Universal MHD Turbulence and Small-Scale Dynamo

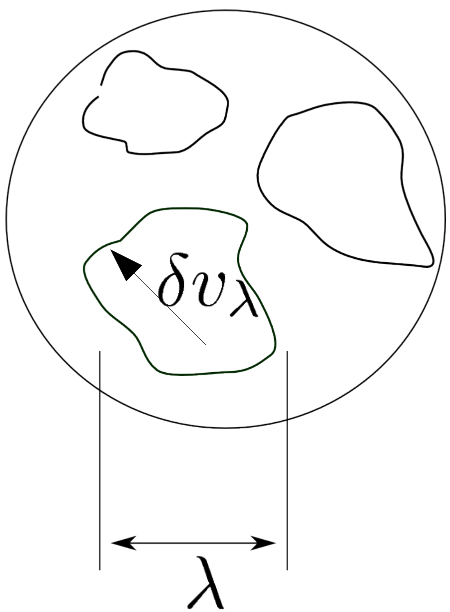
Andrey Beresnyak
Los Alamos Fellow
Theoretical Division

KITP seminar, 2011

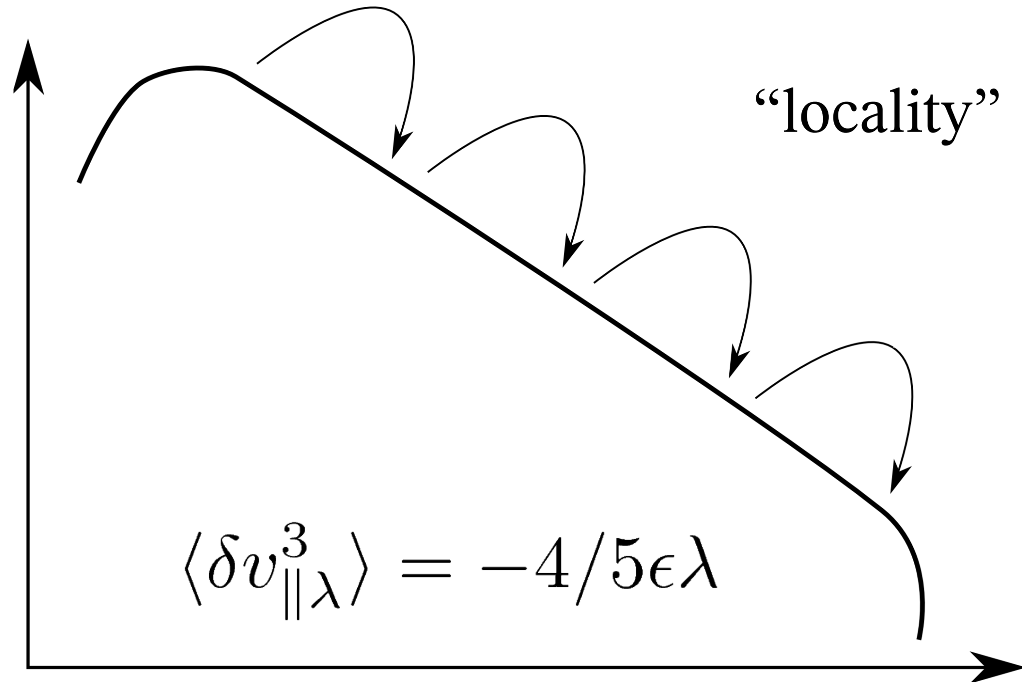
The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers†

By A. N. Kolmogorov

$$\partial_t \mathbf{v} + \hat{S}(\mathbf{v} \cdot \nabla)\mathbf{v} = 0$$



“locality”



“universality”

$$v \to vA, \quad \lambda \to \lambda B, \quad t \to tB/A$$

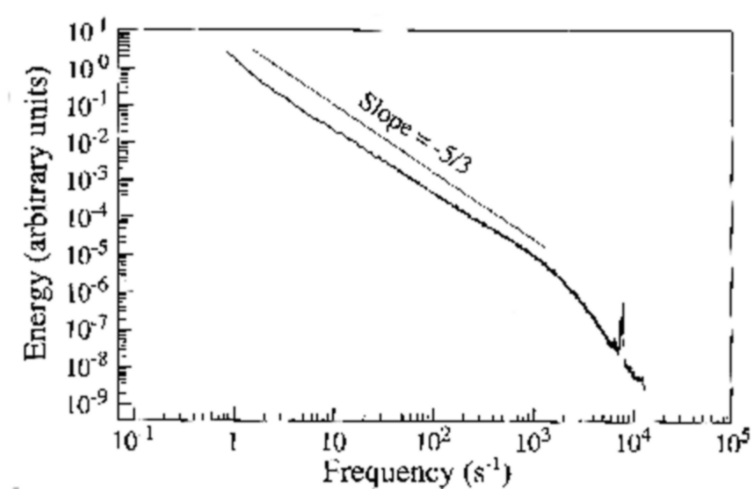
$$\tau_\lambda \sim \lambda / v$$

Universal hydrodynamic Turbulence

$$\epsilon = \frac{\rho v_\lambda^2}{2\tau_\lambda} \sim v_\lambda^3/\lambda \sim \mathrm{const}$$

$$v_\lambda \sim \lambda^{1/3}$$

$$E(k) = C_K k^{-5/3} \epsilon^{2/3}$$



ONERA wind tunnel turbulence

$$C_K \approx 1.64$$

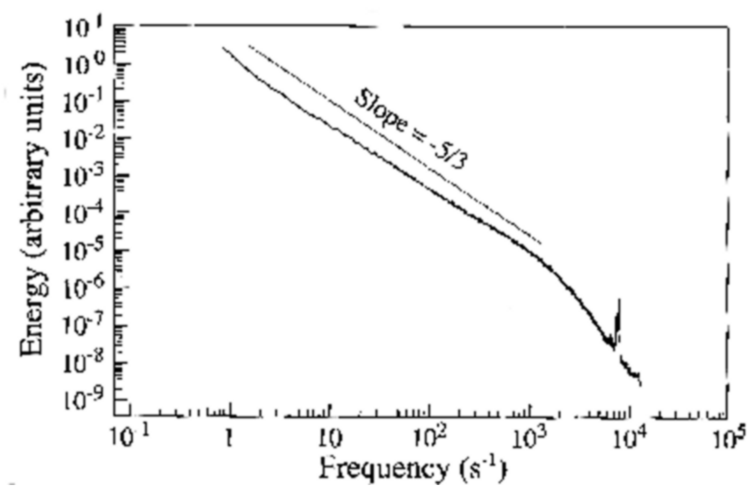
Universal hydrodynamic Turbulence

$$\epsilon = \frac{\rho v_\lambda^2}{2\tau_\lambda} \sim v_\lambda^3/\lambda \sim \mathrm{const}$$

$$v_\lambda \sim \lambda^{1/3}$$

$$E(k) = C_K k^{-5/3} \epsilon^{2/3} (kL)^\alpha$$

$$\alpha \approx -0.035$$



$$C_K \approx 1.64$$

ONERA wind tunnel turbulence

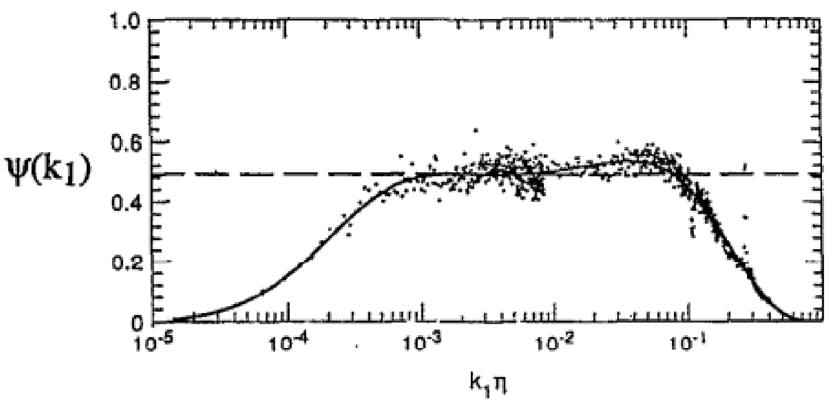


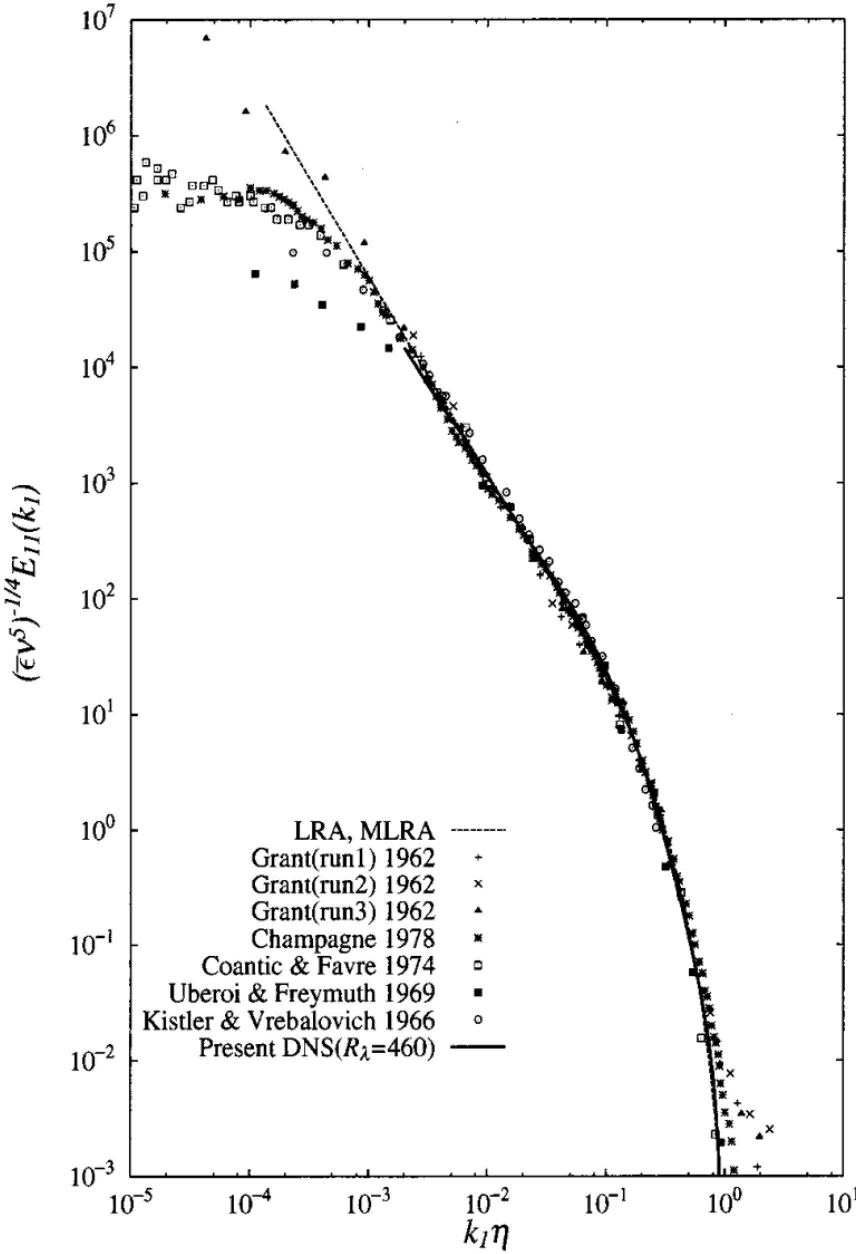
TABLE III. Sources and Reynolds numbers for other laboratory shear flows. These sources are not necessarily exhaustive.

Flow	Source	Position	R_λ	C_κ
Cylinder wake	Champagne[48]	Centerline	138	0.55
	Kailasnath and Sreenivasan[43]	Centerline	130	0.51
	Uberoi and Freymuth[49]	Centerline	93–308	0.45
Wake of a sphere	Gibson *et al.*[50]	Axis	224	0.50
	Uberoi and Freymuth[51]	Axis	36–258	0.50
Mixing layer	Champagne *et al.*[52]	$y/x=-0.015$	330	0.46
	Praskovsky and Oncley[7]		2000	0.62
Round jet	Champagne[48]	Centerline	626	0.48
	Gibson[5]	Centerline	780	0.51
		off-center	710	0.53
Plane jet	Bradbury[53]	$y/\delta=0.5$[a]	350	0.50
Homogeneous shear flow	Champagne *et al.*[54]	$x/h=10.5$[b]	130	0.52
Return channel of wind tunnel	Praskovsky and Oncley[7]		3200	0.58

[a] δ=half-width of the jet.
[b] h=height of the wind-tunnel section in which the shear flow was created.

Katepalli R. Sreenivasan

Sreenivasan 1995



Gotoh et al 2002, see also Kaneda et al 2003

MHD Turbulence

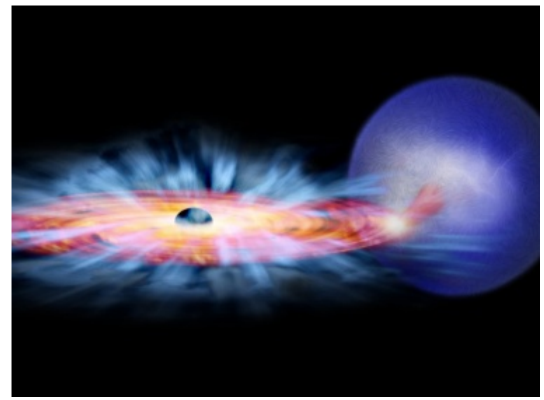
Plasma, cosmic rays and magnetic fields

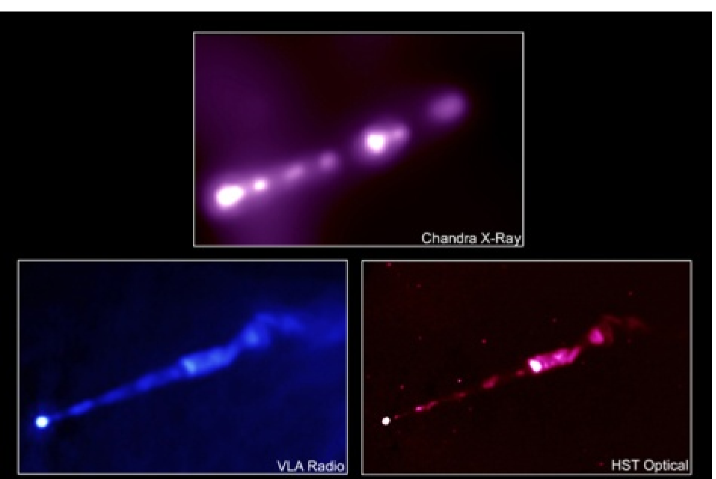
Energy density in our Galaxy

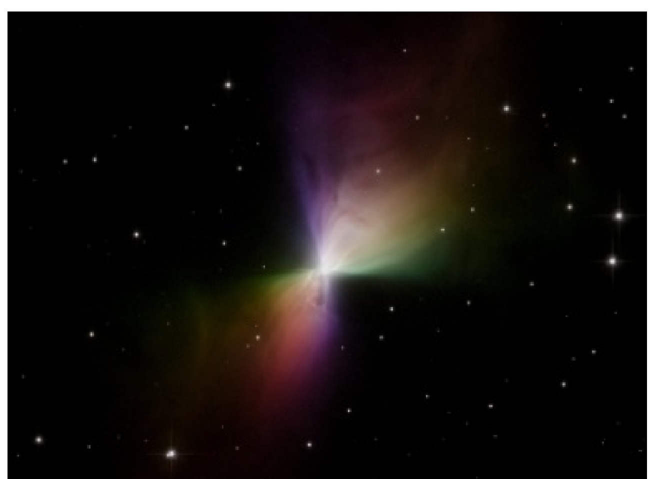
$$w_B \sim w_{CR} \sim w_{\mathrm{kin.}} \sim 1\mathrm{eV}/\mathrm{cm}^3$$

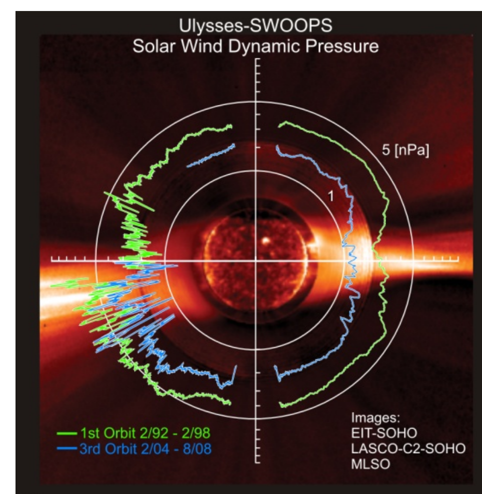
Sources of energy for B and CRs:

1. magneto-rotational instability
2. AGN jets
3. Stellar winds
4. SN explosions









Basic properties of MHD turbulence

$$\partial_t \mathbf{w}^\pm + \hat{S}(\mathbf{w}^\mp \cdot \nabla)\mathbf{w}^\pm = 0$$

Elsasser variables: $\mathbf{w}^\pm = \mathbf{v} \pm \mathbf{B}/\sqrt{4\pi\rho}$ *Solenoidal projection:* $\hat{S}$

$$\langle \delta w^\pm_{\parallel\lambda} (\delta w^\mp_\lambda)^2 \rangle = -4/3\epsilon^\mp \lambda$$

(locality, but there is a caveat, explained later)

Basic properties of MHD turbulence

$$\partial_t \mathbf{w}^{\pm} + \hat{S}(\mathbf{w}^{\mp} \cdot \nabla)\mathbf{w}^{\pm} = 0$$

Elsasser variables: $\mathbf{w}^{\pm} = \mathbf{v} \pm \mathbf{B}/\sqrt{4\pi\rho}$ *Solenoidal projection:* $\hat{S}$

Dynamics is different from hydro, because there is a mean field.

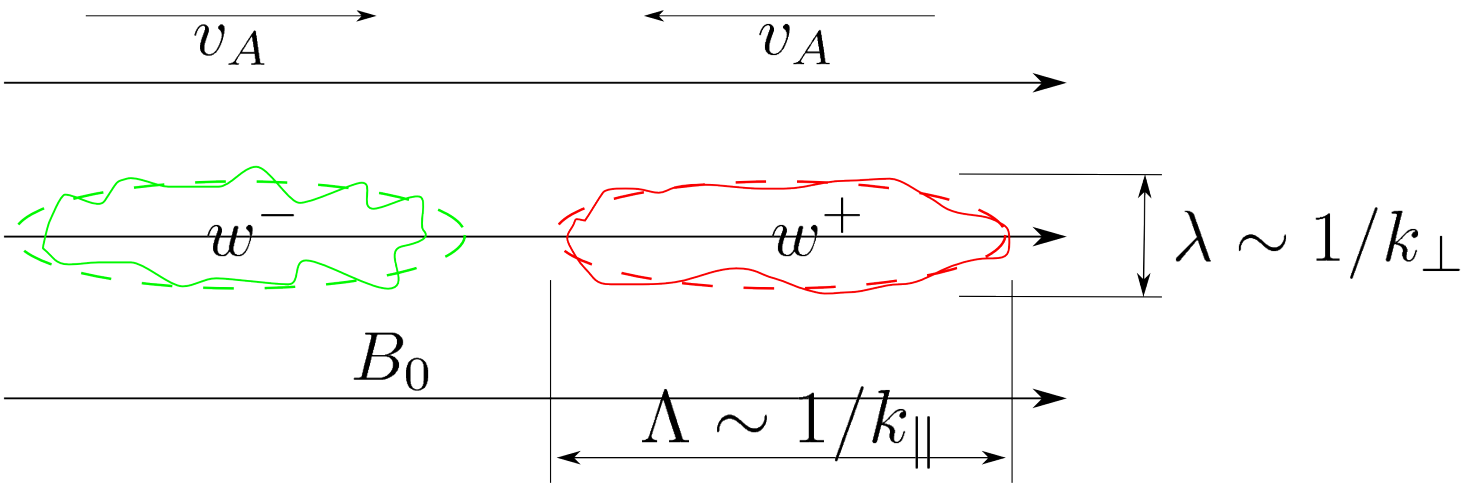
$$\partial_t \delta\mathbf{w}^{\pm} \mp (\mathbf{v_A} \cdot \nabla)\delta\mathbf{w}^{\pm} + \hat{S}(\delta\mathbf{w}^{\mp} \cdot \nabla)\delta\mathbf{w}^{\pm} = 0$$

↑

Mean field (Kraichnan 1965, Iroshnikov 1963)

If universality exists, it is different from hydro.

