# What can Thermal Convection Teach Us about the Nature of Turbulence?

1. **Homogeneous isotropic turbulence** does not provide transports, exhibits vanishing correlations.
2. **Simplest realistic turbulent systems** are homogeneous in two spatial dimensions and in time. They are characterized by finite correlations and by spontaneous coherent structures.
3. **Thermal convection** in extended horizontal fluid layers has become a paradigmatic example for the study of turbulence because of several properties:

- It represents the **simplest mechanism** of hydrodynamic instability.
- It permits **high degrees of visualization** through scalar quantities such as temperature (shadowgraph, radiation etc.) or suspended particles ( thermochromatic liquid crystals, condensed water in clouds etc.).
- It is **not subject to advection** out of the observational frame.

**Friedrich Busse, Univ. Bayreuth**
**KITP, Mar 30, 2011**

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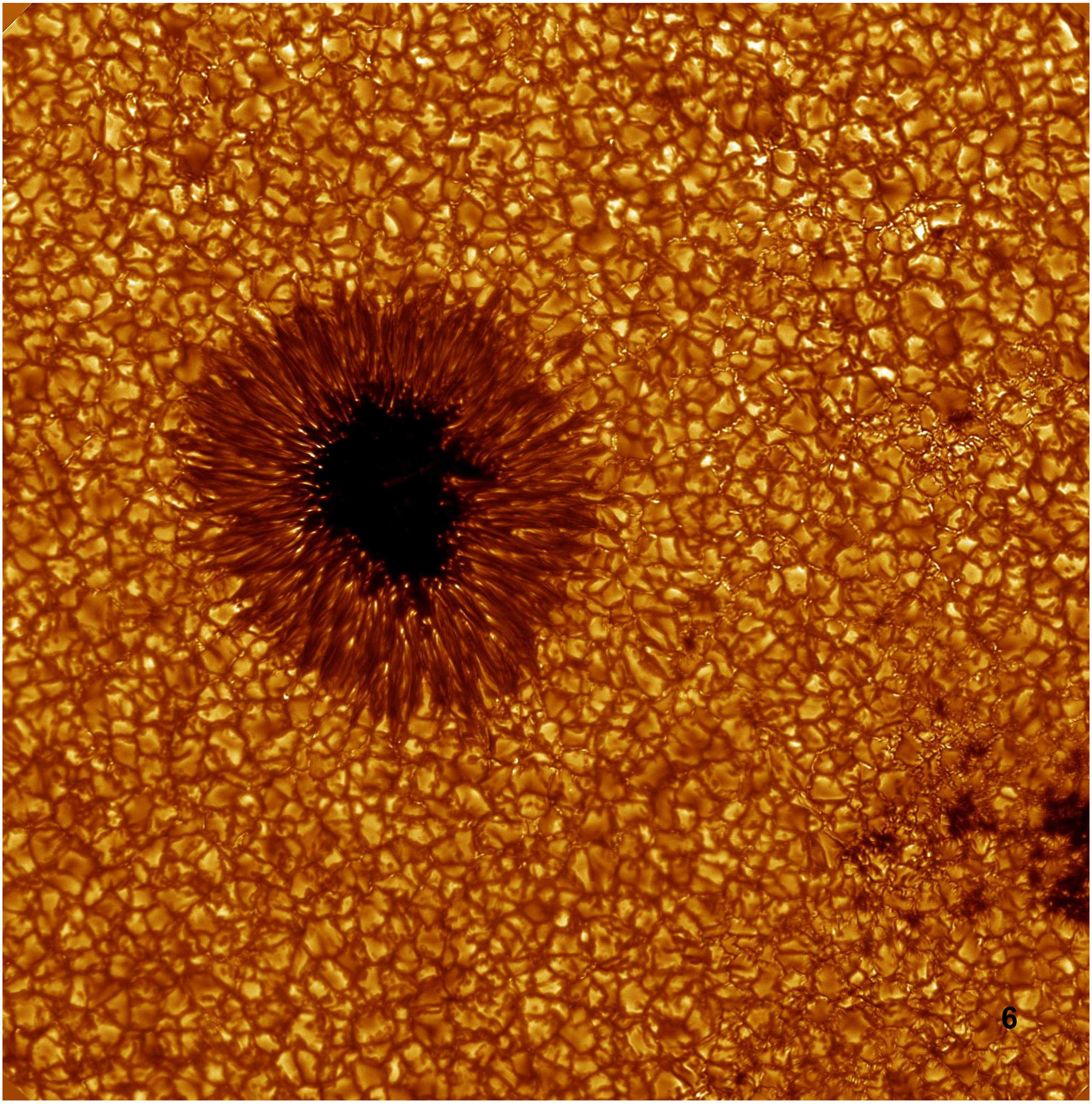
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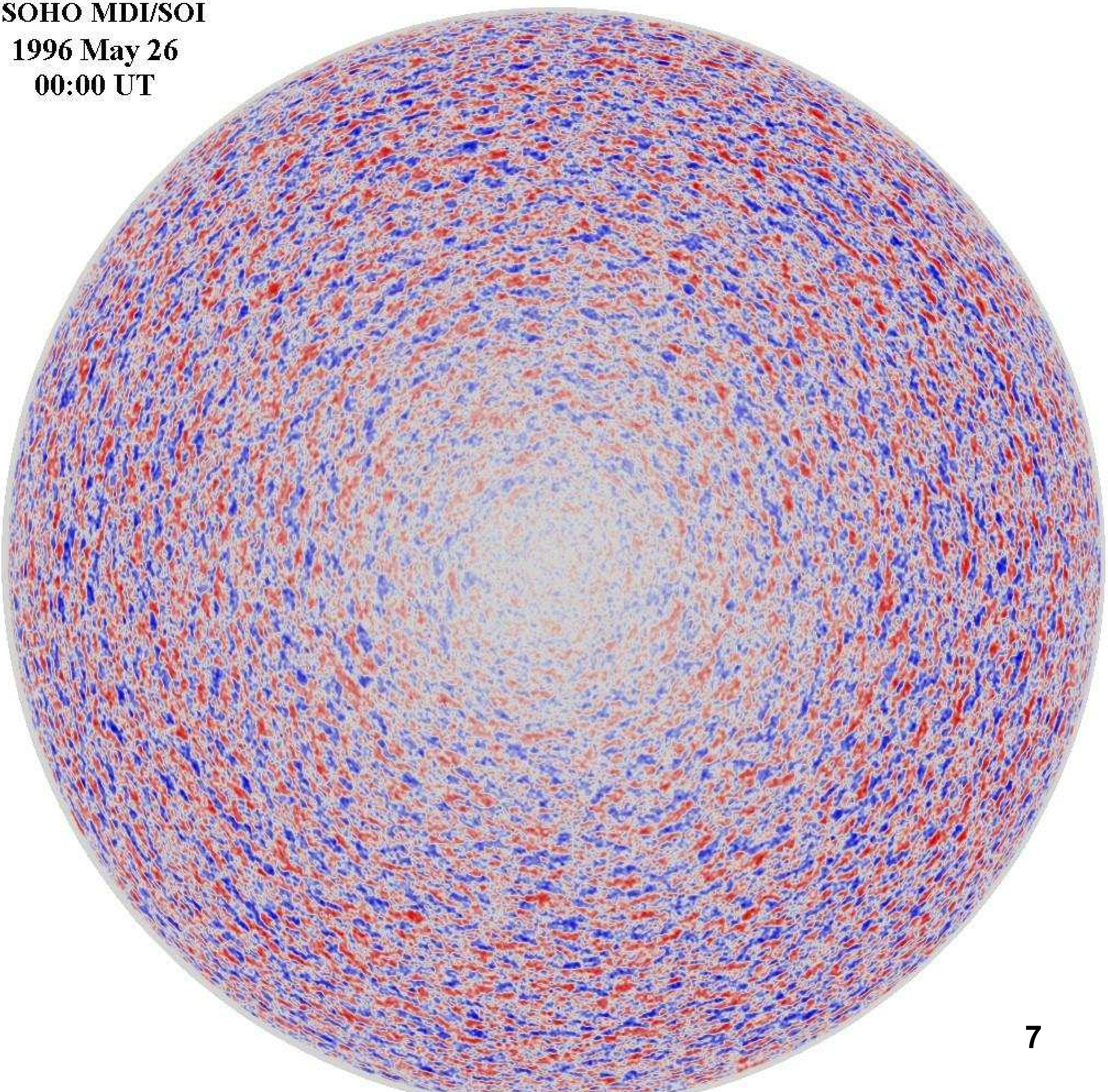




NASA/MSFC Hathaway

# Systems that are Homogeneous in Two Spatial Dimensions and in Time

There are **two routes to complex fluid flow** with increasing control parameter such as Reynolds number or Rayleigh number:

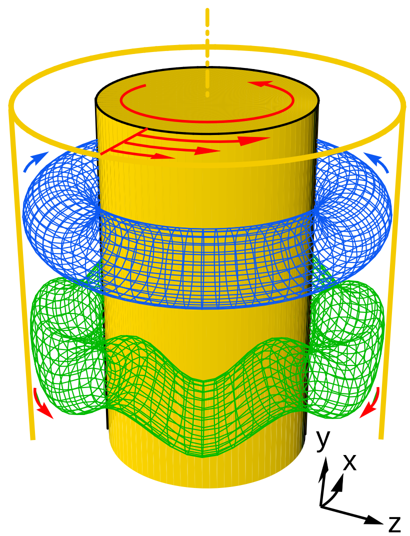
1. **Random initial conditions** lead to increasingly turbulent flows which typically exhibit spontaneous coherent structures.

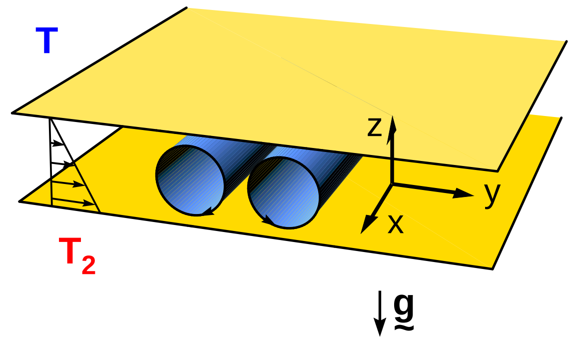
2. **Controlled initial conditions** permit bifurcation sequences of spatially periodic, possibly unstable solutions of the basic Navier-Stokes equations. Some of these solutions embody the characteristic properties of the coherent structures.

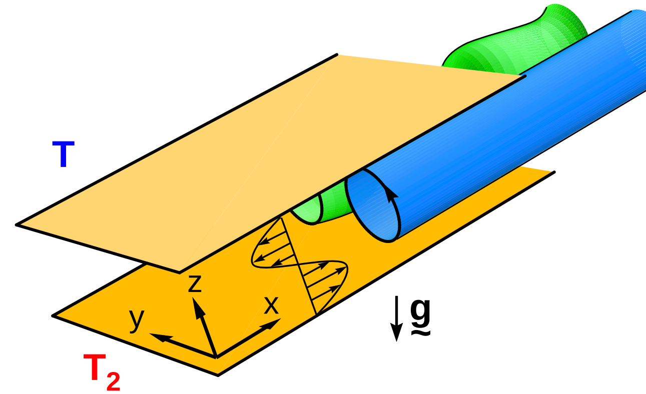
**Nomenclature:**

- **Primary Solution** or **Basic State** reflects all symmetries of external conditions.
- **Secondary Solutions** generically assume the form of "rolls"or "stripes".
- **Tertiary, Quaternary** and **Higher Order Solutions** exhibit large varieties of patterns depending on the physics and on the parameters of the system.

