

# Growth of cloud droplets and raindrops in turbulent clouds

**Wojciech W. Grabowski**

**National Center for Atmospheric Research  
Boulder, Colorado**



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# Droplet growth in warm turbulent clouds

B.J. Devenish<sup>a\*</sup>, P. Bartello<sup>b</sup>, J.-L. Brenguier<sup>c</sup>, L.R. Collins<sup>d</sup>,  
W.W. Grabowski<sup>e</sup>, R.H.A. IJzermans<sup>f</sup>, S.P. Malinowski<sup>g</sup>, M.W. Reeks<sup>f</sup>,  
J.C. Vassilicos<sup>h</sup>, L.-P. Wang<sup>i</sup>, Z. Warhaft<sup>d,j</sup>

<sup>a</sup>*Met Office, Fitzroy Road, Exeter, EX1 3PB, UK*

<sup>b</sup>*McGill University, Montreal H3A 2K6, Canada*

<sup>c</sup>*Météo-France/CNRS, GAME/CNRM, Toulouse, France*

<sup>d</sup>*Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, NY 14853, USA*

<sup>e</sup>*National Center for Atmospheric Research, Boulder, CO, USA*

<sup>f</sup>*School of Mechanical and Systems Engineering, University of Newcastle, Newcastle-upon-Tyne, NE1 7RU, UK*

<sup>g</sup>*University of Warsaw, Institute of Geophysics, Warsaw, Poland*

<sup>h</sup>*Institute for Mathematical Science and Department of Aeronautics, Imperial College London, London SW7 2BY, UK*

<sup>i</sup>*Department of Mechanical Engineering, University of Delaware, Newark, DE 19716, USA*

<sup>j</sup>*Atkinson Center For a Sustainable Future, Cornell University, Ithaca, NY 14853, USA*

\*Correspondence to: Met Office, Fitzroy Road, Exeter, EX1 3PB, UK Email: [ben.devenish@metoffice.gov.uk](mailto:ben.devenish@metoffice.gov.uk)

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[ben.devenish@metoffice.gov.uk](mailto:ben.devenish@metoffice.gov.uk)

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Cloud droplets grow by the diffusion of water vapor (i.e., by condensation) and by collision/coalescence.

For both cloud turbulence is thought to play some role.

Turbulent entrainment and mixing affects the spectrum of cloud droplets as well.

## **Elementary facts about cloud droplets:**

**Radius  $r$  : 5-30 microns ( $r \ll$  Kolmogorov length scale)**

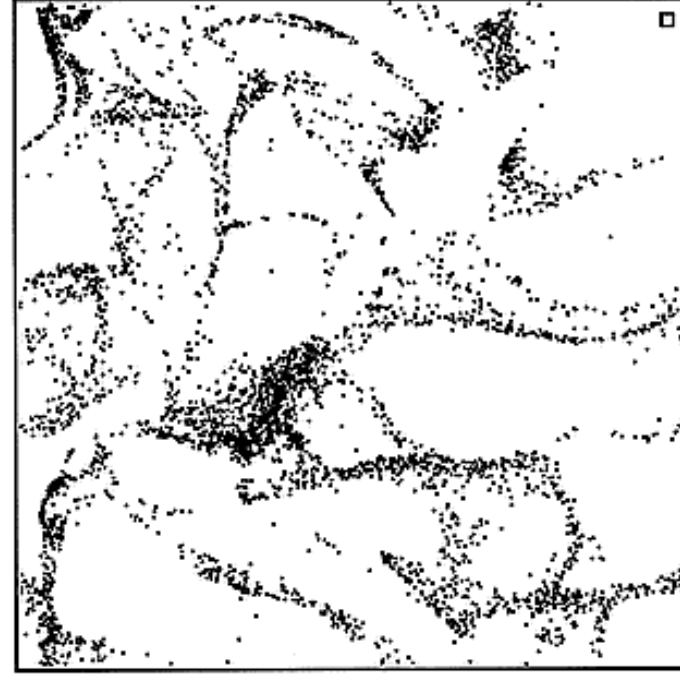
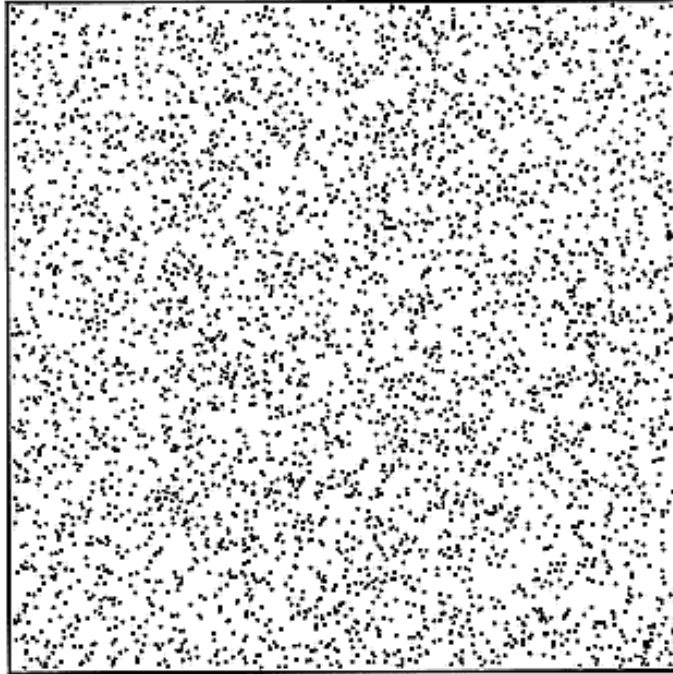
**Concentration: 50-2,000 cm<sup>-3</sup> ( mean separation distance  $\gg r$ )**

**Mass loading: 0.5-5 g kg<sup>-1</sup> (  $\ll 1$ ; no effects on turbulence)**

**Fall terminal velocity  $v_t$ :  $v_t \sim r^2$  ;  $v_t \approx 1$  cm/s for  $r = 10 \mu\text{m}$**

initial conditions

solution at a later time

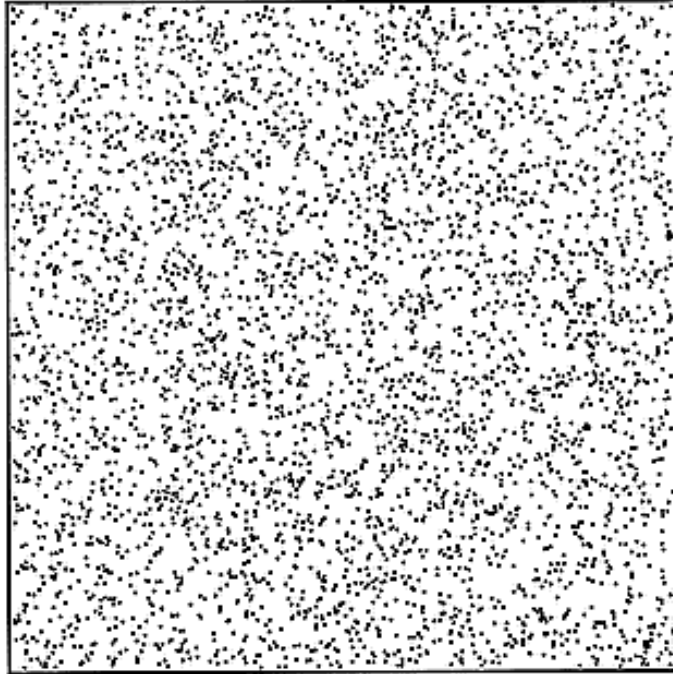


Kolmogorov  
scale

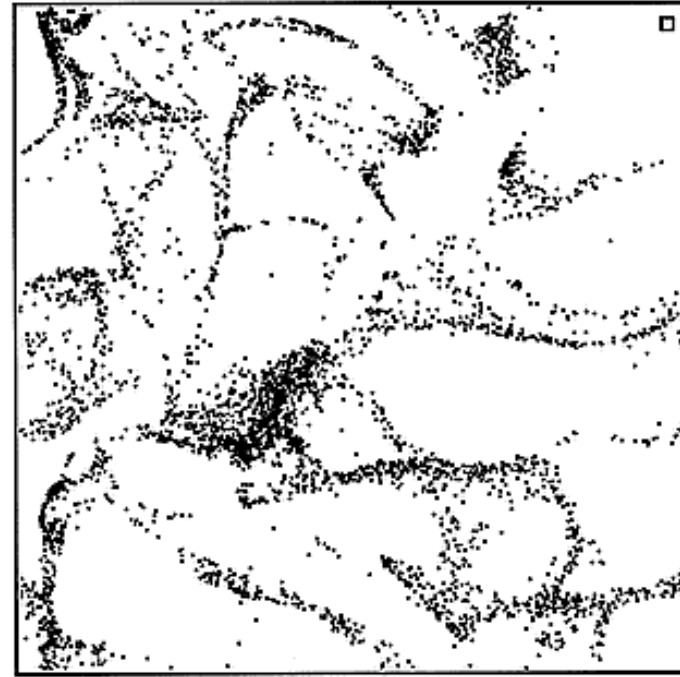
Clustering of nonsedimenting particles for  $St \sim 1$

Shaw et al. JAS 1998

initial conditions



solution at a later time



Kolmogorov  
scale

Clustering of nonsedimenting particles for  $St \sim 1$

Is this how cloud microscale looks like?

Not really!.....

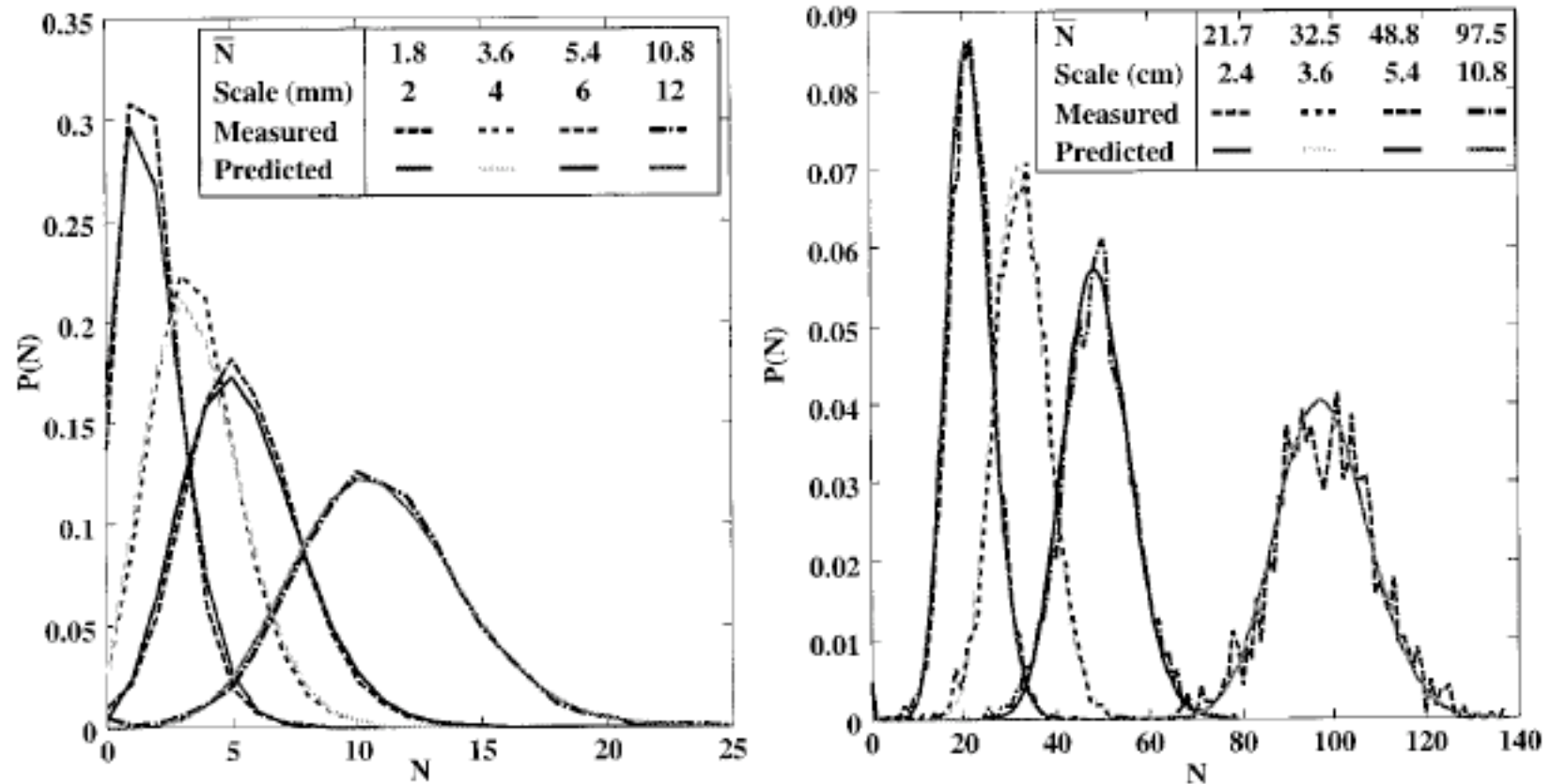


FIG. 1. Probability distribution of the number of counts in actual subsamples (dashed lines), compared to the Poisson distribution (solid lines), for various values of subsample sizes, i.e., of  $\bar{N}$ . The corresponding spatial scale is indicated in the legend. SCMS sample, at 1542:18.6 UTC 10 Aug 1995 (duration: 1.6 s):  $z = 2188$  m,  $C = 329$  cm<sup>-3</sup>,  $\bar{\phi} = 25.7$   $\mu$ m.

Analysis of aircraft observations; Chaumat and Brenguier JAS 2001.

