

# Gapped and gapless phases of the half filled Landau Level in bilayer graphene

**Andrea Young**

UCSB

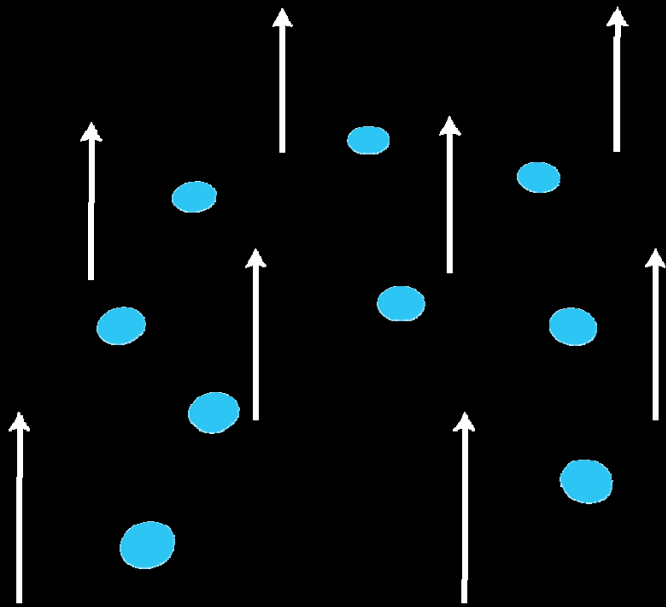
# Outline

- Half filled Landau levels – experimental perspective
- Bilayer graphene introduction: spin, valley and orbital symmetry breaking from layer resolved capacitance measurements
- Fractional quantum Hall effect in all vdW bilayer graphene heterostructures
- Intertwined phases at half filling

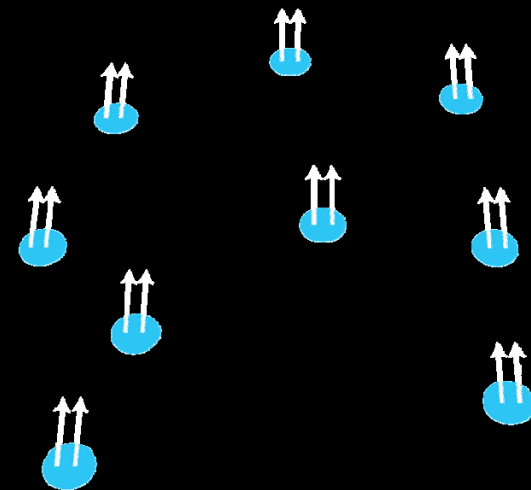
# Composite fermion ansatz

Jain 1989

Electrons in magnetic field  
w/strong interactions



Composite fermions in zero magnetic  
field with weak interactions



Field vanishes at  $\nu = \frac{1}{2}$

$$B_{eff} = B(1-2\nu)$$

What is left over? 'residual' interactions + anisotropies determine fate

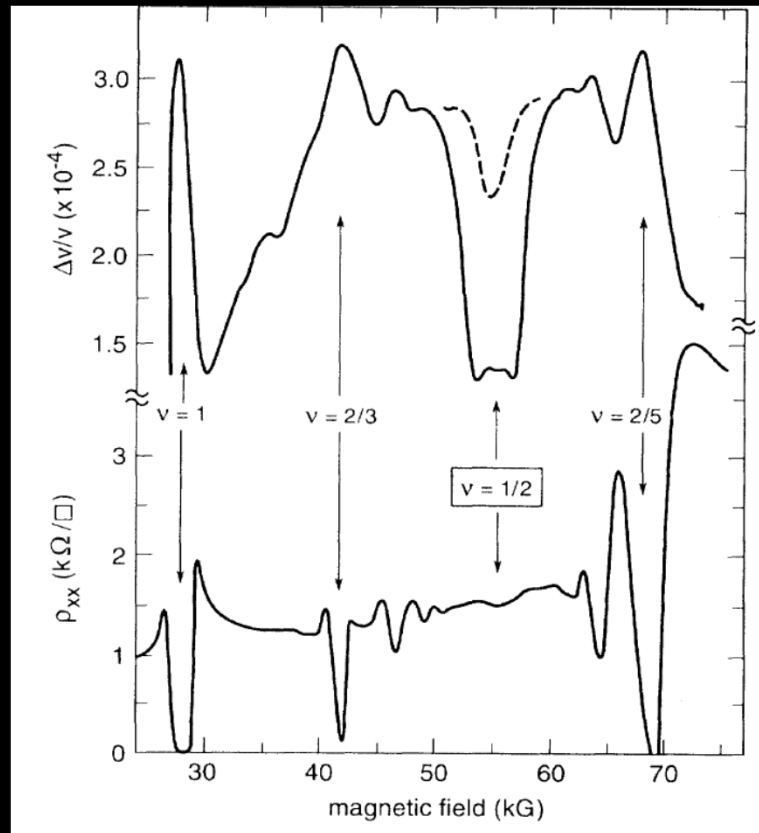
Composite fermion liquid

'superconductor'

CDW

# Half filling: phases

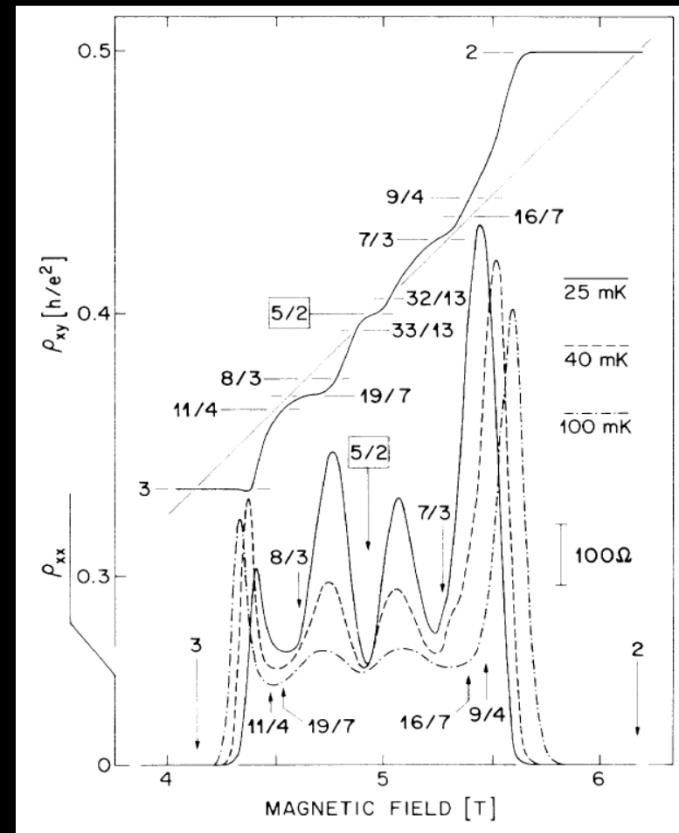
## N=0: Fermi surface



Willet 1993

Halperin, Lee, Read 1993

## N=1: 'paired' QH state



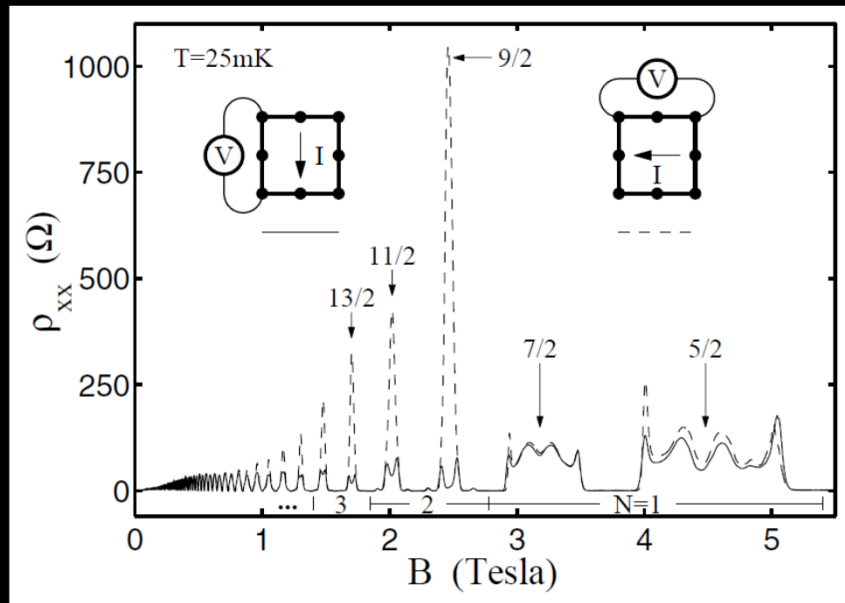
Willet 1987

Moore, Read 1991



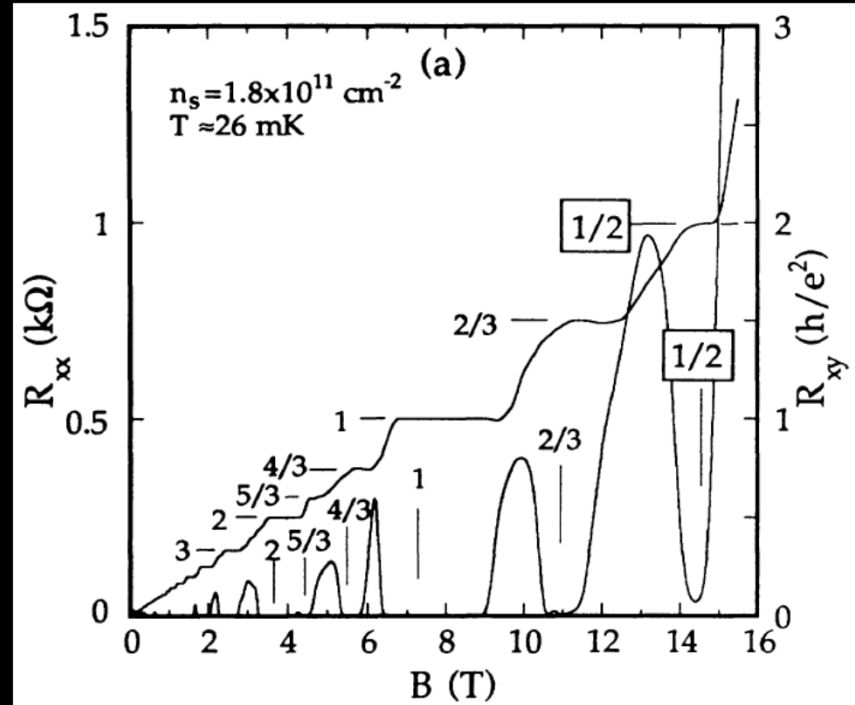
# Half filling: phases

Nematics/stripes :  $N > 1$



Lilly 1999

Multicomponent – ‘331’ QH



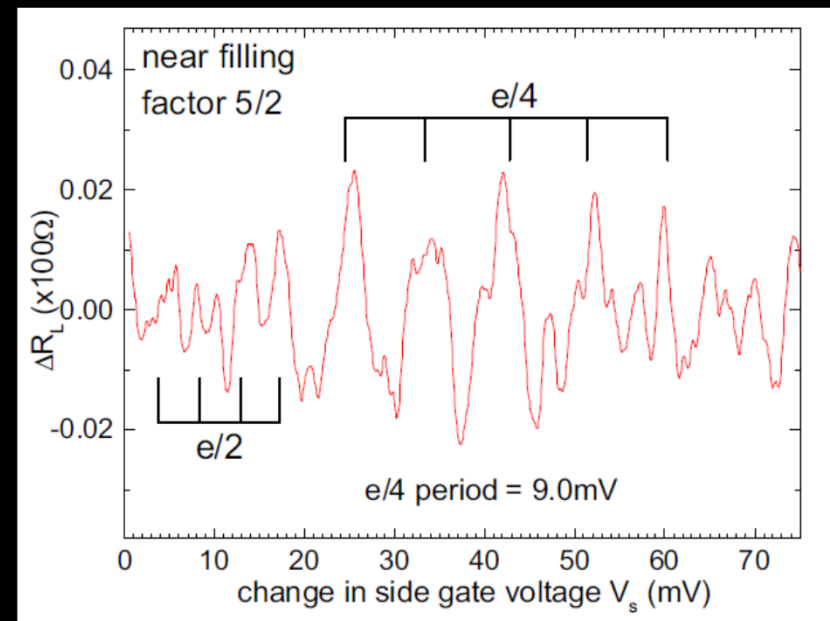
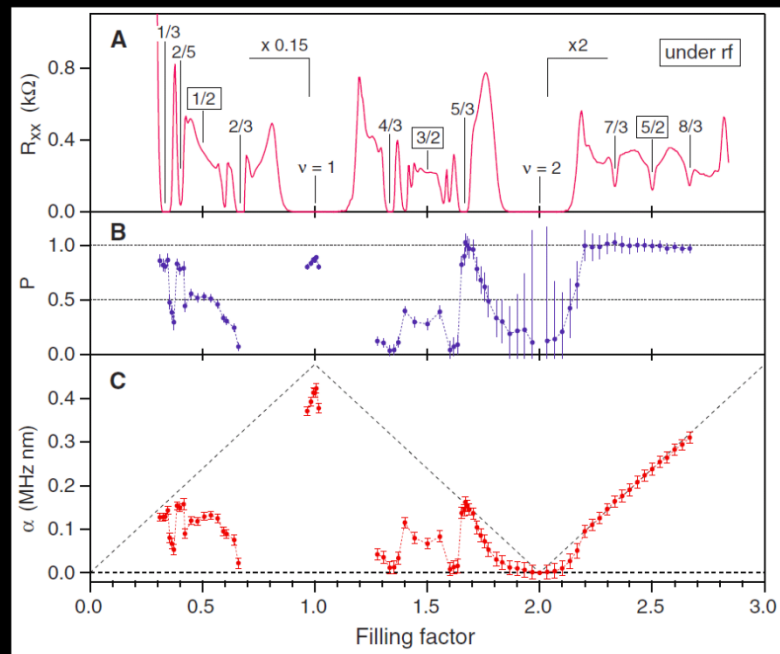
Suen 1992

Eisenstein 1992

Halperin 1983

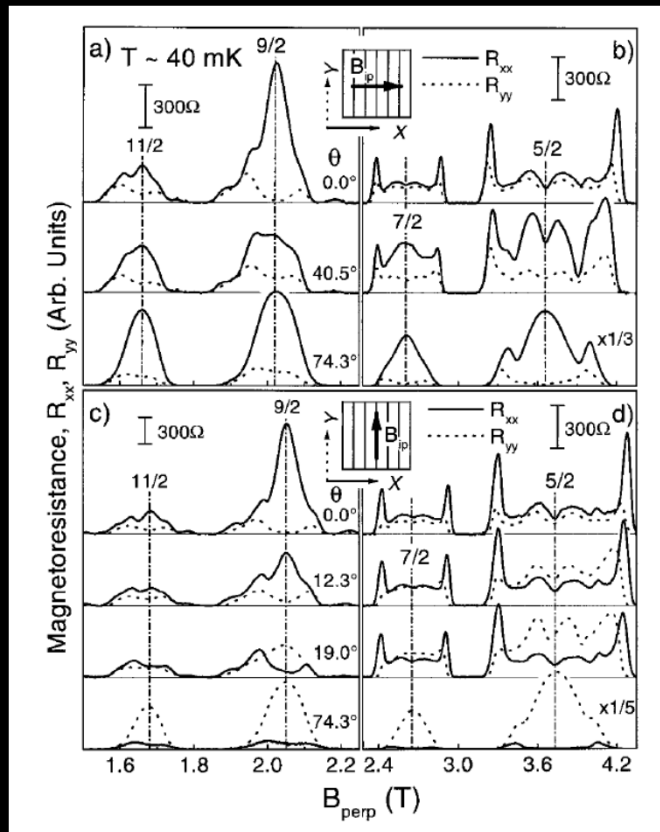
# Pfaffian phase at $\nu = \frac{5}{2}$

- Numerics (Morf 1998)  
N=1 LL, no spin, no mixing:  
Pfaffian,  $\Delta \approx .03 E_c$
- Experiment: spin polarized (Tiemann 2012)
- Charge  $e/4$  consistent  
Heiblum, Yacoby gps
- Quantum dots: no smoking gun...



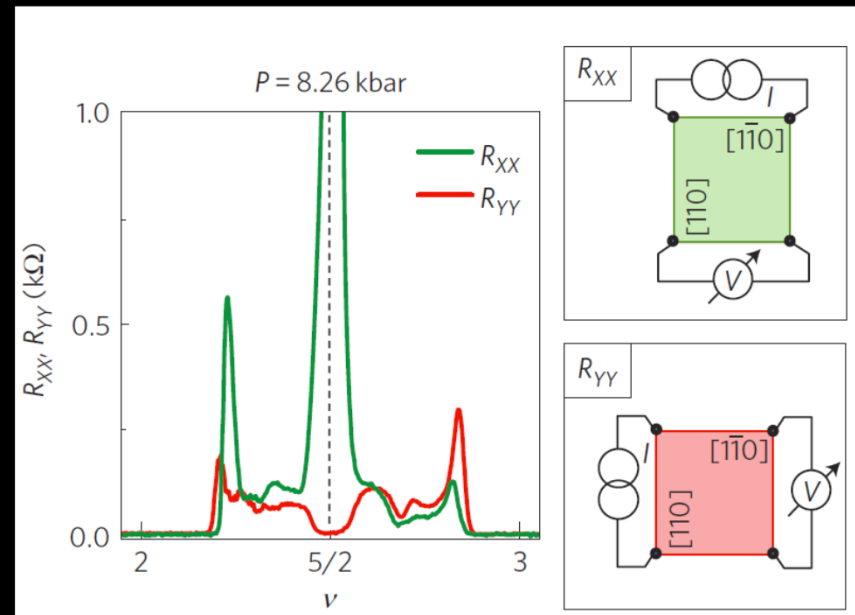
# Complex energy landscape

## In-plane field



Pan 1999

## 'Isotropic' pressure



Samkharadze 2016

# Bernal bilayer Landau levels

- 2x spin, 2x valley  
“degeneracy”

$$H_0 = -\frac{1}{2m} \begin{pmatrix} 0 & \hat{p}^2 \\ (\hat{p}^\dagger)^2 & 0 \end{pmatrix}$$

- $(\Psi_{A,K}, \Psi_{B,K})$  or  $(\Psi_{B,K'}, \Psi_{A,K'})$

$$H_B = -\hbar\omega_c \begin{pmatrix} 0 & \hat{a}^2 \\ (\hat{a}^\dagger)^2 & 0 \end{pmatrix}$$

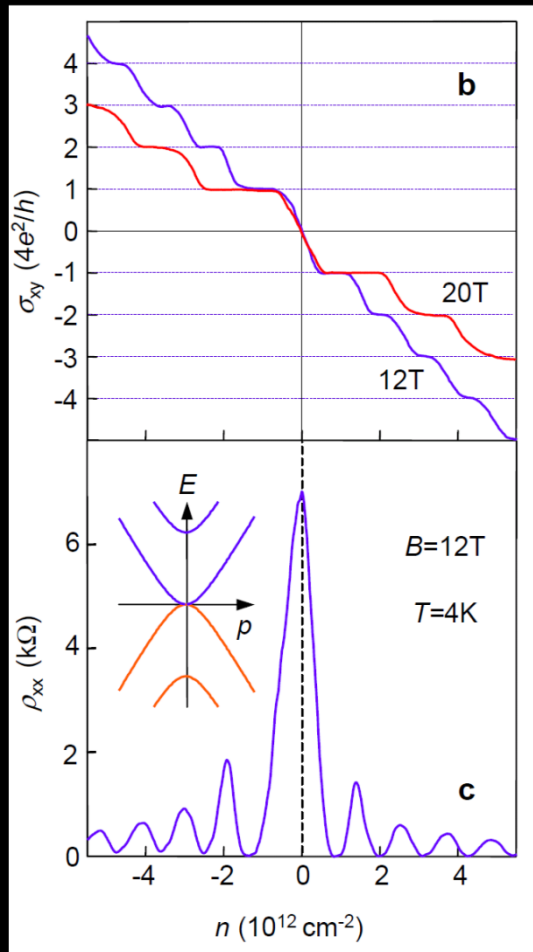
$$\Psi_{n>1} = \frac{1}{\sqrt{2}} \begin{pmatrix} |n-2\rangle \\ |n\rangle \end{pmatrix}$$

$$\epsilon_{n>1} = \hbar\omega_c \sqrt{n(n-1)}$$

$$\Psi_{n=0,1} = \left\{ \begin{pmatrix} 0 \\ |0\rangle \end{pmatrix}, \begin{pmatrix} 0 \\ |1\rangle \end{pmatrix} \right\} \quad \epsilon_{0,1} = 0$$

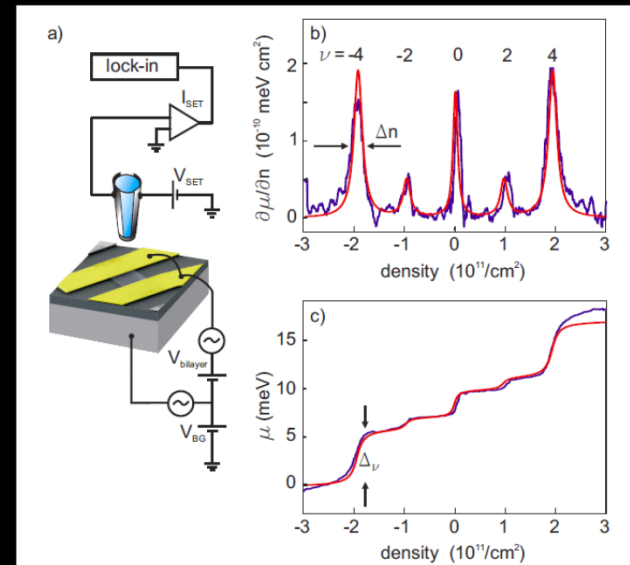
- *Sublattice=valley=layer* in zero energy LL
- *2x orbital degeneracy*  $\rightarrow$  *8x total degeneracy*

