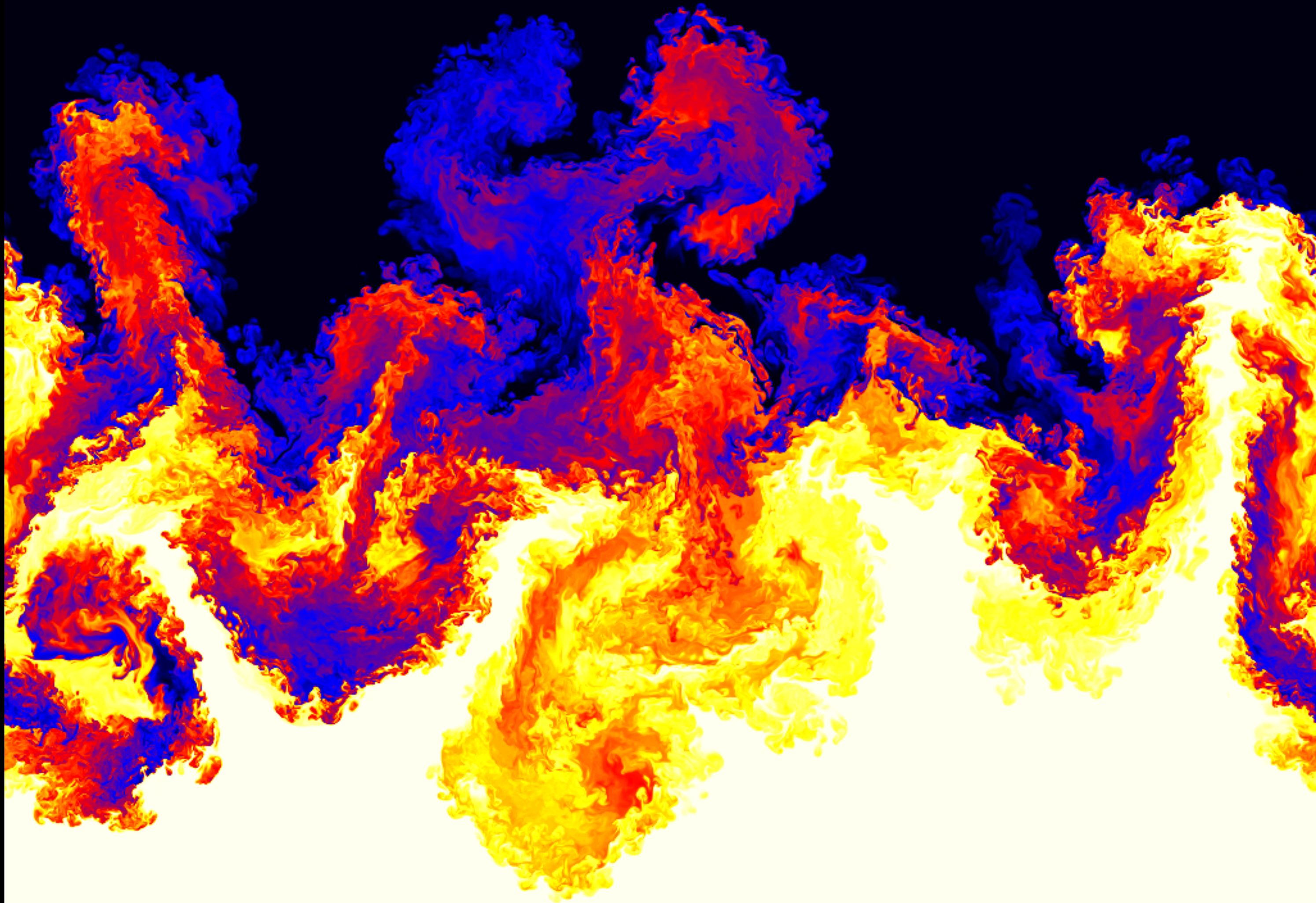
Polymer Heat Transport Enhancement in Thermal Convection: the Case of Rayleigh-Taylor Turbulence



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Outline

* Viscoelastic models (why numerical simulation can be useful)

* Rayleigh-Taylor turbulence

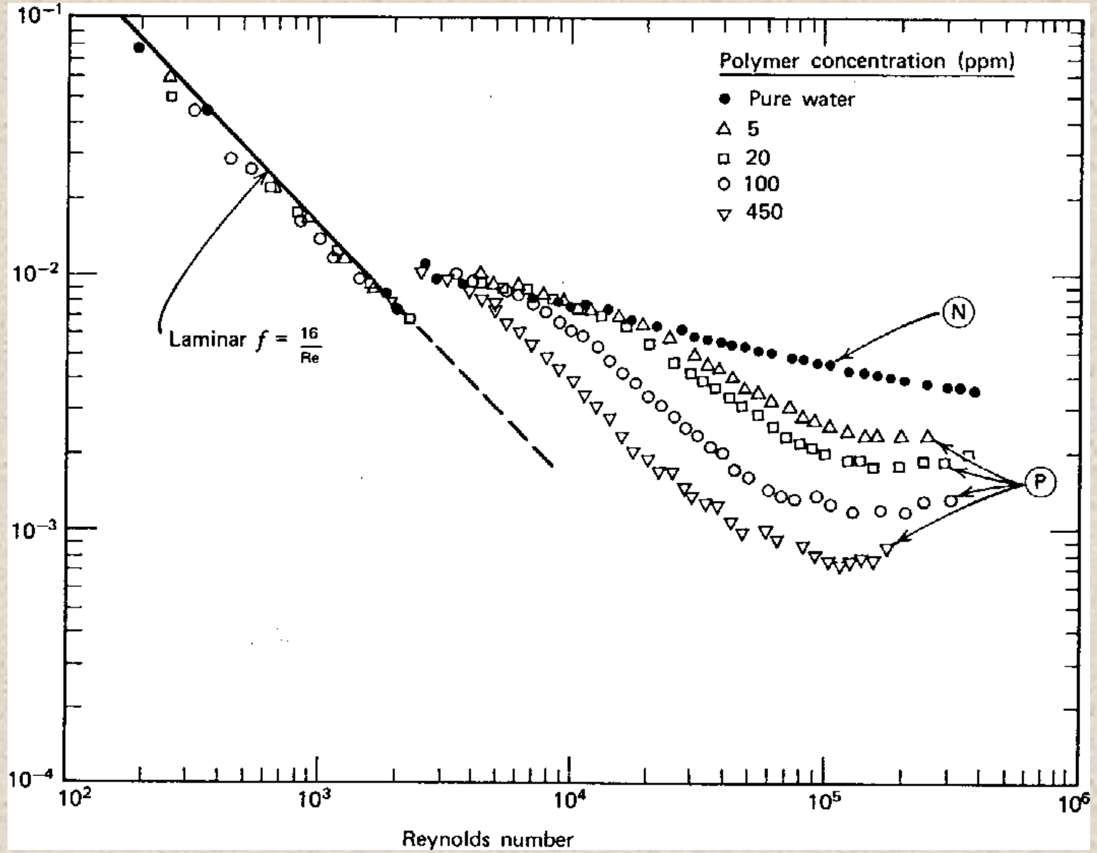
* Viscoelastic RT turbulence

The addition of long-chain polymers has dramatic effects on flowing fluids

Drag reduction:
drag coefficient

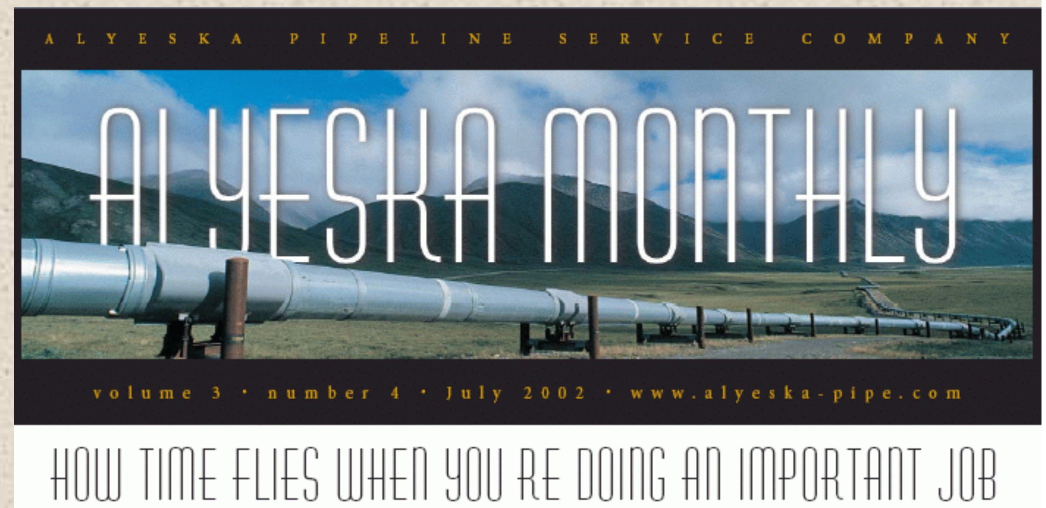
$$f = \frac{\text{work done by force}}{\text{kinetic energy in mean flow}}$$

can be reduced by polymers up to about 80% (Toms 1949)



In a pipe...

$$f = \frac{\Delta p \; R}{\rho U^2 \; L}$$



Numerical simulations of drag reduction

Turbulence drag reduction has been observed in numerical simulations of channel flow based on viscoelastic models of polymer solution (Oldroyd-B, FENE-P) [Sureshkumar et al, POF **9**, (1997), Ptasinski et al, JFM **490** (2003), De Angelis et al, PRE **67** (2003)]