Basic properties of MHD turbulence

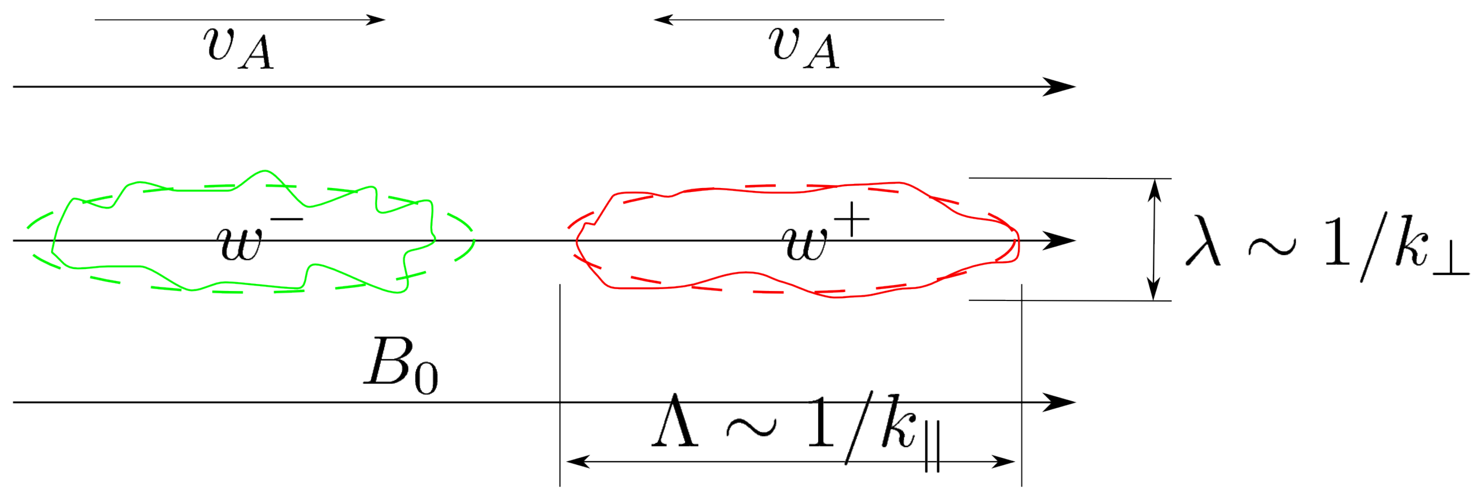


$$\mathbf{k_1} + \mathbf{k_2} = \mathbf{k_3}$$

$$\omega_1 + \omega_2 = \omega_3$$

Some hints from weak interaction case.

Basic properties of MHD turbulence

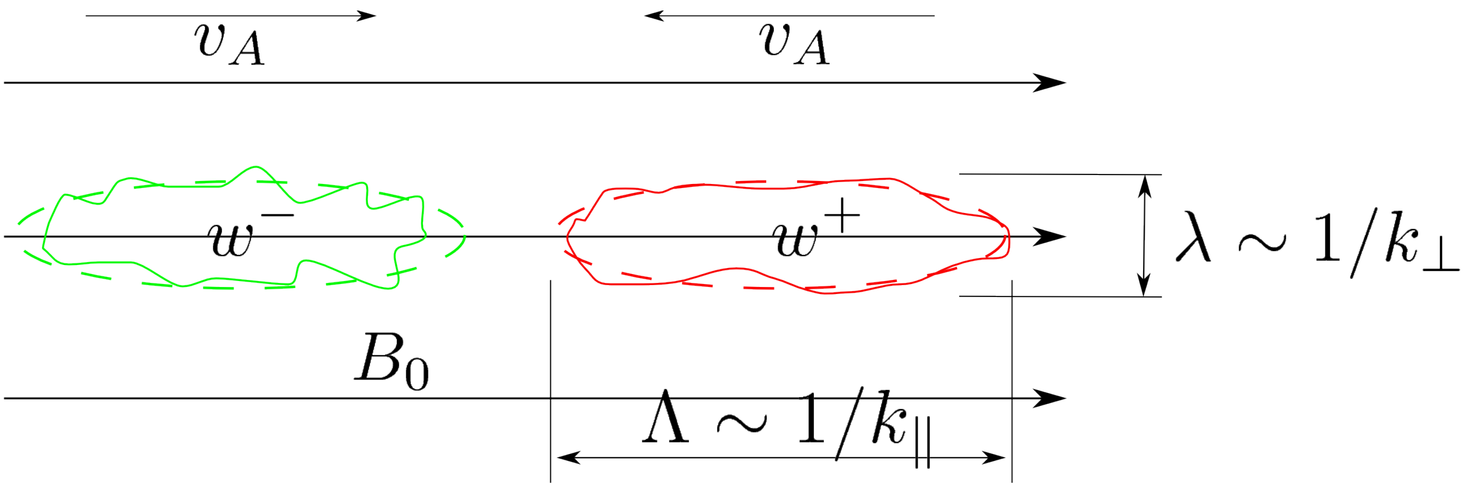


$$\mathbf{k}_{\perp\mathbf{1}} + \mathbf{k}_{\perp\mathbf{2}} = \mathbf{k}_{\perp\mathbf{3}}$$

$$k_{\parallel 1} + k_{\parallel 2} = k_{\parallel 3}$$

$$\omega_1 + \omega_2 = \omega_3$$

Basic properties of MHD turbulence

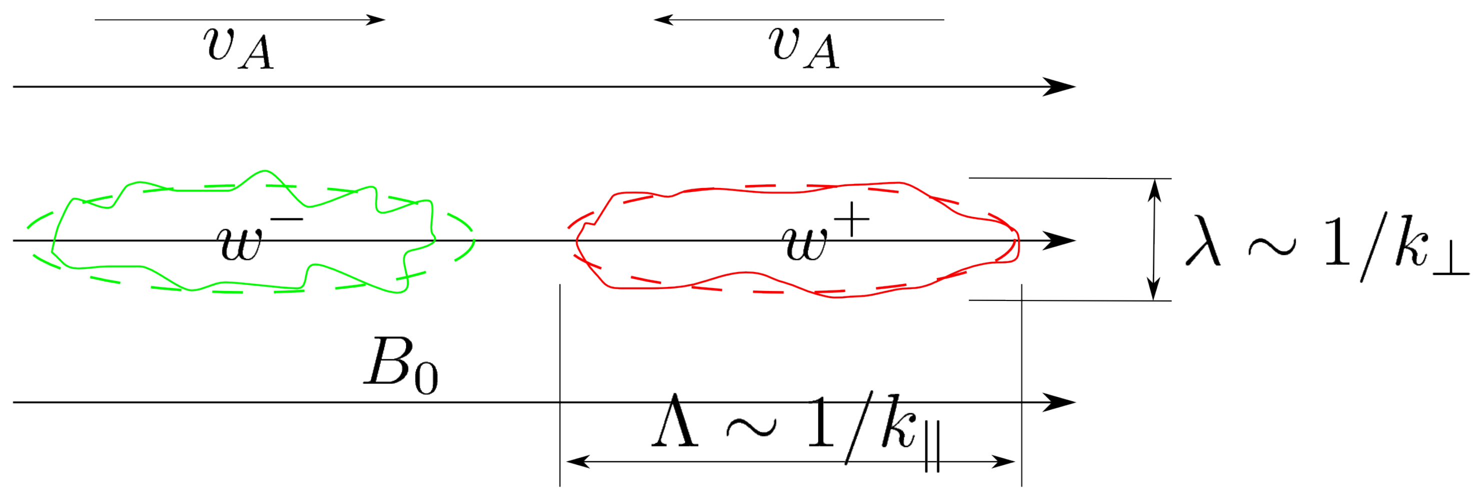


$$\mathbf{k}_{\perp\mathbf{1}} + \mathbf{k}_{\perp\mathbf{2}} = \mathbf{k}_{\perp\mathbf{3}}$$

$$\pm\omega_1 \pm \omega_2 = \pm\omega_3$$

$$\omega_1 + \omega_2 = \omega_3$$

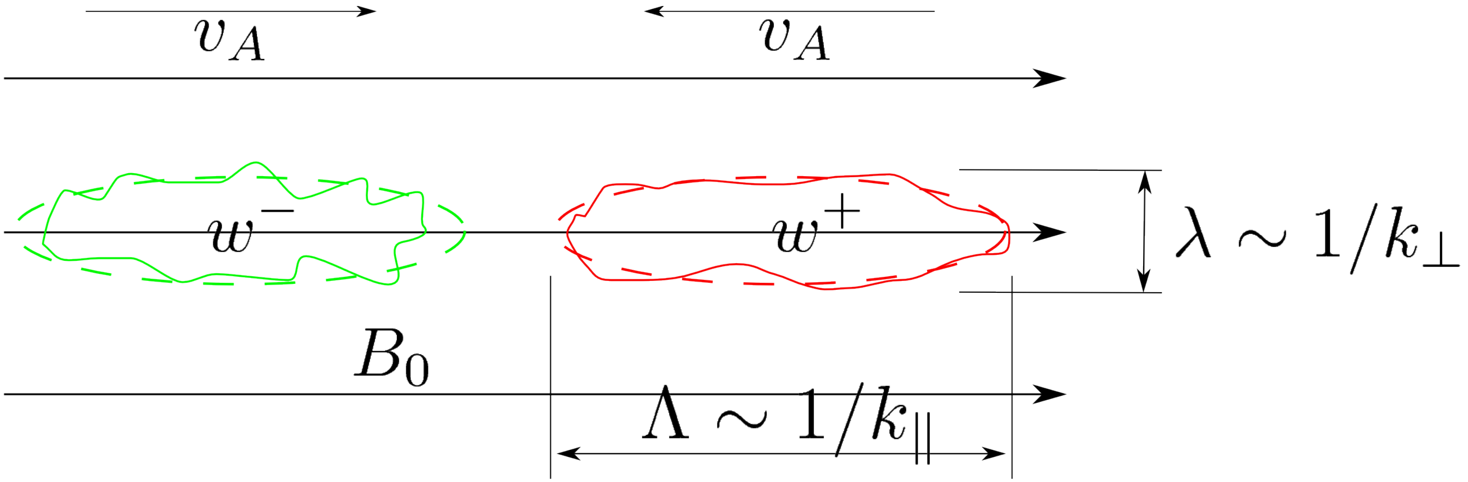
Basic properties of MHD turbulence



$$\mathbf{k}_{\perp \mathbf{1}} + \mathbf{k}_{\perp \mathbf{2}} = \mathbf{k}_{\perp \mathbf{3}}$$

$$\omega_1 = -\omega_2;\ \omega_3 = 0$$

Basic properties of MHD turbulence

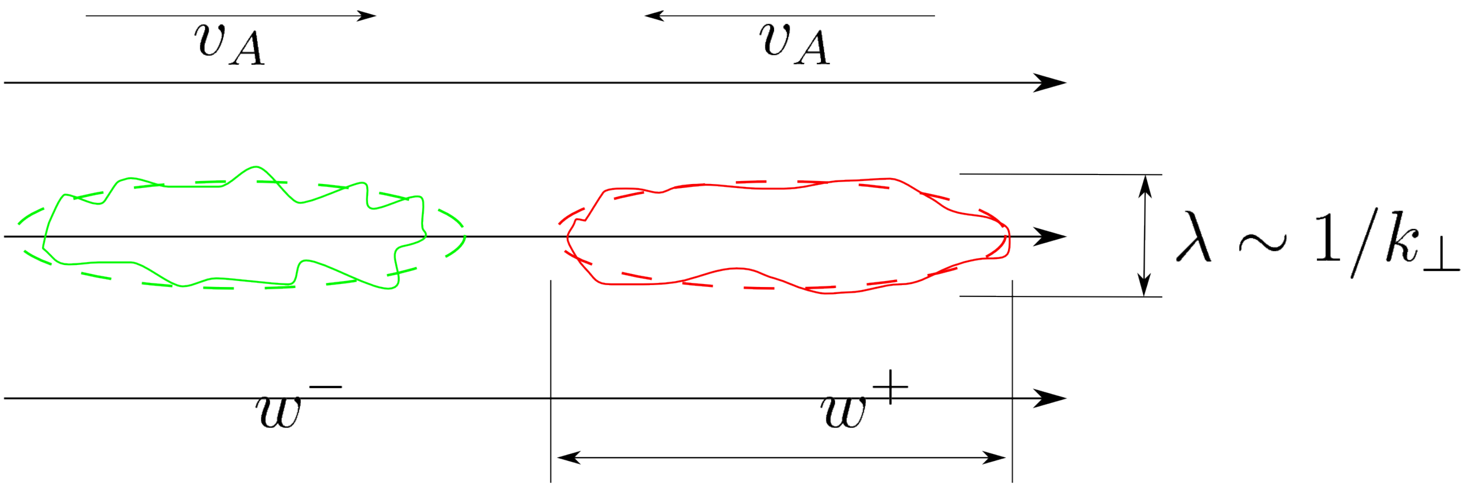


$$\mathbf{k}_{\perp\mathbf{1}} + \mathbf{k}_{\perp\mathbf{2}} = \mathbf{k}_{\perp\mathbf{3}}$$

$$\omega_1 = -\omega_2;\ \omega_3 = 0$$

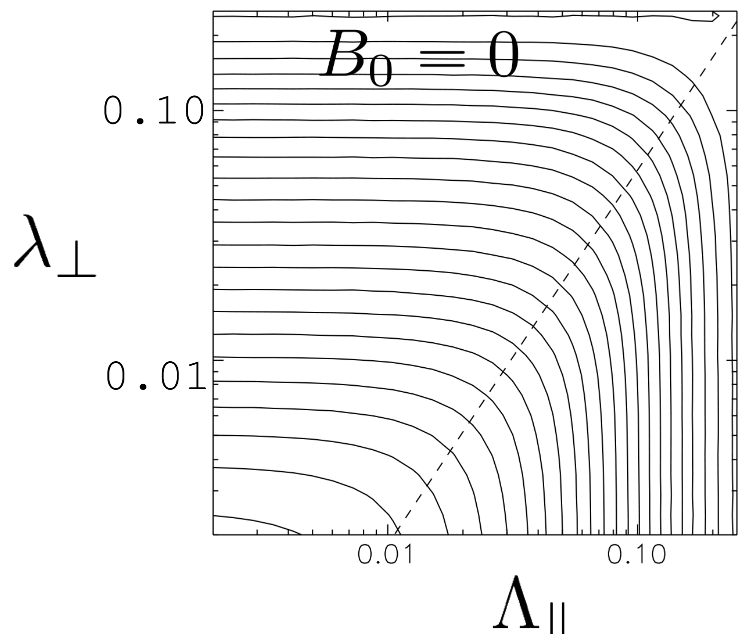
$k_{||}$ is conserved, $k_\perp$ is increasing

Basic properties of MHD turbulence

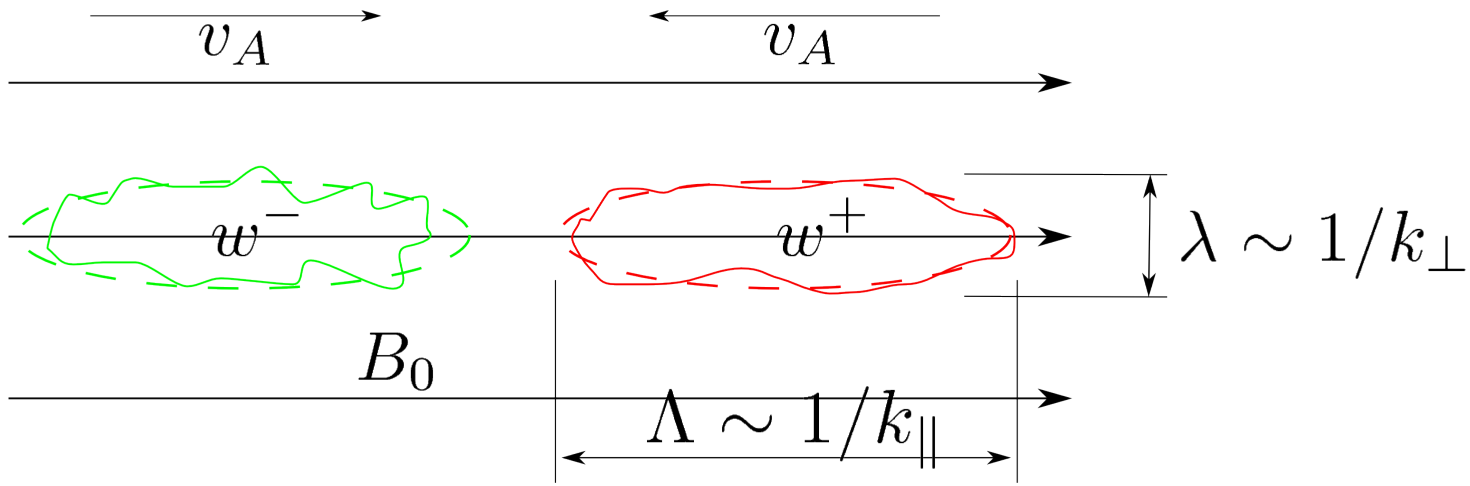


$k_{||}$ is conserved, $k_\perp$ is increasing $\quad k_{||} \ll k_\perp$

Isotropically driven MHD turbulence



Basic properties of MHD turbulence



$k_\parallel$ is conserved, $k_\perp$ is increasing

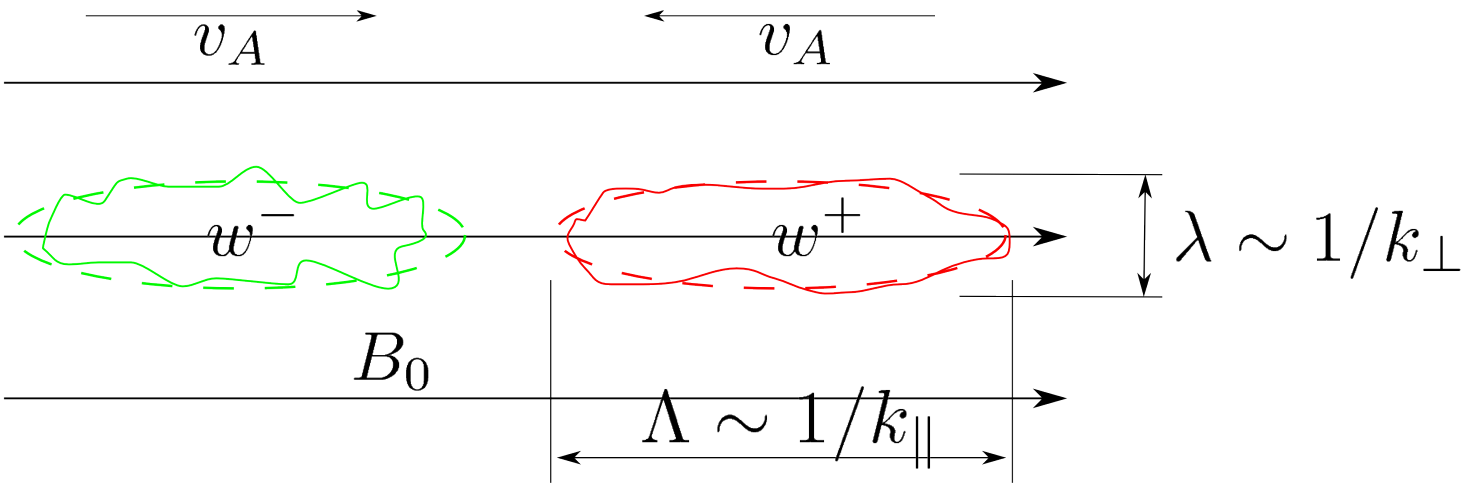
$$k_\parallel \ll k_\perp$$

$$\partial_t \delta\mathbf{w}^\pm \mp (\mathbf{v_A} \cdot \nabla)\delta\mathbf{w}^\pm + \hat{S}(\delta\mathbf{w}^\mp \cdot \nabla)\delta\mathbf{w}^\pm = 0$$