# Bernal bilayer Landau levels



Novoselov 2006

- 8x degenerate ZLL
- Degeneracies lifted in cleaner samples



Feldman 2010

# No *real* degeneracy

## Single particle

- Zeeman effect
  - $\Delta_\sigma = g\mu_B B \approx 1.3K B_{tot}/T$
- Valley splitting by e field
  - $\Delta_V \sim \frac{p_0}{c} \frac{d_{BLG}}{d_{gate}} \approx 10K \frac{p_0/c}{volt}$
- Electron-hole asymmetry  $\gamma_4$ 
  - $\Delta_{10} \approx 3.3K B_\perp/T$

## Many body

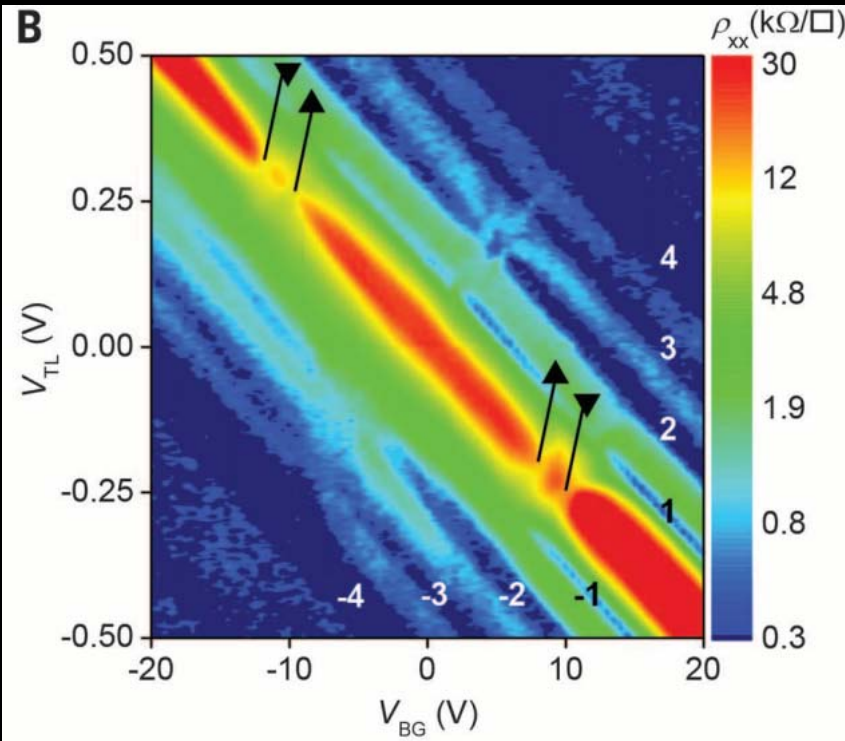
- N=0, N=1: different exchange energies
  - $\zeta_{10} \approx \frac{e^2}{\epsilon \ell_B}$
- SU(4) isospin: short range interactions
  - $u_i \approx \frac{a}{\ell_B} \frac{e^2}{\epsilon \ell_B}$

- Spin-valley: like monolayer, approx SU(4) symmetry
- Orbital: *not a symmetry*

# Degeneracies lifted in experiment

## Chemical potential and quantum Hall ferromagnetism in bilayer graphene

Kayoung Lee,<sup>1</sup> Babak Fallahazad,<sup>1</sup> Jiamin Xue,<sup>1</sup> David C. Dillen,<sup>1</sup> Kyoungwan Kim,<sup>1</sup> Takashi Taniguchi,<sup>2</sup> Kenji Watanabe,<sup>2</sup> Emanuel Tutuc<sup>1\*</sup>

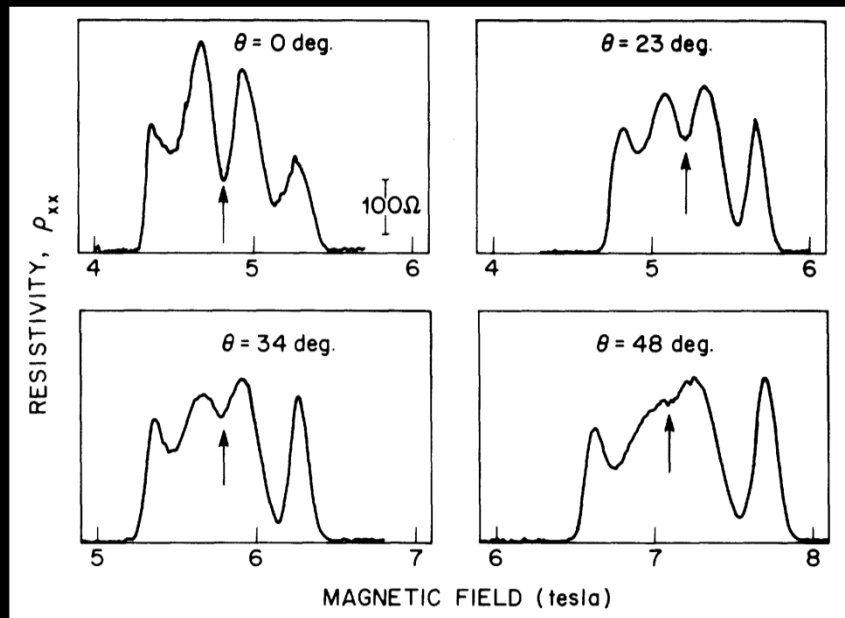


- In what “order?”
- Conventional wisdom: Fill  $SU(4)$  polarized  $|0\rangle$
- Next fill  $|1\rangle$  same isospin
- $\sim SU(4)$  QHFM of orbital doublets

Barlas 2008  
Abanin 2009

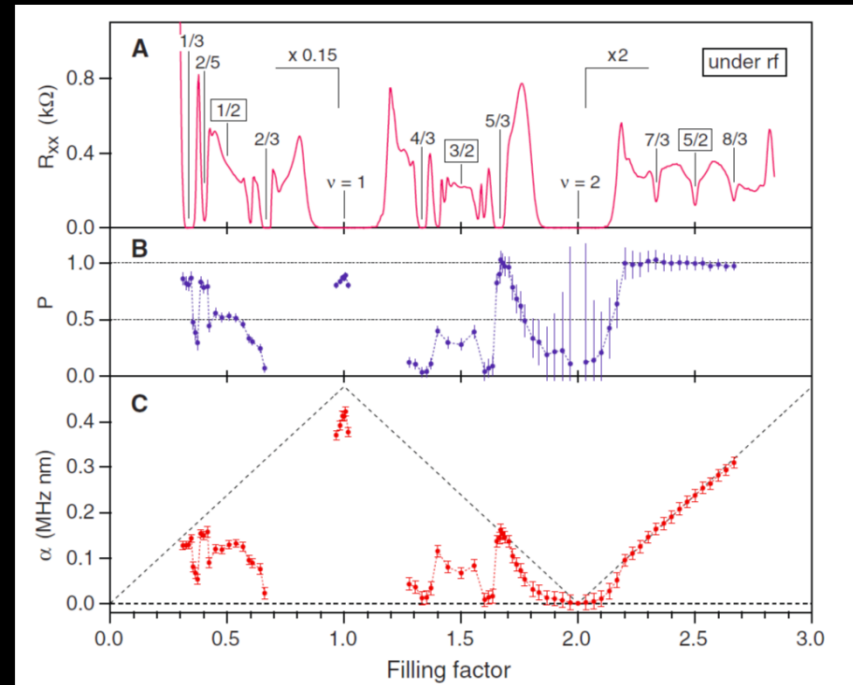
# Transport vs. direct probes: 5/2

Transport: unpolarized



Eisenstein 1988

Direct probe: polarized

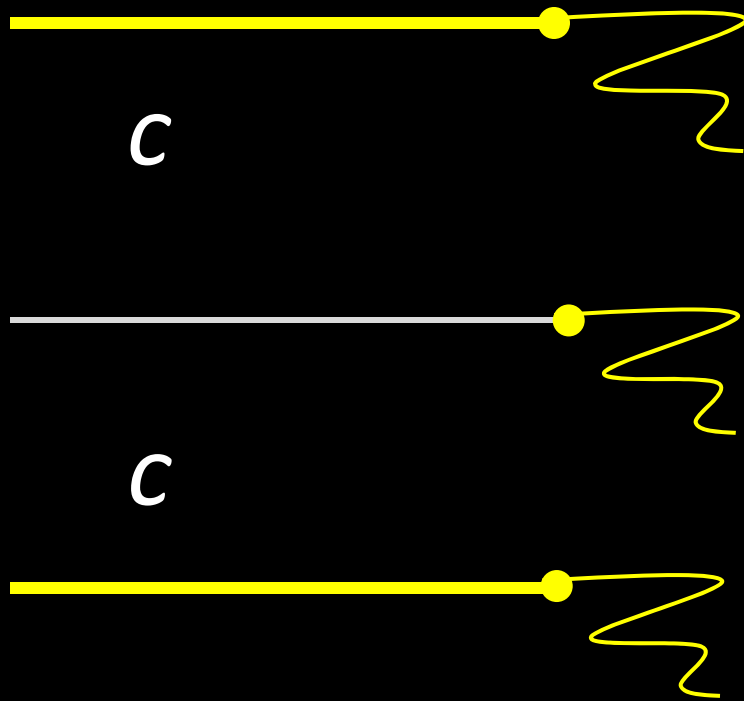


Tiemann 2012



# Capacitance in a 2DES

- Geometric capacitance & electronic compressibility

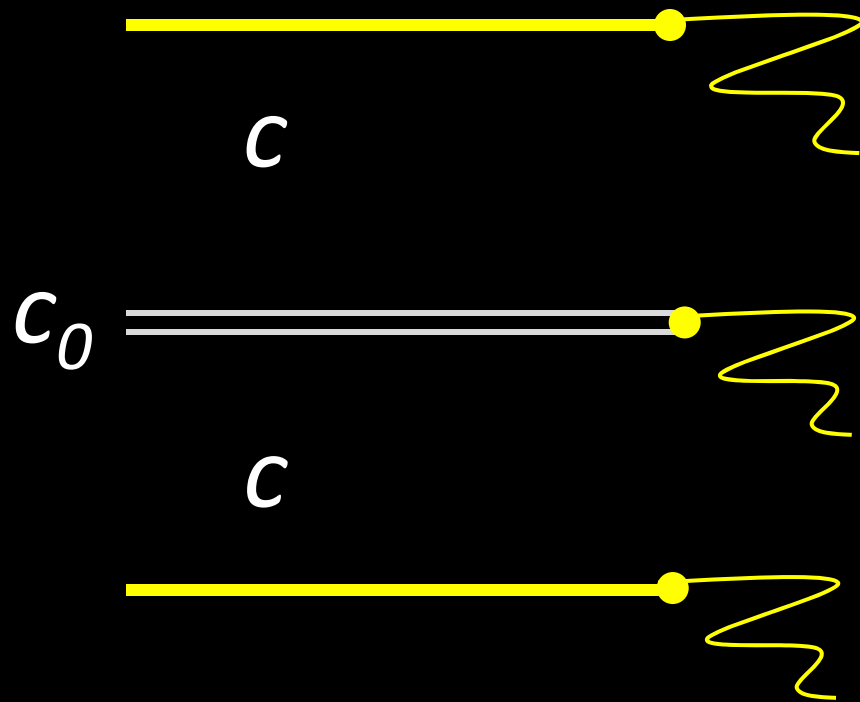


$$C_T \sim \frac{c\nu}{2c + \nu}$$

$$C_B \sim \frac{c\nu}{2c + \nu}$$

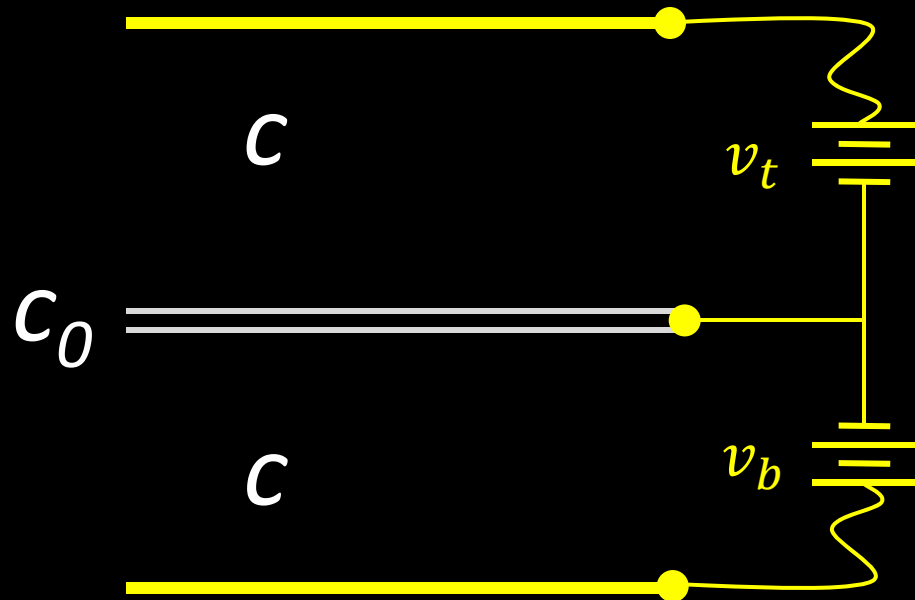
$$C_P \sim \frac{c^2}{2c + \nu}$$

# Capacitance in a bilayer



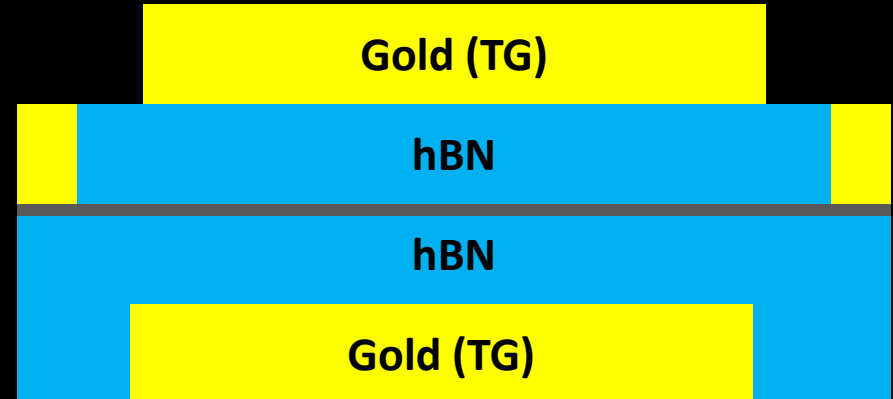
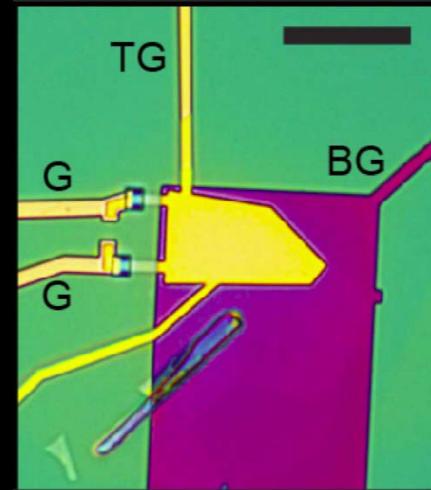
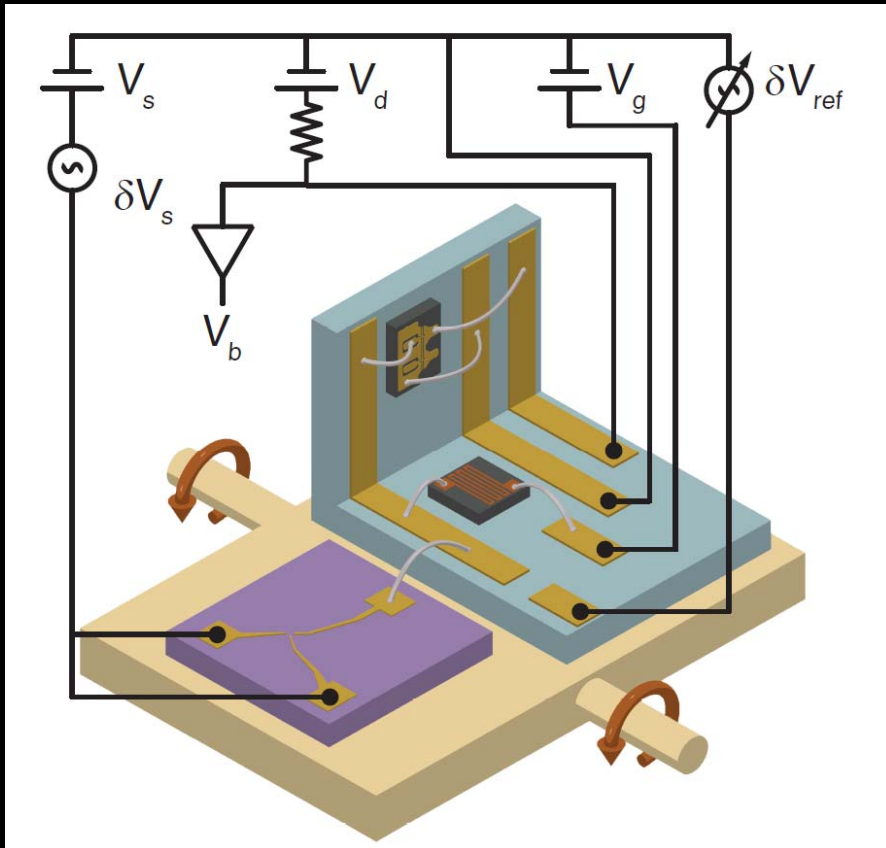
- Geometric capacitance & electronic compressibility
- *Interlayer capacitance:* charge can rearrange
- Layers:  $\mu_1 + \varphi_1 = \mu_2 + \varphi_2$   
 $\varphi_1 \neq \varphi_2, \mu_1 \neq \mu_2$
- $C_T, C_B,$  and  $C_P$  probe different physics

# Capacitance in a bilayer



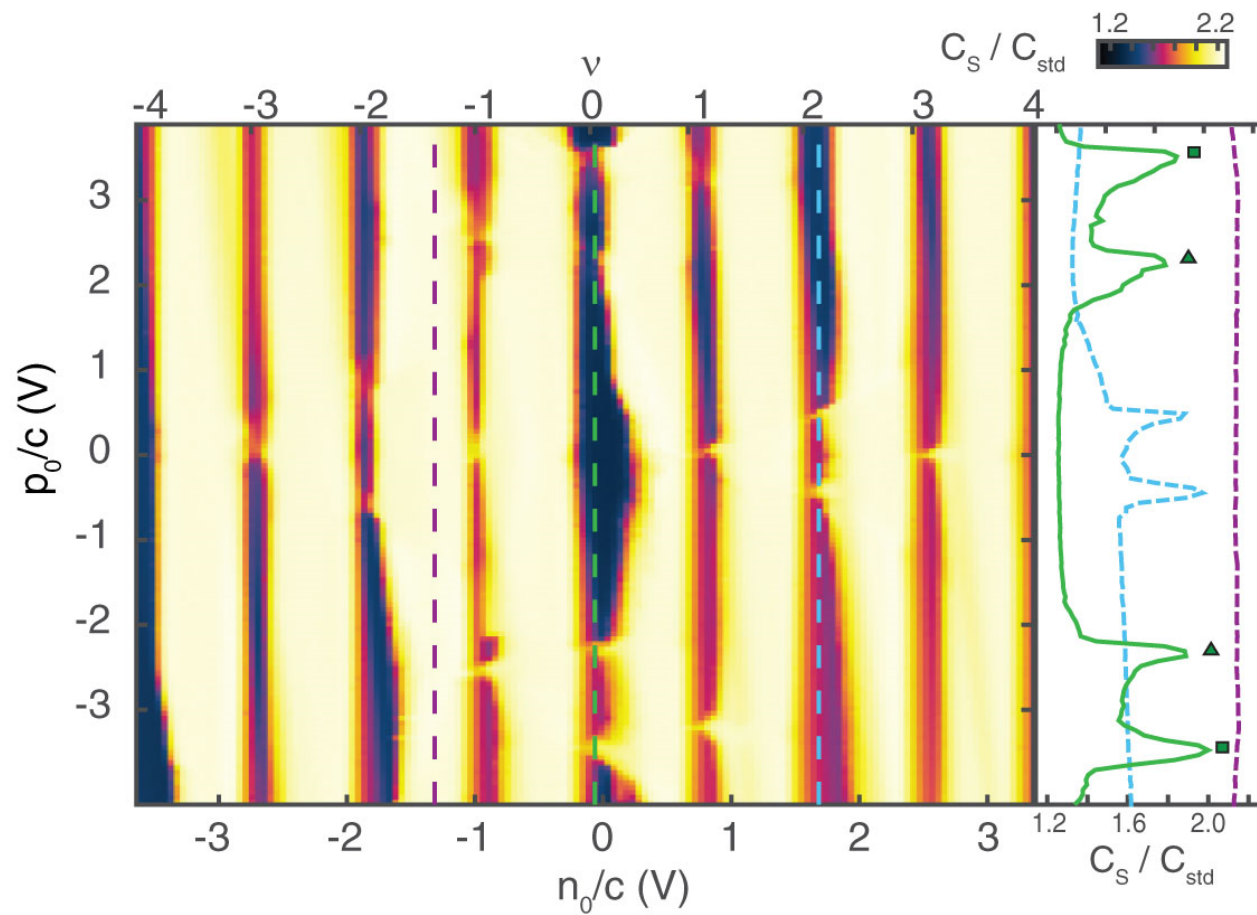
- $$\begin{cases} n_0 \equiv c(v_t + v_b) \\ p_0 \equiv c(v_t - v_b) \end{cases}$$
- $$\frac{\partial n}{\partial n_0} = C_S$$
- $$\frac{\partial p}{\partial n_0} \propto \frac{\partial n}{\partial p_0} = C_A$$
- $$\frac{\partial p}{\partial p_0} = C_S + 4C_P$$