**Parameters describing interaction of cloud droplets with turbulence for the case with gravity:**

**Stokes number:  $St = \tau_p / \tau_\eta$**

**$\tau_p$ - droplet response time**

**$\tau_\eta$  – Kolmogorov timescale**

**Nondimensional sedimentation velocity:  $Sv = v_p / v_\eta$**

**$v_p$  - droplet sedimentation velocity ( $g\tau_p$  for small droplets)**

**$v_\eta$  – Kolmogorov velocity scale**



# Nondimensional parameters (*St* and *Sv*) for typical cloud conditions: ***St* << *Sv***

			Dissipation rate	Kologorov velocity scale	Kolmogorov time scale	
$R$ $\mu\text{m}$	$v_t$ $\text{cm s}^{-1}$	$t_p$ $\text{s}$	$\epsilon \text{ m}^2 \text{ s}^{-3}$ $v_\eta \text{ cm s}^{-1}$ $t_\eta \text{ s}$			
			$10^{-4}$	$10^{-3}$	$10^{-2}$	
			0.64	1.10	2.00	
			0.41	0.13	$4.1 \times 10^{-2}$	
5	0.32	$3.3 \times 10^{-4}$	St $8.0 \times 10^{-4}$	$2.5 \times 10^{-3}$	$8.0 \times 10^{-3}$	
			$S_v$ 0.50	0.28	0.16	
15	2.7	$2.9 \times 10^{-3}$	St $7.0 \times 10^{-3}$	$2.2 \times 10^{-2}$	$7.0 \times 10^{-2}$	
			$S_v$ 4.2	2.4	1.3	
25	7.5	$8.2 \times 10^{-3}$	St $2.0 \times 10^{-2}$	$6.3 \times 10^{-2}$	0.20	
			$S_v$ 12	6.6	3.7	

droplet radius      sedimentation velocity      response time

# Nondimensional parameters (*St and Sv*) for typical engineering applications: ***St >> Sv***

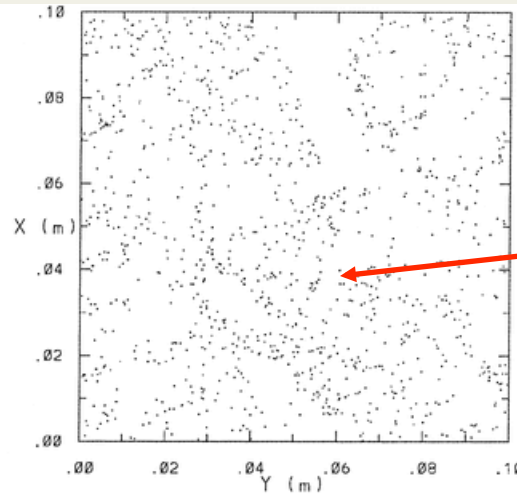
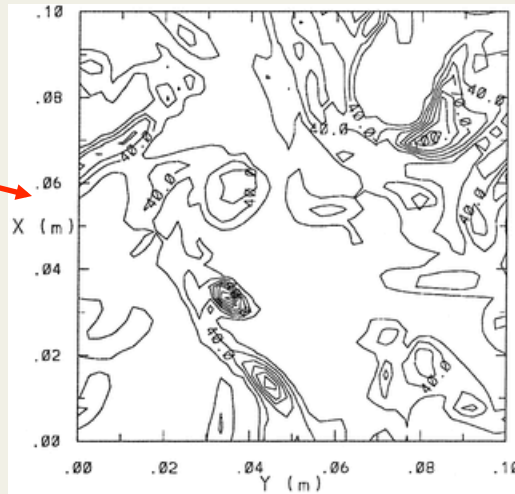
$R$ $\mu\text{m}$	$v_t$ $\text{cm s}^{-1}$	$t_p$ $\text{s}$	$\epsilon \text{ m}^2 \text{ s}^{-3}$ 10 $u_\eta \text{ cm s}^{-1}$ 11.00 $t_\eta \text{ s}$ $1.3 \times 10^{-3}$	
5	0.32	$3.3 \times 10^{-4}$	St	0.25
			$S_v$	$2.8 \times 10^{-2}$
15	2.7	$2.9 \times 10^{-3}$	St	2.2
			$S_v$	0.24
25	7.5	$8.2 \times 10^{-3}$	St	6.3
			$S_v$	0.66

$$St / Sv \sim \epsilon^{3/4} \quad (\text{for Stokes limit: } \mathbf{v_p = g\tau_p})$$

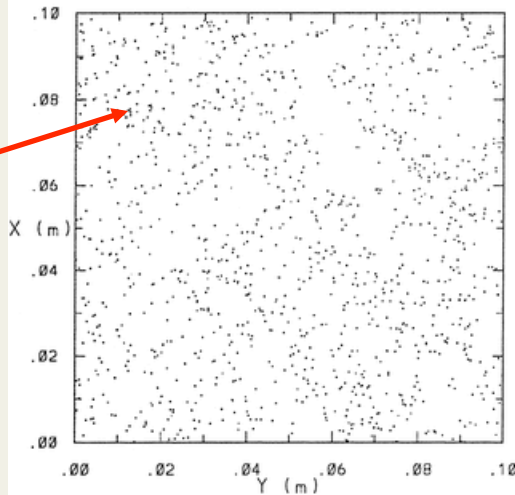
The key point: turbulence in clouds is significantly weaker than in laboratory experiments. This makes droplet sedimentation significantly more important.

DNS simulations with sedimenting droplets for conditions relevant to cloud physics ( $\epsilon=160 \text{ cm}^2\text{s}^{-3}$ )

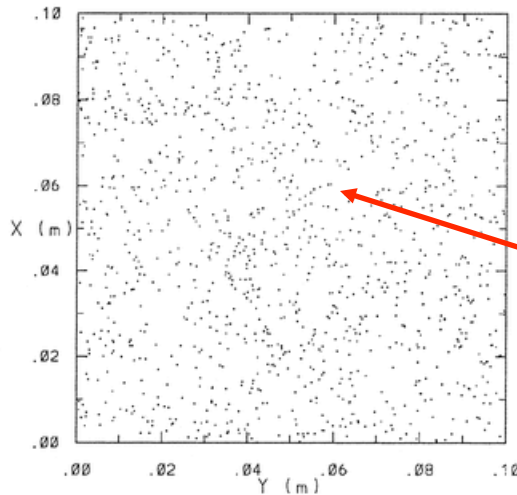
Vorticity  
(contour  $15 \text{ s}^{-1}$ )

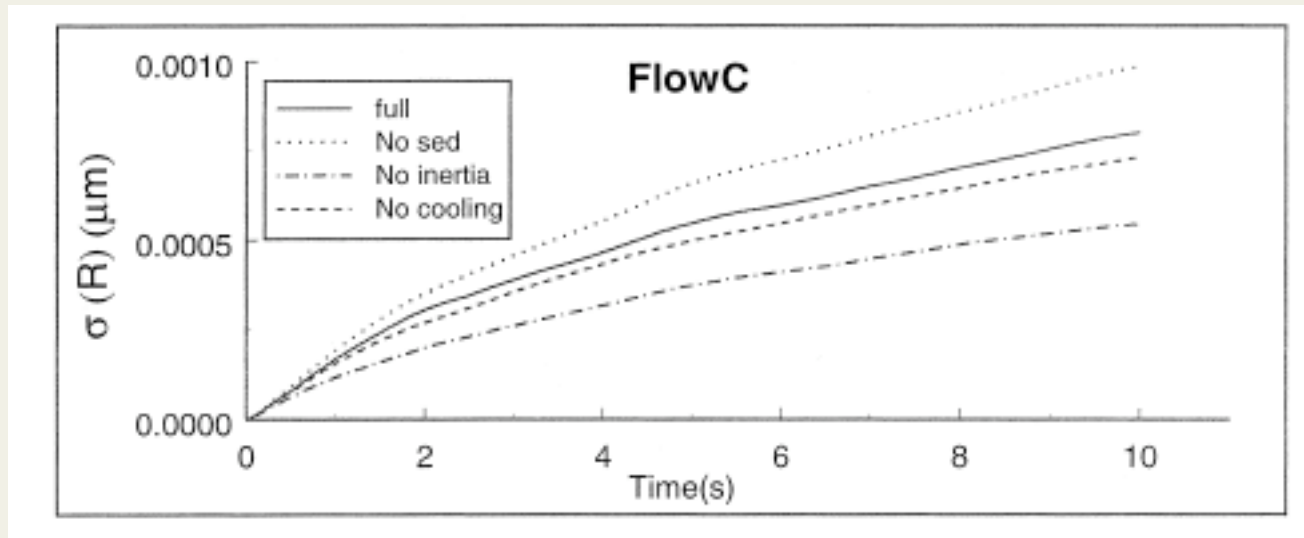


$r=15 \text{ micron}$



$r=10 \text{ micron}$





Main conclusion: small-scale turbulence has a very small effect...

What about those DNS limitations?

Argument: if  $Re$  increases (i.e., the range of scales involved increases), can supersaturation fluctuation increase as well?

Yes, but only to some point...

# The **brake** on supersaturation fluctuations:

$$\frac{dS}{dt} = \alpha w - \frac{S}{\tau_{qe}}$$

$$\tau_{qe} \sim 1\text{sec}$$

$$\frac{dS}{dt} \equiv 0 \rightarrow S_{qe} = \alpha w \tau_{qe}$$

TABLE 1. Time constant characterizing supersaturation.  
(Values of  $\tau = 1/(a_2 I)$  s for  $p = 771$  mb,  $T = 4.3^\circ\text{C}$ )

Radius ( $\mu\text{m}$ )	Droplet concentration ( $\text{cm}^{-3}$ )			
	100	300	500	1000
2	14.1	4.7	2.8	1.4
3	8.7	2.9	1.7	0.87
5	4.9	1.6	0.98	0.49
10	2.3	0.77	0.46	0.23

Politovich and Cooper, JAS 1988

For eddies with time-scale  
larger than  $\tau_{qe}$ ,  $S$  is limited  
to  $S_{qe}$  !!!

***So within a uniform cloud (e.g., the adiabatic core), fluctuations of the supersaturation have a small effect.***



Another way to think about this problem:

Condensational growth is reversible: droplets grow more in higher  $S$ , and then less in lower  $S$ , and the two situations change rapidly...

*But if you think about the collisional growth, then the story is different: growth is not reversible...*

*So within a uniform cloud (e.g., the adiabatic core), fluctuations of the supersaturation have a small effect. But what about the impact of mixing with the dry air from cloud environment (entrainment)?*

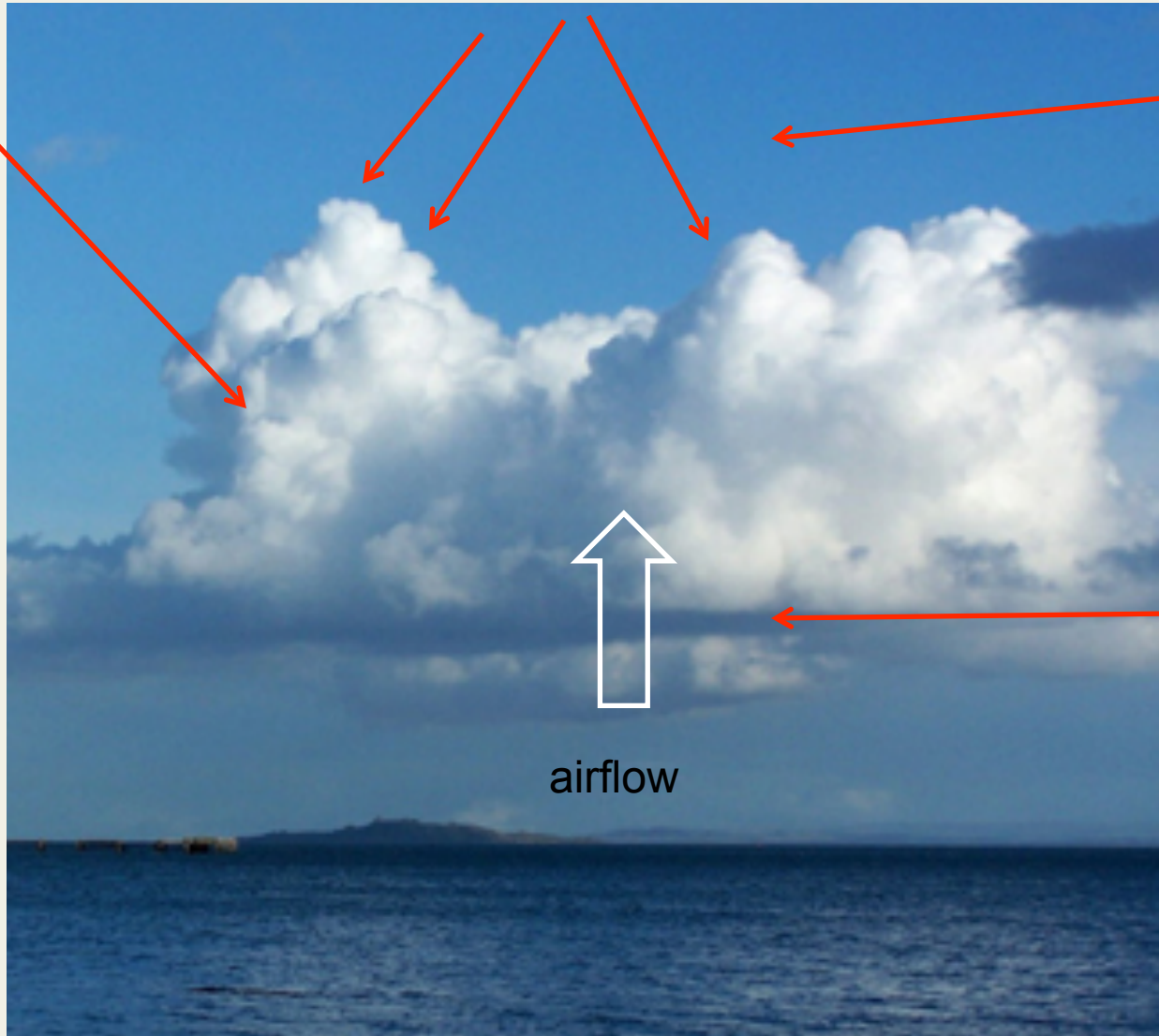
turbulent  
cloud

interfacial  
instabilities

calm (low-  
turbulence)  
environment

cloud base  
(activation of  
cloud droplets)

airflow



## Entrainment/mixing and the cloud droplet spectra:

1. Entrainment/mixing typically leads to partial evaporation of cloud water. How this evaporation affects the spectrum of cloud droplets?
2. Entrainment/mixing may lead to activation of cloud droplets above the cloud base.
3. Entrainment/mixing provides large-scale source of cloud turbulence (this may seem irrelevant...).