$\rightarrow$ ~~$k_\parallel$~~, $k_\perp$

could be split in two equations

Basic properties of MHD turbulence



$k_\parallel \ll k_\perp$

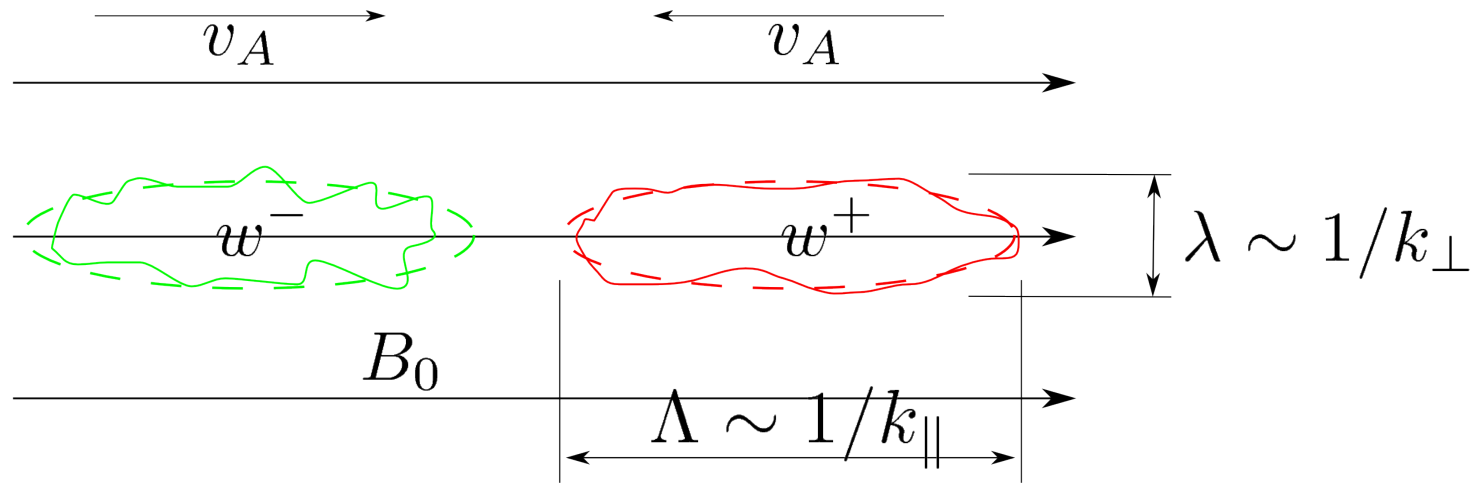
Alfvenic dynamics (a.k.a. "reduced MHD") has essential nonlinearity:

$$\partial_t \mathbf{w}_\perp^\pm \mp (\mathbf{v_A} \cdot \nabla_\parallel)\mathbf{w}_\perp^\pm + \hat{S}(\mathbf{w}_\perp^\mp \cdot \nabla_\perp)\mathbf{w}_\perp^\pm = 0$$

Slow mode is passively mixed:

$$\partial_t w_\parallel^\pm \mp (\mathbf{v_A} \cdot \nabla_\parallel) w_\parallel^\pm + \hat{S}(\mathbf{w}_\perp^\mp \cdot \nabla_\perp) w_\parallel^\pm = 0$$

Basic properties of MHD turbulence



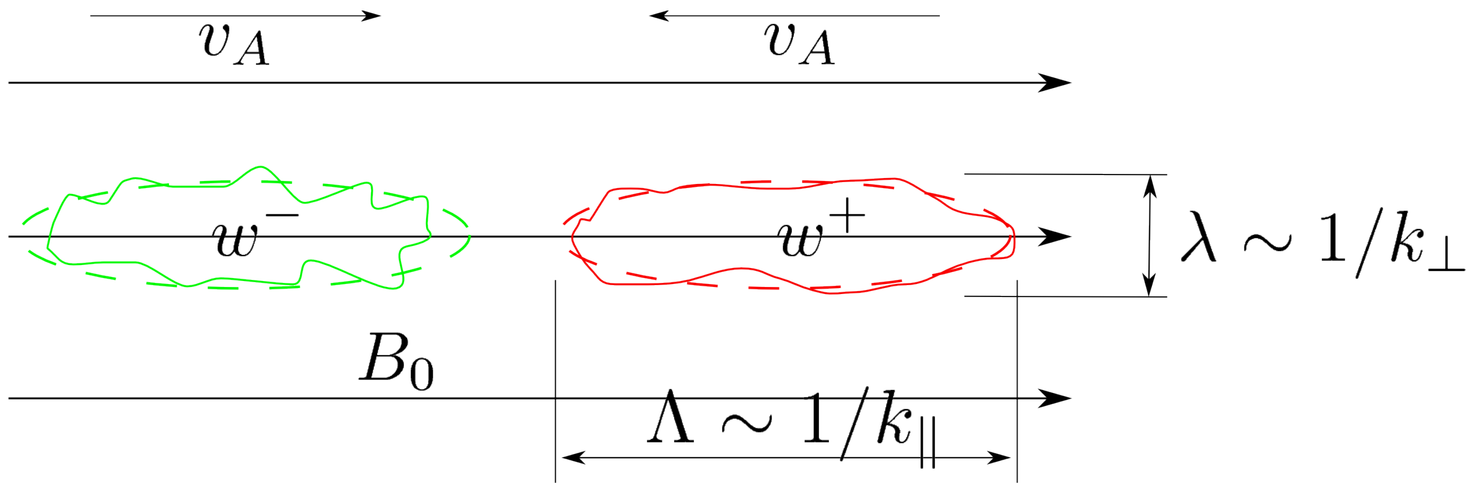
$$k_{||} \ll k_\perp$$

$$\partial_t \mathbf{w}_\perp^\pm \mp (\mathbf{v_A} \cdot \nabla_{||})\mathbf{w}_\perp^\pm + \hat{S}(\mathbf{w}_\perp^\mp \cdot \nabla_\perp)\mathbf{w}_\perp^\pm = 0$$

A new universality is possible:

$$w \to wA, \quad \lambda \to \lambda B, \quad t \to tB/A, \quad \Lambda \to \Lambda B/A$$

Basic properties of MHD turbulence



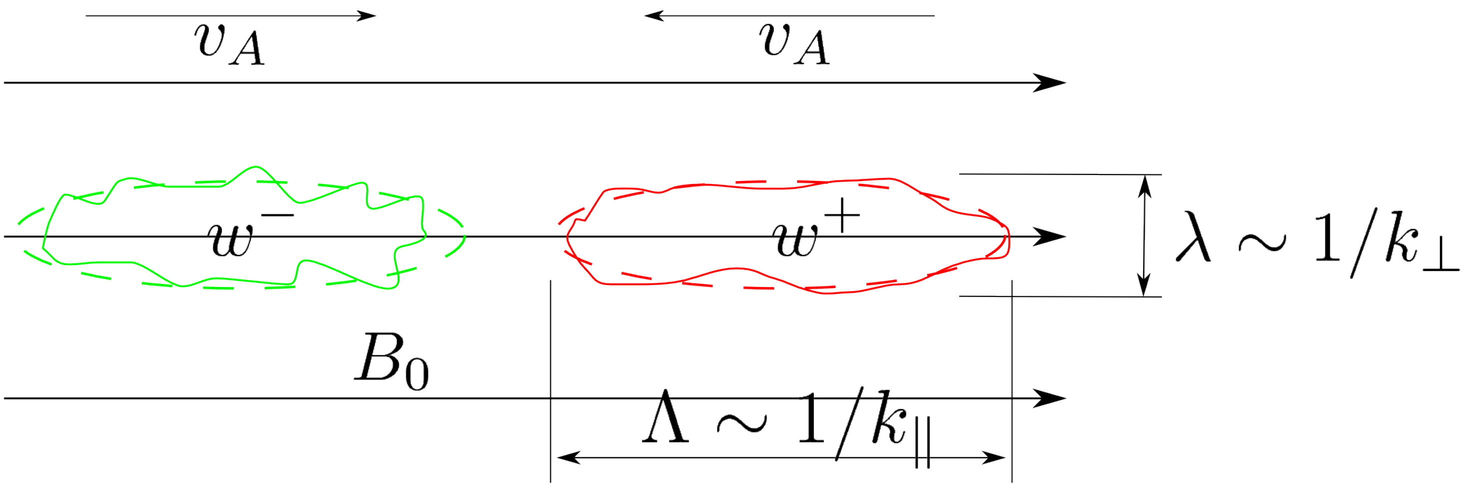
$$k_{||} \ll k_\perp$$

$$\partial_t \mathbf{w}_\perp^\pm \mp (\mathbf{v_A} \cdot \nabla_{||})\mathbf{w}_\perp^\pm + \hat{S}(\mathbf{w}_\perp^\mp \cdot \nabla_\perp)\mathbf{w}_\perp^\pm = 0$$

Contribution of nonlinear term has a tendency to increase, thus leading to "strong turbulence", despite a strong mean field, i.e. $v_A >> w$.

$$v_A k_{||} / \delta w k_\perp \sim 1$$

Basic properties of MHD turbulence



Goldreich-Sridhar (1995) model:

critical balance, an uncertainty relation $\omega \tau_{\rm cas} \sim 1$

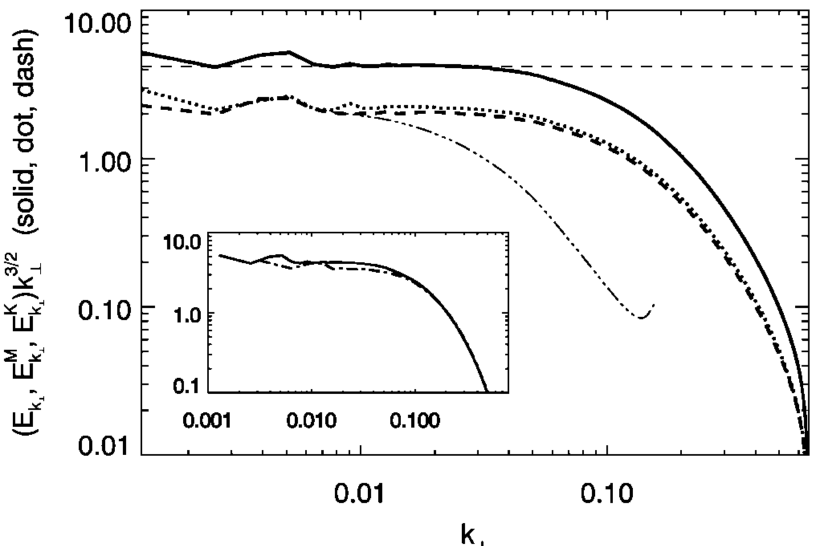
Strong cascading, -5/3 spectra:

$$\epsilon^+ = \frac{(w^+_\lambda)^2 w^-_\lambda}{\lambda}; \qquad \epsilon^- = \frac{(w^-_\lambda)^2 w^+_\lambda}{\lambda}.$$

Energy spectral slopes: -5/3 or -3/2?

Goldreich-Sridhar model predicts -5/3 but shallower slopes are often observed in simulations.

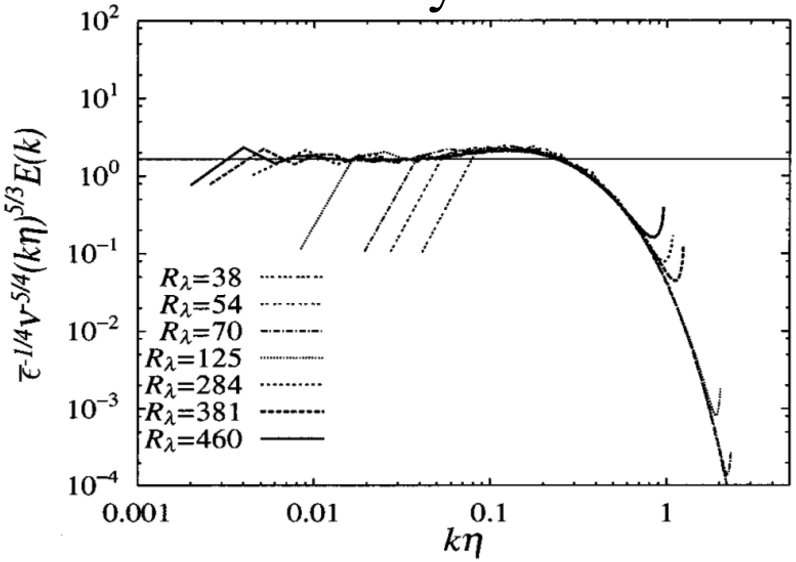
MHD, strong mean field



Muller & Grappin 2005

(same paper claims -5/3 without bottleneck for B0=0 case)

Hydro

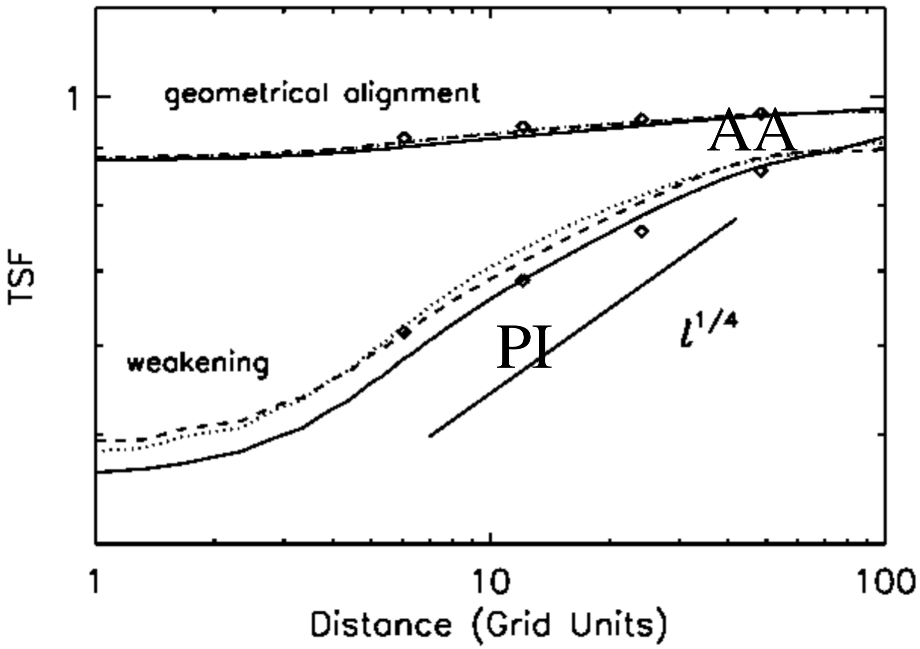


Gotoh et al 2002

Dynamic alignment (Boldyrev, 2005)

Boldyrev (2005) proposed "dynamic alignment" which will weaken the interaction and produce -3/2 slope.

Beresnyak & Lazarian (2005):



$$\mathrm{AA} = \langle |\sin\theta| \rangle$$

$$\mathrm{PI} = \langle |\delta\mathbf{w}^+ \times \delta\mathbf{w}^-| \rangle / \langle |\delta w^+ \delta w^-| \rangle$$

$$\mathrm{DA} = \langle |\delta\mathbf{v} \times \delta\mathbf{b}| \rangle / \langle |\delta v \delta b| \rangle$$

note that $\delta\mathbf{w}^+ \times \delta\mathbf{w}^- = -2\delta\mathbf{v} \times \delta\mathbf{b}$

Mason, Cattaneo & Boldyrev (2006) measured DA, similar to PI, and claimed precise correspondence with Boldyrev model and a new universal -3/2 spectral slope for MHD.

What is the physics behind alignment?

Boldyrev (2006) proposed that alignment is dynamically created on each scale and is limited by the field wandering. This gives alignment proportional to the amplitude.

But this directly contradicts the above-mentioned precise symmetry

$$w \to wA, \quad \lambda \to \lambda B, \quad t \to tB/A, \quad \Lambda \to \Lambda B/A$$

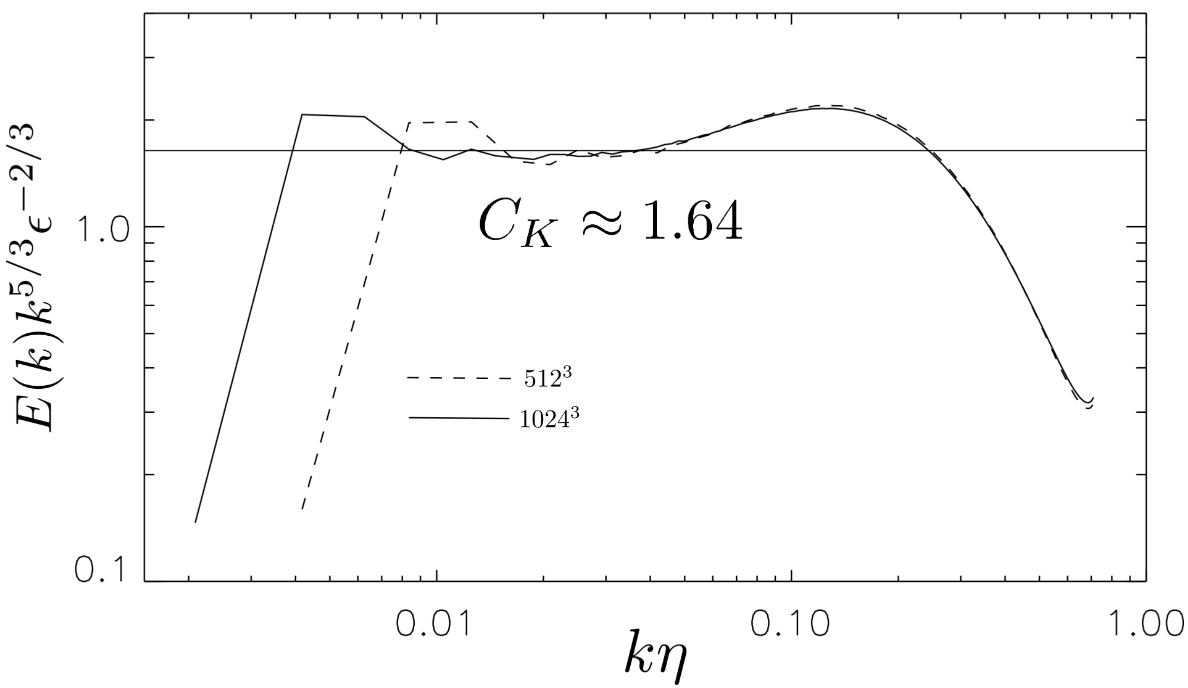
-3/2 slope is not well justified

But why alignment is scale-dependent?