# Device and measurement



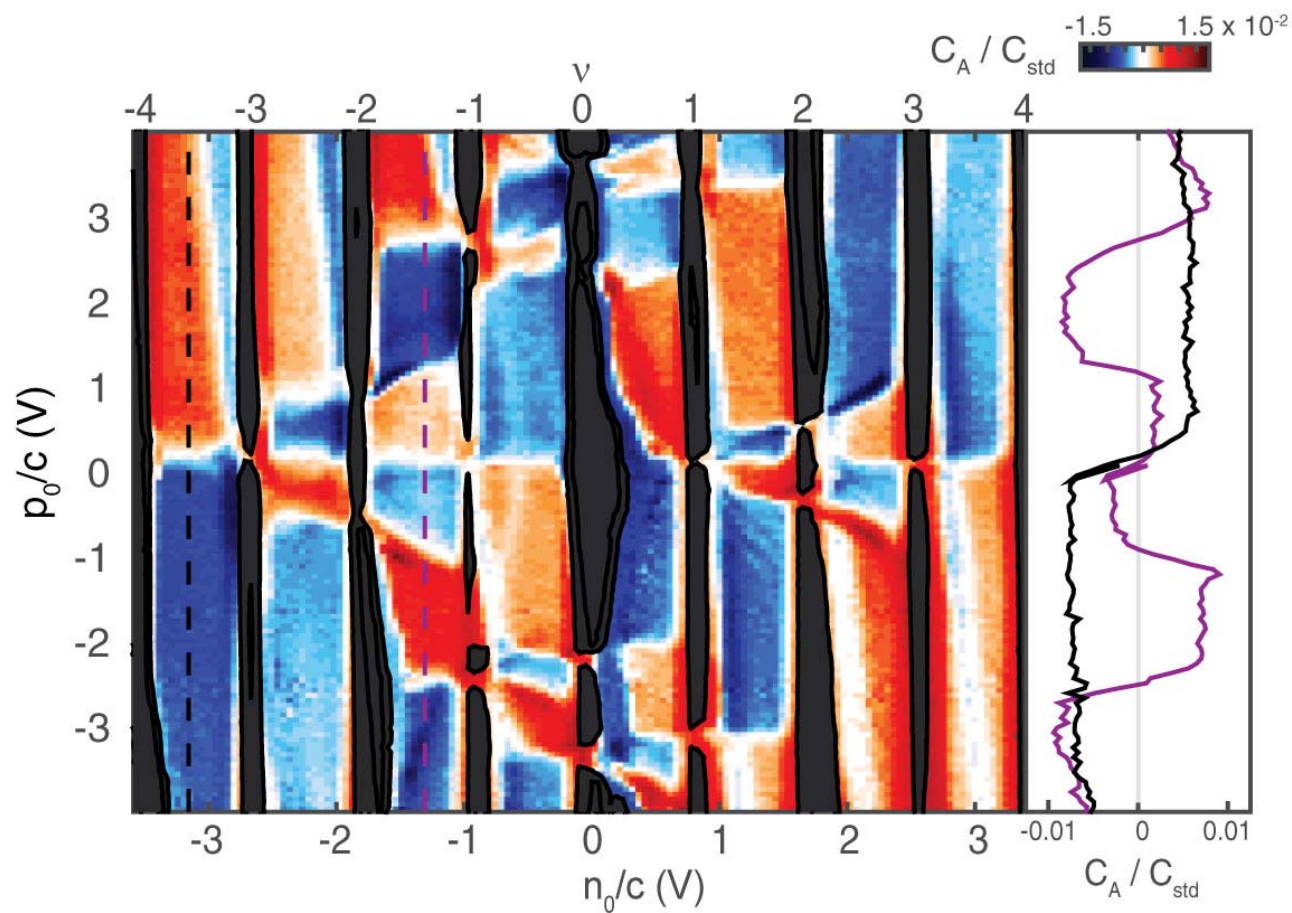
*HEMT Cryogenic amplifiers*  
Ashoori *PRL* (1991)

# Bilayer graphene: 31T



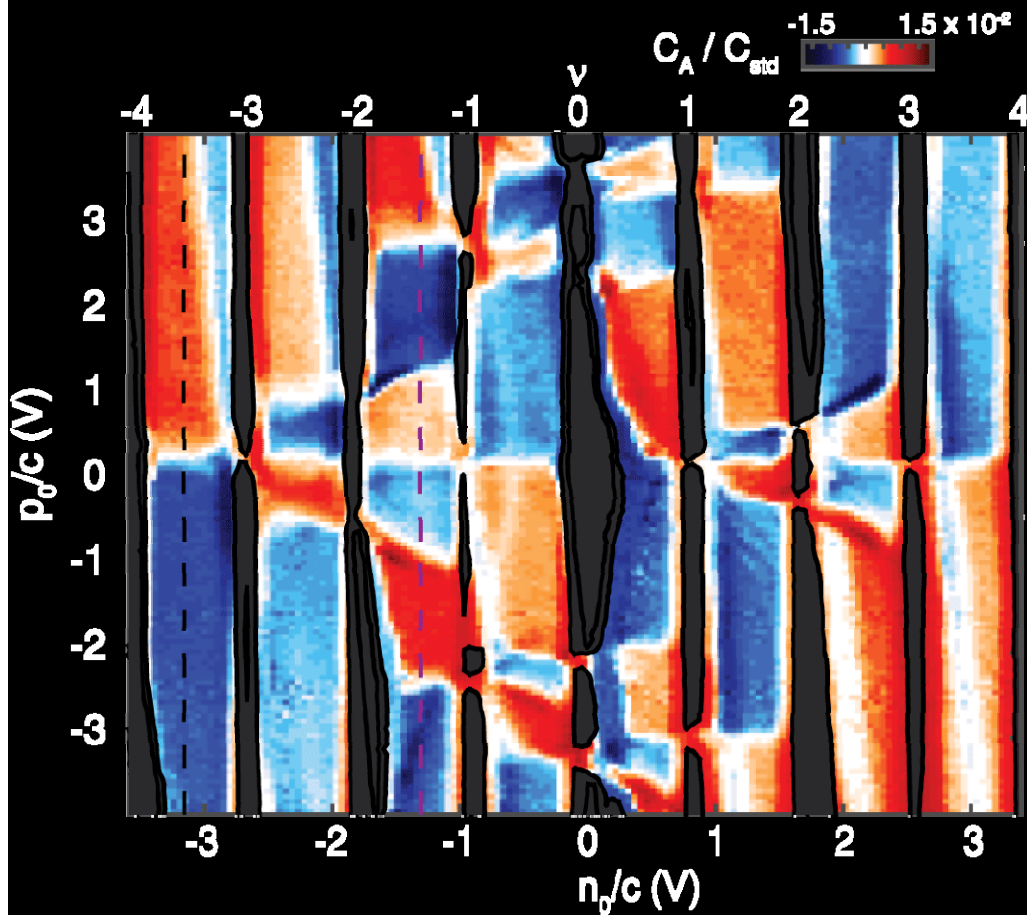
16 compressibility transitions

# Bilayer graphene: 31T



- $C_A$ : richer phase diagram
- red/blue  
→ top/bottom
- orange/red;  
blue/cyan → ???

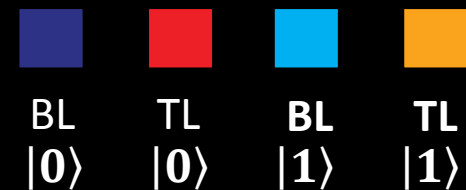
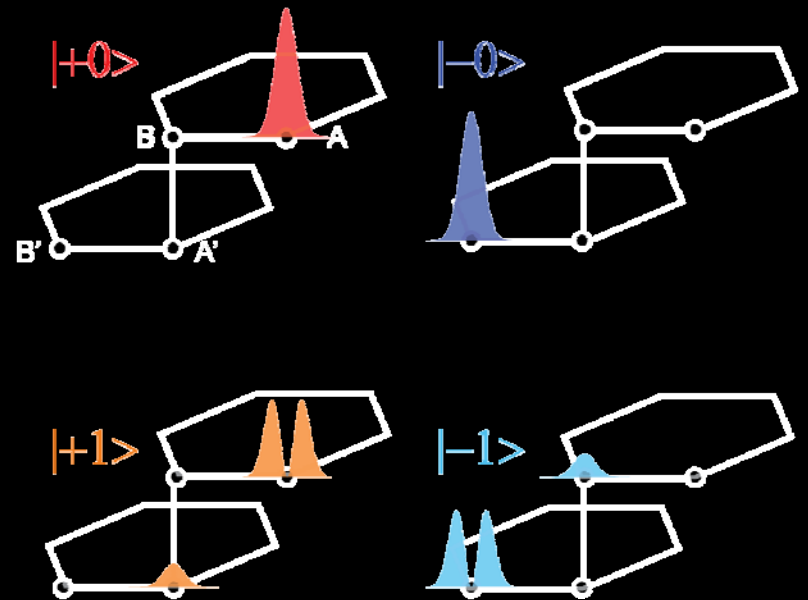
# BLG, B=31T (ZLL)



## 4 Band Orbital Wavefunctions

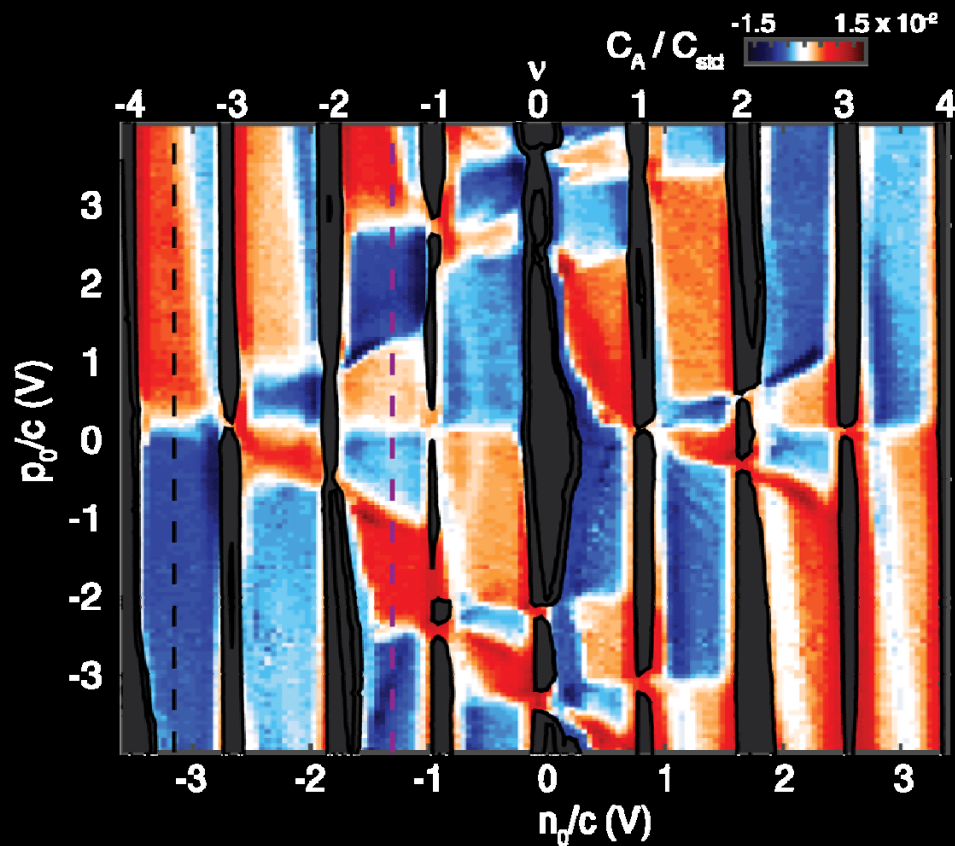
$$\psi_0 = \begin{pmatrix} |0\rangle \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \psi_1 = \begin{pmatrix} \alpha|1\rangle \\ 0 \\ \beta|0\rangle \\ \gamma|0\rangle \end{pmatrix} \quad \begin{matrix} A \\ B' \\ B \\ A' \end{matrix}$$

$$p = (\alpha^2 + \beta^2) - \gamma^2 \sim 0.6 @ 31 T$$



$C_A = \frac{\partial p}{\partial n_0}$ : where did the last electron added go?

# Bilayer graphene: 31T

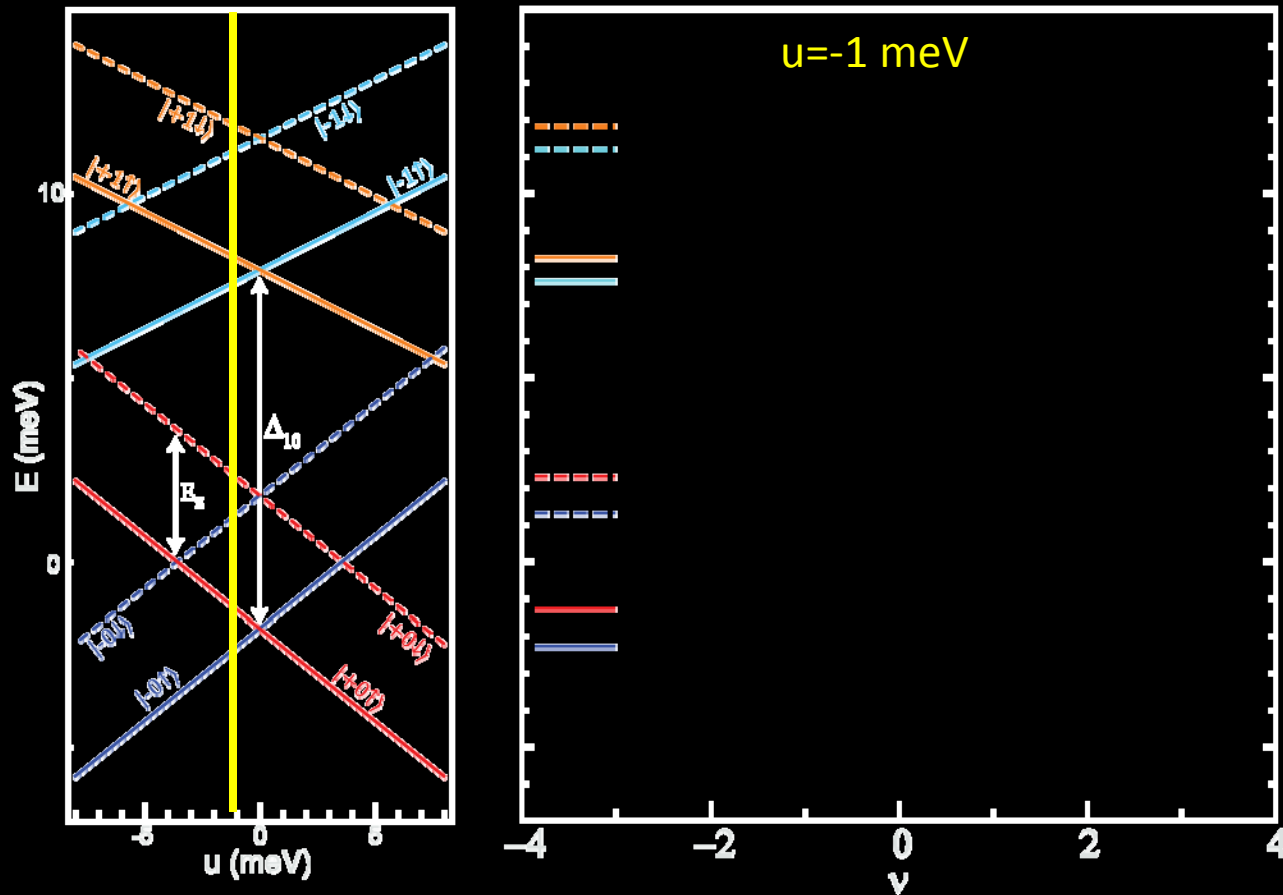


- High  $p_0$ :  $|0 +\rangle, |1 +\rangle$  order respected
- BUT low  $p_0$ :  $|0 +\rangle, |0 -\rangle$
- TL;DR:  
 $|NK\sigma\rangle$  levels fill in some order, but *one component at a time*



# Constructing a model

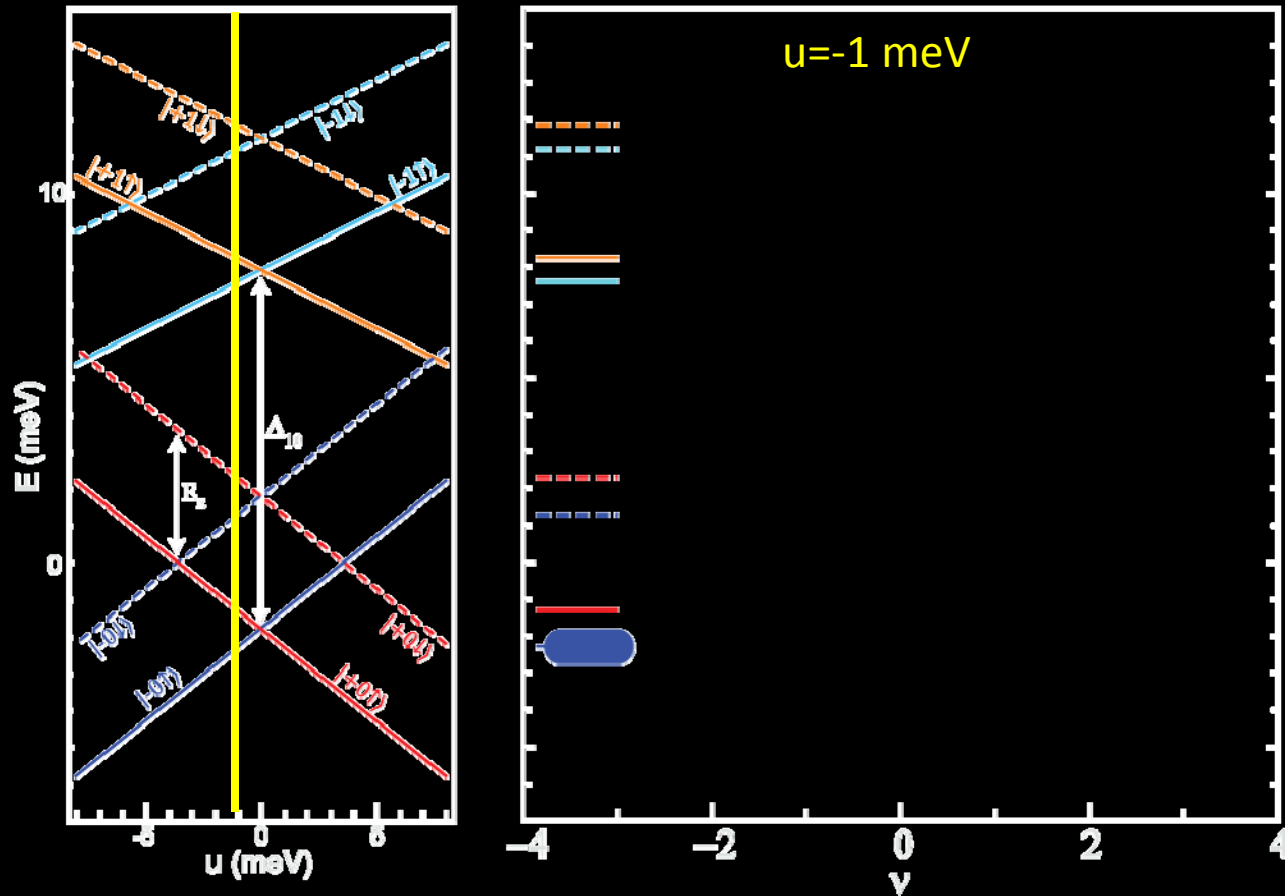
- Single particle energies:  $E_Z$ ,  $\Delta_{10}$ , and  $u$



