulates a long cylindrical geometry

momentum discretized $k_y = 0, \frac{2\pi}{L_y}, \cdots, 2\pi$



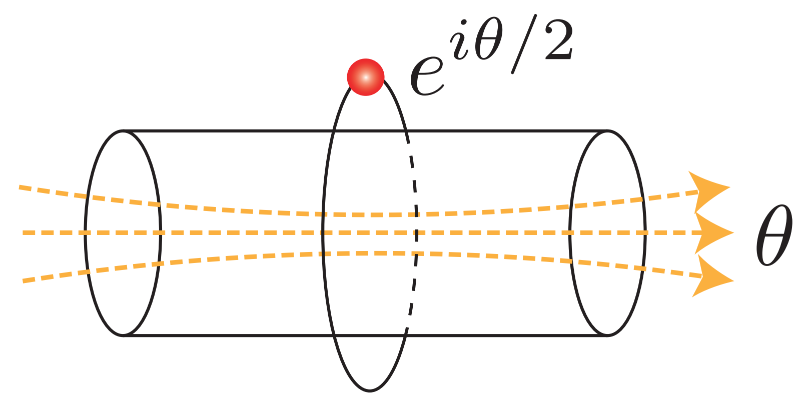
or



Dirac spin liquid, more subtle

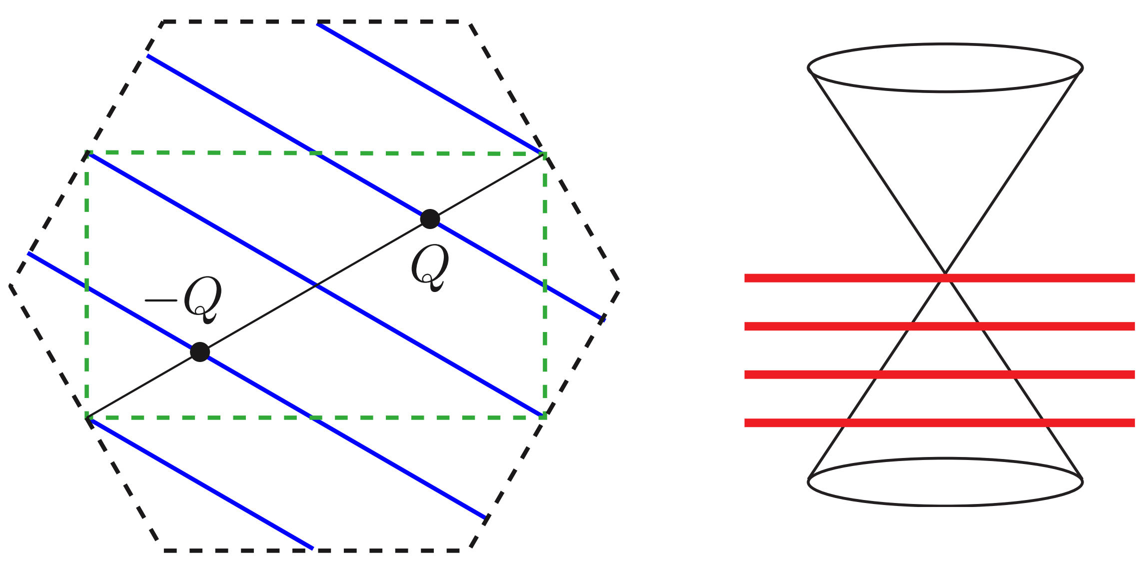
Dirac spin liquid may be gapped on "any" small cylinder/torus

Spinons' boundary condition has ambiguity



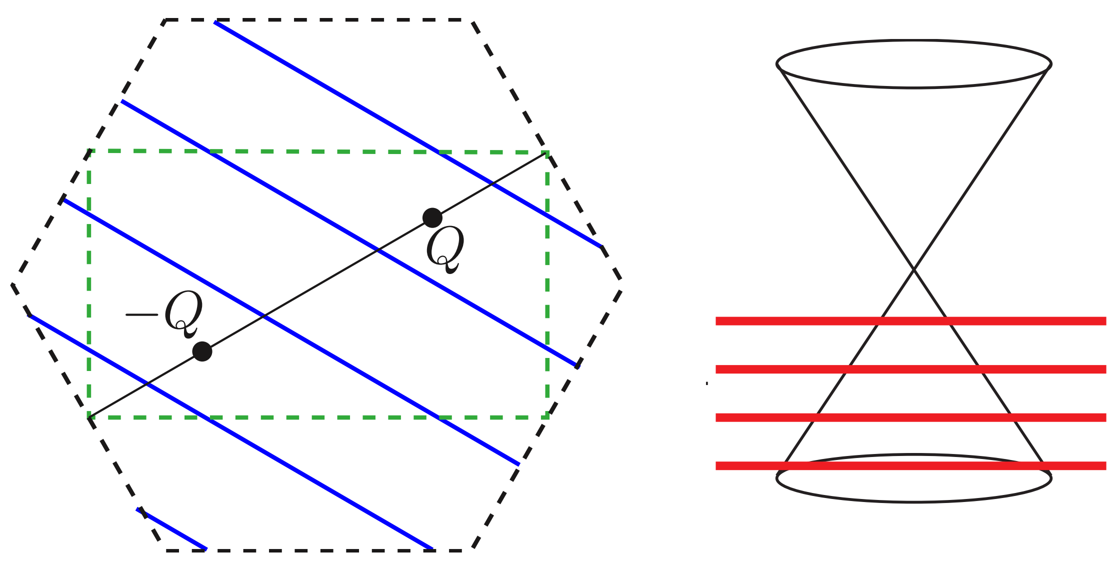
Two topological sectors

$\theta = 0$



spinons have PBC

$\theta = 2\pi$

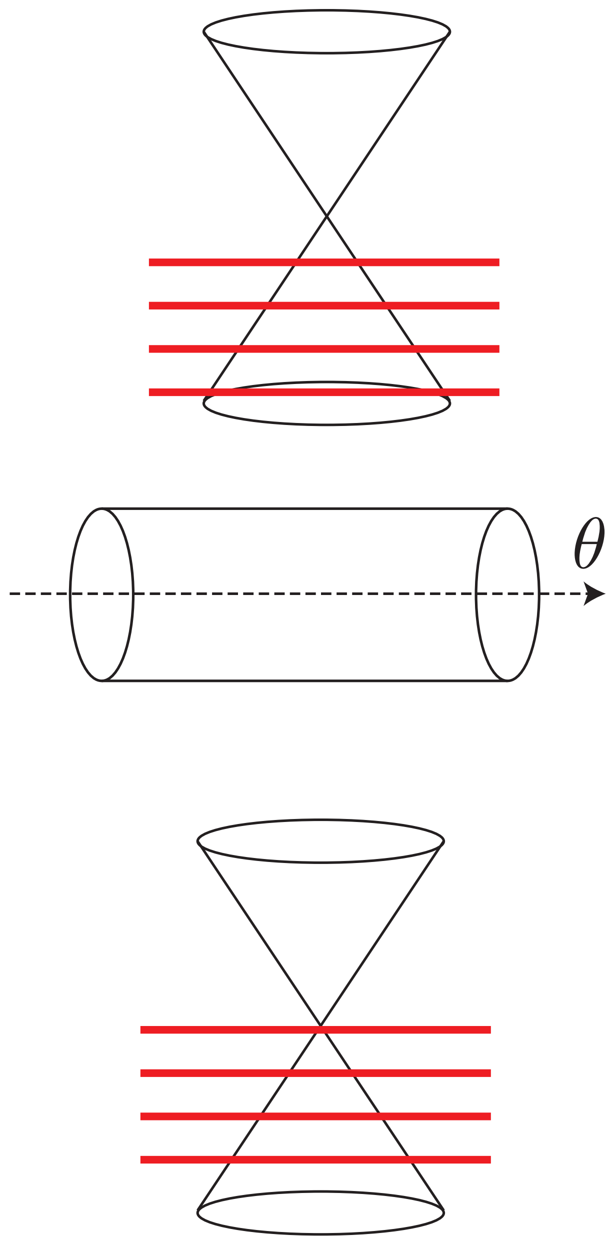


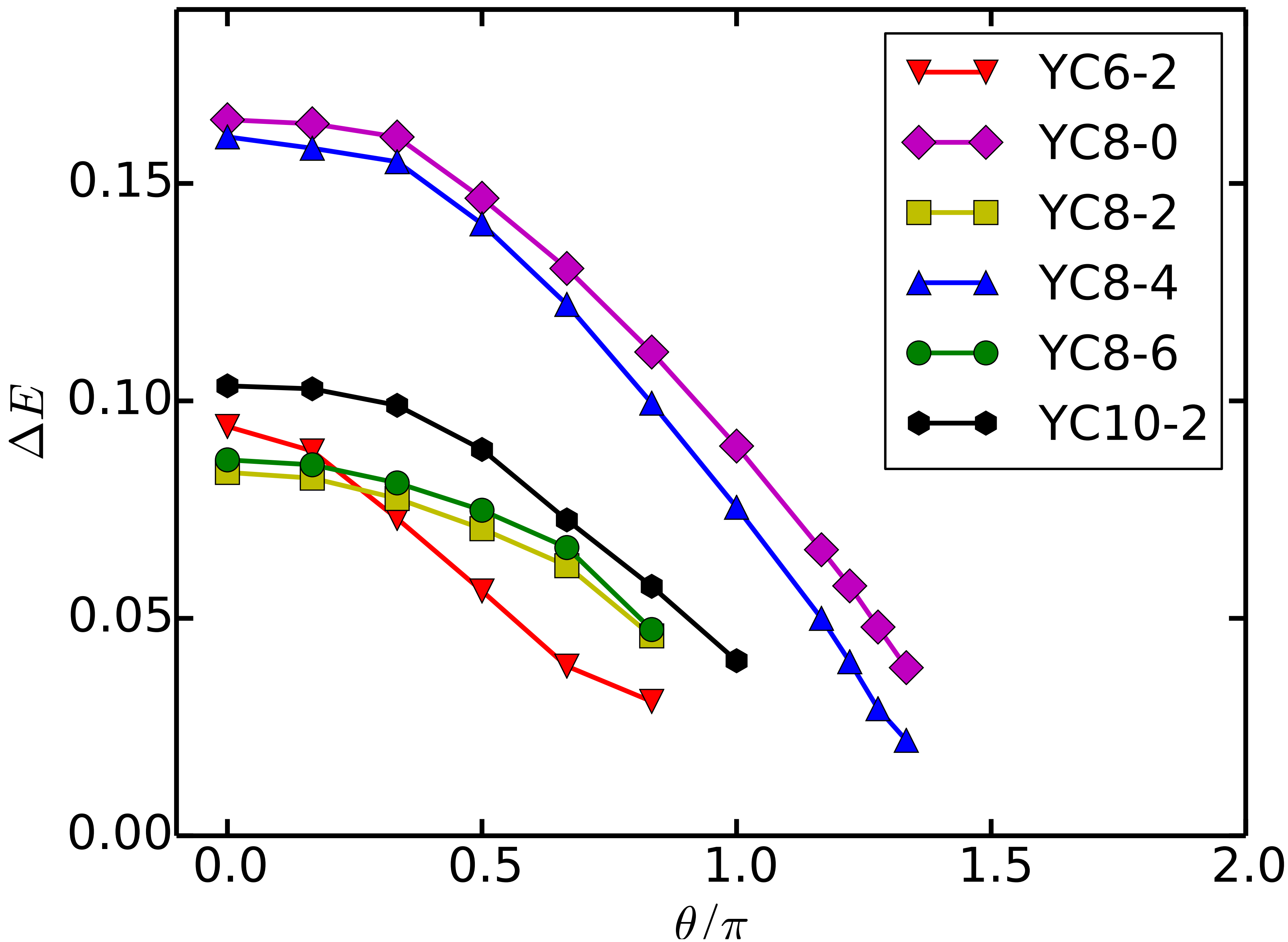
spinons have APBC

Spin gap under twist

DMRG

YCH, Zaletel, Oshikawa, Pollmann (to appear)





"Excitation Spectrum" from DMRG!