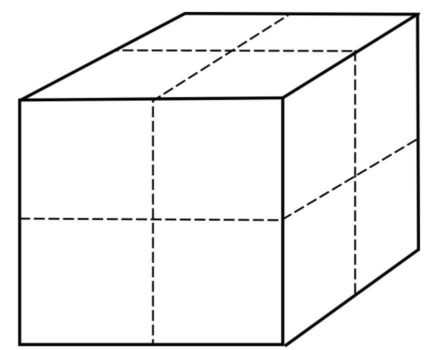
Resolution study

Universality helps us to understand both hydro and MHD

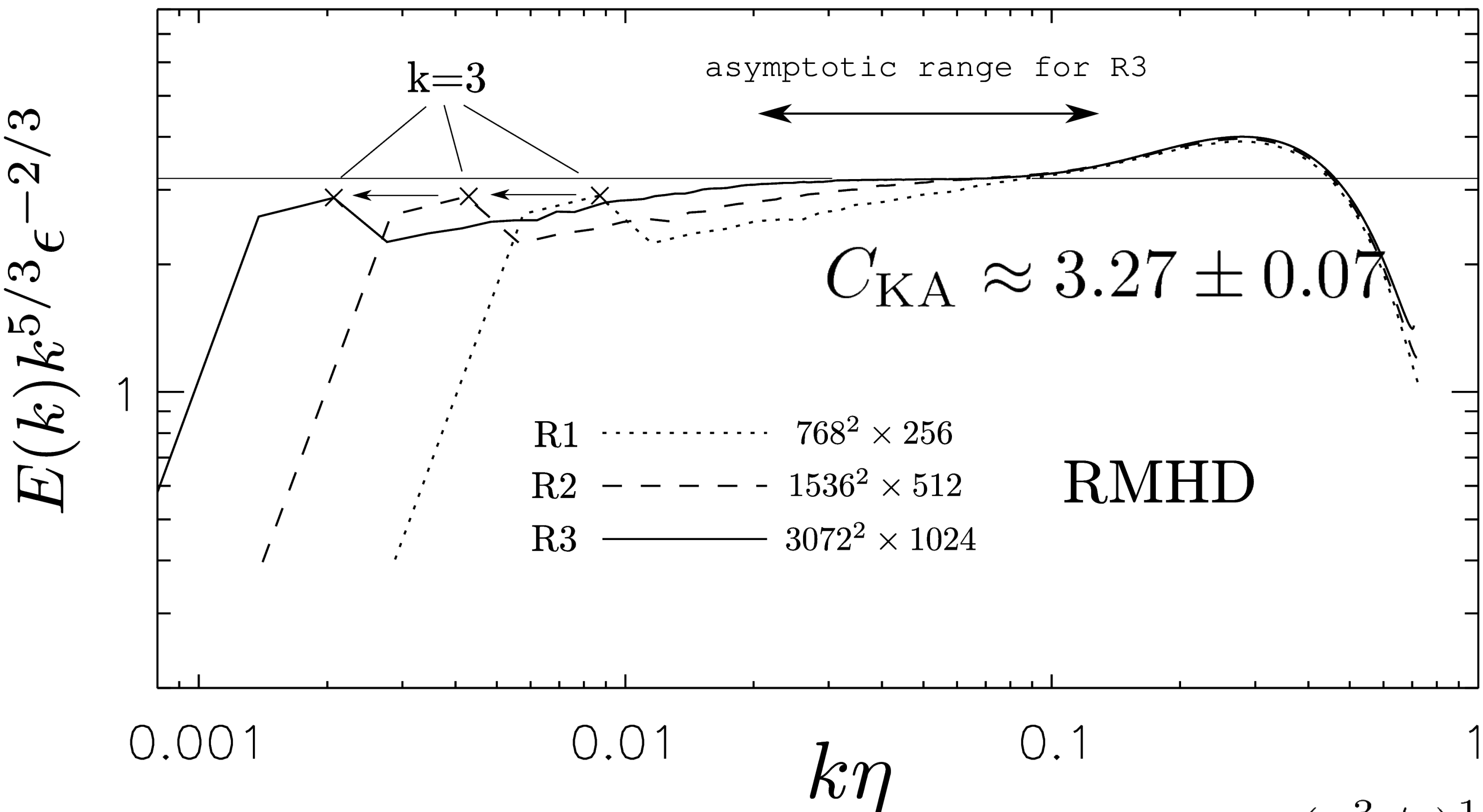
Hydro:



Larger resolution could mean larger scales



Resolution study (Alfvenic turbulence)



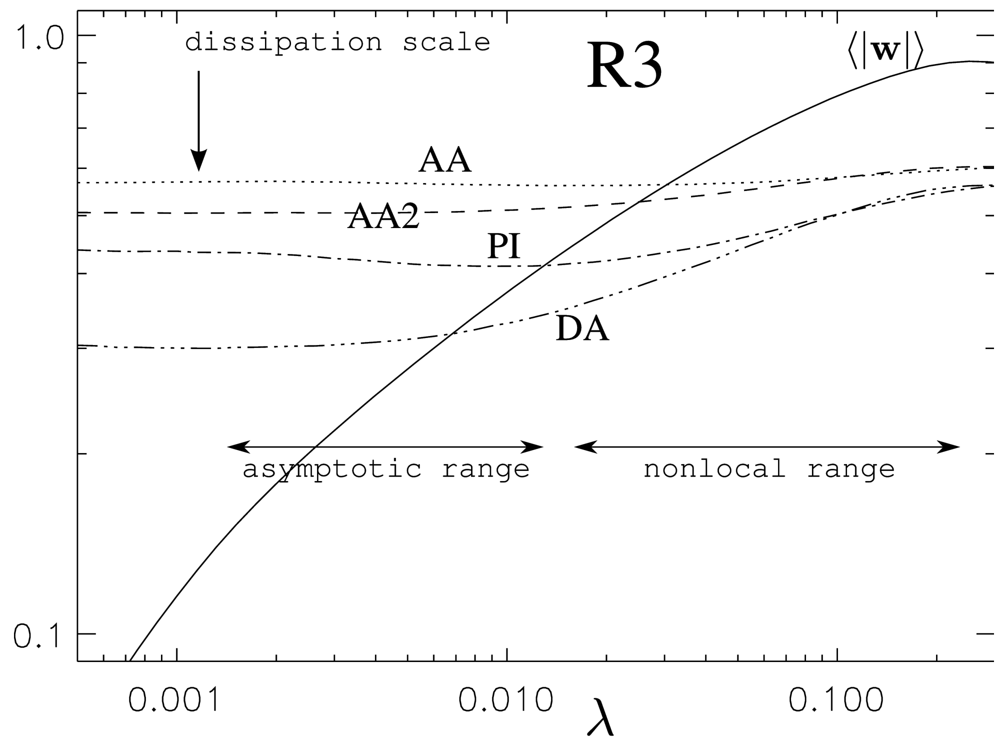
$$\eta = (\nu_n^3/\epsilon)^{1/(3n-2)}$$

Due to the presence of the slow mode (1-1.3 energy of the Alfvenic mode), the full Kolmogorov constant will be:

$$C_K(\text{MHD}) \approx 4.2 \pm 0.2$$

Alignment effects are limited to nonlocal range, and do not modify the -5/3 slope of MHD turbulence

$$3072^2 \times 1024$$

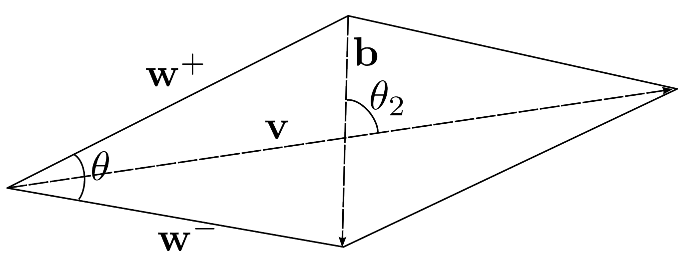


$$\mathrm{AA} = \langle |\sin\theta| \rangle$$

$$\mathrm{AA2} = \langle |\sin\theta_2| \rangle$$

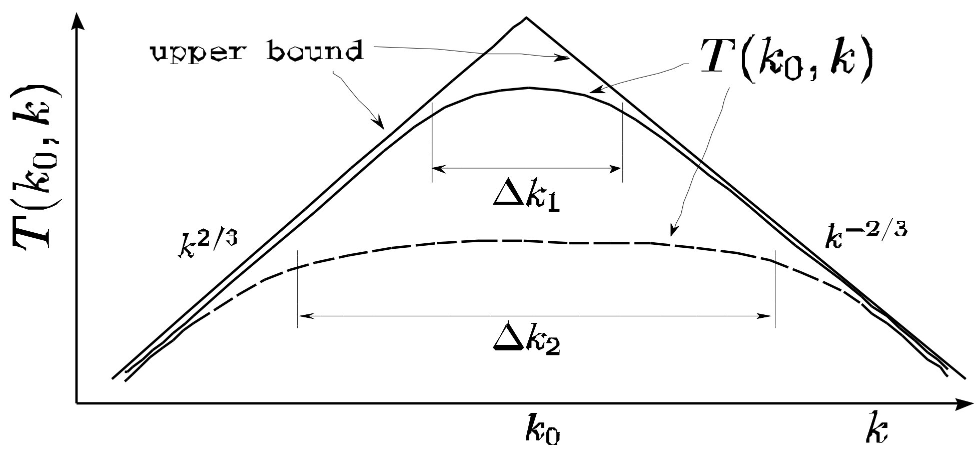
$$\mathrm{PI} = \langle |\delta w^+ \delta w^- \sin\theta| \rangle / \langle |\delta w^+ \delta w^-| \rangle$$

$$\mathrm{DA} = \langle |\delta v \delta b \sin\theta_2| \rangle / \langle |\delta v \delta b| \rangle$$



Resolution study shows convergence to 3072 data.

Locality of energy transfer

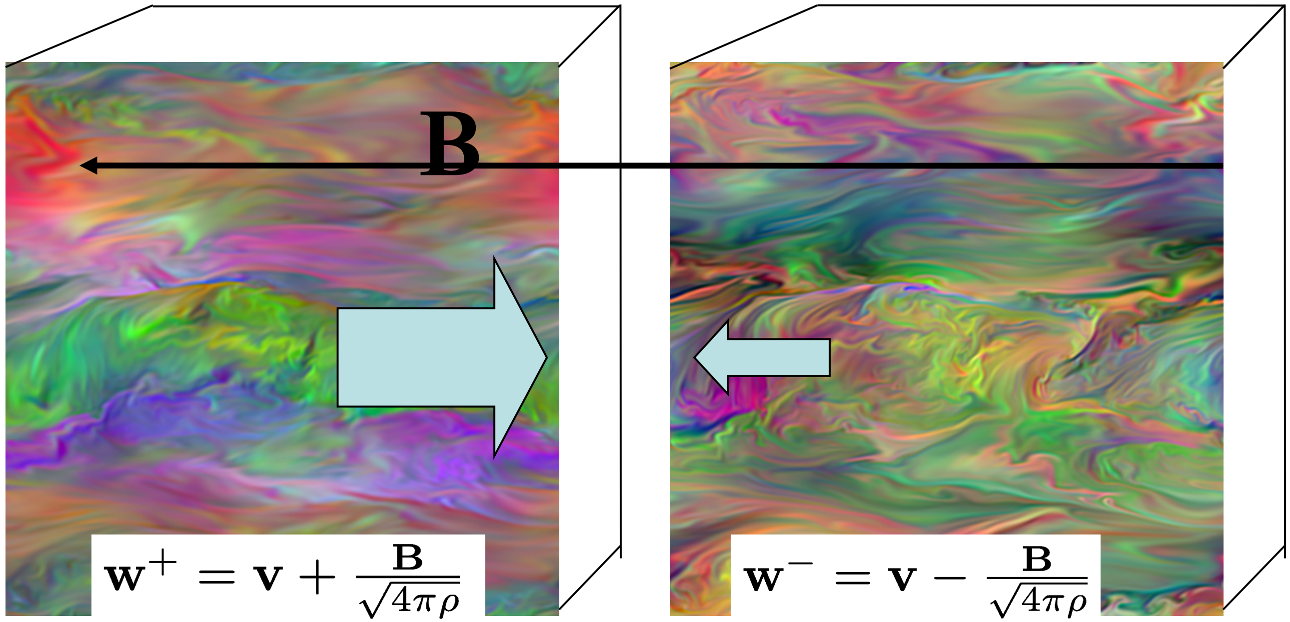


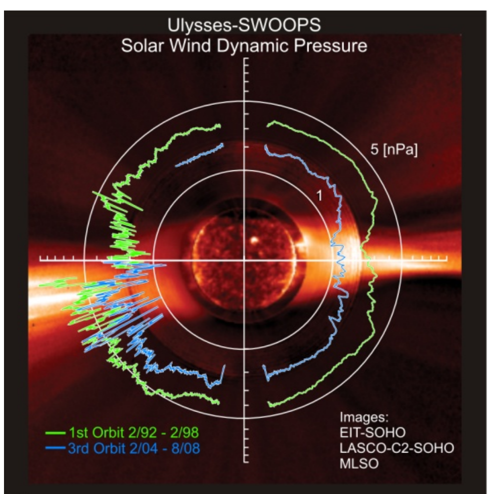
Diffuse locality of MHD turbulence is consistent with high value of the Kolmogorov constant

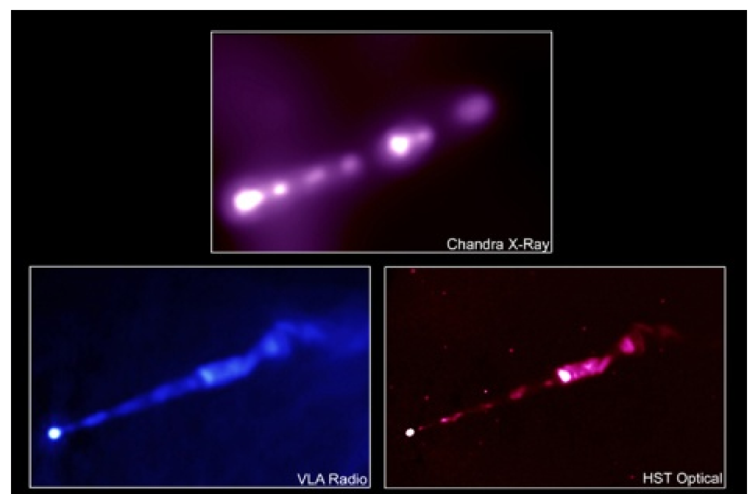
Summary for balanced turbulence

- MHD turbulence has a -5/3 spectrum and a Kolmogorov constant which is much higher than hydrodynamic constant, i.e., in MHD turbulence the energy transfer is much less efficient.
- This has implications for turbulence decay times and turbulence heating rates. E.g., the turbulent heating rate calculated using the measured energy spectrum and hydrodynamic value of the constant will be off by a factor of (4.1/1.6)^1.5 ~ 4.
- More details in Phys. Rev. Lett. 106, 075001

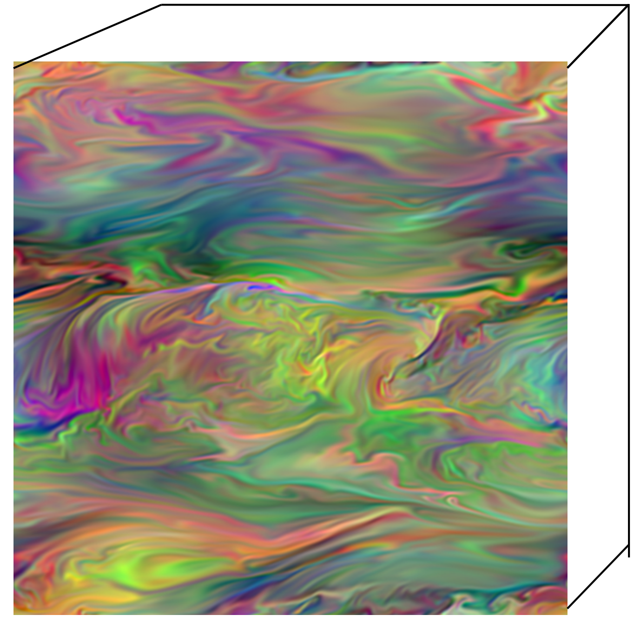
Imbalanced turbulence







Numerical simulations



Pseudospectral code solves MHD/hydro/RMHD equations on a grid, three-dimensional, stationary driven turbulence, explicit dissipation, up to 3072^2x1024 (balanced), 1536^3(imbalanced).