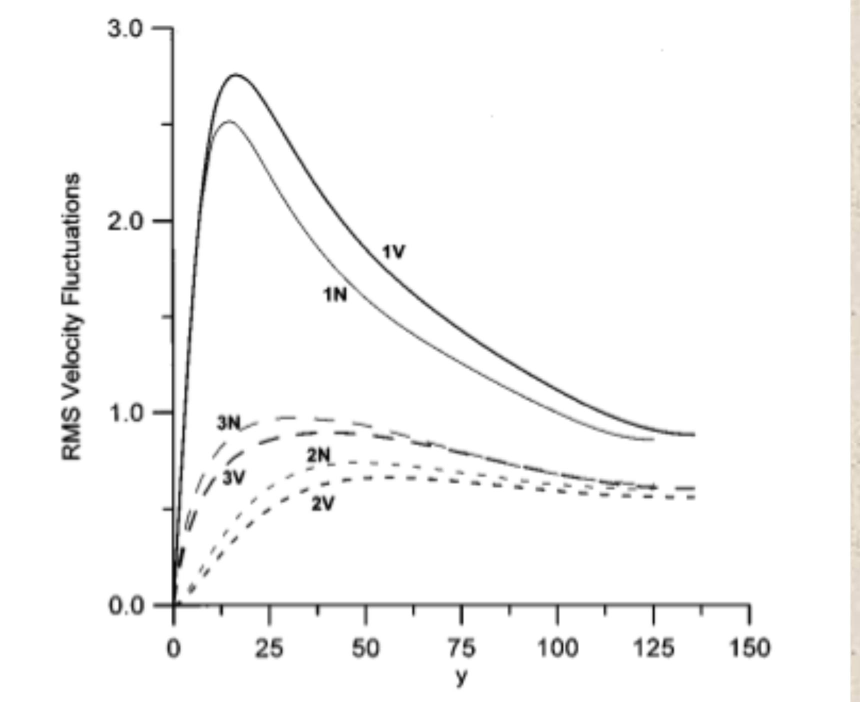
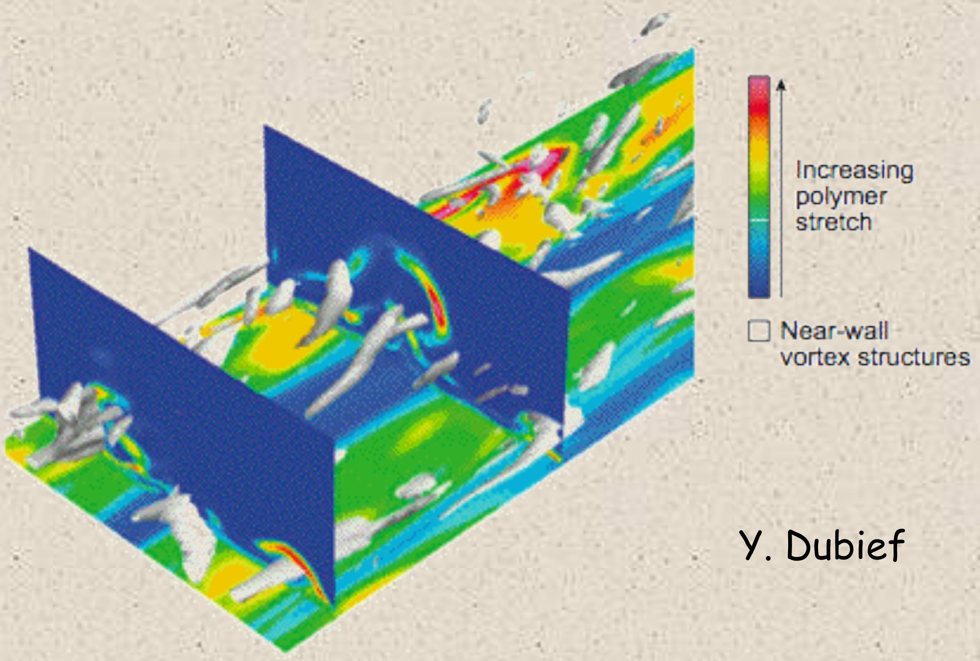
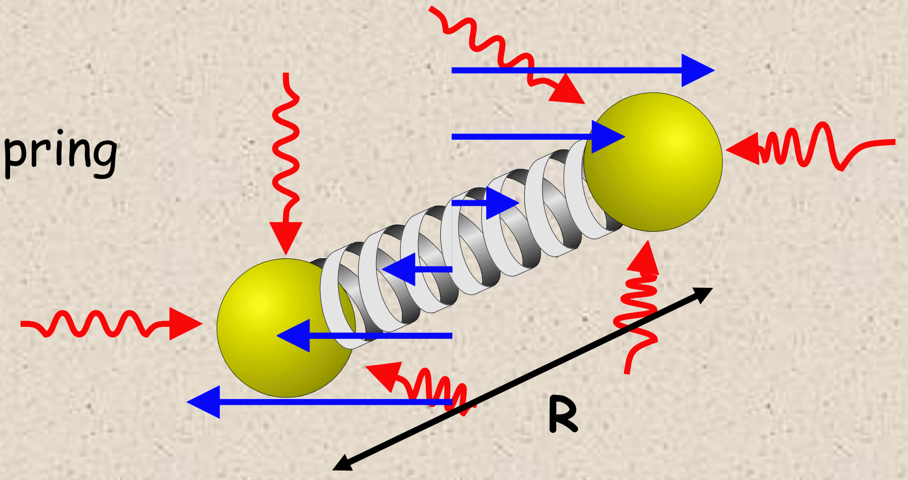


FIG. 3. The rms velocity fluctuations as a function of the distance from the wall. Curves 1N, 2N and 3N show the streamwise, the wall-normal and the spanwise components for the Newtonian flow, respectively; curves 1V, 2V and 3V show corresponding data for the viscoelastic flow.



Y. Dubief

Modeling polymers: Oldroyd-B



Dumbbell: 2 massless beads connected by a spring
one relaxation time
no polymer-polymer interaction
always smaller than Kolmogorov scale

$$\frac{d\mathbf{R}}{dt} = \left(\mathbf{R}\cdot\nabla\right)\mathbf{u} - \frac{1}{\tau}\mathbf{R} + \left(2R_0^2/\tau\right)^{1/2}\xi$$

From single polymer to elastic field
Polymer conformation tensor $\sigma_{ij} = R_0^{-2}\left\langle R_i R_j \right\rangle$

$\tau = f/K_0$

$R_0^2 = kT/K_0$

f – friction coef
K_0 – elastic modulus

$$\partial_t \sigma + \left(\mathbf{u}\cdot\nabla\right)\sigma = \left(\nabla\mathbf{u}\right)^T\cdot\sigma + \sigma\cdot\left(\nabla\mathbf{u}\right) - 2\frac{\sigma - \mathbf{1}}{\tau}$$

Feedback on the flow: elastic stress tensor $T_{ij}^P = nK_0\left\langle R_i R_j \right\rangle = nK_0 R_0^2 \sigma_{ij}$

$$\partial_t \mathbf{u} + \left(\mathbf{u}\cdot\nabla\right)\mathbf{u} = -\frac{\nabla p}{\rho} + \nu\Delta\mathbf{u} + \mathbf{f} + \frac{2\eta\nu}{\tau}\nabla\cdot\sigma$$

$\eta = \dfrac{nK_0 R_0^2 \tau}{2\mu}$ zero-shear contribution to total viscosity: $\mu(1+\eta)$

[Boffetta, Celani, Mazzino, PRE **71** (2005)]

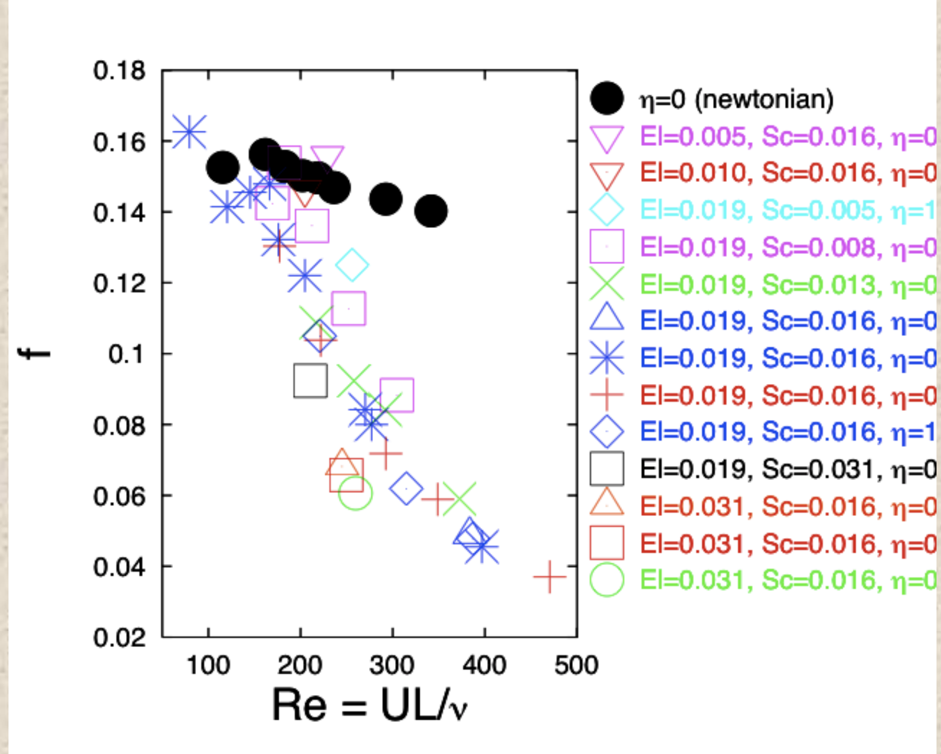
Drag reduction in the absence of boundaries

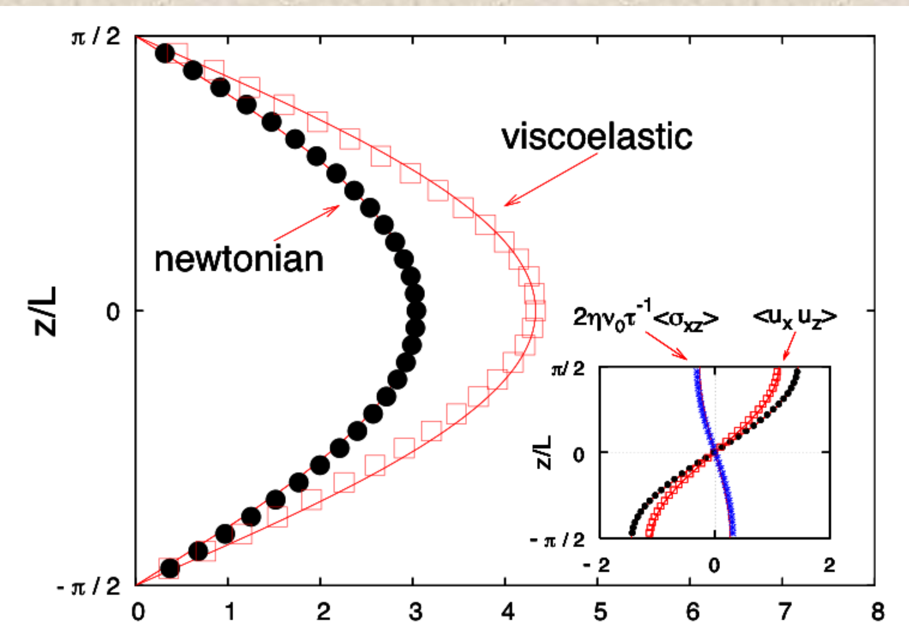
Numerical simulations of viscoelastic Kolmogorov flow show DR phenomenology

Navier-Stokes forced with **F**=(Fcos(kz),0,0) ,   mean velocity: <**u**>=(Ucos(kz),0,0)

Drag coefficient $f = \frac{F\,L}{U^2}$

Laminar: $F=\nu U/L^2$ and $f = \frac{1}{Re}$





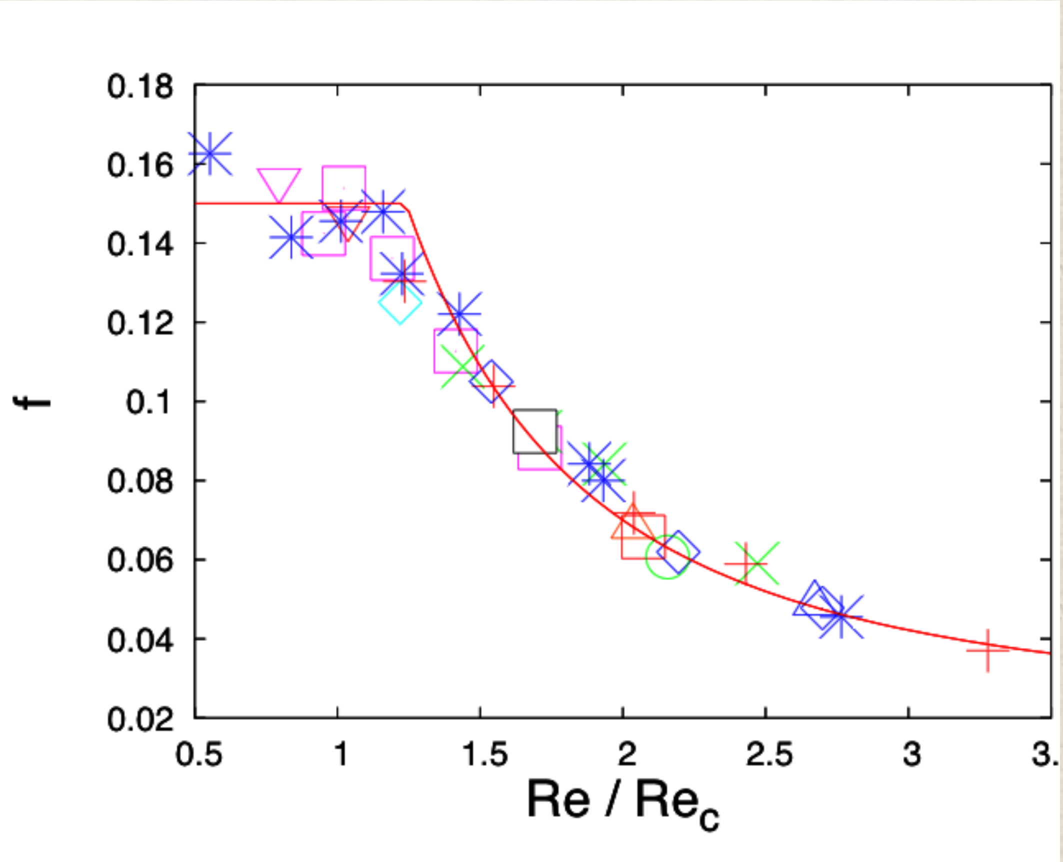
mean velocity increases
Reynolds stress decreases

