Forecasting Turbulence: Experiments, Theory, and Numerics

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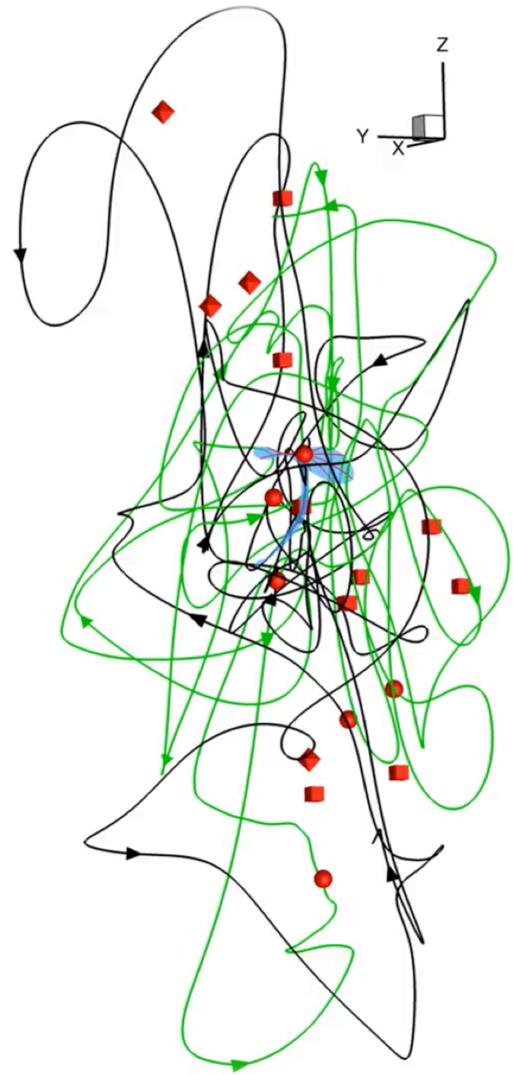

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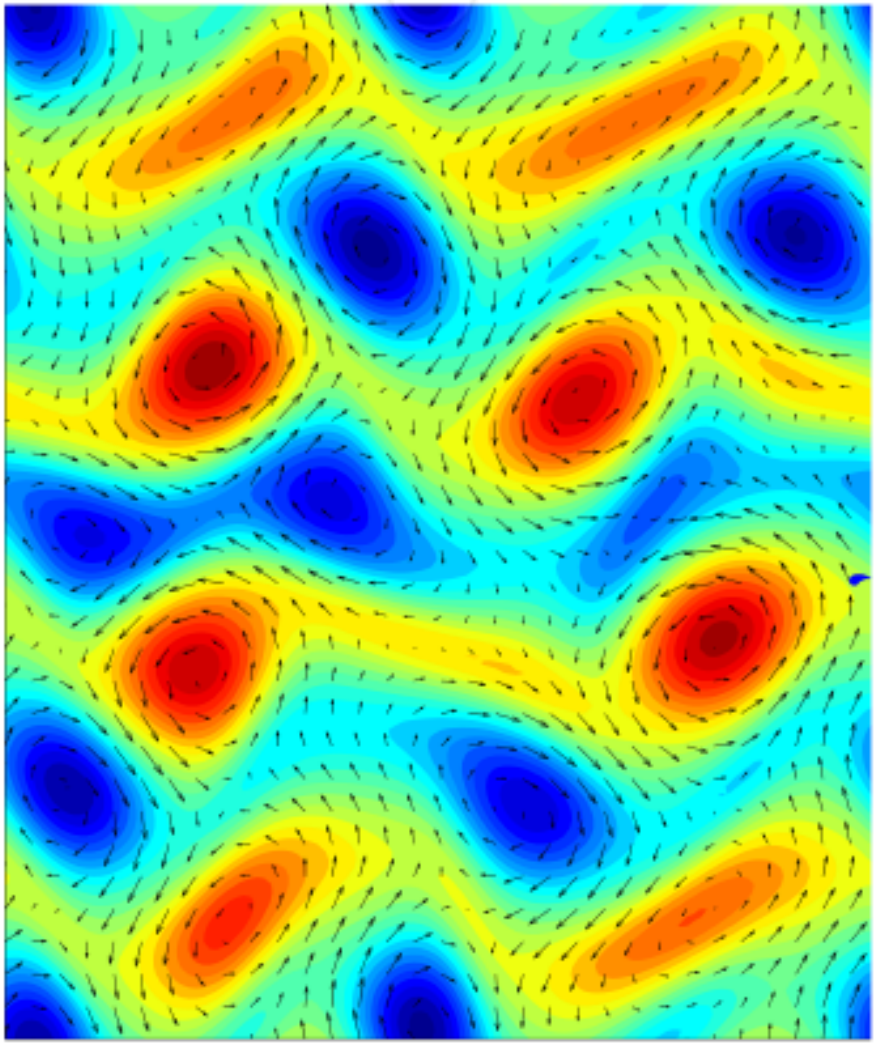
CBET-0900018, DMS-1125302)

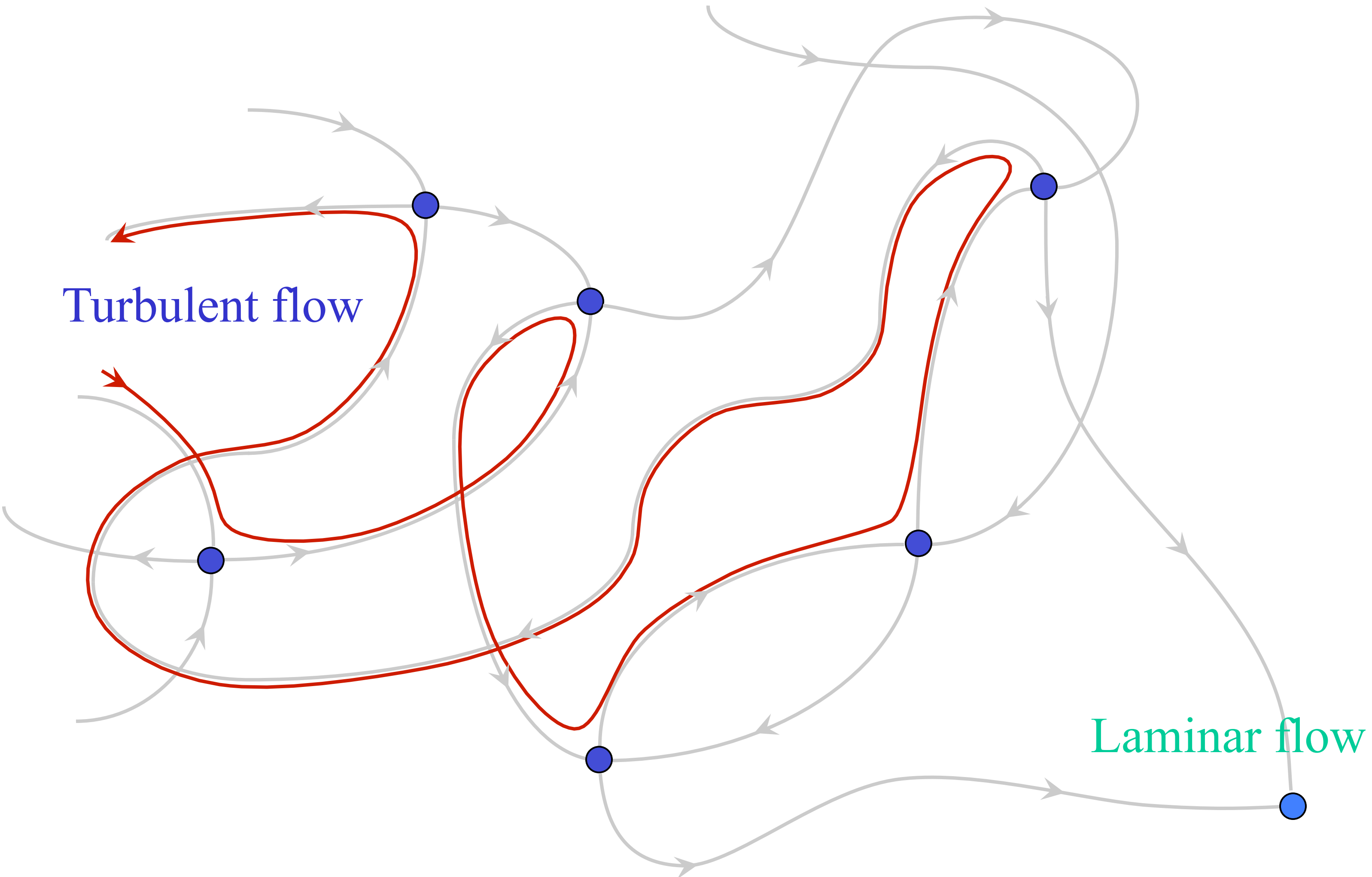
State Space Picture



→

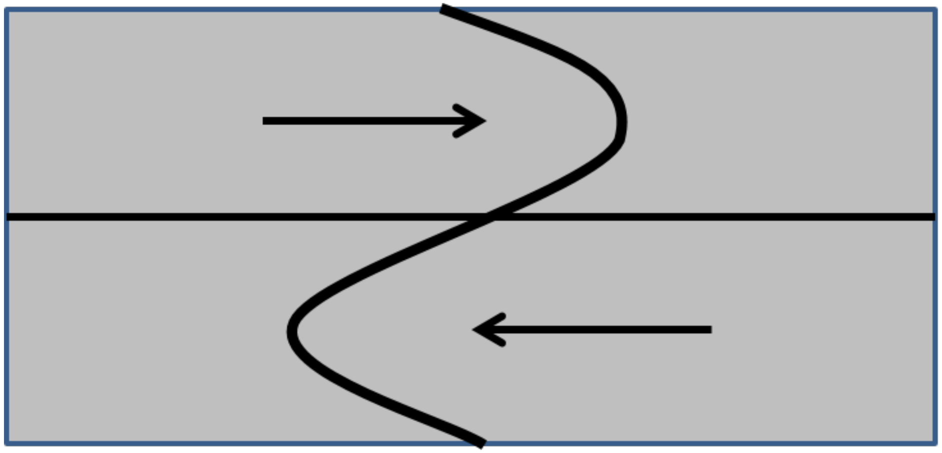
$$\vec{V} = \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \\ \cdot \\ \cdot \\ \cdot \\ v_{21} \\ v_{22} \\ \cdot \\ \cdot \\ \cdot \\ v_{nm} \end{bmatrix}$$

State Space Picture



Kolmogorov Flow

- A strictly 2D flow driven by a sinusoidal forcing



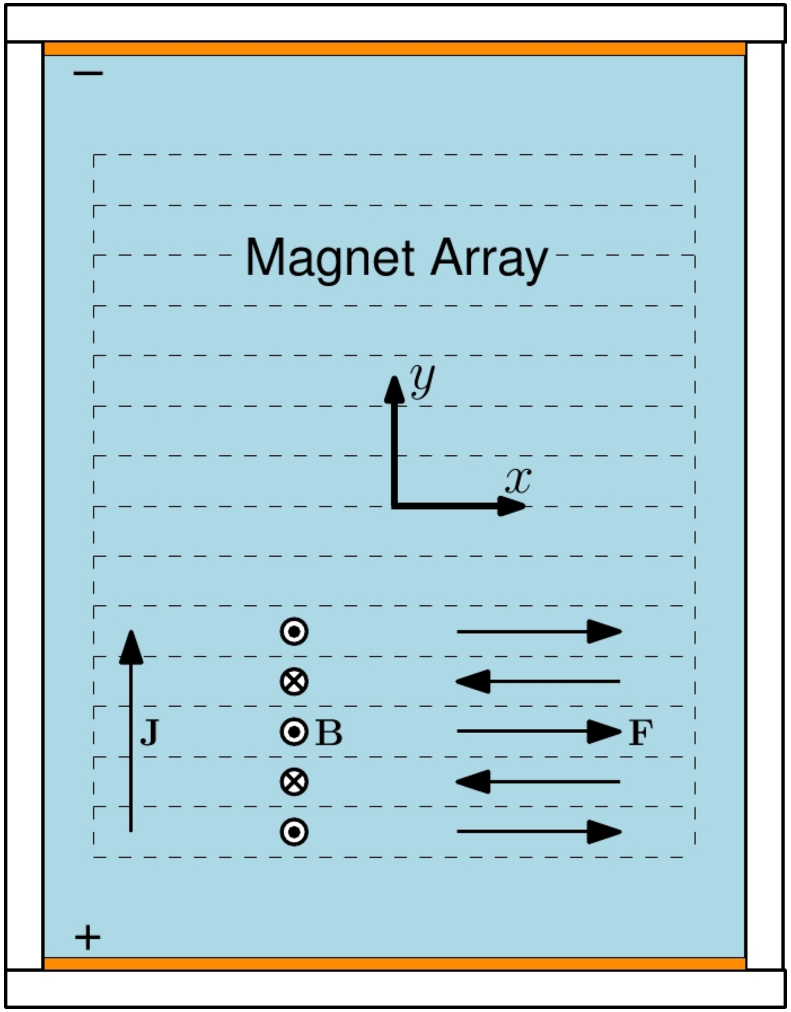
- Previously studied in the context of ECS

    2D: Chandler & Kerswell (2013); Lucas & Kerswell (2015)

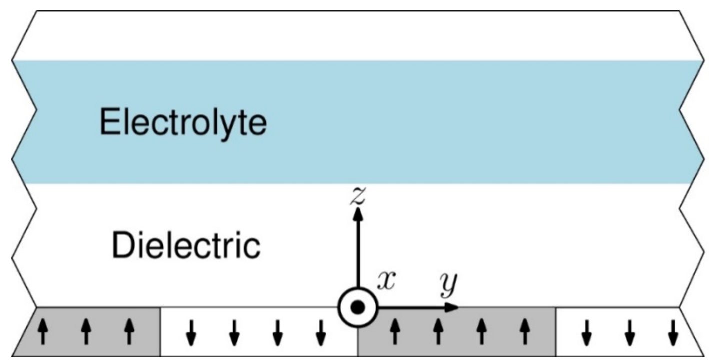
    3D: van Veen and Goto (2016)

Quasi-2D (Q2D) Kolmogorov Flow

Bondarenko et al. (1979)