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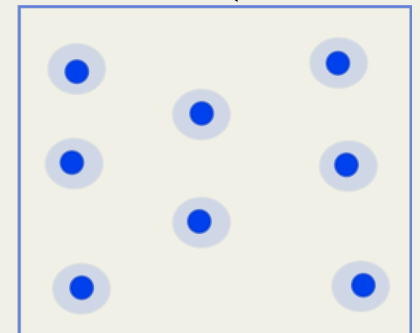
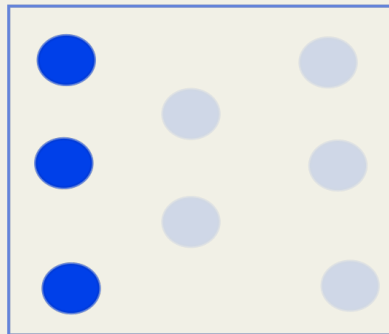
# Turbulent mixing



extremely  
inhomogeneous  
mixing

homogeneous  
mixing

Microphysical  
transformations due to  
subgrid-scale mixing  
cover a wide range of  
mixing scenarios.



Homogeneity of mixing depends on the ratio of two relevant time scales:

***-Droplet evaporation time scale:***

$$\tau_{\text{evap}} \equiv r \left( \frac{dr}{dt} \right)^{-1} = \frac{r^2}{A(1 - \text{RH})},$$

$$A \approx 10^{-10} \text{ m}^2 \text{ s}^{-1}$$

***-Turbulent mixing time scale***

$$\tau_{\text{mix}} \equiv \frac{L}{U(L)} \sim \left( \frac{L^2}{\epsilon} \right)^{1/3},$$

$$U(L) \sim (\epsilon L)^{1/3}$$





droplet  
spectra

vertical and  
along-track  
velocity

liquid water  
content

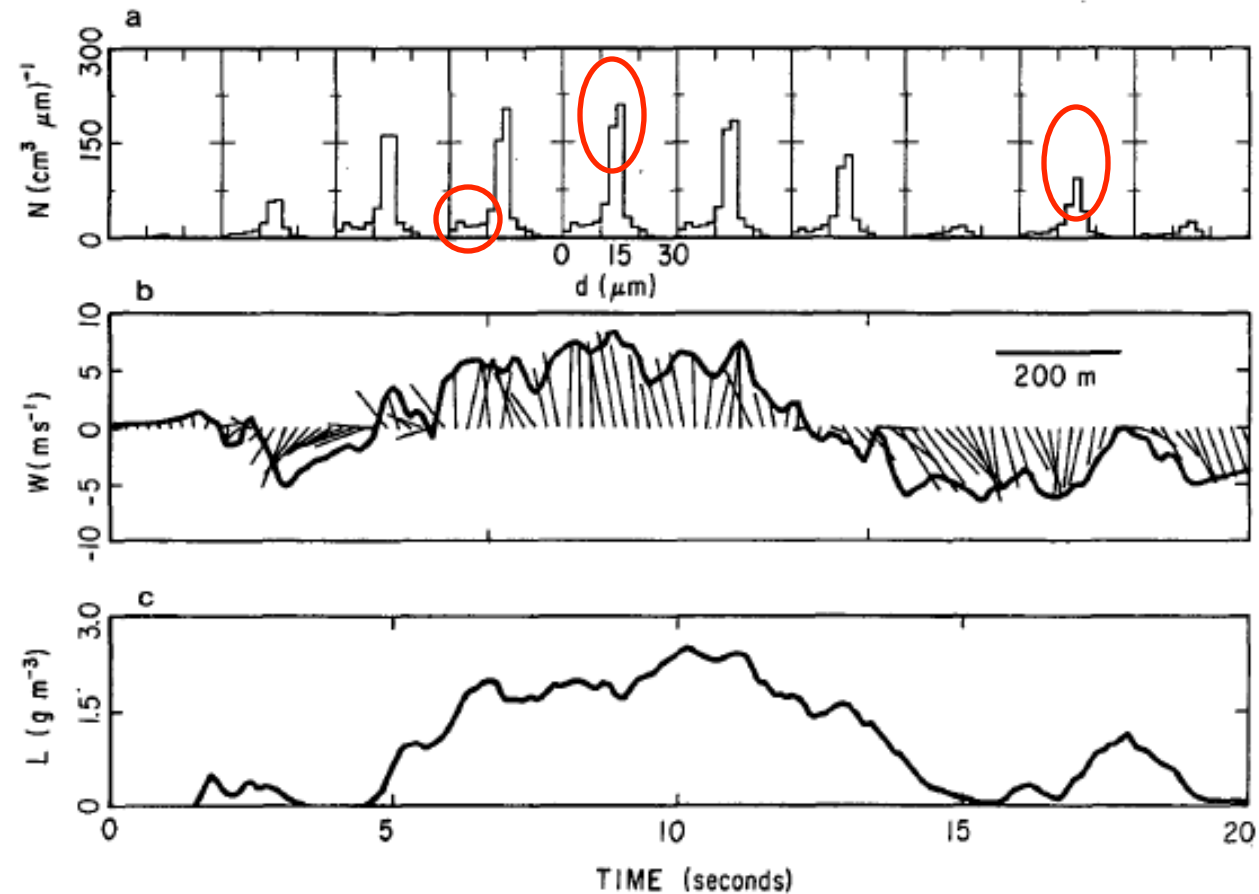
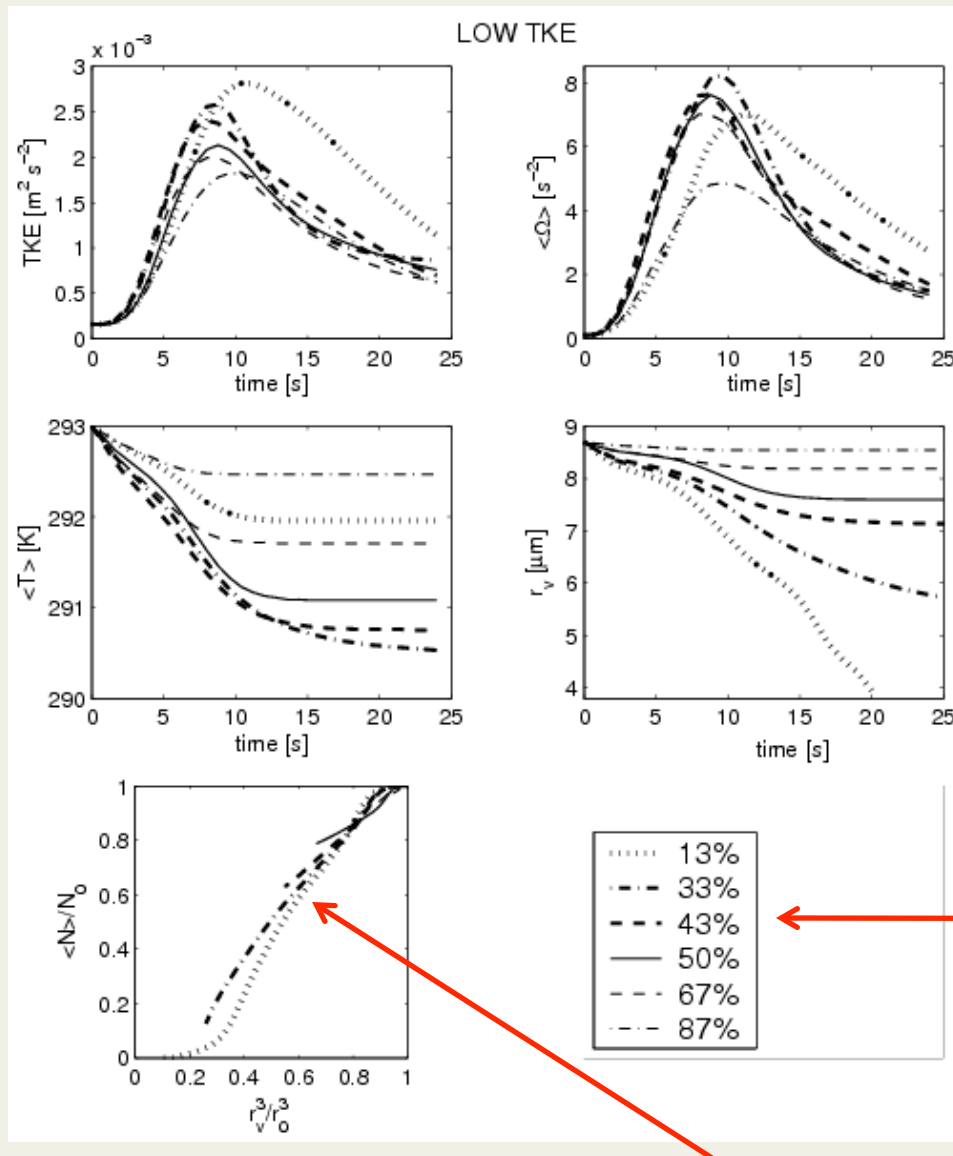


FIG. 3. Penetration at 600 mb, 6 June: (a) two-second averaged droplet spectra (sizes for diameter bins are those given by the manufacturer); (b) wind velocity, the lines represent wind vectors formed from the vertical wind and the wind along the flight path; (c) liquid water density measured by the Johnson-Williams device. All H-2 measurements.

(Austin et al. JAS 1985)

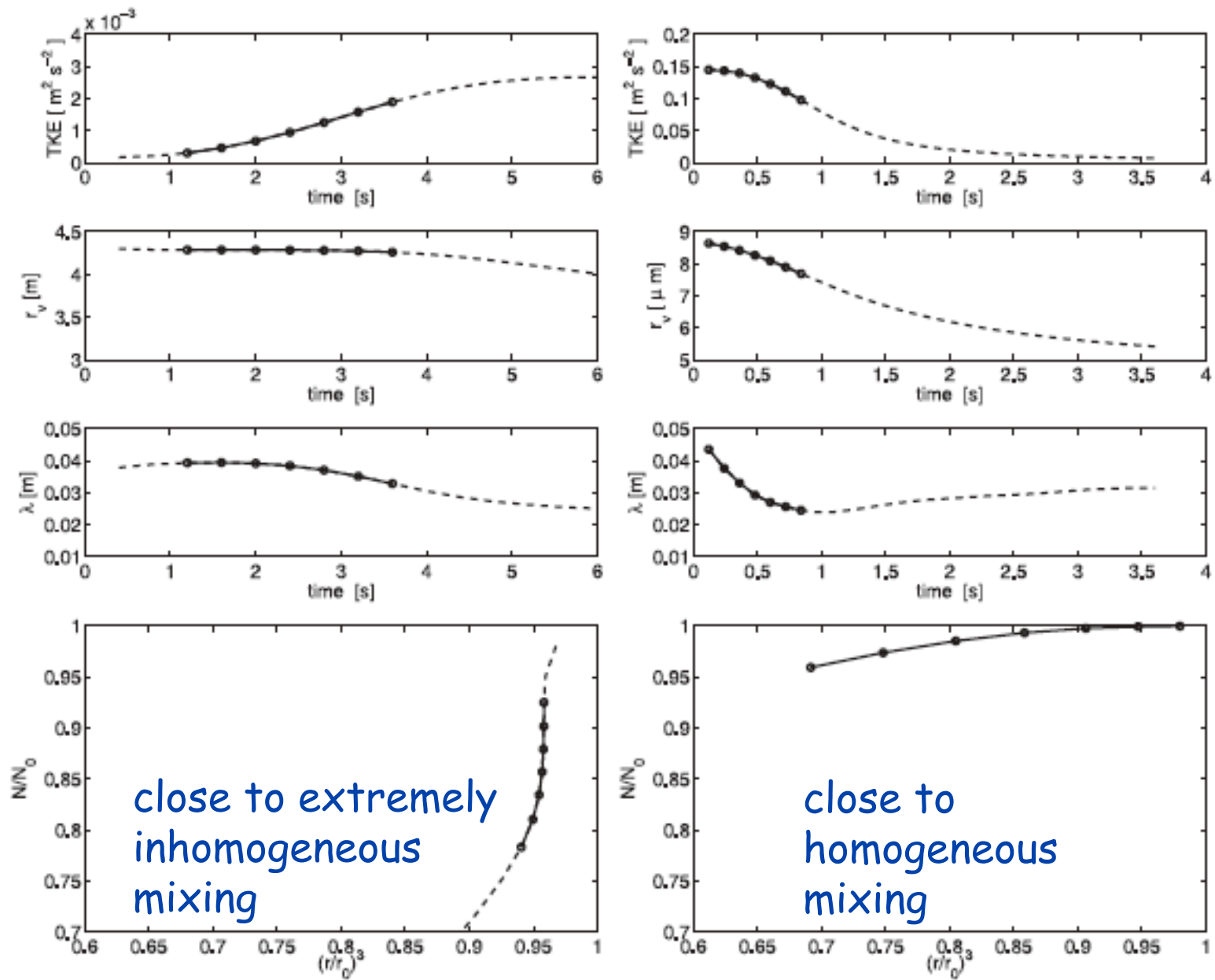
# DNS simulations of microscale homogenization of initially separate filaments of cloudy and cloud-free air (decaying turbulence setup).

Andrejczuk et al JAS 2006



The percentage represents the initial volume fraction of cloudy air.