Collapse of drag coefficients at different elasticity

Lumley's time criterion:
onset of drag reduction when $\nabla u \sim \tau^{-1}$

$Re_c \approx El^{-2/3}$

Balkovsky, Fouxon, Lebedev, PRE **64** (2001)

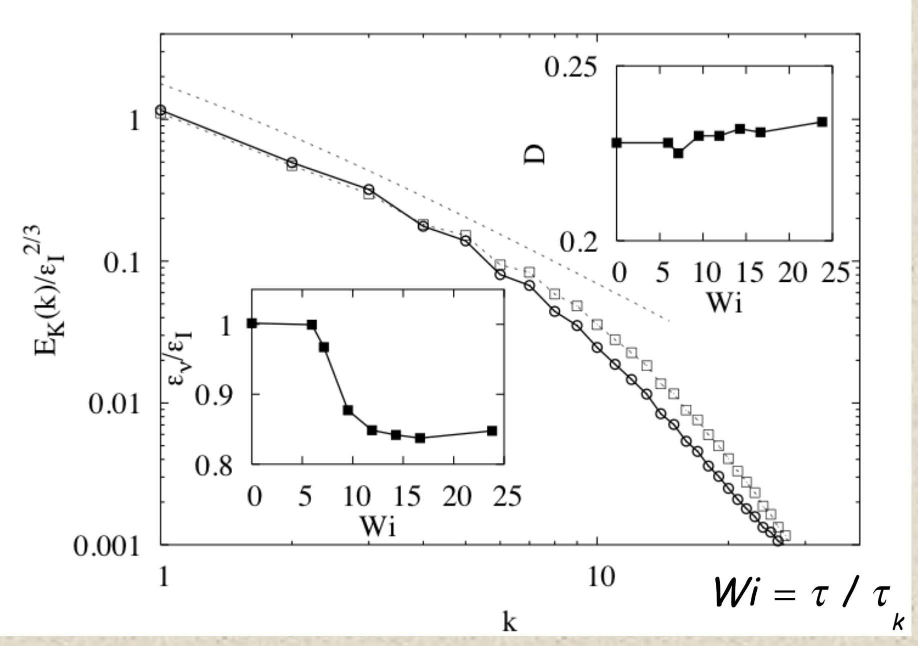


"Drag reduction" in absence of mean flow

Effects of polymers on homogeneous-isotropic turbulence:
* reduction of turbulent fluctuations below the Lumley scale $\ell_L = (\varepsilon\tau)^{1/2}$
* steeper energy spectrum at small scales
* reduced viscous dissipation
* increased turbulent fluctuations at large scale (not always)

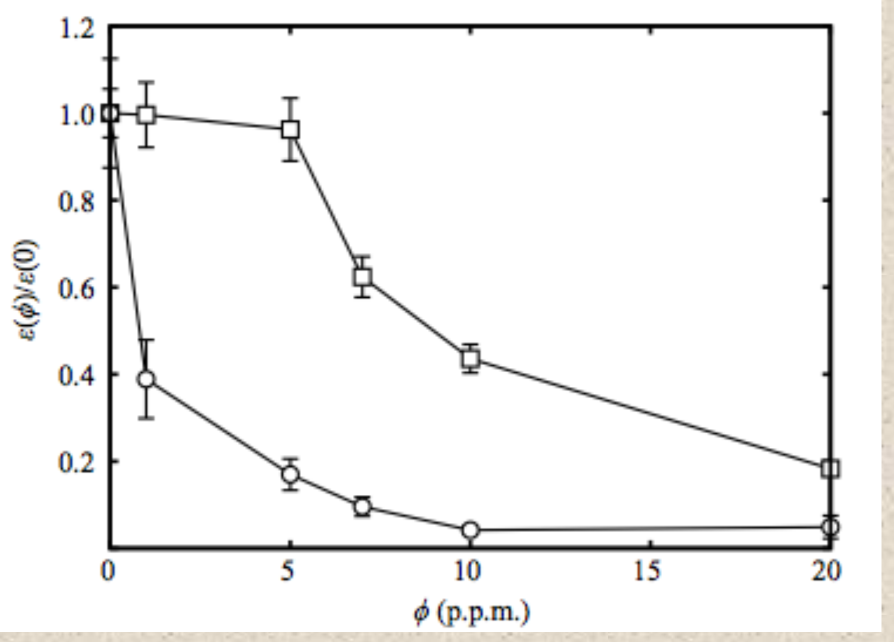
[De Angelis et al, JFM **531** (2005), Liberzon et al, POF **17** (2005), Berti et al, EPL **76** (2006), Ouelette et al, JFM **629** (2009)]

simulation



Berti et al, EPL **76** (2006)

experiment



Ouelette et al, JFM **629** (2009)

Is there any effect of polymers on turbulent convection ?

Does polymers make heat transfer more efficient ?

Not clear:

Ahlers and Nikolaenko, Rayleigh-Benard experiment: negative
[PRL **104**, 034503 (2010)]

Benzi et al, shell model and mean temperature gradient: positive
[PRL **104**, 024502 (2010)]

Numerical study of the problem in Rayleigh-Taylor turbulence

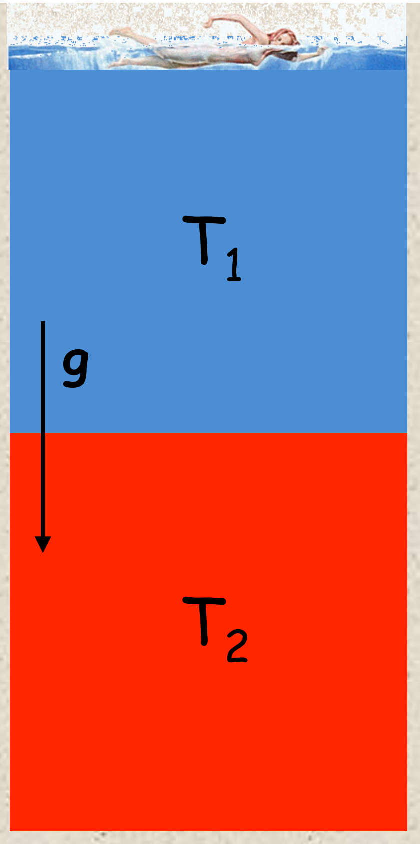
Boffetta, Mazzino, Musacchio and Vozella
JFM **643**, 127 (2010)
PRL **104**, 184501 (2010)
PRE **83**, 056318 (2011).

Rayleigh-Taylor setup and instability

Instability on the interface of two fluids of different densities with relative acceleration.

Single fluid with temperature jump: $\theta_0 = T_2 - T_1$

Atwood: $$A \equiv \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \simeq \frac{1}{2}\beta\theta_0$$ (β: thermal expansion coef.)



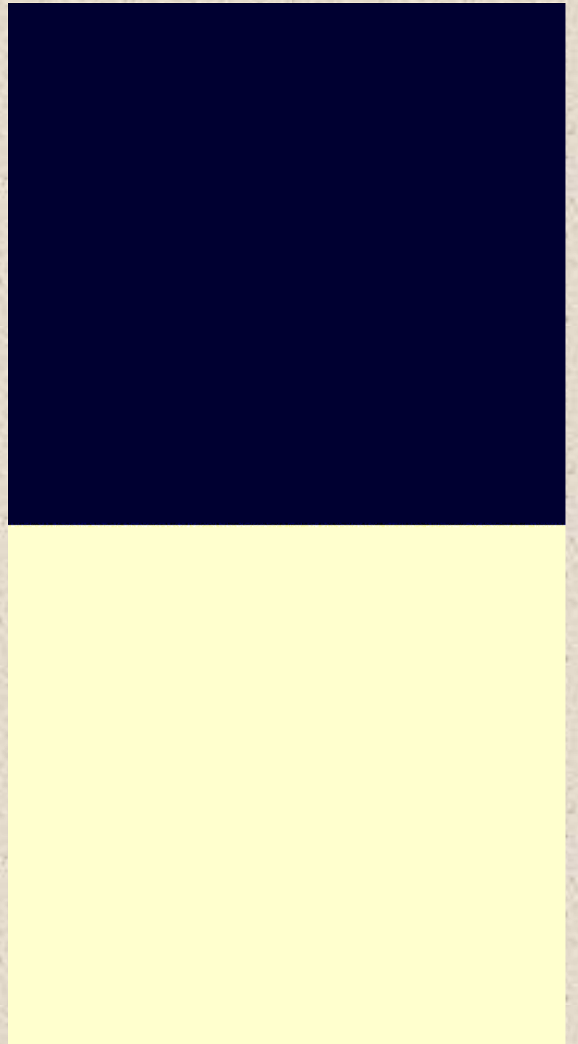
For small A the Boussinesq approximation for an incompressible fluid holds:

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta\mathbf{g}T \\ \partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T \end{cases}$$

Time evolving turbulence with initial conditions:

$$\begin{cases} \mathbf{u}(\mathbf{x},0) = 0 \\ T(\mathbf{x},0) = -(1/2)\theta_0 \,\mathrm{sgn}(z) \end{cases}$$

Phenomenology of RT turbulence



Energy balance:
turbulent kinetic energy $E=(1/2)\langle u^2\rangle$ produced from potential energy $P=-\beta g\langle zT\rangle$

$$\varepsilon = \nu\left\langle (\nabla u)^2 \right\rangle$$

$$\frac{dE}{dt} = \beta g \left\langle wT \right\rangle - \varepsilon = -\frac{dP}{dt} - \varepsilon$$

Dimensional balance: $\dfrac{du^2_{rms}}{dt} \simeq \beta g \theta_0 u_{rms}$ therefore

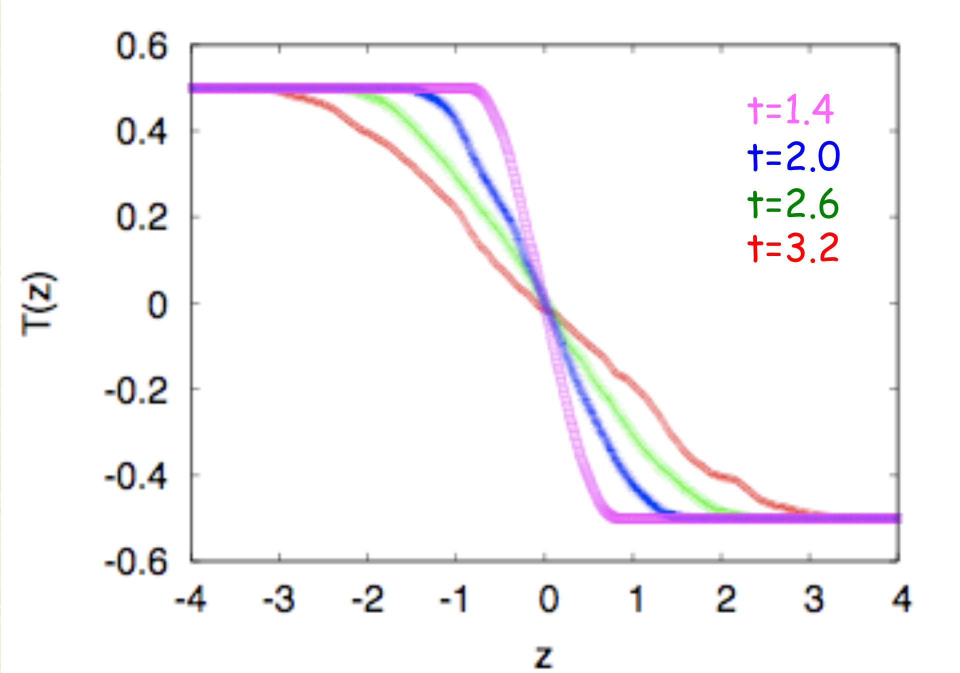
Large scale velocity fluctuations $u_{rms}(t) \approx Agt$