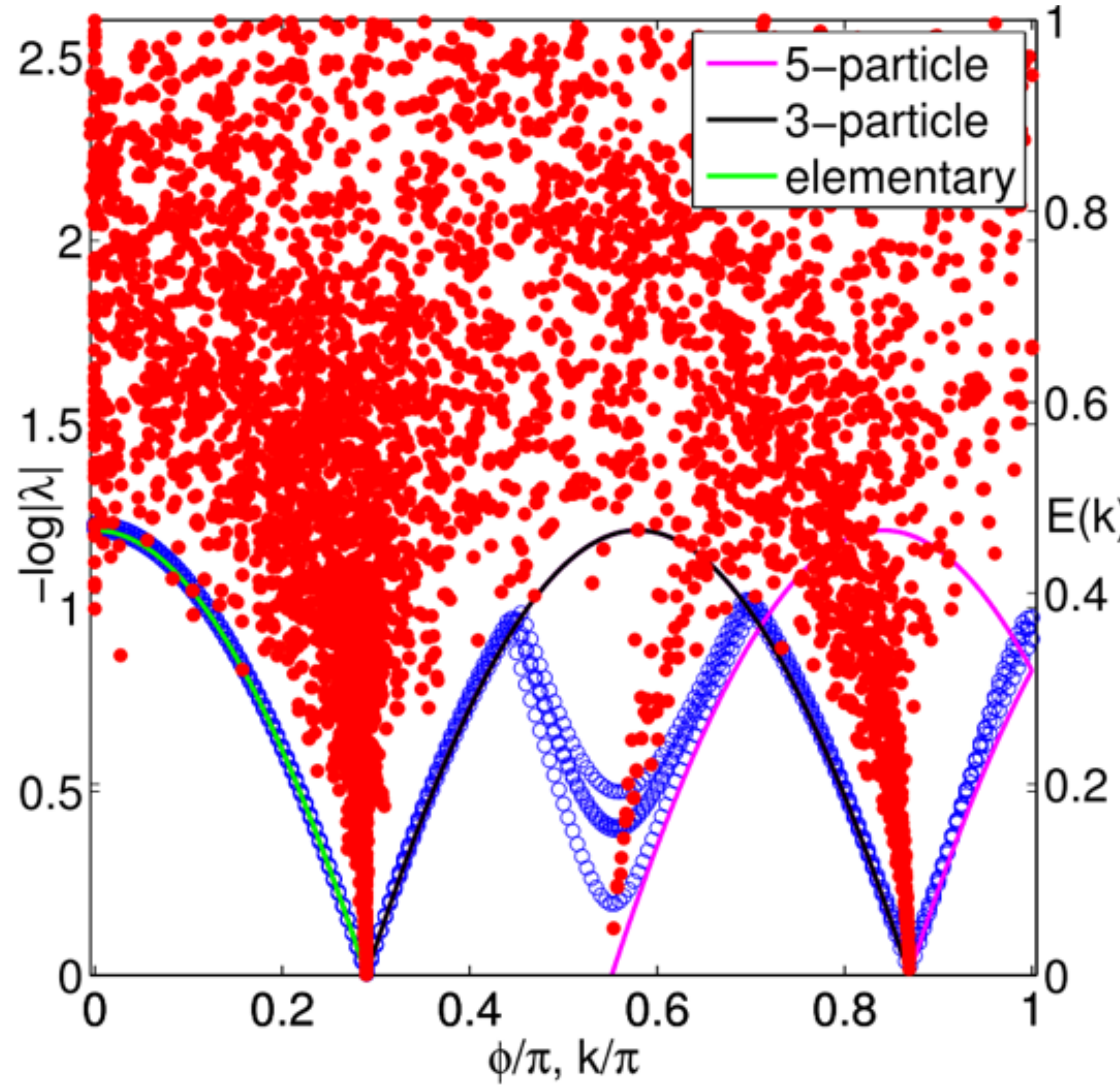
Zauner, et. al., Verstraete, arXiv:1408.5140