**Taylor-Couette-System** | **Rayleigh-Bénard-System** | **Inclined Convection Layer**




**Taylor-Couette-System:** Couette Flow → Taylor-Vortices → Wavy Vortices → Doubly Wave Vortices; Taylor-Vortices → Twists → Domai States

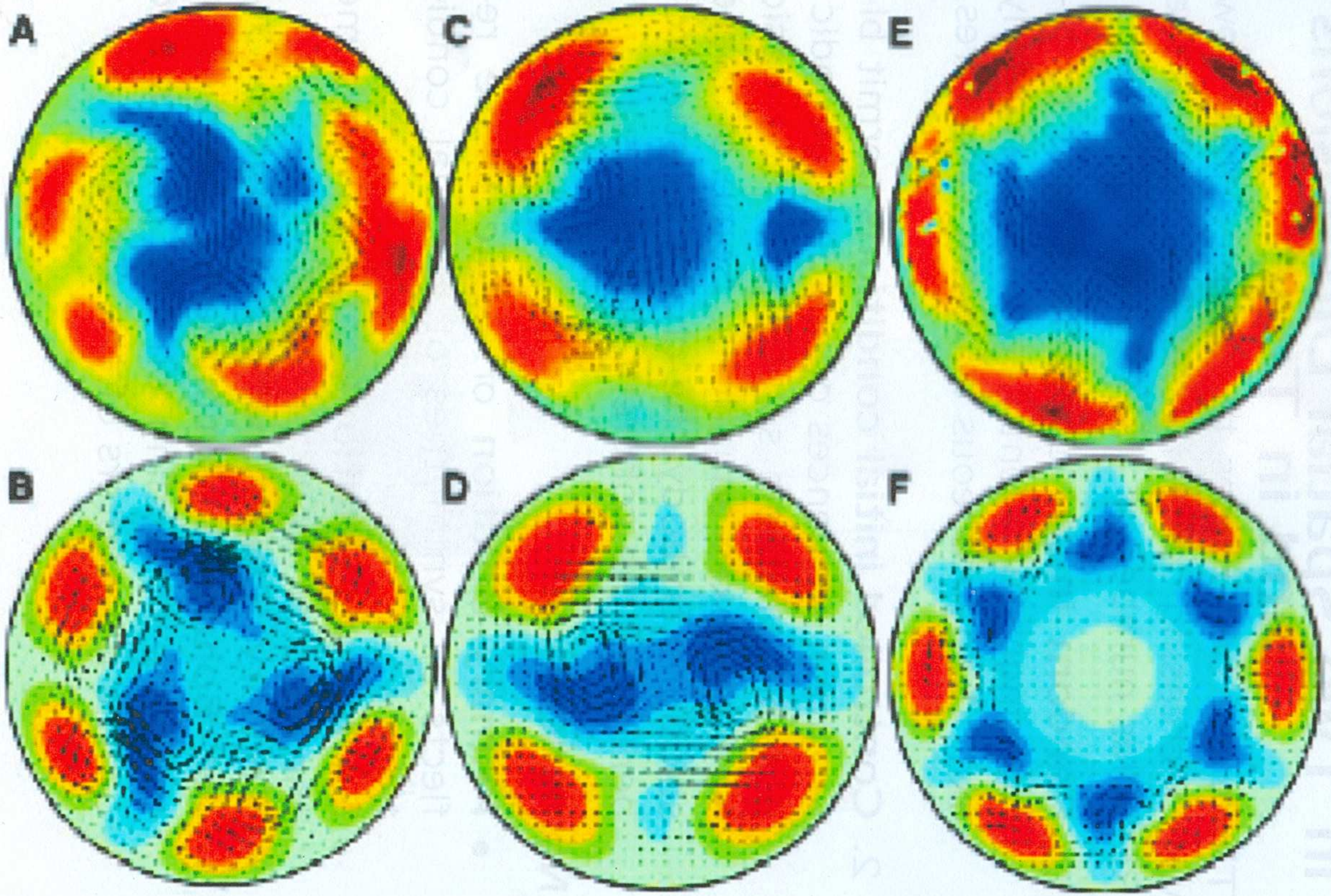
**Rayleigh-Bénard-System:** Static State → Convection Rolls → Knot Convection → Oscillating Knot Convection; Convection Rolls → Bimodal Convection → Oscillating Bimodal Convection

**Inclined Convection Layer:** Cubic Profile Flow → Longitudinal Rolls → Wavy Rolls → Transversely Drifting Wavy Rolls g

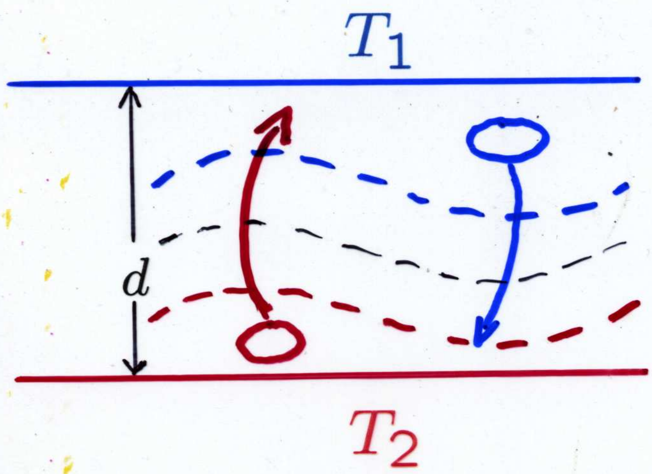
# Tertiary Solutions in Pipe Flow

Hof et al.
2004

Faisst &
Eckhardt
2003



# Rayleigh number




$$\rho = \rho_0\big(1-\gamma(T-T_0)\big)$$

**Energy gain:** $L_P = C_1 \rho_0 \gamma (T_2 - T_1) g V \frac{V}{\kappa/d}$

$\kappa/d = thermal\,velocity$

**Dissipation:** $L_D = C_2 \rho_0 \nu \frac{V}{d^2} V$

**For onset of convection:** $L_P \geq L_D$

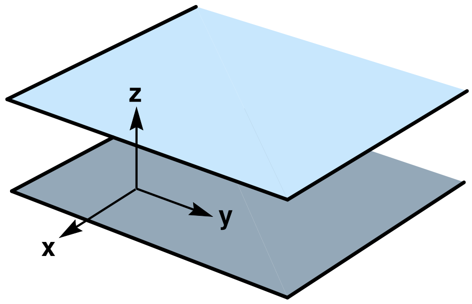
**or:** $R \equiv \frac{L_P C_2}{L_D C_1} \geq \frac{C_2}{C_1} \equiv R_c$

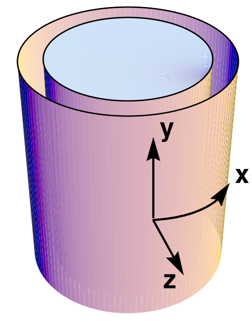
**Rayleigh number:** $R \equiv \frac{\gamma(T_2 - T_1) g d^3}{\nu\kappa}$

**Prandtl number:** $P \equiv \frac{\nu}{\kappa}$

# Secondary Solutions

Physical conditions are homogeneous in two spatial dimensions and in time t.




## Basic equation

$$L\varphi - RB\varphi = N\varphi\varphi + V\frac{\partial}{\partial\tau}\varphi$$

Linearized problem : $\varphi_0 \propto \exp\{\, i\mathbf{q}\cdot\mathbf{x} + \sigma t \,\}$

Typical case : $\sigma_i = 0$ , $R_{min}$ nondegenerate

## Steady Rolls :

$$\varphi = \sum_{m,n>0} a_{mn} \exp\{i m\alpha y\}\, f_n(z) + \dots$$

$$\alpha \equiv |\mathbf{q}_c|, \qquad f_n(z) = (-1)^{n-1} f_n(-z)$$

for symmetry about midplane

## Symmetry Properties

| | | |
|---|---|---|
| translation in time | $\frac{\partial}{\partial\tau}\varphi = 0$ | defining rolls |
| translation in longit. direct. | $\frac{\partial}{\partial x}\varphi = 0$ | defining rolls |
| transverse periodicity | $\varphi(y+\frac{2\pi}{\alpha}, z) = \varphi(y, z)$ | defining rolls |
| transverse reflection | $\varphi(-y, z) = \varphi(y, z)$ or $a_{-mn} = a_{mn}$ | |
| inversion about roll axis* | $\varphi(\frac{\pi}{\alpha} - y, z) = -\varphi(y, z)$ or $a_{mn} = 0$ for m+n = odd | |

* narrow gap approximation + nearly corotating cylinders in the case of Taylor vortices
requires Bussinesq approximation for Rayleigh-Bénard problem

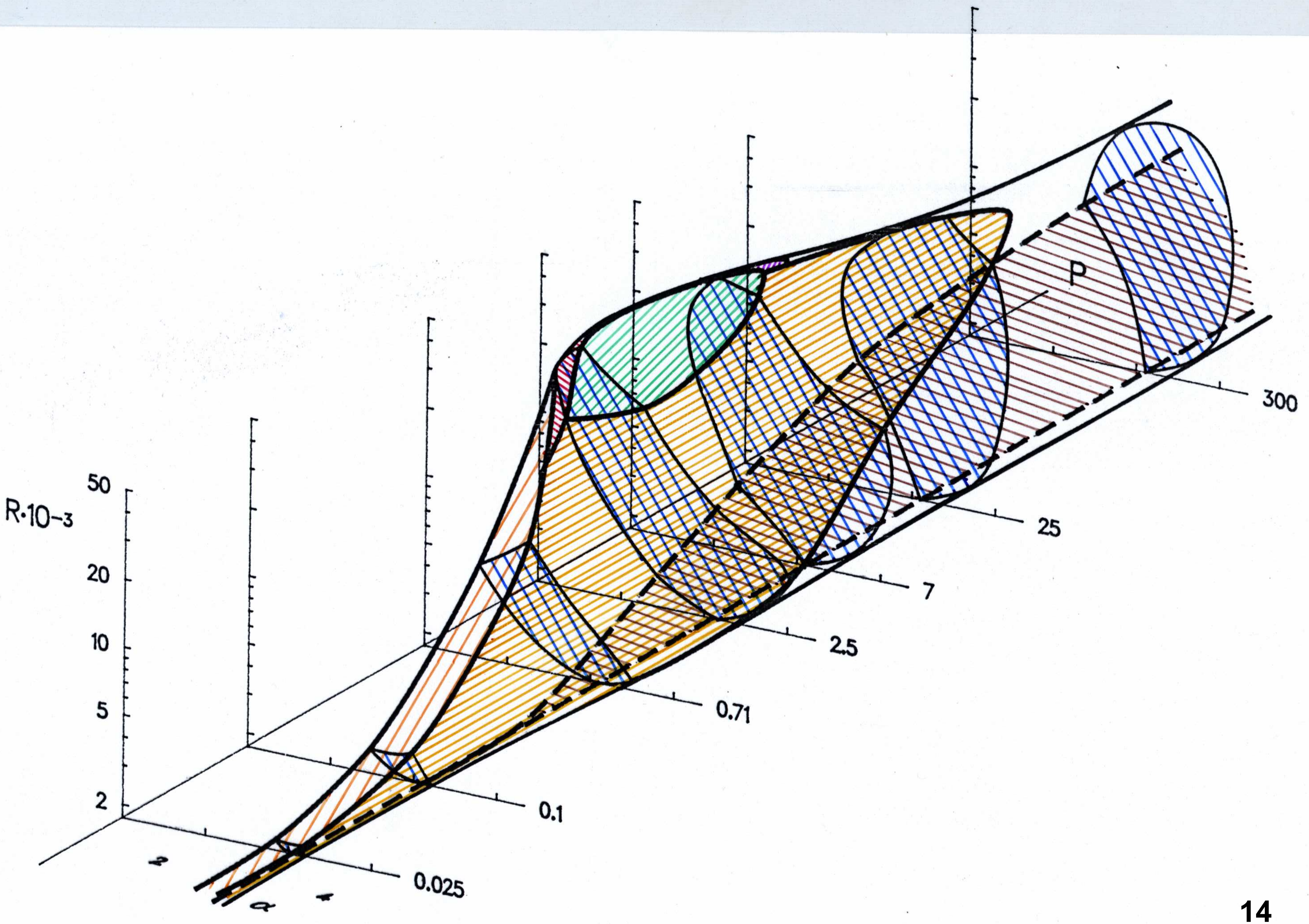
## Instabilities of Rolls :

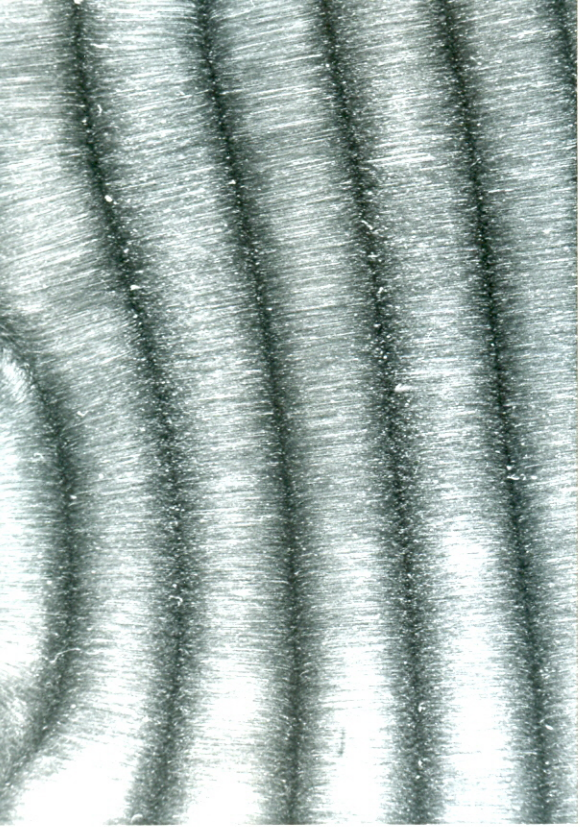
$$\tilde{\varphi} = \left\{ \sum_{m,n>0} \tilde{a}_{mn} \exp\{i m\alpha y\}\, f_n(z) \right\} e^{ibx + idy + \sigma t}$$