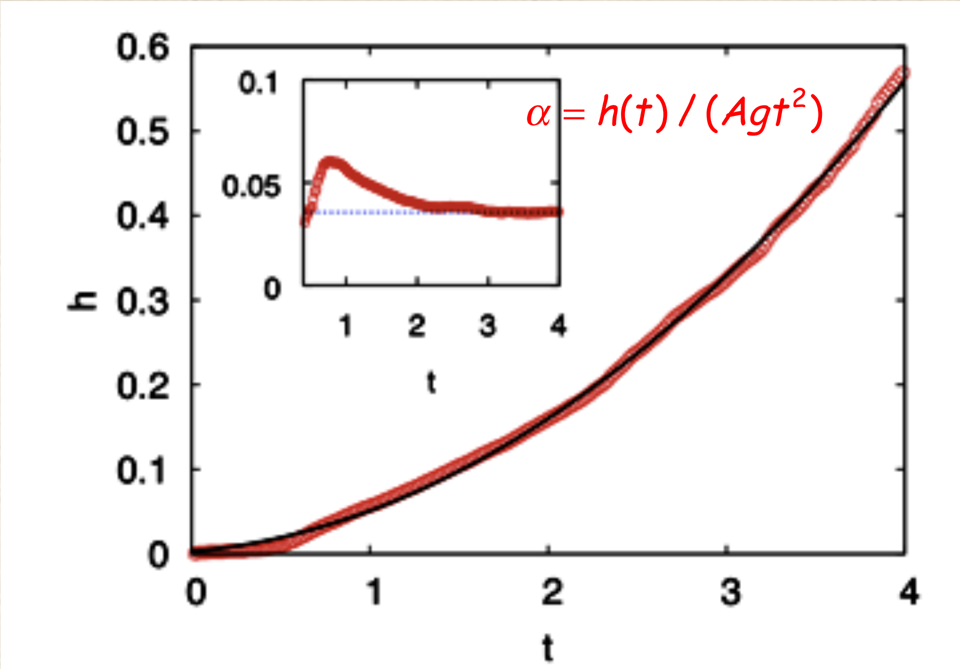
Turbulent mizing layer of width h(t) $h(t) \approx Agt^2$

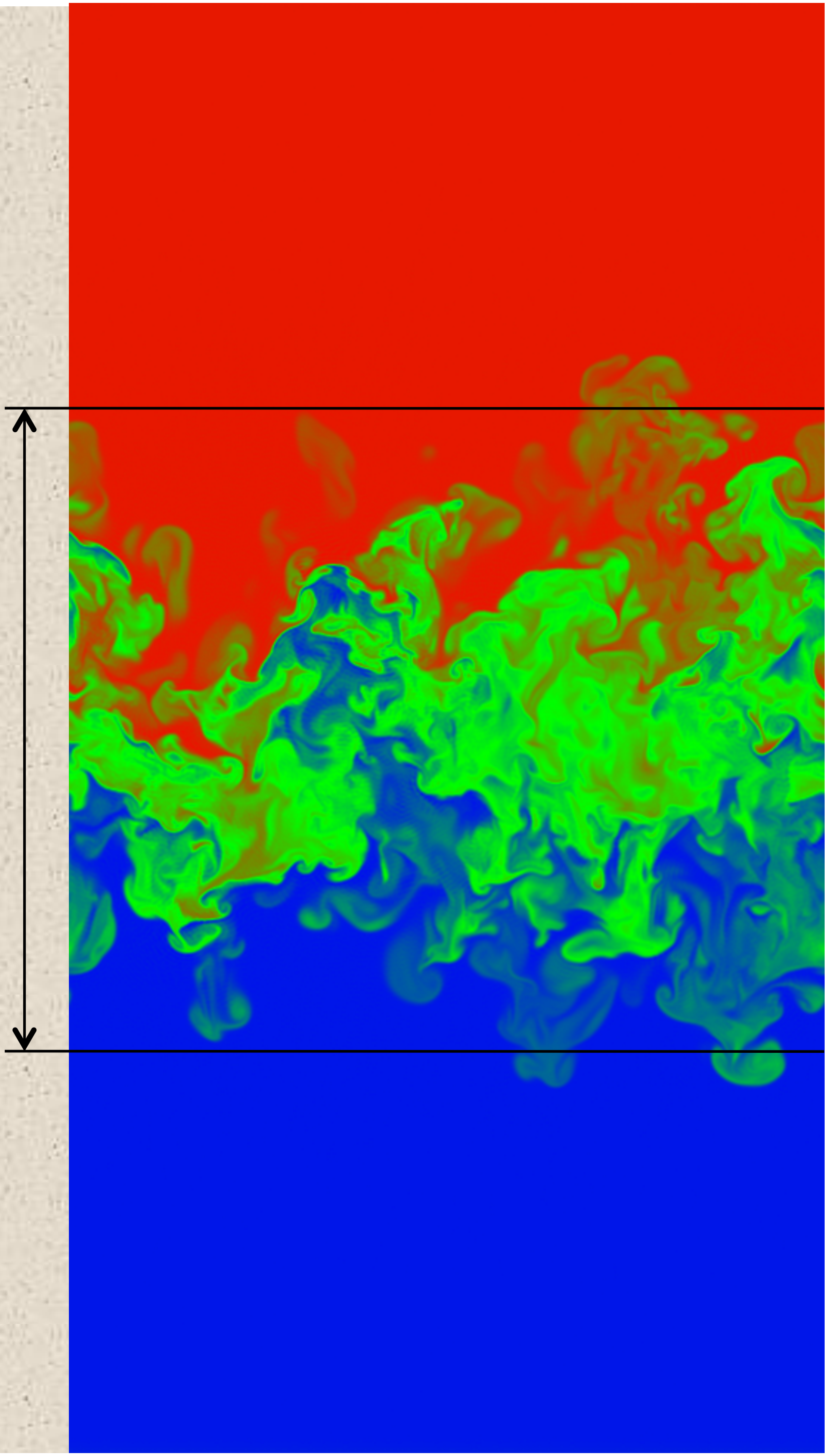
Kinetic energy pumped in the system at a rate $\varepsilon_I \simeq \dfrac{u^3}{h} \simeq (Ag)^2 t$

-> time evolving turbulence

Evolution of the mixing layer







Small scale theory of RT turbulence

M. Chertkov, PRL **91** (2003)

Ansatz: buoyancy negligible at small scales

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta\mathbf{g}T \\ \partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T \end{cases}$$

$$\beta g \delta_r T \ll \frac{\delta_r u^2}{r}$$

(small Richardson number)

passive temperature in turbulent flow with time dependent flux

$$\varepsilon(t) \approx (Ag)^2 t$$

small scale fluctuations follow Kolmogorov-Obukhov scaling

$$\delta_r u(t) \simeq u_L(t)\left(\frac{r}{h(t)}\right)^{1/3} \simeq (\beta g \theta_0)^{2/3} t^{1/3} r^{1/3}$$

$$\delta_r T(t) \simeq \theta_0 \left(\frac{r}{h(t)}\right)^{1/3} \simeq \frac{\theta_0}{(\beta g \theta_0)^{1/3}} t^{-2/3} r^{1/3}$$

consistency:

$$Ri = \frac{\beta g \delta_r T(t)}{\delta_r u^2(t)/r} \simeq \left(\frac{r}{L(t)}\right)^{2/3} \rightarrow 0$$

Inconsistent in 2D where the energy flows to large scale (buoyancy dominated)

RT turbulence in 2D

M. Chertkov, PRL **91** (2003)

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta \mathbf{g} T \\ \partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T \end{cases}$$

Buoyancy balances inertia at all scales

$$\beta g \delta_r T \approx \frac{\delta_r u^2}{r} \qquad \text{(Ri=O(1))}$$

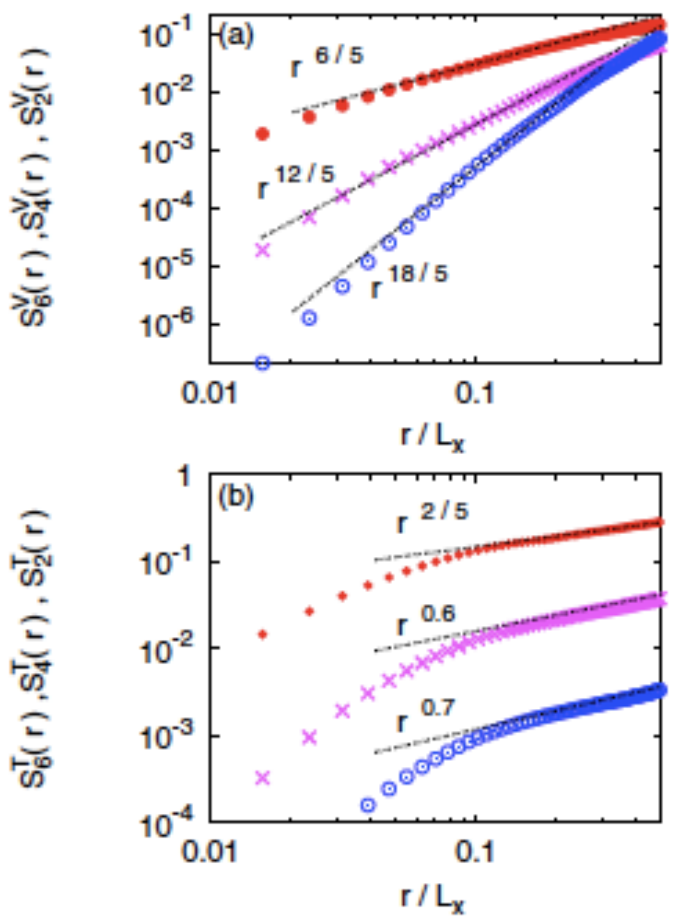
direct cascade of temperature fluctuations

$$\varepsilon_T(t) \simeq \frac{\delta_r u \delta_r T^2}{r} \simeq \frac{\delta_r u^5}{r^3(\beta g)^2} \simeq \frac{u_L^5}{h^3(\beta g)^2}$$

small scale fluctuations follow Bolgiano scaling

$$\delta_r u(t) \simeq u_L(t)\left(\frac{r}{h(t)}\right)^{3/5} \simeq (\beta g \theta_0)^{2/5} t^{-1/5} r^{3/5}$$

$$\delta_r T(t) \simeq \theta_0 \left(\frac{r}{h(t)}\right)^{1/5} \simeq \frac{\theta_0}{(\beta g \theta_0)^{1/5}} t^{-2/5} r^{1/5}$$