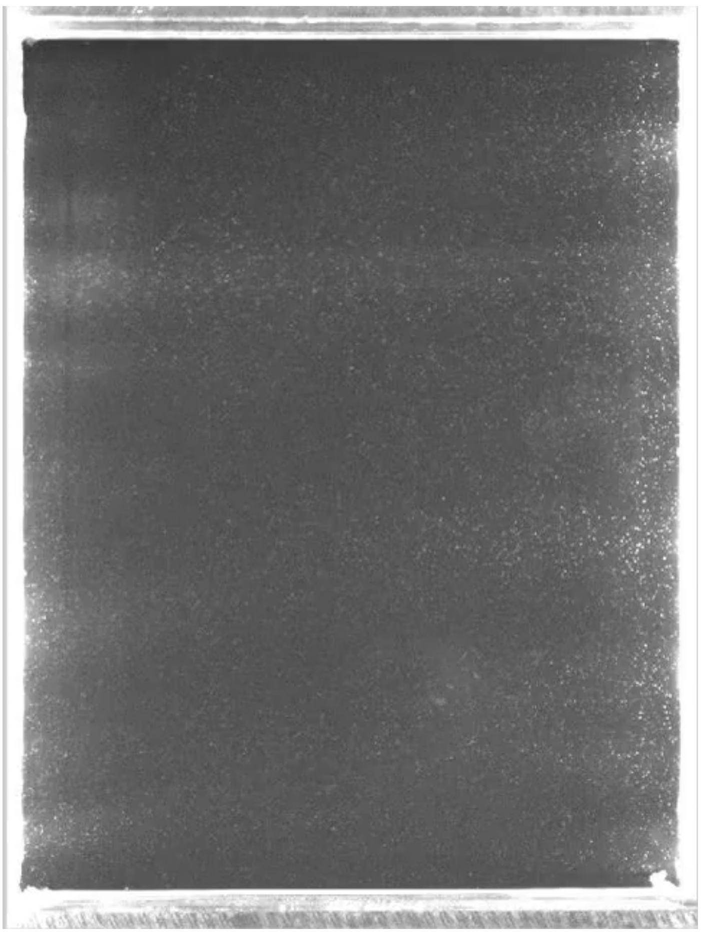
Top View



Cross Section

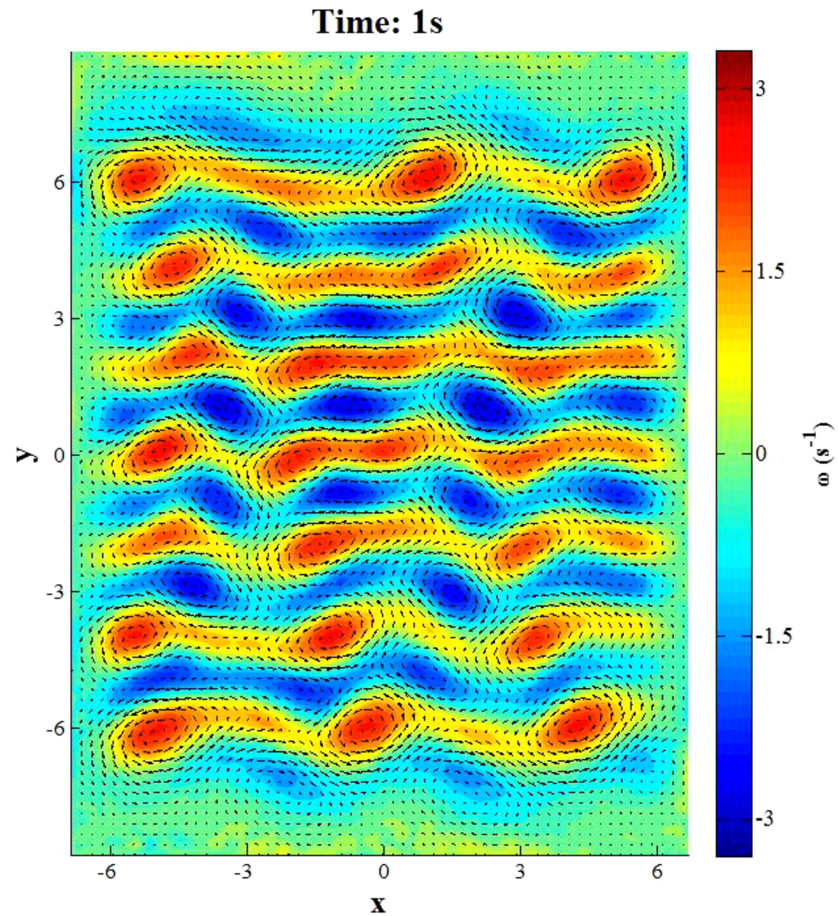
Experimental Data

Raw Images at 15 fps



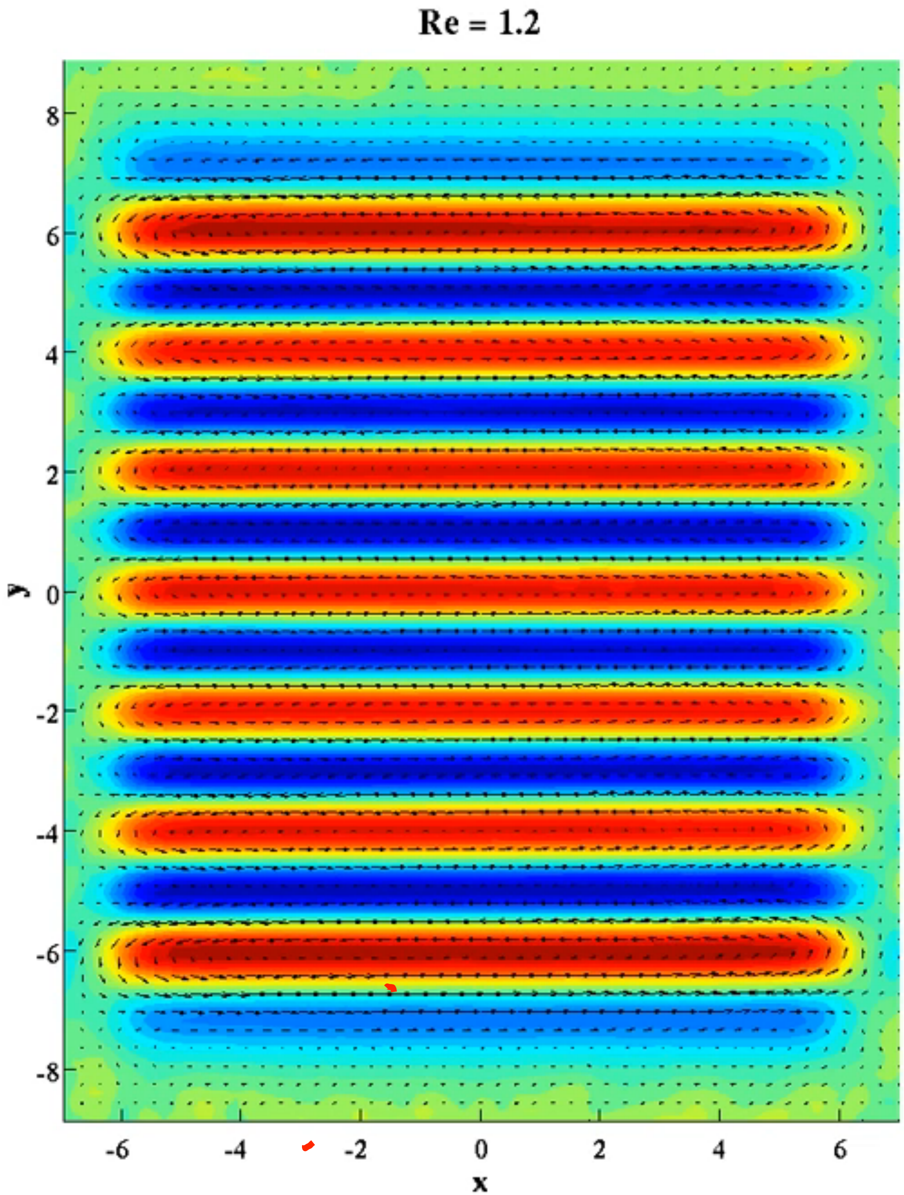
PIV

Processed Data (Sped up 30x)

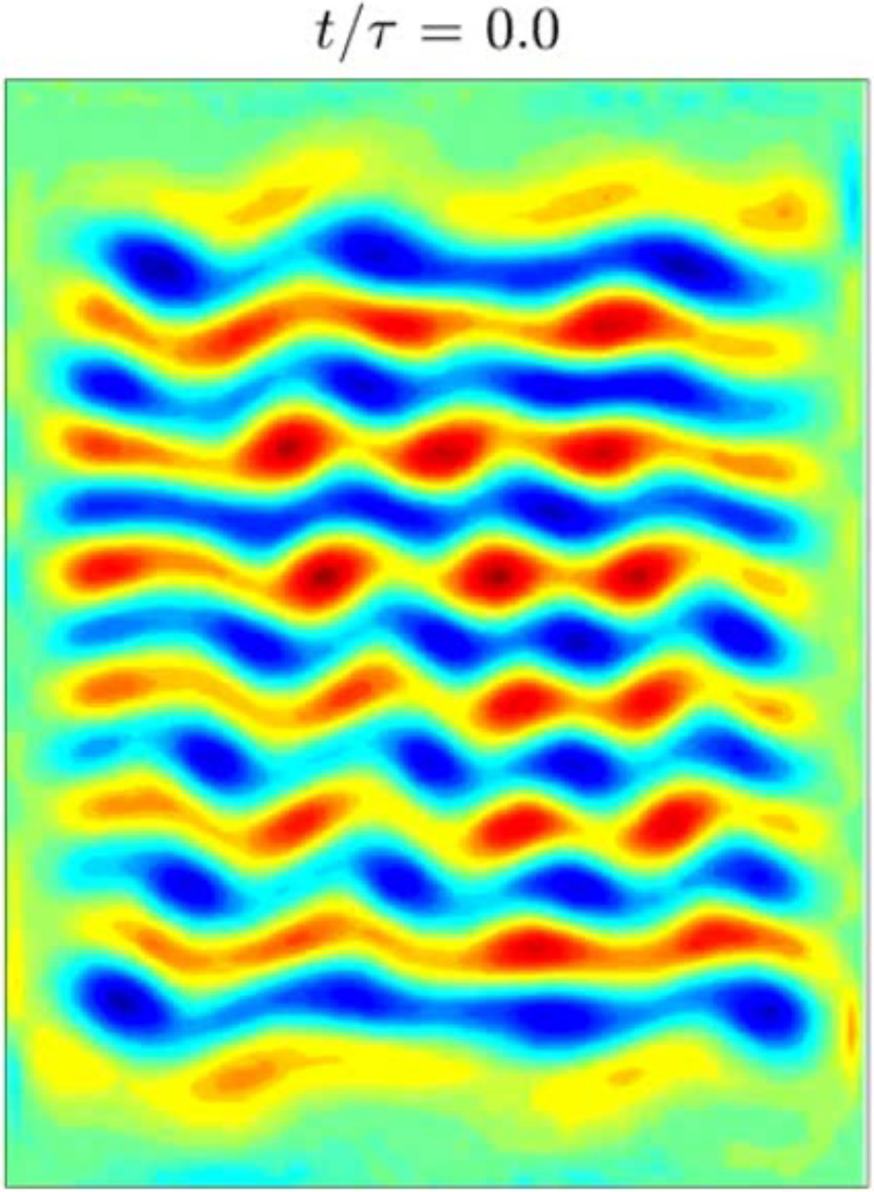


PIV Software: Prana, freely available at: http://sourceforge.net/projects/qi-tools/files/

Transition to Turbulence

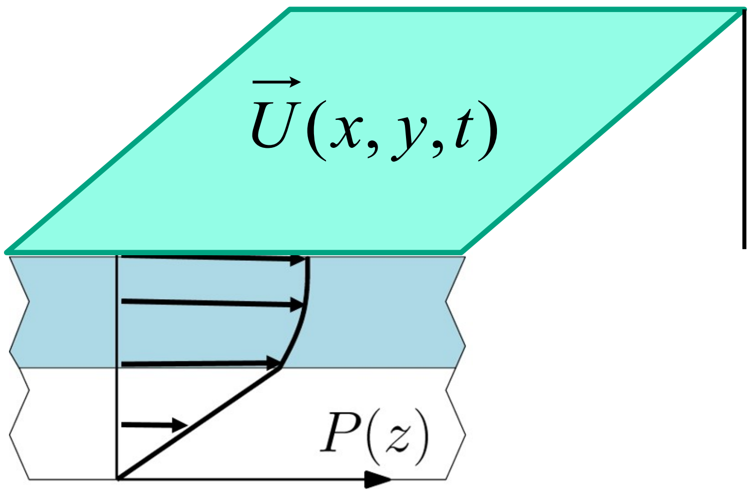


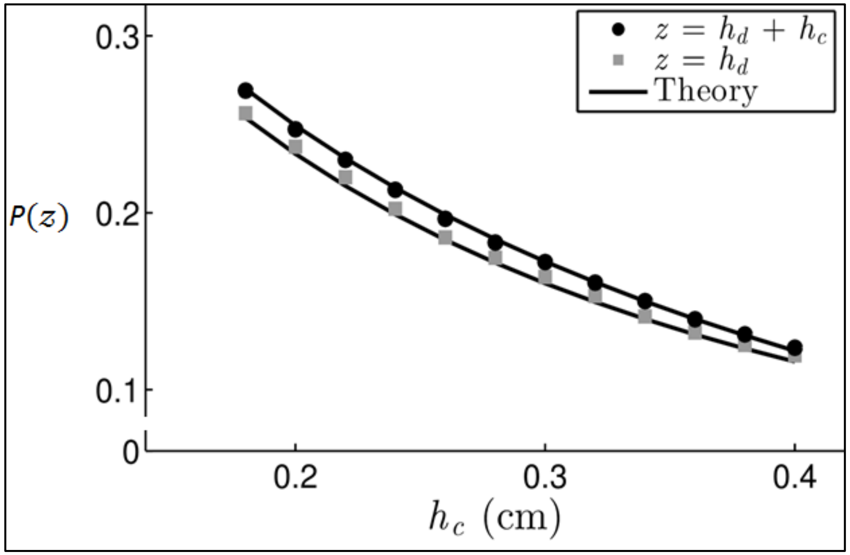
Turbulence (*Re* = 22.5)



2D Model

- Quasi-2D Approximation: $\vec{V}(x,y,z,t) = P(z)\,\vec{U}(x,y,t)$

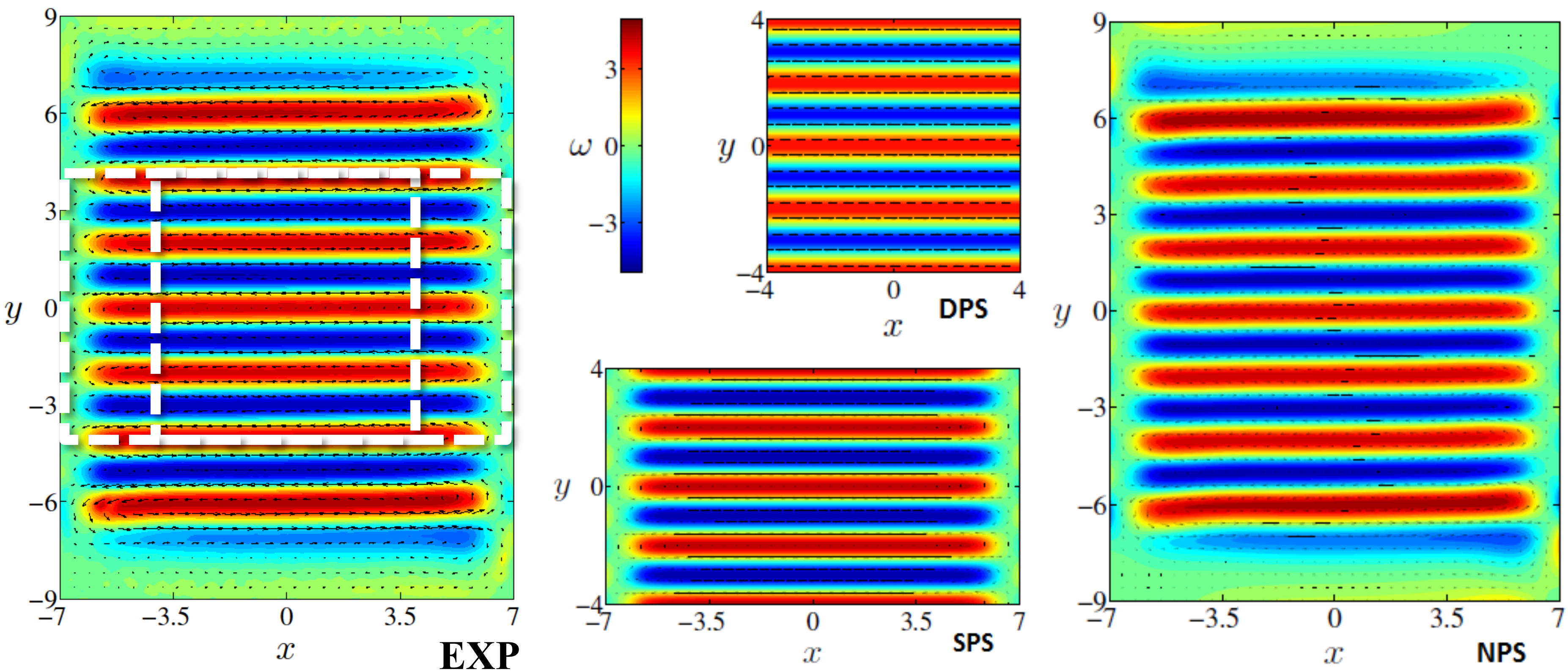




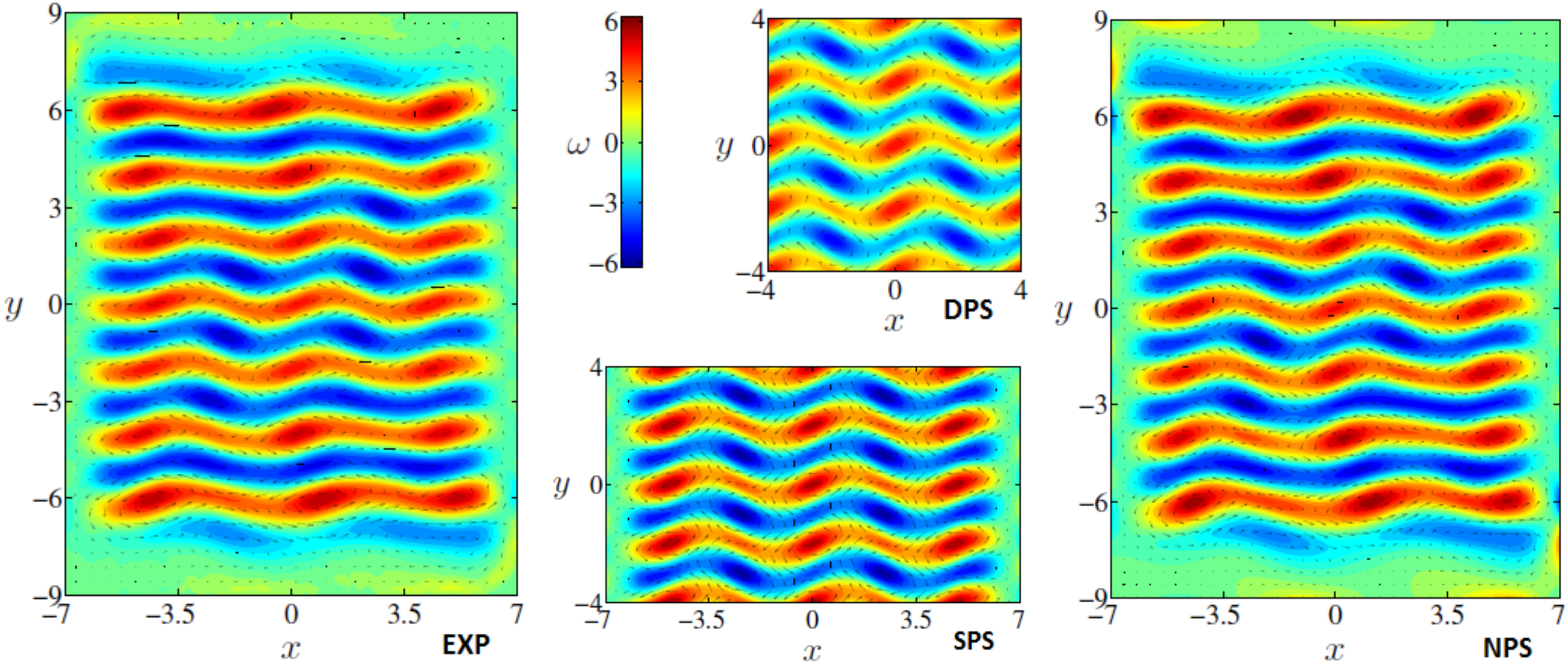
- Depth-Averaged 2D Model (*Suri et al. 2014*):

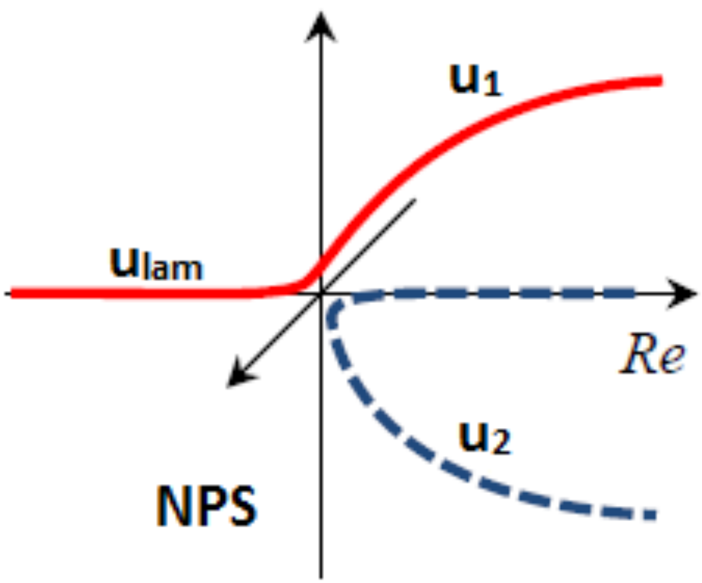
$$\partial_t \vec{U} + \beta \vec{U} \cdot \nabla \vec{U} = -\frac{1}{\rho} \nabla p + \bar{\nu}\, \nabla^2 \vec{U} - \alpha \vec{U} + \langle \vec{F} \rangle_z, \quad \nabla \cdot U = 0$$