- Basic physics: competition btw.  $\Delta_{10}$ ,  $\xi_{10}$

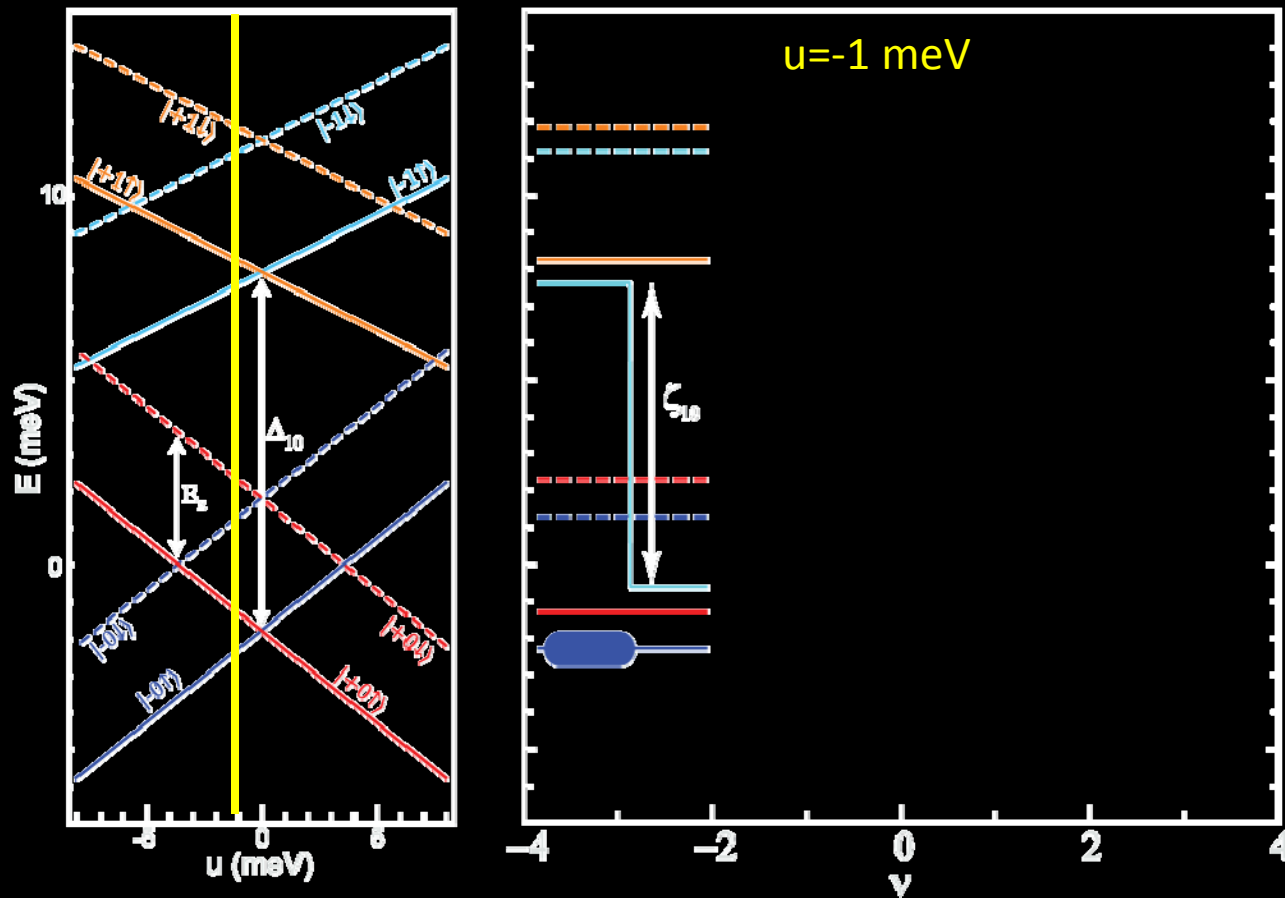
# Constructing a model

- Fill lowest energy level first



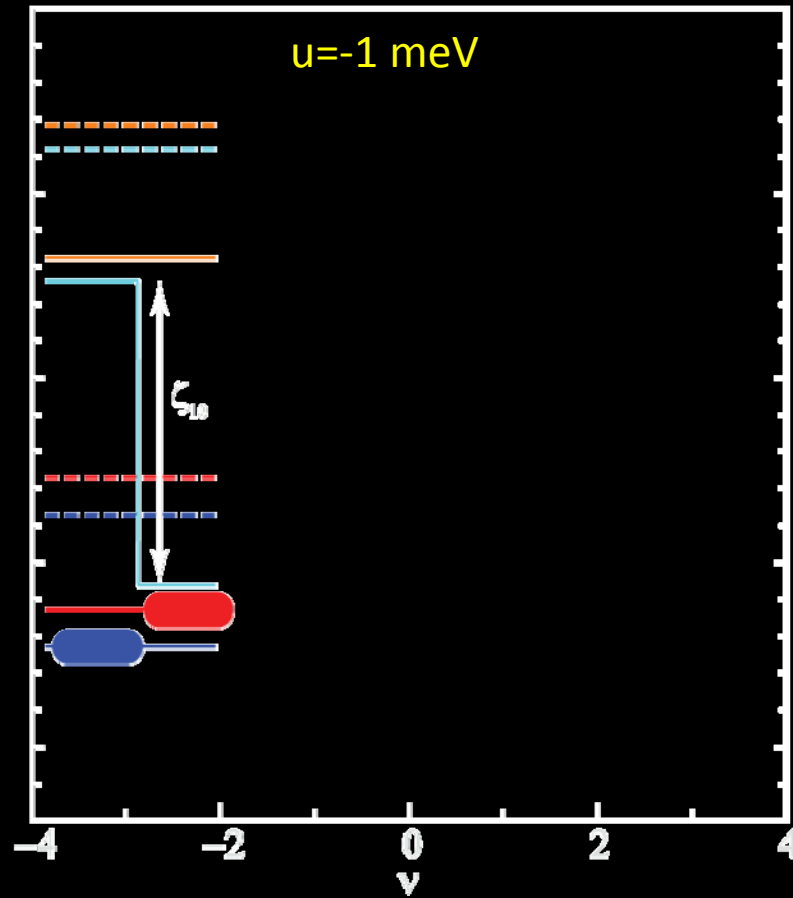
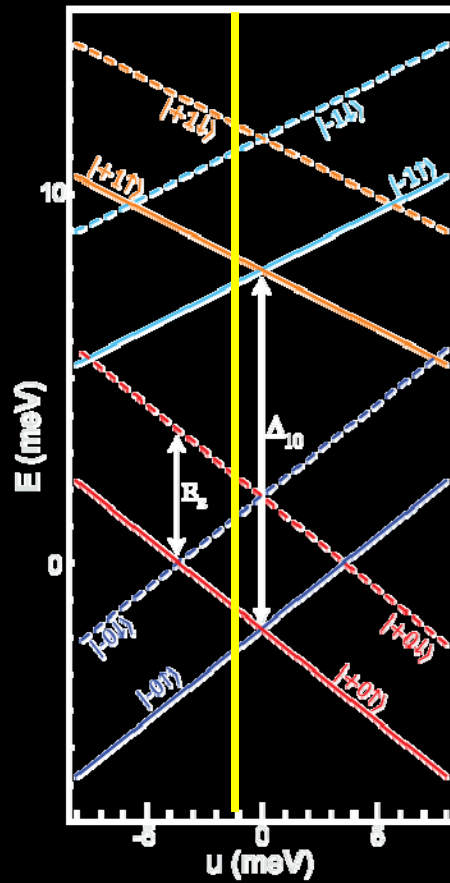
# Constructing a model

- Filled  $|0\rangle$  lowers empty  $|1\rangle$

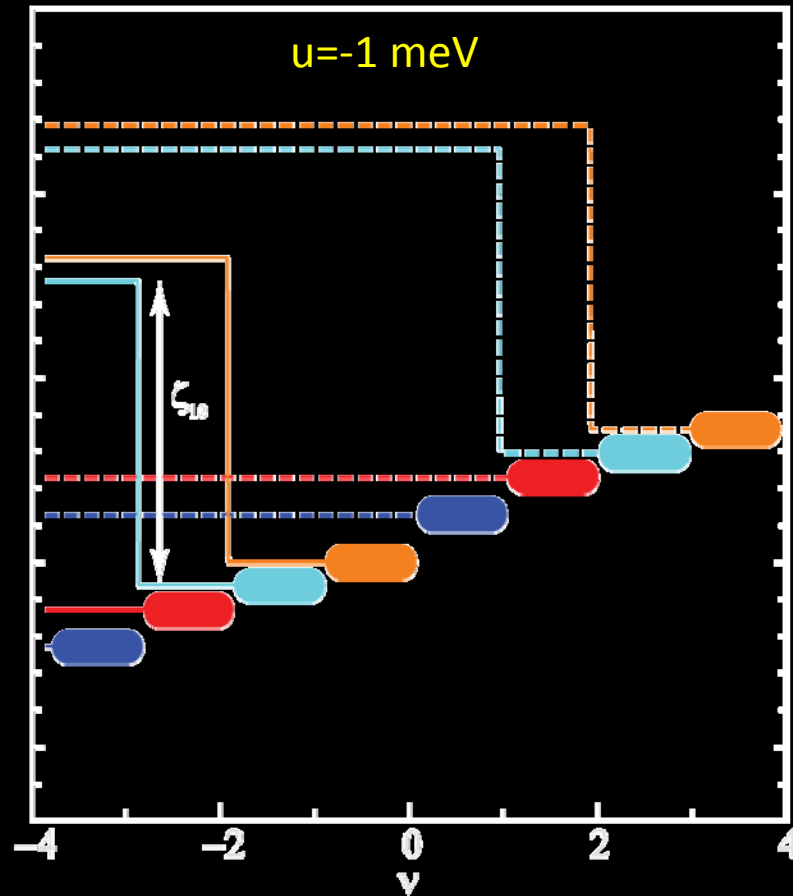
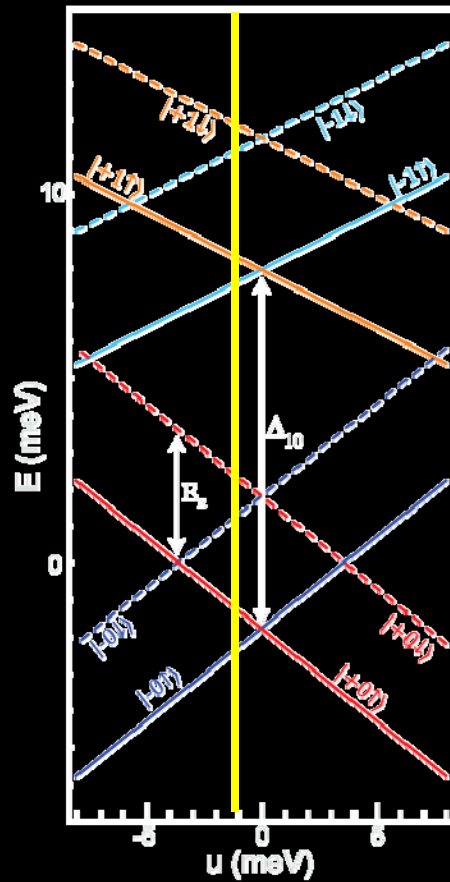


- Enough to overcome  $\Delta_{10}$  anisotropy?

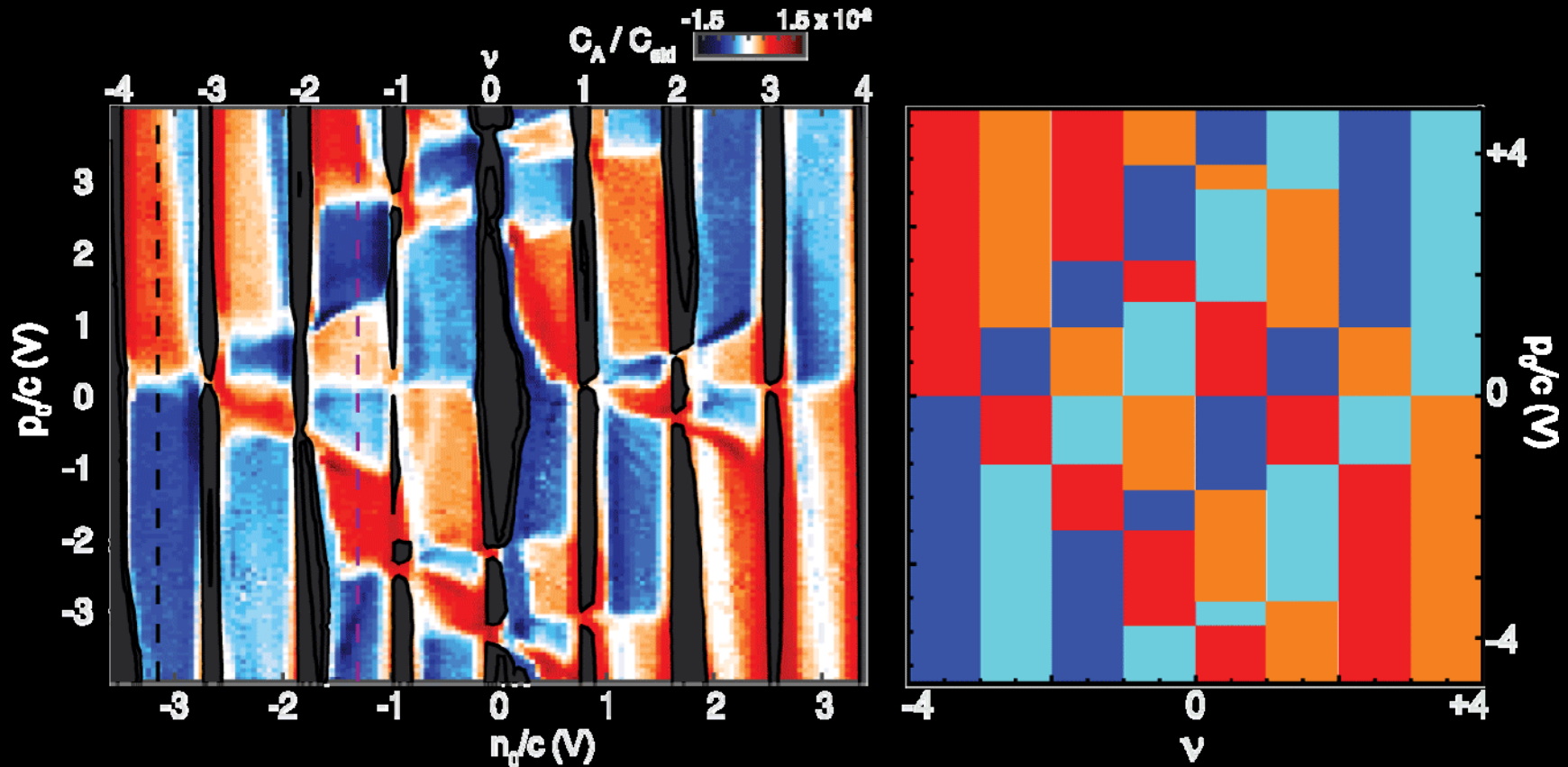
# Constructing a model



# Constructing a model



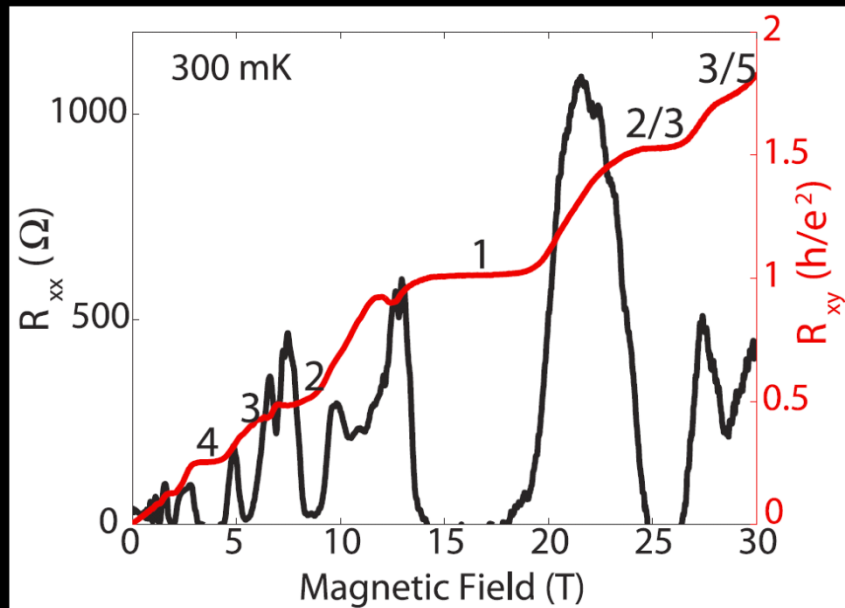
# Phase diagram



- Large regions where isospin, orbital physics 'inert'
- Others with dense orbital/isospin crossings
- Need large  $\epsilon \approx 11$  to fit

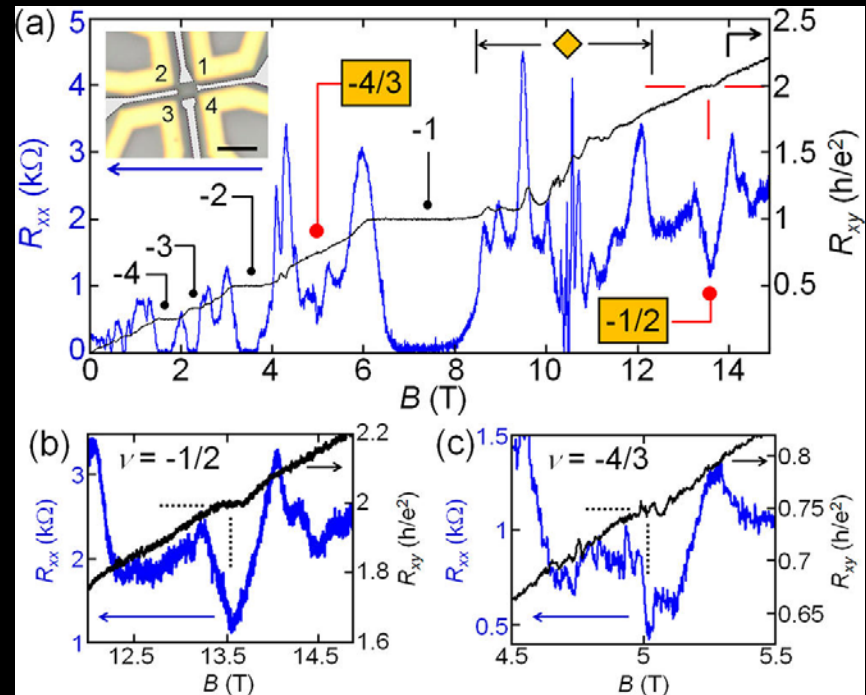
# Bilayer graphene FQH: literature

FQHE: 1/3's and 1/5's



Maher 2014  
Kou 2014

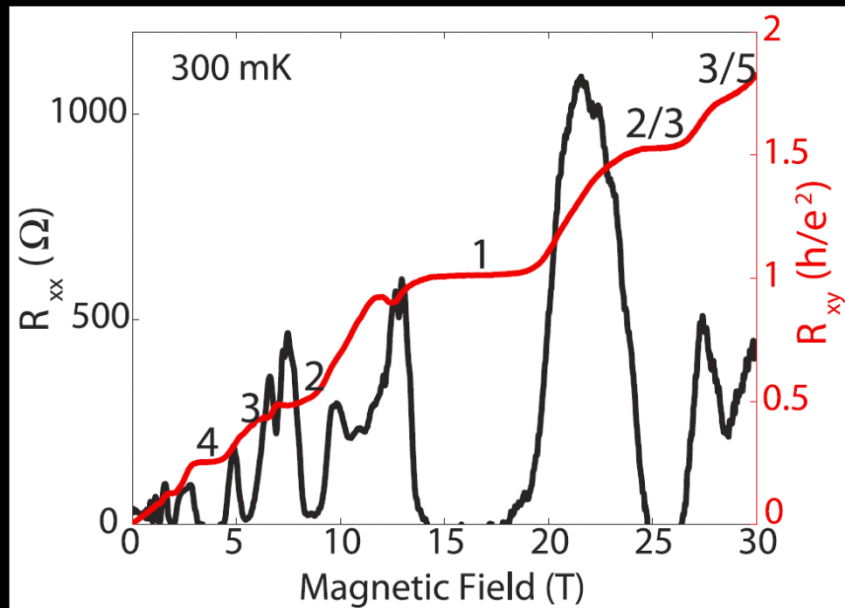
Even denominator – Pfaffian?