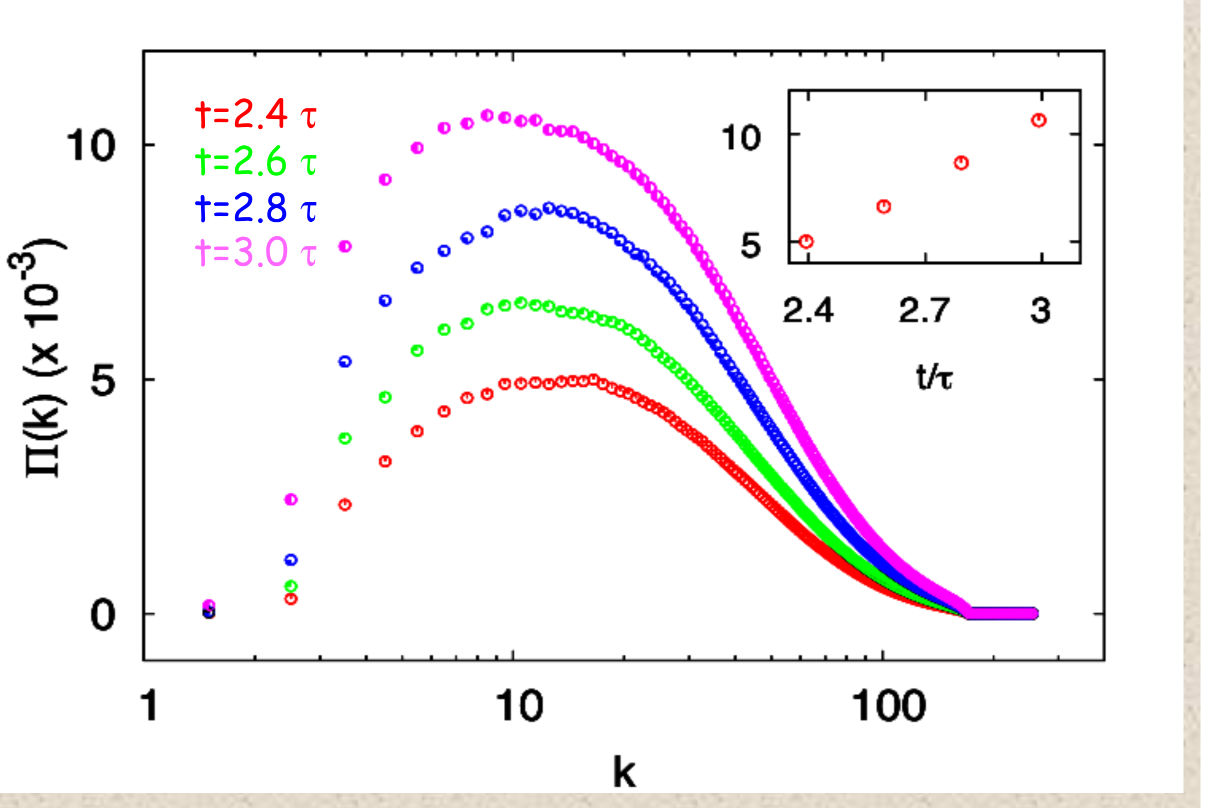


A.Celani, A.Mazzino, L.Vozella, PRL **96** (2006)

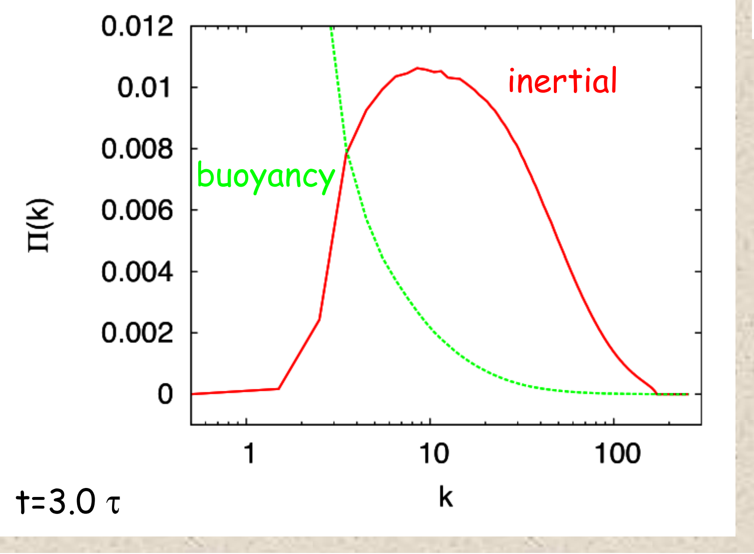
Energy flux

Inertial range of scale-independent kinetic energy flux $\Pi(k,t)$

$$\varepsilon = \Pi(k,t) \approx \frac{u_L^3}{h} \simeq t$$



RT as an adiabatically evolving turbulence (small scales adapt to large scale variations)



Buoyancy contribution to the energy flux becomes negligible at small scales

$$Ri = \frac{\beta g \delta_r T(t)}{\delta_r u^2(t) / r} \simeq \left(\frac{r}{L(t)} \right)^{2/3} \to 0$$

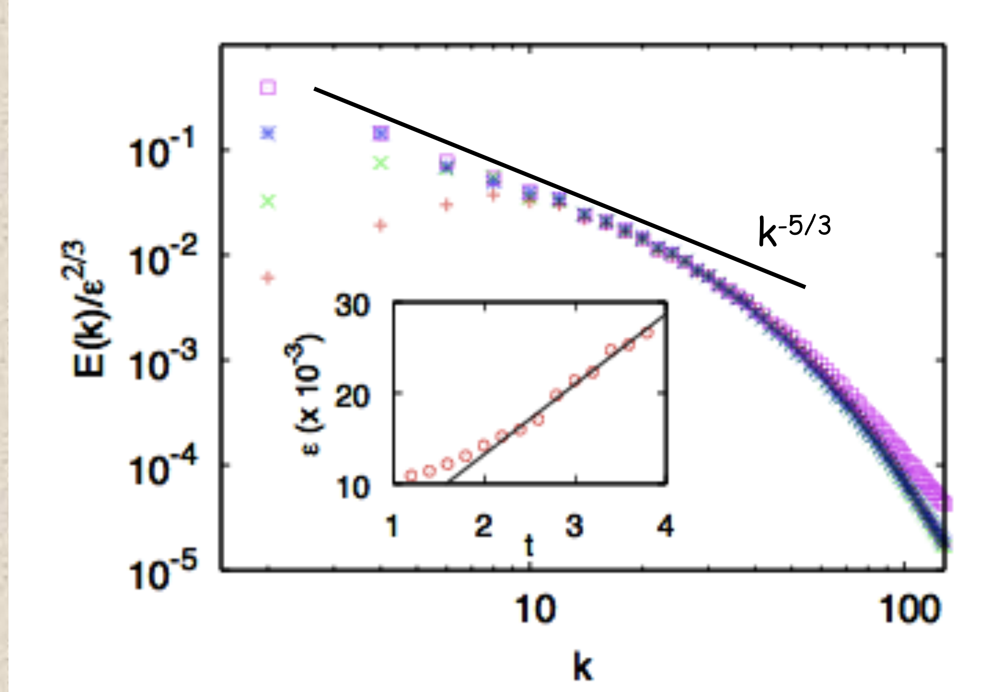
Spectra collapse
(Kolmogorov scaling)

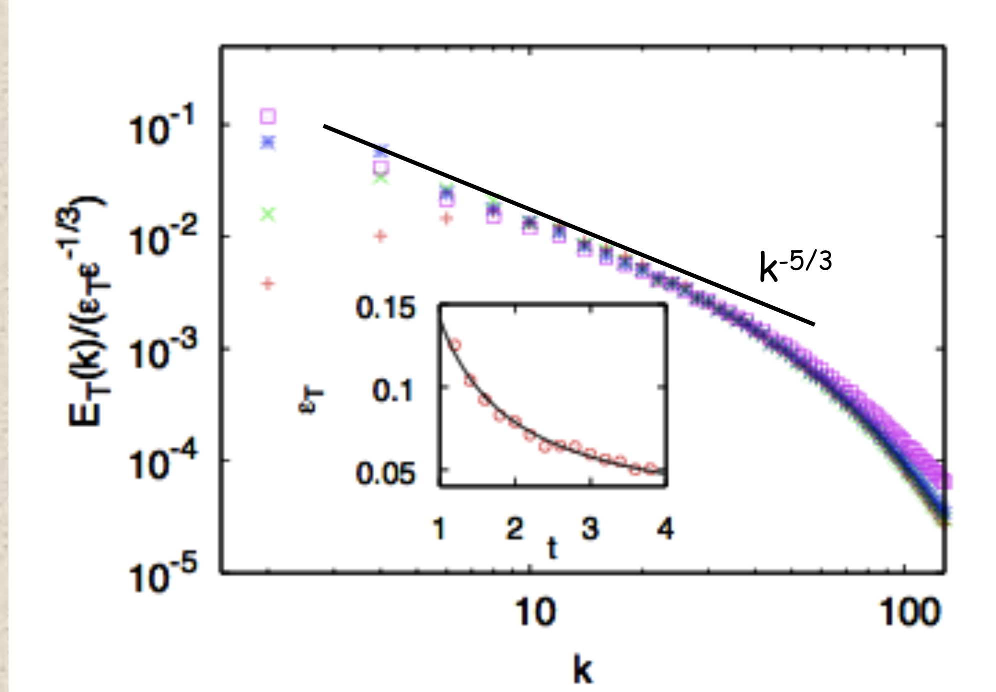
Collapse of kinetic energy and temperature variance spectra at t/τ=1.0, 1.4, 1.8, 3.8

Insets: time evolution of kinetic energy dissipation $\varepsilon \approx t$ and temperature variance dissipation $\varepsilon_T \approx t^{-1}$

Spatial-temporal scaling in agreement with dimensional theory

$$E(k,t) \simeq t^{2/3} k^{-5/3}$$
$$E_T(k,t) \simeq t^{-4/3} k^{-5/3}$$





A remark on the evolution of the spectrum

Time dependent input $\varepsilon_I(t)$ $$\frac{dE}{dt} = \varepsilon_I(t) - \varepsilon_\nu$$

Adiabatic evolution of a Kolmogorov spectrum $E(k,t) = C\varepsilon^{2/3}(t)k^{-5/3}$
is compatible with energy balance only if the integral scale L evolves according to

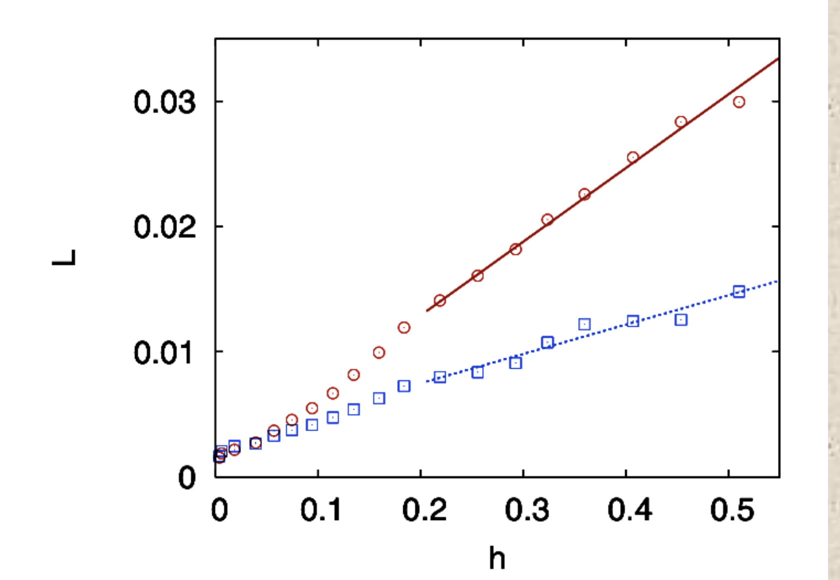
$$\frac{d}{dt}(L\varepsilon)^{2/3} = \varepsilon$$

In RT turbulence

$$\varepsilon_I(t) = -\frac{dP}{dt} \simeq t$$

which gives

$$L(t) \simeq t^2 \propto h(t)$$



Velocity correlation scale L(t) vs mixing layer width h(t)

Mixing efficiency: turbulent heat transport

In Rayleigh-Taylor turbulence dimensionless numbers Re, Ra, Nu grow in time

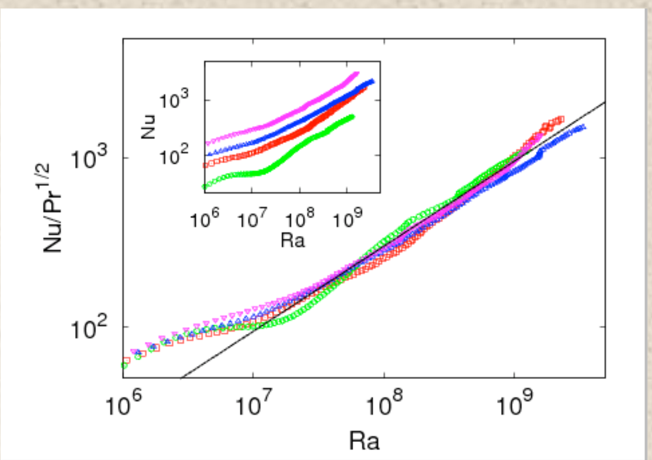
$Ra = \dfrac{\beta g \theta_0 h^3}{\kappa \nu}$	$Pr = \dfrac{\nu}{\kappa}$
$Re = \dfrac{u_{rms} h}{\nu}$	$Nu = \dfrac{< u_z T > h}{\kappa \theta_0}$