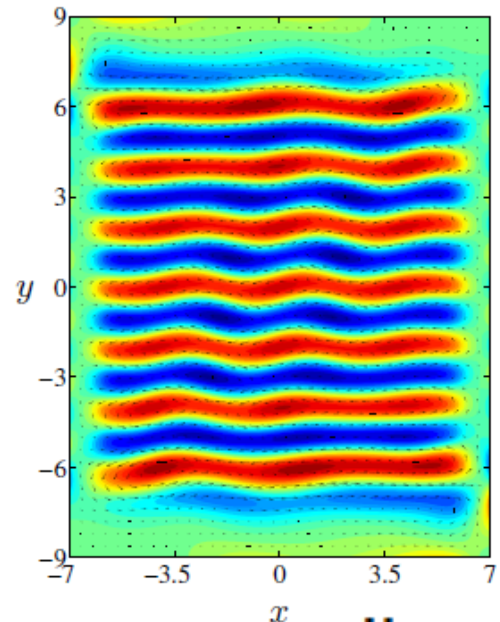
Laminar Flow (*Re* = 8)



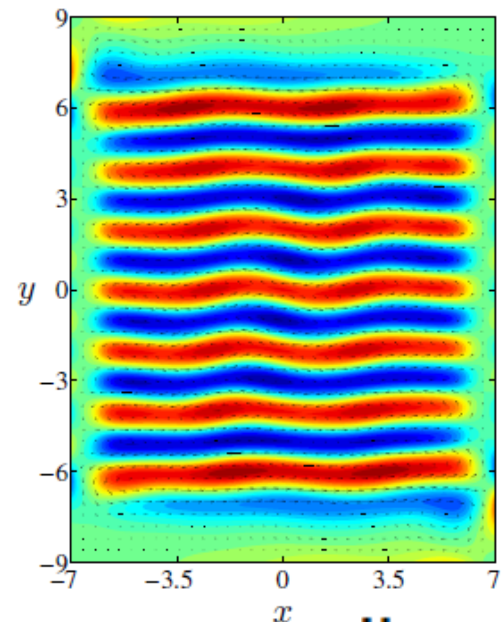
Modulated Flow (*Re* = 14)



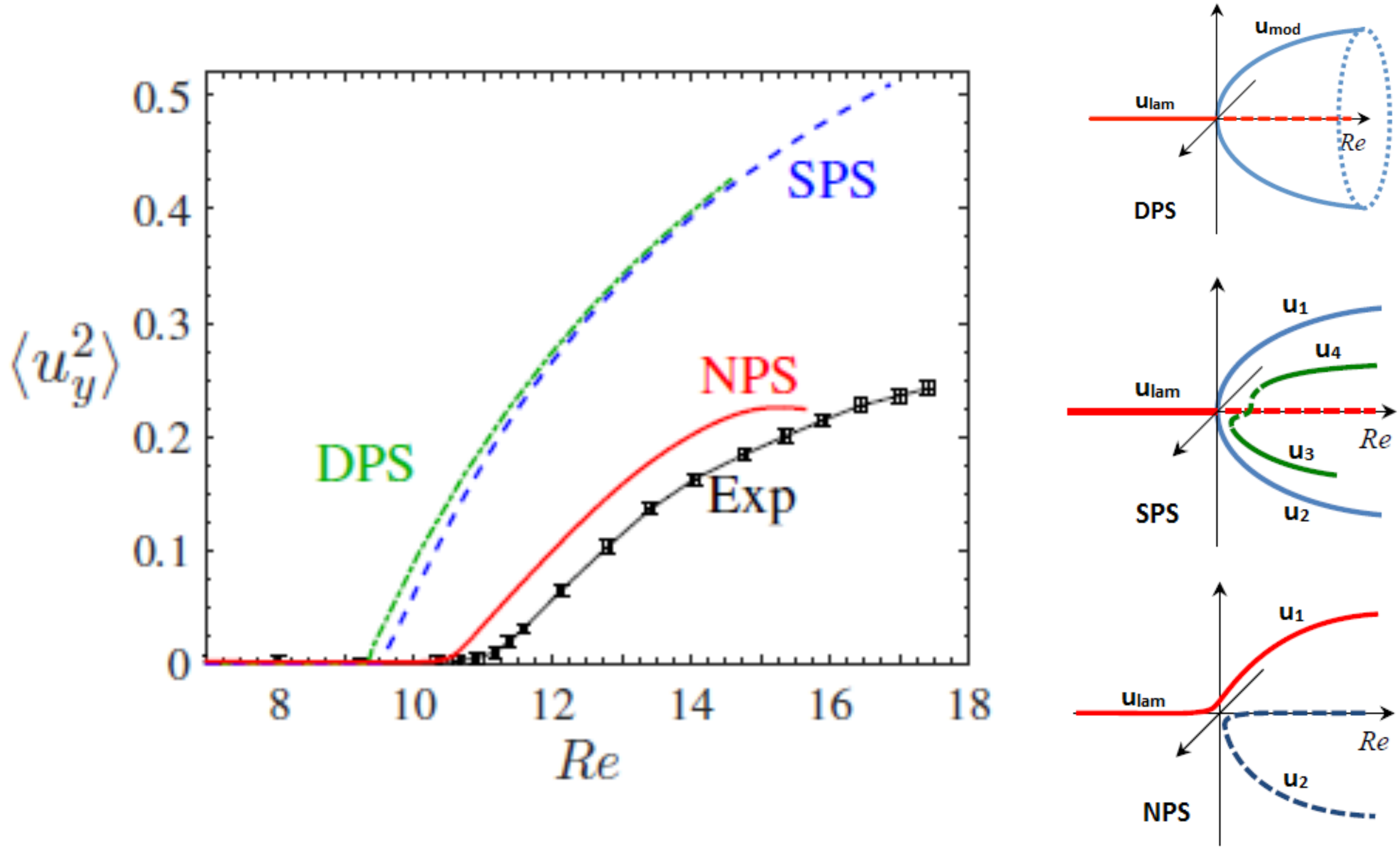




u_1

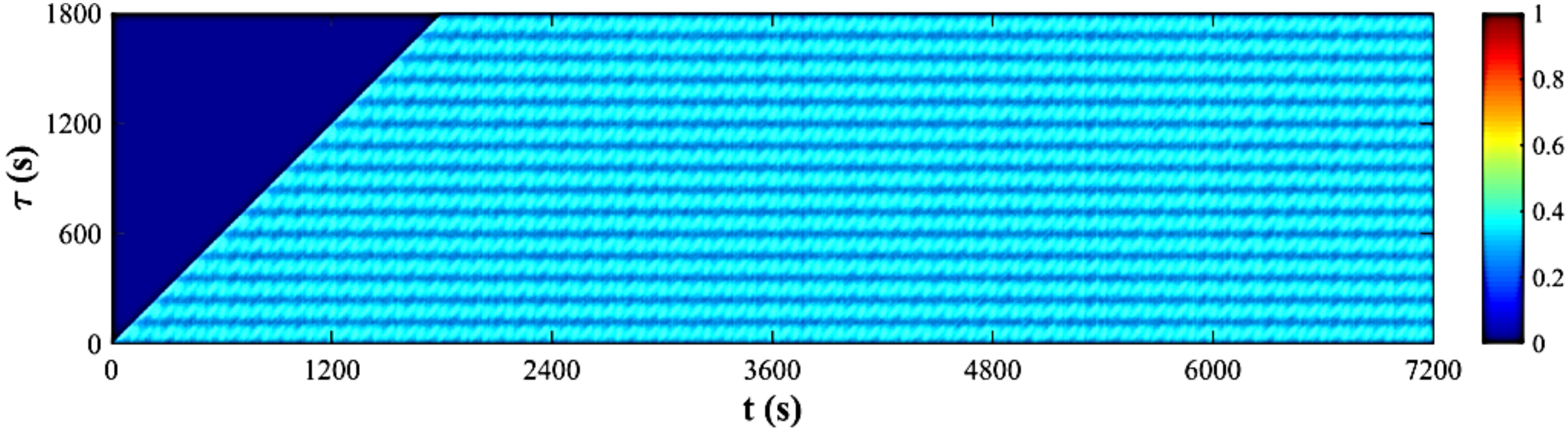


u_2



0.5
0.4
0.3
0.2
0.1
0
8
10
12
14
16
18
Re
$\langle u_y^2 \rangle$
SPS
NPS
DPS
Exp
u_mod
u_lam
Re
DPS
u_1
u_4
u_lam
Re
u_3
SPS
u_2
u_1
u_lam
Re
u_2
NPS

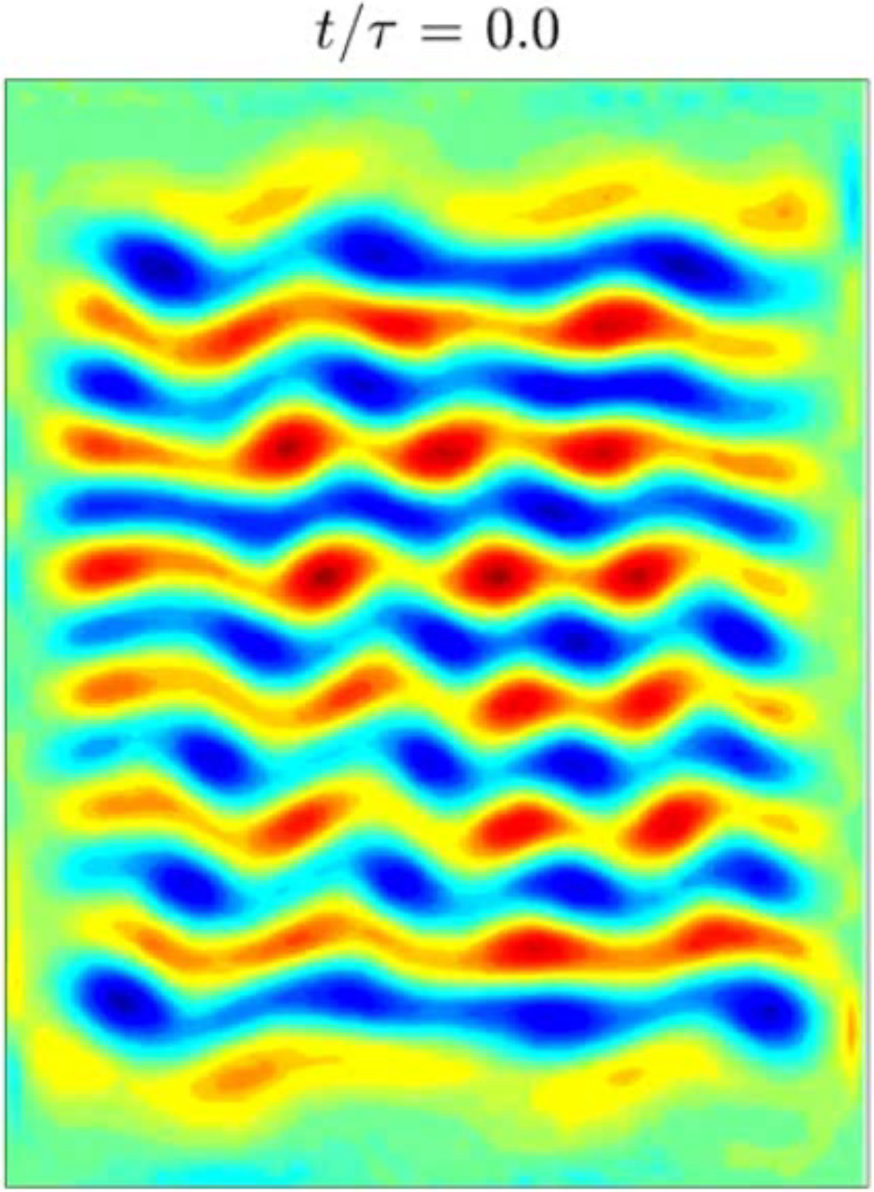
Periodic Orbit
(Supercritical Hopf Bifurcation)



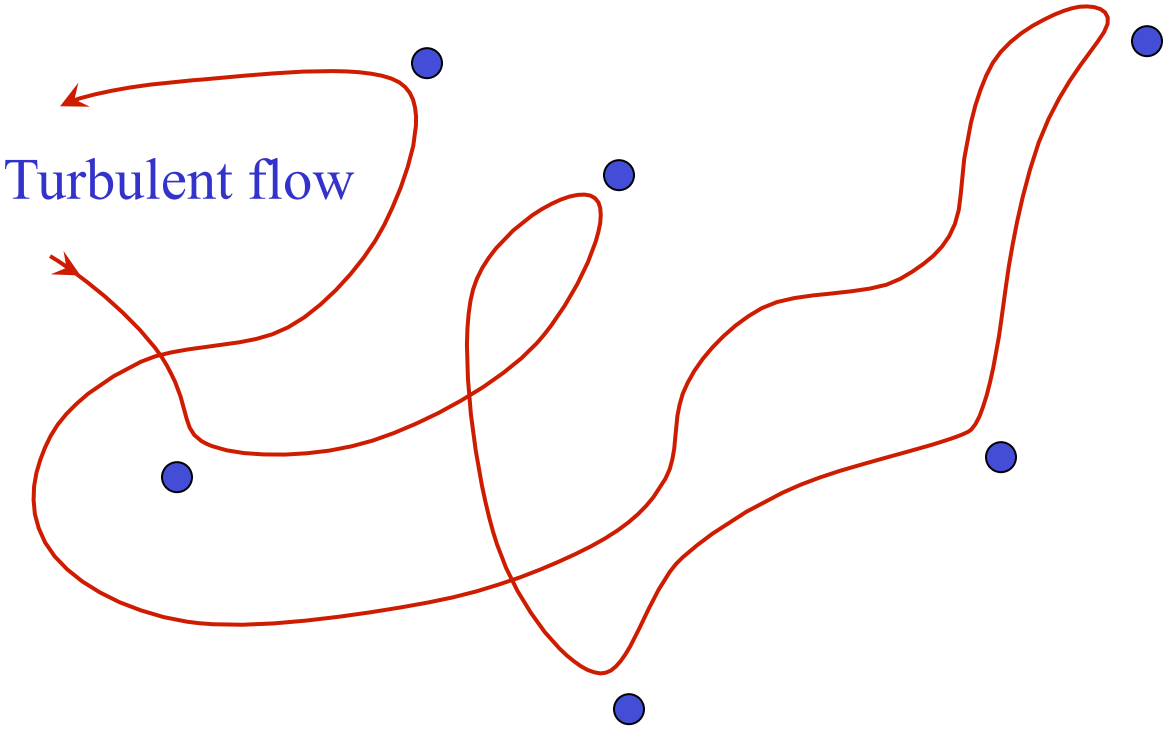
Experiment: Re_c=17.6 (+/- 0.1); T =120 s (+/- 1s)

Depth Averaged Model: Re_c=15.6; T =137 s

Turbulence (*Re* = 22.5)



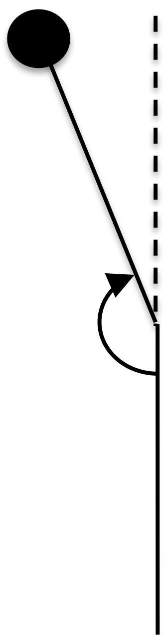
Identifying Exact Coherent Structures

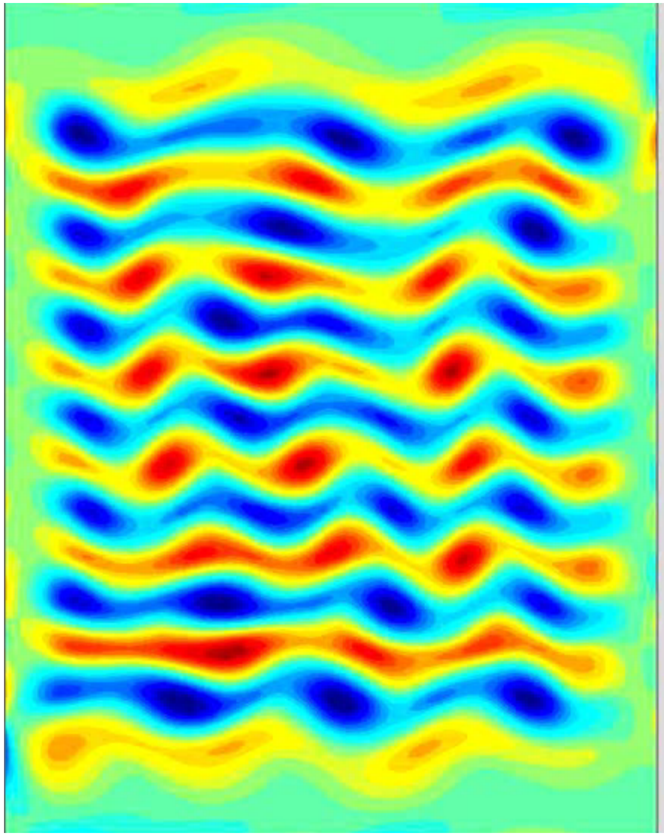


Signatures of Exact Coherent Structures (ECS)

- ECS are *not* known *a priori*
- Dramatic slow-down in the evolution indicates possible role of unstable equilibria in evolution