Basic idea: $\Delta \propto 1/\xi$

correlation-length spectrum

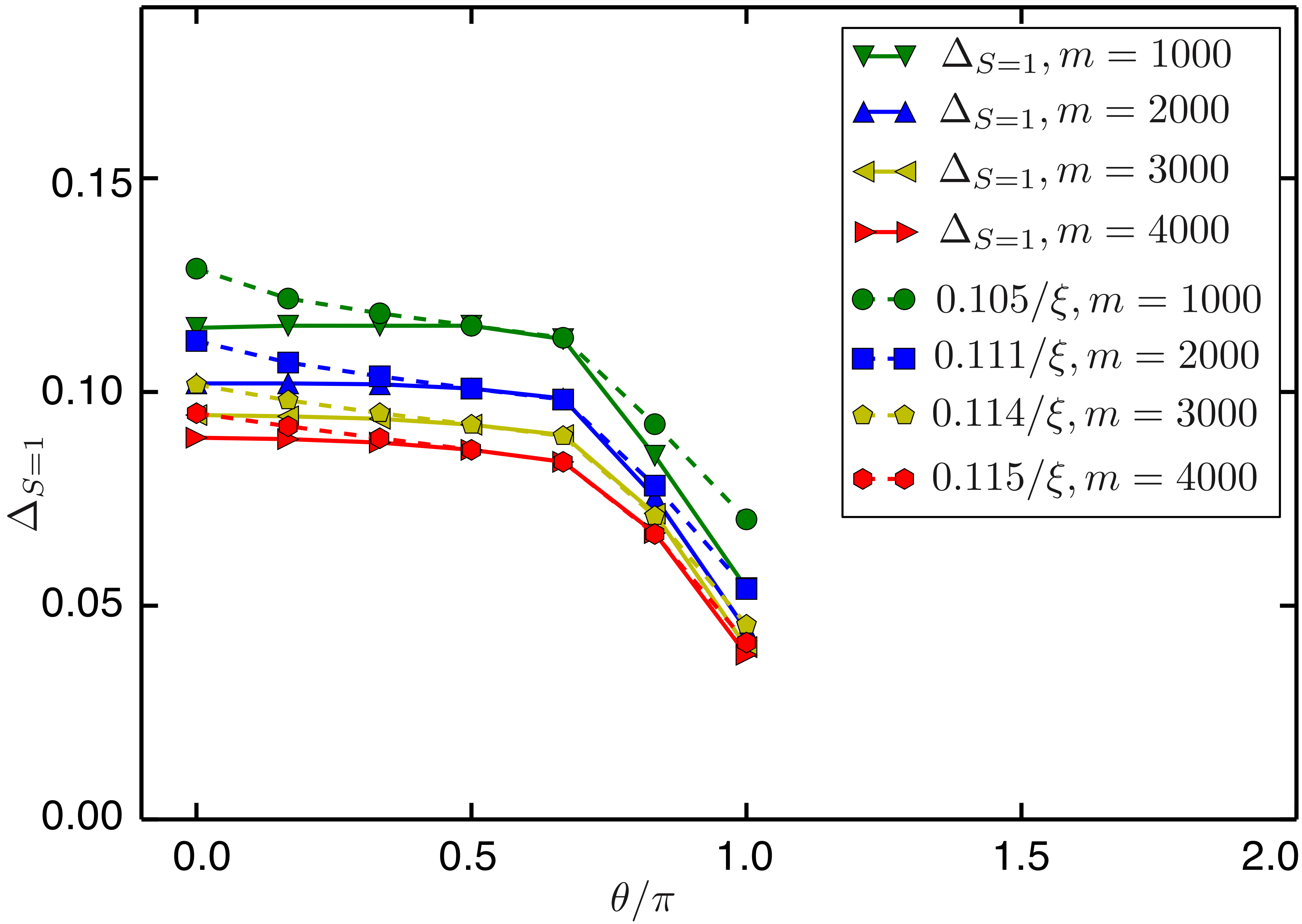
Eigenvalues of transfer matrix

$$\lambda_i = e^{ik-1/\xi}$$



Correlation length versus spin gap

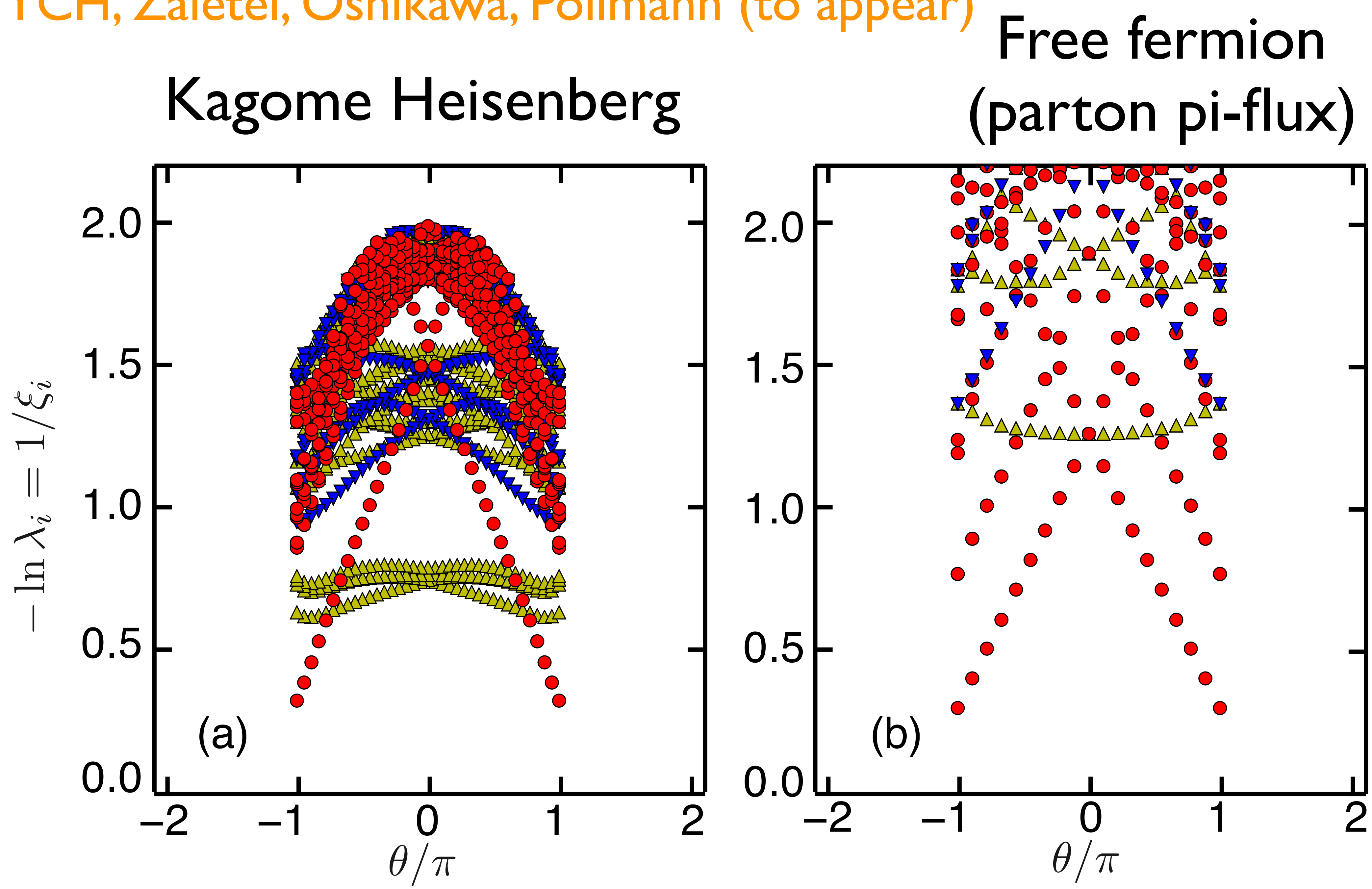
$\Delta \propto 1/\xi$ works perfectly



Kagome Heisenberg model, YC8-2 cylinder

Spectrum of triplet excitation

YCH, Zaletel, Oshikawa, Pollmann (to appear)



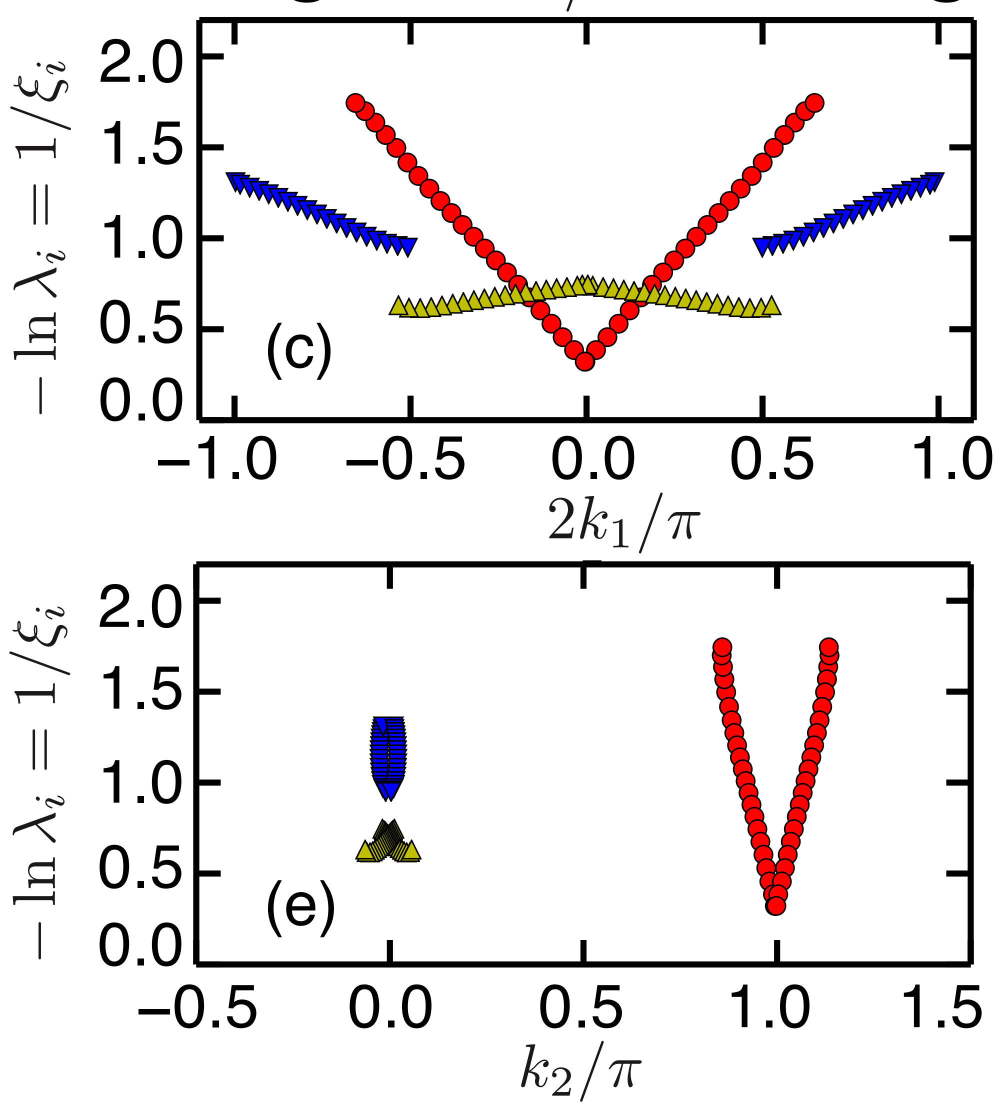
DMRG data

$$S^+(p-q) = f_\uparrow^\dagger(p) f_\downarrow(q)$$

Spectrum of triplet excitation

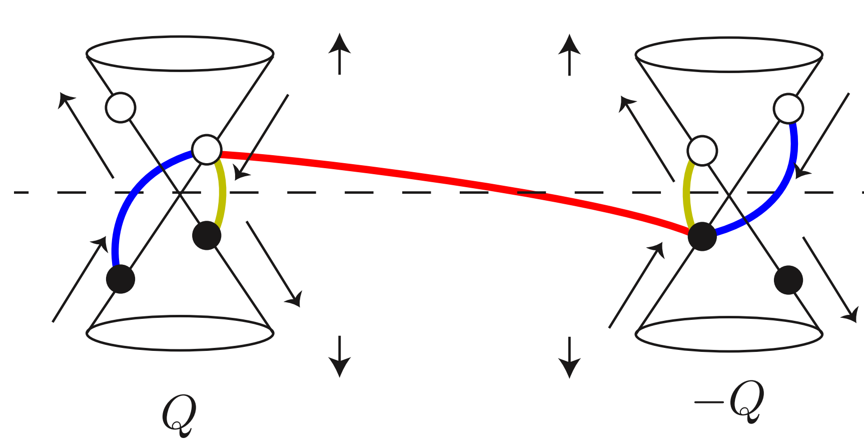
YCH, Zaletel, Oshikawa, Pollmann (to appear)

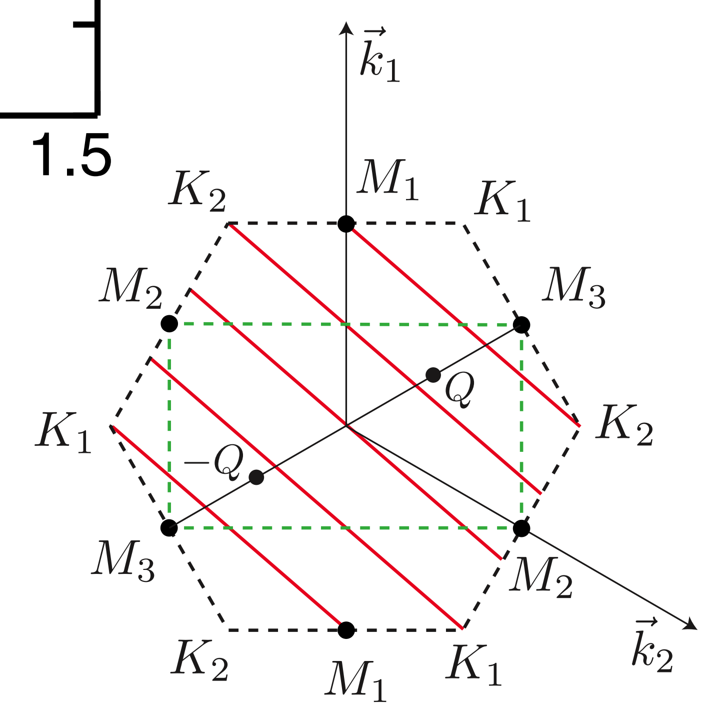
Kagome Heisenberg | Free fermion (parton pi-flux)



DMRG data

$$S^+(p-q) = f_\uparrow^\dagger(p) f_\downarrow(q)$$





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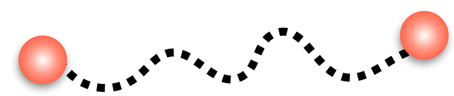
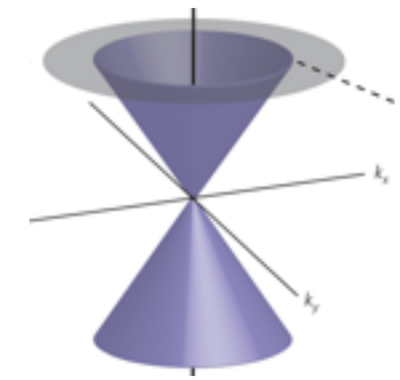
How to solve?