Evolution of the number of droplets  $N$  and their mean volume radius  $r_v$ , both normalized by the initial values



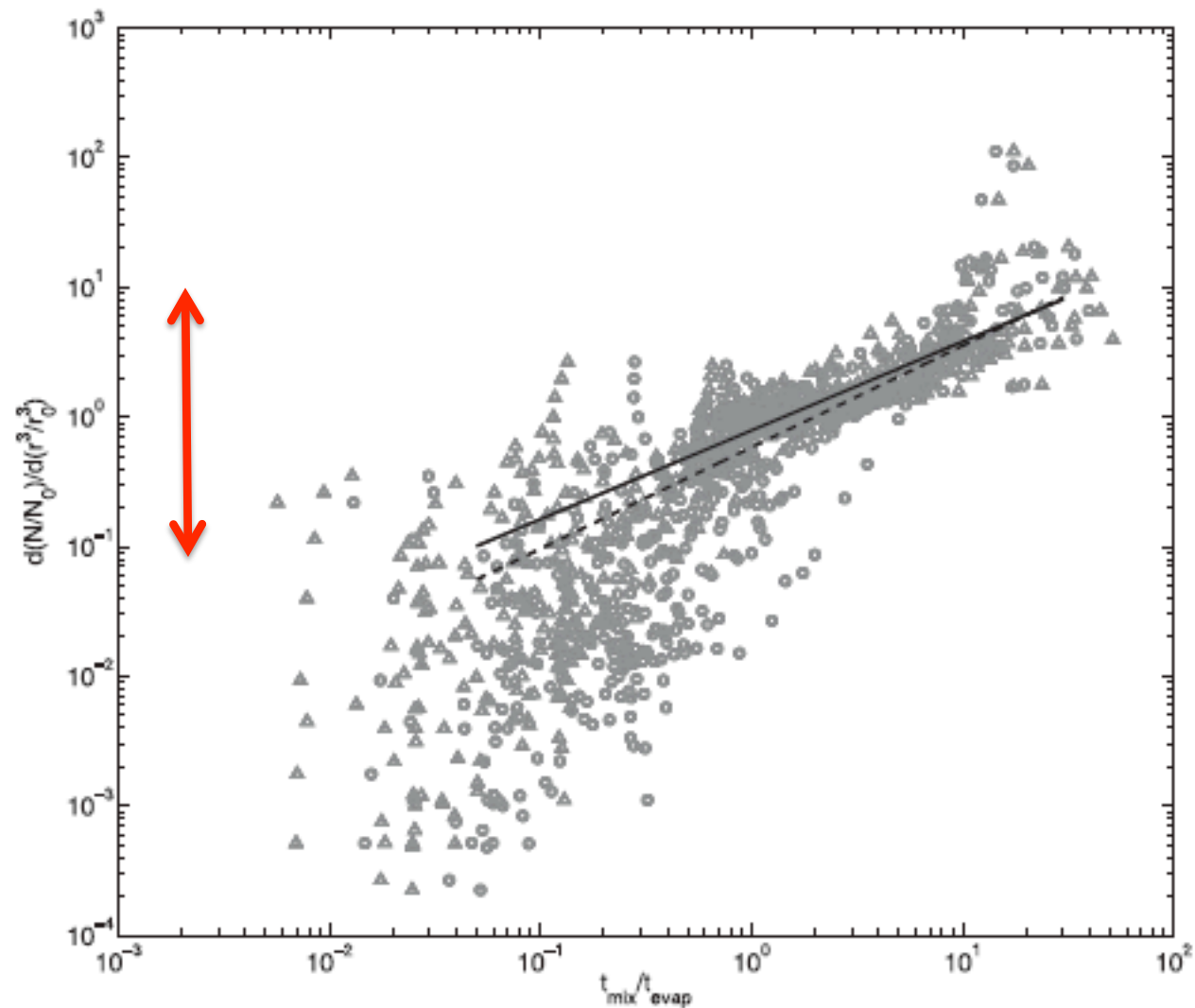


FIG. 2. Scatterplot of the slope of the mixing line on the  $r$ - $N$  diagram vs the ratio between the turbulent mixing and the droplet evaporation time scales. Each data point represents analysis of instantaneous DNS data as explained in text, with triangles (circles) depicting data points with the mixing time scale calculated using TKE (enstrophy). The solid and dashed lines represent linear fits for either triangles or circles. See text for details.

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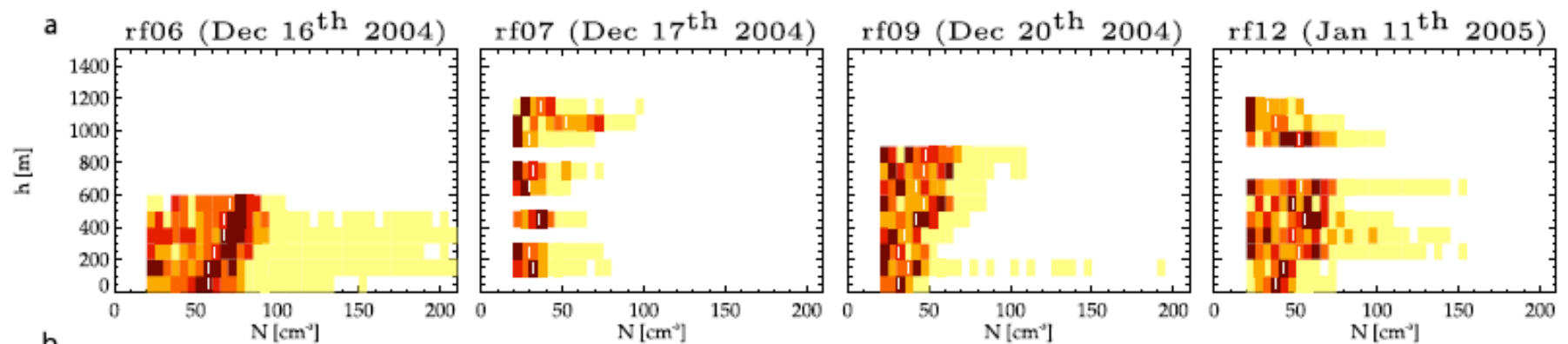
## Gerber et al. *JMSJ* 2008

Table 3. Microphysics of the seven Cu at five different levels shown in Fig. 2, with mean values of LWC (liquid water content) and its sample standard deviation for three horizontal data resolutions, total droplet concentration  $N$ , and mean volume radius  $r_v$ . The latter two parameters correspond to 10-m resolution data. The subscript a indicates expected adiabatic values.

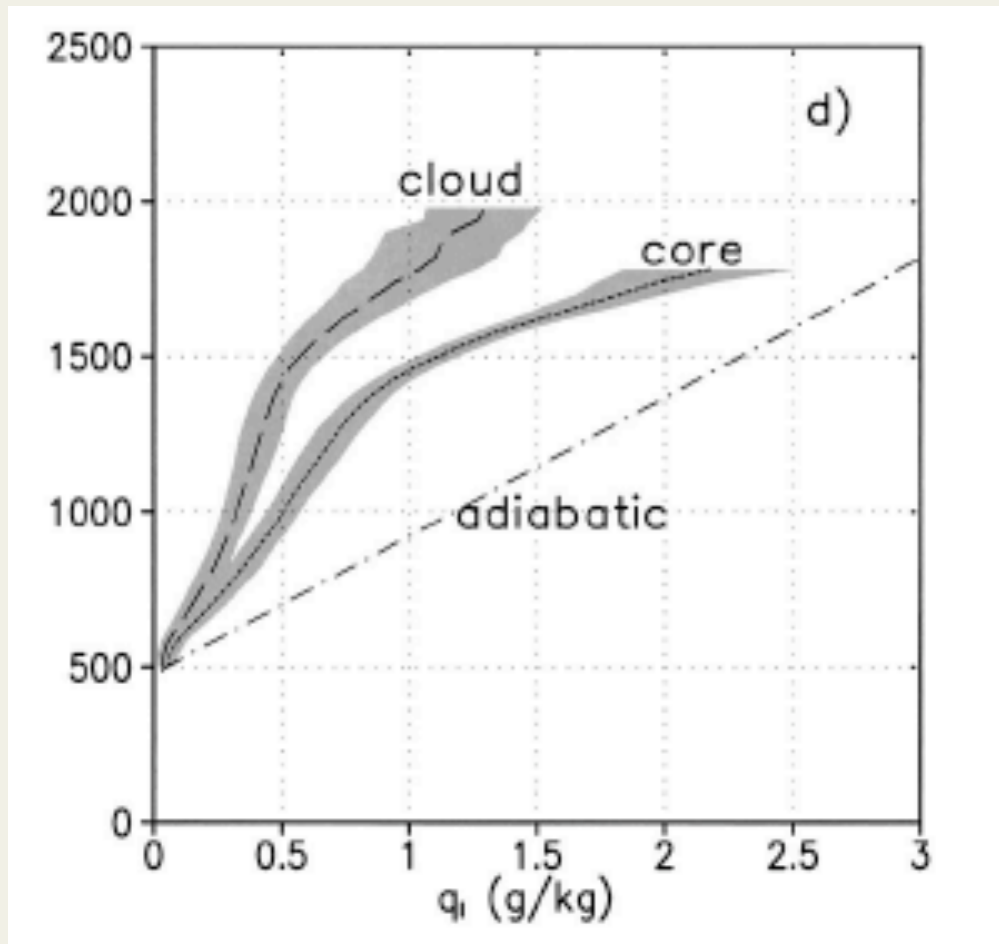
Level	$LWC_g$ (g/m <sup>3</sup> )	$LWC$ (g/m <sup>3</sup> )	s (10 cm) (g/m <sup>3</sup> )	s (50 cm) (g/m <sup>3</sup> )	s (1000 cm) (g/m <sup>3</sup> )	$N$ (No/cc)	s [ $N$ ] (No/cc)	$r_{va}$ ( $\mu$ m)	$r_v$ ( $\mu$ m)	s ( $r_v$ ) ( $\mu$ m)
1	.605	.284	.084	.078	.063	95	12	11.4	9.2	2.0
2	1.00	.427	.142	.136	.128	97	22	13.5	10.6	3.1
3	1.42	.520	.160	.153	.145	112	25	15.2	10.2	1.7
4	2.11	.536	.196	.184	.173	116	11	17.3	10.6	2.4
5	2.46	.331	.142	.135	.125	54	35	18.2	11.9	3.7

## Arabas et al. *GRL* 2009

### ARABAS ET AL.: OBSERVATIONS OF CU MICROPHYSICS



***How is it possible that the dilution of the cloud water content is NOT accompanied by the dilution of the droplet concentration?***



***How is it possible that the dilution of the cloud water content is NOT accompanied by the dilution of the droplet concentration?***

**In-cloud activation (i.e., activation above the cloud base)!**

Slawinska et al. (*J. Atmos. Sci.*; in review)

Wyszogrodzki et al. (*Acta Geophysica*; in review)



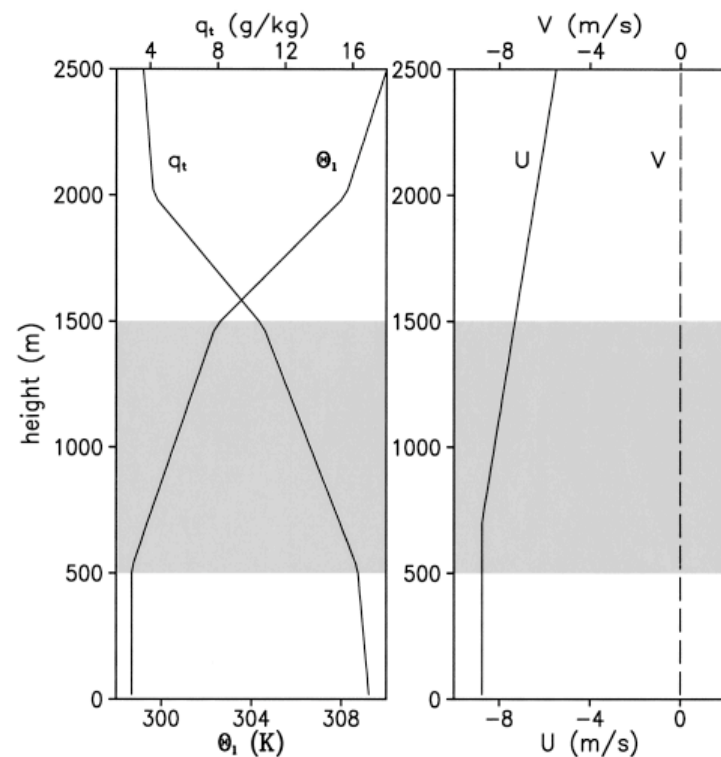
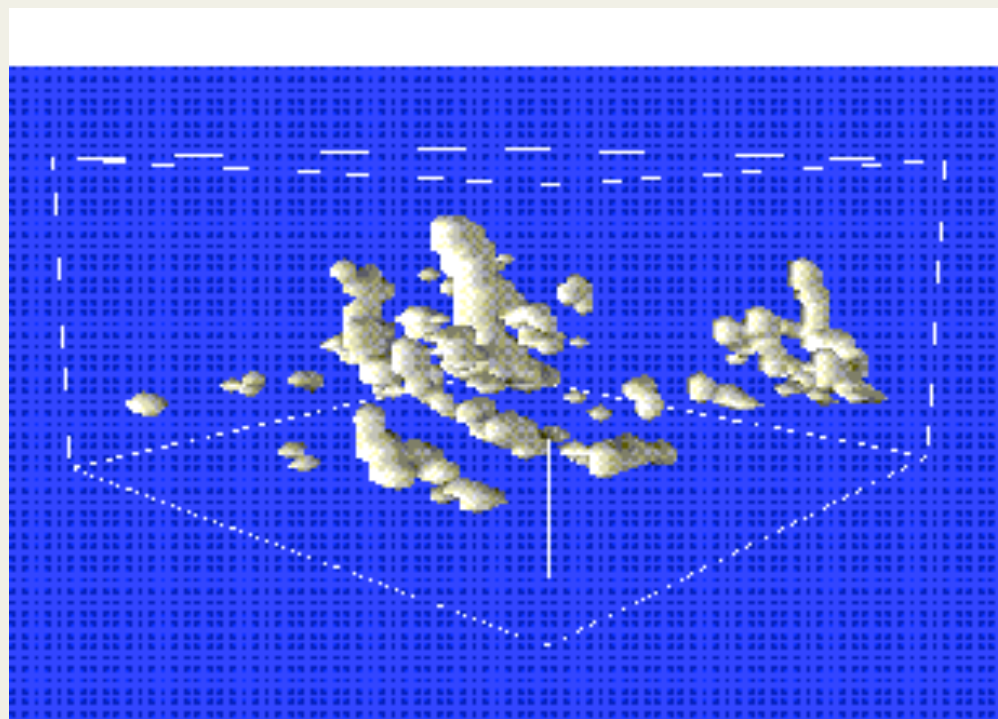
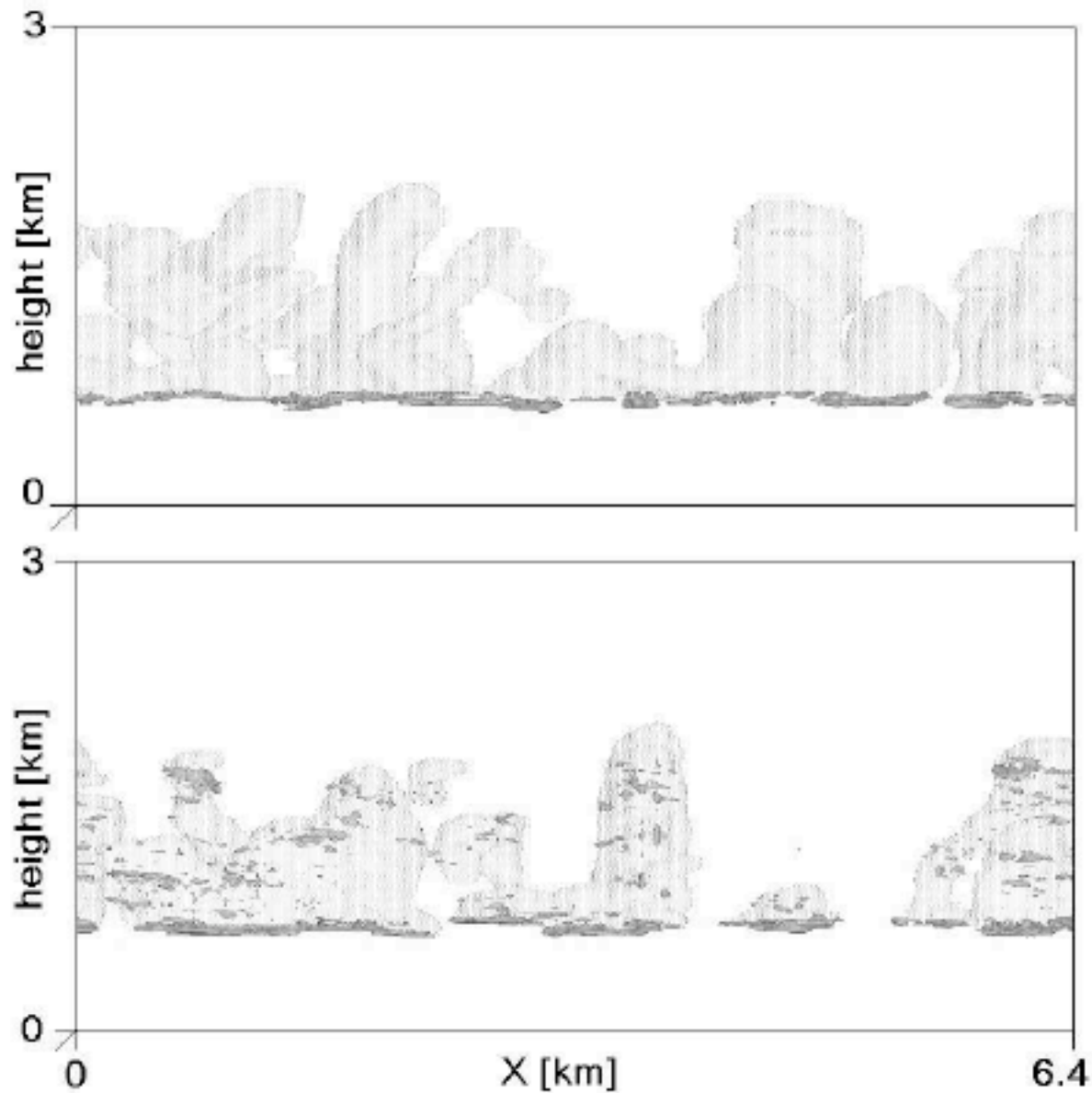