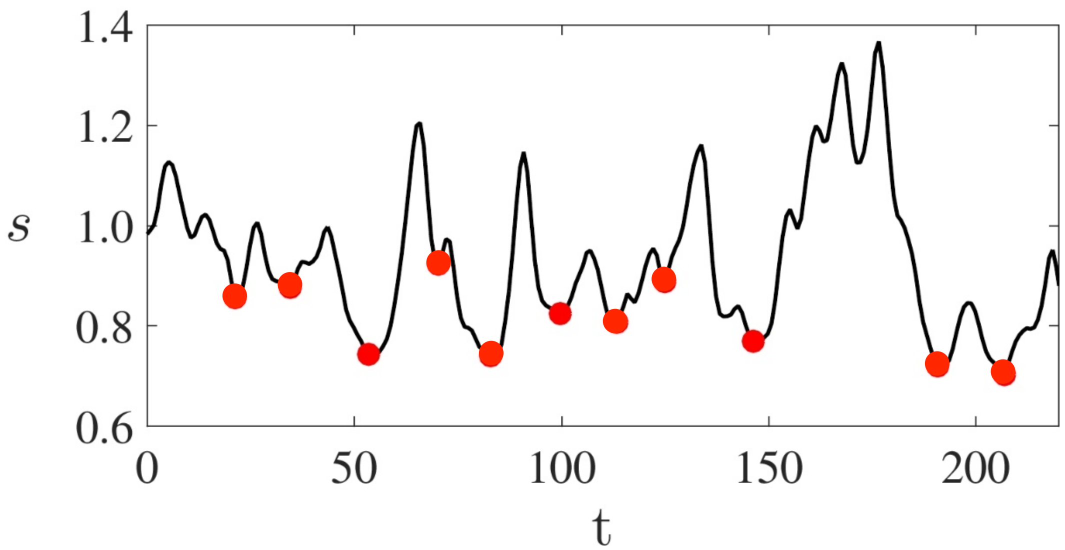
Simple Analogy: A Pendulum





Rate of Evolution

- Local minima correspond to possible close passes to unstable equilibria

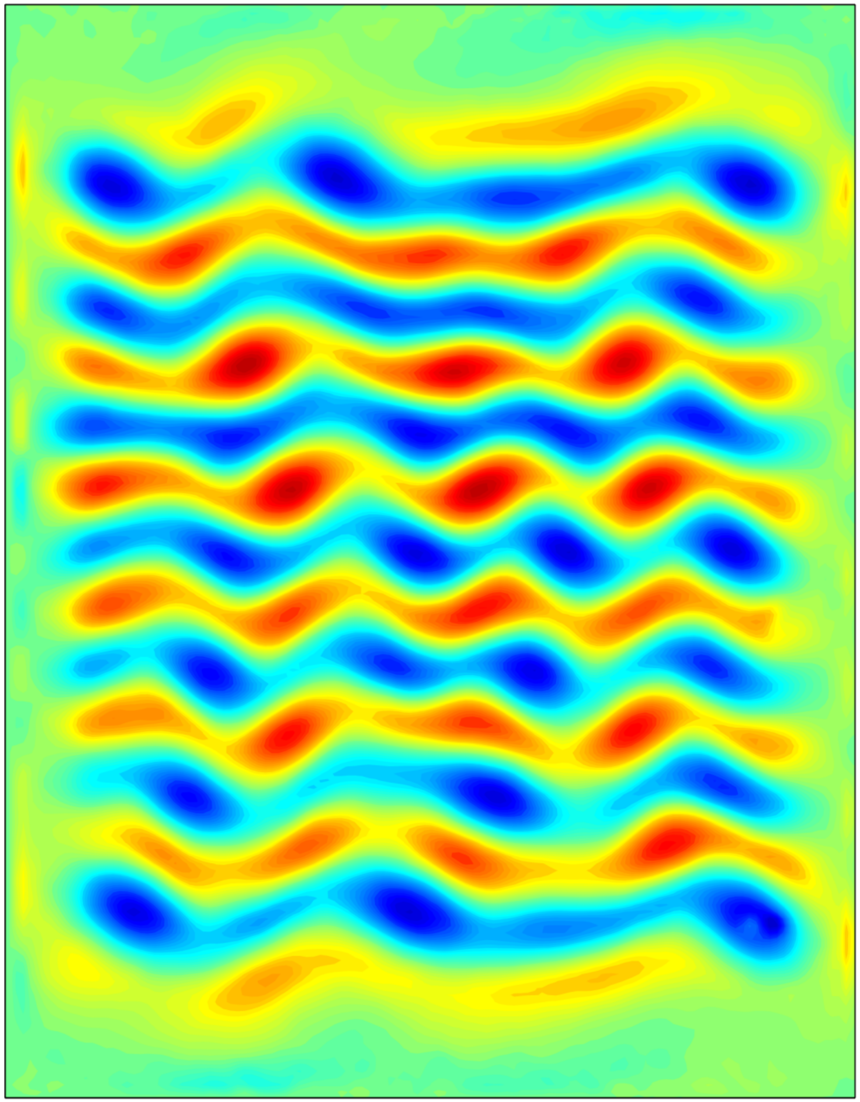


$$s = \left\| \frac{U(t+\Delta t) - U(t)}{\Delta t} \right\|$$

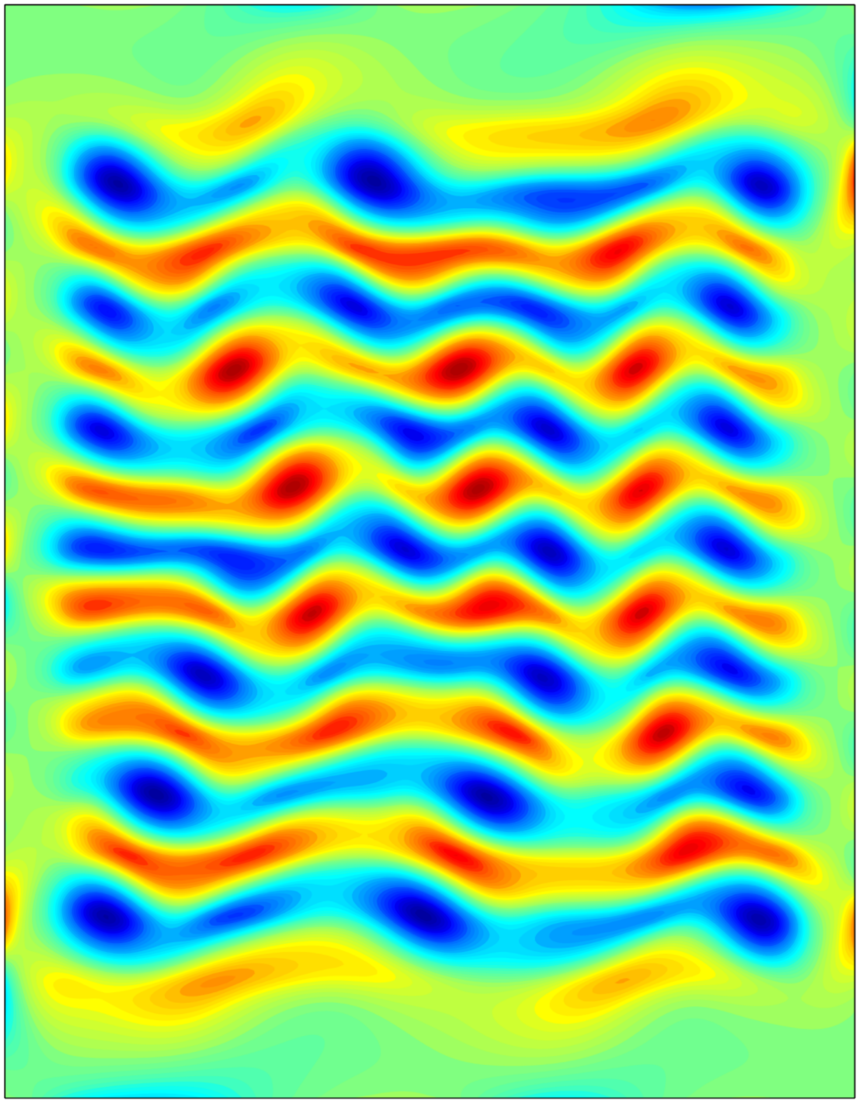
- Serve as initial conditions to a Newton-Krylov solver

ECS from Experiment

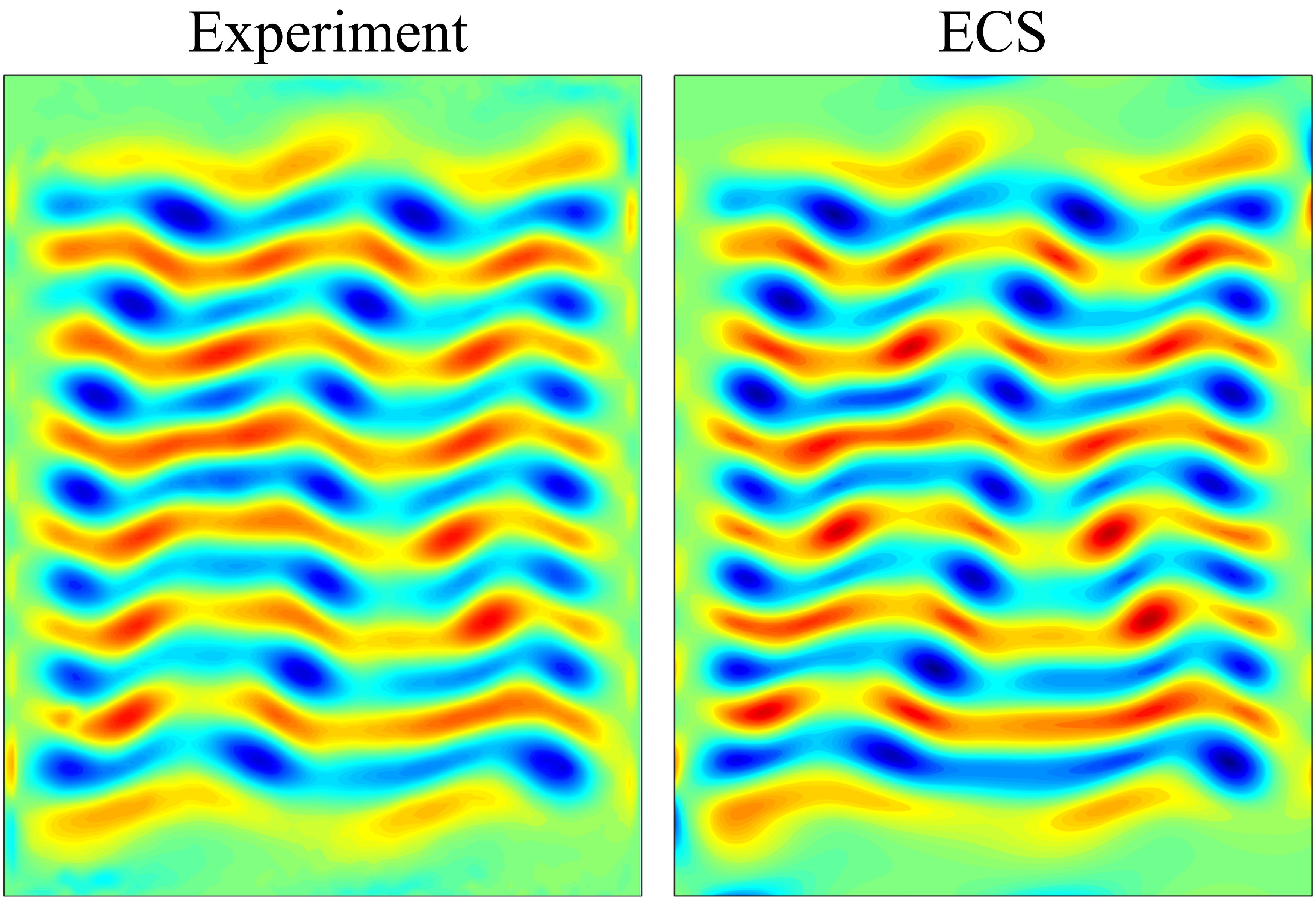
Experiment



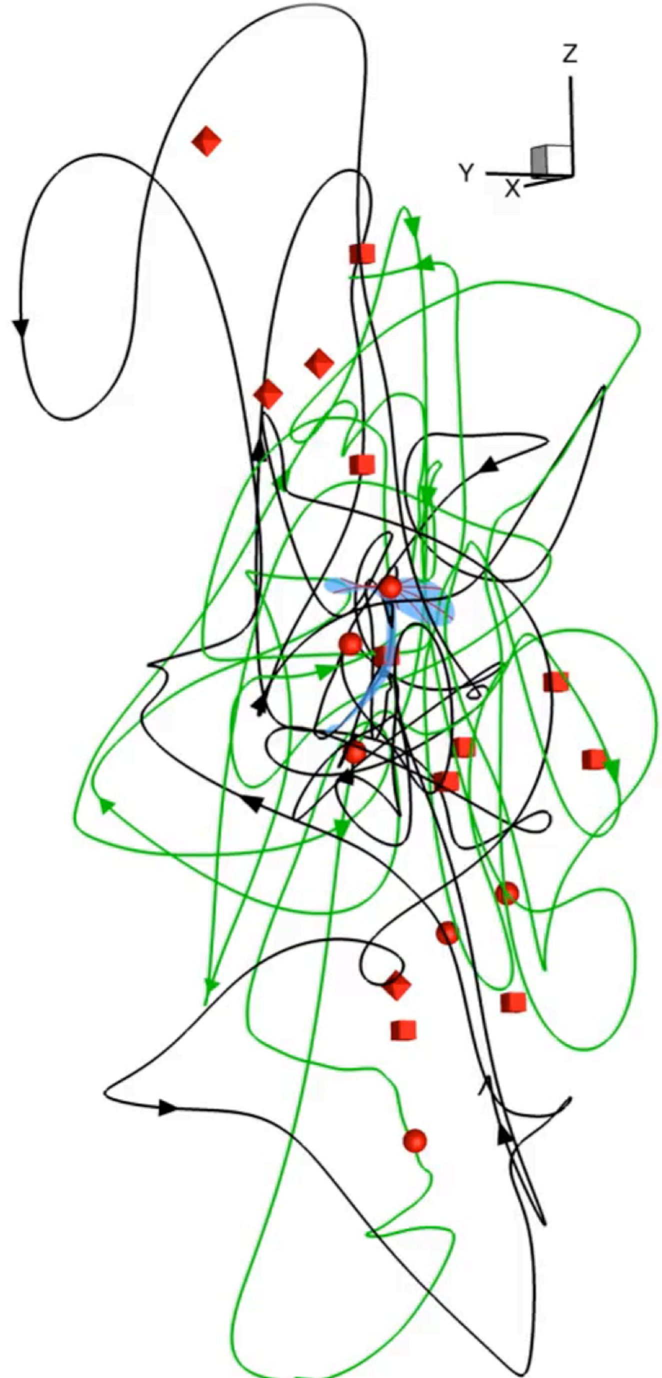
ECS



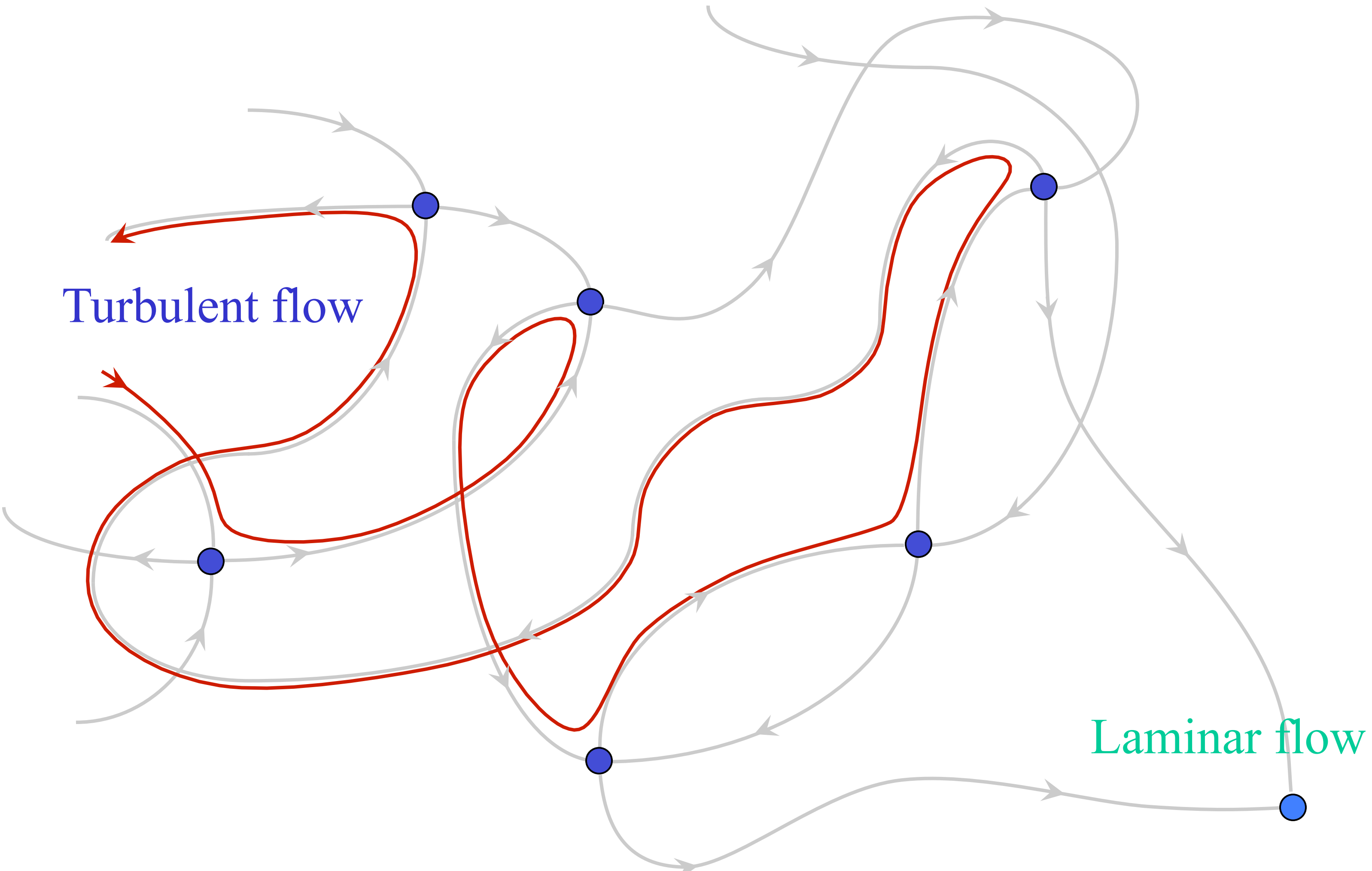
ECS from Experiment



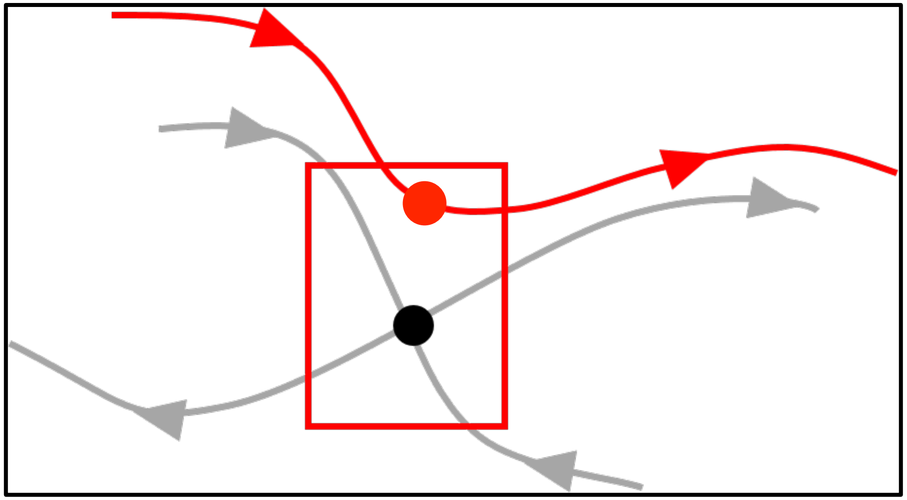
Projection of ECS & Turbulent Trajectories