Ki 2014

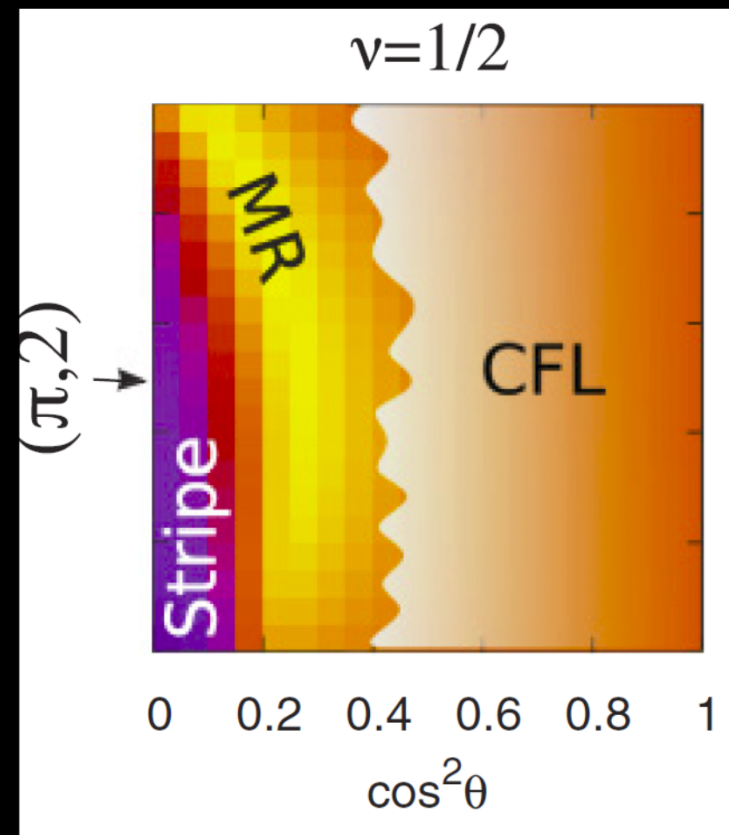
# Bilayer graphene FQH: literature

## FQHE: 1/3's and 1/5's



Maher 2014  
Kou 2014

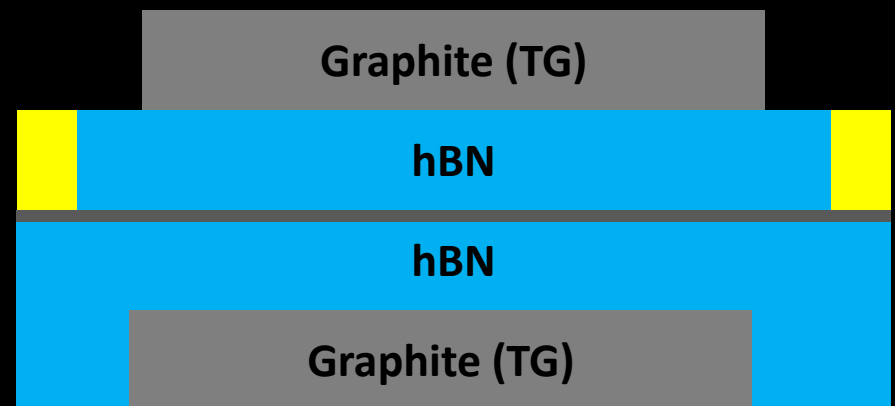
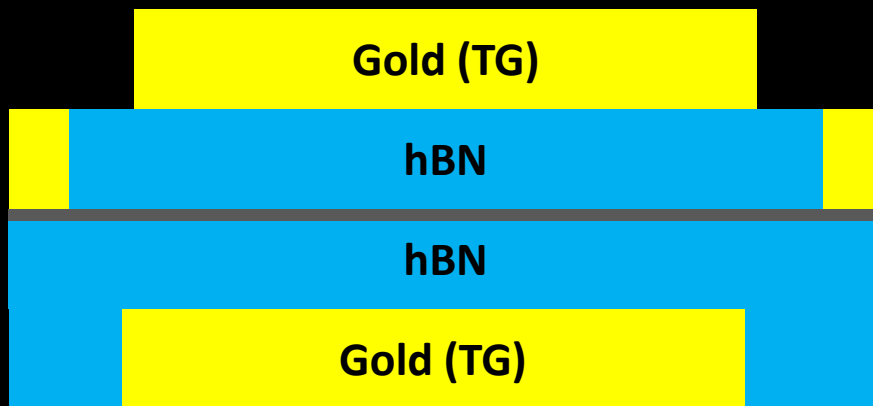
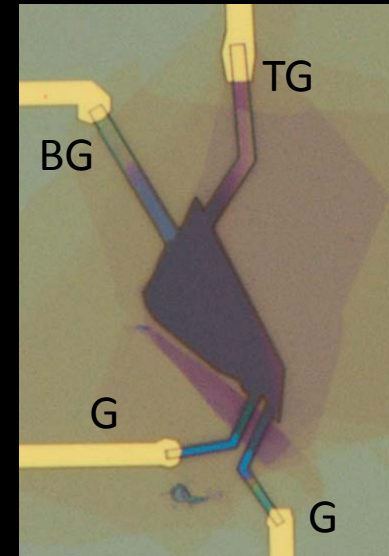
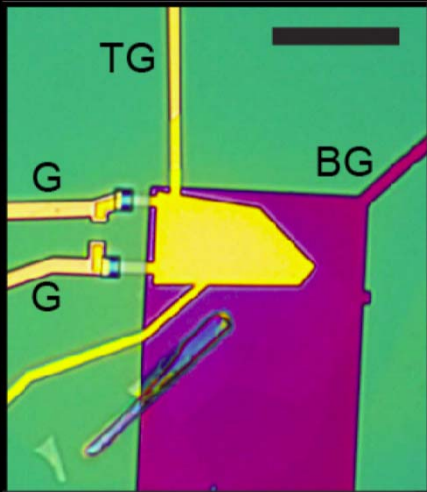
## Even denominator – Pfaffian?



Papic 2011  
Apalkov 2011  
Papic 2014

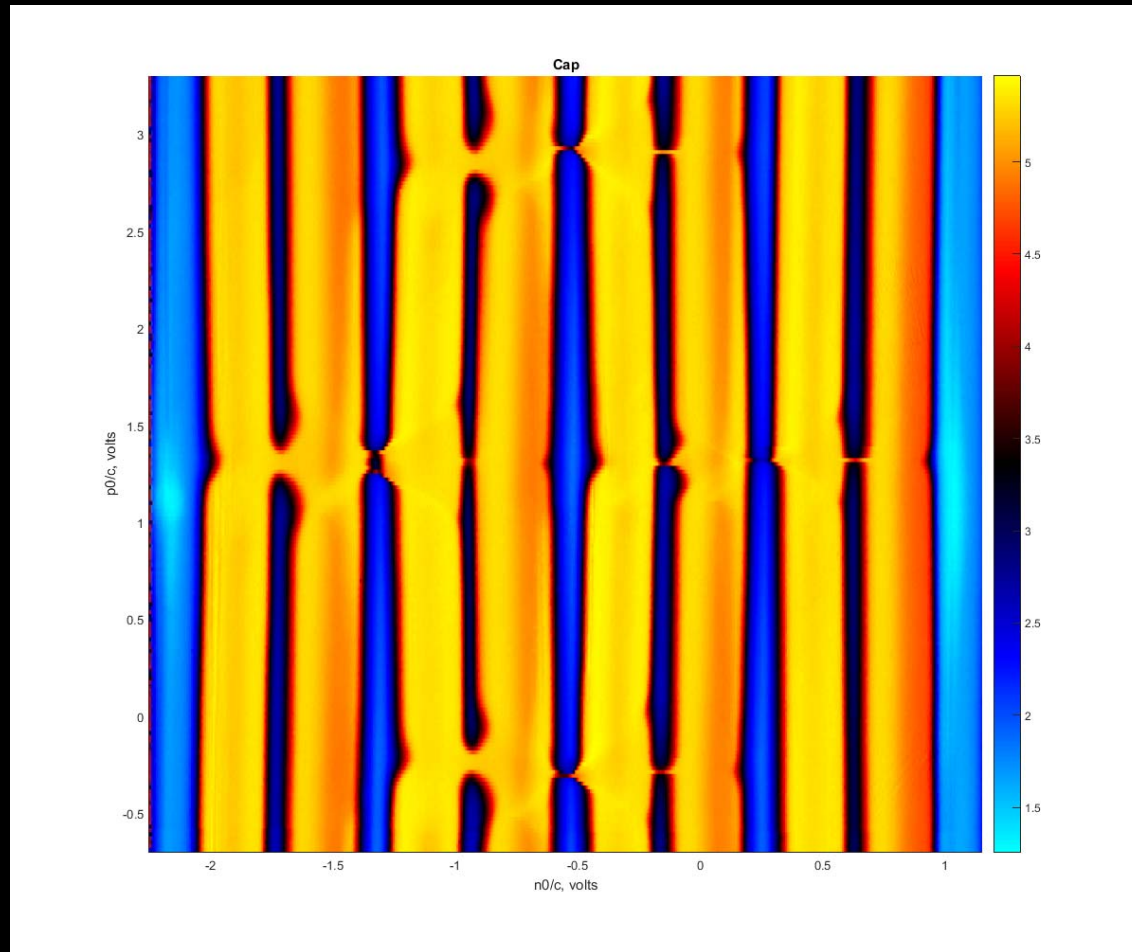


# What limits sample quality?

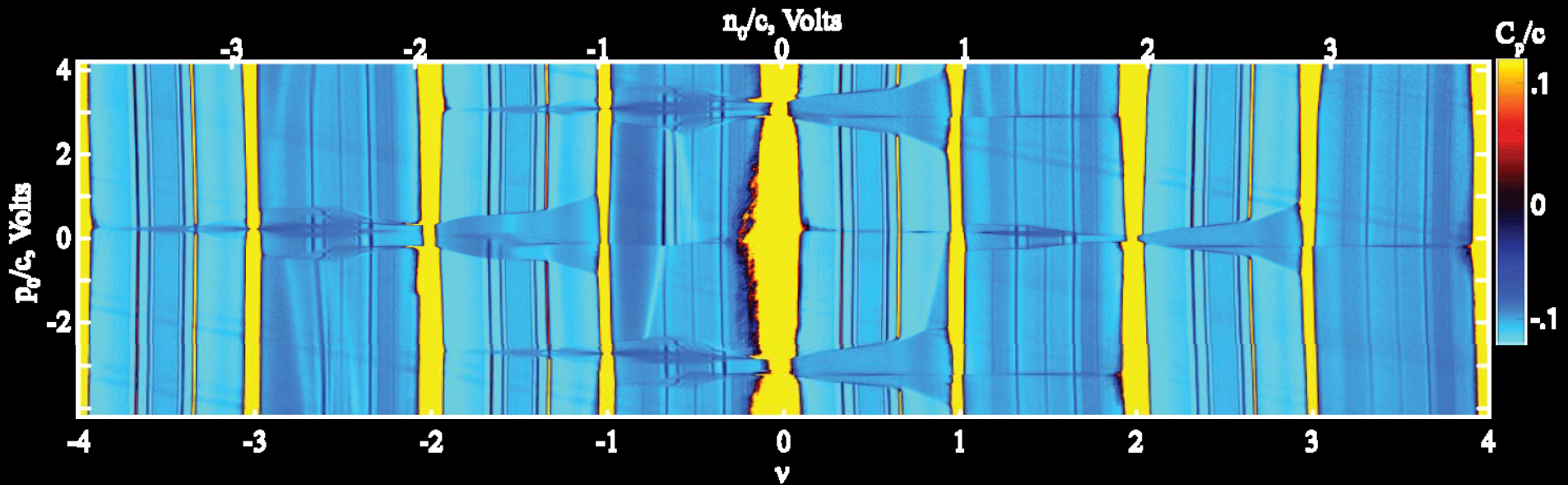


# Metal gates – IQHE gaps

$B=10\text{T}; T \approx 50 \text{ mK}, C_S$

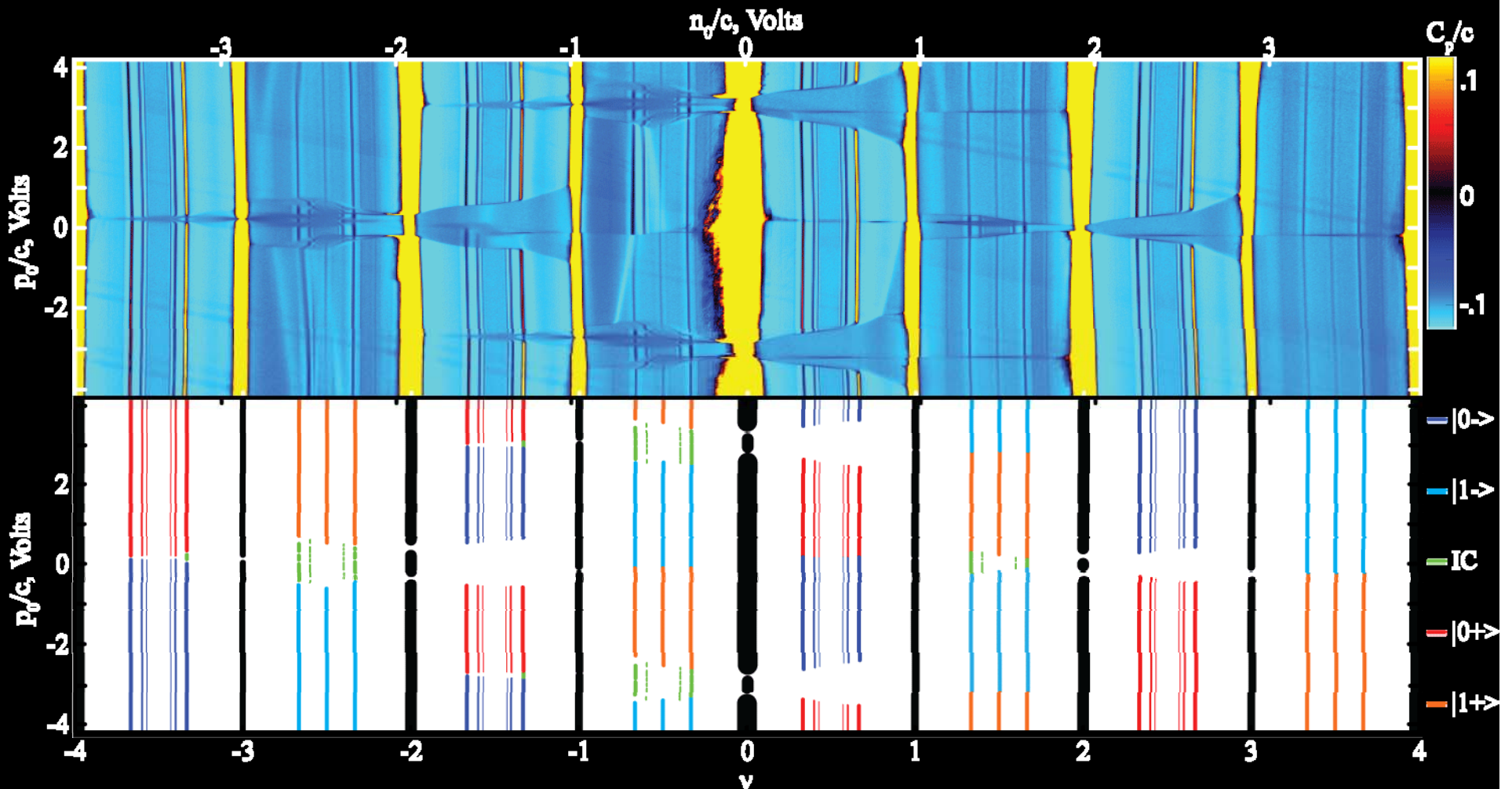


# Graphite gates – FQHE!



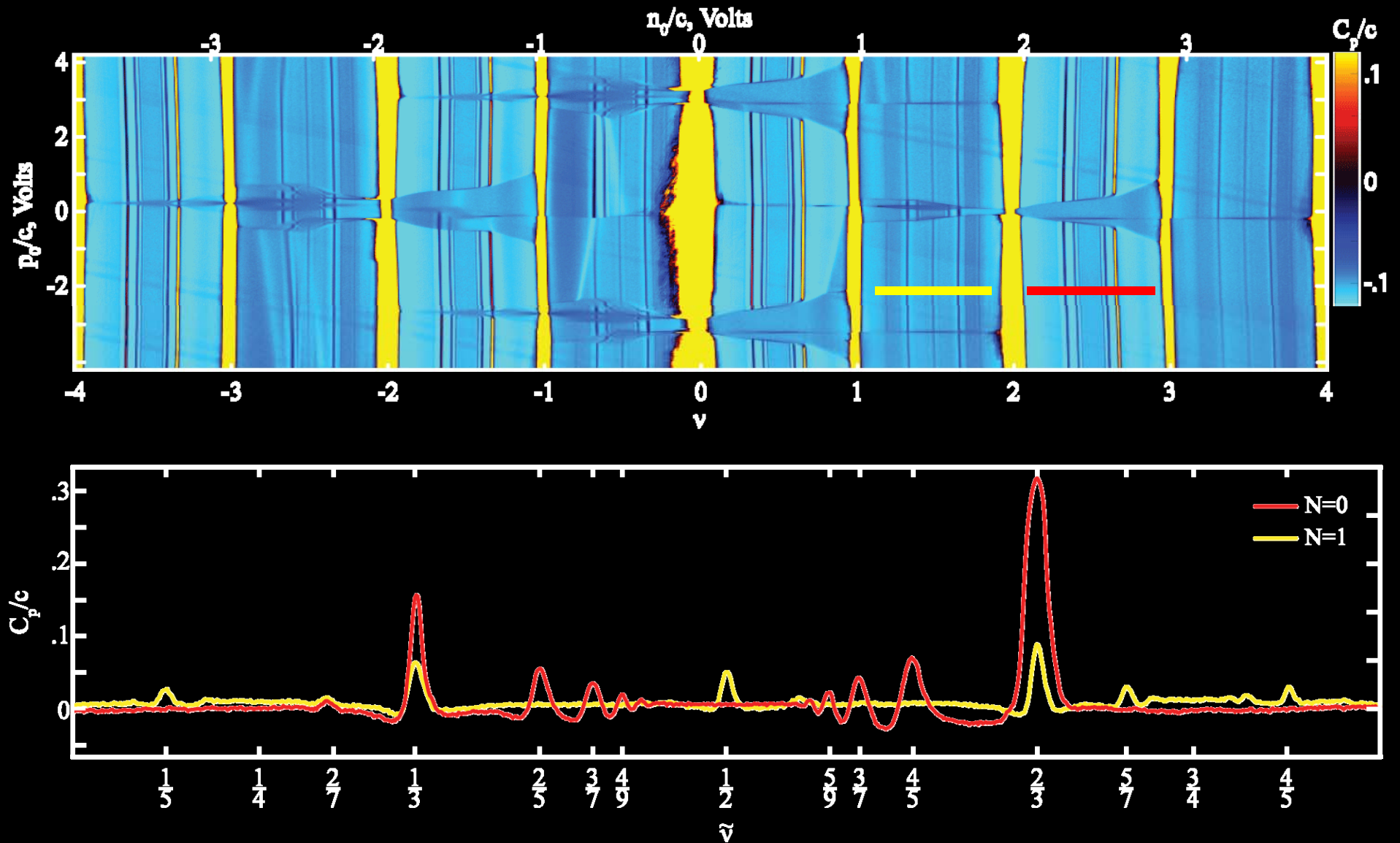
- Incompressible states at fractional filling throughout ZLL
- Many phase transitions – same # at integer  $\nu$ , many more at fractional  $\nu$

# Fractional quantum Hall effect



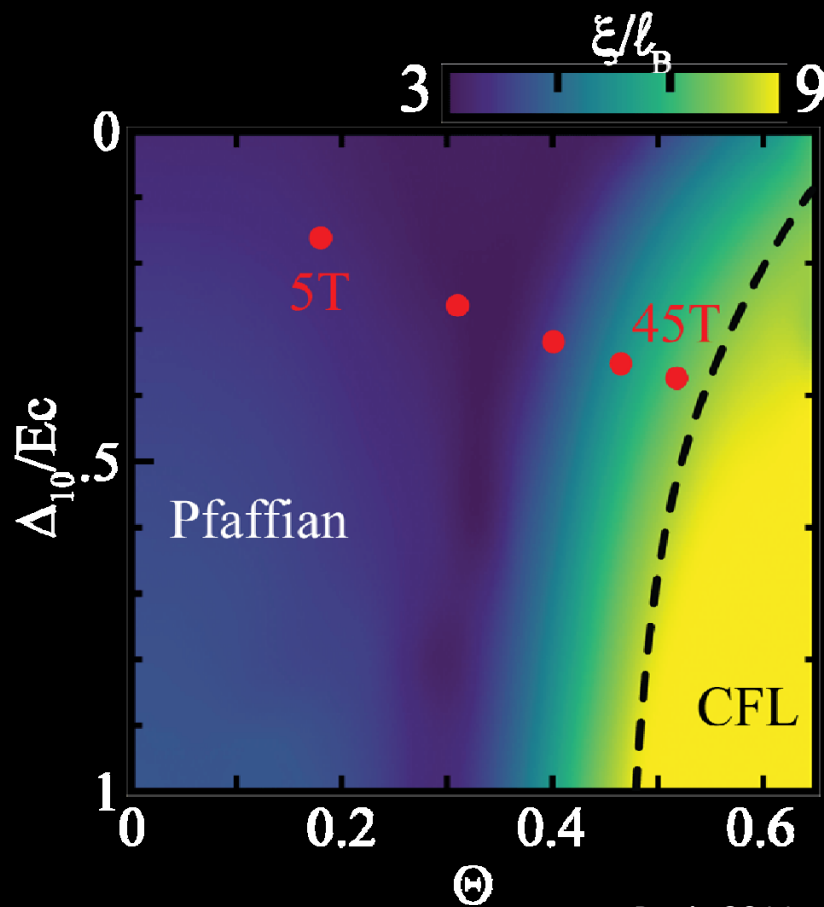
- Three generic 'regimes':  $|0\rangle$ ,  $|1\rangle$ , interlayer correlated

# Fractional quantum Hall effect





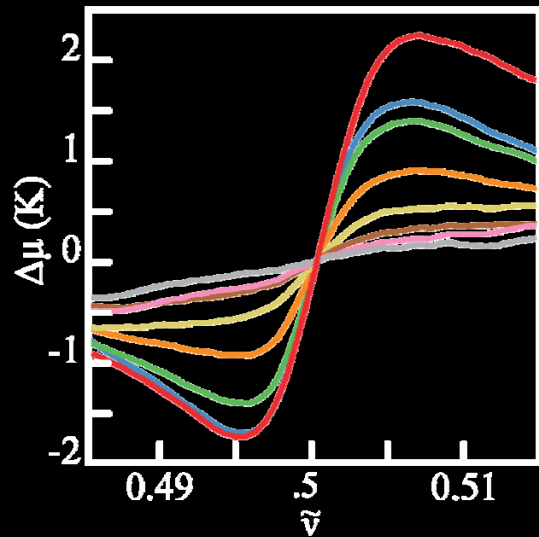
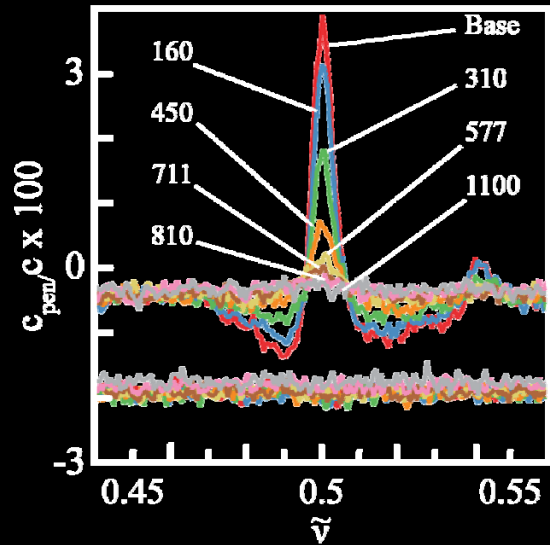
# Pfaffian