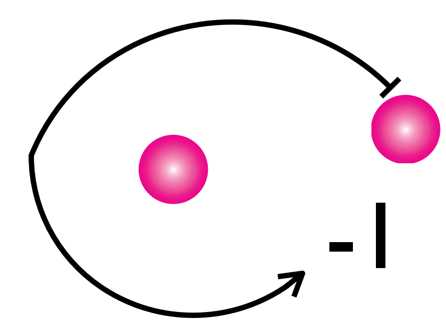
gauge field

spinon





$J \sum \vec{S}_i \cdot \vec{S}_j$ — Spin model — Unbiased? → $t \sum e^{i\mathcal{A}_{ij}} c_i^\dagger c_j$ Effective lattice gauge model → $\bar{\psi} i \gamma_\mu (\partial_\mu + i\mathcal{A}_\mu)\psi$ Deconfined spin liquid



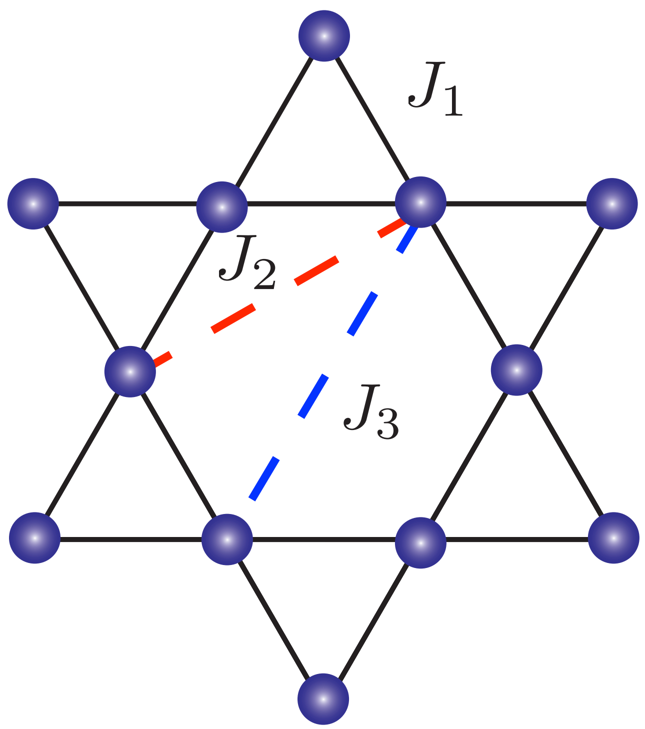
~~biased mean field~~

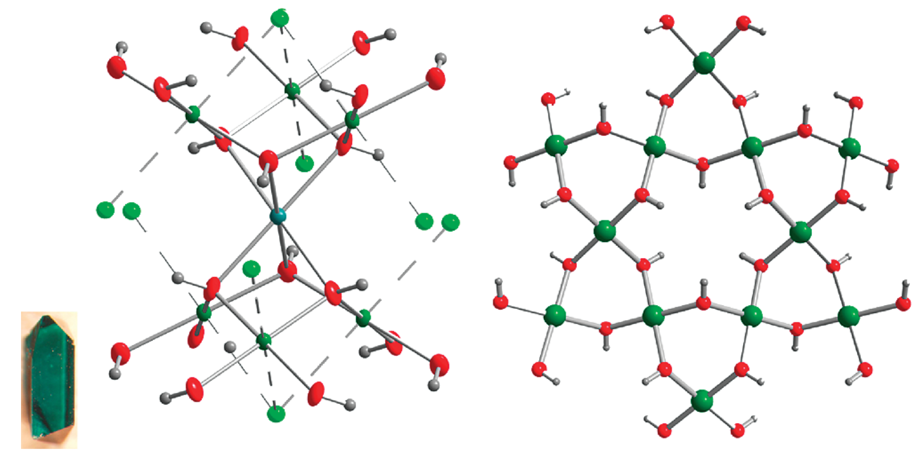
~~slave particle~~

$\vec{S}_i = c_{i,s}^\dagger \vec{\sigma}_{s,s'} c_{i,s'}$

$\langle c_i^\dagger c_j \rangle \neq 0$

Make it more general





XXZ anisotropy
second neighbor
third neighbor
DM interaction
interlayer coupling
impurity
....

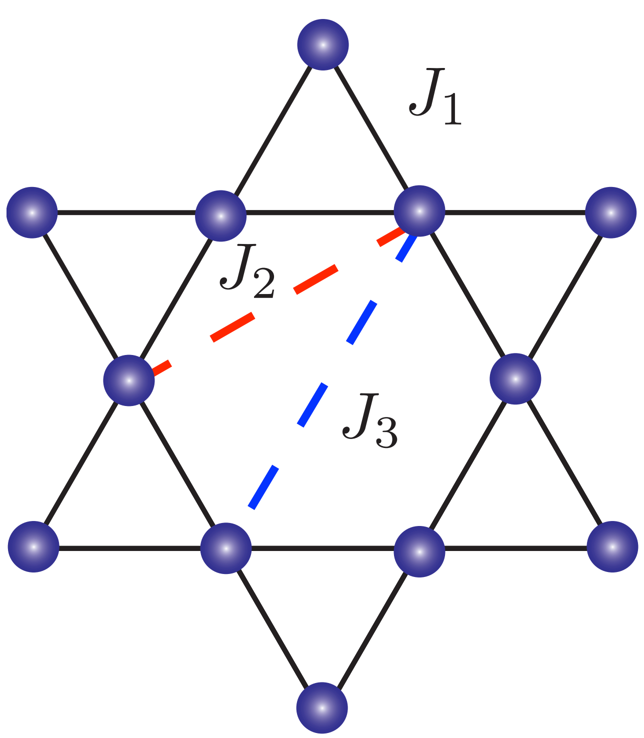
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j, \quad J > 0$$

Extended kagome model

YCH & Chen, PRL 2015

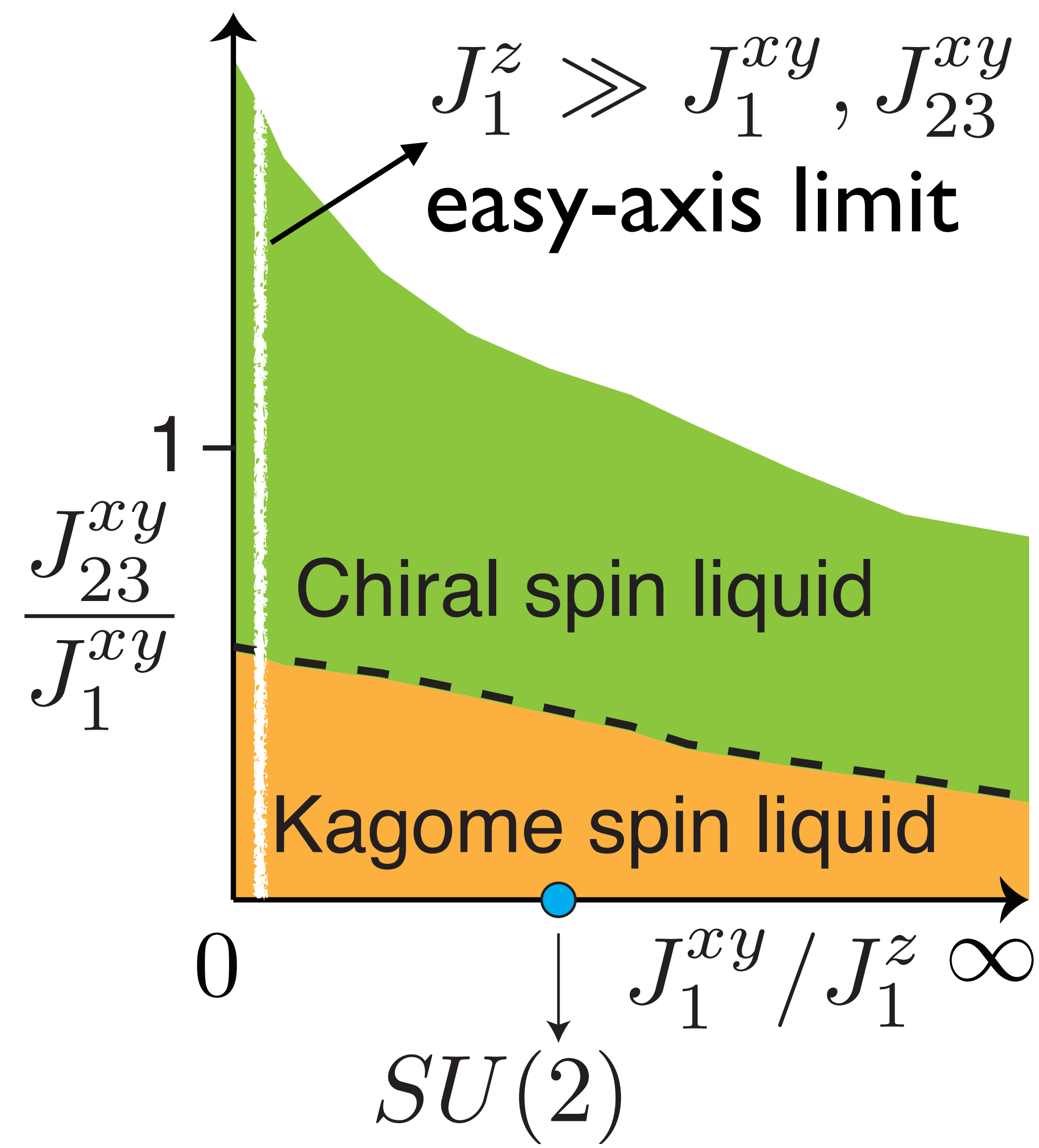
$$H_{XXZ} = J_1^z \sum_{\langle pq \rangle} S_p^z S_q^z + \frac{J_1^{xy}}{2} \sum_{\langle pq \rangle} (S_p^+ S_q^- + h.c.)$$ 1st XXZ

$$+ \frac{J_{23}^{xy}}{2} \left(\sum_{\langle\langle pq \rangle\rangle} + \sum_{\langle\langle\langle pq \rangle\rangle\rangle} \right) (S_p^+ S_q^- + h.c.)$$ 2nd, 3rd XY



DMRG results

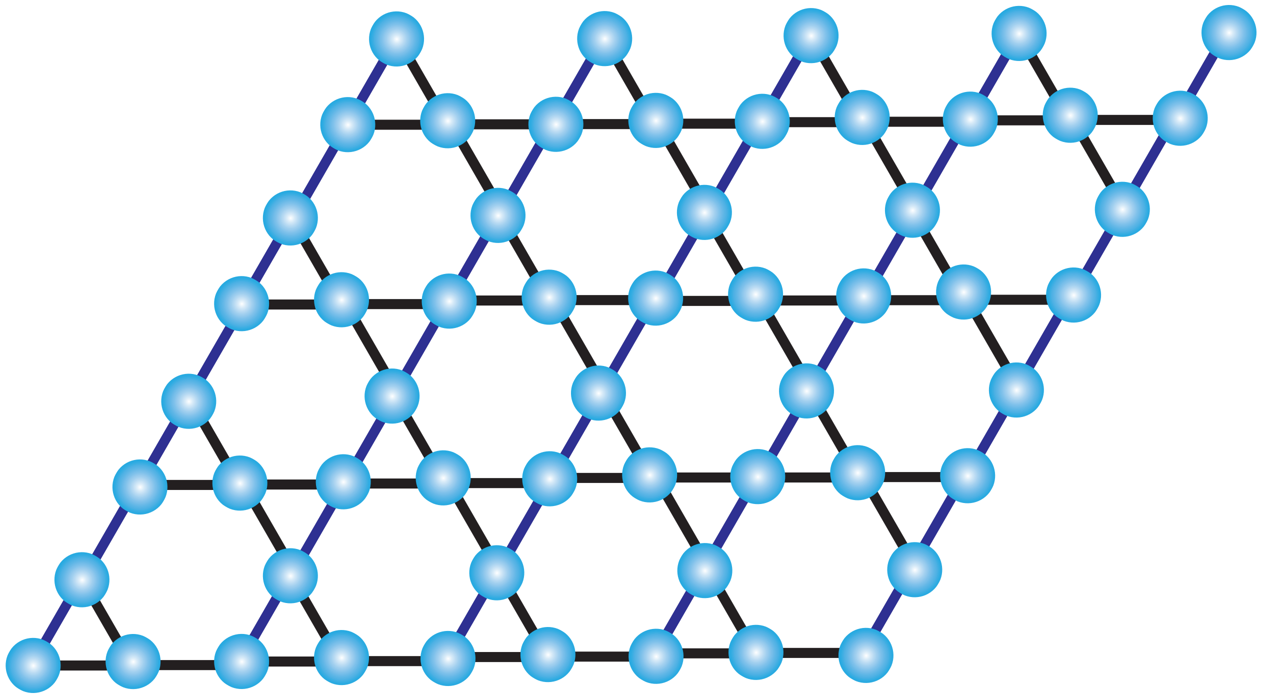
independent of XXZ anisotropy



also see ED calculation:

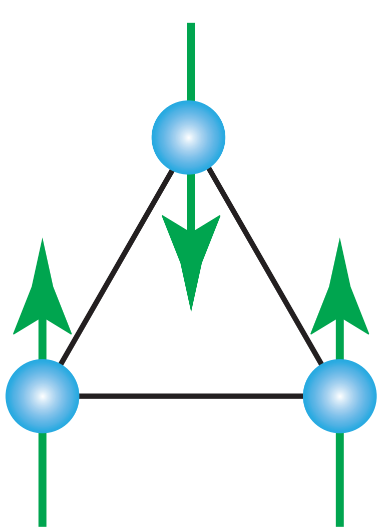
Lauchli & Moessner, arXiv (2015)

Easy axis kagome

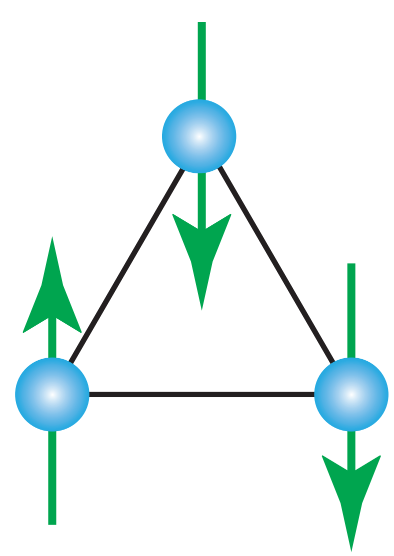


$$H = J_1^z \sum_{1\mathrm{st}} S_i^z S_j^z + \lambda H_1$$

$$J_z \gg \lambda > 0$$



$$\sum S_i^z = \frac{1}{2}$$



$$\sum S_i^z = -\frac{1}{2}$$

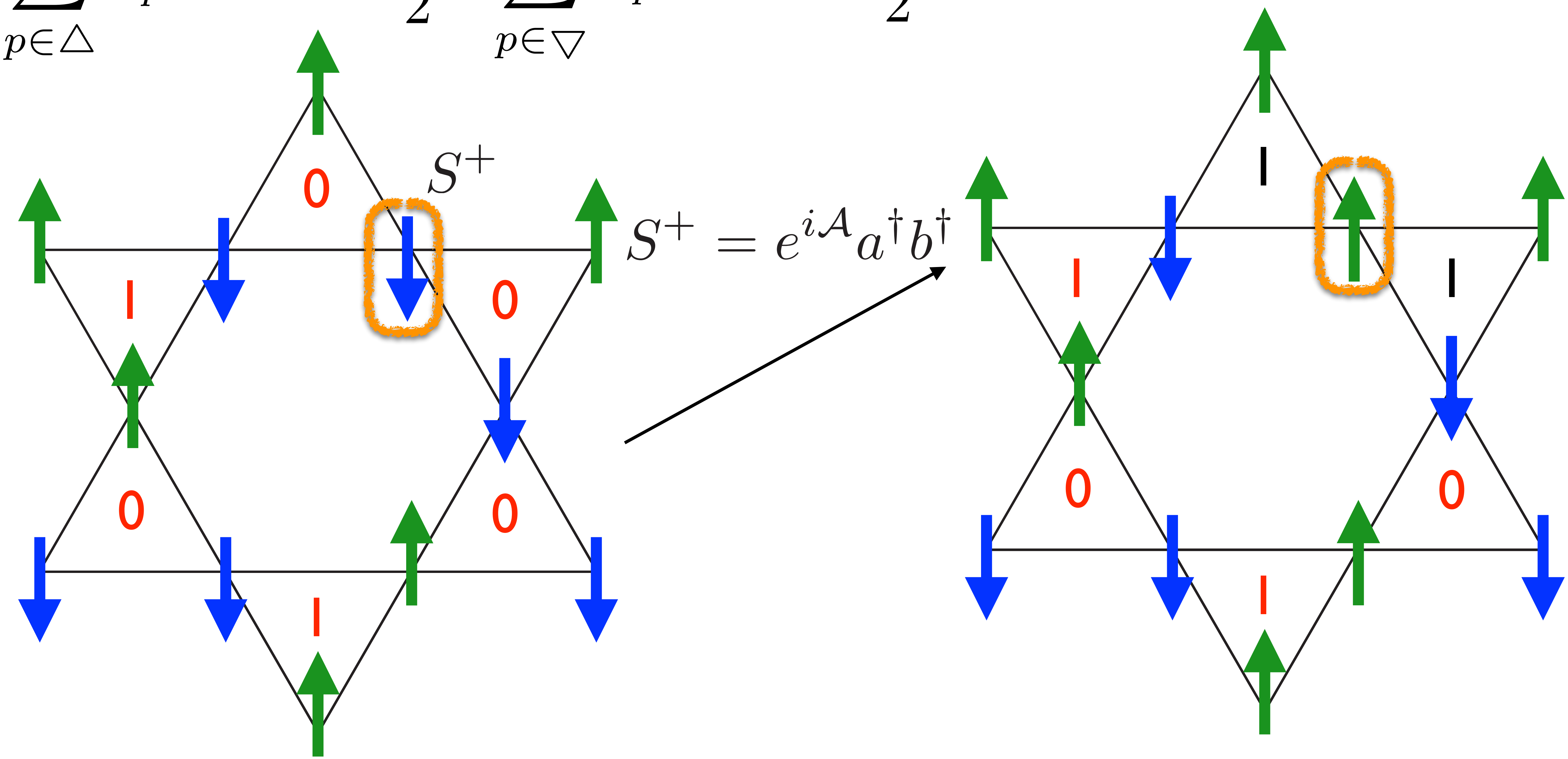
extensive classical degeneracy

H_1 lifts the classical degeneracy

Lattice gauge mapping

$$\sum_{p \in \triangle} S_p^z = a^\dagger a - \frac{1}{2} \qquad \sum_{p \in \nabla} S_p^z = b^\dagger b - \frac{1}{2}$$

Nikolic & Senthil 2005 PRB



similar system: quantum dimer model, pyrochlore lattice

Fradkin & Kivelson, 1990

Hermele, Fisher & Balents 2004

Castelnovo, Moessner & Sondhi 2008

Lattice gauge mapping: XXZ kagome

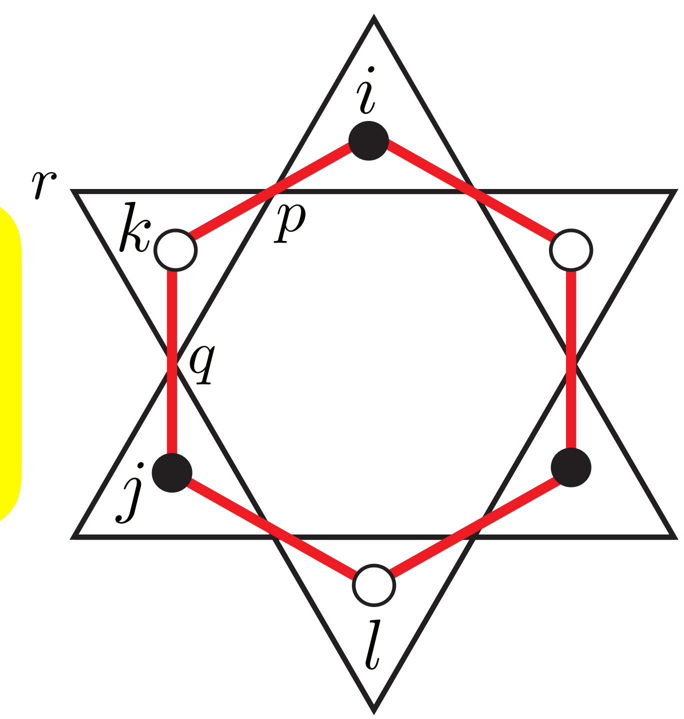
$$H = J_1^z \sum_{\langle pq\rangle} S_p^z S_q^z + \frac{J_1^{xy}}{2} \sum_{\langle pq\rangle} (S_p^+ S_q^- + h.c.)$$

$$+ \frac{J_{23}^{xy}}{2} \sum_{\langle\langle pq\rangle\rangle} (S_p^+ S_q^- + h.c.) + \frac{J_{23}^{xy}}{2} \sum_{\langle\langle\langle pq\rangle\rangle\rangle} (S_p^+ S_q^- + h.c.)$$

Unbiased

$$S_p^+ = e^{i\mathcal{A}_{ik}} a_i^\dagger b_k^\dagger$$

lattice gauge mapping

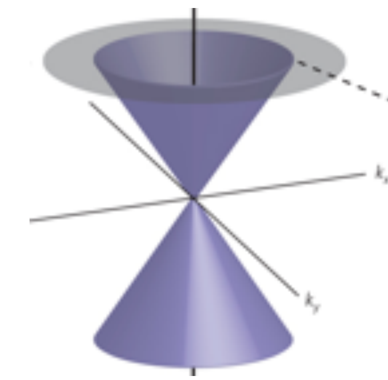


$$H^{\mathrm{LGT}} = J_1^{xy} \Big[\textstyle\sum_{\langle\langle ij\rangle\rangle} e^{i\mathcal{A}_{ij}} a_i^\dagger a_j + \sum_{\langle\langle kl\rangle\rangle} e^{i\mathcal{A}_{lk}} b_k^\dagger b_l + h.c. \Big]$$

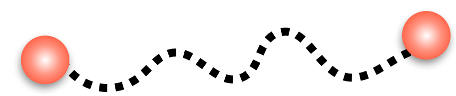
$$+ J_{23}^{xy} \textstyle\sum_{\langle ik\rangle, \langle jl\rangle \in \hexagon} \Big[(e^{i\mathcal{A}_{ik}} a_i^\dagger b_k^\dagger)(e^{i\mathcal{A}_{lj}} b_l a_j) + h.c. \Big]$$

$$+ \kappa \sum E_{ik}^2 + 1/\kappa \sum \cos(\textstyle\sum \mathcal{A}_{ik}) \qquad \kappa \sim \kappa_{\mathrm{SL}}$$