Dynamics Near ECS



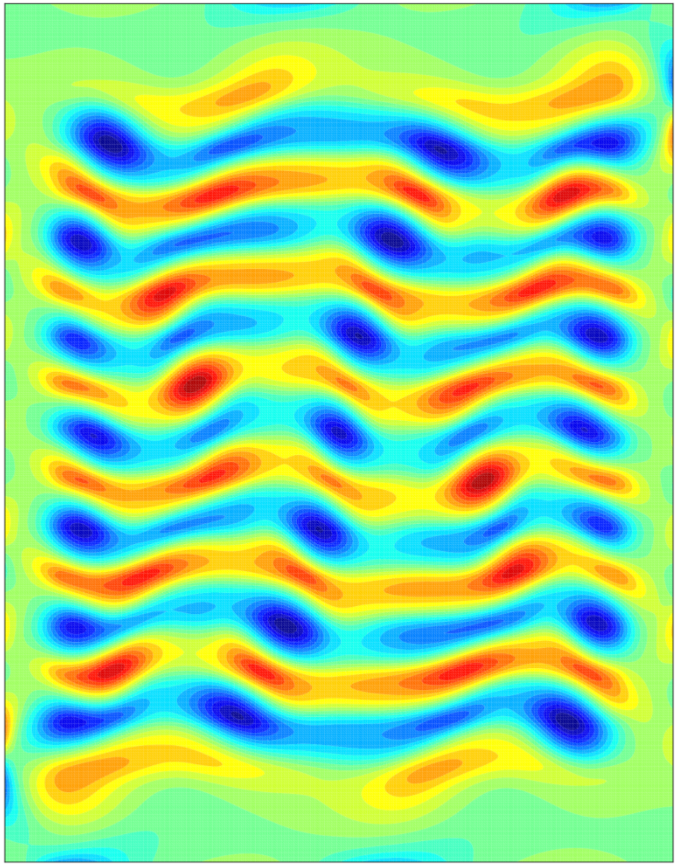
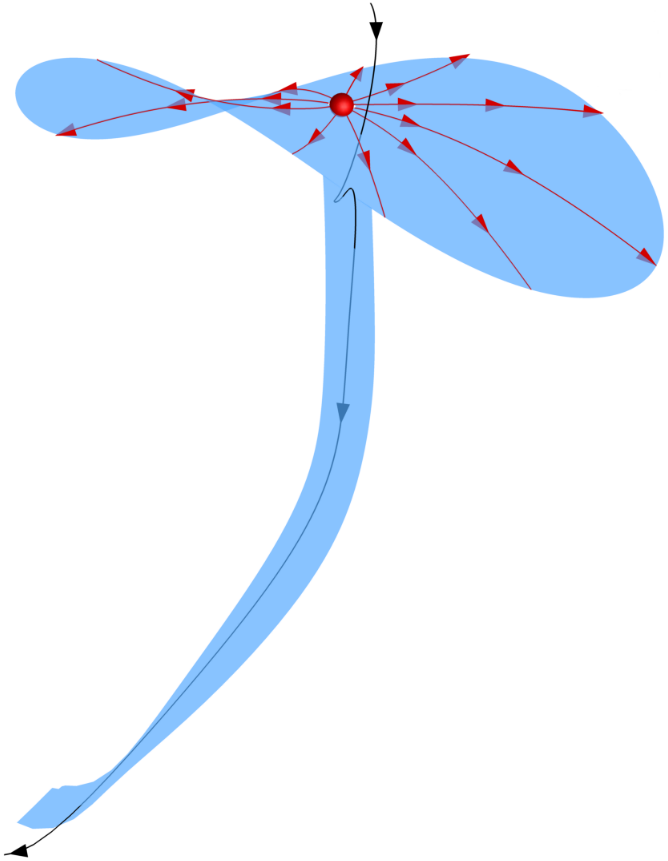
The Neighborhood



- Stable and unstable eigendirections/manifolds
- Typically 10 or fewer unstable directions
- Unstable manifold guides the departure of the turbulent trajectory from the vicinity of the ECS

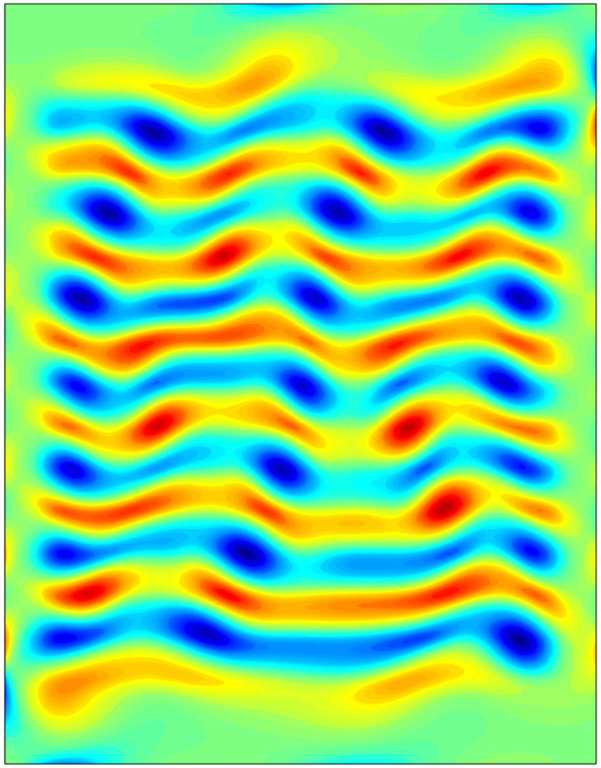
Example: 2D Manifold

- Comparable Eigenvalues ➡ Construct the 2D manifold using numerical integration

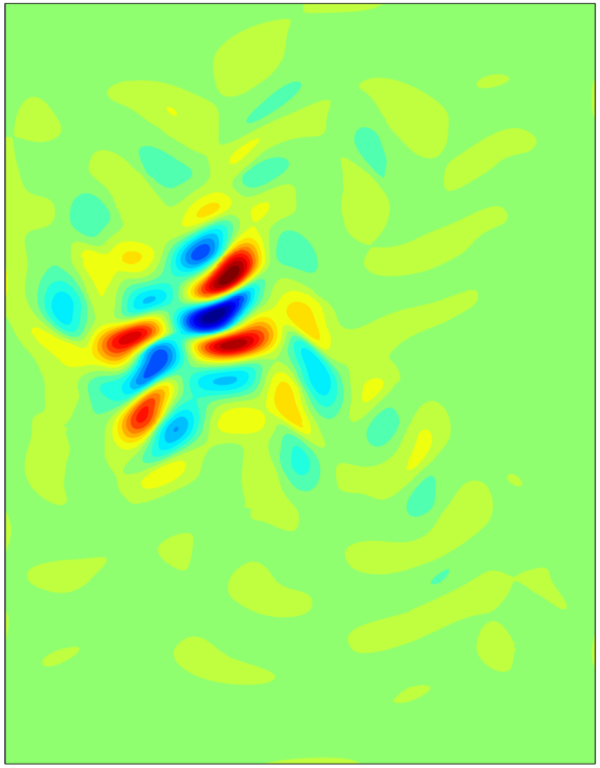



Example: ~1D Unstable Manifold

- A 7D unstable manifold
- $\lambda_1 > 10x\{\lambda_2, \lambda_3, \ldots\ldots\}$ Effectively 1D manifold



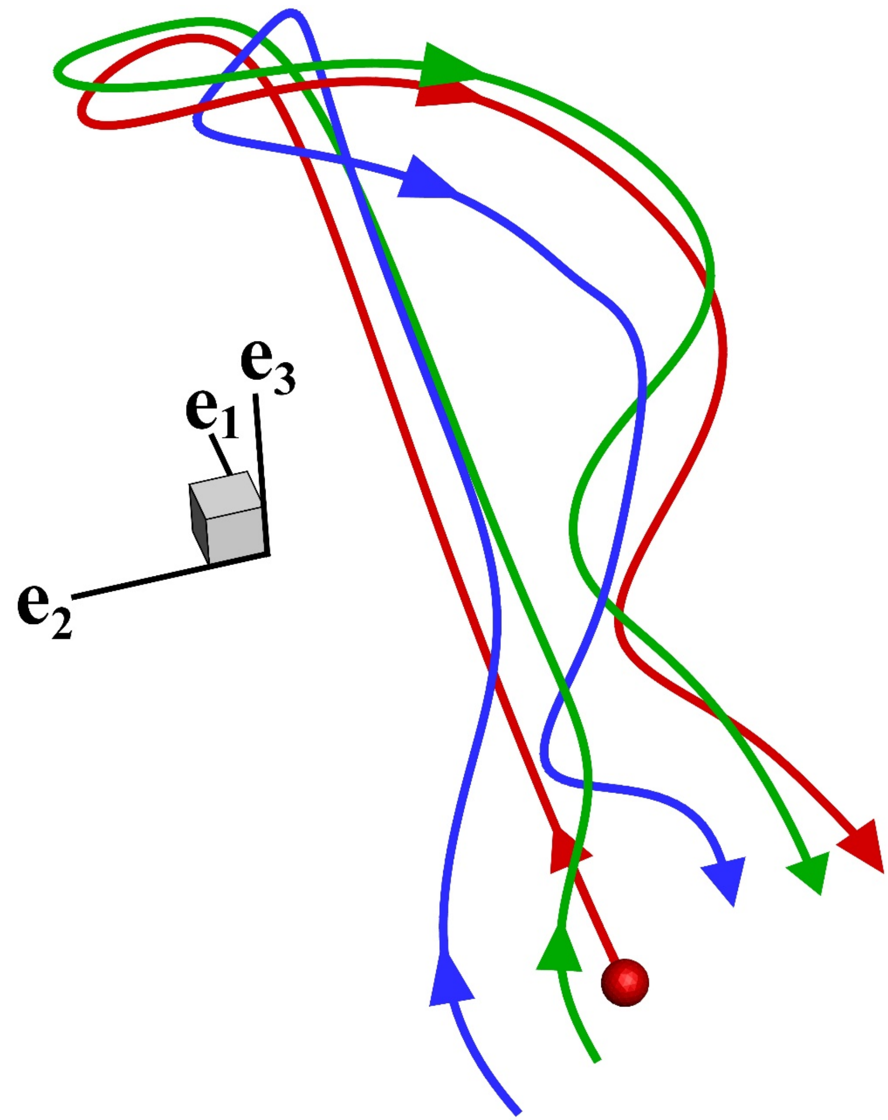
ECS

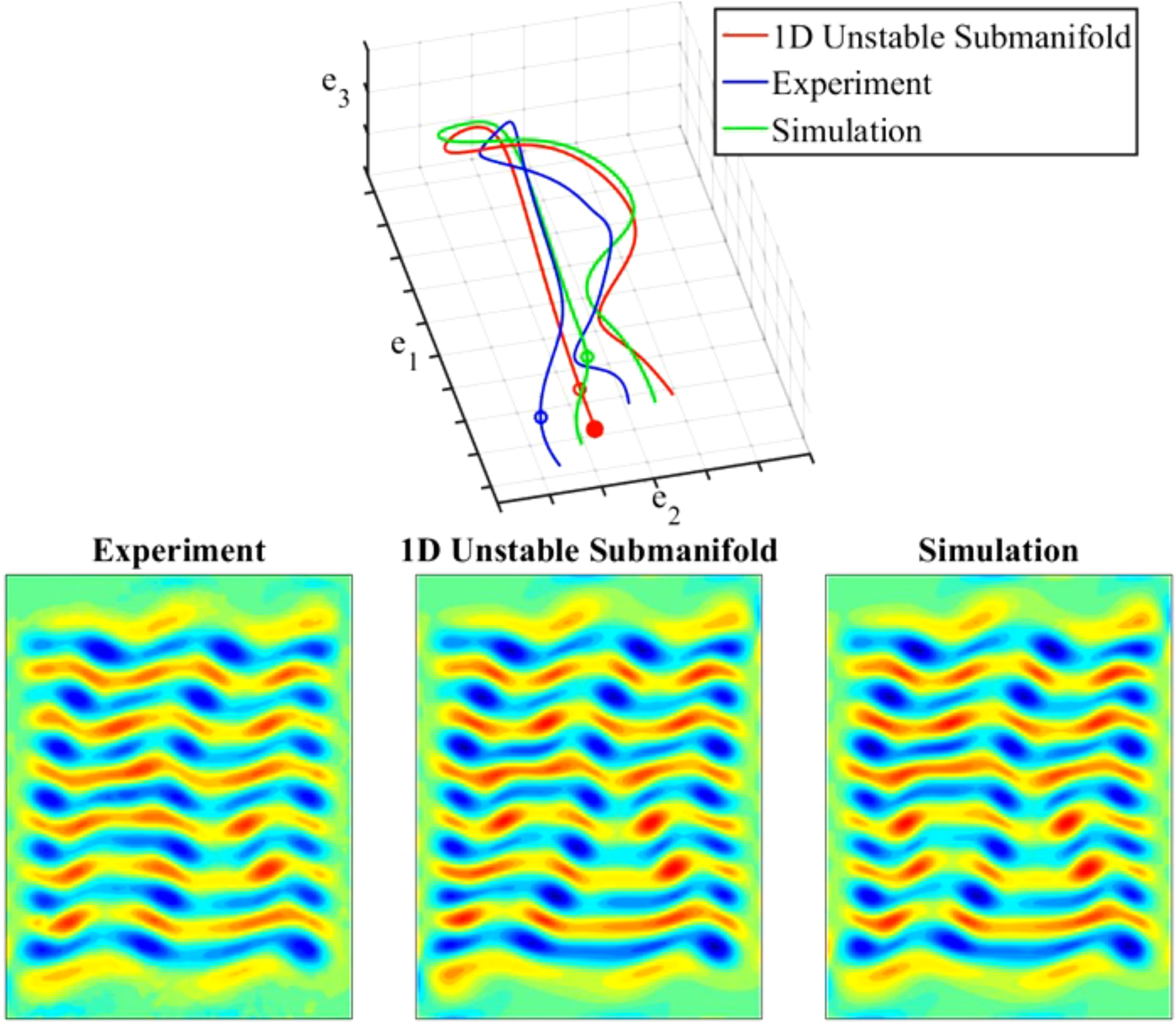


Eigenvector

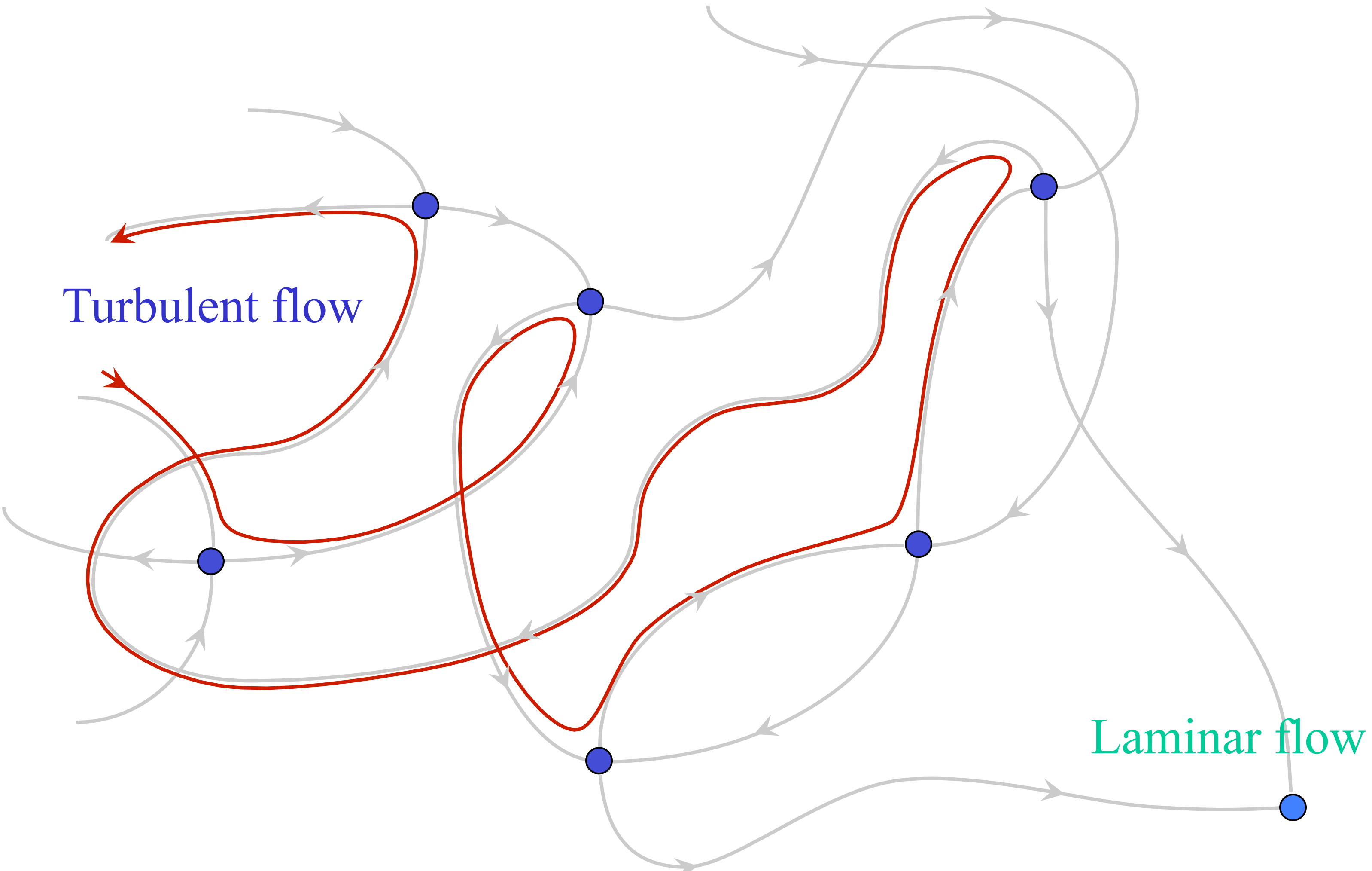
Forecasting Turbulence in Experiments

— (blue)	Experimental turbulent trajectory
— (green)	Numerical turbulent trajectory
— (red)	1D Unstable Submanifold
● (red)	Unstable equilibrium