Notation

$$w^{\pm} = z^{\pm} = v \pm b \qquad \text{Elsasser variables}$$

w's – used in Goldreich's papers
z's – used in Biskamp book

$(w^{\pm})^2$ – Elsasser energy

$\tau^{\pm}$ – nonlinear timescale

$\epsilon^{\pm}$ – dissipation rate

Energy cascade

$\lambda^{\pm}$ – perpendicular scale

$\Lambda^{\pm}$ – parallel scale

Geometry

Basic Measurements

$(w^{\pm})^2 = E(1 \pm \sigma_C)/2$ – Elsasser energy

$\tau^{\pm}$ – nonlinear timescale

$\epsilon^{\pm}$ – dissipation rate

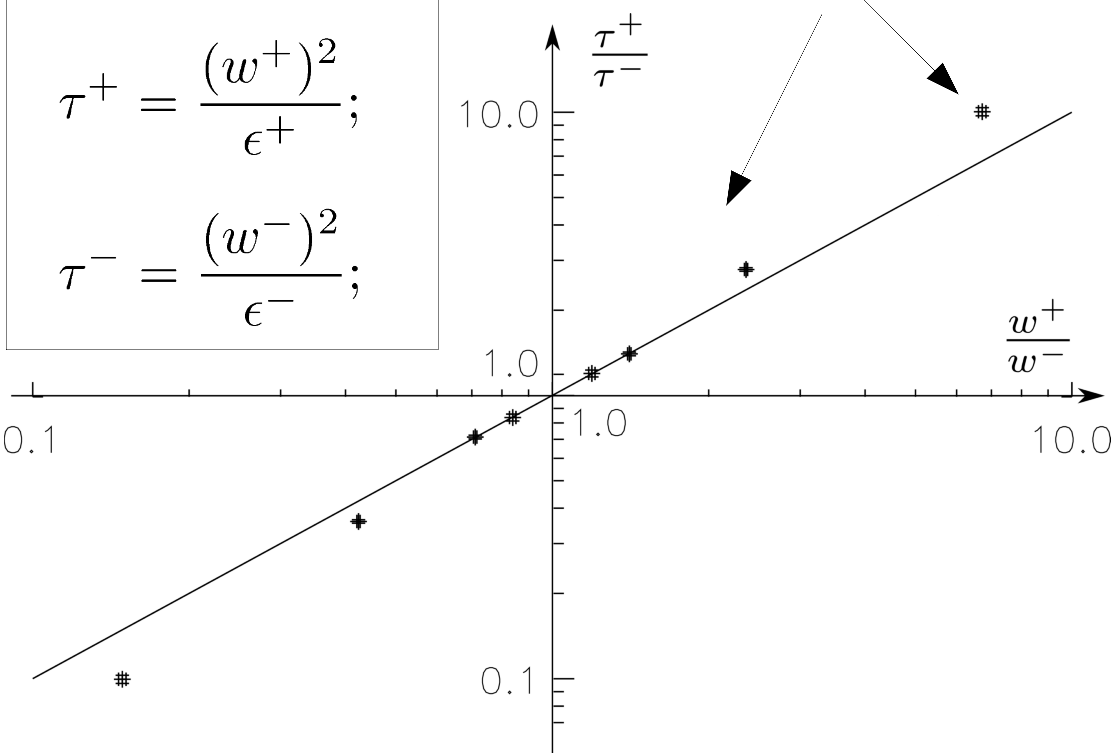
Energy cascade

obtained in simulations

$$\tau^{+} = \frac{(w^{+})^2}{\epsilon^{+}};$$

$$\tau^{-} = \frac{(w^{-})^2}{\epsilon^{-}};$$

Data from 1024x1536² simulations

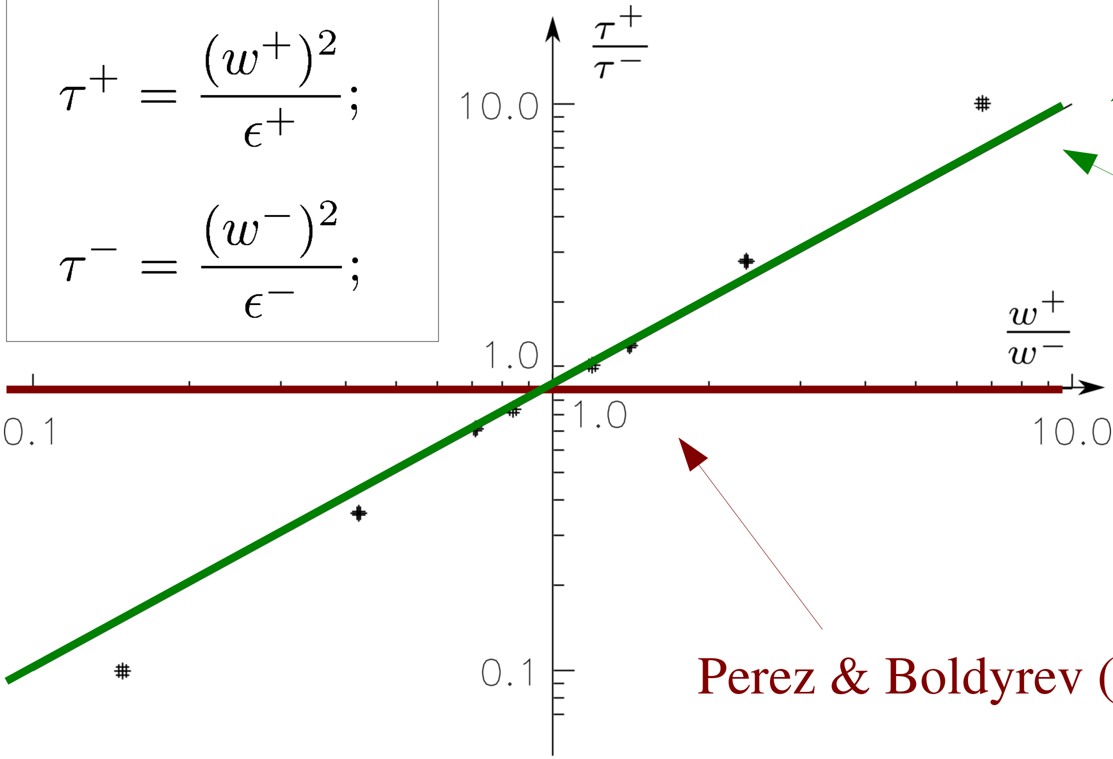


Energy cascade

obtained in simulations

$$\tau^+ = \frac{(w^+)^2}{\epsilon^+};$$

$$\tau^- = \frac{(w^-)^2}{\epsilon^-};$$



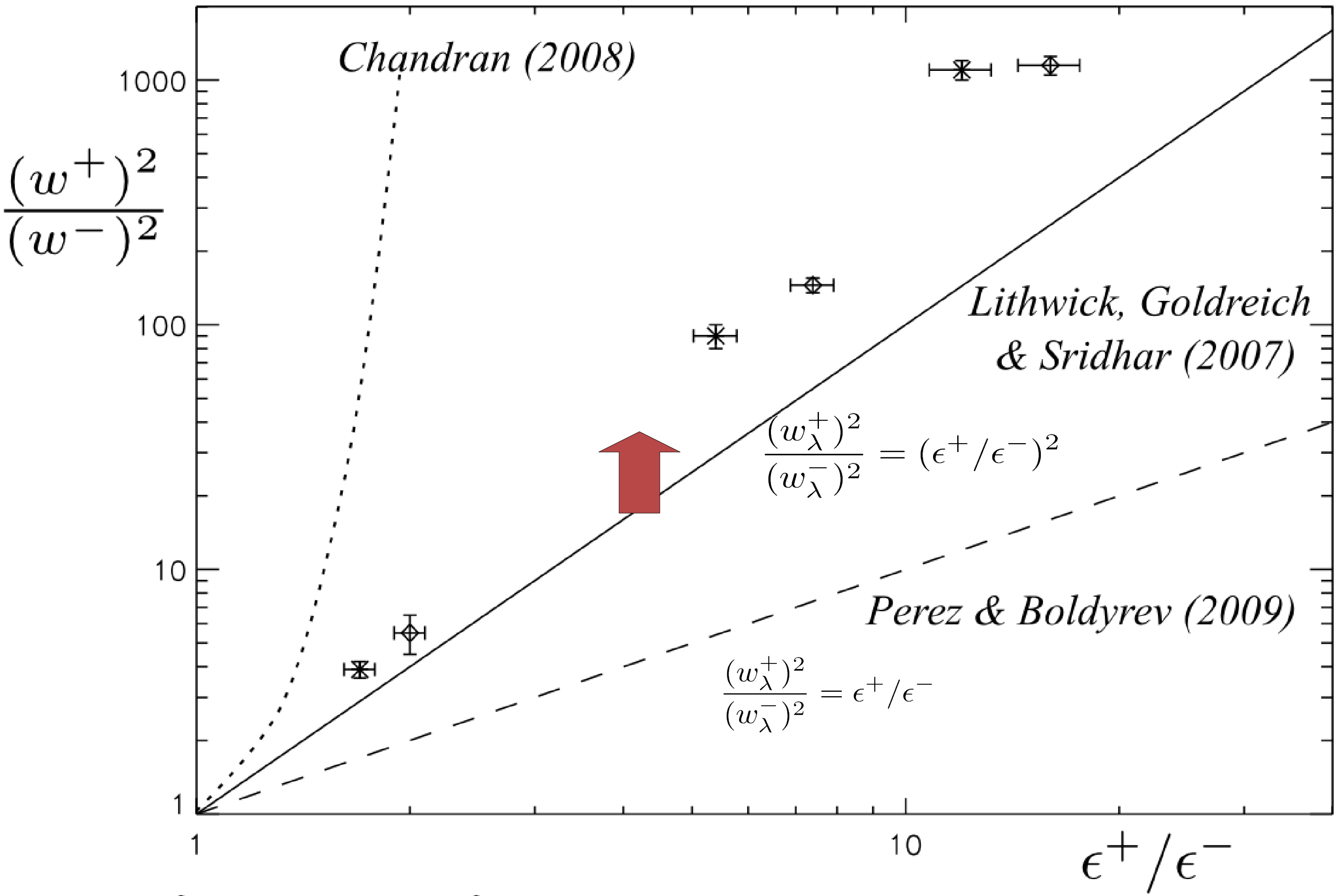
GS95,
Lithwick et al (2007)

Strong cascading

$$\tau^+ = \lambda / w_\lambda^-;$$

$$\tau^- = \lambda / w_\lambda^+;$$

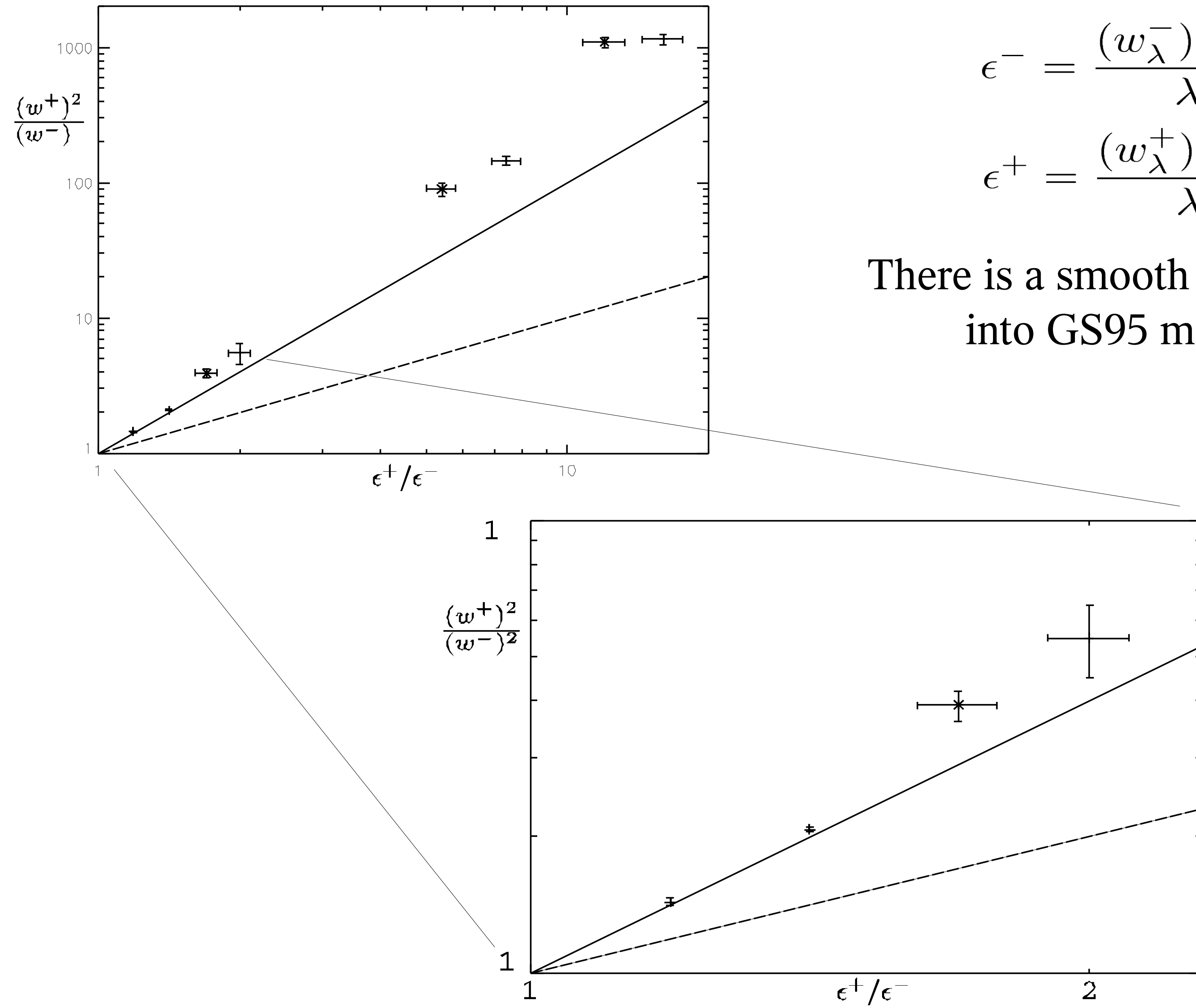
Comparison of predictions of *local* models with numerics



$E^+ = (w^+)^2,\ E^- = (w^-)^2,\ \epsilon^+,\ \epsilon^-$ are very easy to measure

$\frac{(w_\lambda^+)^2}{(w_\lambda^-)^2} \geq \left(\frac{\epsilon^+}{\epsilon^-}\right)^2$ is generally satisfied

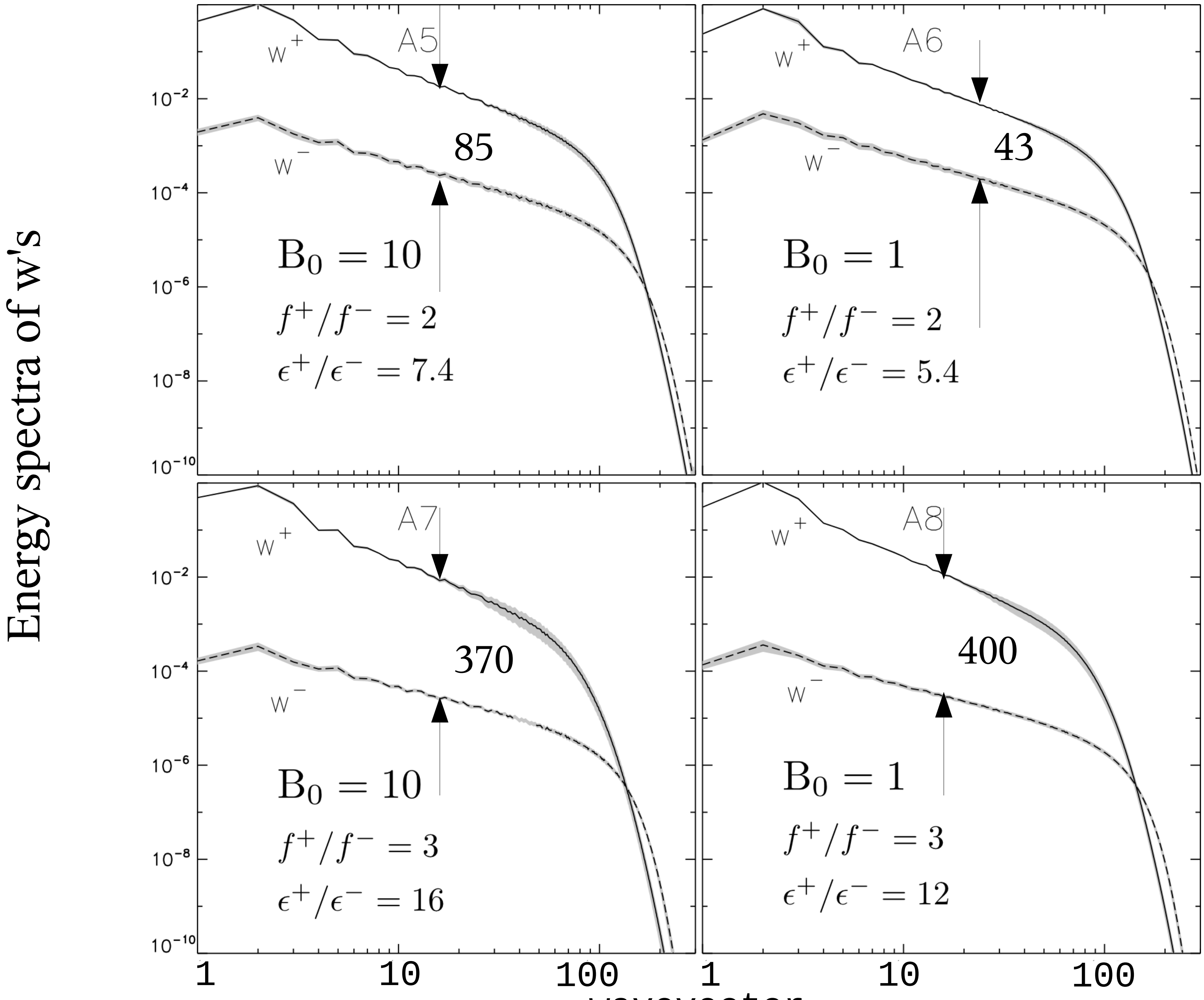
Limit of very small imbalances



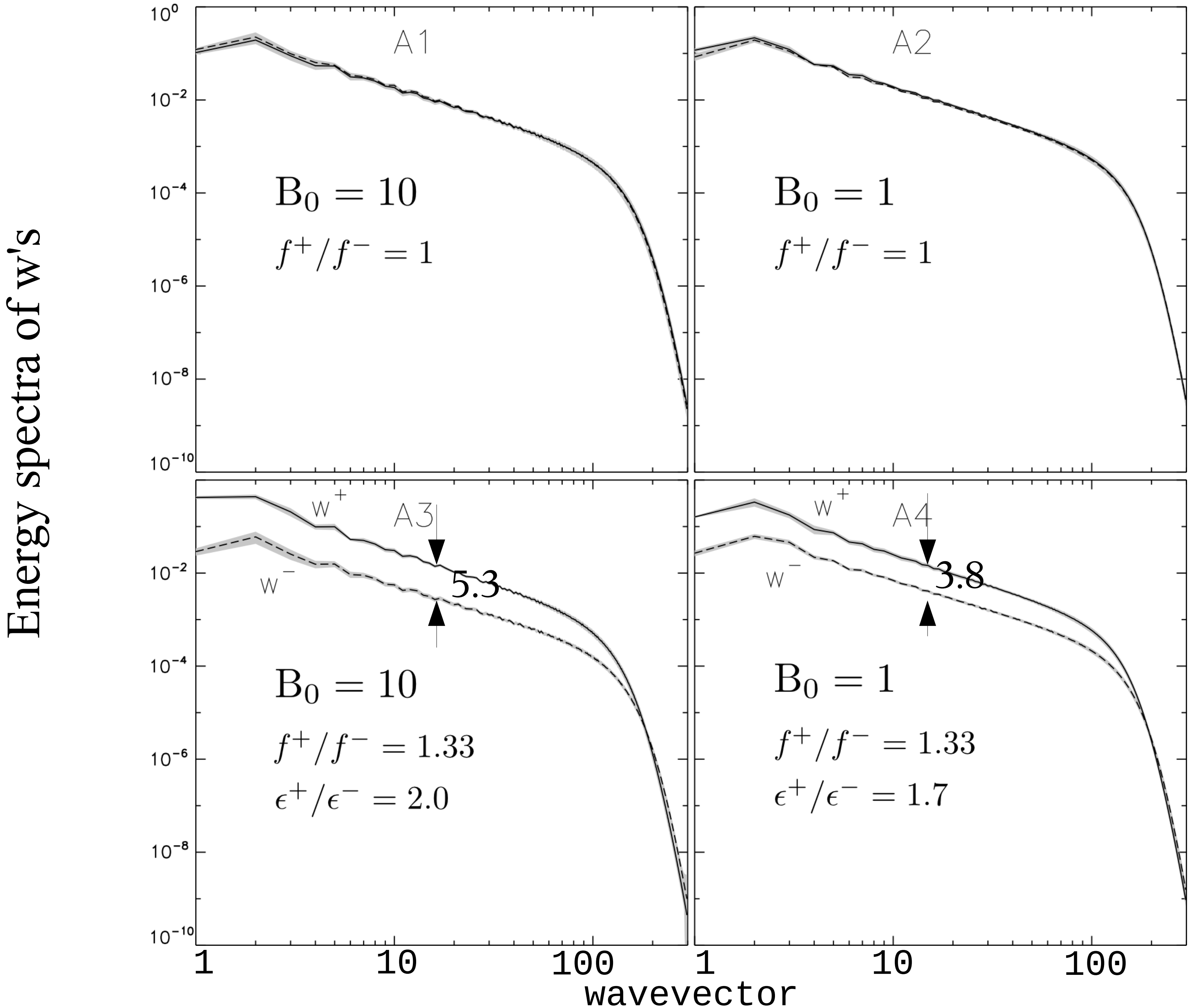
$$\epsilon^- = \frac{(w_\lambda^-)^2 w_\lambda^+}{\lambda};$$

$$\epsilon^+ = \frac{(w_\lambda^+)^2 w_\lambda^-}{\lambda}$$

There is a smooth transition into GS95 model



Energy spectra of w's
wavevector
A5
A6
A7
A8
w+
w−
85
43
370
400
B0 = 10
B0 = 1
f+/f− = 2
f+/f− = 3
ϵ+/ϵ− = 7.4
ϵ+/ϵ− = 5.4
ϵ+/ϵ− = 16
ϵ+/ϵ− = 12
10-2
10-4
10-6
10-8
10-10
1
10
100



Energy spectra of w's
wavevector
A1
$B_0 = 10$
$f^+/f^- = 1$
A2
$B_0 = 1$
$f^+/f^- = 1$
A3
w^+
w^-
5.3
$B_0 = 10$
$f^+/f^- = 1.33$
$\epsilon^+/\epsilon^- = 2.0$
A4
w^+
w^-
3.8
$B_0 = 1$
$f^+/f^- = 1.33$
$\epsilon^+/\epsilon^- = 1.7$
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1
10
100

Powerful message from numerics:

$$\frac{(w_\lambda^+)^2}{(w_\lambda^-)^2} \geq \left(\frac{\epsilon^+}{\epsilon^-}\right)^2,$$

which is also makes sense from theory.