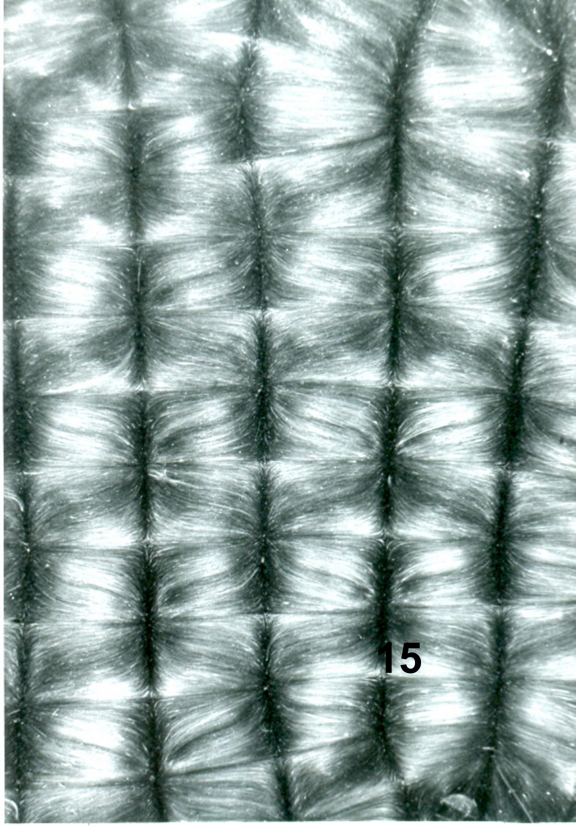
# Symmetries Broken by Bifurcations from Steady Rolls

| Analytical properties / Symmetries broken | $\sigma_i \neq 0$ translation in time | $b \neq 0$ longitud. translation | $d \neq 0$ transverse periodicity | $\tilde{a}_{mn} \neq \tilde{a}_{-mn}$ transverse reflection | $\tilde{a}_{mn} \neq 0$ inversion about axis | Other Examples |
|---|---|---|---|---|---|---|
| **Eckhaus Inst.** | | | ✖ | ✖ | | |
| **Cross roll Inst. Knot Inst.** | | ✖ | | | ✖ | |
| **Oscillatory Inst.** | ✖ | ✖ | | ✖ | | Wavy Inst. in presence of Poisseuille flow |
| **Blob Instab. E** | ✖ | ✖ | | | | |
| **Blob Instab. O** | ✖ | ✖ | | | ✖ | |
| **Mon. Skewed Varicose** | | ✖ | ✖ | ✖ | ✖ | Küppers-Lortz-Inst. |
| **Osc. Skewed Varicose** | ✖ | ✖ | ✖ | ✖ | | |
| **Zig-Zag Inst.** | | ✖ | | ✖ | | Wavy Inst. in the presence of Couette flow |

***basic roll symmetries***


R·10-3
50
20
10
5
2
0.025
0.1
0.71
2.5
7
25
300
P





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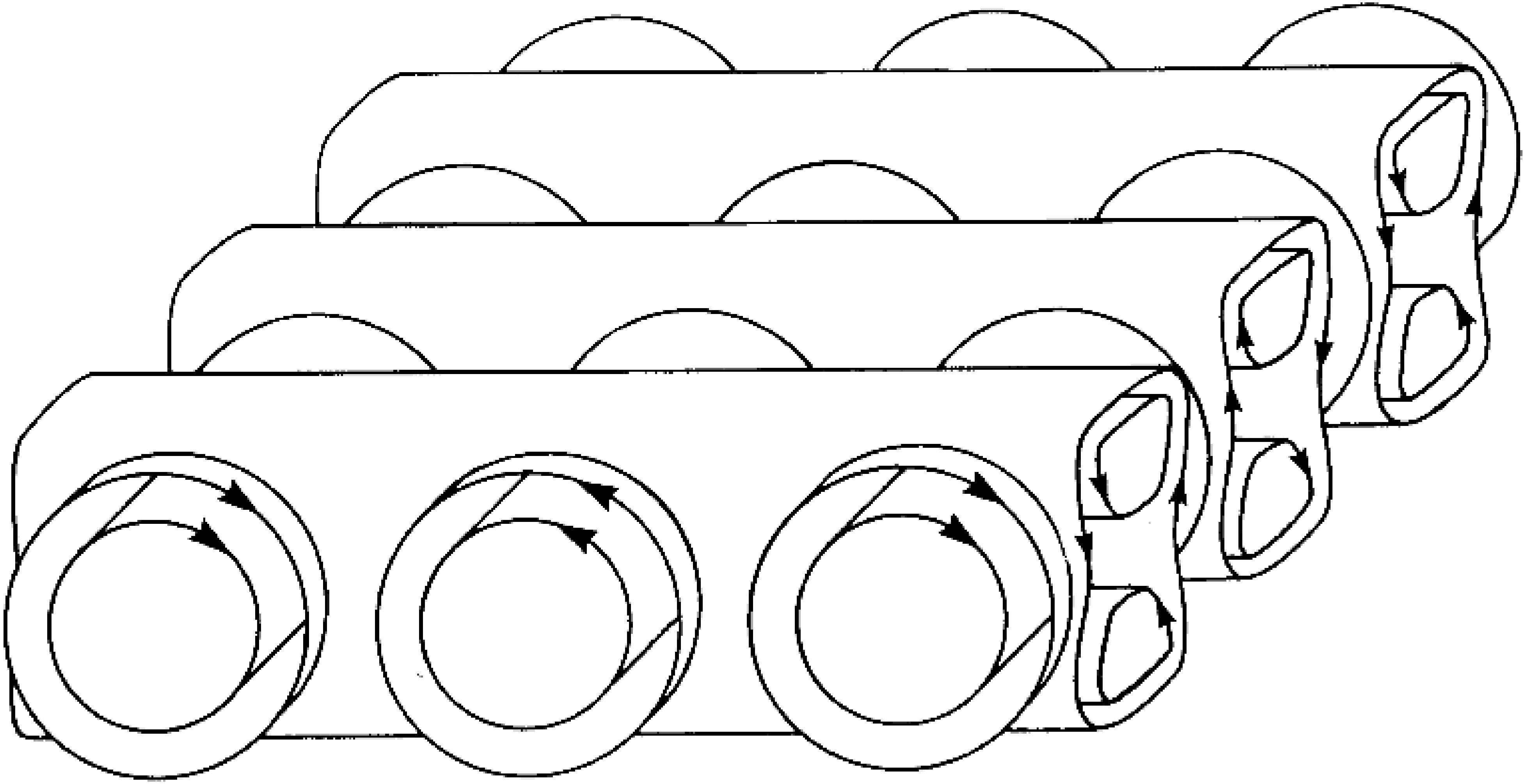
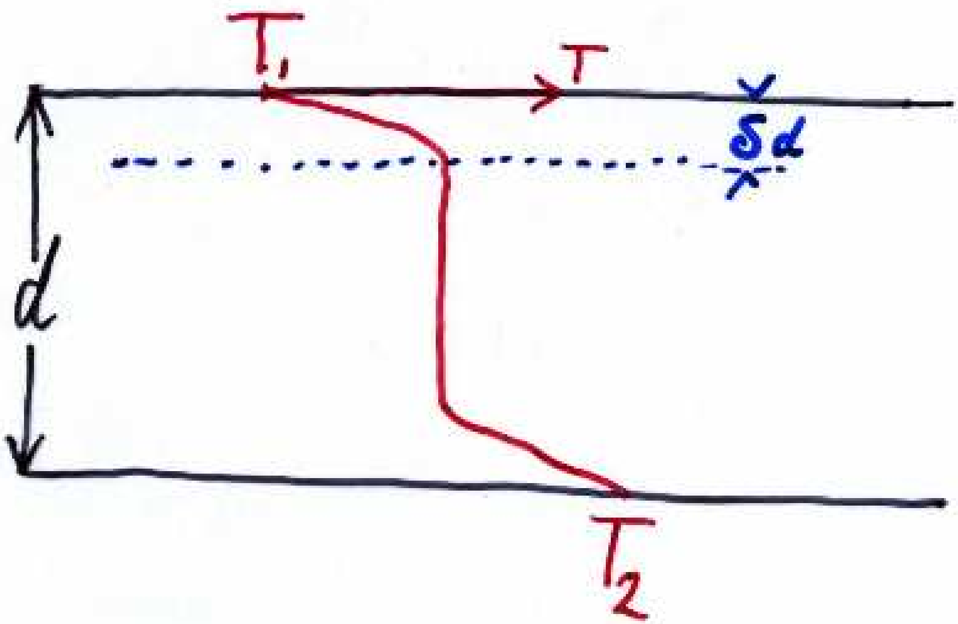


# Transition to Bimodal Convection
## through the instability of thermal boundary layer

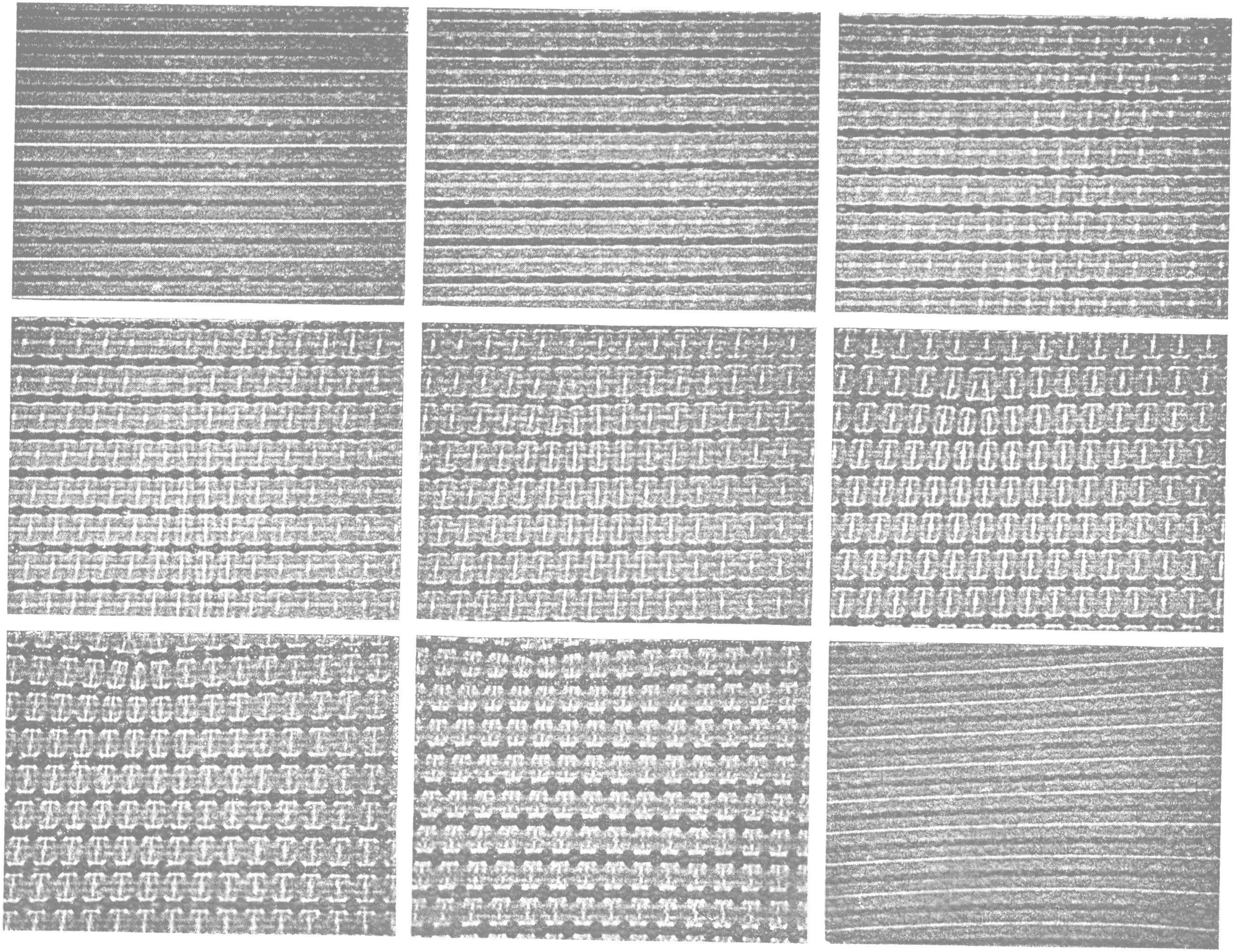



$$\frac{R}{2}\delta^3 > R_c \text{ for instability}$$

$$H \approx \frac{R}{2\delta} \quad \text{or} \quad \delta = \frac{R}{2H}$$

$$\left(\frac{R}{2}\right)^4 \frac{1}{H^3} > R_c \text{ for instability}$$

$$H < \frac{R}{2}\left(\frac{R}{2R_c}\right)^{\frac{1}{3}} \text{ for instability}$$