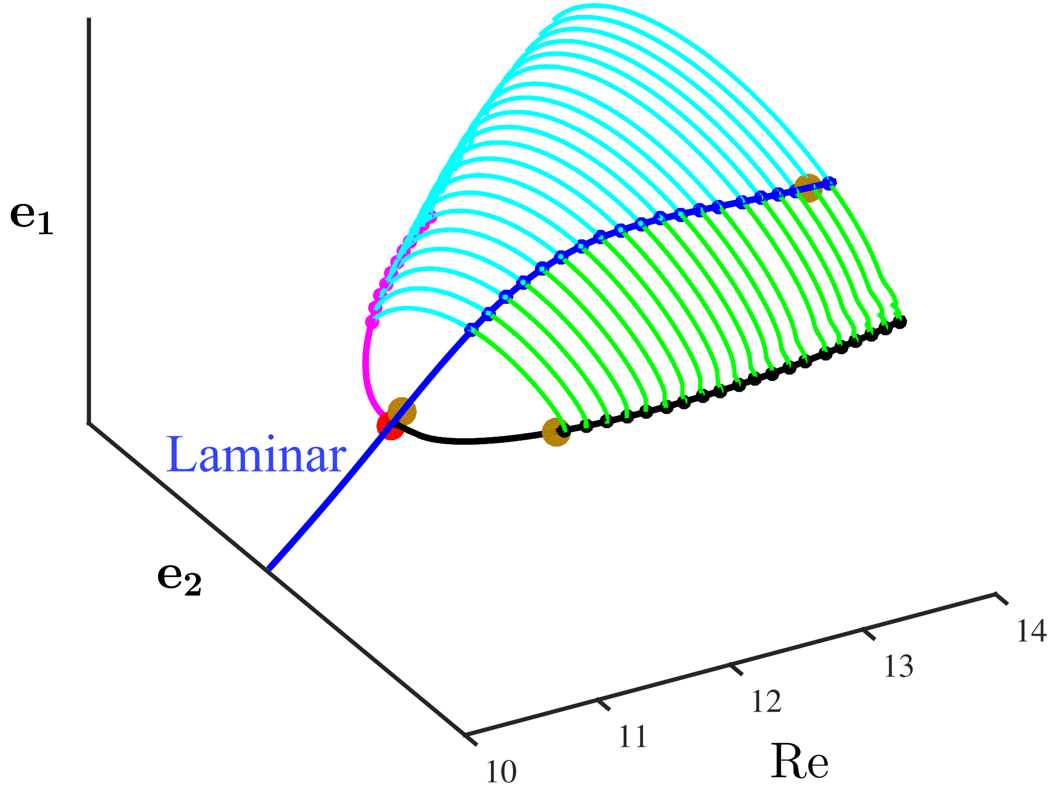

1D Unstable Submanifold
Experiment
Simulation
e3
e1
e2
Experiment
1D Unstable Submanifold
Simulation

Dynamical Connections

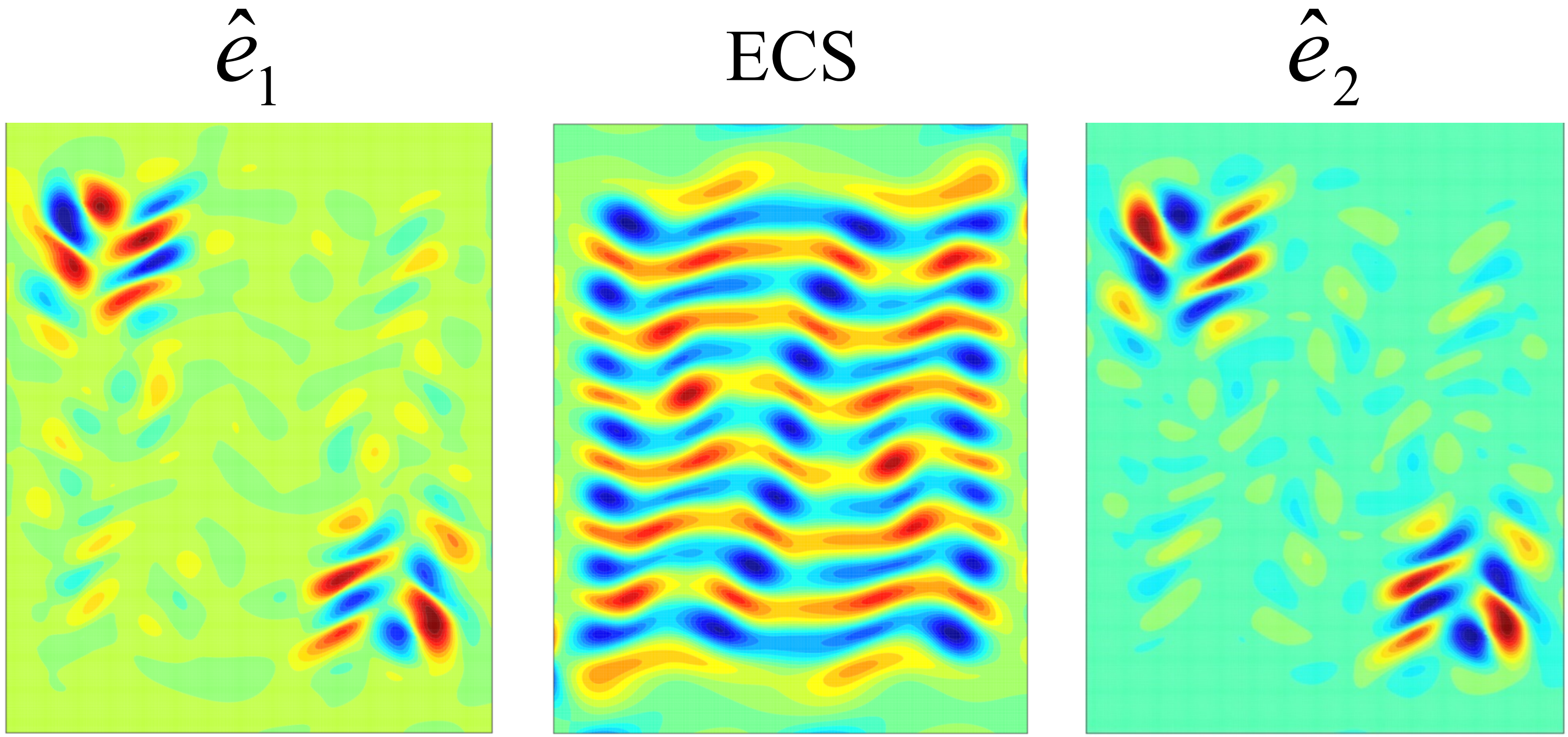


Exploit Bifurcations

- Calculating heteroclinic connections by continuation



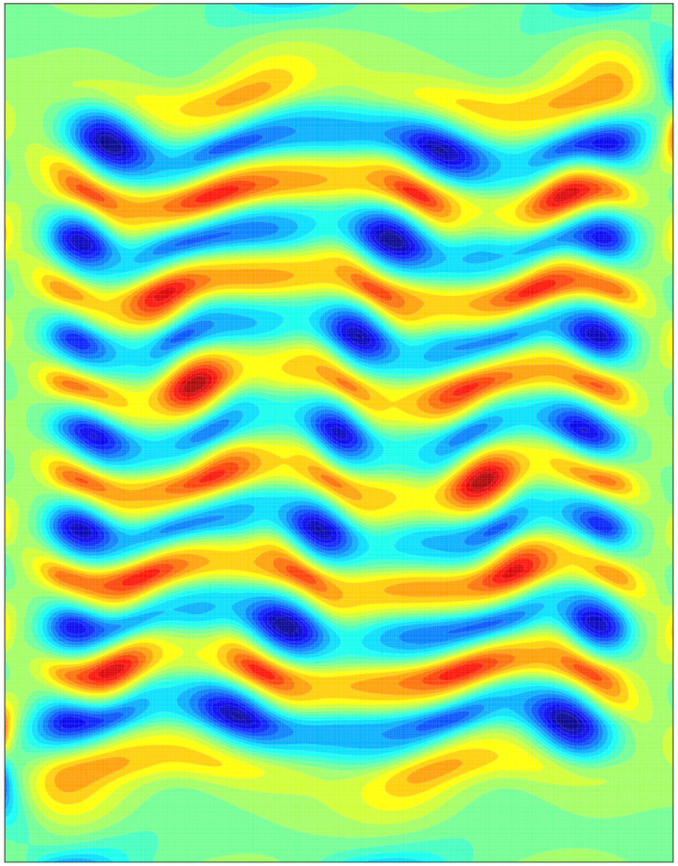
Symmetries of Eigenvectors

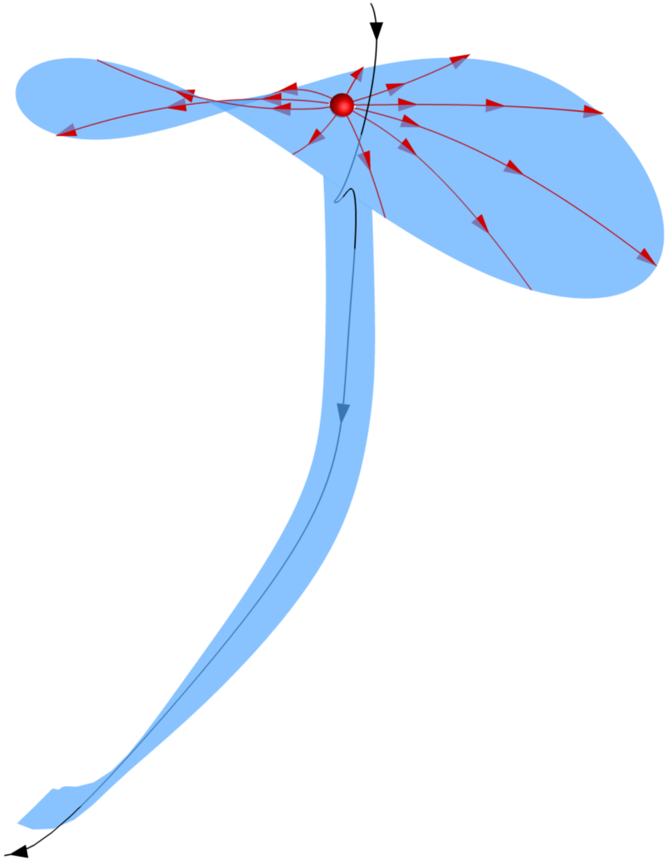


- Governing equation is equivariant under rotation

Exploit Symmetry

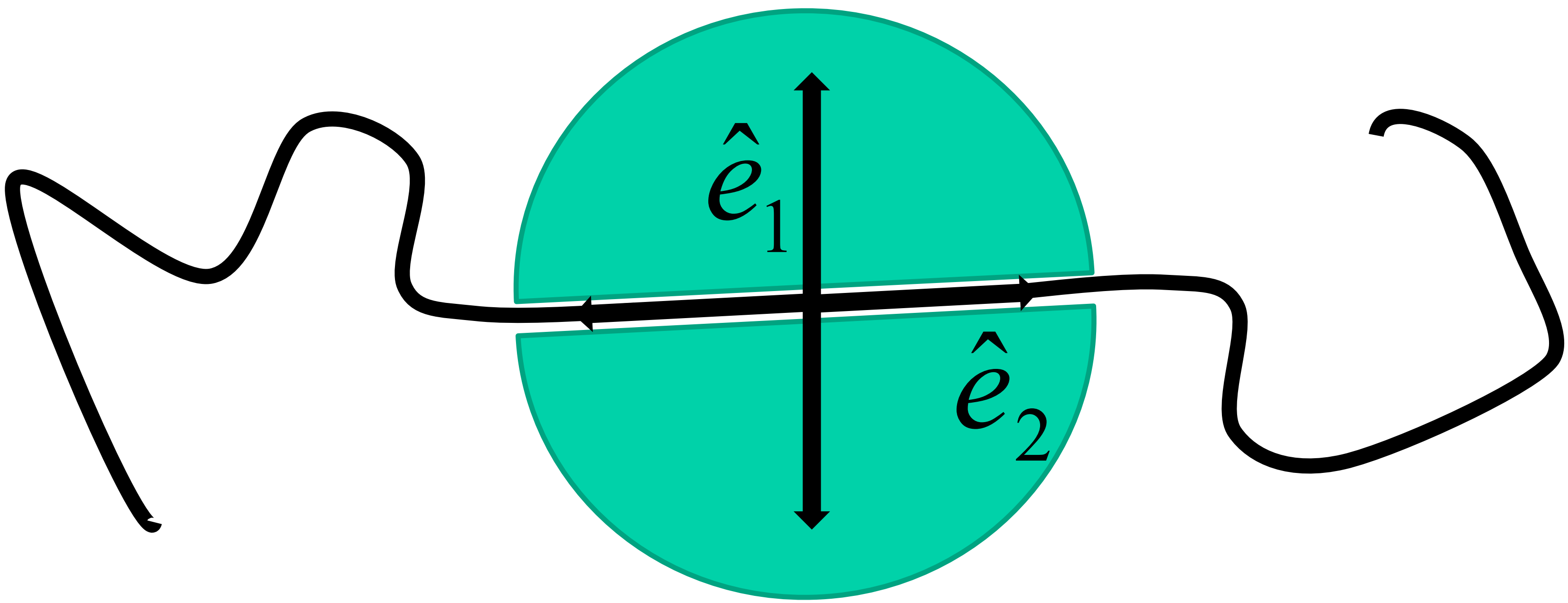
- Rotationally symmetric ECS and eigenvector



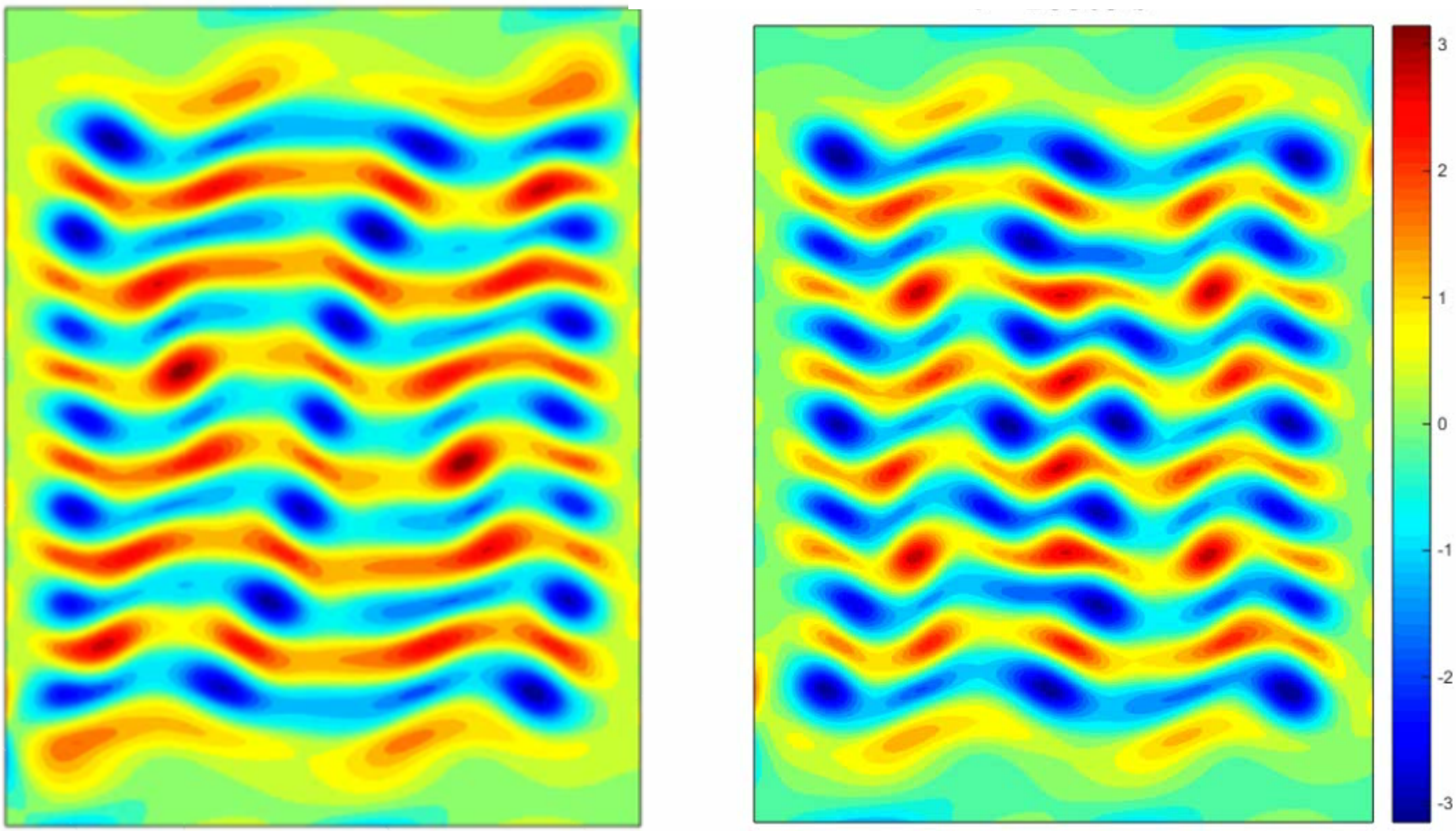


Equivariance Under Rotation

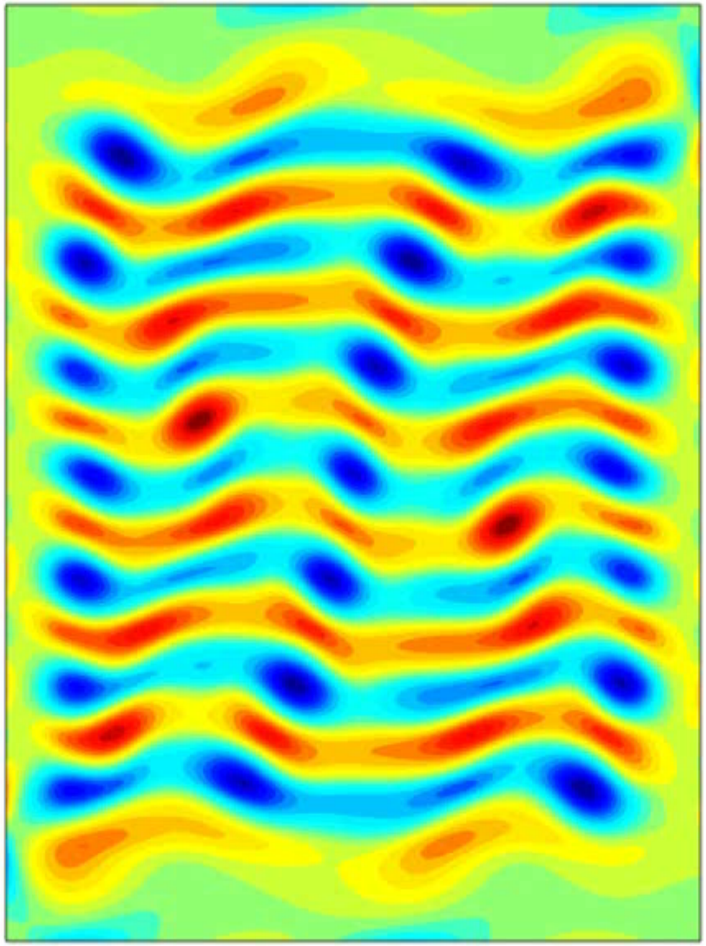
- Governing equation is equivariant under Rotation $(x, y) \rightarrow (-x, -y)$
- Trajectories starting in a rotationally symmetric subspace remain invariant under rotation