Solving the kagome spin liquid phase



gauge field



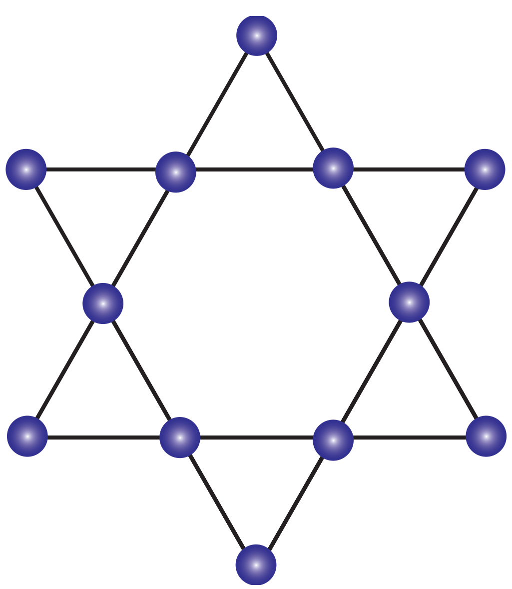
spinon

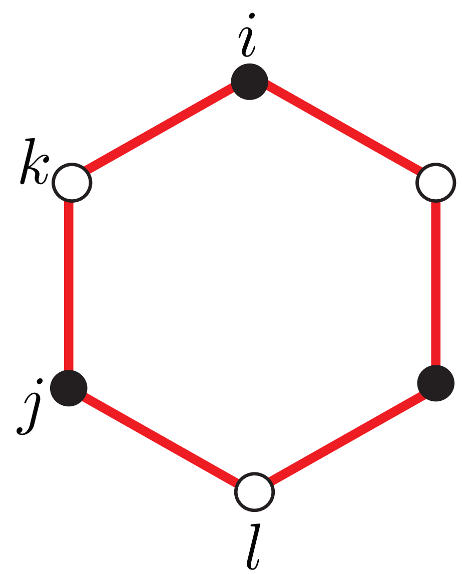
$J\sum \vec{S}_i \cdot \vec{S}_j$

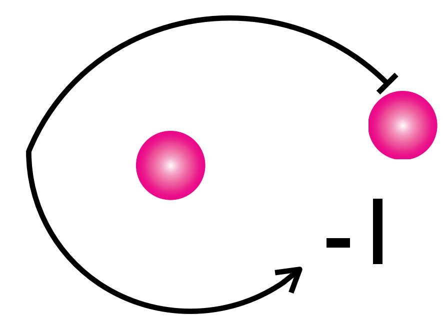
$t\sum e^{i\mathcal{A}_{ij}} c_i^\dagger c_j$

$\bar{\psi} i\gamma_\mu(\partial_\mu + i\mathcal{A}_\mu)\psi$

Spin model — **Unbiased** → Effective lattice gauge model — **?** → Deconfined spin liquid







No "free" spin liquid

U(1) compact gauge field + Dynamical bosonic spinons

can beat confinement

Fradkin & Shenker, PRD 1979

confinement in 2+1D

Polyakov

The Z2 spin liquid has been realized

Moessner & Sondhi 2001

No "free" spin liquid

U(1) compact gauge field + Dynamical bosonic spinons

...nement

...dkin & Shenker, PRD 1979

con... ...e Z2 spin liquid has been realized

...yakov

Moessner & Sondhi 2001

Beyond the Z2 spin liquid?

Phases in a lattice gauge model

gauge field + matter

interacting bosonic matter

gauge field

gauging

spinon

Higgs phase		Superfluid
Z2 spin liquid	Fradkin & Shenker	Charge-2 superfluid
Chiral spin liquid	Levin & Gu Barkeshli YCH et al.	U(1) SPT (Bosonic integer QH)

Symmetry protected topological phase!

Classified by cohomology group $H^{d+1}[G, U(1)]$

Chen, Gu, Liu & Wen, PRB 2012

SPT protected by U(1) charge conservation

⇕

Bosonic integer quantum Hall

Senthil & Levin, PRL 2013
Lu & Vishwanath, PRB 2012

gauging ↓ $$\frac{2}{4\pi}\varepsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

chiral spin liquid

YCH, Bhattacharjee, Pollmann, and Moessner, PRL 2015

See also:
Barkeshli, arxiv 2013

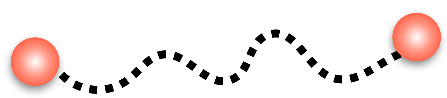
Phases in a lattice gauge model

gauge field + matter

interacting bosonic matter

gauge field

gauging

spinon 

Higgs phase

Superfluid

Z2 spin liquid

Fradkin & Shenker

Charge-2 superfluid

Chiral spin liquid

Levin & Gu
Barkeshli
YCH et al.

U(1) SPT
(Bosonic integer QH)

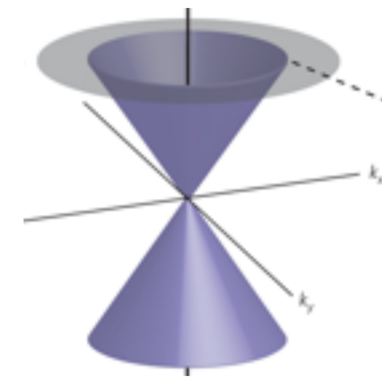
Critical spin liquid

Deconfined critical point

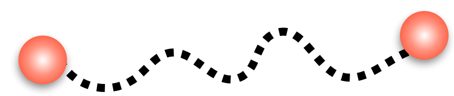
YCH, Fuji & Bhattacharjee

Solving the kagome spin liquid phase

gauge field

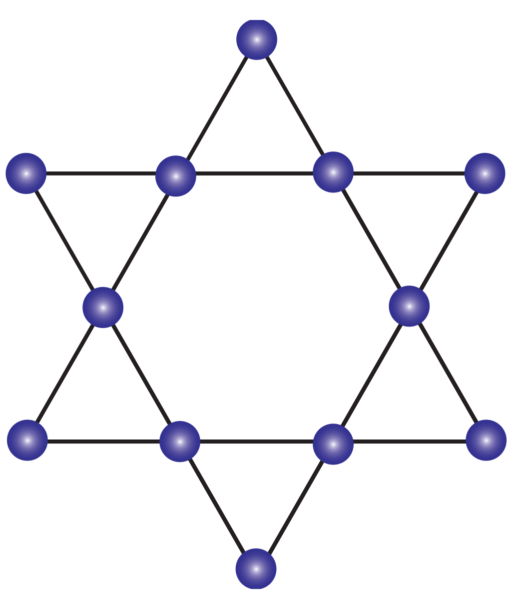


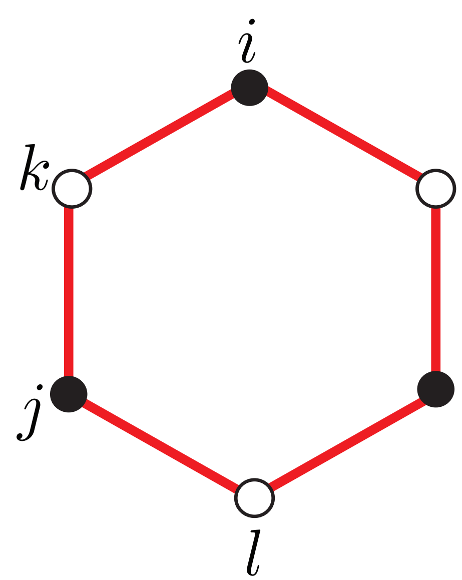
spinon

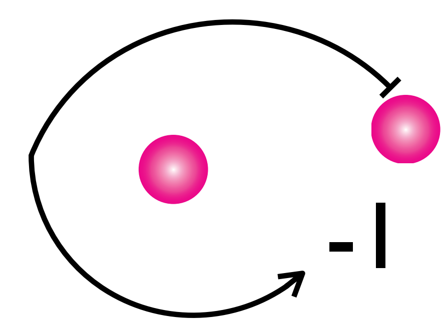


$J \sum \vec{S}_i \cdot \vec{S}_j$ $\quad$ $t \sum e^{i\mathcal{A}_{ij}} c_i^\dagger c_j$ $\quad$ $\bar{\psi} i \gamma_\mu (\partial_\mu + i\mathcal{A}_\mu)\psi$

Spin model — **Unbiased** → Effective lattice gauge model — **?** → Deconfined spin liquid



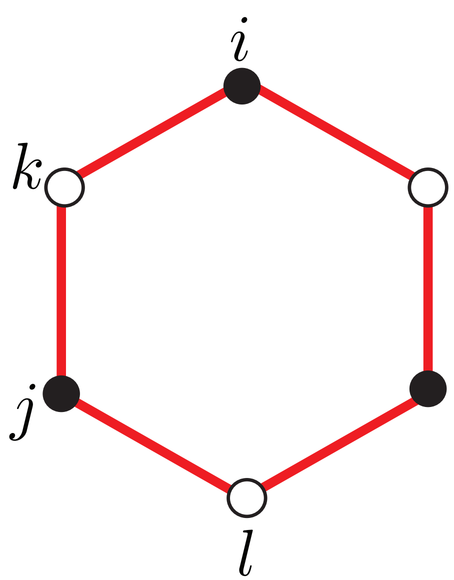


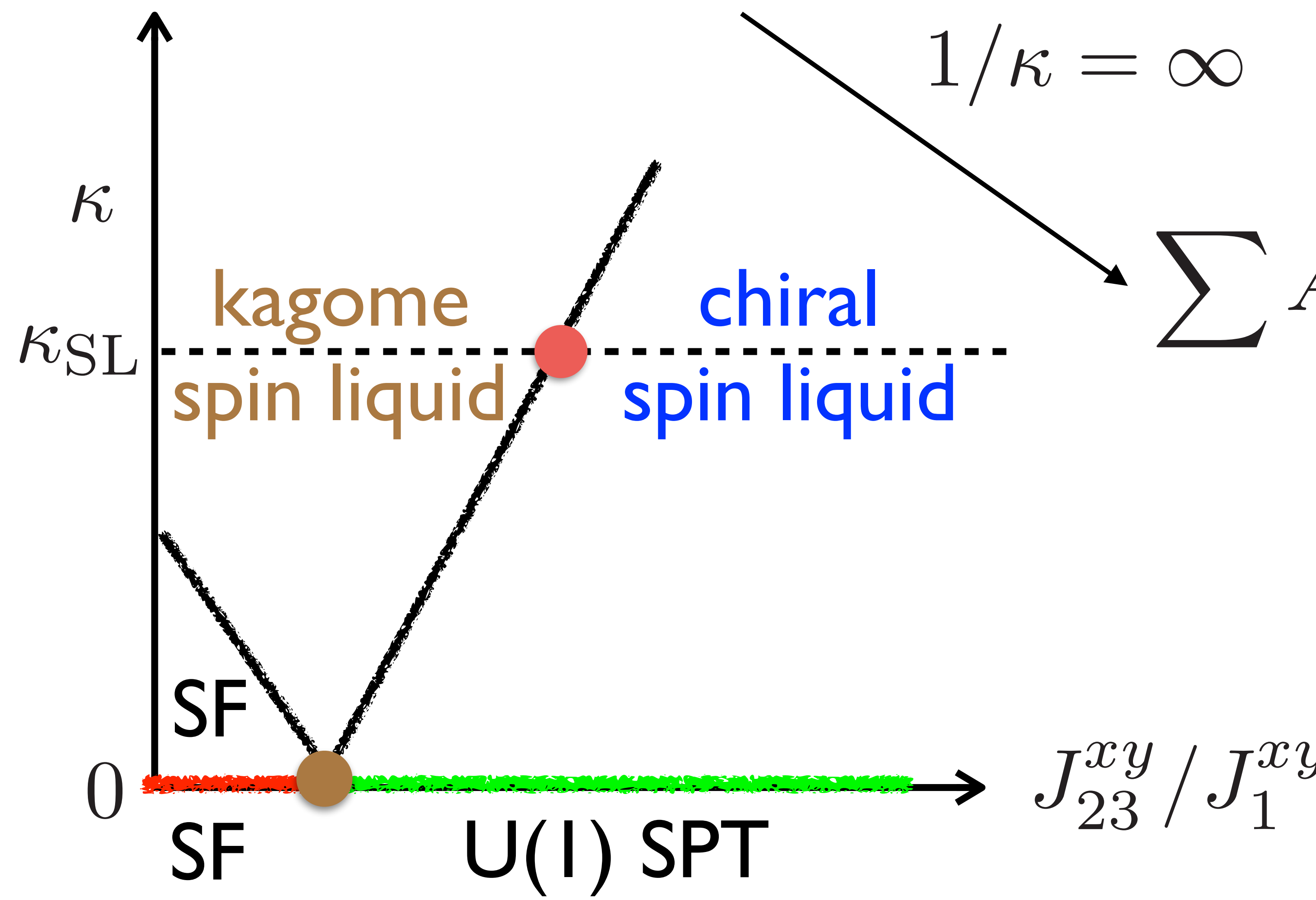


Lattice gauge model

YCH, Fuji, and Bhattacharjee, arXiv:1512.05381 (2015).