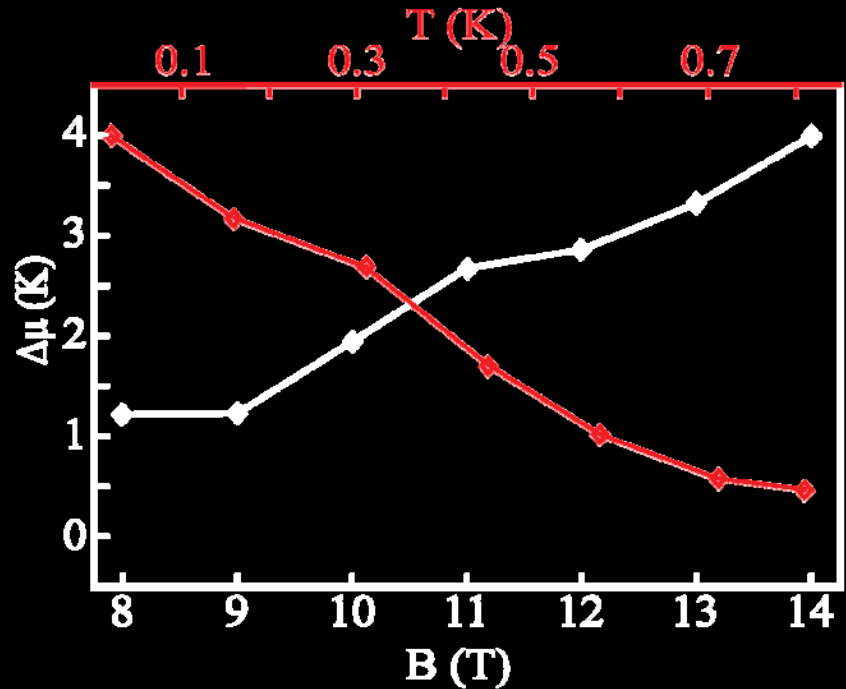
Papic 2011  
Papic 2014

- Even denominator observed in most single component  $|1\rangle$  regimes
- Pfaffian strongly favored in numerics
- Accurate LL mixing, wavefunction form factors

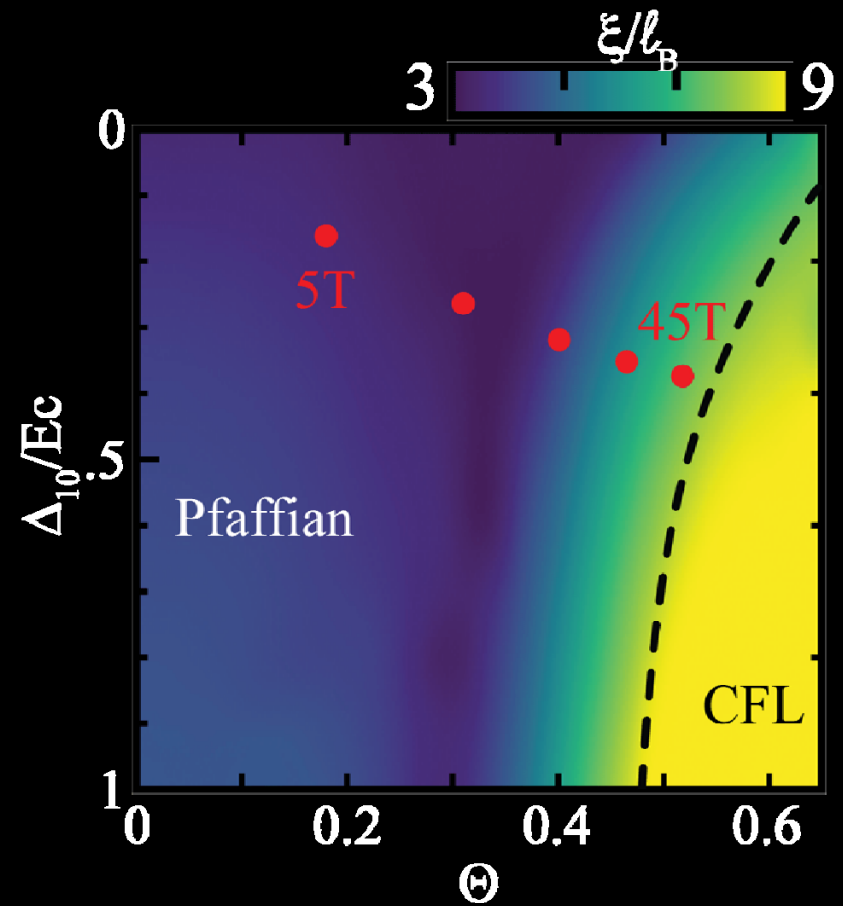
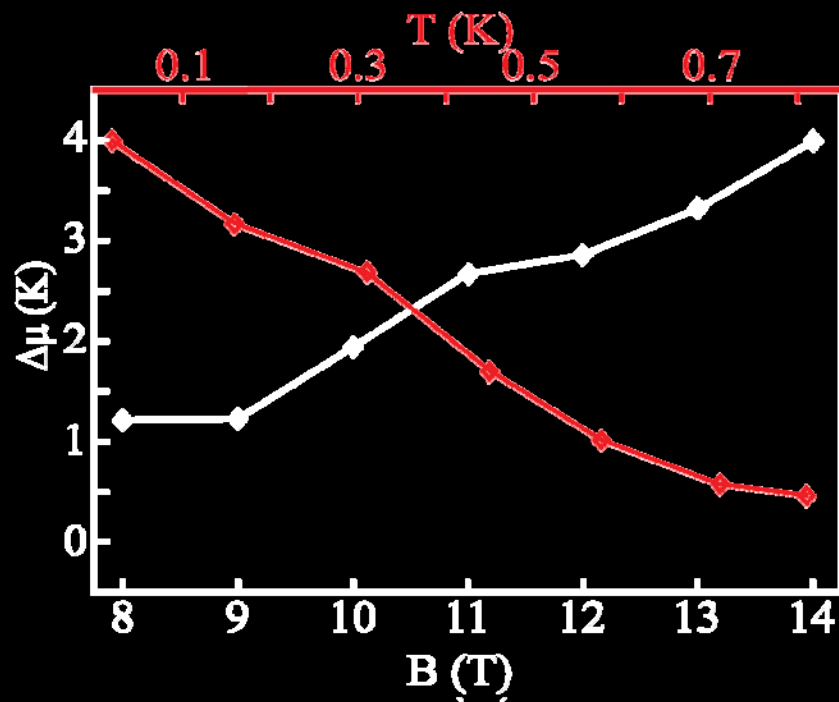
# Pfaffian gap



- $\frac{C_P}{c} \approx \frac{\partial \mu}{\partial n}$
- $\Delta\mu = \int \frac{\partial \mu}{\partial n} dn_0$
- $\frac{e}{q} \Delta\mu \simeq 4 \times \Delta_{act}$

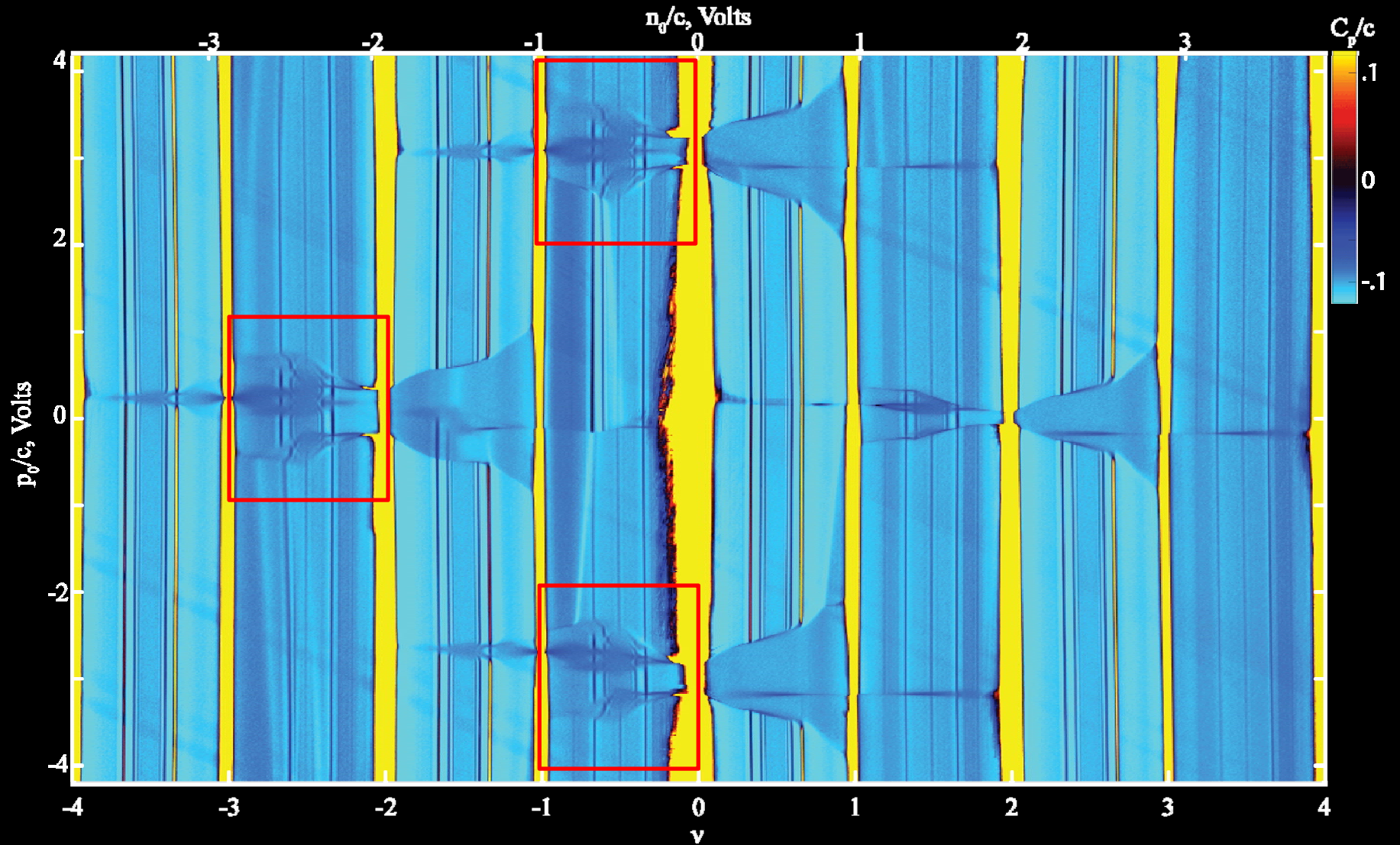


# Pfaffian gap

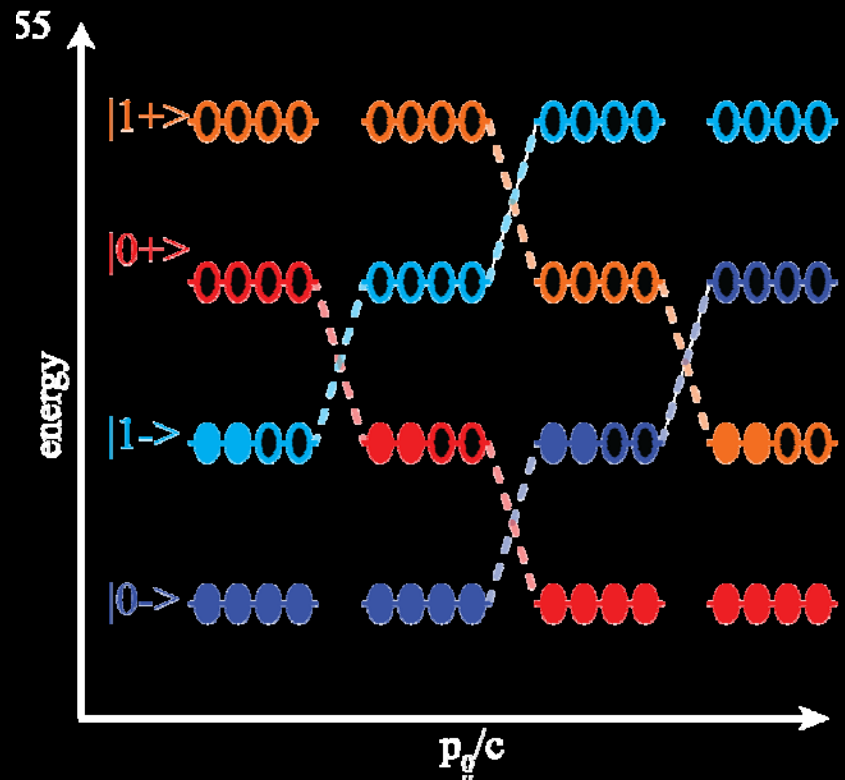
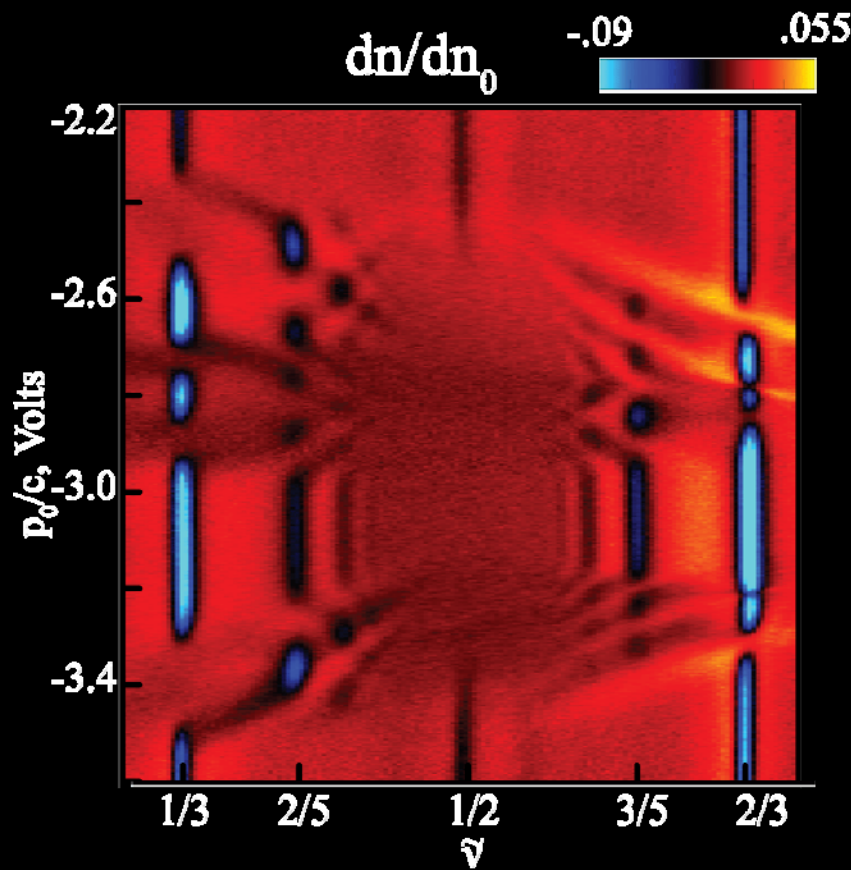




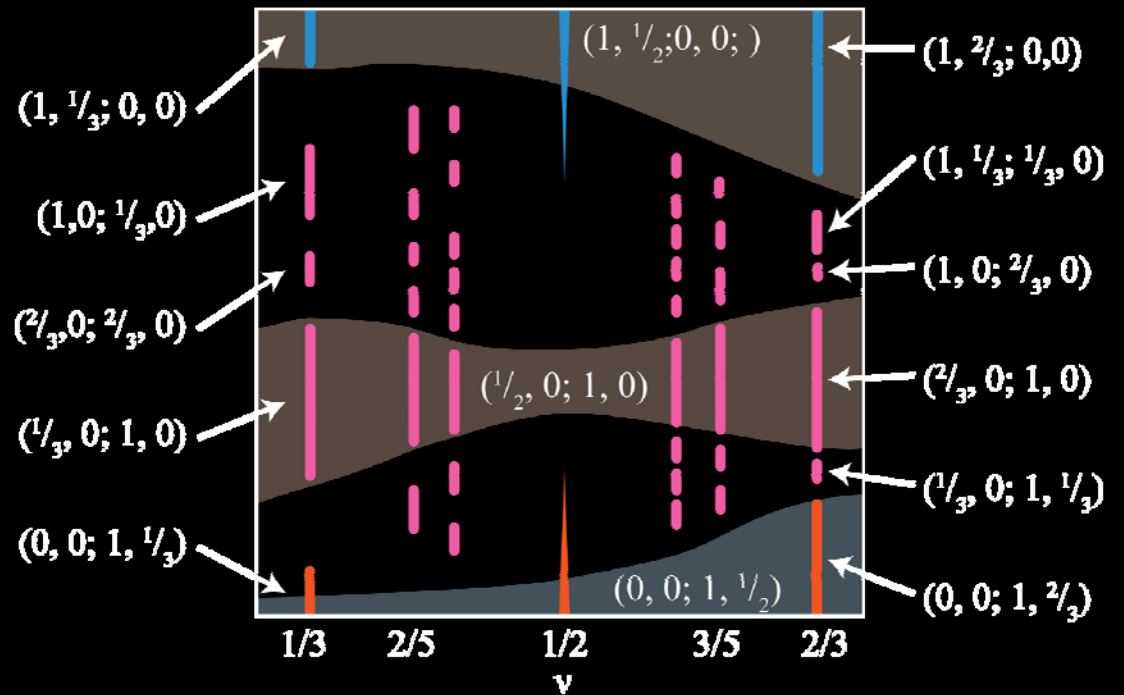
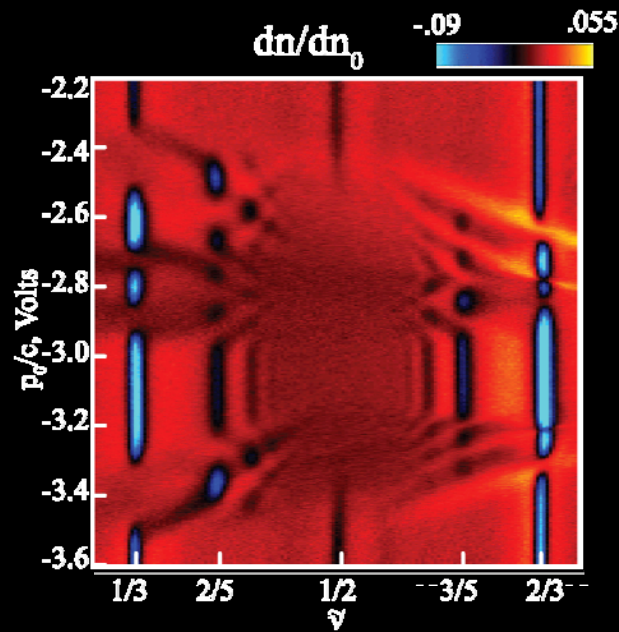
# FQHE near level crossings



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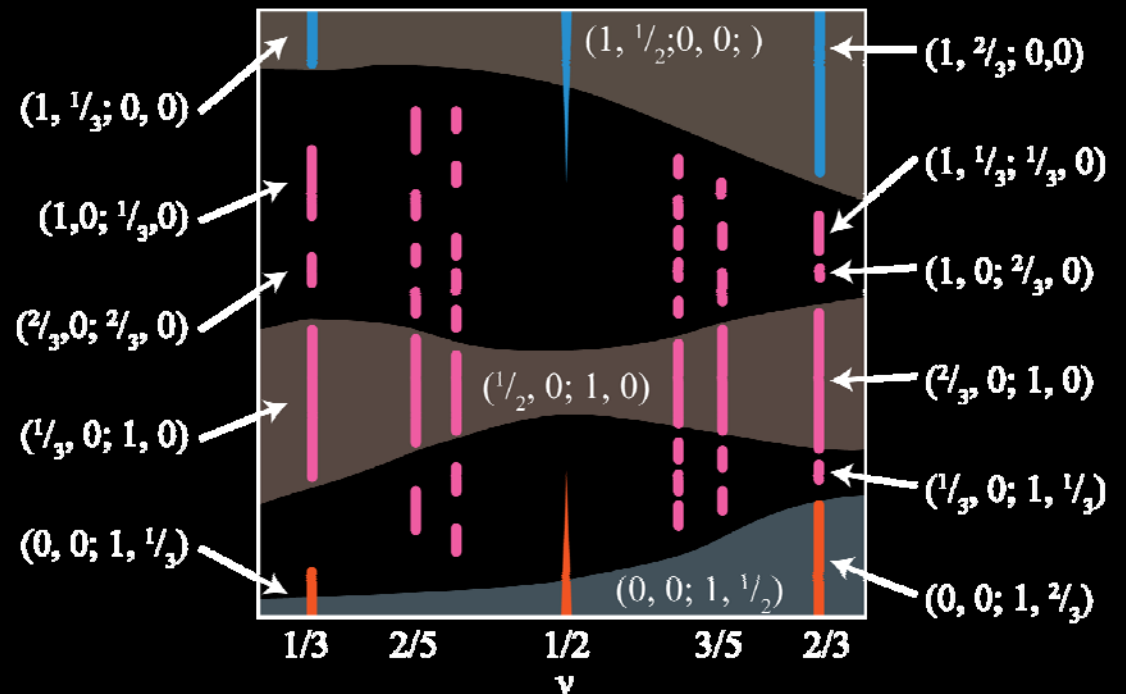
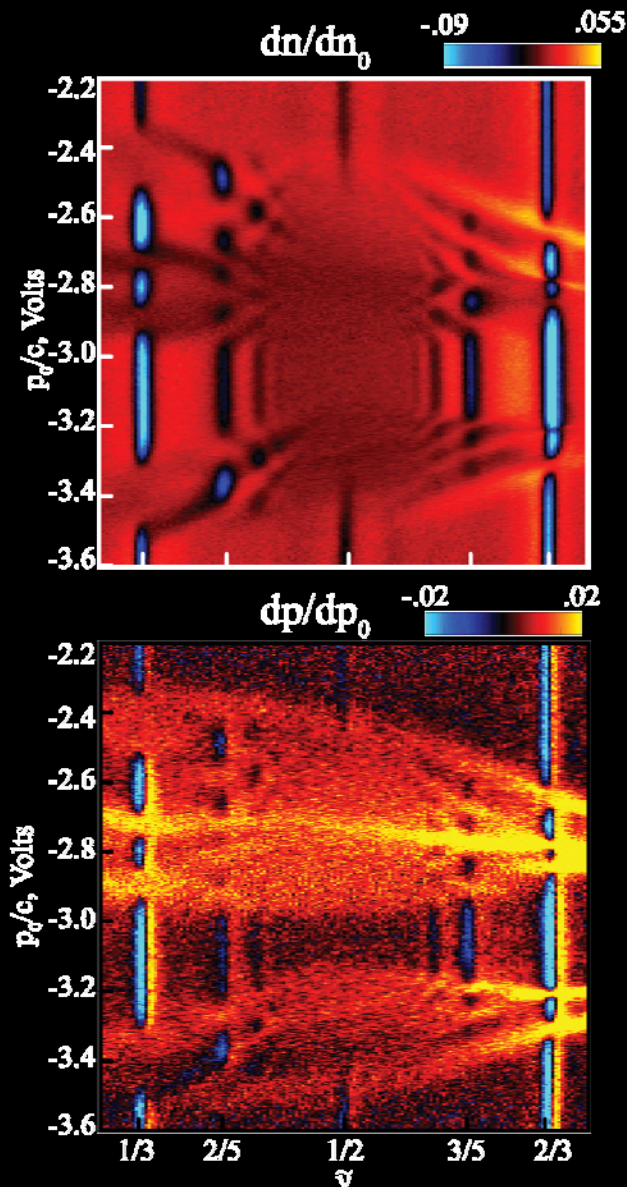