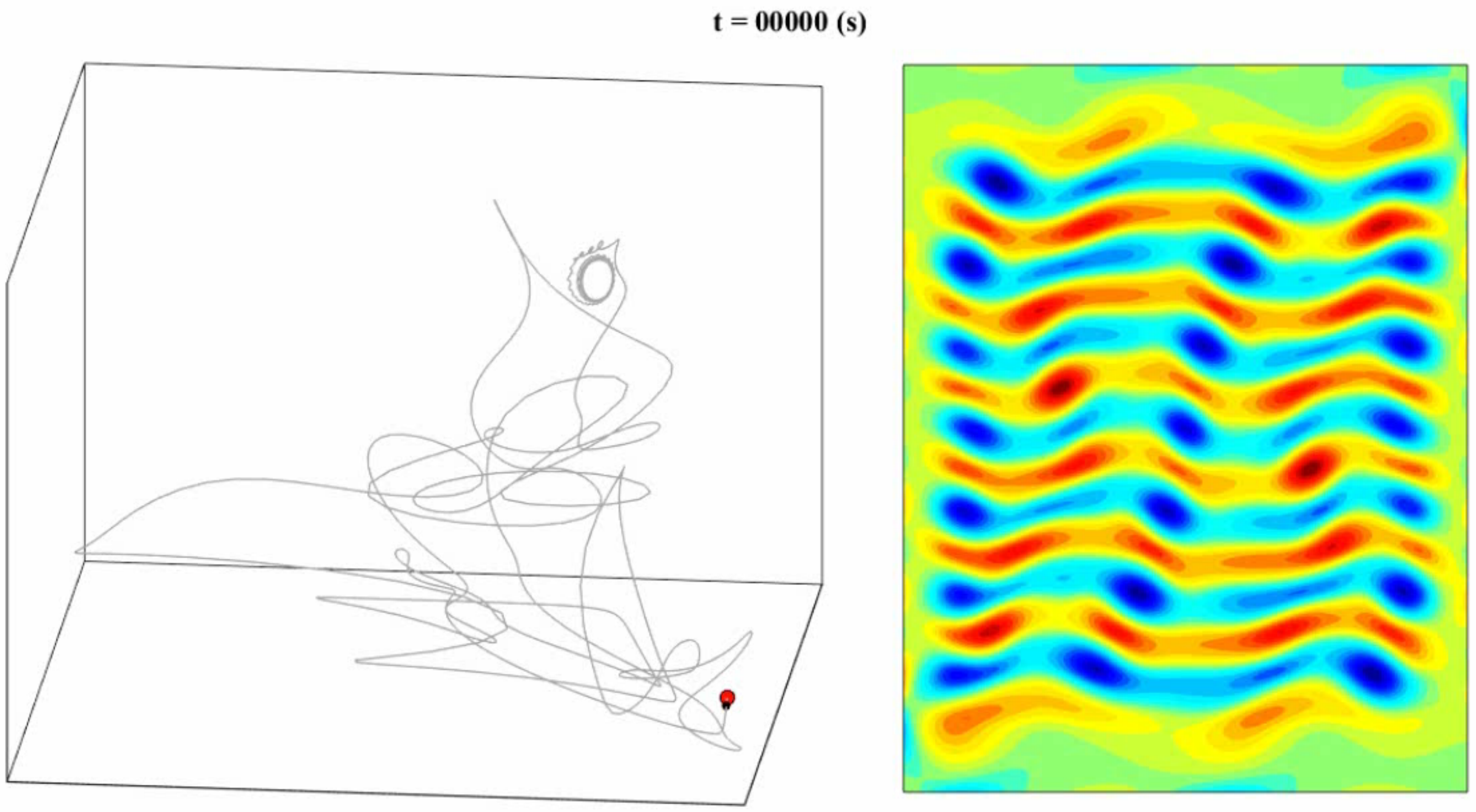
Two Unstable Solutions (Re = 23.0)



Dynamical Connections:Exploit Symmetry



Dynamical Connection (Full State Space)



Complementary Approaches

- **Lyapunov Spectrum**

  Estimate fractal dimension ~20

  Xu and Paul (Virginia Tech), unpublished

- **Persistent Homology**

  Dynamics in symmetry reduced space

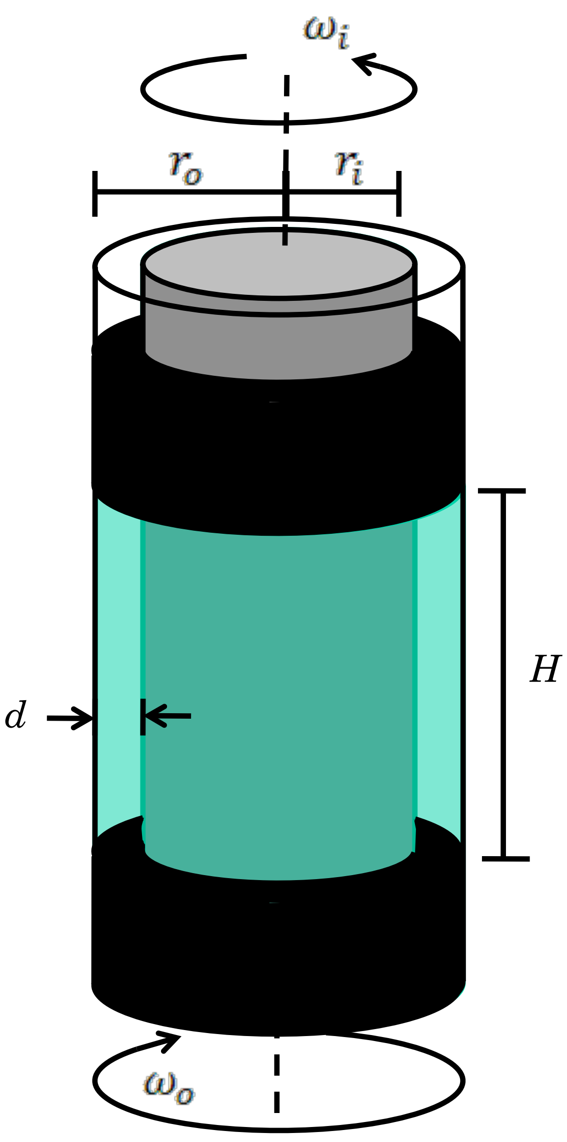
  Kramar, et al., *Physica D* **334,** 82 (2016)

- **Lagrangian Coherent Structures**

  First experimental test in Q2D flow

  Voth, et al., *Phys. Rev. Lett.* (2016)

Taylor–Couette Flow



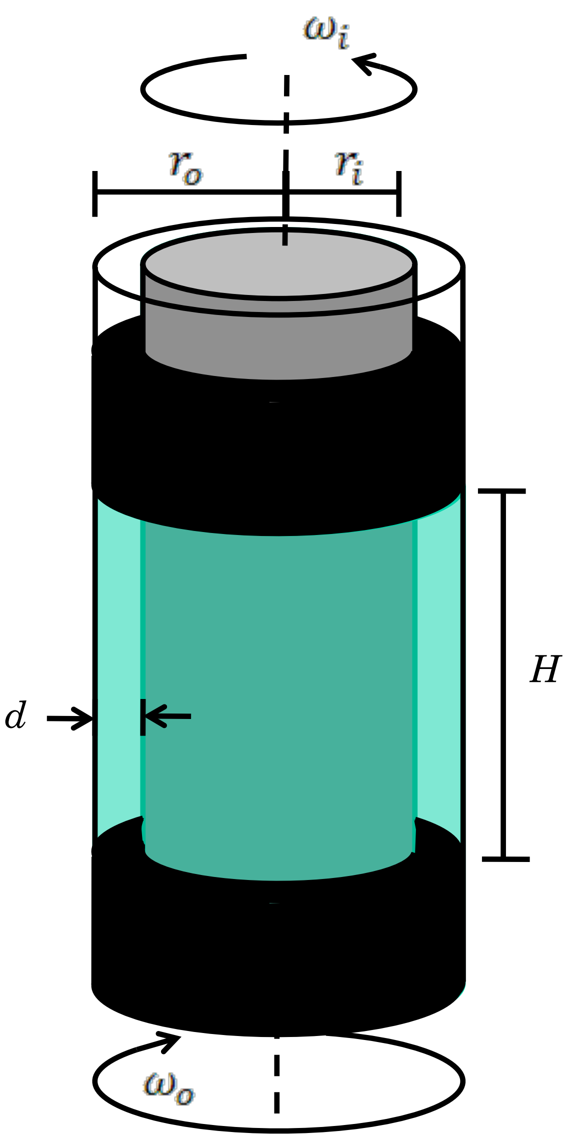
Radius ratio $$\eta = \frac{r_i}{r_0}$$

Aspect Ratio $$\Gamma = \frac{H}{d}$$

$$Re_i = \frac{\omega_i r_i d}{\nu}$$

$$Re_o = \frac{\omega_o r_o d}{\nu}$$

Taylor–Couette Flow



Radius ratio $\eta = \dfrac{r_i}{r_0} = 0.905$

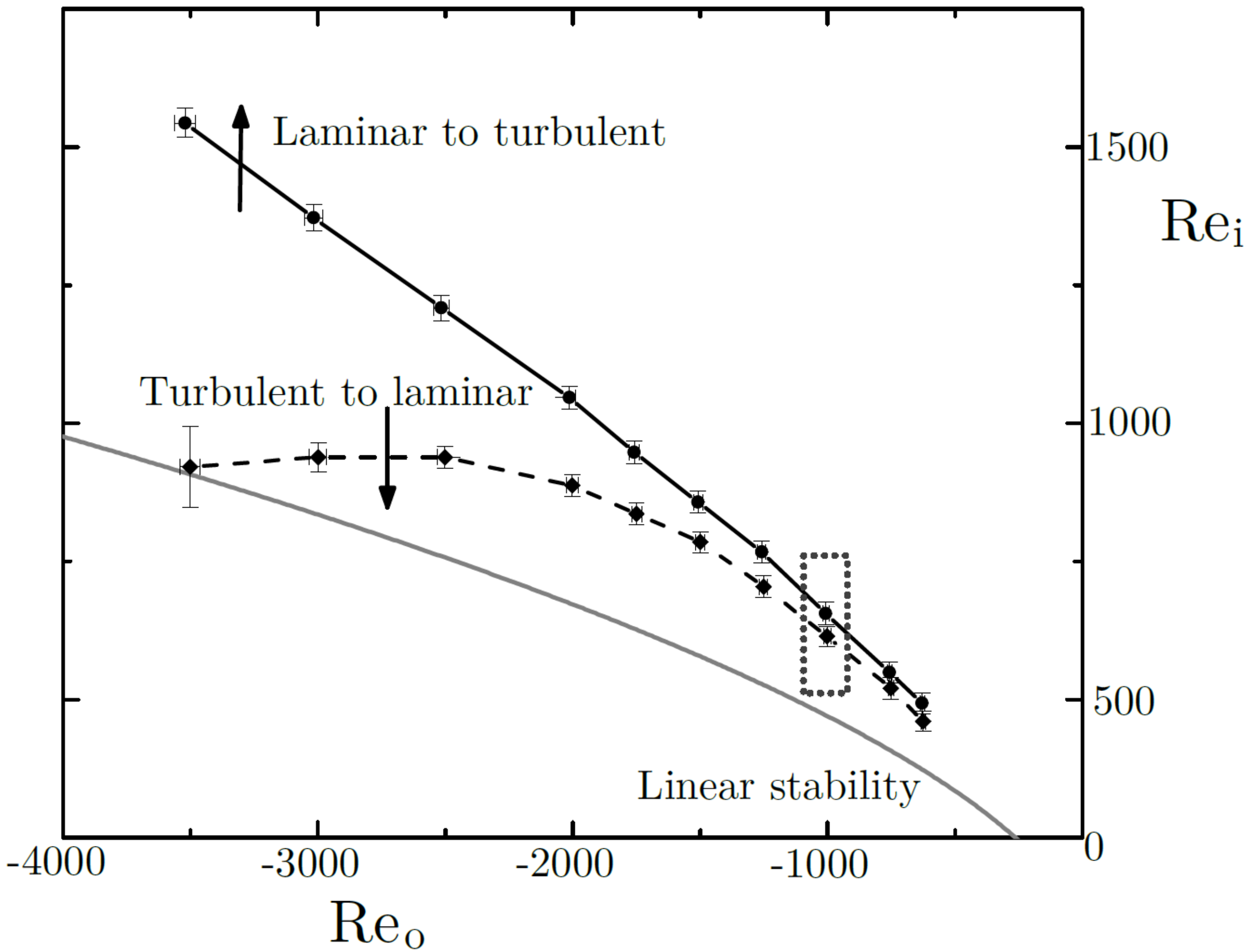
Aspect Ratio $\Gamma = \dfrac{H}{d} = 5.24$

$$Re_i = \frac{\omega_i r_i d}{\nu} = 600$$

$$Re_o = \frac{\omega_o r_o d}{\nu} = \text{-}1000$$

Subcritical Transition

$\Gamma = 5.24 \qquad \eta = 0.905$

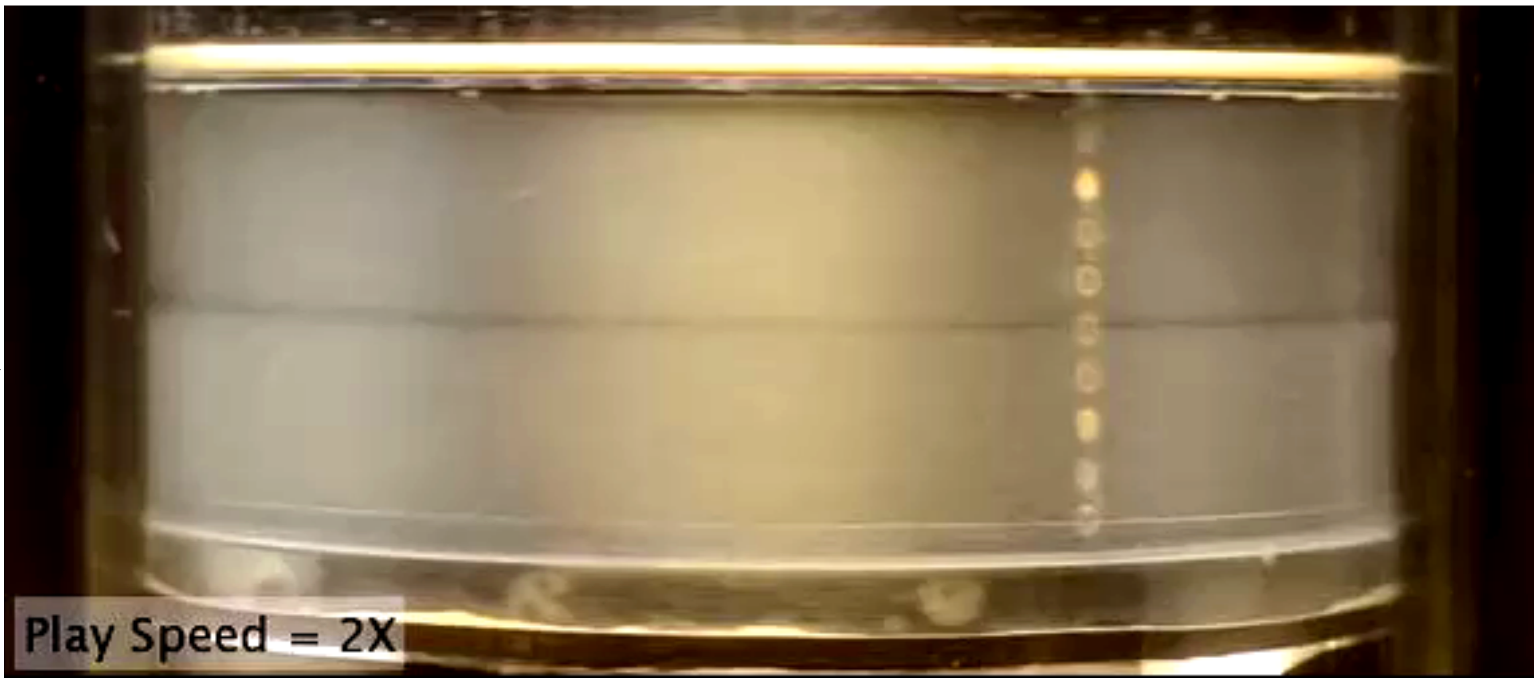


Direct Transition to Turbulence

Just after increasing Re_{in} by 0.13
(that's 1 in 10^4)