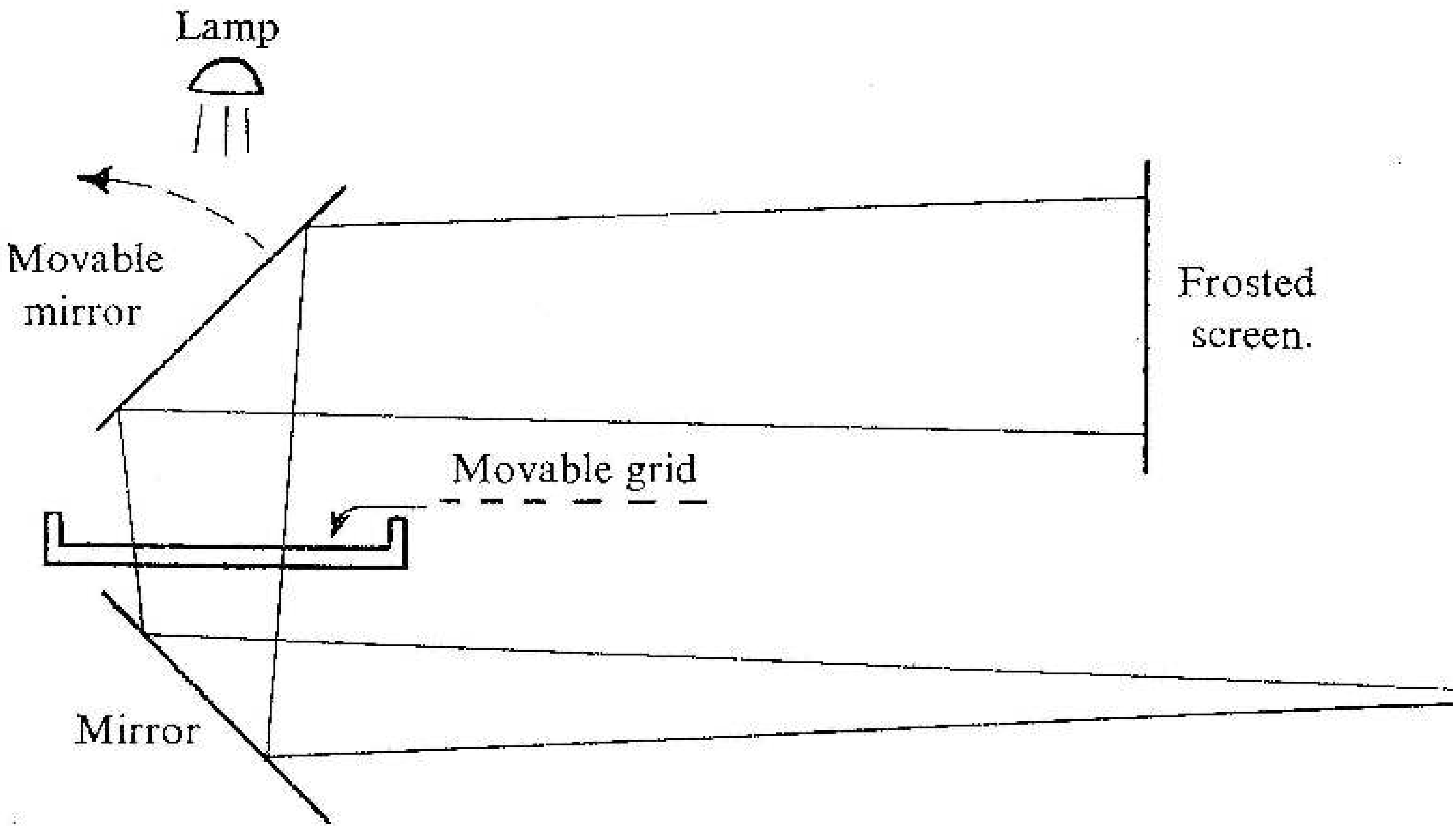
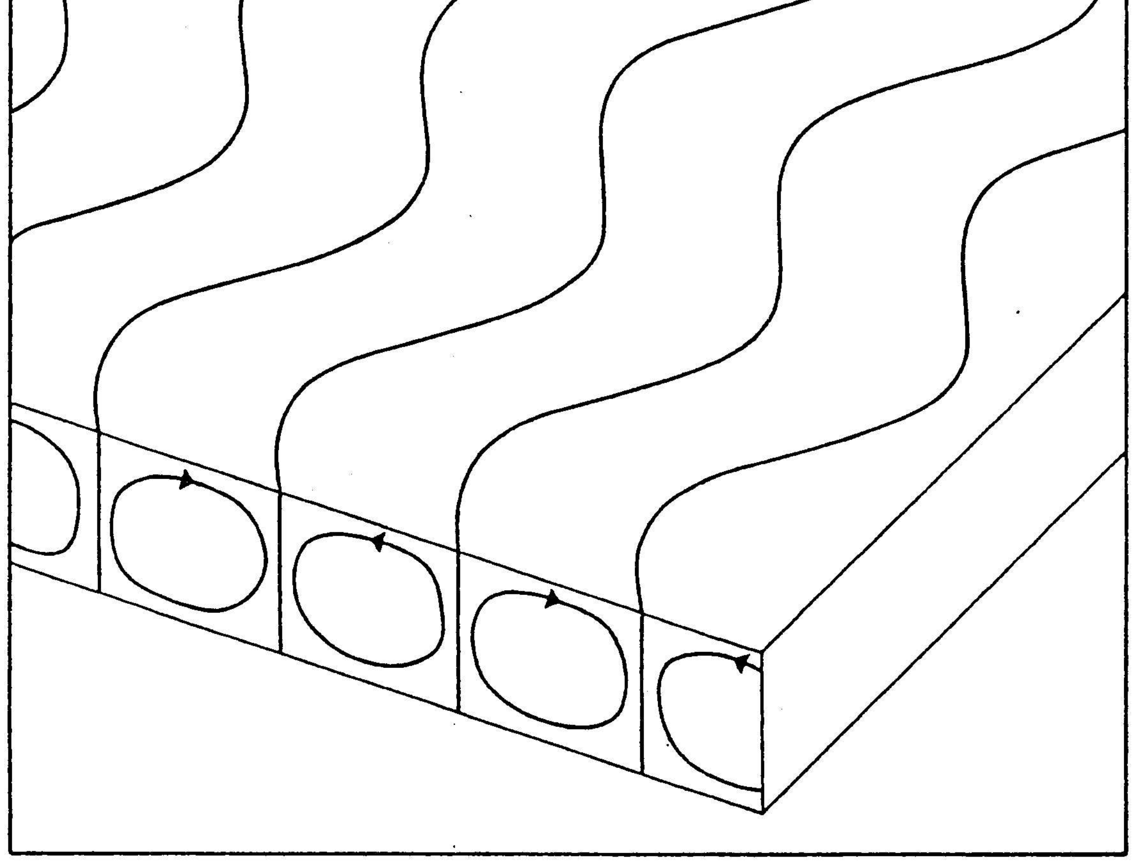
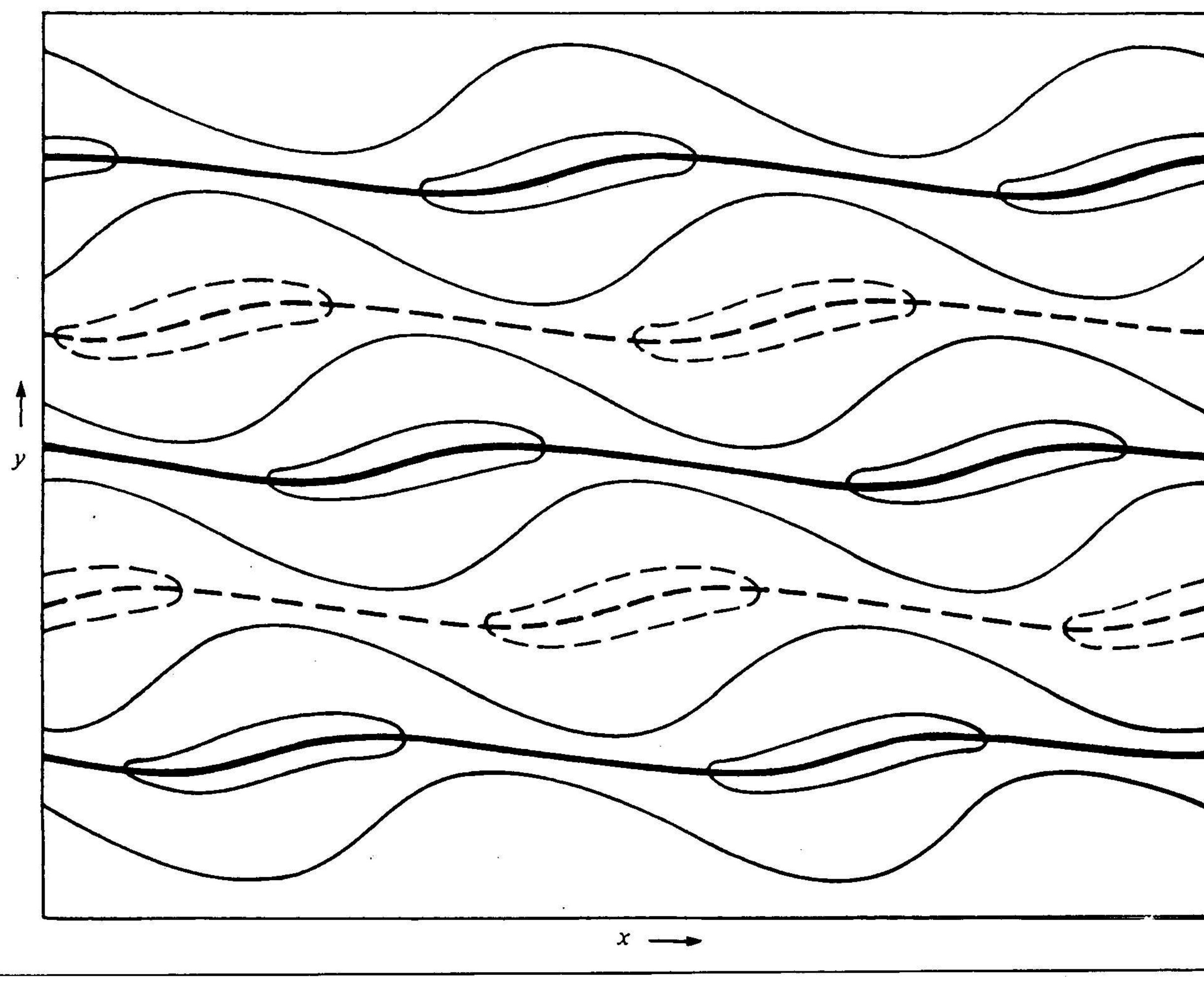
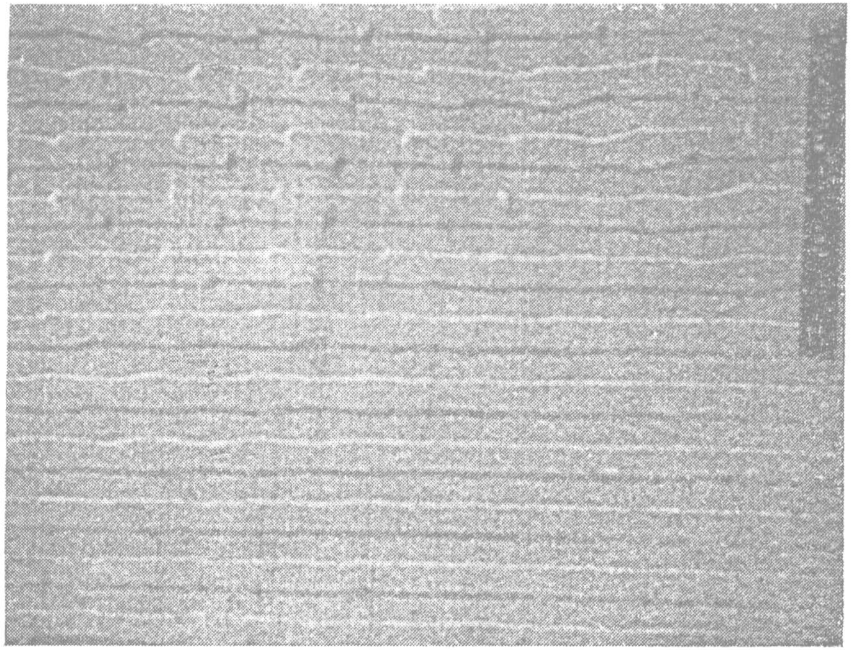



Diagram of the observational technique showing mov
mirror and grid used for inducing rolls.