FIG. 1. Initial profiles of the total water specific humidity  $q_t$ , the liquid water potential temperature  $\theta_t$ , and the horizontal wind components  $u$  and  $v$ . The shaded area denotes the conditionally unstable cloud layer.



The Barbados Oceanographic and Meteorological Experiment (BOMEX) case (Holland and Rasmusson 1973)

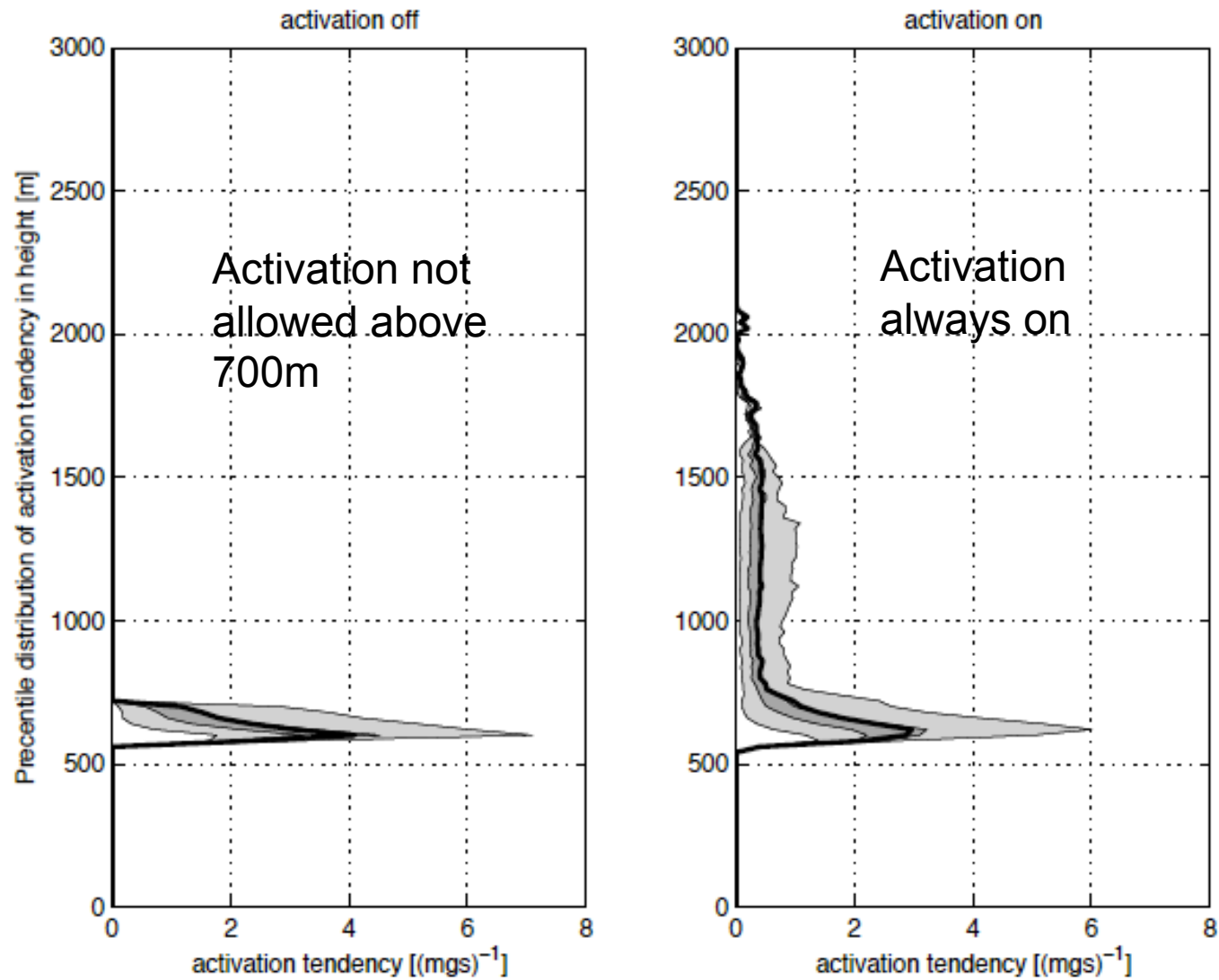
gray – cloud water; dark gray – positive activation tendency

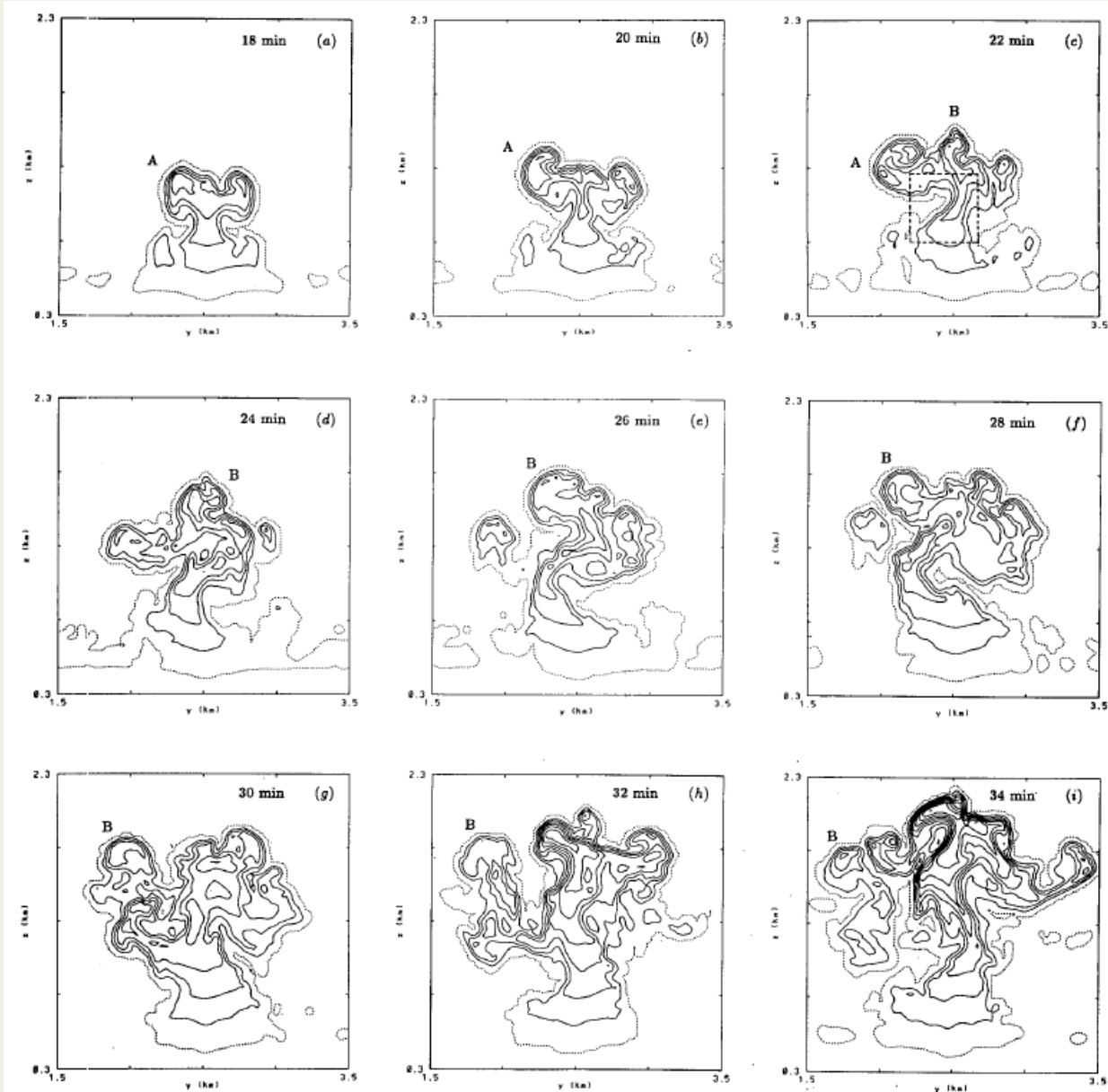


Activation not  
allowed above  
700m

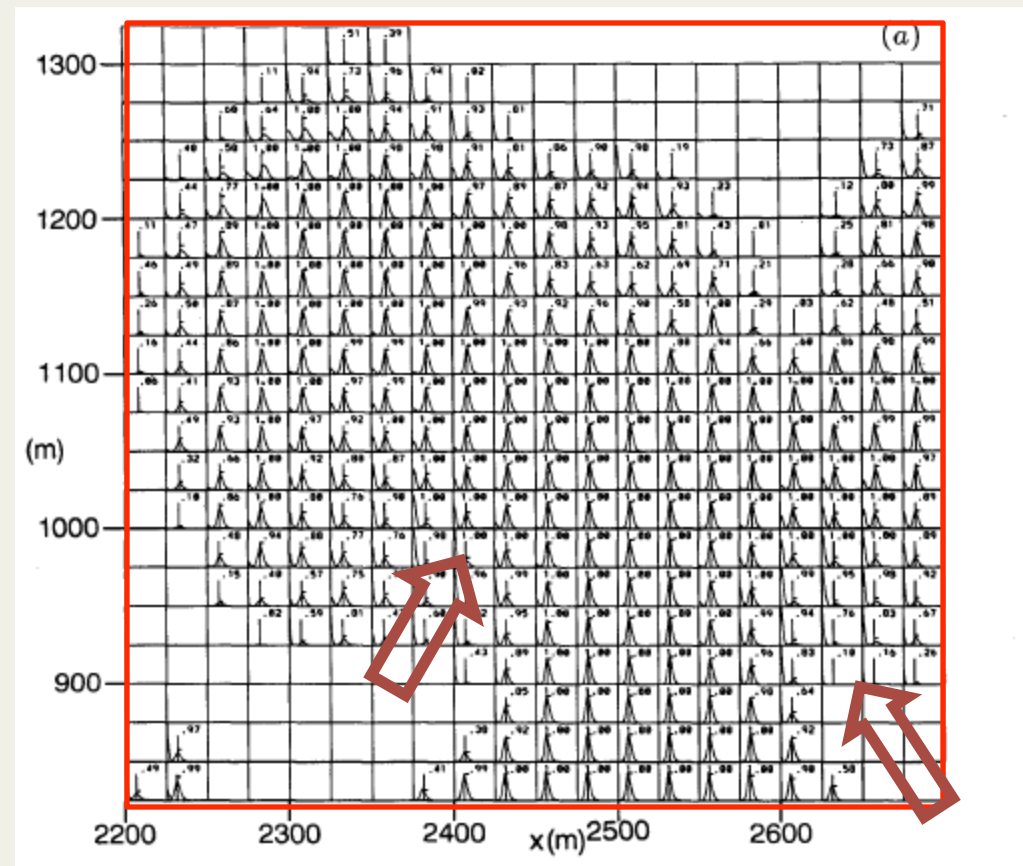
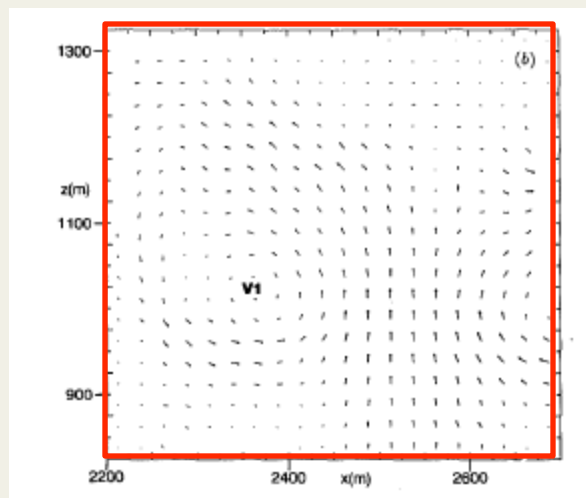
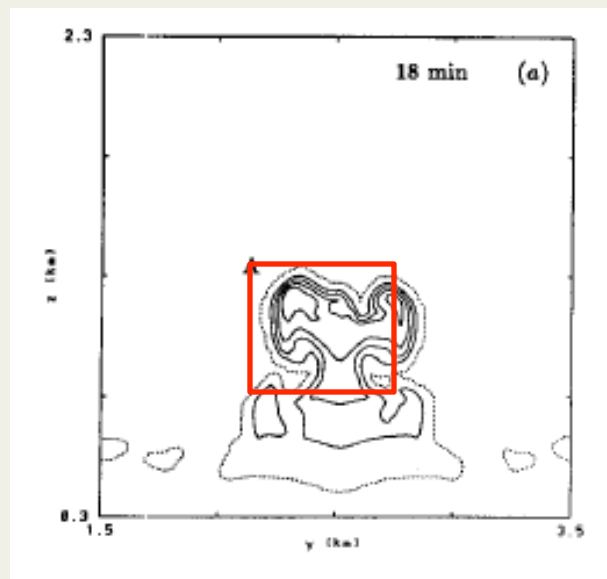
Activation  
always on