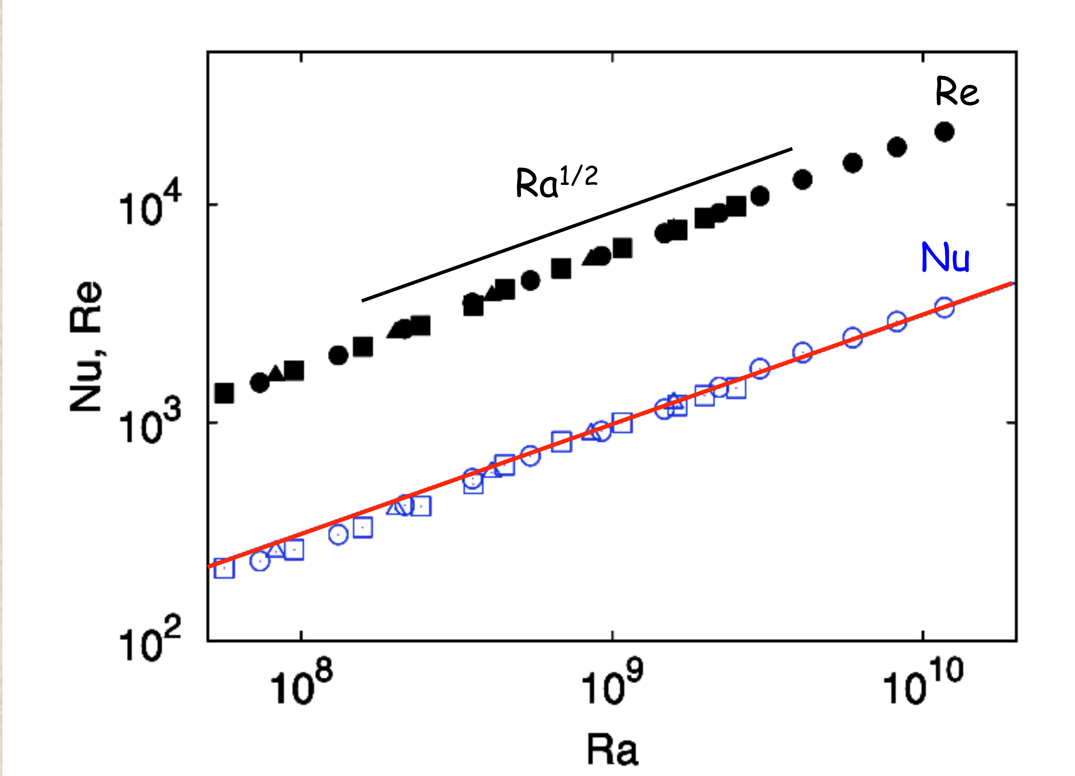
Dimensionally, because $u \simeq \beta g \theta_0 t$ and $h \simeq \beta g \theta_0 t^2$ one has

$$Re \simeq Pr^{-1/2} Ra^{1/2}$$

$$Nu \simeq Pr^{1/2} Ra^{1/2}$$

ultimate state regime
[Kraichnan, 1962]





256x256x1024 (□) 512x512x2048 (O) 1024x1024x2048 (Δ)

Viscoleastic Rayleigh-Taylor turbulence

Numerical model: Oldroyd-B model + temperature within Boussinesq approximation

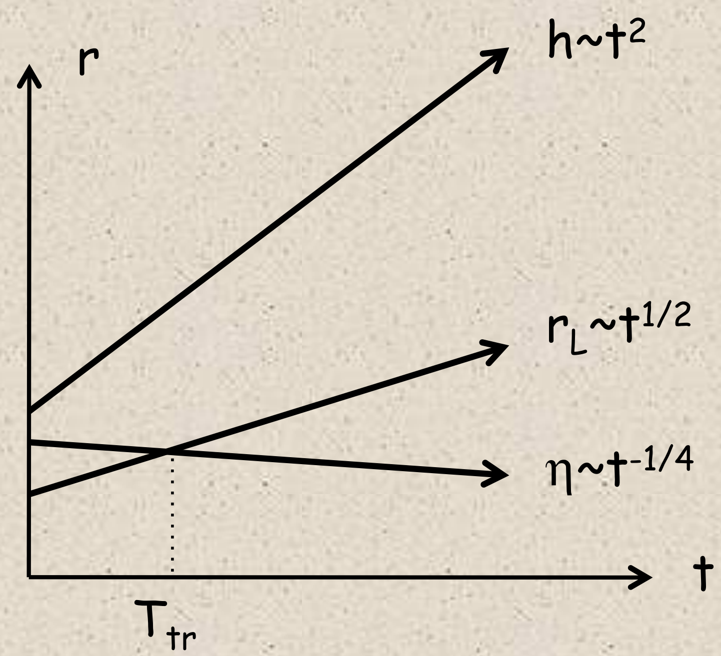
$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \Delta \mathbf{u} - \beta \mathbf{g} T + (2\nu\eta / \tau_p) \nabla \cdot \sigma \\ \partial_t T + \mathbf{u} \cdot \nabla T = \kappa \Delta T \\ \partial_t \sigma + \mathbf{u} \cdot \nabla \sigma = (\nabla \mathbf{u})^T \cdot \sigma + \sigma \cdot (\nabla \mathbf{u}) - (2 / \tau_p)(\sigma - \mathbf{1}) \end{cases}$$

small Atwood: no direct effect of temperature fluctuations on polymers

TABLE I. Parameters of the simulations.

Run	$N_{x,y}$	N_z	$L_{x,y}$	L_z	θ_0	βg	$\nu = \kappa$	κ_p	γ	τ_p
N	512	1024	2π	4π	1	0.5	3×10^{-4}	—	—	—
A	512	1024	2π	4π	1	0.5	3×10^{-4}	1×10^{-3}	0.2	1
B	512	1024	2π	4π	1	0.5	3×10^{-4}	1×10^{-3}	0.2	2
B2	512	1024	2π	4π	1	0.5	3×10^{-4}	3×10^{-3}	0.2	2
C	512	1024	2π	4π	1	0.5	3×10^{-4}	1×10^{-3}	0.2	10

Coil-stretch transition in RT turbulence



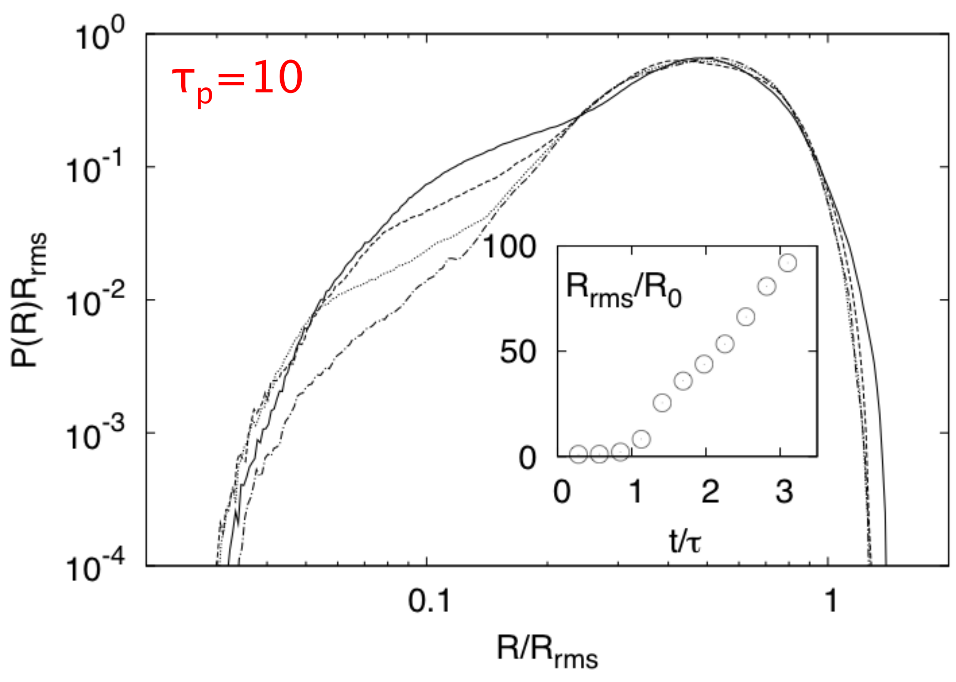
Kolmogorov timescale decreases in time

$$\tau_\eta \simeq \nu^{1/2}(Ag)^{-1}t^{-1/2}$$

Weissenberg number increases and coil-stretch transition at time

$$T_{tr} \simeq \frac{\nu}{(\tau_p Ag)^2}$$

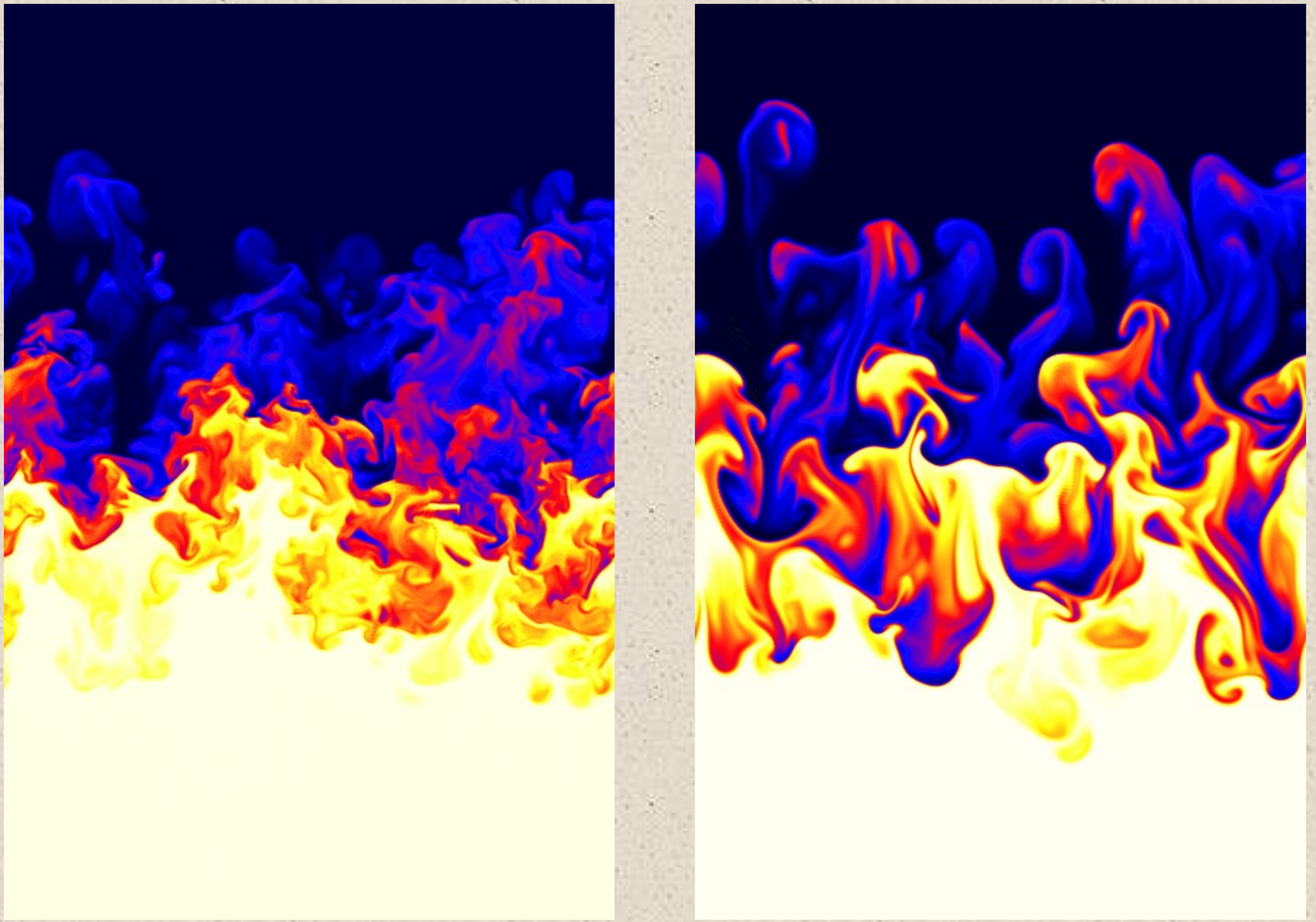
Lumley scale: $\tau_p/\tau(r_L)=1$



For $t>T_{tr}$:

* linear growth of mean elongation (from energy balance)
* self-similar evolution of the right tail of pdf (viscoelastic effect: polymers reduce velocity gradients)

temperature field



Newtonian viscoelastic

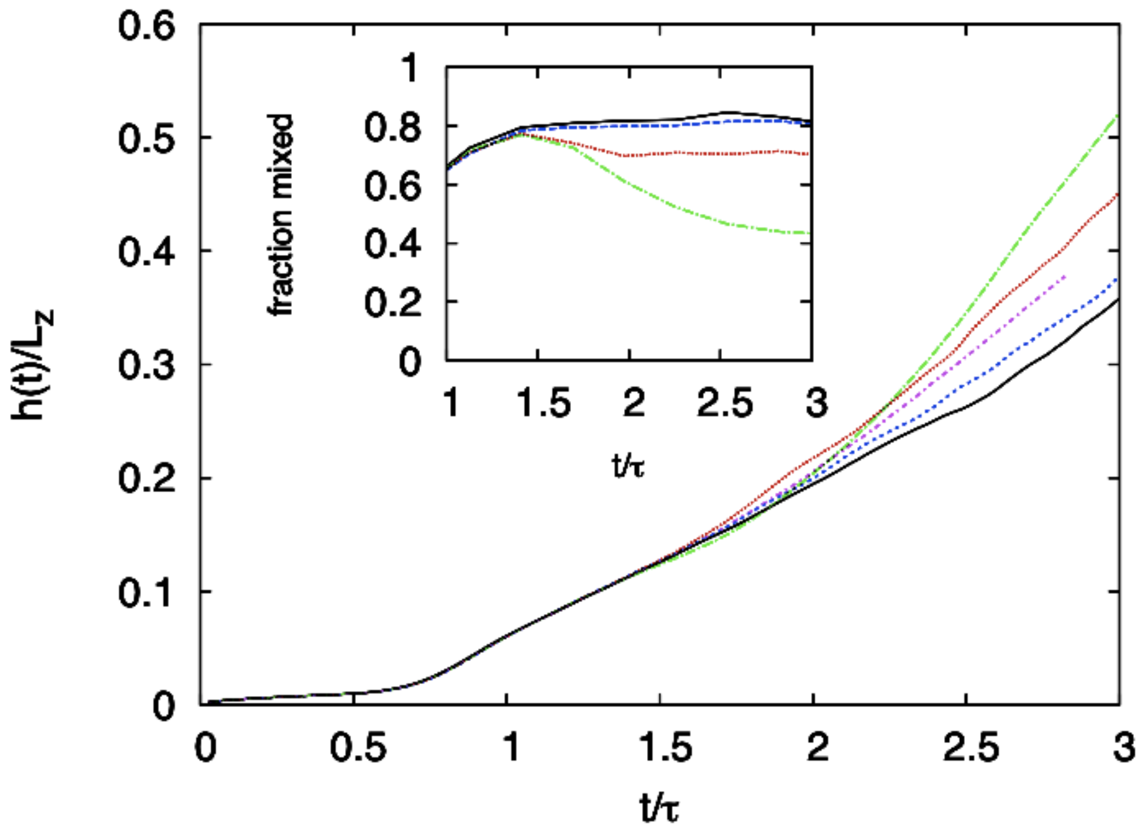
Two effects induced by polymers:

- faster thermal plumes: acceleration of mixing layer growth ("drag" reduction)
- small scale turbulence reduced: heat transfer enhancement

Accelerated growth of the mixing layer

When polymers are stretched mixing layer grows faster than in the Newtonian case (up to 50% larger at final time).

Consistent with the speed-up of RT instability (linear analysis)
G.Boffetta, A.Mazzino, S.Musacchio, L.Vozella, JFM **643** (2010)



Interpretation in terms of drag reduction

Assuming a linear temperature profile: $\Delta P = P(0) - P(t) \approx \frac{1}{6} Agh(t)$

Kinetic energy of large scale plumes: $K \approx \frac{1}{2} \dot{h}^2(t)$

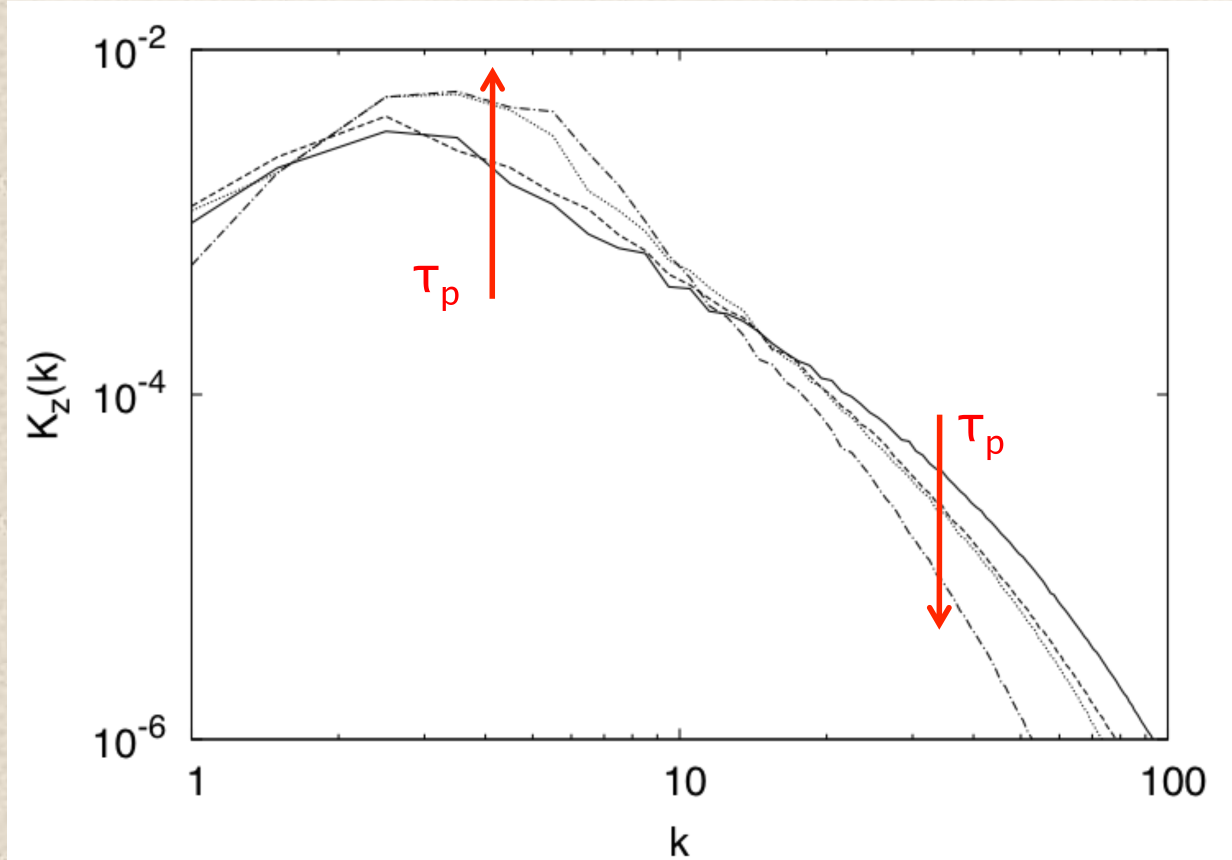
Drag coefficient

$$f = \frac{\Delta P}{K} = \frac{1}{3}\frac{h}{\dot{h}^2} = \frac{1}{12\alpha}$$

where $h(t) = \alpha Agt^2$

30% reduction of drag coefficient f

Energy spectra



more energy at large scales (mixing layer accelerates)

less energy at small scales (reduced mixing)

consistent with what observed in homogeneous-isotropic turbulence

Energy balance and asymptotic behavior

$$E = P + K + \Sigma = -\beta g < zT > + \frac{1}{2} < u^2 > + \frac{\nu\eta}{\tau}[< tr(\sigma) > -3]$$

Energy balance

$$\frac{dP}{dt} = \beta g < wT > = \frac{d}{dt}\frac{1}{2} < u^2 > + \frac{d}{dt}\frac{\nu\eta}{\tau} < tr(\sigma) > - \varepsilon_\nu - \varepsilon_\Sigma$$

with $\varepsilon_\Sigma = \frac{2\nu\eta}{\tau^2} < tr(\sigma) >$

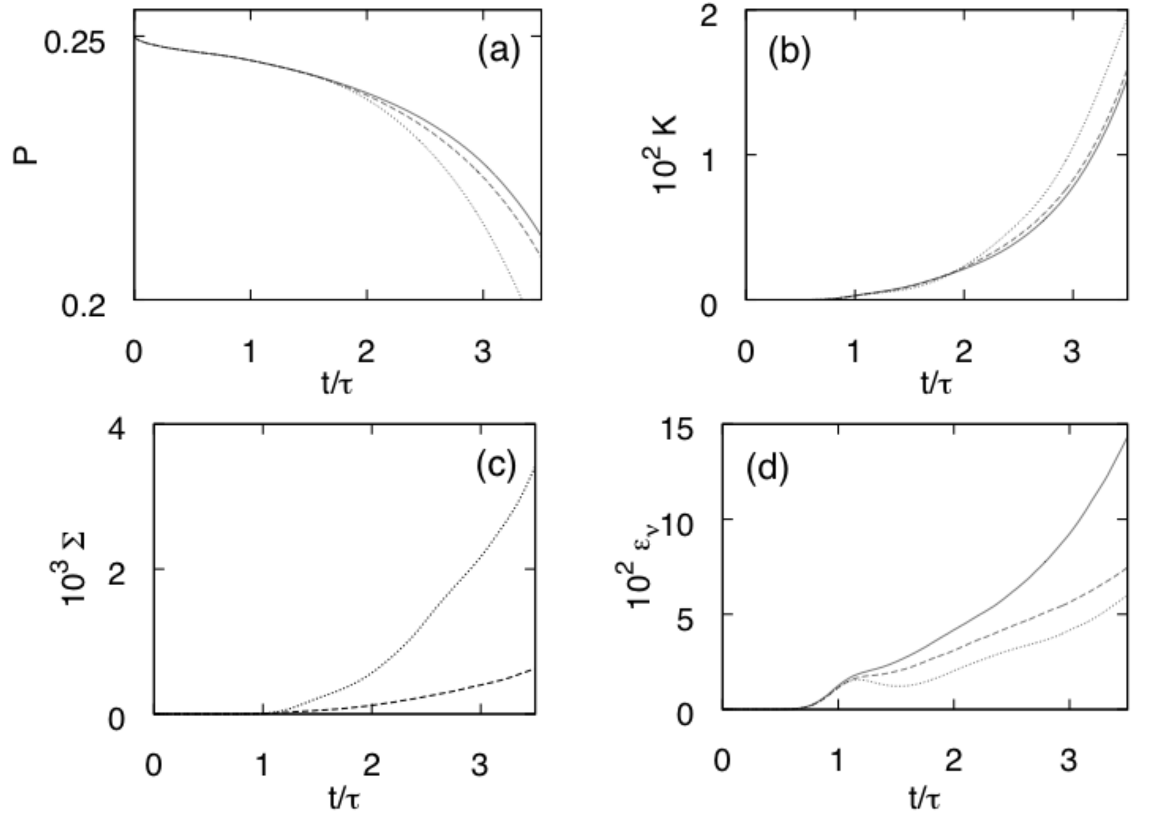
At variance with Newtonian case not all terms in energy balance can have same temporal scaling