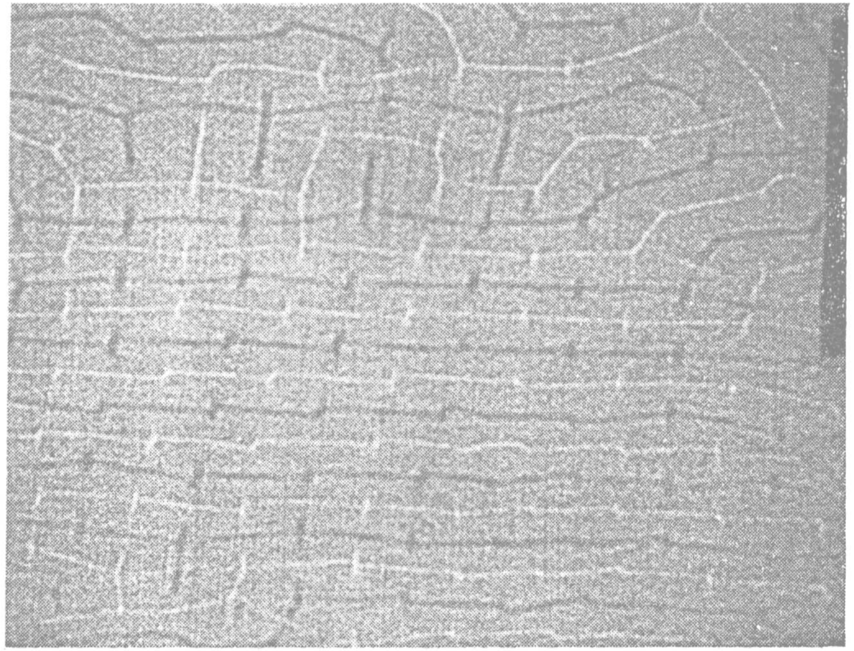




y
x



(a)



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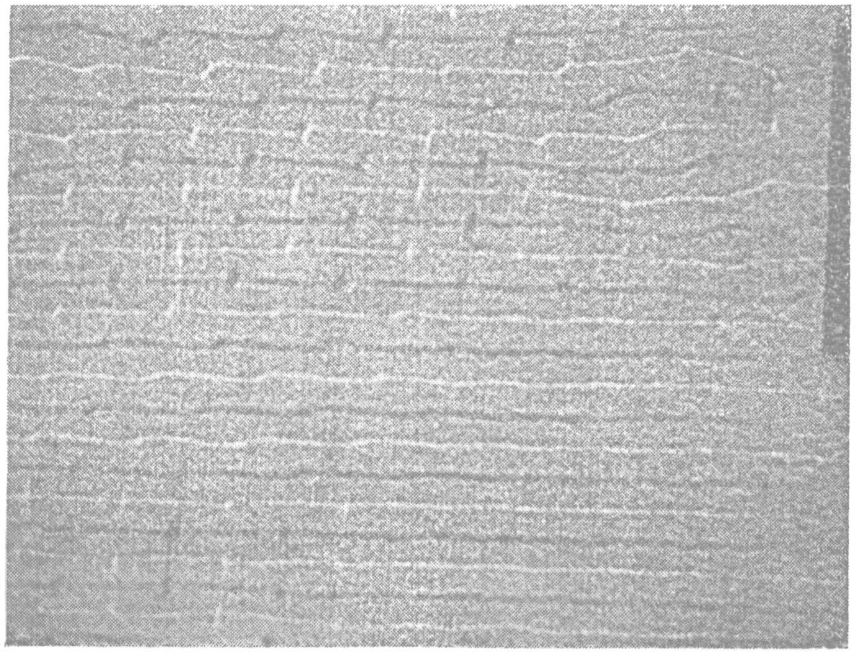
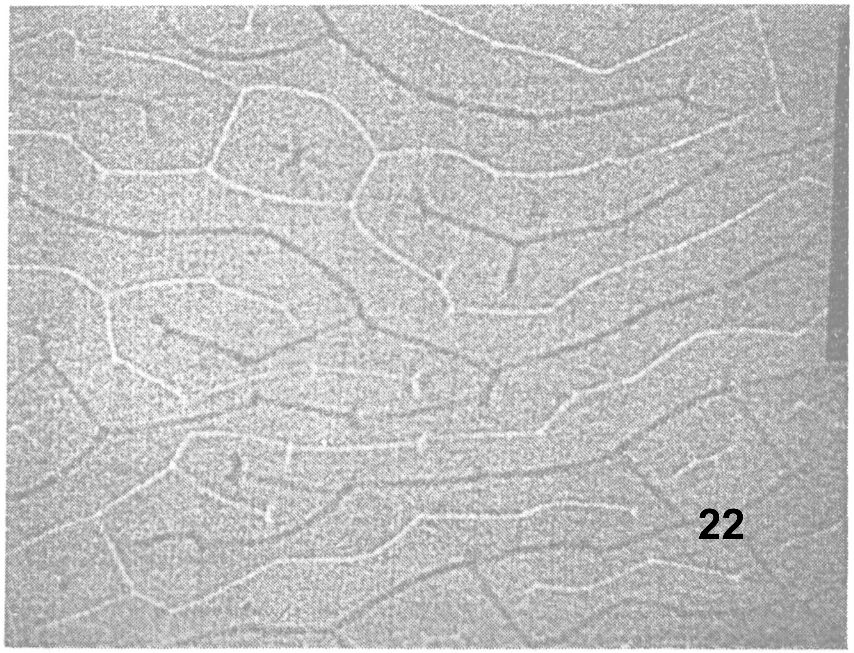




# Tertiary Solutions

**steady state or travelling wave**

$$\varphi = \sum_{l,m,n} a_{lmn} \exp\{il\alpha_x x + im\alpha_y y\} f_n(z)$$

with $a_{-l-mn} = a^+_{lmn}$, $x \to \hat{x} \equiv x - ct$,

for travelling wave

Symmetries of twice spatially periodic flows

| Examples | reflection symmetries | inversion symmetry |
|---|---|---|
| bimodal convection Knot convection | $a_{l-mn} = a_{-lmn} = a_{lmn}$ | $a_{lmn} = 0$ for $l+m+n = \text{odd}$ |
| travelling blob convection | $a_{l-mn} = a_{lmn}$ | $a_{lmn} = 0$ for $l+m+n = \text{odd}$ |
| wavy rolls wavy Taylor vortices | $a_{lmn} = (-1)^l a_{l-mn} = (-1)^{m+n} a_{-lmn}$ | |

**stability analysis with respect to arbitrary infinitesimal disturbances**

$$\tilde{\varphi} = \exp\{ibx + idy + \sigma t\} \sum_{l,m,n} \tilde{a}_{lmn} \exp\{il\alpha_x x + im\alpha_y y\} f_n(z)$$