$$H^{\mathrm{LGT}} = J_1^{xy}\Big[\sum_{\langle\langle ij\rangle\rangle} e^{i\mathcal{A}_{ij}} a_i^\dagger a_j + \sum_{\langle\langle kl\rangle\rangle} e^{i\mathcal{A}_{lk}} b_k^\dagger b_l + h.c.\Big]$$
$$+J_{23}^{xy} \sum_{\langle ik\rangle,\langle jl\rangle\in\hexagon} \Big[(e^{i\mathcal{A}_{ik}} a_i^\dagger b_k^\dagger)(e^{i\mathcal{A}_{lj}} b_l a_j) + h.c.\Big]$$
$$+\kappa \sum E_{ik}^2 + 1/\kappa \sum \cos\left(\sum \mathcal{A}_{ik}\right)$$





$\kappa = 0$ or

$1/\kappa = \infty$

$$\sum A^0 = \pi$$

Kagome spin liquid:

"gauged" deconfined critical point

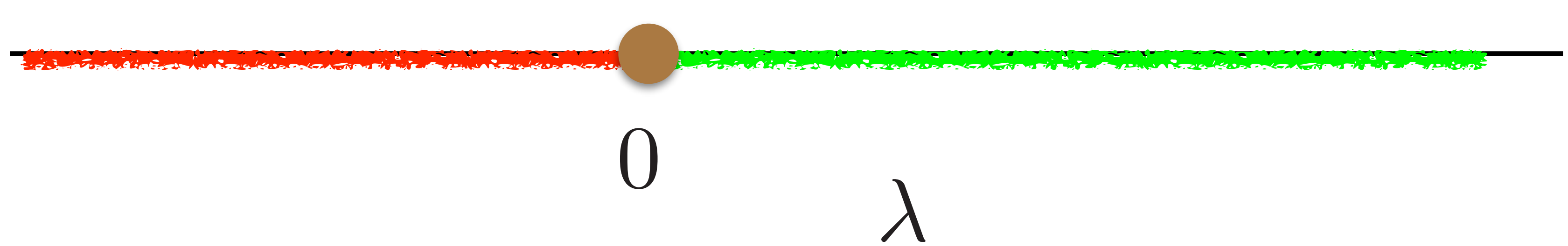
Field theory for zero gauge fluctuation

$$
\begin{aligned}
\mathcal{L} = &\sum_{\sigma=\pm} \bar{\psi}^f_\sigma [i\gamma^\mu(\partial_\mu - i a^f_\mu - i\sigma A^c_\mu)]\psi^f_\sigma - \frac{1}{2\pi}\varepsilon_{\mu\nu\rho}A^s_\mu \partial_\nu a^f_\rho \\
&+ \sum_{\sigma=\pm} \bar{\psi}^g_\sigma [i\gamma^\mu(\partial_\mu - i a^g_\mu - i\sigma A^c_\mu)]\psi^g_\sigma + \frac{1}{2\pi}\varepsilon_{\mu\nu\rho}A^s_\mu \partial_\nu a^g_\rho \\
&+ \sum_{\sigma=\pm} \phi(\bar{\psi}^f_\sigma \psi^f_\sigma + \bar{\psi}^g_\sigma \psi^g_\sigma) + 2\lambda\phi^2 - u\phi^4 + (\partial_\mu\phi)^2 + \cdots
\end{aligned}
$$

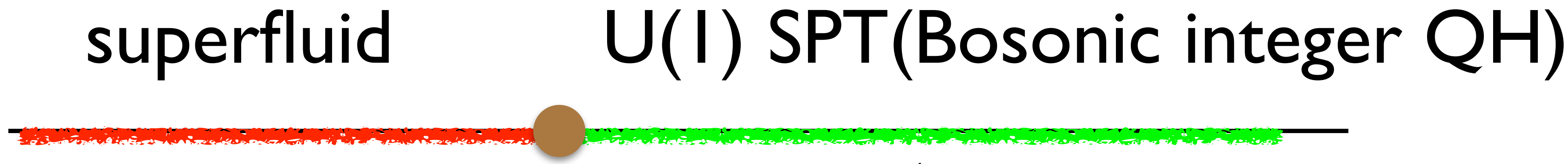
YCH, Fuji, and Bhattacharjee, arXiv:1512.05381 (2015).

U(1) charge A^c U(1) pseudospin A^s

superfluid U(1) SPT(Bosonic integer QH)



Deconfined criticality



$$\mathcal{L} = \sum_{\sigma=\pm} \bar{\psi}^f_\sigma [i\gamma^\mu(\partial_\mu - ia^f_\mu - i\sigma A^c_\mu)]\psi^f_\sigma - \frac{1}{2\pi}\varepsilon_{\mu\nu\rho}A^s_\mu \partial_\nu a^f_\rho$$

$$+ \sum_{\sigma=\pm} \bar{\psi}^g_\sigma [i\gamma^\mu(\partial_\mu - ia^g_\mu - i\sigma A^c_\mu)]\psi^g_\sigma + \frac{1}{2\pi}\varepsilon_{\mu\nu\rho}A^s_\mu \partial_\nu a^g_\rho$$

$$+ \sum_{\sigma=\pm} \phi(\bar{\psi}^f_\sigma \psi^f_\sigma + \bar{\psi}^g_\sigma \psi^g_\sigma) + 2\lambda\phi^2 - u\phi^4 + (\partial_\mu \phi)^2 + \cdots$$

YCH, Fuji, and Bhattacharjee, arXiv:1512.05381 (2015).

- Emergent Dirac fermions and U(1) gauge field

  Grover, Vishwanath (2013); Lu, Lee (2014)

- Can be derived using the coupled wire construction

  Mross, Alicea, Motrunich

- Related with the particle-vortex duality of Dirac fermions

  Wang, Senthil; Metliski, Vishwanath

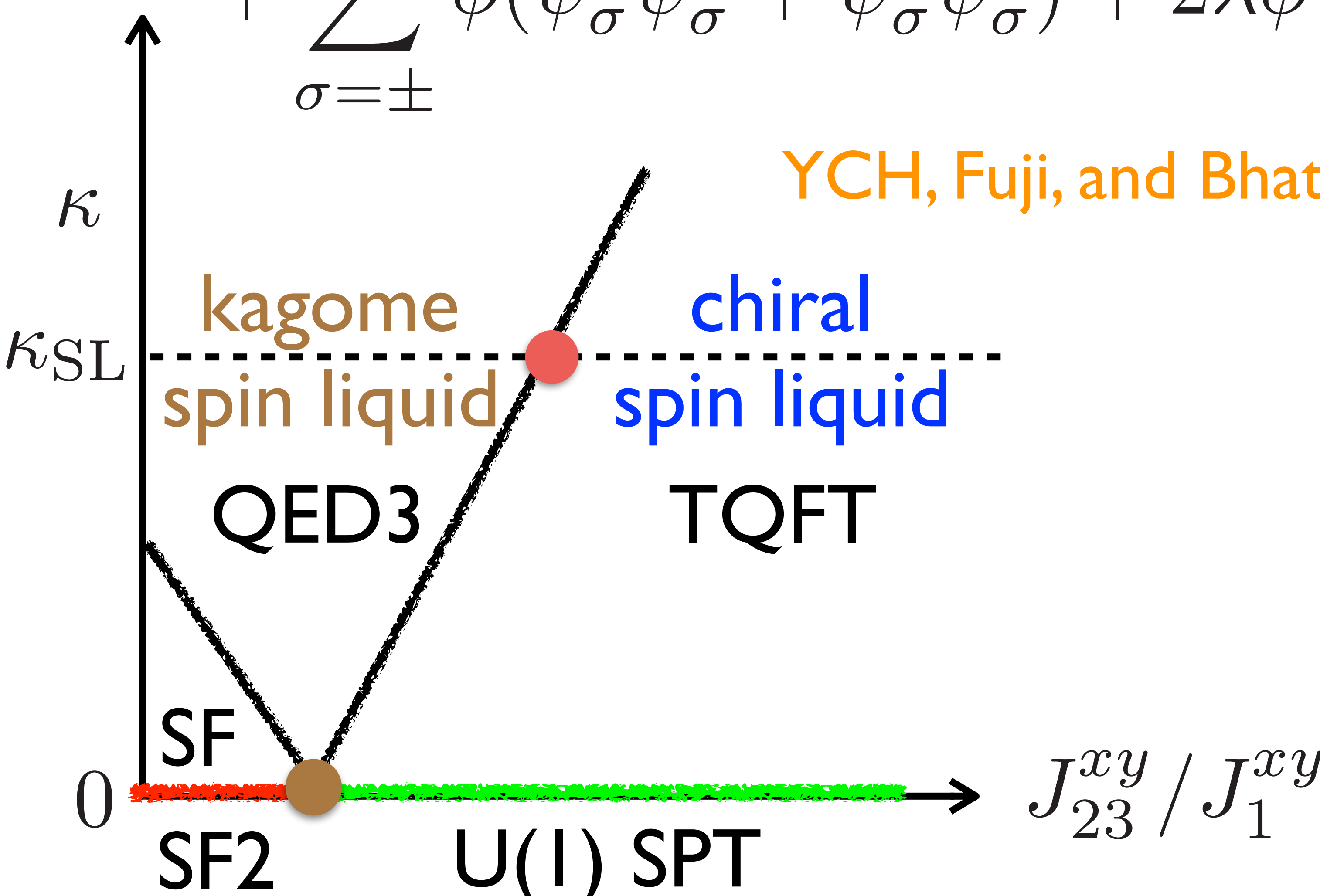
Field theory for finite gauge fluctuation

$$\mathcal{L} = \sum_{\sigma=\pm} \bar{\psi}^f_\sigma [i\gamma^\mu(\partial_\mu - ia^f_\mu - i\sigma A^c_\mu)]\psi^f_\sigma - \frac{1}{2\pi}\varepsilon_{\mu\nu\rho}A^s_\mu \partial_\nu a^f_\rho$$

$$+ \sum_{\sigma=\pm} \bar{\psi}^g_\sigma [i\gamma^\mu(\partial_\mu - ia^g_\mu - i\sigma A^c_\mu)]\psi^g_\sigma + \frac{1}{2\pi}\varepsilon_{\mu\nu\rho}A^s_\mu \partial_\nu a^g_\rho$$

$$+ \sum_{\sigma=\pm} \phi(\bar{\psi}^f_\sigma \psi^f_\sigma + \bar{\psi}^g_\sigma \psi^g_\sigma) + 2\lambda\phi^2 - u\phi^4 + (\partial_\mu\phi)^2 + \cdots$$