We observe that

$$\Sigma \approx K \approx t^2$$

and elastic dissipation dominates

$$\frac{\varepsilon_\Sigma}{\varepsilon_\nu} \approx t$$



Energy balance and asymptotic behavior

$$E = P + K + \Sigma = -\beta g < zT > + \frac{1}{2} < u^2 > + \frac{\nu\eta}{\tau}[< tr(\sigma) > -3]$$

Energy balance

$$\frac{dP}{dt} = \beta g < wT > = \frac{d}{dt}\frac{1}{2} < u^2 > + \frac{d}{dt}\frac{\nu\eta}{\tau} < tr(\sigma) > -\varepsilon_\nu - \varepsilon_\Sigma$$

with $\varepsilon_\Sigma = \frac{2\nu\eta}{\tau^2} < tr(\sigma) >$

At variance with Newtonian case not all terms in energy balance can have same temporal scaling

We observe that

$$\Sigma \approx K \approx t^2$$

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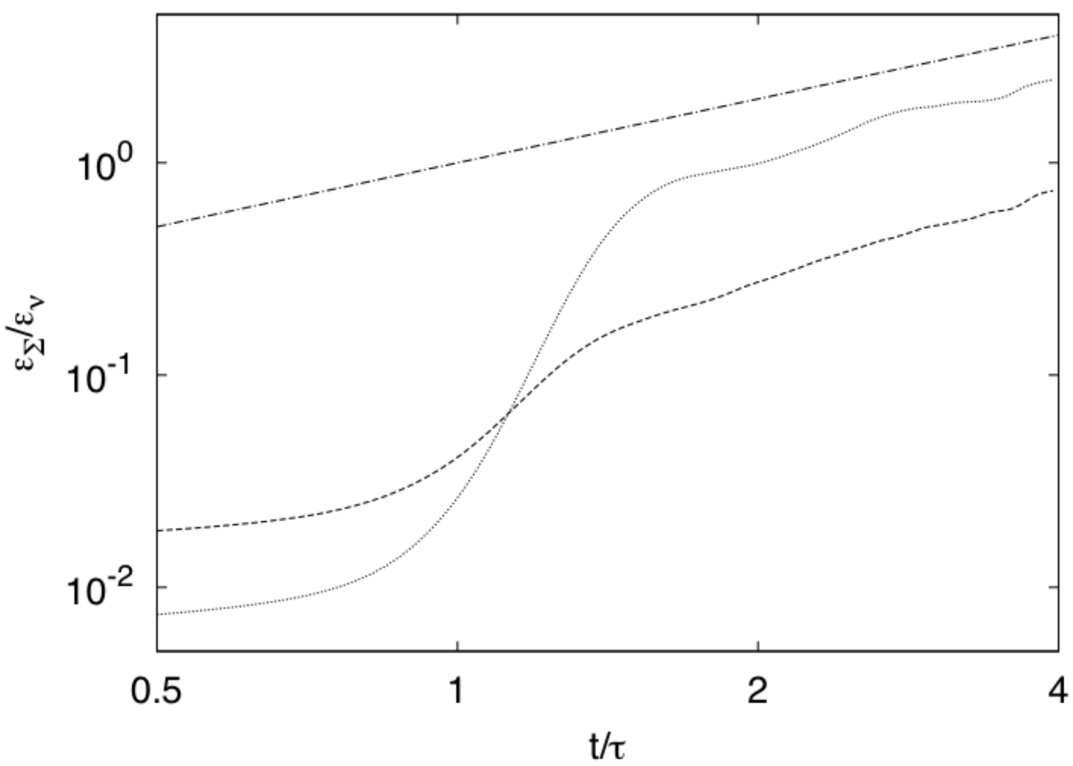
$$\frac{\varepsilon_\Sigma}{\varepsilon_\nu} \approx t$$



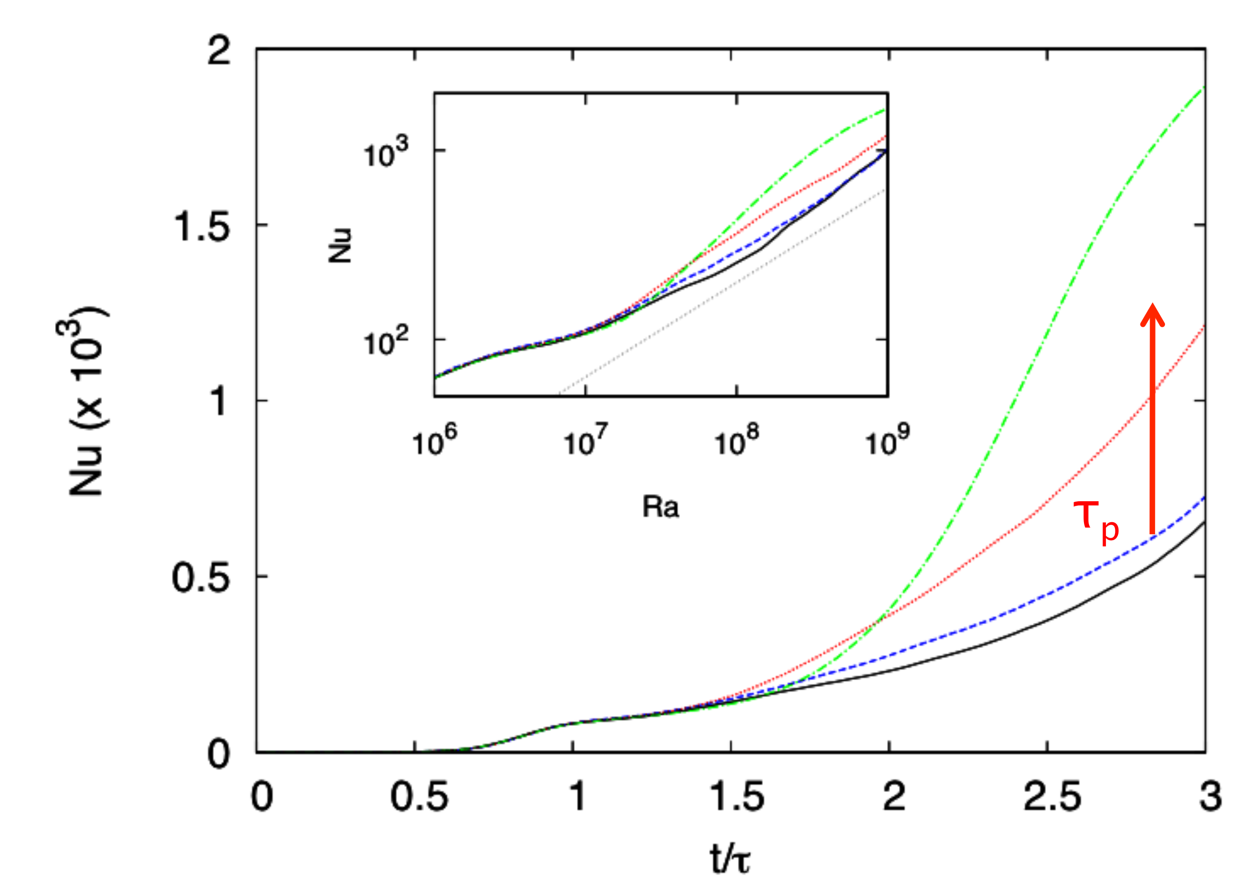
Heat transfer enhancement

Nu is larger in presence of polymers.

Nu increases more than Ra therefore

$$Nu = C\,Pr^{1/2}\,Ra^{1/2}$$

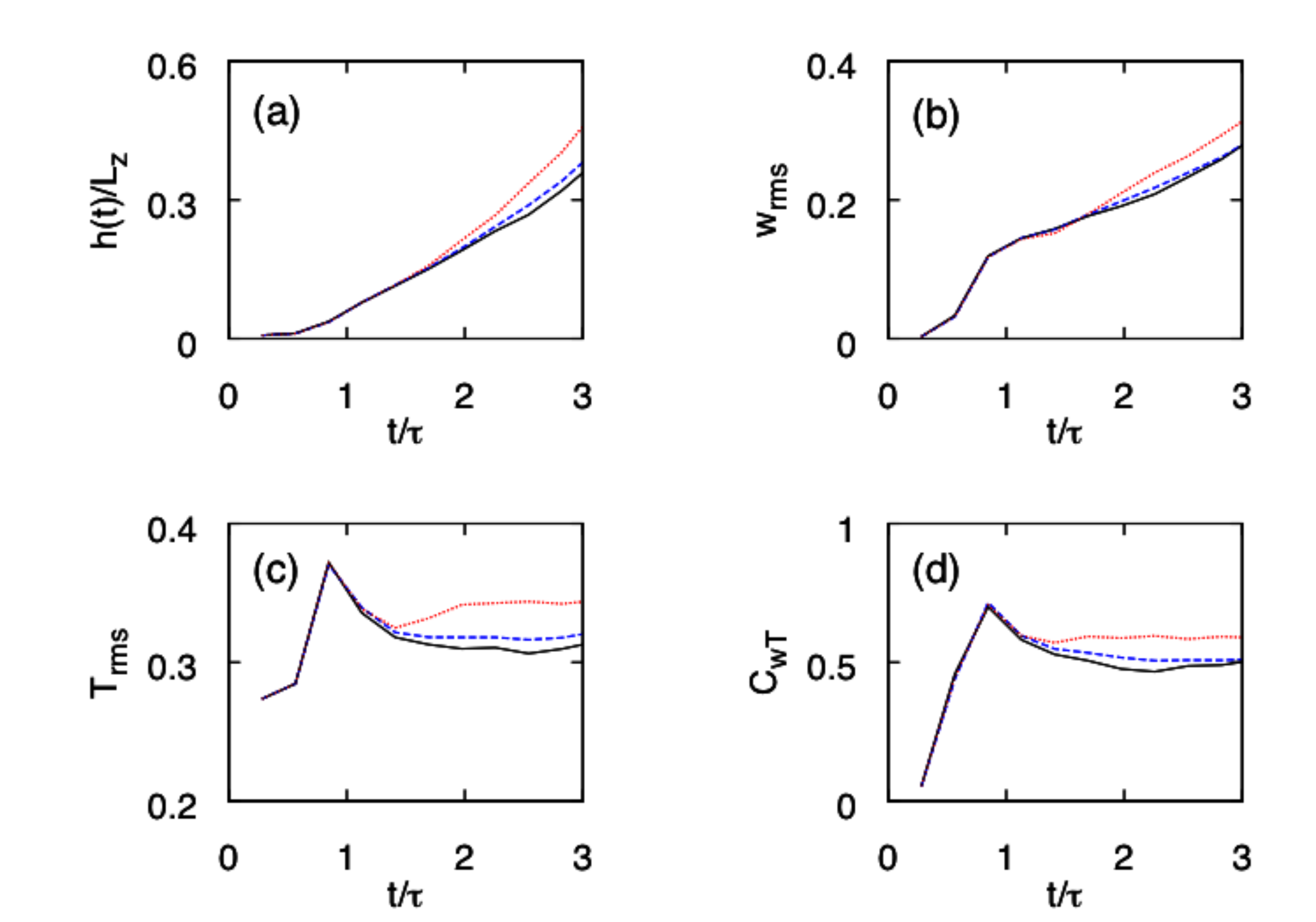
C increases with polymers (about 50%)



$\tau_p = 0$
$\tau_p = 1$
$\tau_p = 2$
$\tau_p = 10$

The origin of heat transfer enhancement ?

$$Nu = \frac{1}{k\theta_0} h w_{rms} T_{rms} C(w,T) \qquad C(w,T) = \frac{\langle wT \rangle}{w_{rms} T_{rms}}$$



faster thermal plumes (h and w), reduced turbulent mixing (T) and stronger correlation between T and w (more coherent plumes)

$\tau_p=0$
$\tau_p=1$
$\tau_p=2$

Effects of polymers: 30% increase of h, 15% increase for w_{rms}, 10% increase for T_{rms}, 20% increase for C_{wt}