Cascade in Imbalanced Turbulence?

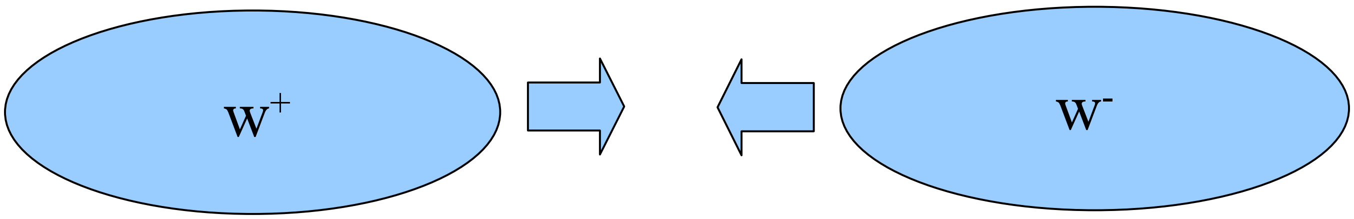
$$\epsilon^- = \frac{(w_\lambda^-)^2 w_\lambda^+}{\lambda}; \qquad \epsilon^+ = \frac{(w_\lambda^+)^2 w_\lambda^-}{\lambda} \cdot o(1)$$

Suppose, one of the waves is cascaded somewhat weaker than strong. If “-” wave have insufficient amplitude to provide strong cascading, then:

$$\frac{(w_\lambda^+)^2}{(w_\lambda^-)^2} \geq \left(\frac{\epsilon^+}{\epsilon^-}\right)^2$$

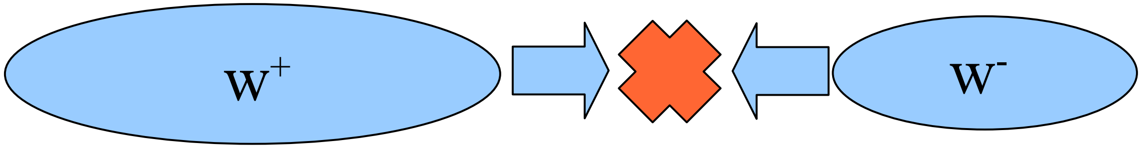
Serious problem!

GS95: uncertainty relation $\tau_{cas}\omega \sim 1$, i.e. $\Lambda \sim \lambda v_A / \delta w$.

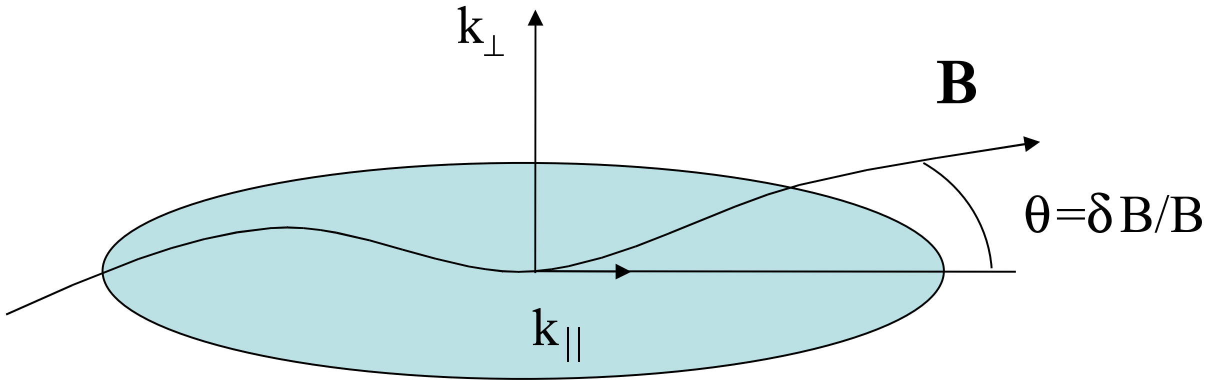


For weak interaction $\Lambda \sim$const.

If for w^+ $\Lambda \sim$const, but for w^- Λ is decreasing, cascade stops?



Uncertainty in Λ associated with local field uncertainty

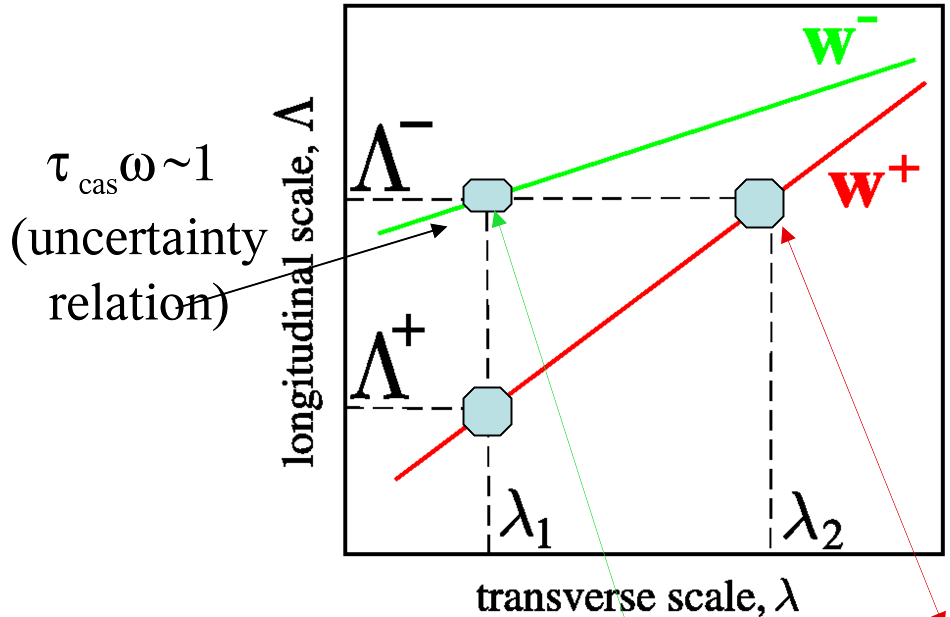


$k_{||} \sim 1/\Lambda$; $\quad\quad k_\perp \sim 1/\lambda$;

$\delta k_{||} \sim k_\perp \theta$; $\quad\quad \delta\Lambda \sim \lambda B/\delta B \sim \lambda v_A/\delta w$.

From the point of interacting eddies,
mean field is not well-defined.
This is the unique feature of strong turbulence.

Imbalanced cascade is different from GS cascade

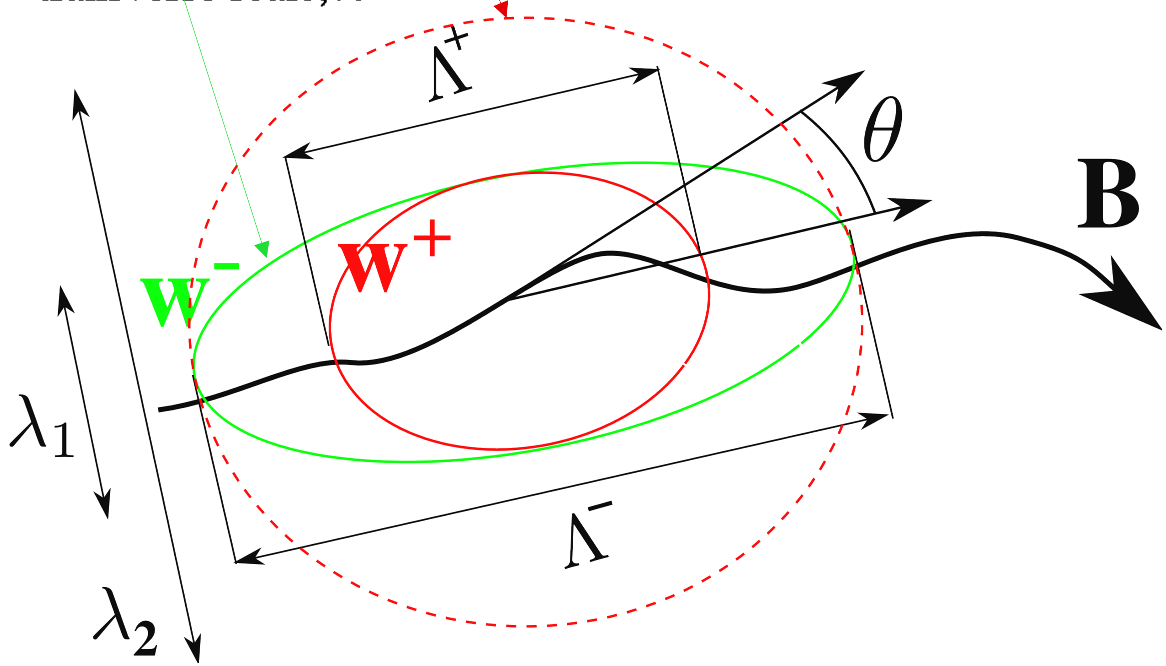


Difference in local field direction

$$\theta \sim \delta b^+(\lambda_2)/v_A$$

correspond to anisotropy

$$\lambda_1 = \theta \Lambda^+$$



A model of strong imbalanced turbulence

(*Beresnyak & Lazarian, ApJ, 2008*)

Old critical balance (causality) $\Lambda^- = v_A \left(\frac{w^+(\lambda_1)}{\lambda_1}\right)^{-1}$; $\left(\frac{\Lambda^+}{\lambda_1}\right)^{-1} = \frac{w^+(\lambda_2)}{v_A}$ New critical balance (field wandering)

energy — shear rate

$$\epsilon^- = \frac{(w^-(\lambda_1))^2 w^+(\lambda_1)}{\lambda_1}$$

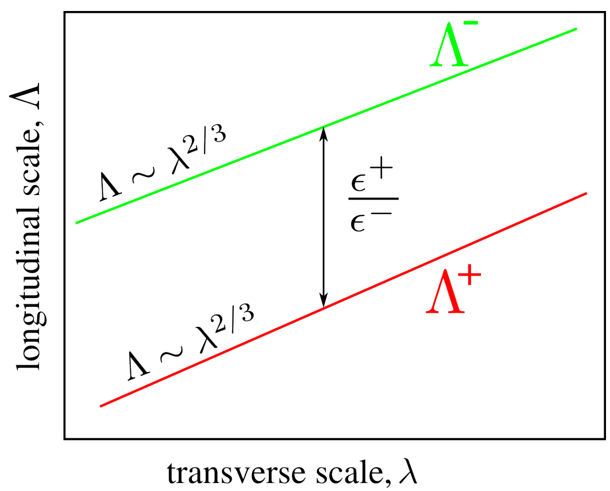
Strong cascading of weak wave

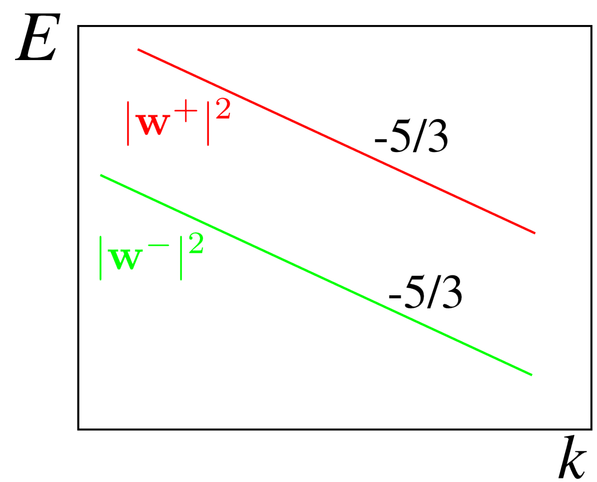
$$\epsilon^+ = \frac{(w^+(\lambda_2))^2 w^-(\lambda_1)}{\lambda_1} \cdot \frac{w^-(\lambda_1)\Lambda^-}{v_A \lambda_1} \cdot f(\lambda_1/\lambda_2)$$

weakening factor

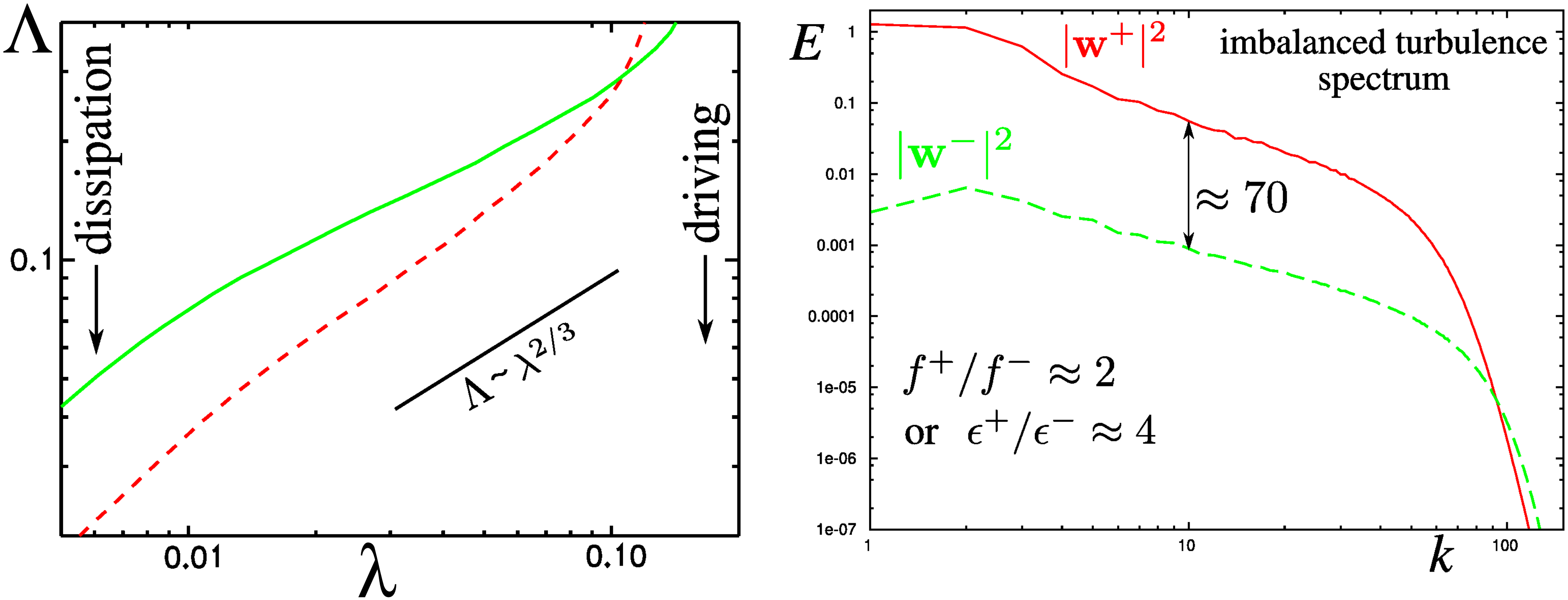
Weak cascading of strong wave

Asymptotic power-law solutions:

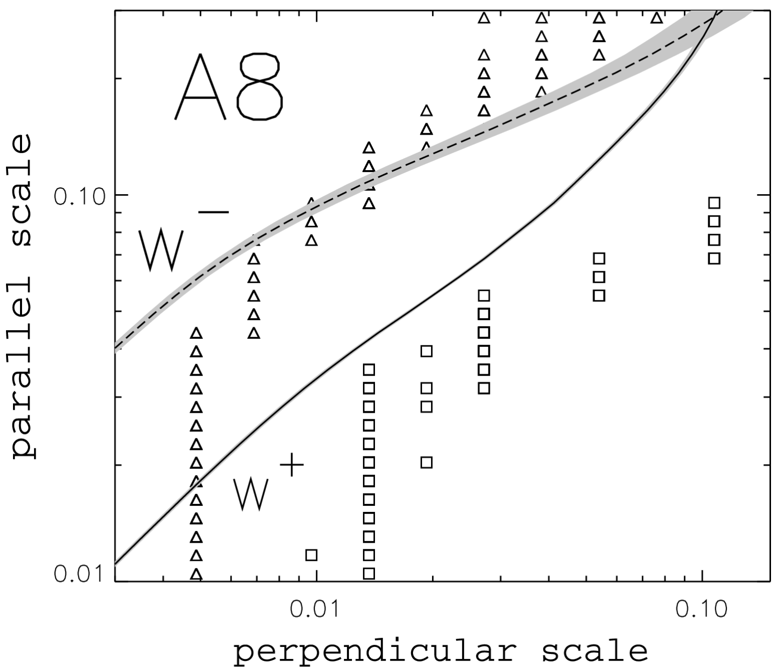


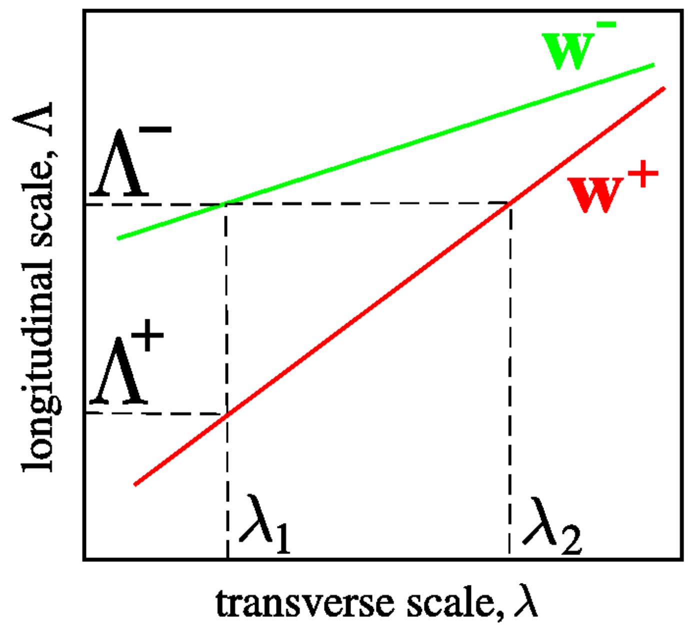


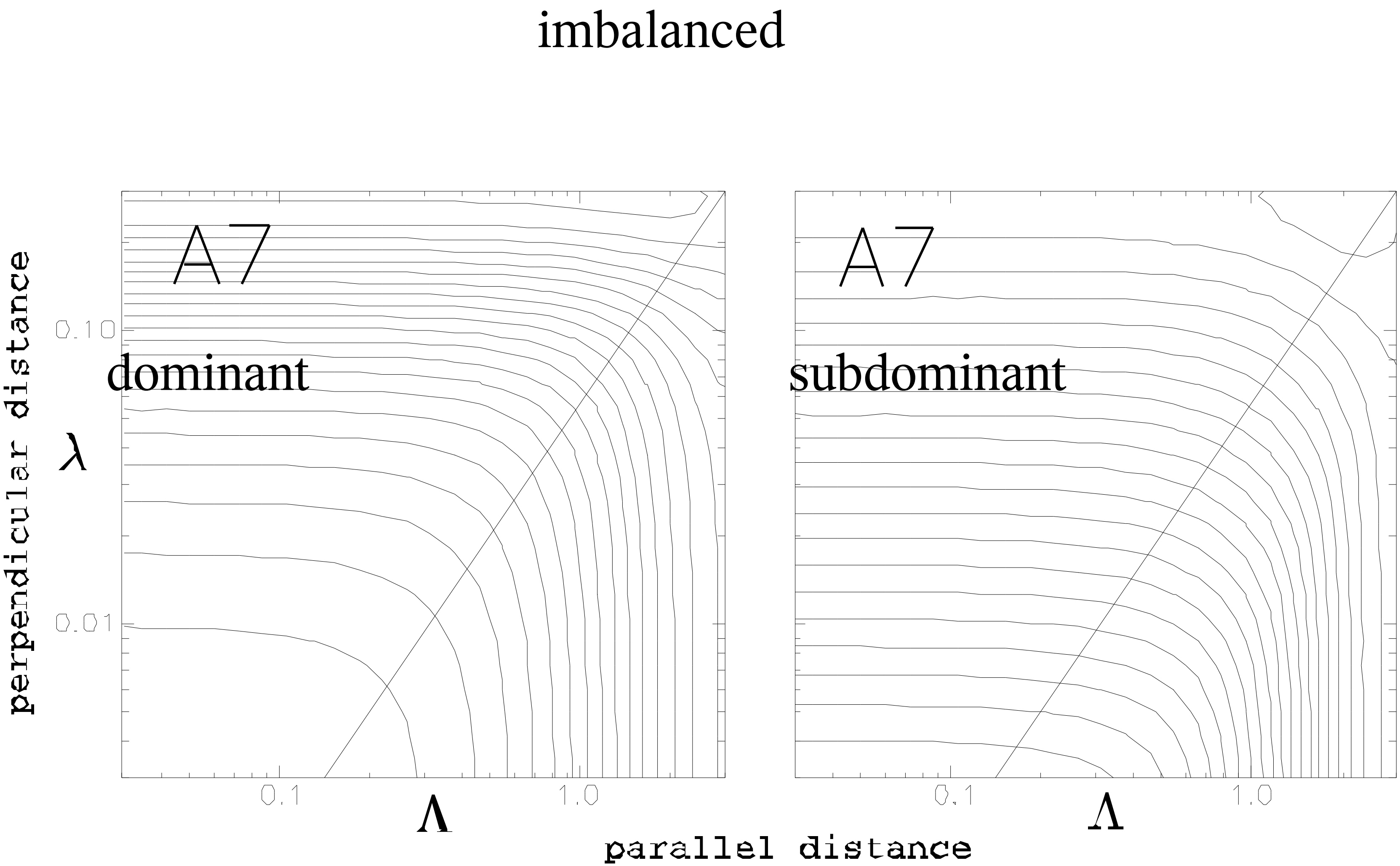
Numerical data support this model



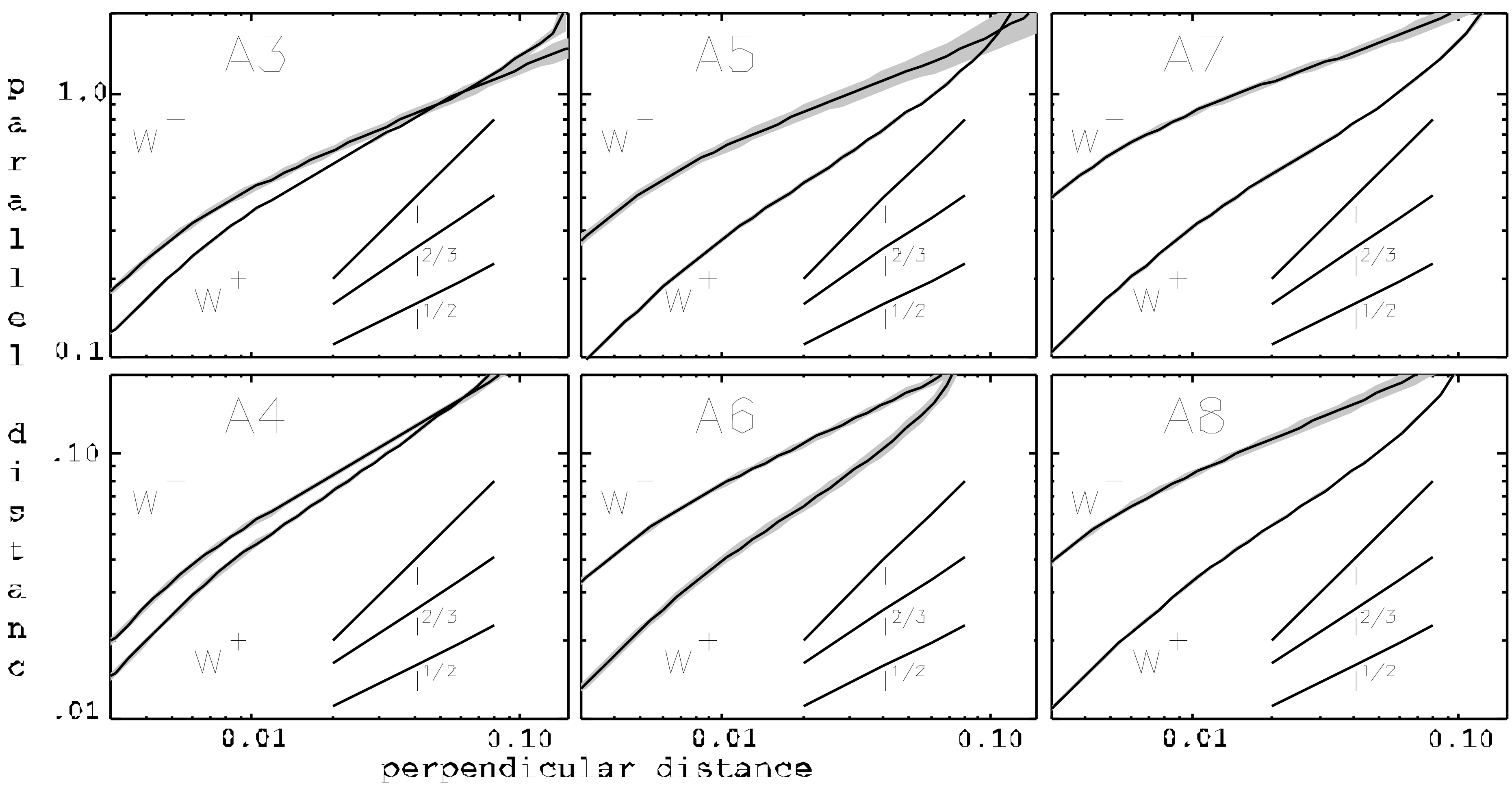
Strong eddies are aligned with respect to larger-scale field






imbalanced
A7
dominant
A7
subdominant
0.10
0.01
perpendicular distance
λ
0.1
1.0
Λ
0.1
1.0
Λ
parallel distance

Imbalanced turbulence



Model vs numerics:

a) the energy imbalance is higher than in the case when both waves are cascaded strongly, which suggest that dominant wave is cascaded weakly
b) Time evolution of spectra suggests that strong wave have a longer dissipation timescale
c) the anisotropies are different and the strong wave anisotropy is smaller
d) subdominant wave eddies are aligned with respect to the local field, while dominant wave eddies are aligned with respect to larger-scale field
e) the inertial range of the dominant wave is shorter
f) there is no “pinning” on dissipation scale, which suggest nonlocal cascading

models / numerics	*Lithwick et al (2007)*	*Beresnyak & Lazarian (2008)*	*Chandran (2008)*	*Perez & Boldyrev (2009)*
cascading timescales	✔✘	✔	✘	✘
spectral slopes	-	-	✘	-
anisotropies	✘	✔	✘	✘?
time evolution	✔	✔	✘	✘
dissipation scale	✔	✔	✘	✘

Summary

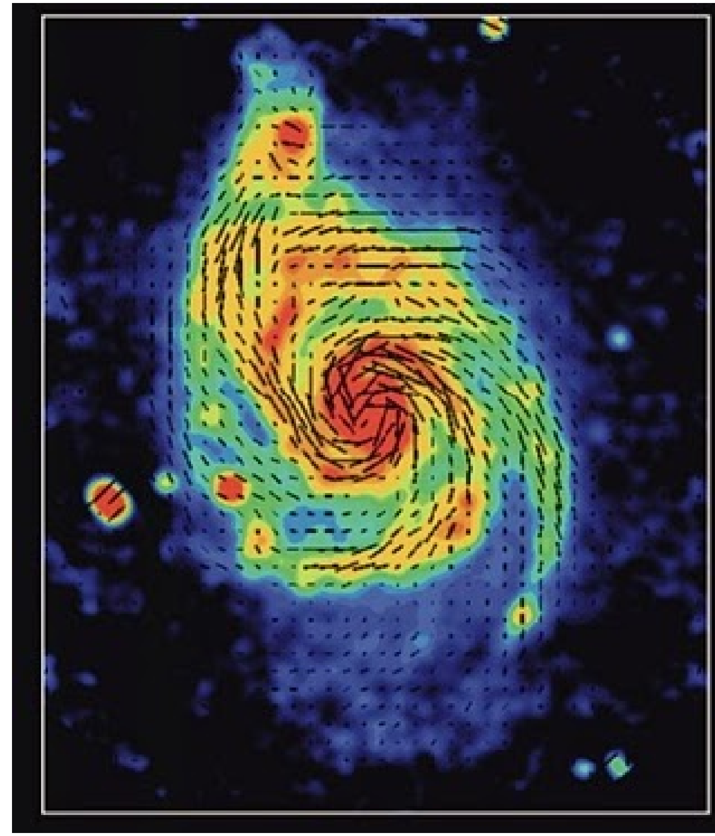
- MHD turbulence has a *universal cascade,* although different from hydrodynamic cascade.
- For the first time, we were able to measure the Kolmogorov constant, i.e. the efficiency of the energy transfer in MHD turbulence and explained the lack of bottleneck effect in earlier MHD simulations.
- We now have a good idea how cascading happens in the general case, i.e., in *imbalanced turbulence*. In nature, imbalanced turbulence is more common than the balanced one, as sources and sinks of energy exist in a large scale mean magnetic field.
- Numerics is an efficient tool to discriminate between models, by both qualitative and quantitative means.

MHD Dynamo

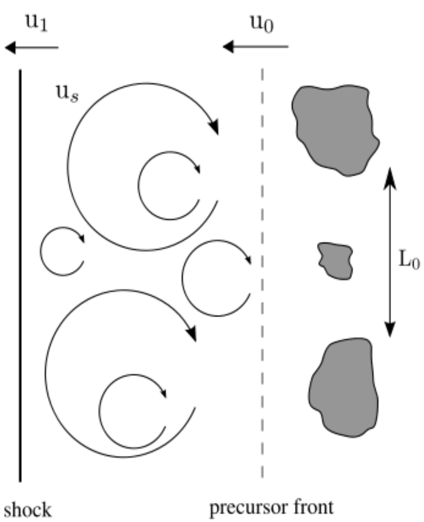
Large-scale dynamo L(B)>L_t
require special conditions and slow.

Small-scale dynamo L(B)<L_t
very generic in three-dimensional dynamics and fast

Disk galaxies:

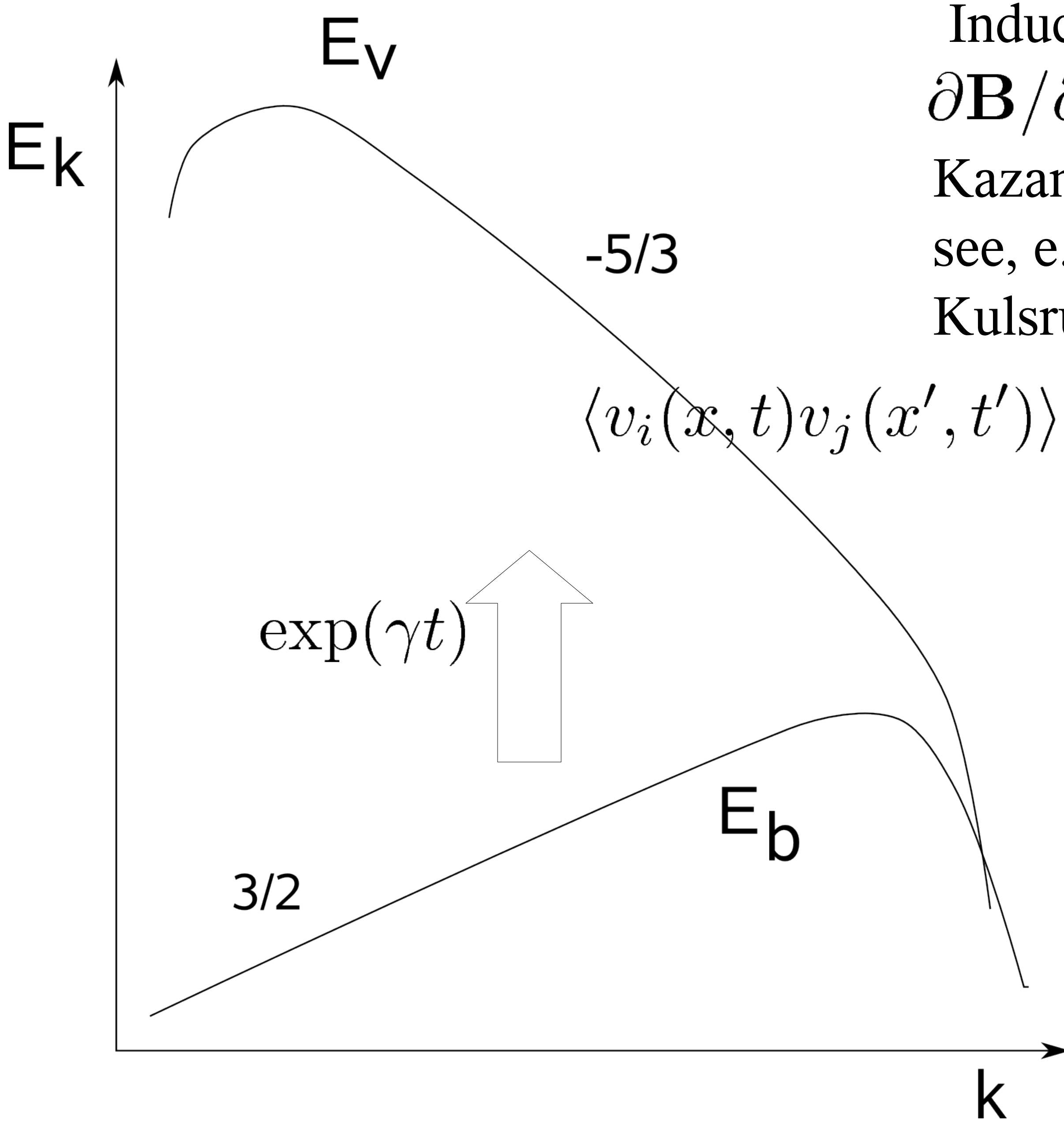


CR shocks:



Clusters

Kinematic Dynamo



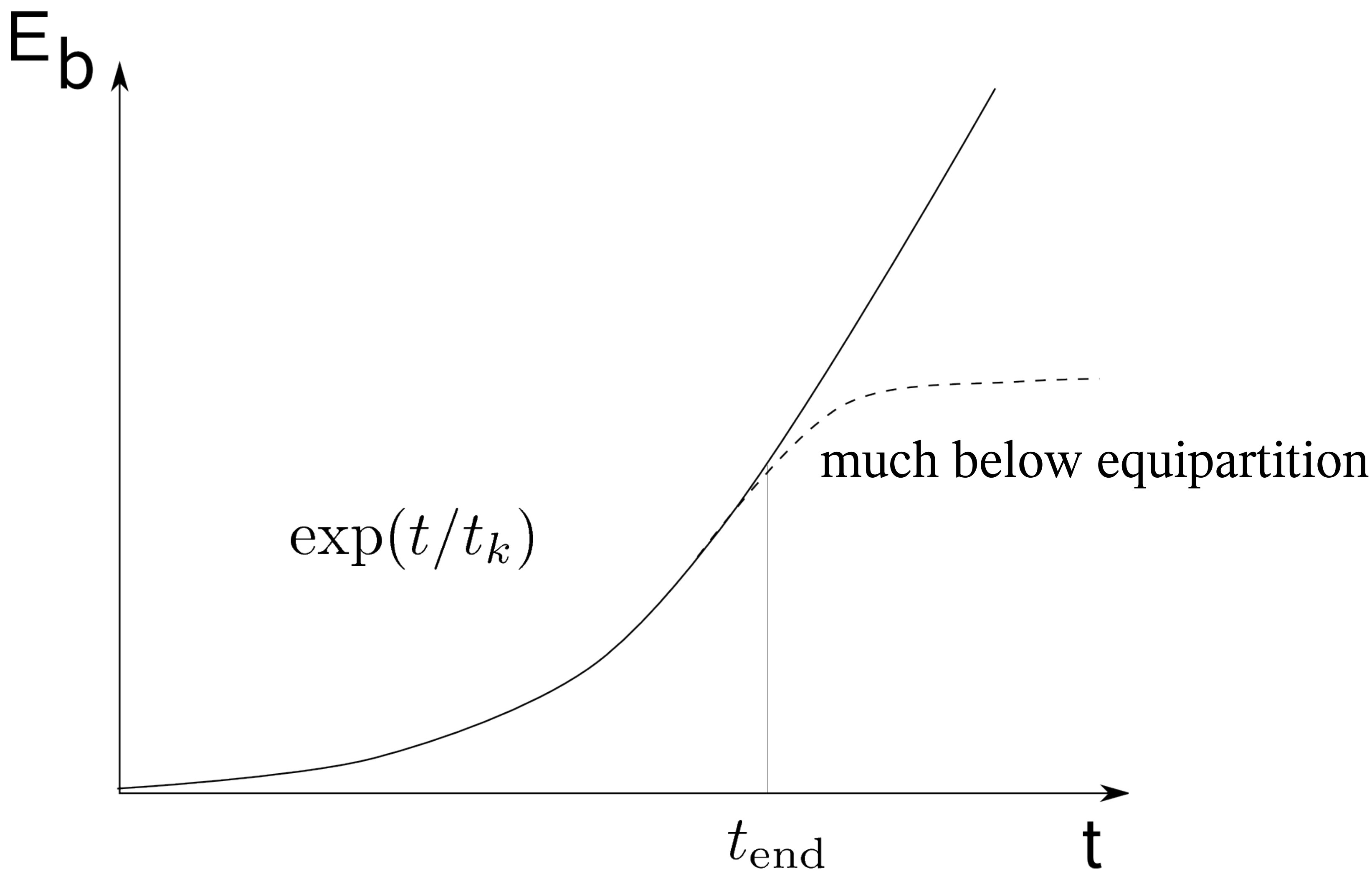
Induction eq-n is linear

$$\partial \mathbf{B}/\partial t = \nabla \times (\mathbf{v} \times \mathbf{B})$$