D = 15 1/4 cm

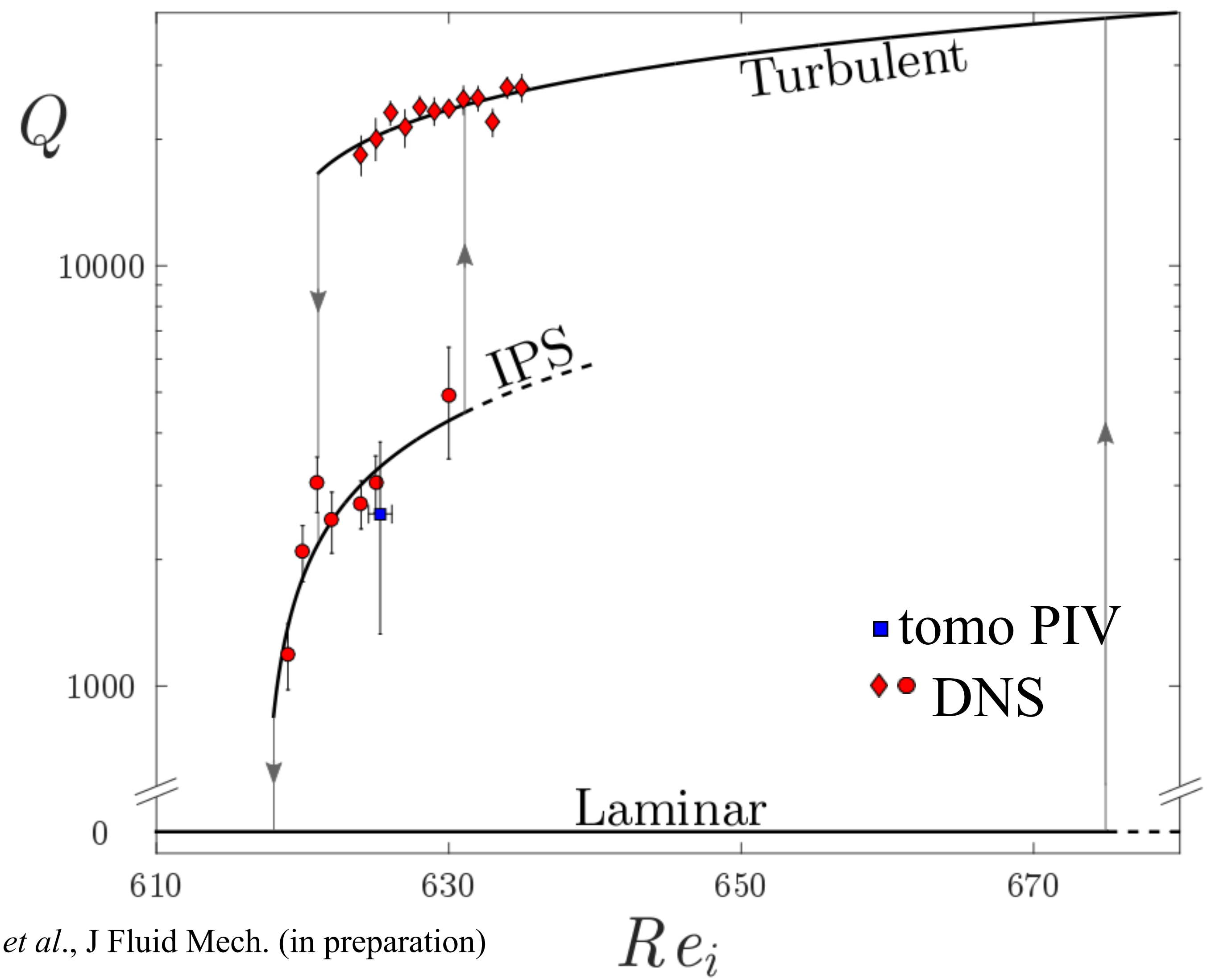
H = 4 cm



$$Re_{out} = -1000 \qquad Re_{in} = 650$$

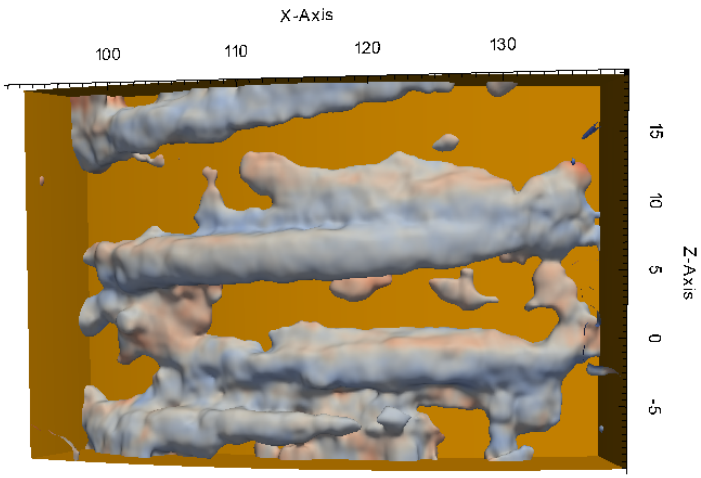
Bifurcation Diagram

$$Q = \frac{1}{AT}\int_0^T dt \iint_A dA \left(\mathbf{v}_\theta(t) - \langle \mathbf{v}_\theta \rangle_t\right)^2$$

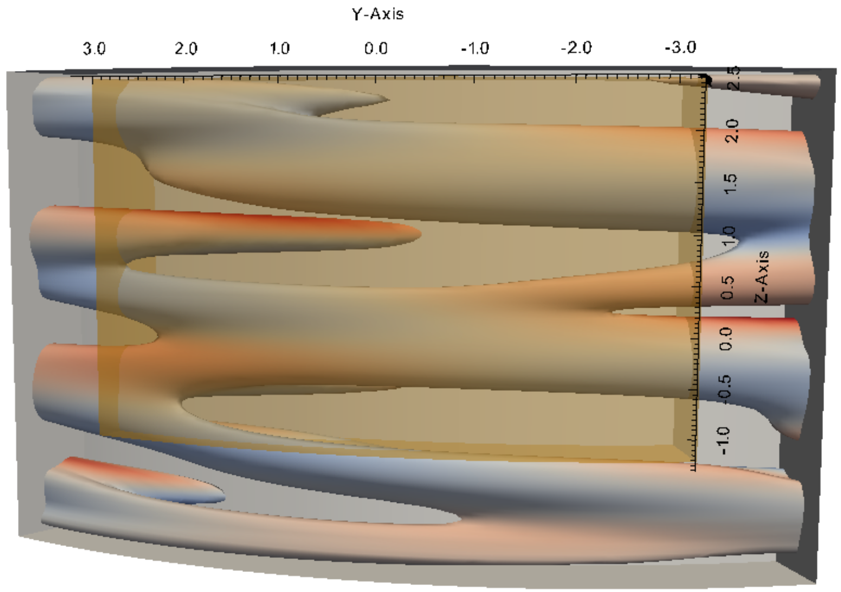


C. J. Crowley *et al.*, J Fluid Mech. (in preparation)

Interpenetrating Spirals (Ro = -1000, Ri = 625)



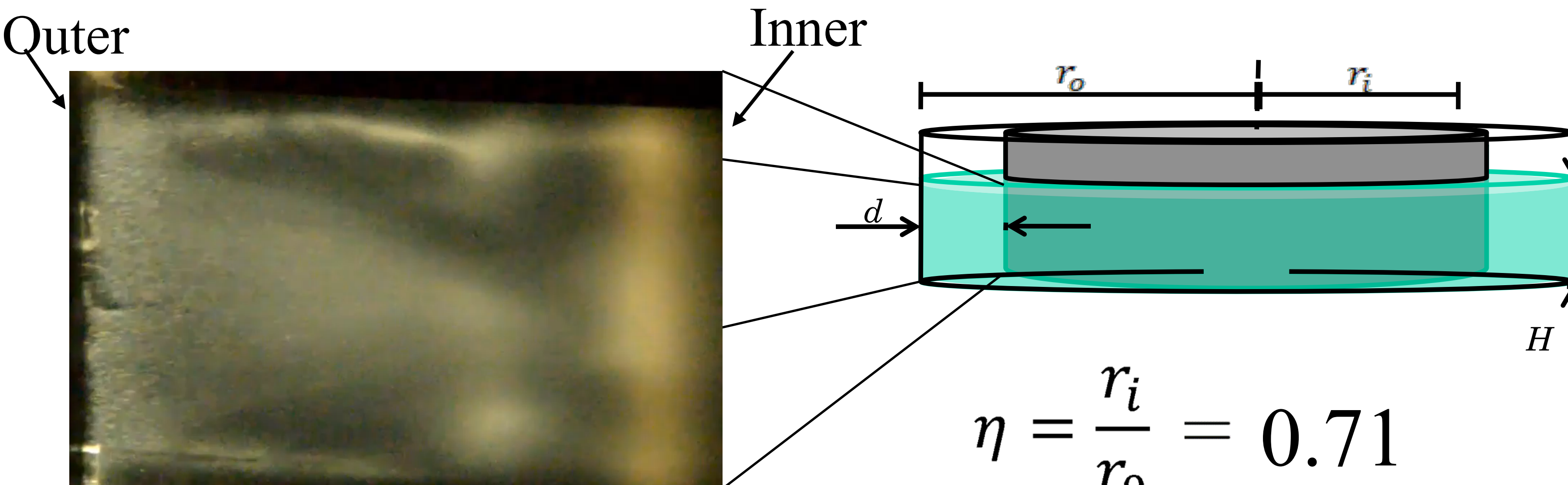
Experiment (tomo-PIV)



Direct Numerical Simulation

C. J. Crowley *et al*., J Fluid Mech. (in preparation)

Γ=1 TCF Experiment



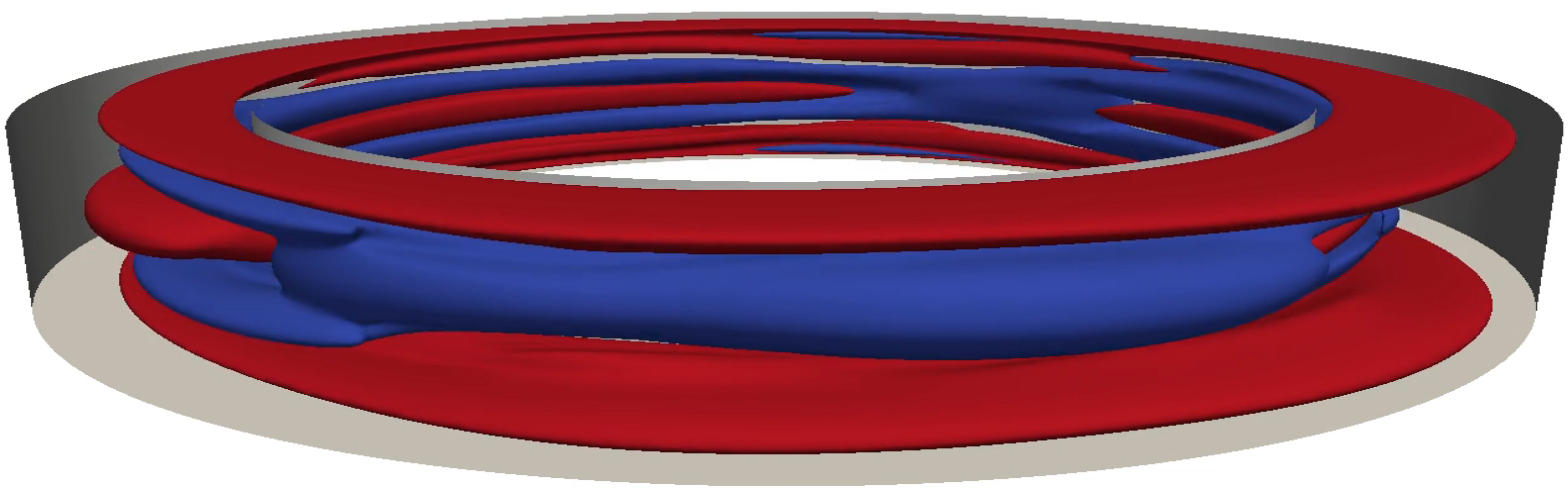
$$\eta = \frac{r_i}{r_0} = 0.71$$

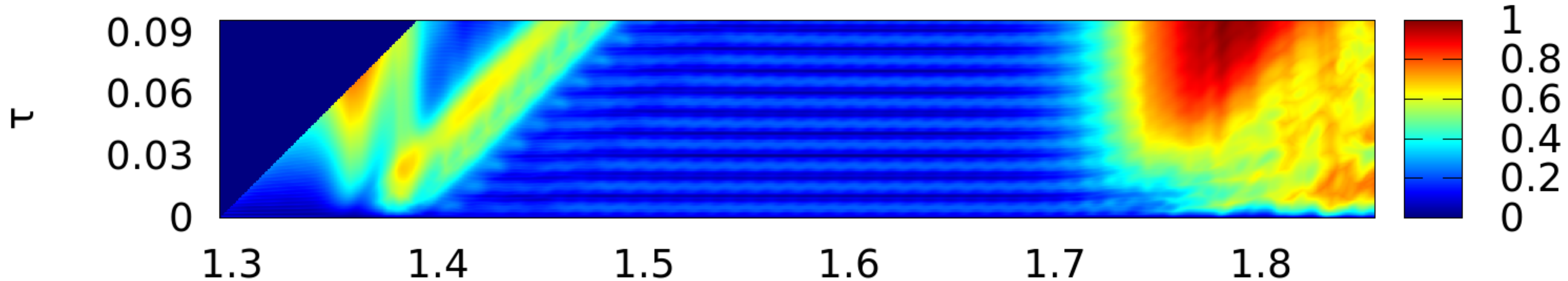
Play speed =
32X

$$\Gamma = \frac{H}{d} = 1$$

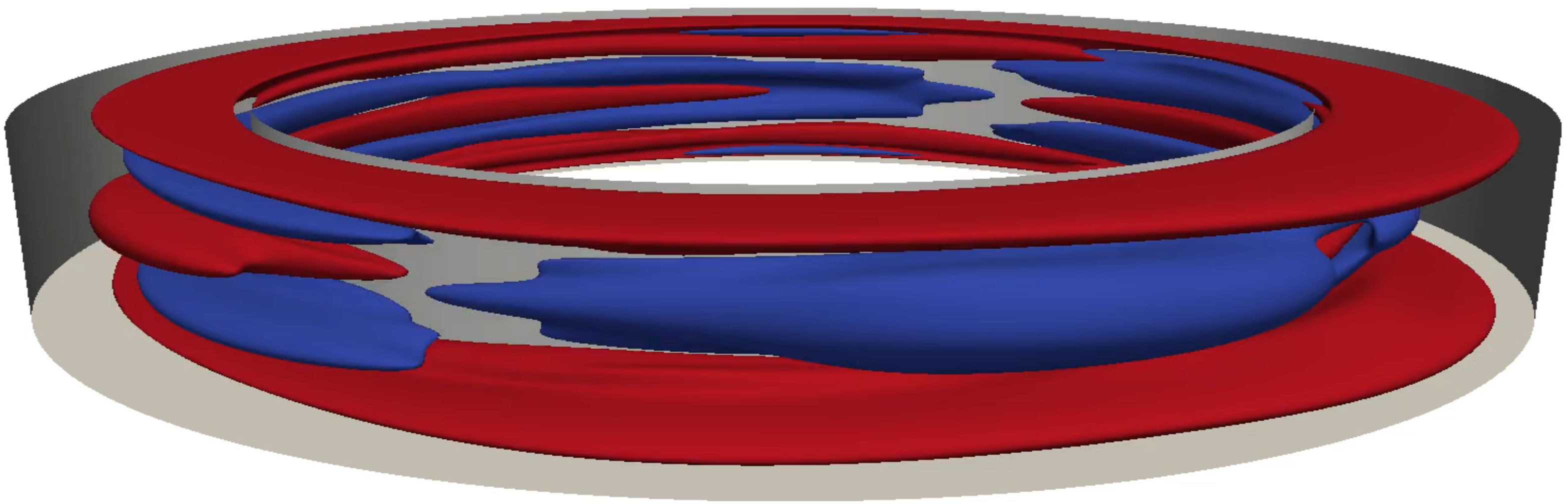
Re_o = -700 Re_i = 710

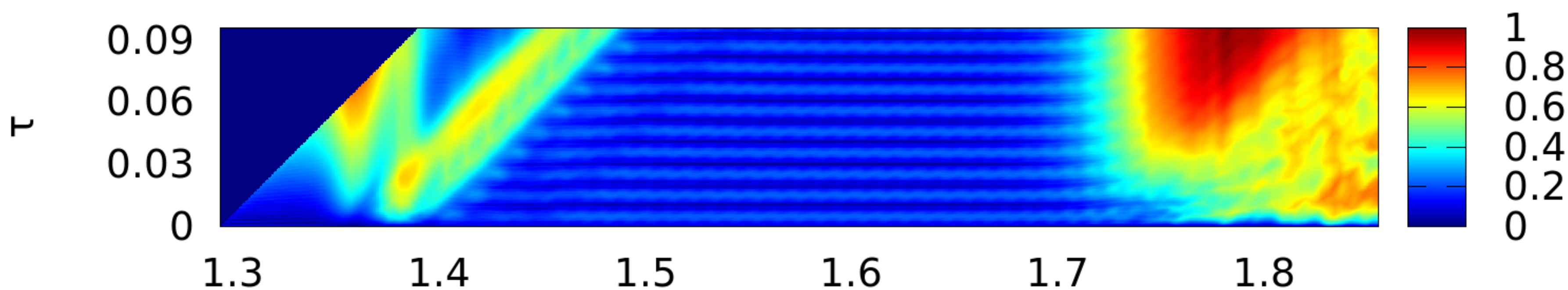
Γ=1 TCF (Simulation)





Γ=1 TCF (ECS)





Summary and Future Directions

- Experimental evidence for dynamical relevance of invariant solutions in Q2D Kolmogorov flow.
- To Do: Identify sufficient number of ECS/dynamical connections to enable extended prediction of dynamics in experiment and simulation.
- To Do: Explore ECS/dynamical connections in small-aspect-ratio (Γ=1) Taylor-Couette flow.

Acknowledgements

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+ Supported by:
ARO (W911NF-15-1-047, W911NF-16-1-0281),
DARPA (HR0011-16-2-0033),
NSF (CMMI-1234436, CBET-0853691, CBET-0900018, DMS-1125302)

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