## Conditionally-sampled activation tendency

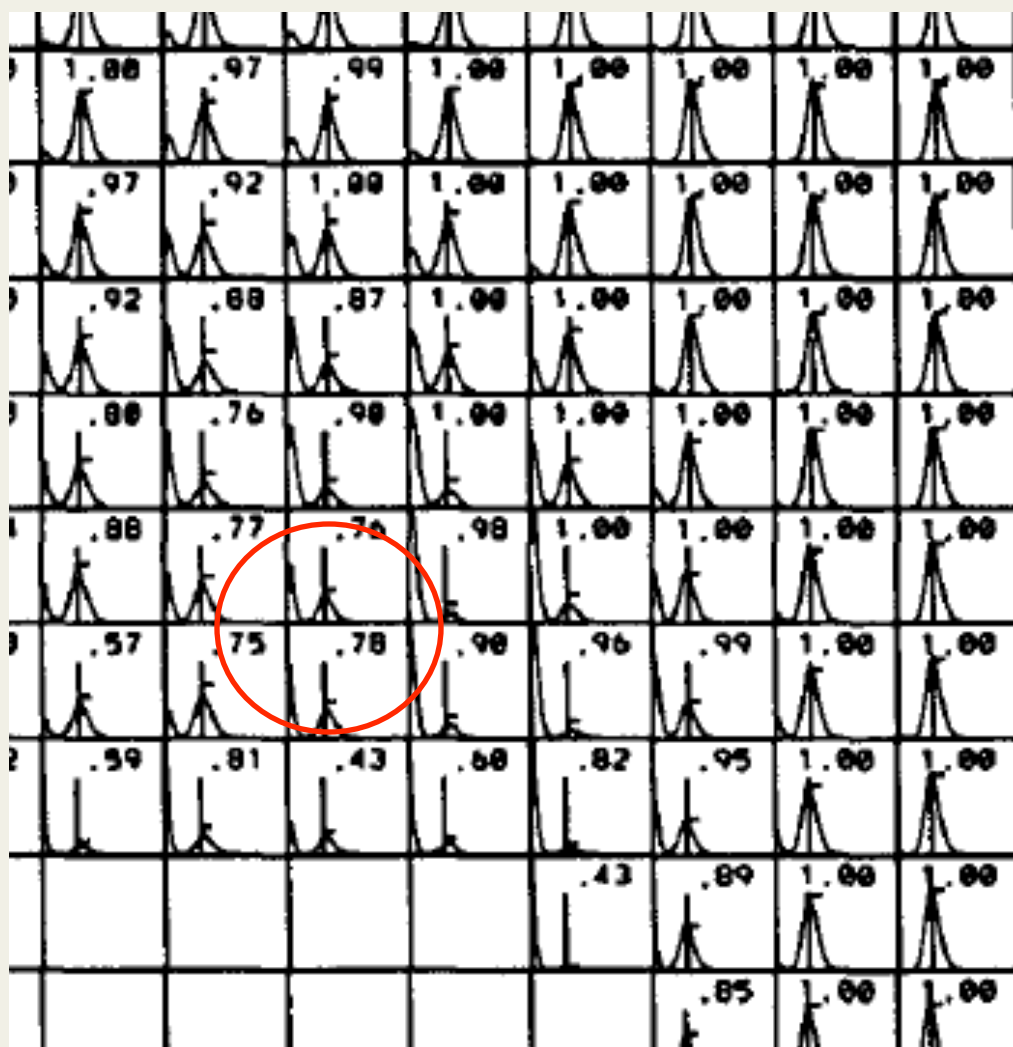




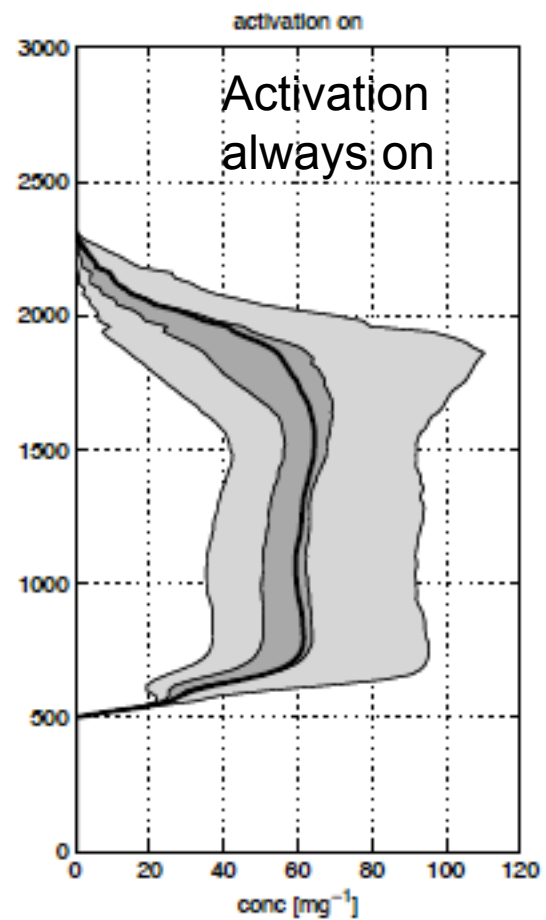
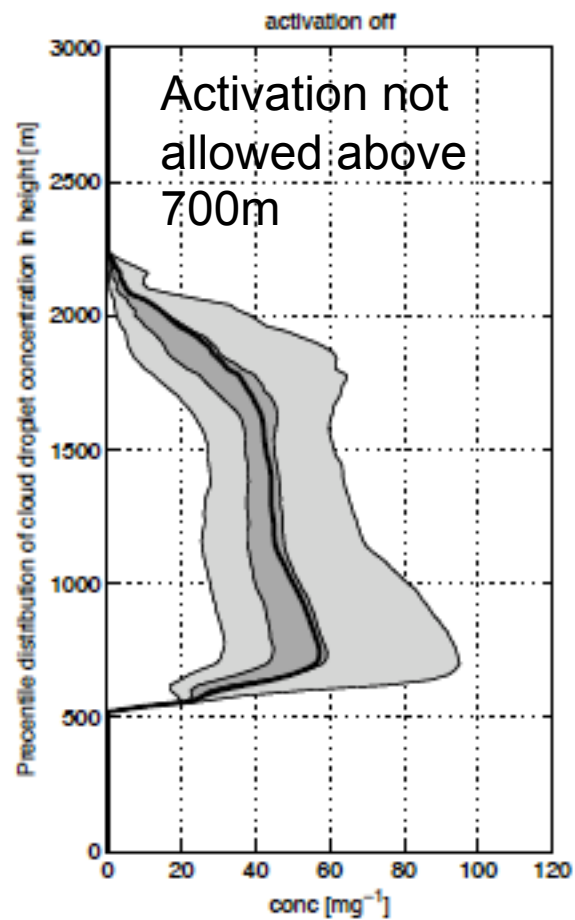
Brenguier and Grabowski (JAS 1993)



Brenguier and Grabowski (JAS 1993)



Brenguier and Grabowski (JAS 1993)



traditional view



view suggested by  
model simulations





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## ***“Large-eddy hopping”***

(Al Cooper, NCAR; Sonia Lasher-Trapp, Purdue; Alan Blyth, Leeds):

**Droplets observed in a single location within a cloud arrive along variety of fluid trajectories:**

- large scales are needed to provide different droplet activation/growth histories;*
- small scales needed to allow hopping from one large eddy to another.*

[see also Sidin et al. (*Phys. Fluids* 2009) for idealized 2D synthetic turbulence simulations]

# The Initiation of Coalescence in a Cumulus Cloud

## A Beneficial Influence of Entrainment and Mixing

Al Cooper

NCAR EOL

Collaborators:

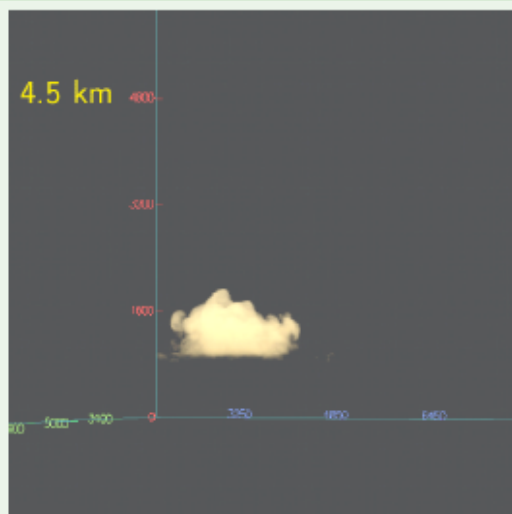
Sonia Lasher-Trapp, Purdue University

Alan Blyth, University of Leeds

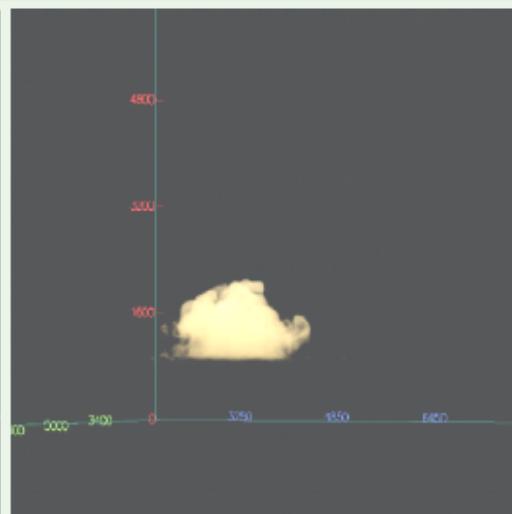
April 21, 2011: EOL/MMM Seminar

courtesy of Al Cooper, NCAR

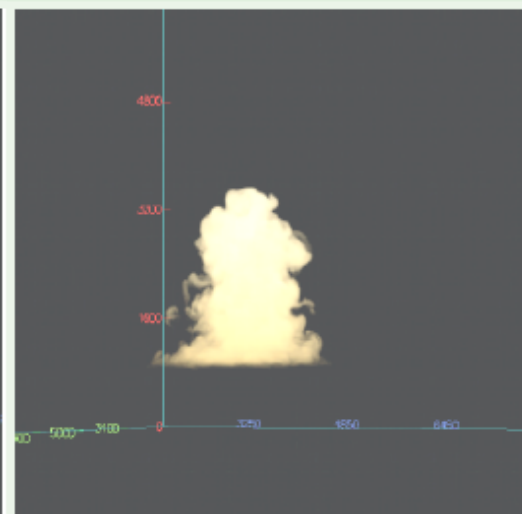
2 min



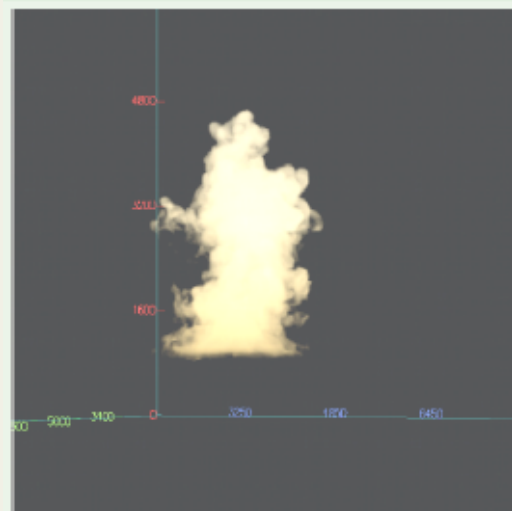
4 min



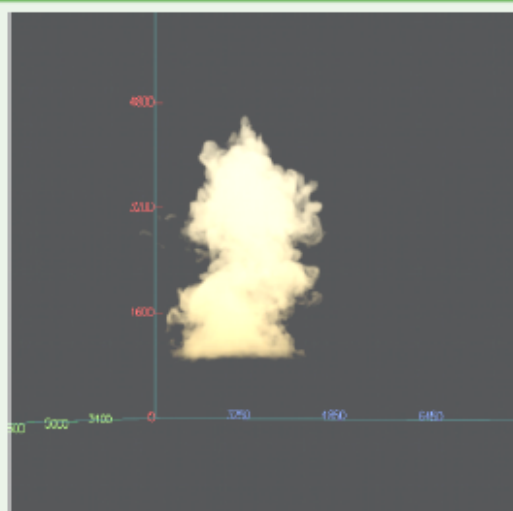
10 min



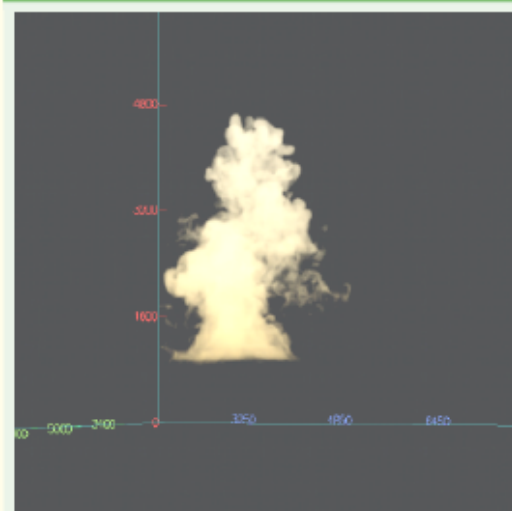
16 min



18 min



21.75 min

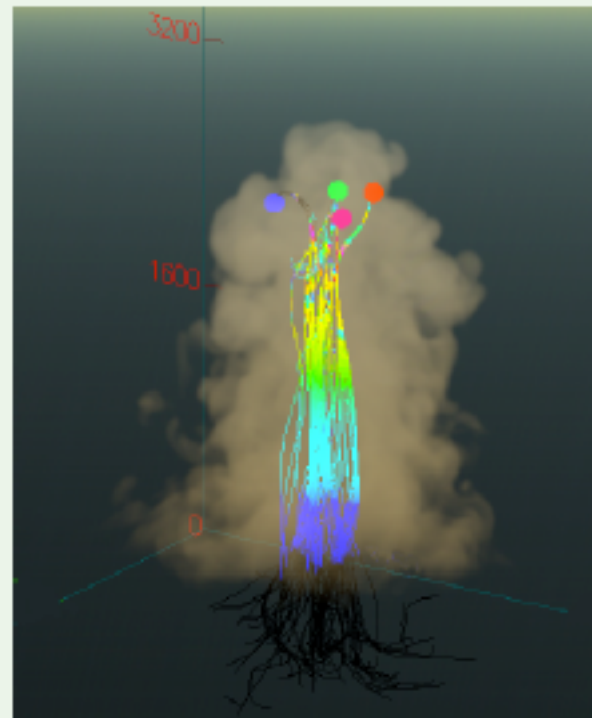


courtesy of Al Cooper, NCAR

### Model of Rain Formation

- 1 Dynamical cloud model
- 2 **Generate trajectories**
- 3 Lagrangian microphysical model
- 4 Combine droplets from trajectories; continue growth
- 5 Inject resulting embryos into cloud-water fields
- 6 Allow continued growth until decay of the cloud

### Trajectories through the cloud



courtesy of Al Cooper, NCAR

# ADIABATIC ASCENT

## Test Case For Comparison

### Results

ascent to 3 km (254 s)

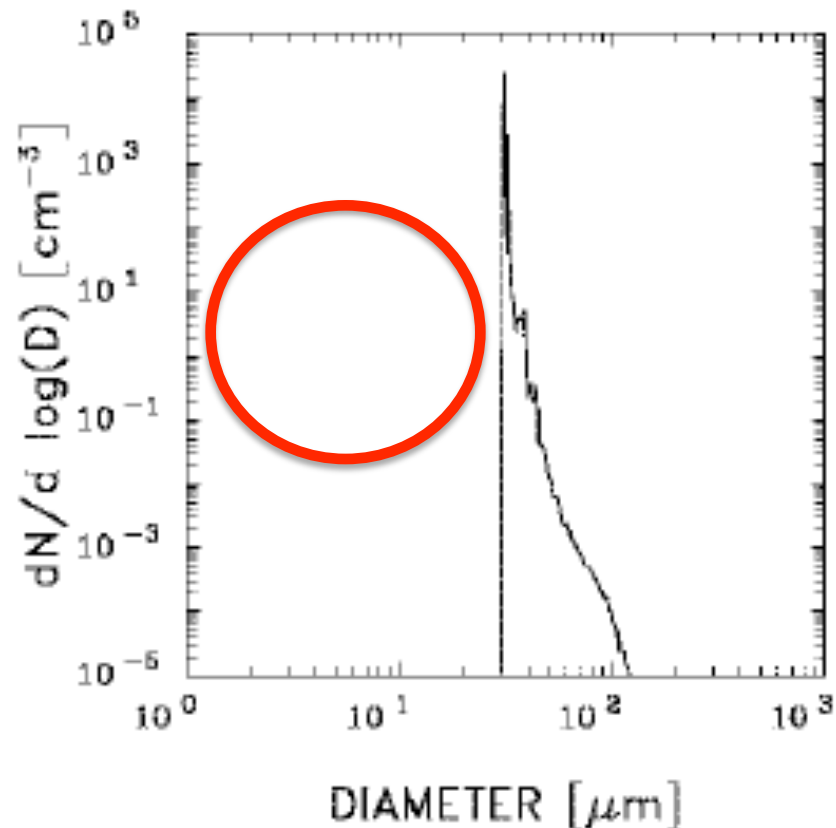
$N = 272 \text{ cm}^{-3}$

$LWC = 4.31 \text{ g m}^{-3}$

$\bar{d} = 31.2 \mu\text{m}$ ,  $\sigma = 0.30 \mu\text{m}$

### Noteworthy Aspects:

- 1 very narrow:  
 $\sigma/d < 0.01$
- 2 peaks, multiples of modal mass



courtesy of Al Cooper, NCAR

# ENSEMBLE CONTRIBUTIONS

Result of Variability Along Trajectories

## Results (vs. adiabatic)

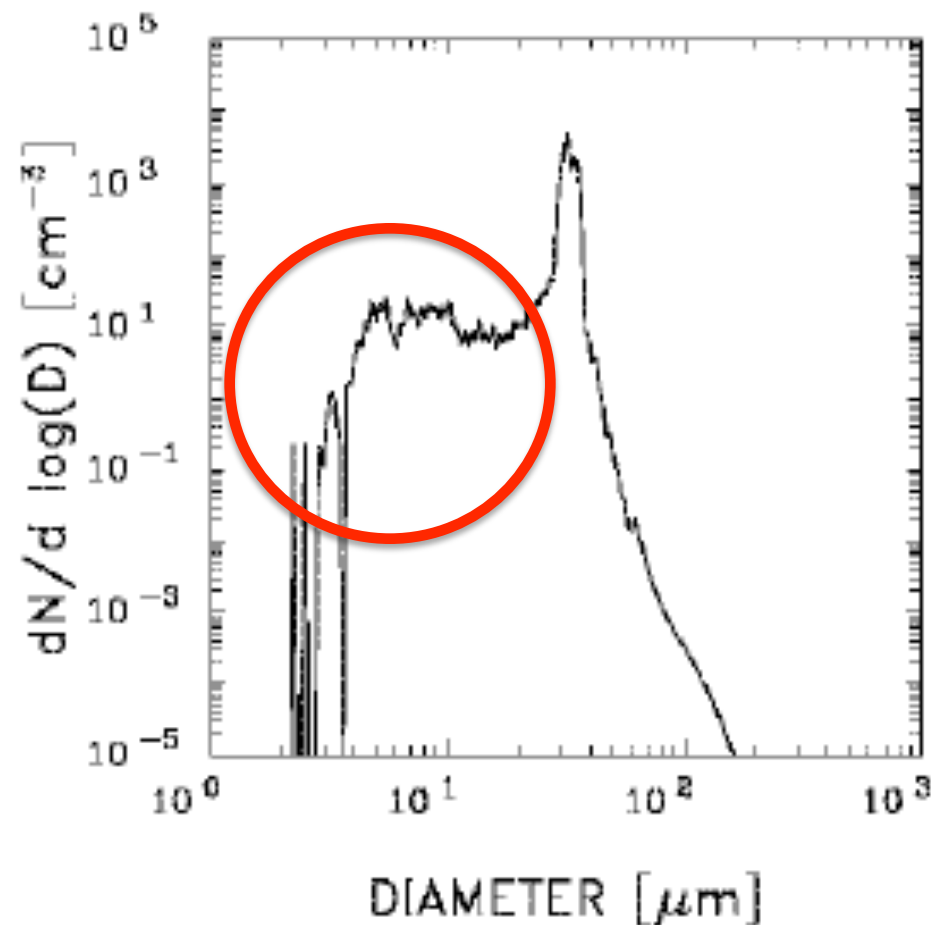
$N = 220 \text{ cm}^{-3}$  [80%]

LWC: 78%

$\bar{d} = 29.4 \mu\text{m}$ ,  $\sigma = 7.0 \mu\text{m}$

## Noteworthy Aspects:

- 1 Realistic shape:
  - broad, bimodal
  - dispersion  $\simeq 0.24$
- 2 many more large drops



courtesy of Al Cooper, NCAR

Condensational growth is reversible: droplets grow more in higher  $S$ , and then less in lower  $S$ , and the two situations change rapidly...

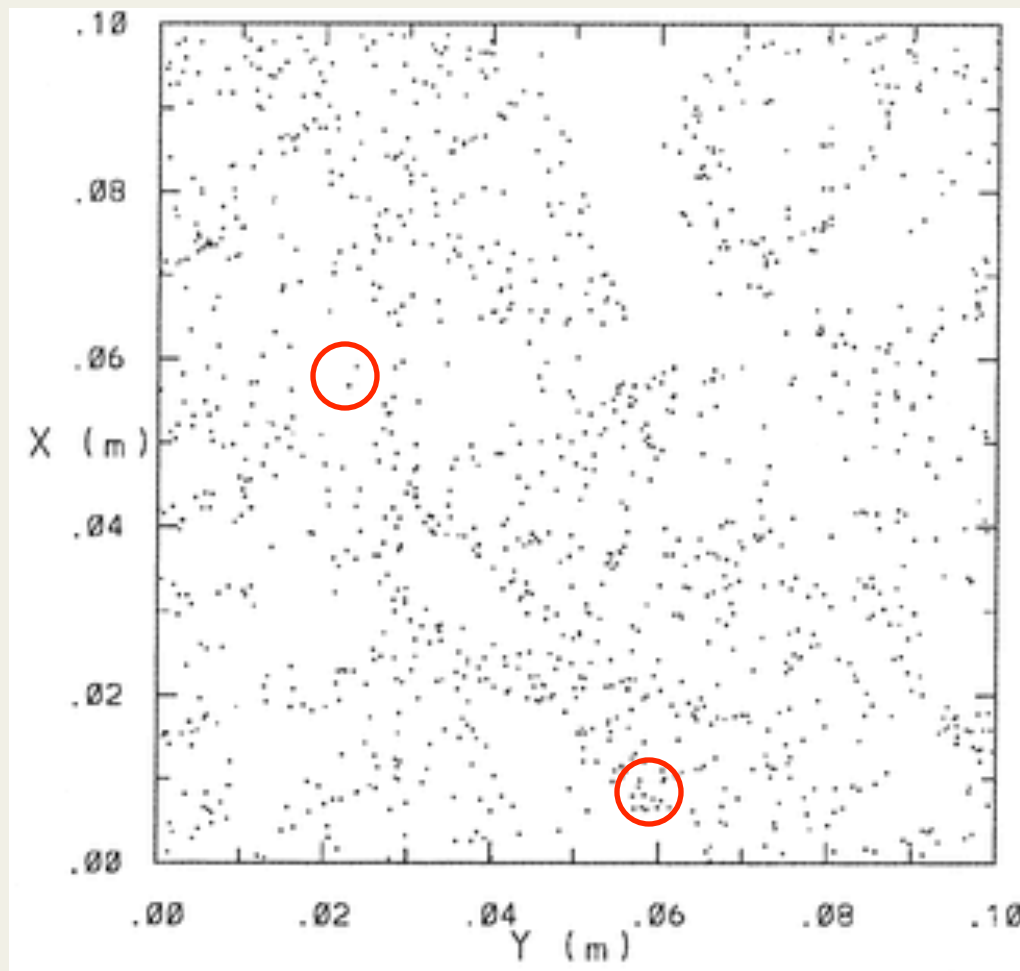
*... but entrainment/mixing and the “large-eddy hopping” provides additional effects contributing to spectral width.*

But if you think about the collisional growth, then the story is different: growth is not reversible...