YCH, Fuji, and Bhattacharjee, arXiv:1512.05381 (2015)



$$A^c \longrightarrow \frac{A^{S^z}}{2}$$

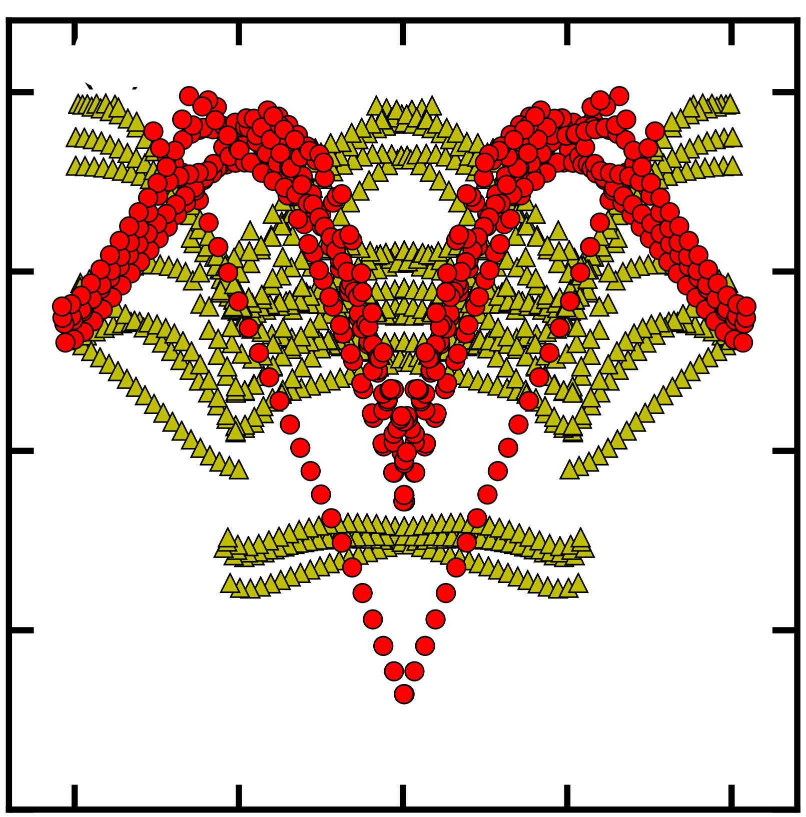
$$A^s \longrightarrow \mathcal{A}$$

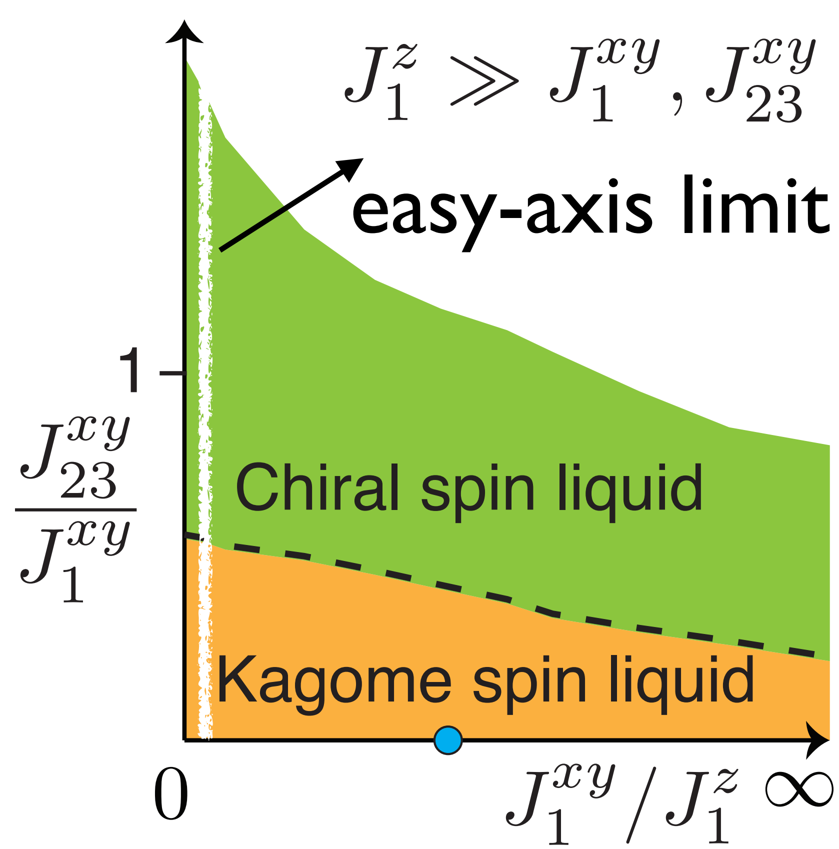
dynamical

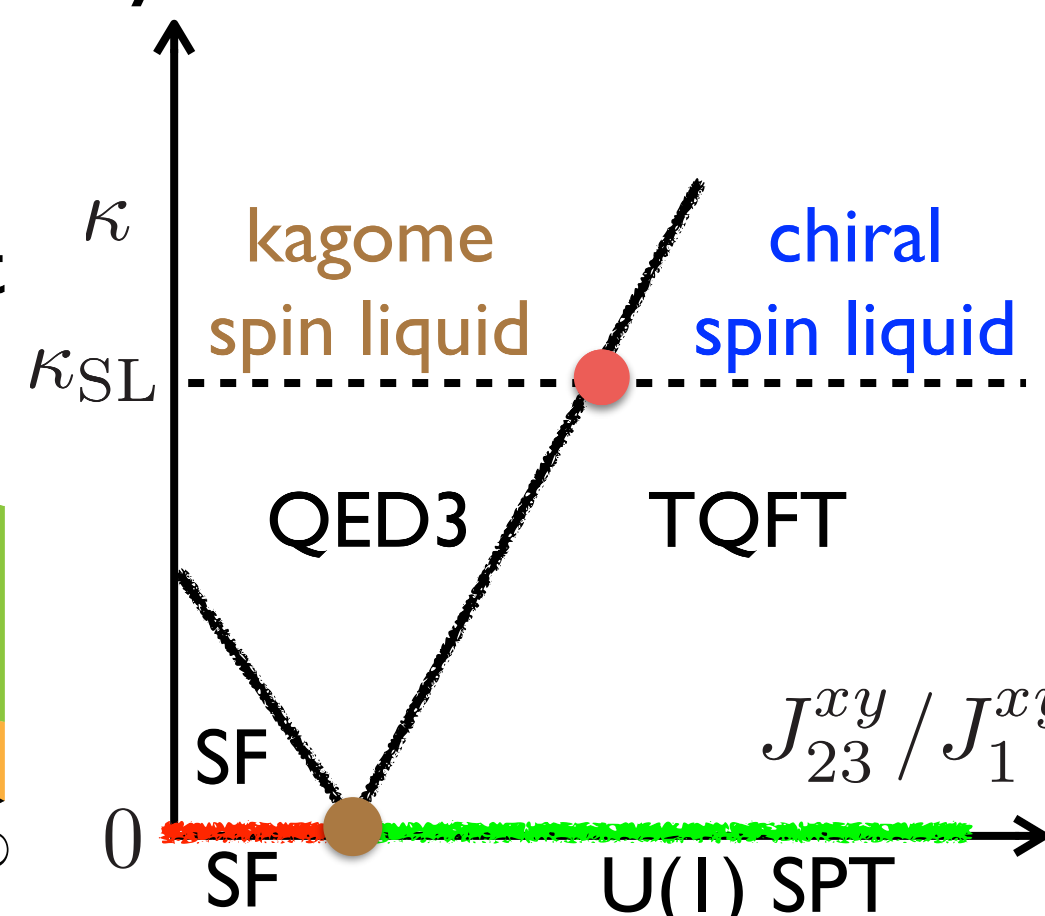
$$a^f = a^g$$

Summary

1. Numerical evidence that kagome spin liquid is a Dirac spin liquid.
2. Spin liquids on kagome lattice are independent of the XXZ anisotropy.
3. An unbiased theoretical study of spin liquids under a lattice gauge mapping.
4. Make a concrete connection between topological order, critical spin liquid, SPT phase, deconfined criticality.

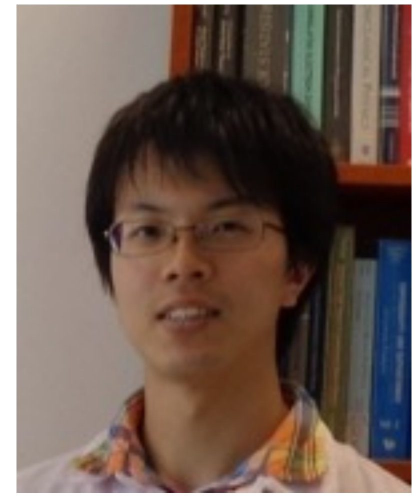


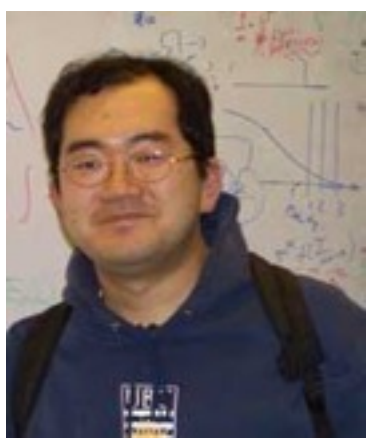


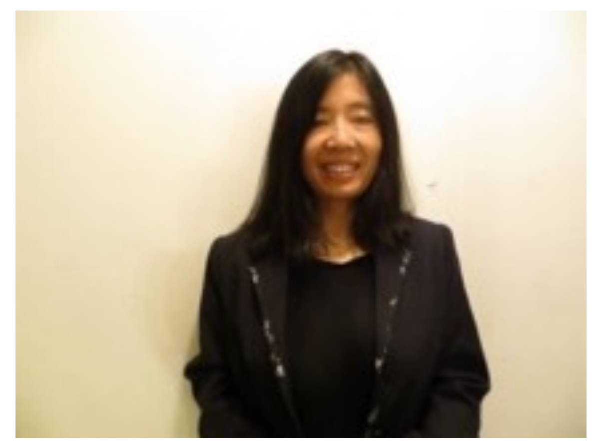


Thanks for your attention!







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