# Quarternary Solutions

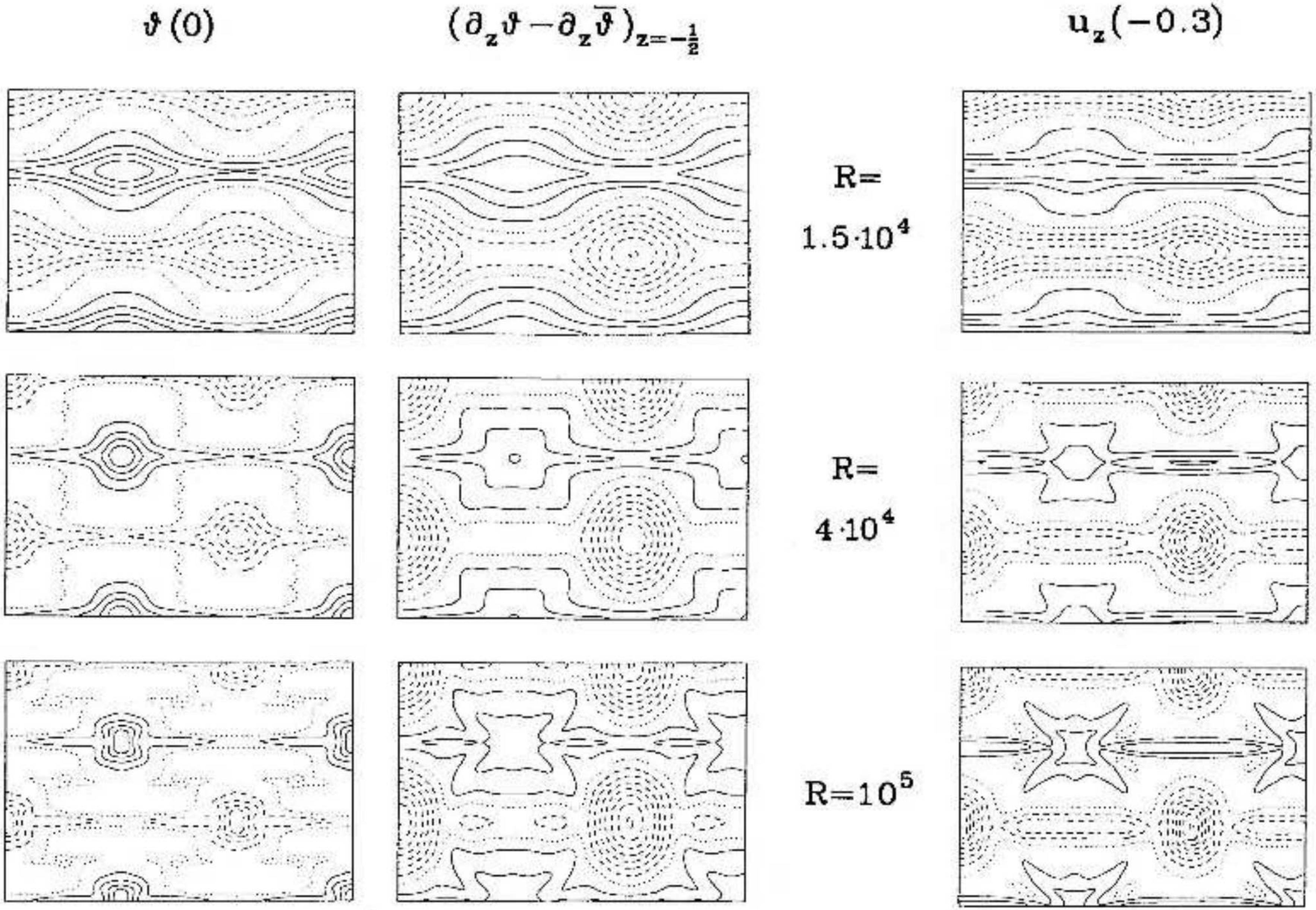
**time periodic or steady state**

$$\varphi = \sum_{l,m,n} a_{lmn}(t) \exp\{il\alpha_x x + im\alpha_y y\} f_n(z)$$

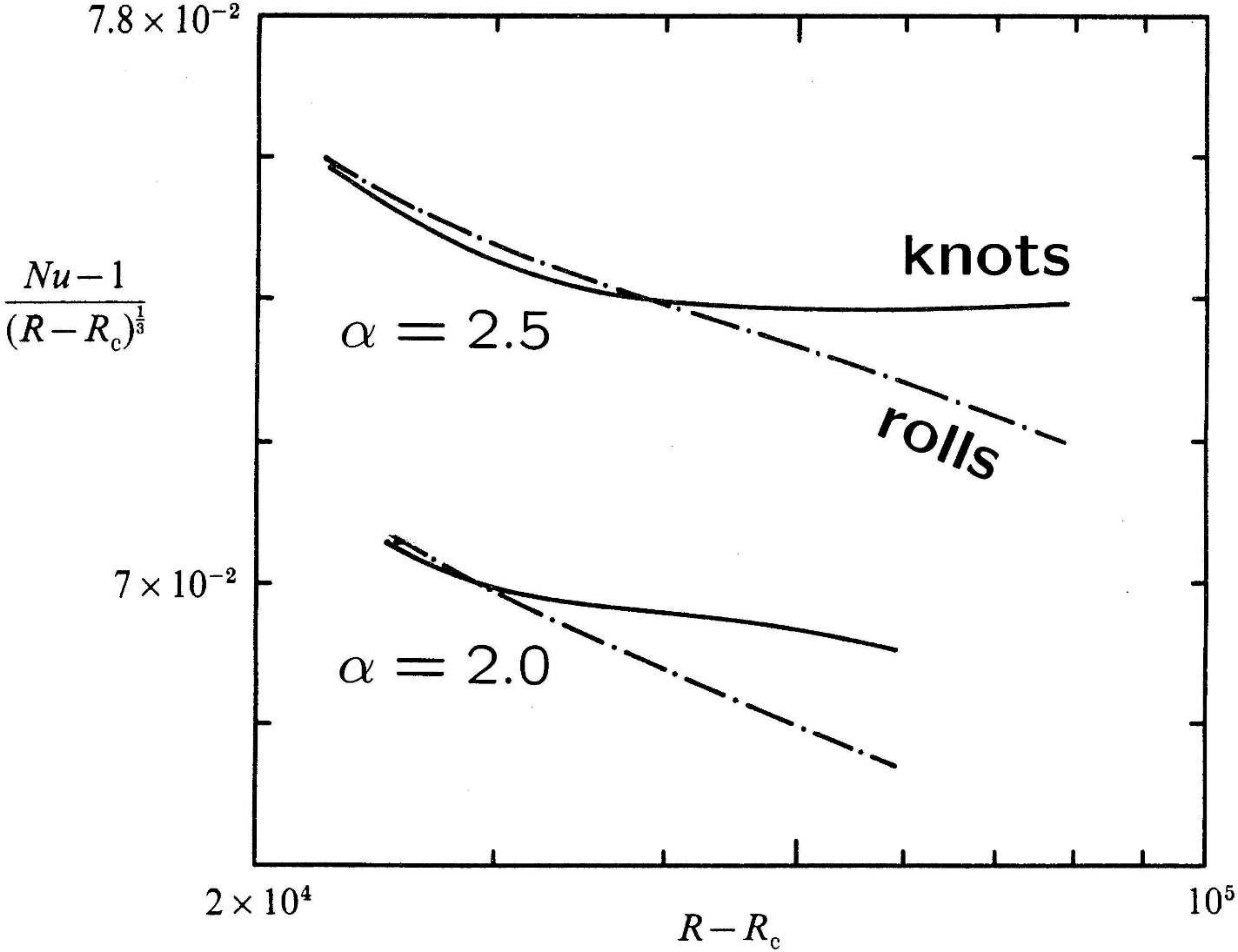
## Examples

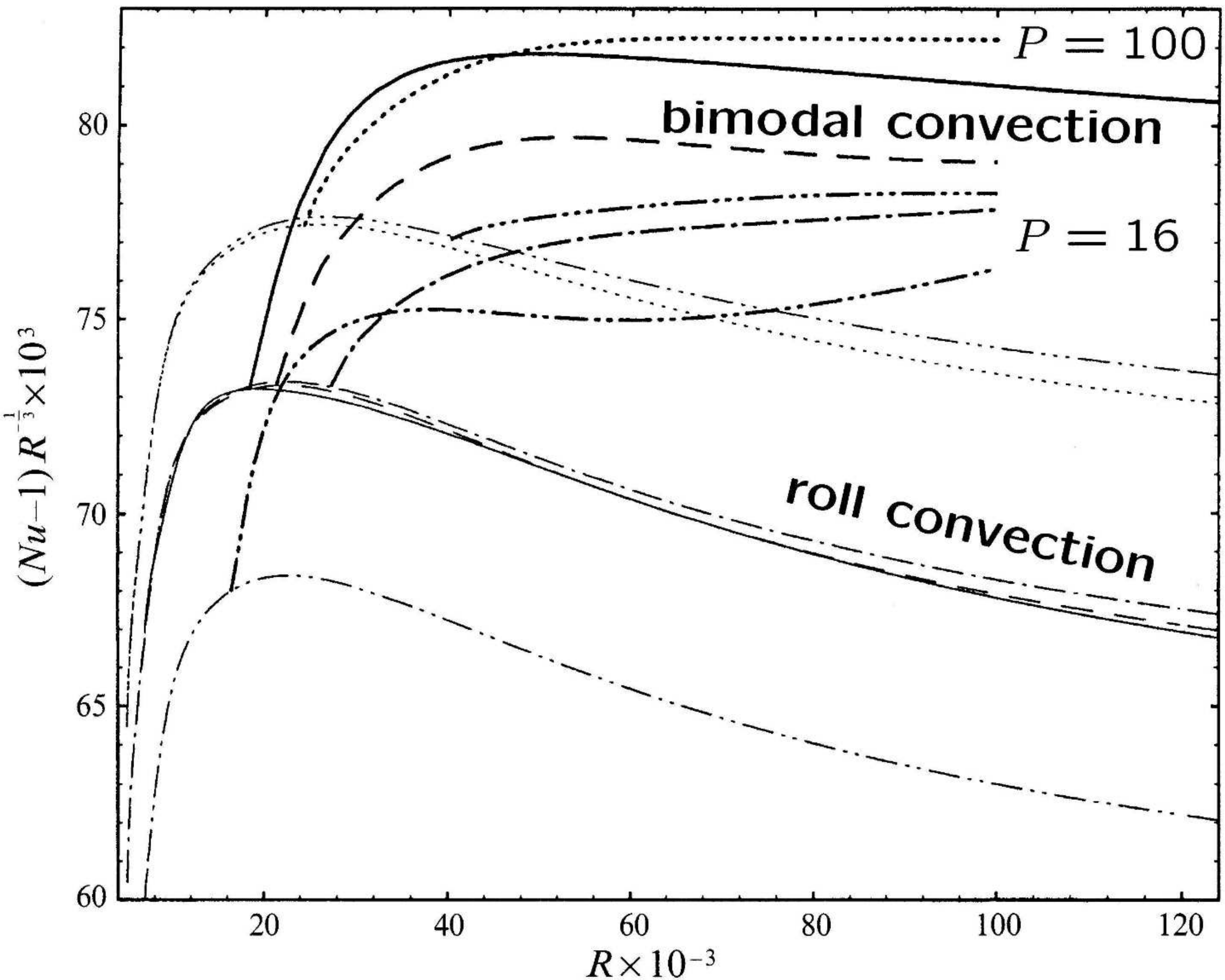
oscillatory bimodal convection, oscillatory knot convection, pulsating travelling blob convection

# Steady Knot Convection



# Nusselt number at P=7





$P = 100$
bimodal convection
$P = 16$
roll convection
$(Nu-1)\,R^{-\frac{1}{3}}\times 10^3$
$R\times 10^{-3}$
80
75
70
65
60
20
40
60
80
100
120