# FQHE near level crossings



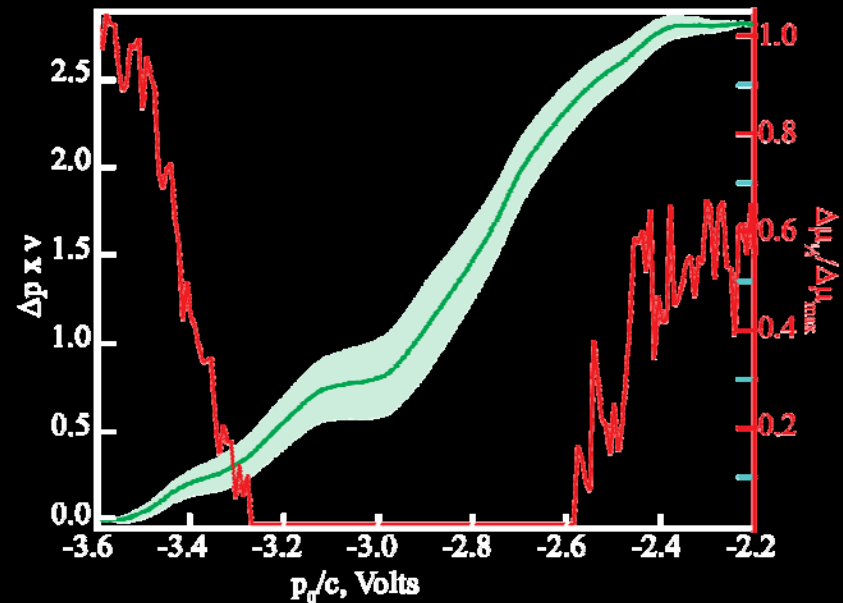
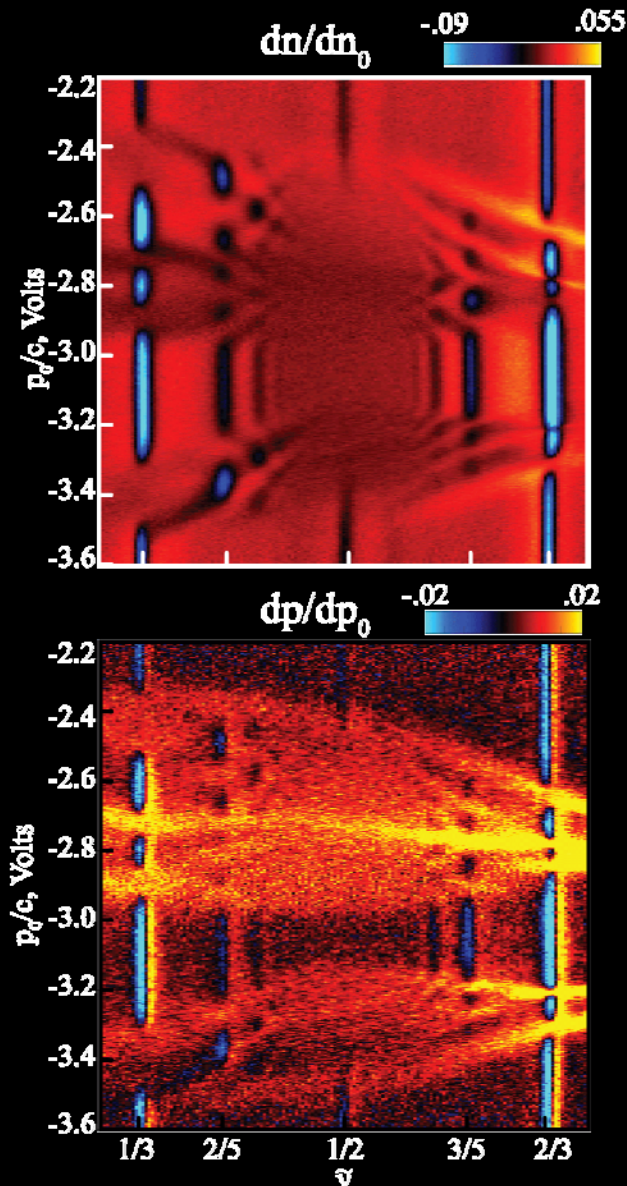


# FQHE near level crossings



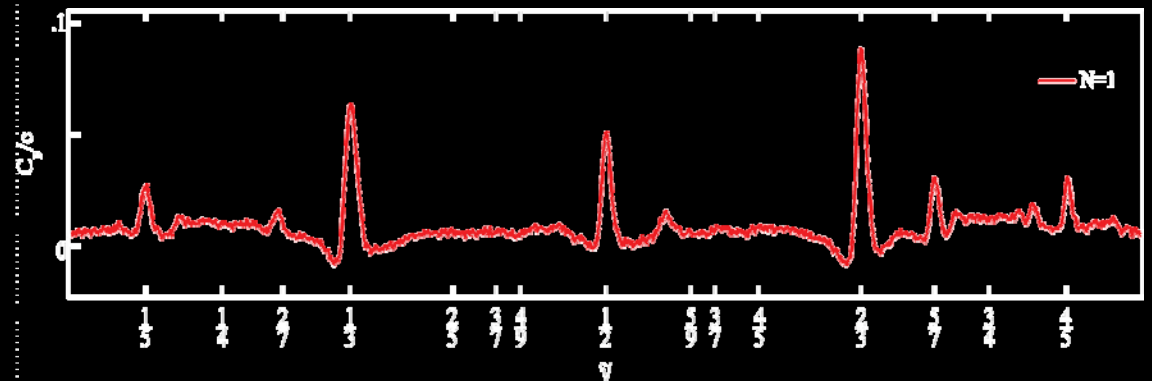
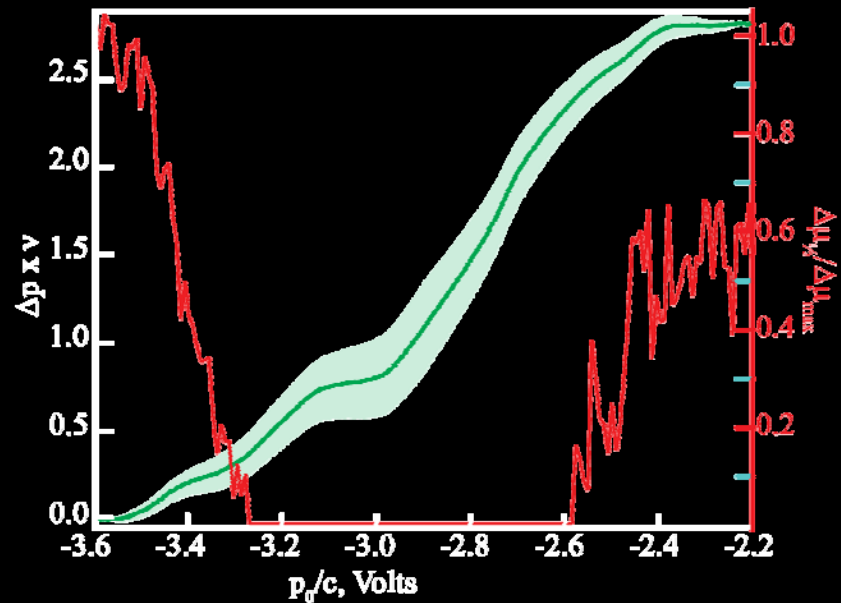
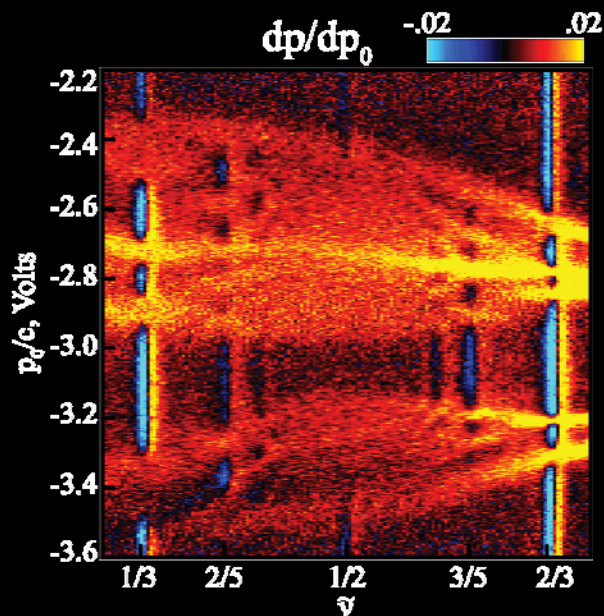
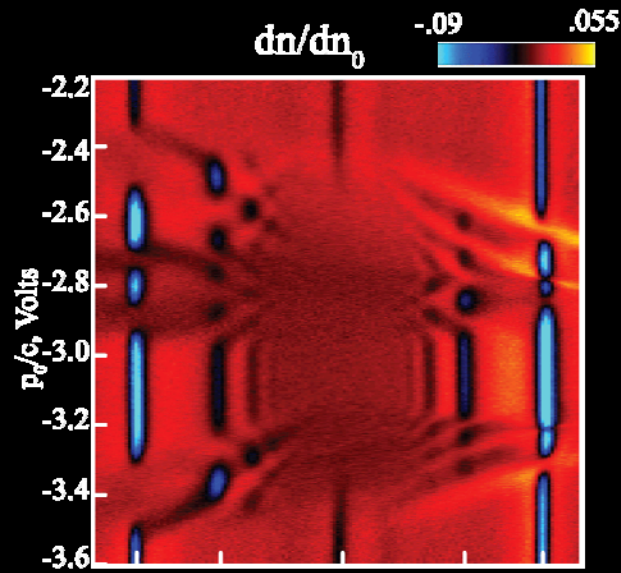
- Odd denominators: sharp phase transitions, hierarchy
- Even denominator: smooth?

# Depolarization at 1/2



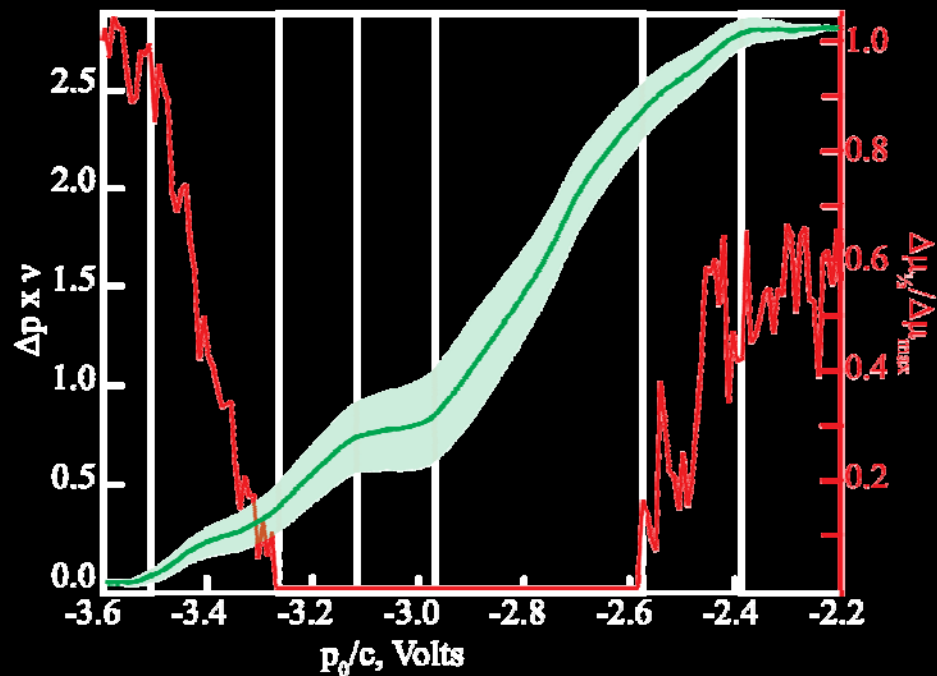
- Gap survives from  $(\nu_+, \nu_-) = (1.5, 0) \rightarrow (1.33, .17)$

# Depolarization at 1/2

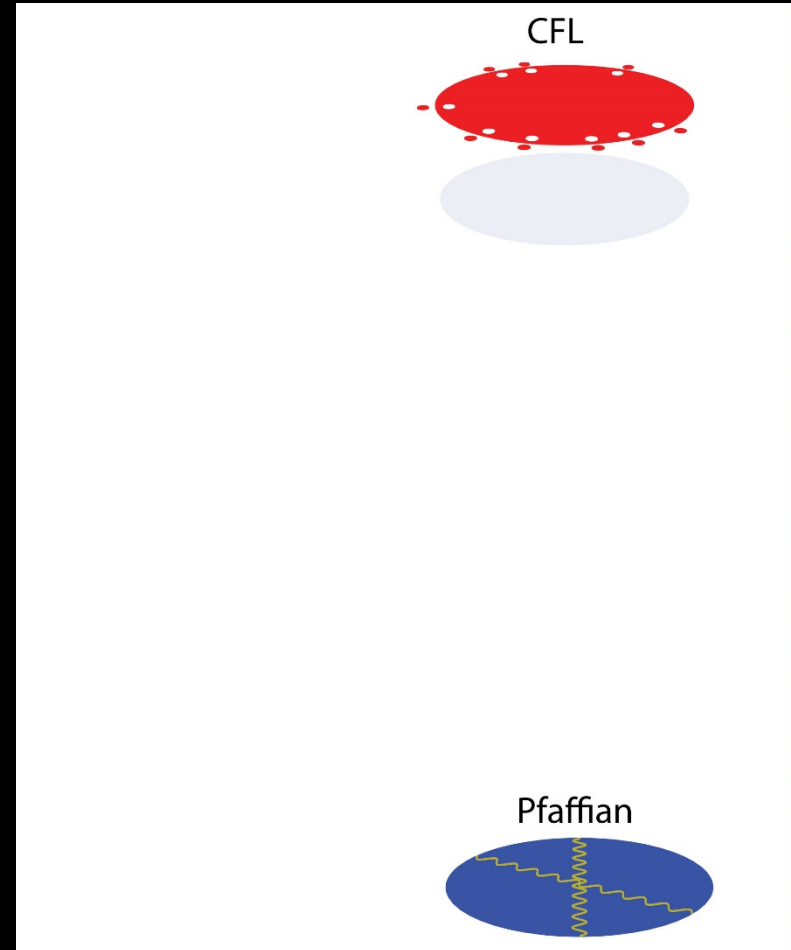
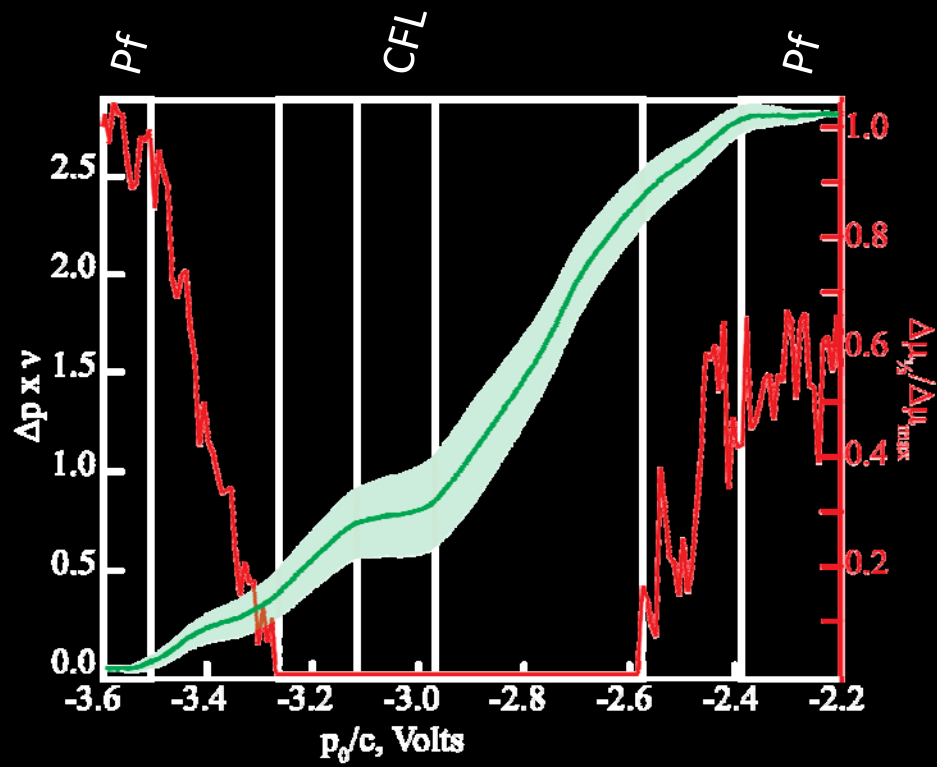


# Depolarization at 1/2

- Four phases:
  - Gapped, polarized  $|1\rangle$
  - Gapped, mixed
  - Gapless, mixed
  - Gapless, polarized  $|0\rangle$

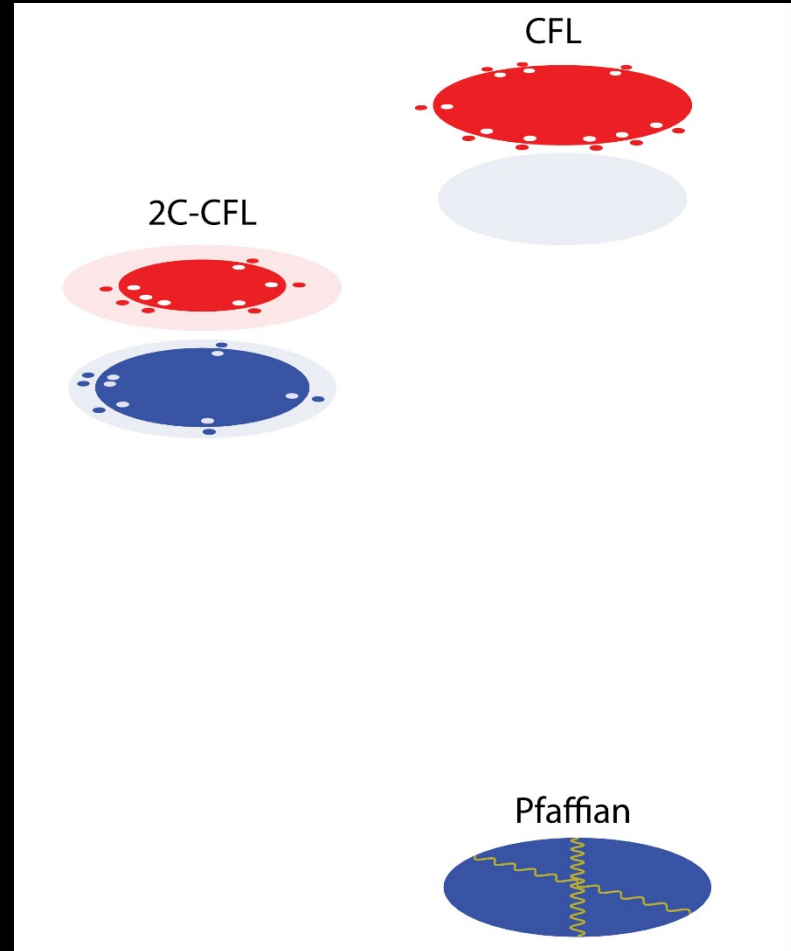
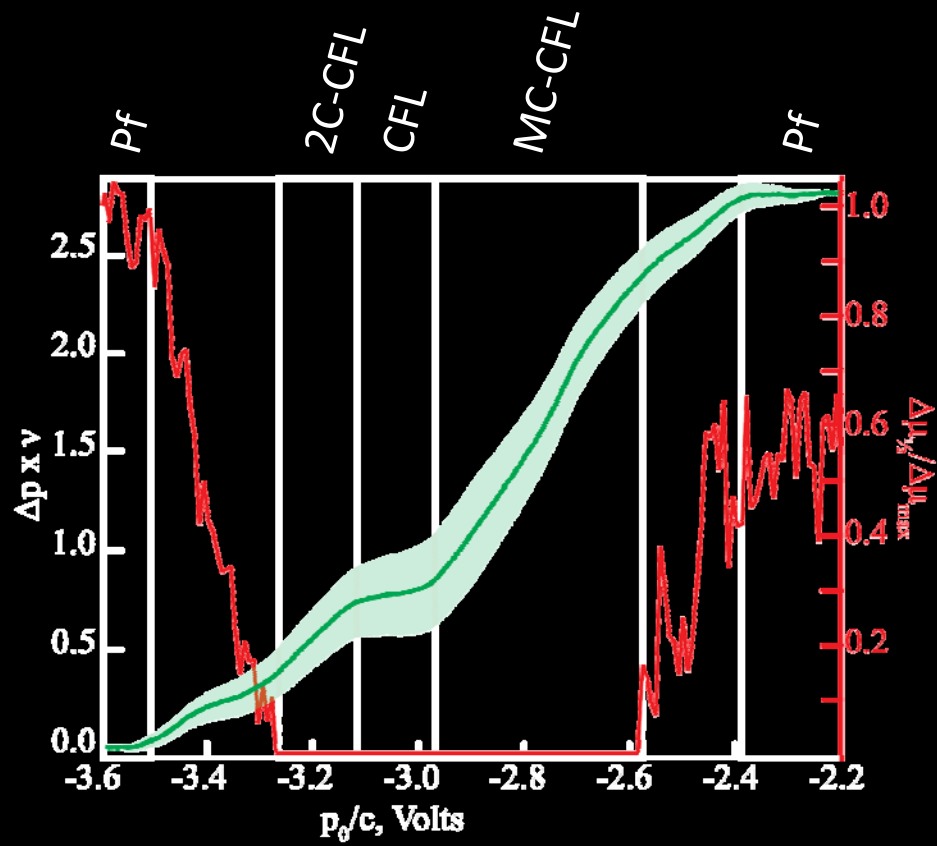