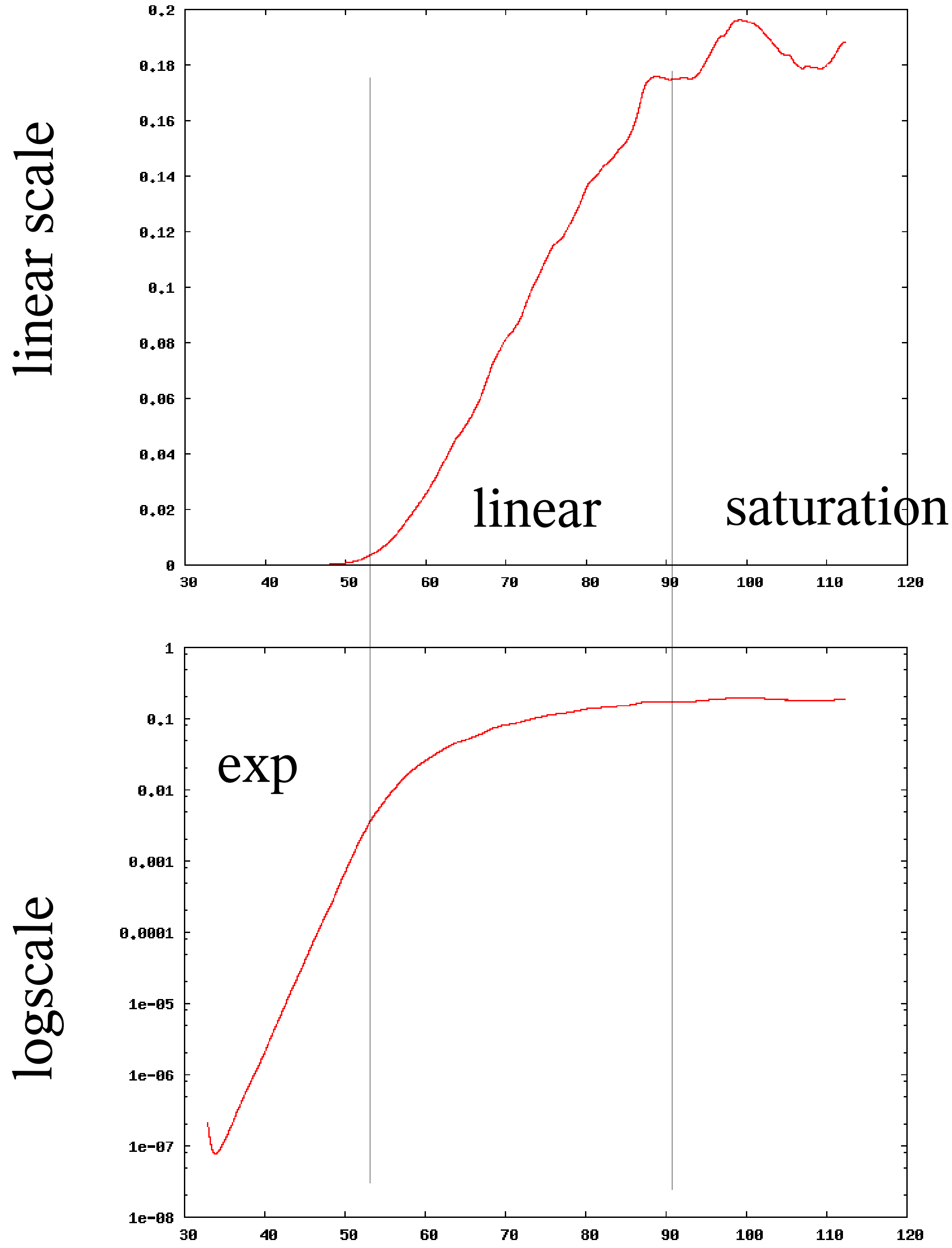
Kazantsev-Krachnan model, see, e.g. Kazantsev, 1967, Kulsrud & Anderson 1992:

$$\langle v_i(x,t) v_j(x',t') \rangle = \kappa_{ij}(x-x')\delta(t-t')$$

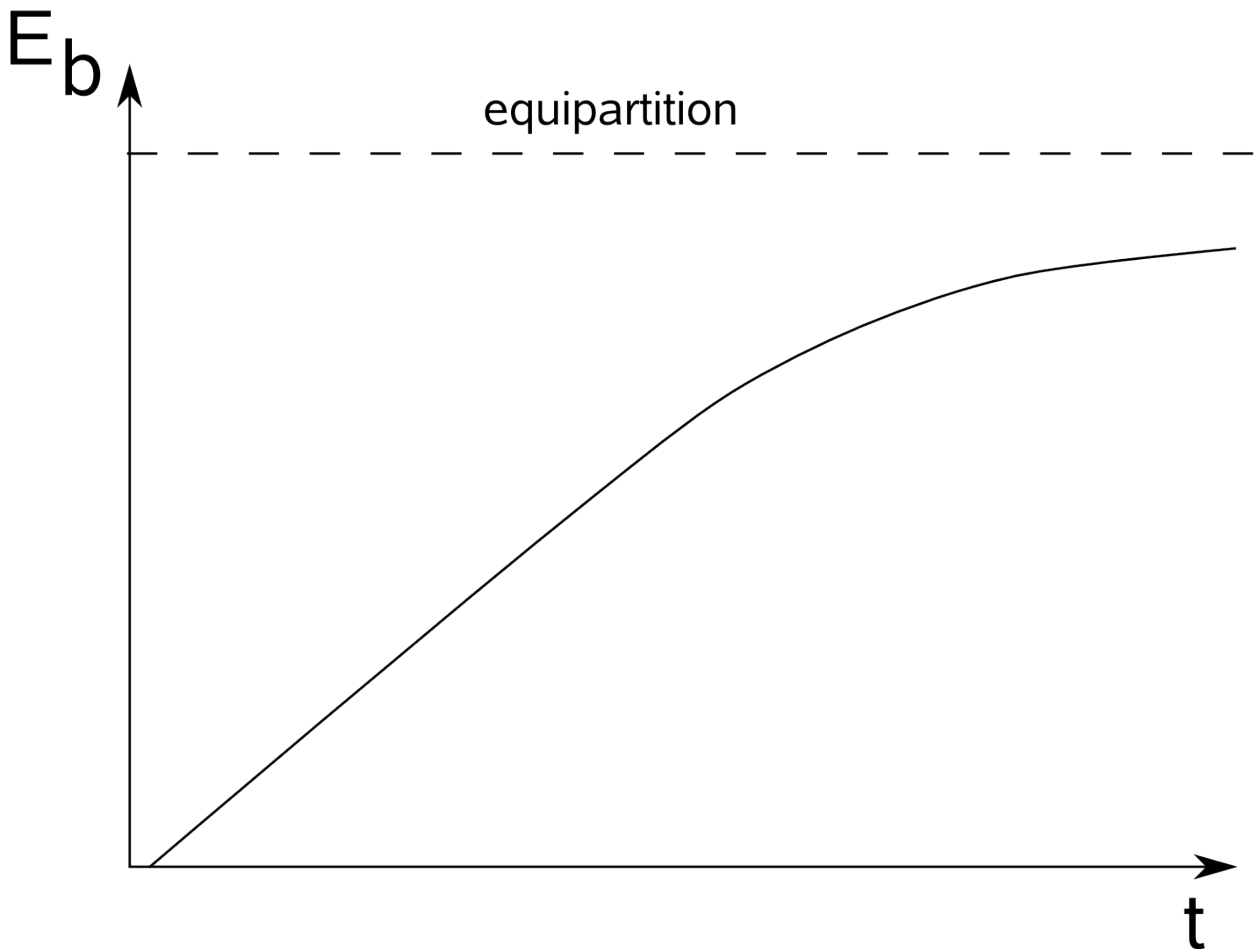
Nonlinear Dynamo



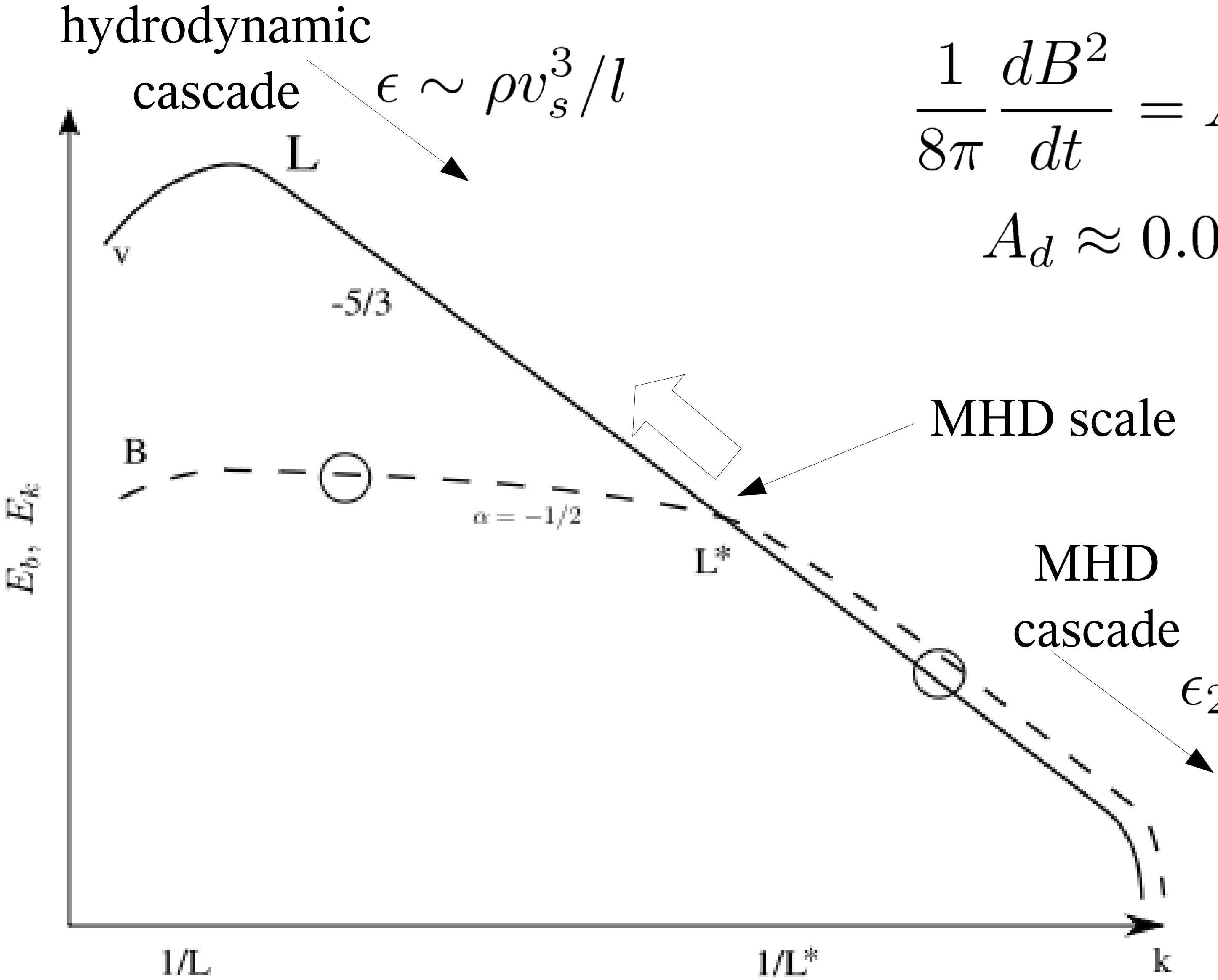
Evolution of total magnetic energy



Astrophysical Small-Scale Dynamo



Astrophysical Small-Scale Dynamo



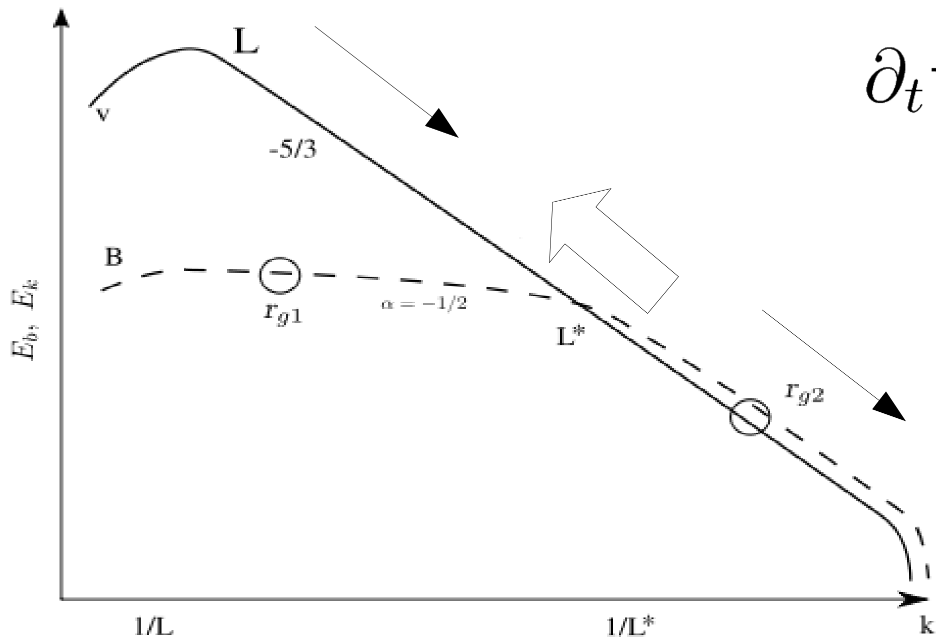
$$\frac{1}{8\pi}\frac{dB^2}{dt} = A_d \epsilon t$$

$$A_d \approx 0.06$$

Why A_d is so small (0.06)?

Possible answers:

-- small-scale dynamo is a decorrelation of Elsasser fields that propagates upscale, while the cascade direction is downscale



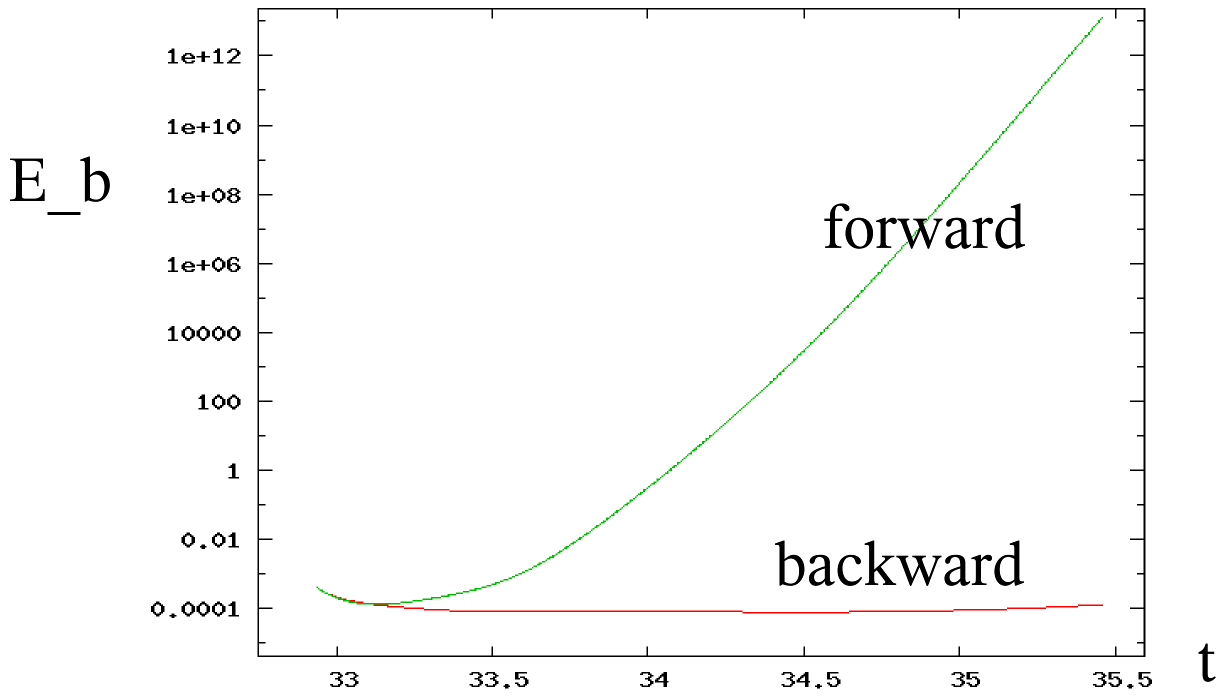
$$\partial_t \mathbf{w}^{\pm} + \hat{S}(\mathbf{w}^{\mp} \cdot \nabla)\mathbf{w}^{\pm} = 0$$

$$\mathbf{w}^{\pm} = \mathbf{v} \pm \mathbf{B}/\sqrt{4\pi\rho}$$

-- may be our understanding of kinematic dynamo is naive?

Such as taking $\langle v_i(x,t)v_j(x',t')\rangle = \kappa_{ij}(x-x')\delta(t-t')$ is bad?

Simulations of kinematic dynamo forward and backward in time



Solved Euler's equations (no dissipation and no Lorenz force), and induction equation with diffusivity, i.e. kinematic dynamo.

Unexpected things!

compared to Kazantsev-Kraichnan model:

- growth rate is off by a factor of several,
- velocity backward in time doesn't produce any growth