# Symmetric oscillatory bimodal convection

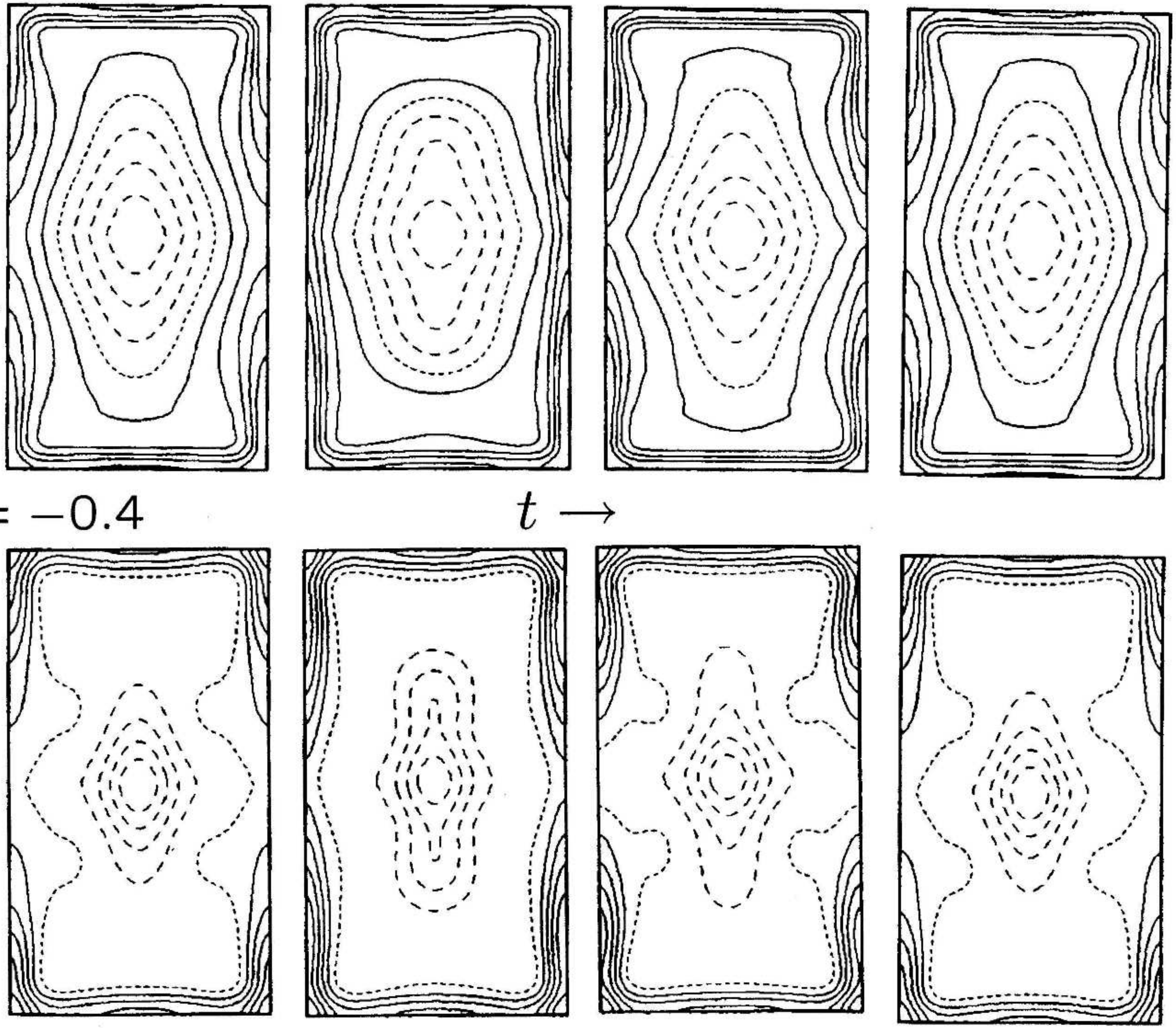
$P = 60$

$R = 14 \cdot 10^4$

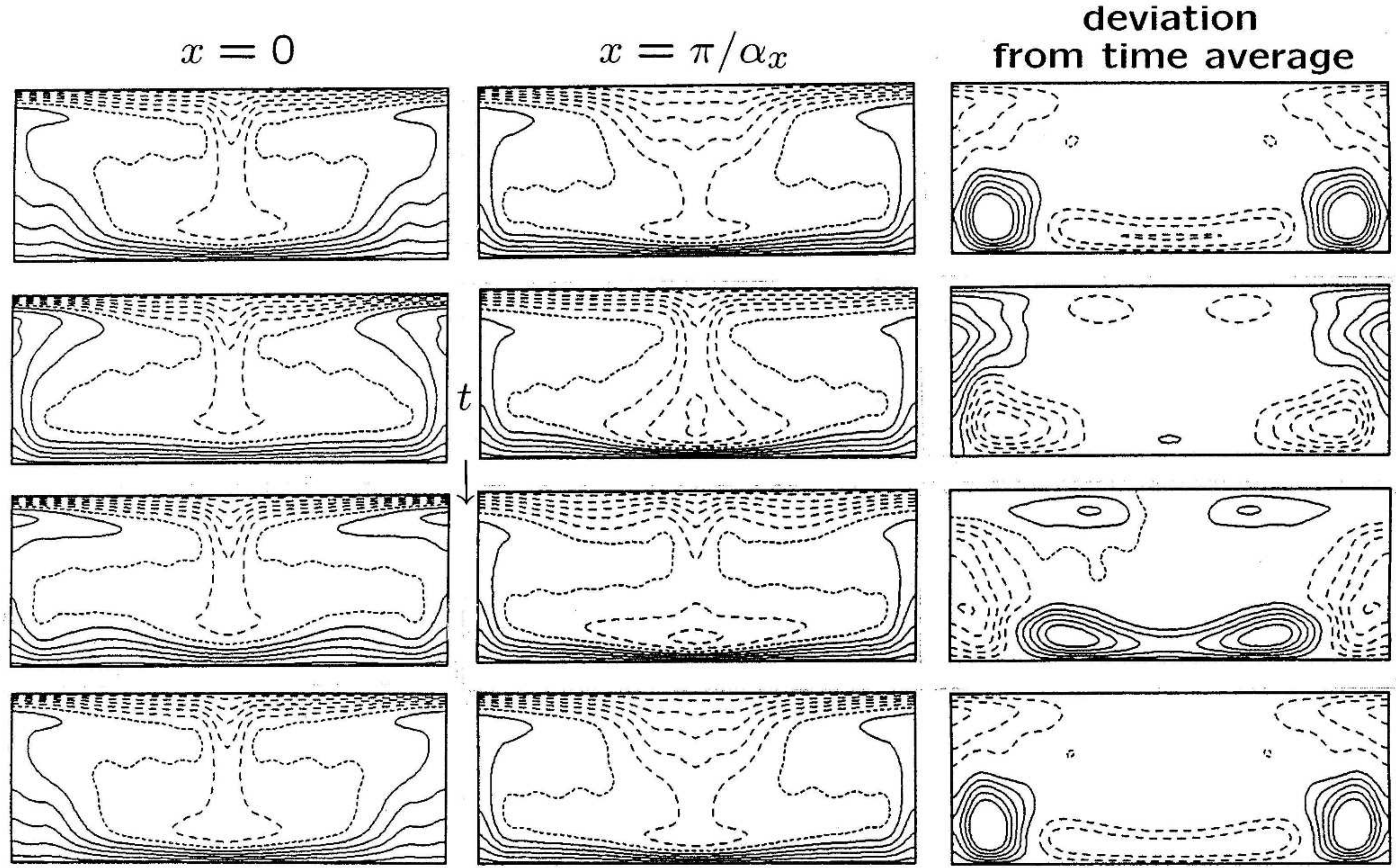
$\theta$ at $z = -0.4$

$t \longrightarrow$

$u_z$



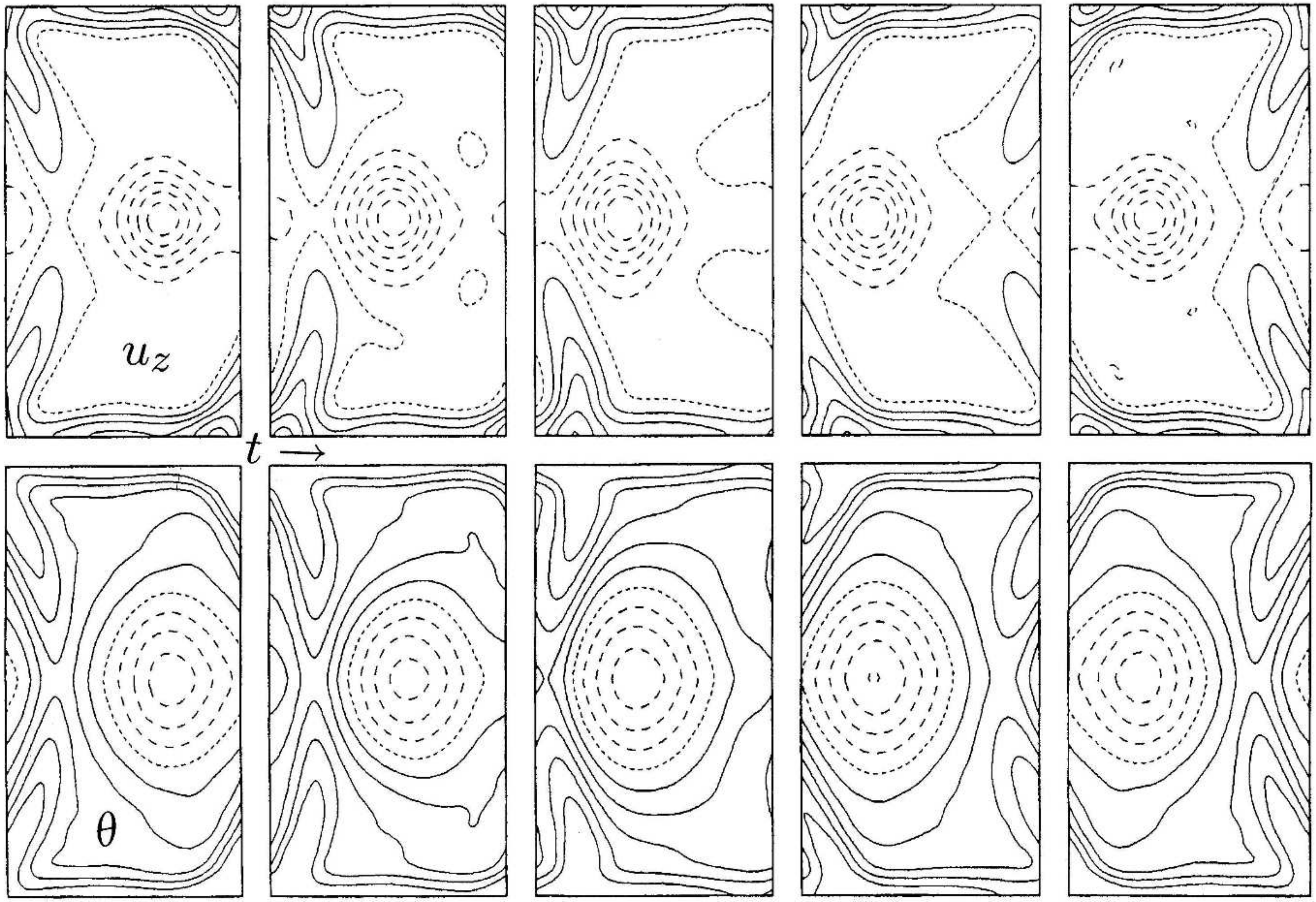
# Oscillatory bimodal convection



# Wavy oscillatory bimodal convection

$P = 16$

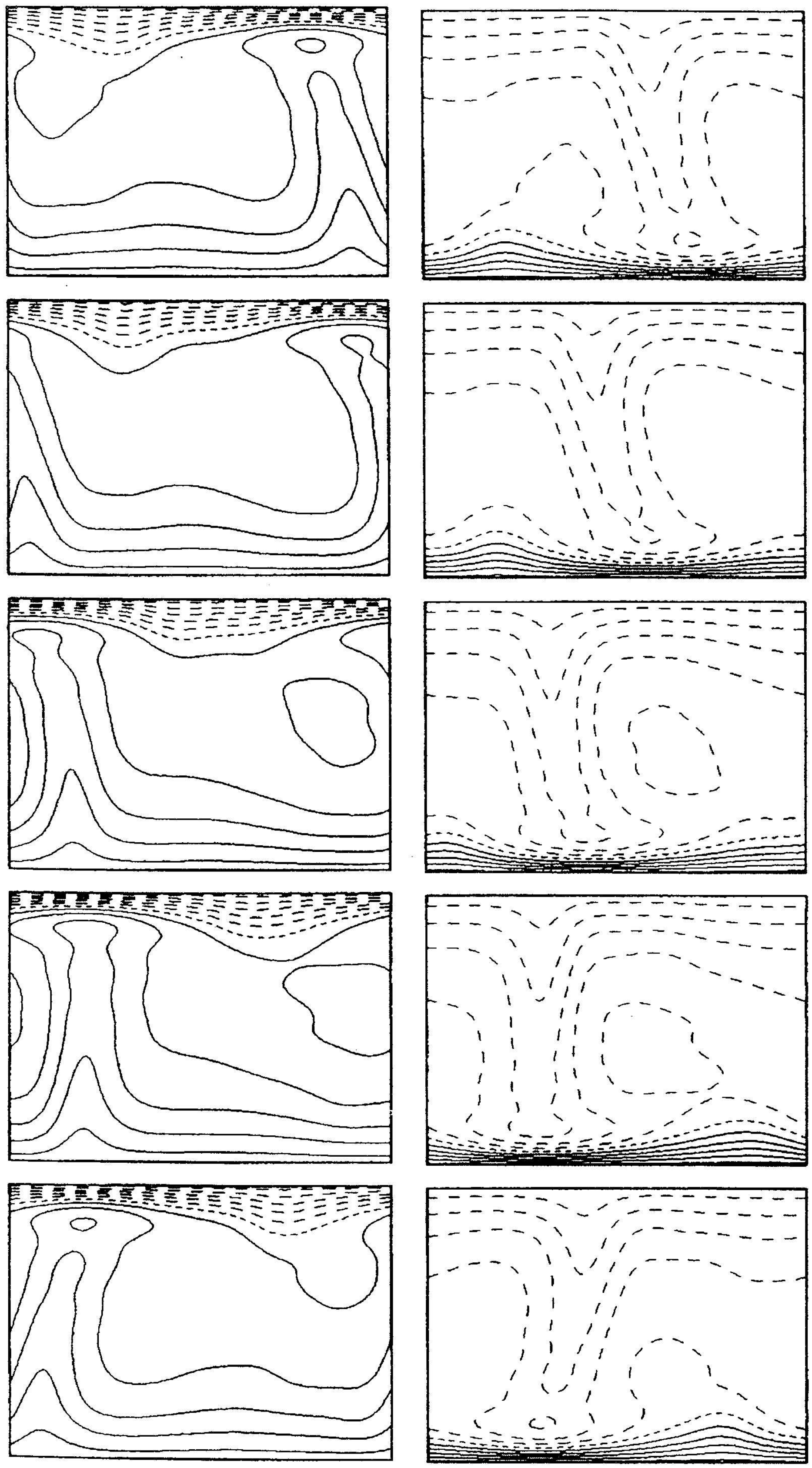
$R =$
$7 \cdot 10^4$

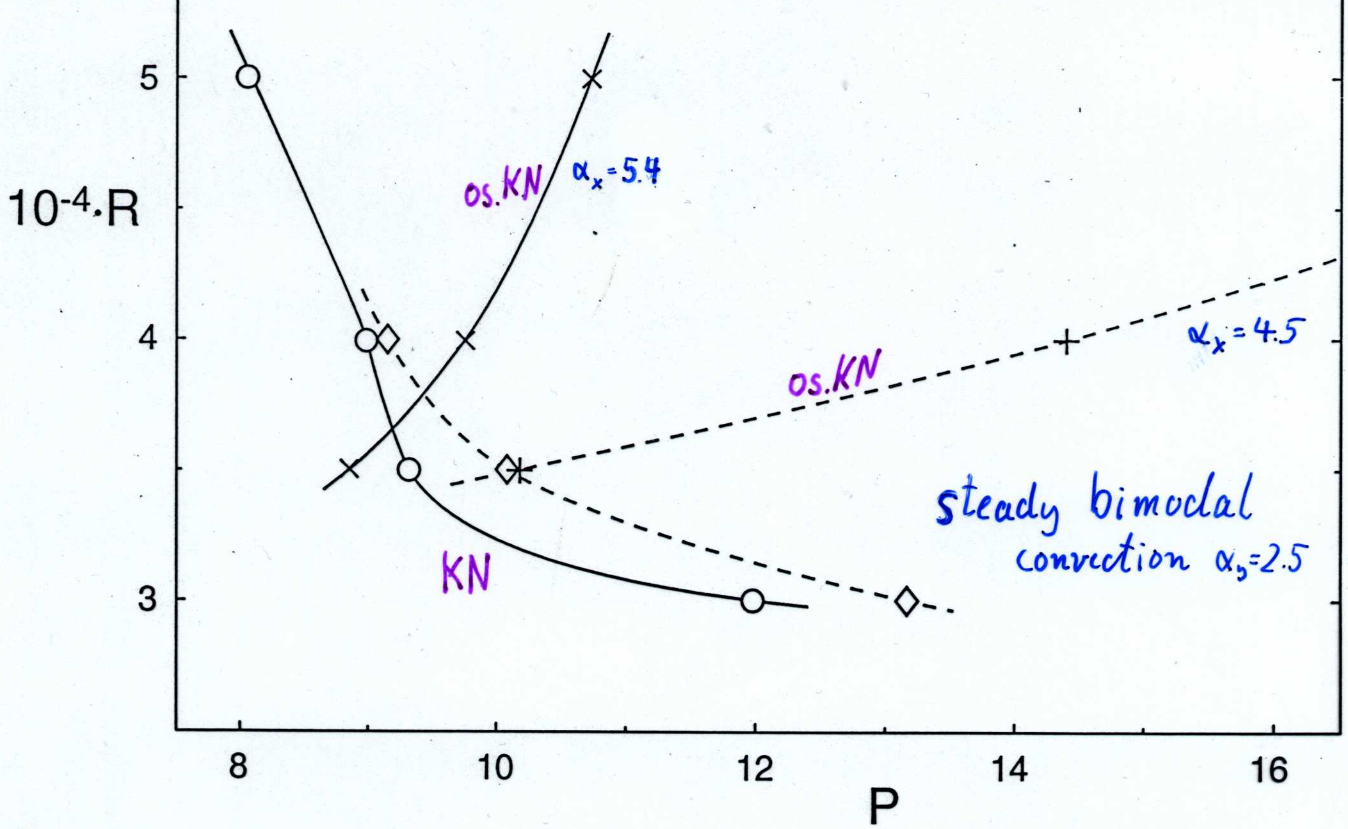


# Wavy oscillatory bimodal convection

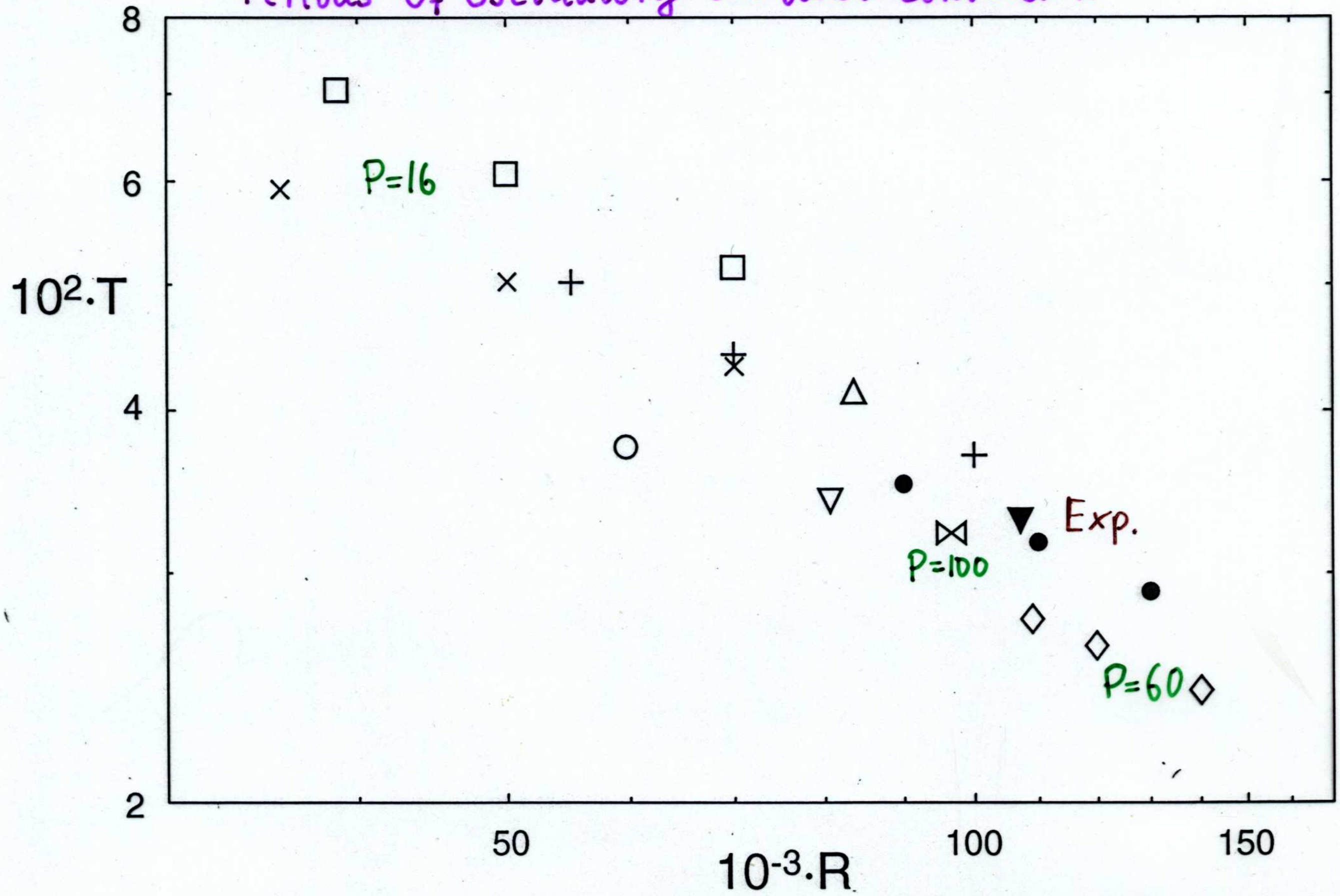
$t$

$\downarrow$






Periods of oscillatory bimodal convection

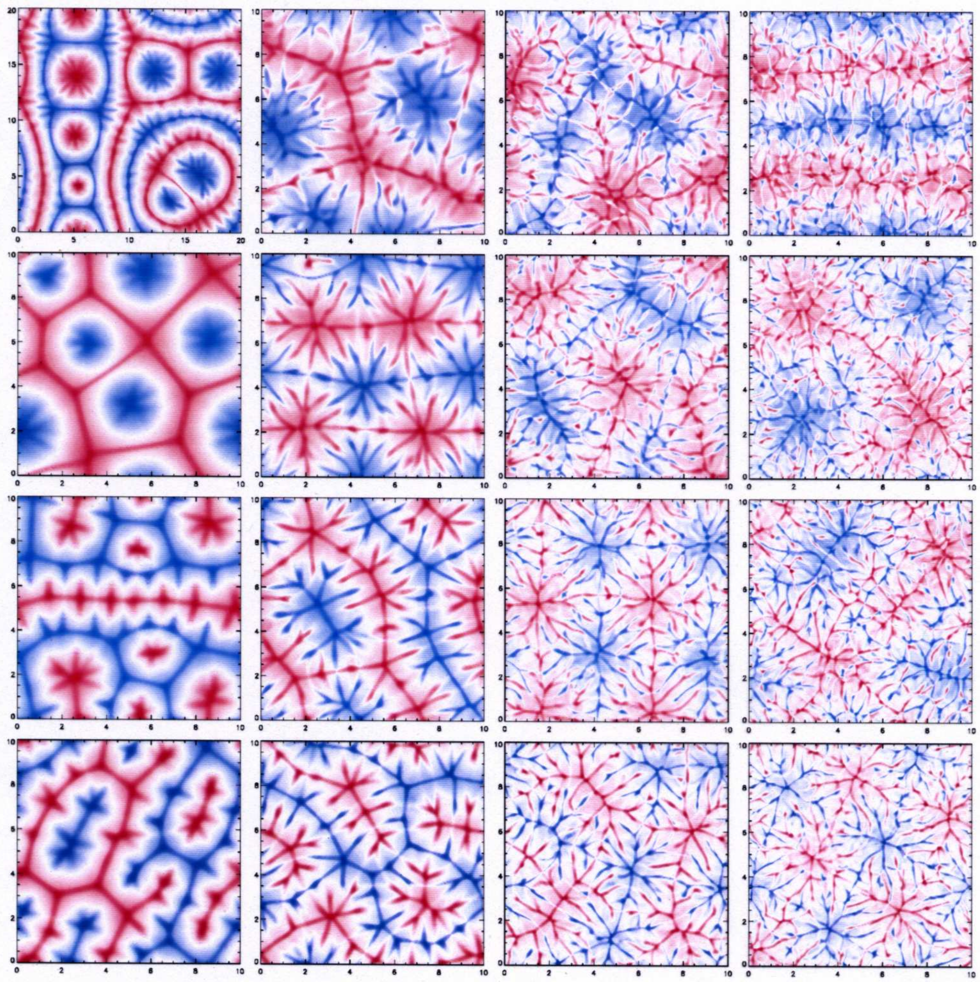


T. Hartlep, A. Tilgner & F.H.B, JFM 2005

# vertically averaged temperature, $\Gamma = 10$

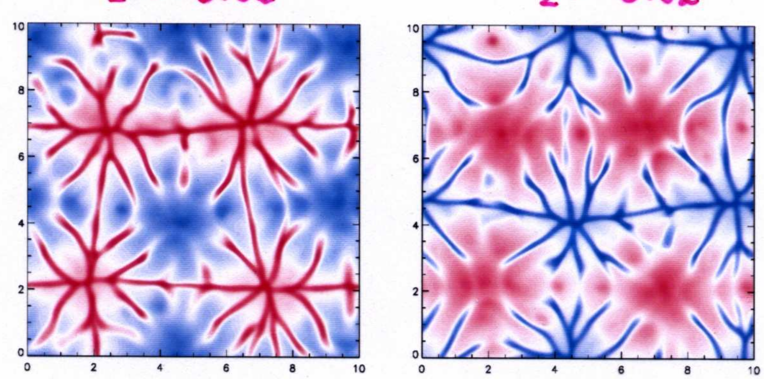
$\Gamma = 20$ → $R = 3.2 \cdot 10^4$ | $10^5$ | $5 \cdot 10^5$ | $10^6$

$P = 7$, 15, 30, 60



temperature at

$z = -0.32$ | $z = 0.32$

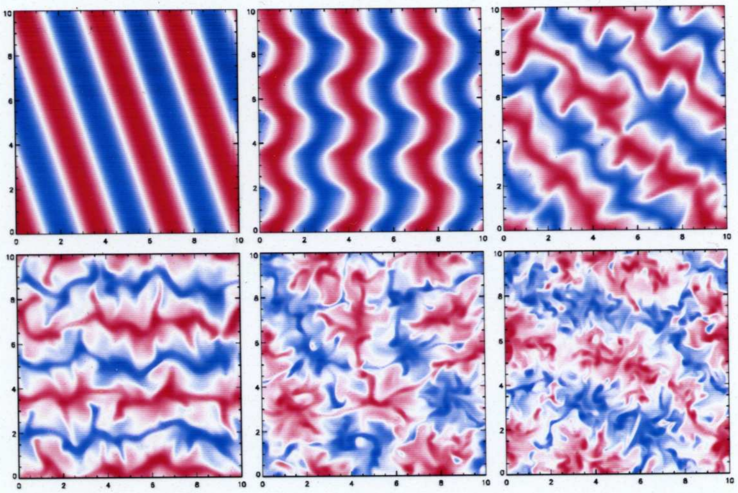


$R = 10^5$, $P = 15$, $\Gamma = 10$

# Convection in Air ($P = 0.7$)

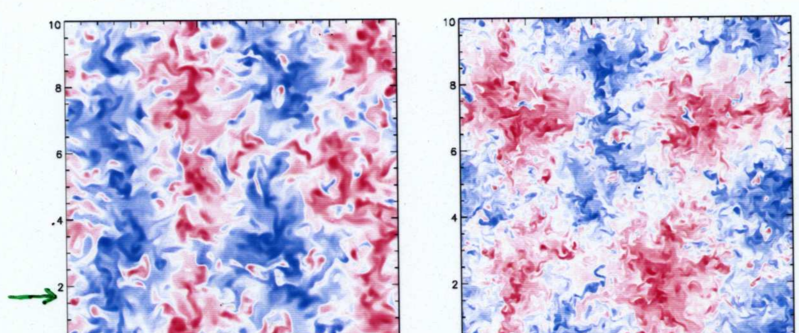
## Temperature at the midplane, $\Gamma = 10$

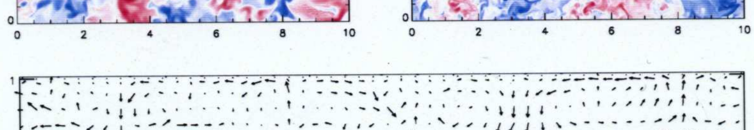
$R = 4000$    $8000$    $16000$



$R = 3.2 \cdot 10^4$    $10^5$    $2.5 \cdot 10^5$

$R = 10^6$    $R = 10^7$





velocity field in vertical crosssection at $y = 1.75$

# Conclusions

- **Tertiary and higher order solutions** of bifurcation sequences describe dynamic mechanisms that operate as coherent structures in turbulent systems.
- **Controlled initial conditions** permit the realizations of spatially periodic higher order solutions in computer simulations or laboratory experiments. These solutions may be unstable or their basins of attraction are so small that they have no chance to be realized without controlled conditions.
- **Instantaneous two-dimensional visualizations** are best suited for identifying coherent structures in turbulent systems.
- Similarities between structures of fully turbulent systems and those of their laminar equivalents at the laboratory scale lend support to the **concept of eddy diffusivities**.