# Depolarization at 1/2

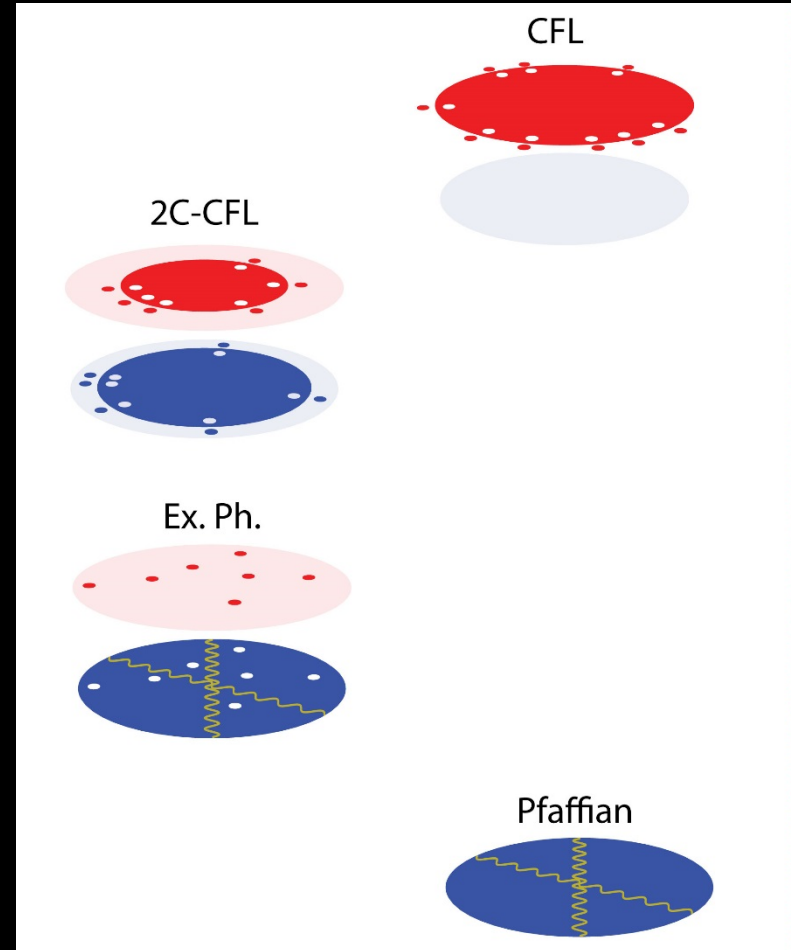
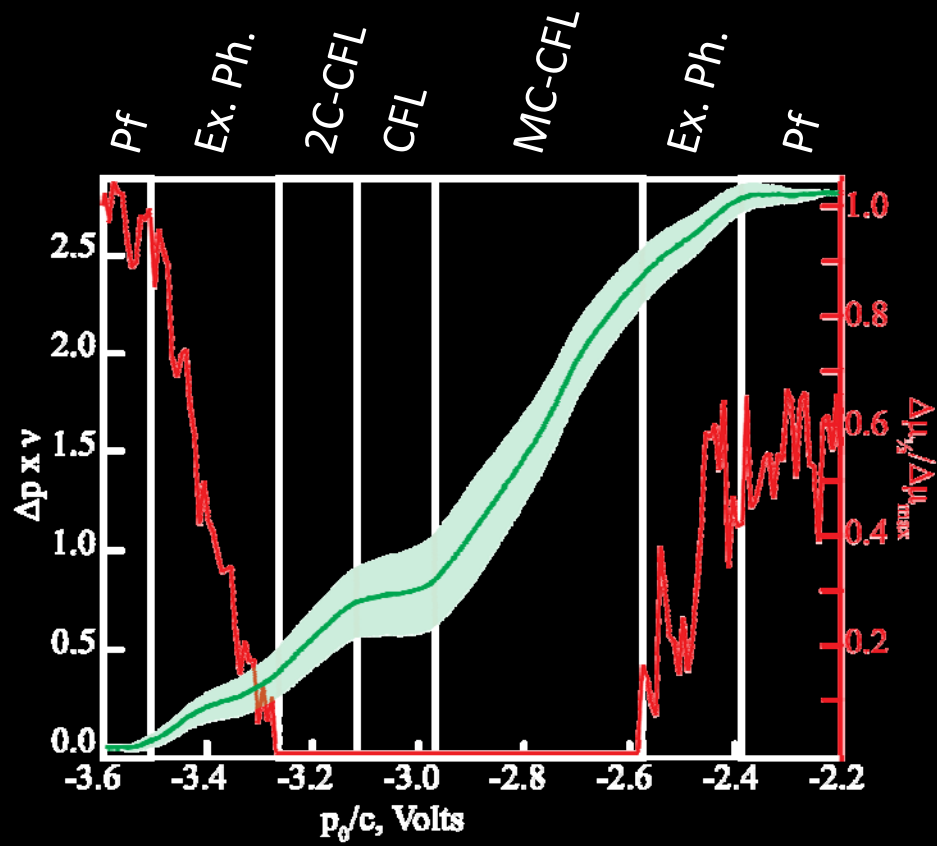




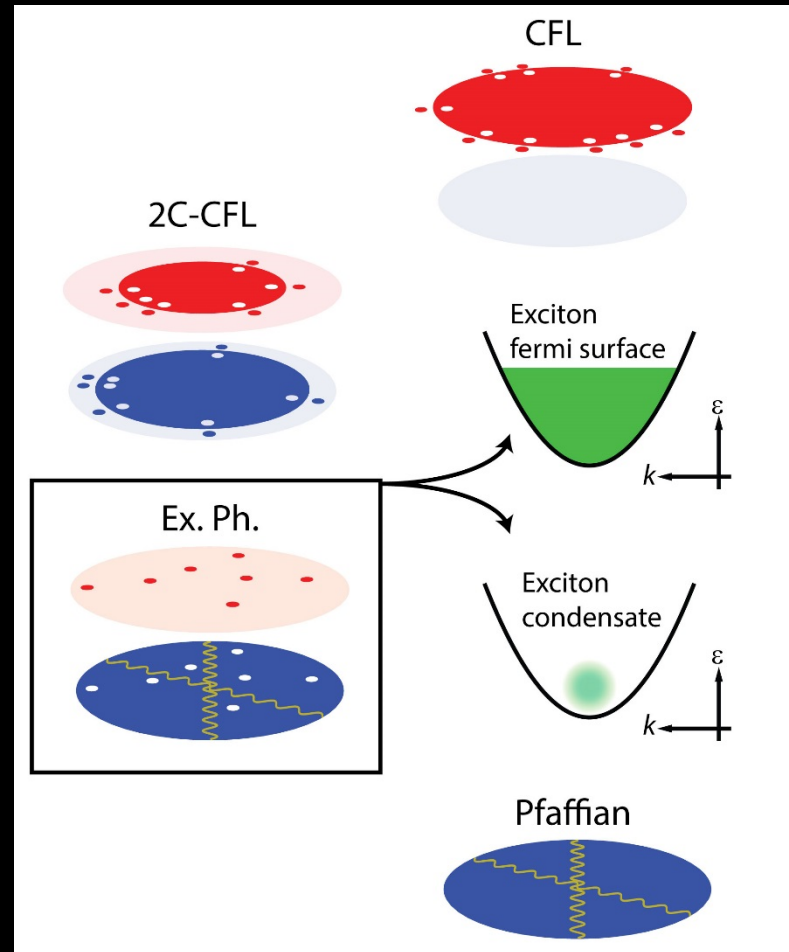
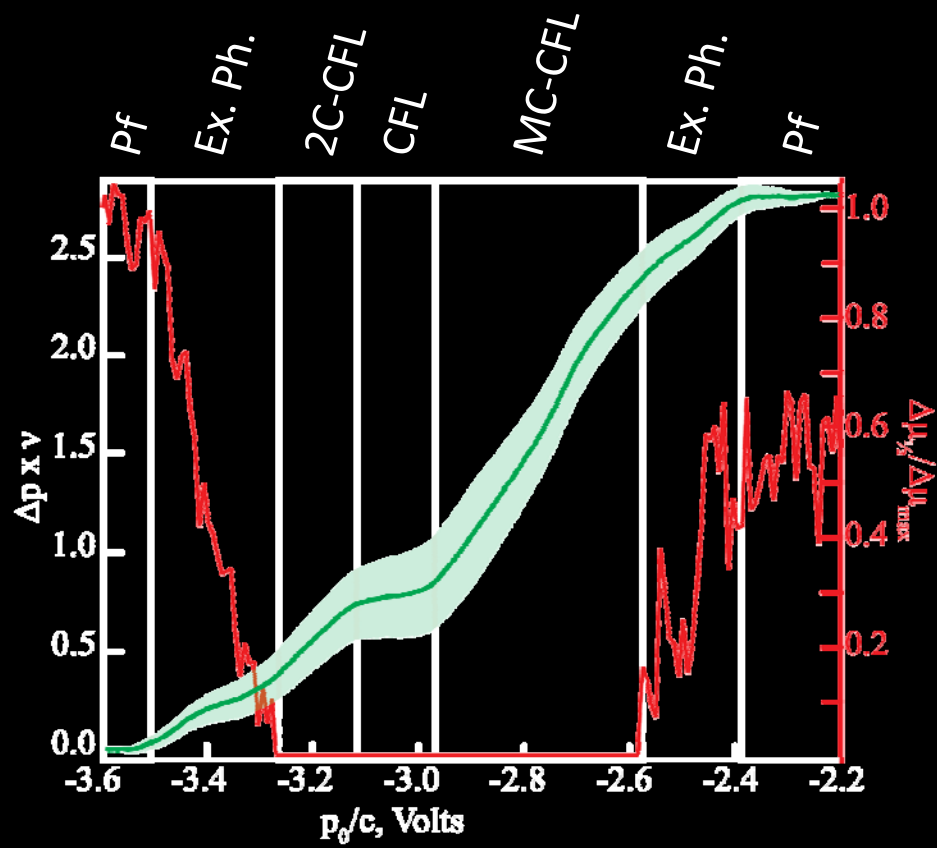
# Depolarization at 1/2



# Depolarization at 1/2



# Depolarization at 1/2



# Outlook

- Bilayer hosts robust Pfaffian ground states
- All vdW architecture allows gating, locally
  - Time to revisit interferometry?
- T-dep  $\mu \rightarrow$  n-dep  $S$ 
  - Detect entropy of topological degeneracy?
- $|1\rangle \rightarrow |0\rangle$  transition at  $\nu = \frac{1}{2}$ : excitonic phase?
  - Anisotropic transport?
  - Double bilayers: drag, counterflow, etc.
  - Detect exciton FS?

# Acknowledgements

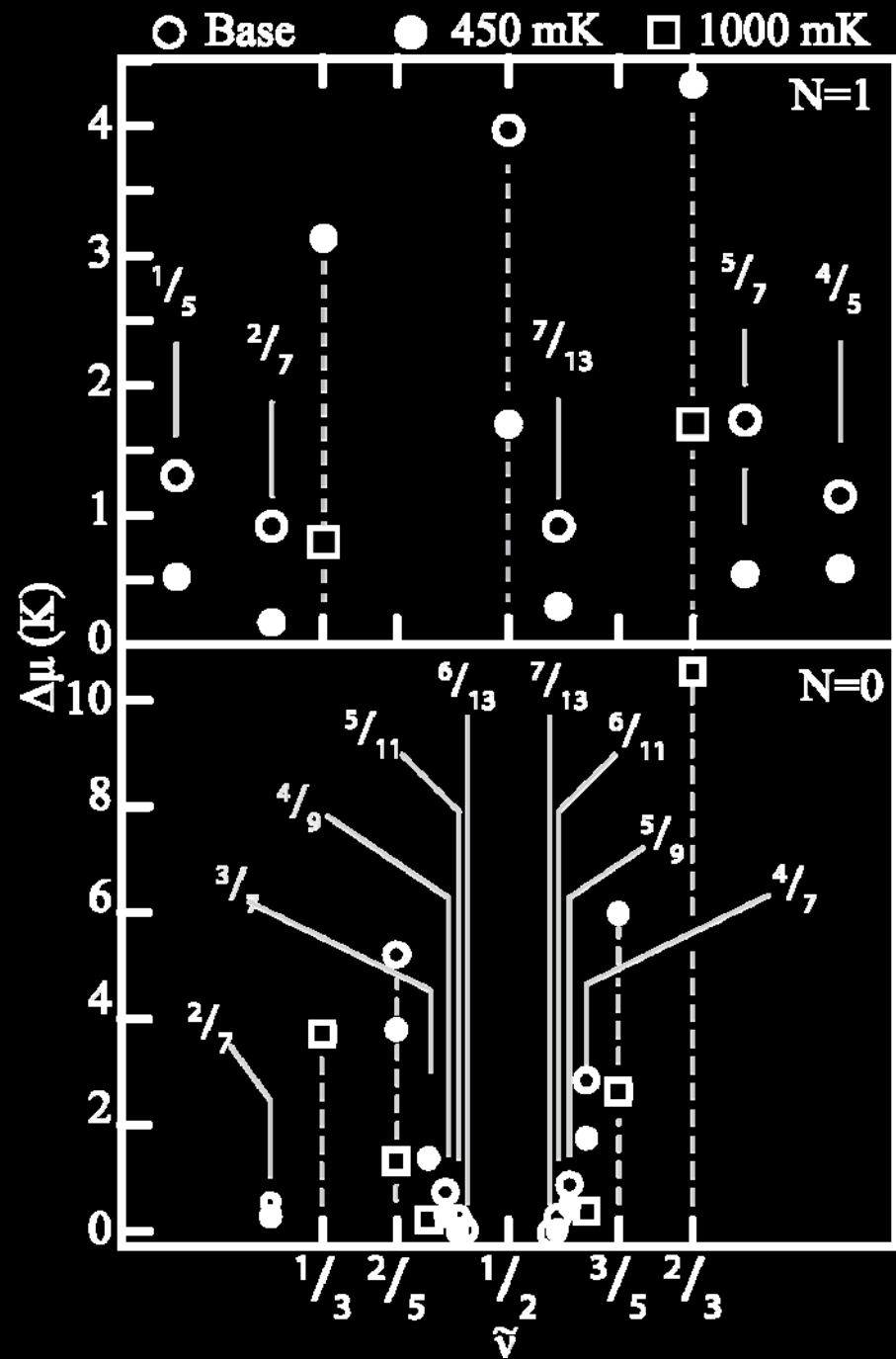
## Integer QH

- **MIT**
  - **B. Hunt (CMU)**, R. Ashoori, L. Levitov
- **Columbia**
  - L. Wang, J. Li, C. Dean, J. Hone
- **NIMS**
  - T. Taniguchi, K. Watanabe
- **UCSB**
  - Sasha Zibrov
- **Station Q**
  - **Mike Zalatel**

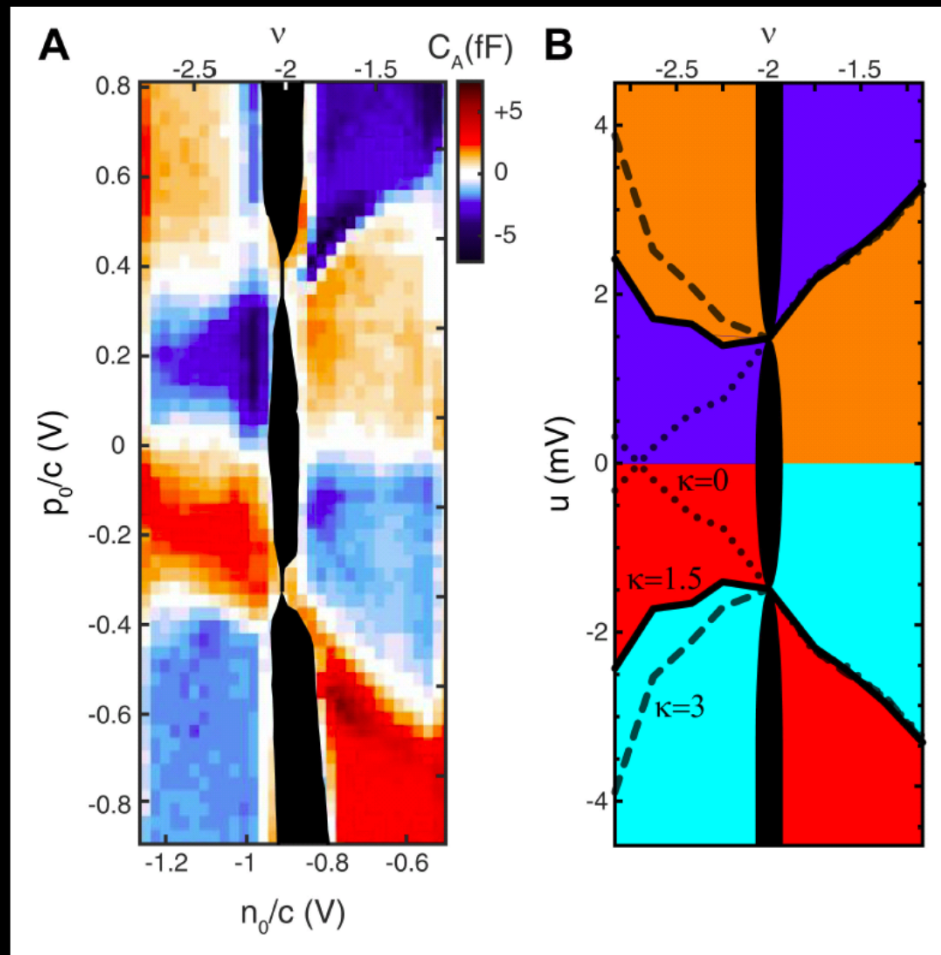
## Fractional QH

- **UCSB**
  - **Sasha Zibrov**, Carlos Kometter
- **Station Q**
  - **Mike Zalatel**, **Maissam Barkeshli**, Chetan Nayak
- **NIMS**
  - T. Taniguchi, K. Watanabe



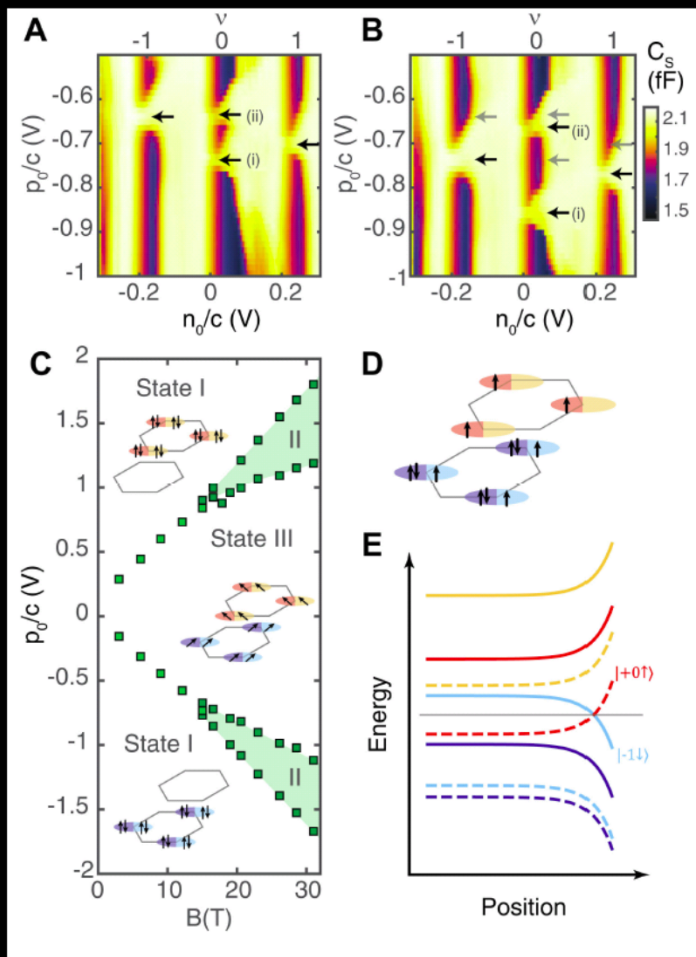


# Tilted phase transitions



- Phase transition: two degenerate states
- Tilt: energy per particle depends on particle number
- Correlation effect, DMRG says: very sensitive to details

# Spin structure of ZLL



- Tilted field measurements: change Zeeman, nothing else
- Consistent with CAF state at  $\nu = 0$ , but new state appears
- Spin polarized: should be helical phase
- Need transport....