*The impact on collisional growth is being studied in a joint project with Prof. Lian-Ping Wang (U. of Delaware).*



**Growth by collision/coalescence:** nonuniform distribution of droplets in space affects droplet collisions...



## Three basic mechanisms of turbulent enhancement of gravitational collision/coalescence:

- Turbulence modifies local droplet concentration (preferential concentration effect)*
- Turbulence modifies relative velocity between colliding droplets (e.g., small-scale shears, fluid accelerations)*
- Turbulence modifies hydrodynamic interactions when two droplets approach each other*

## Three basic mechanisms of turbulent enhancement of gravitational collision/coalescence:

geometric collisions (no hydrodynamic interactions)

***-Turbulence modifies local droplet concentration (preferential concentration effect)***

***-Turbulence modifies relative velocity between colliding droplets (e.g., small-scale shears, fluid accelerations)***

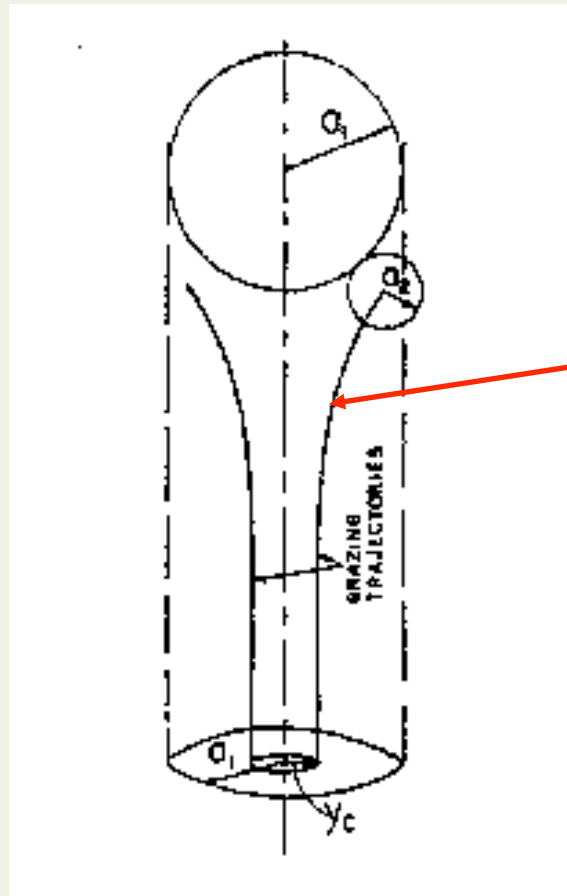
***- Turbulence modifies hydrodynamic interactions when two droplets approach each other***

## Three basic mechanisms of turbulent enhancement of gravitational collision/coalescence:

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collision efficiency

Collision efficiency  $E_c$  for the gravitational case:



Grazing trajectory

$$E_c = \frac{y_c^2}{(a_1 + a_2)^2}$$

**The hybrid DNS approach: including disturbance flows due to droplets**

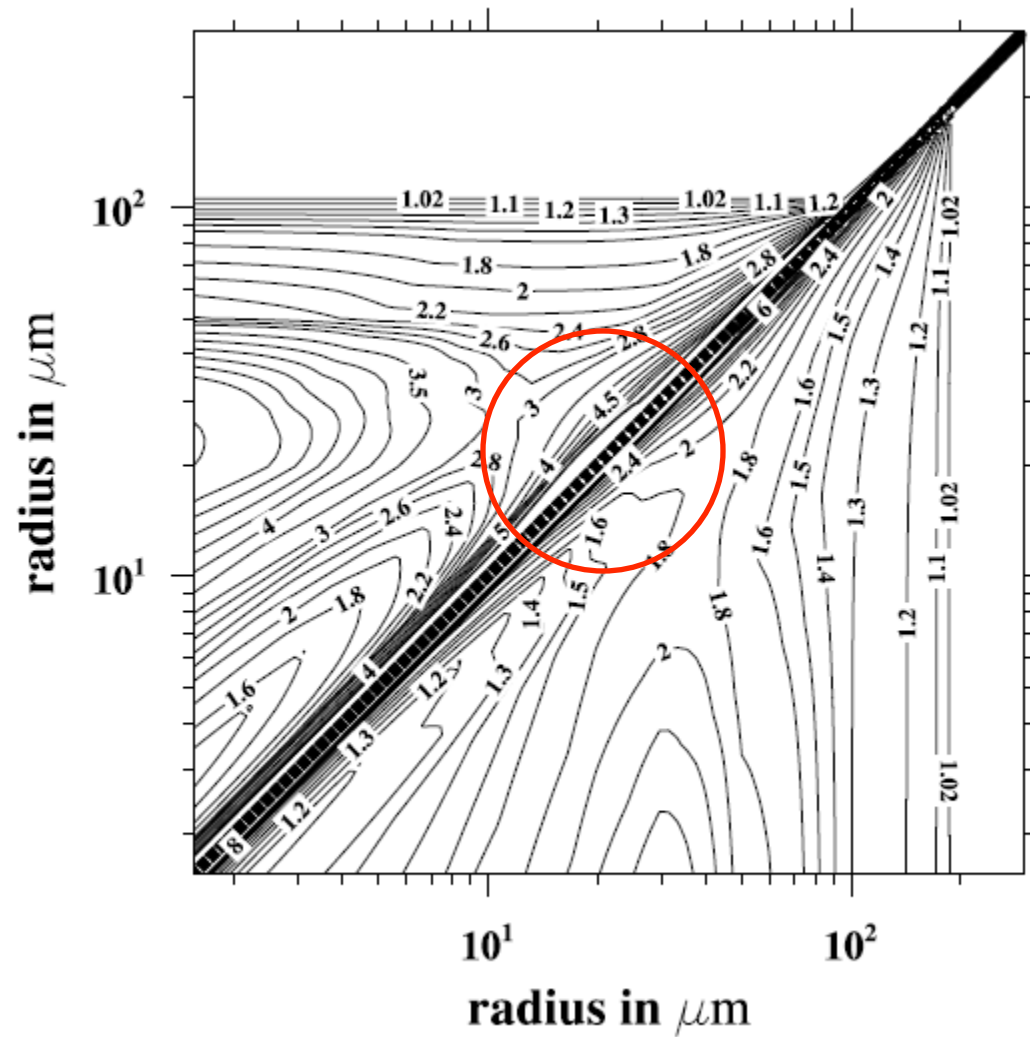
$$\vec{U}(\vec{x}, t) + \sum_{k=1}^{N_p} \vec{u}_s(\vec{r}_k; a_k, \vec{V}_k - \vec{U}(\vec{Y}_k, t) - \vec{u}_k)$$

Background turbulent flow

Disturbance flows due to droplets

Features: Background turbulent flow can affect the disturbance flows;  
 No-slip condition on the surface of each droplet is satisfied on average;  
 Both near-field and far-field interactions are considered.

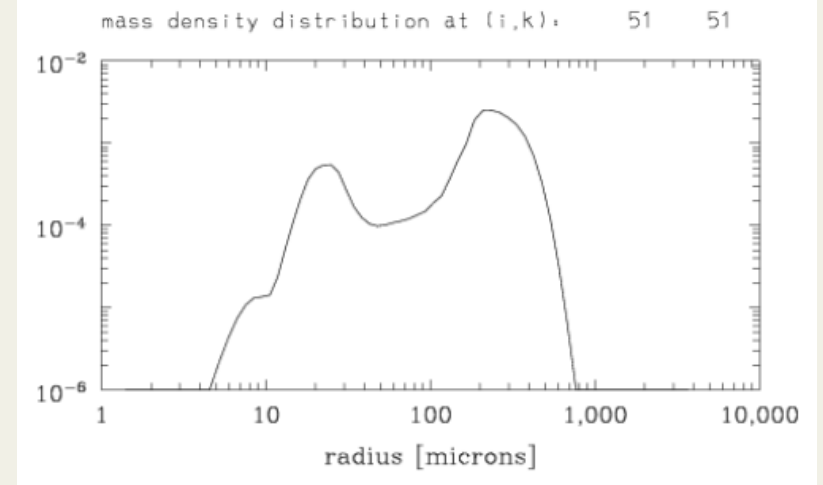
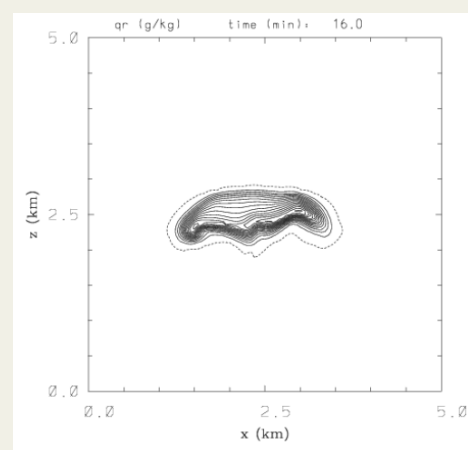
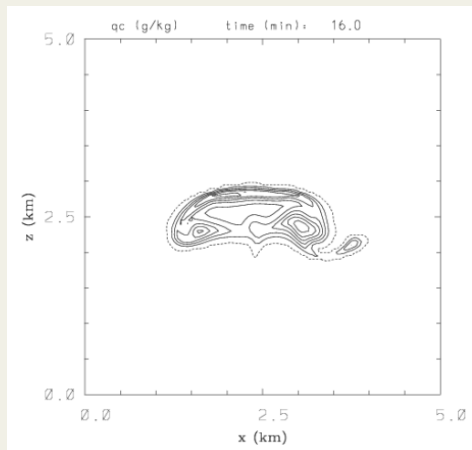
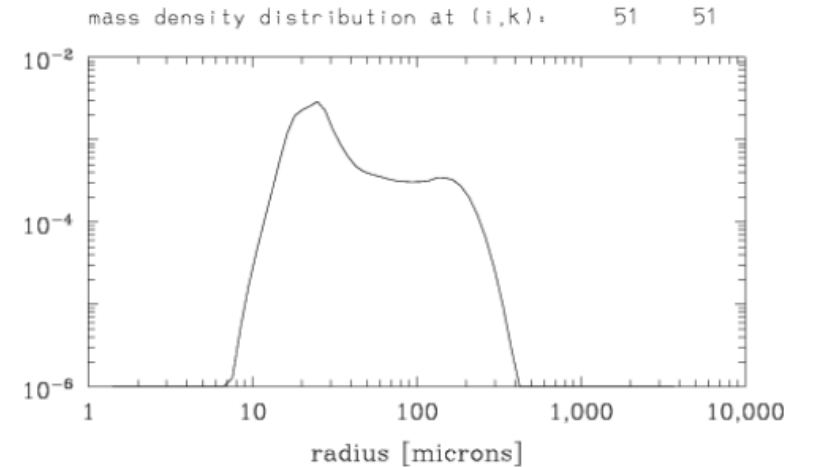
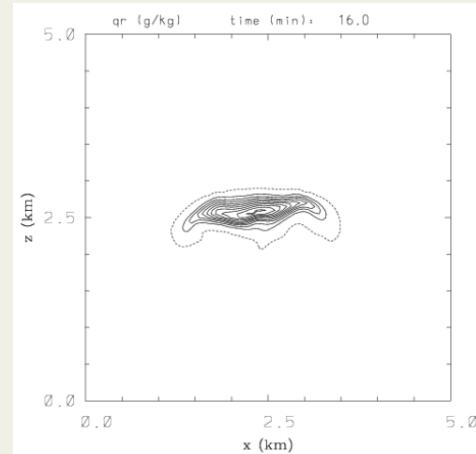
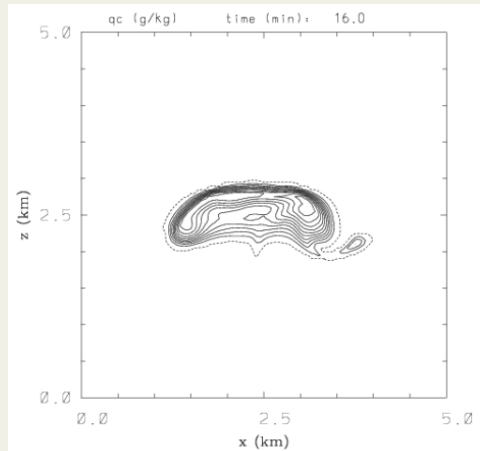
Wang, Ayala, and Grabowski, J. Atmos. Sci. 62: 1255-1266 (2005).  
 Ayala, Wang, and Grabowski, J. Comp. Phys. 225: 51-73 (2007).



Enhancement factor for the collision kernel (the ratio between turbulent and gravitation collision kernel in still air) **including turbulent collision efficiency**;  $\varepsilon = 100$  and  $400 \text{ cm}^2 \text{ s}^{-3}$ .

## 2D simulation of a small precipitating cloud: $t=16$ min

*no turbulence*

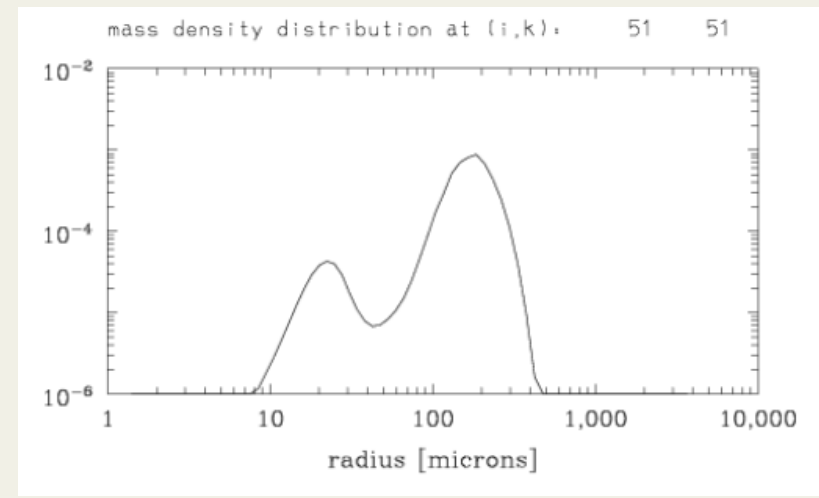
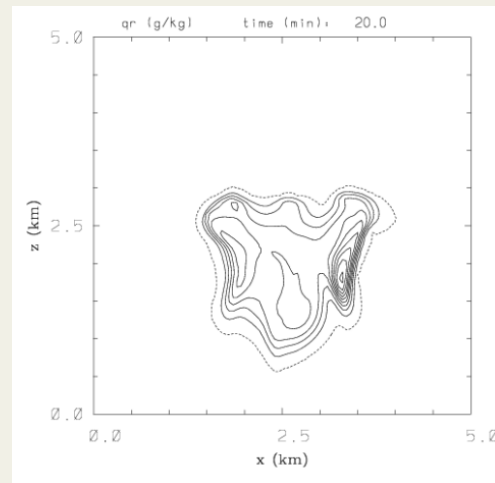
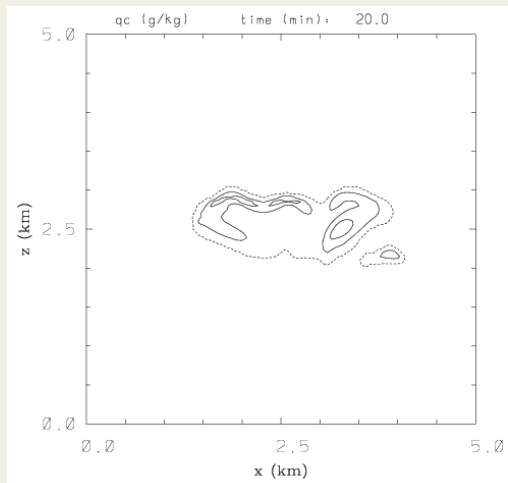
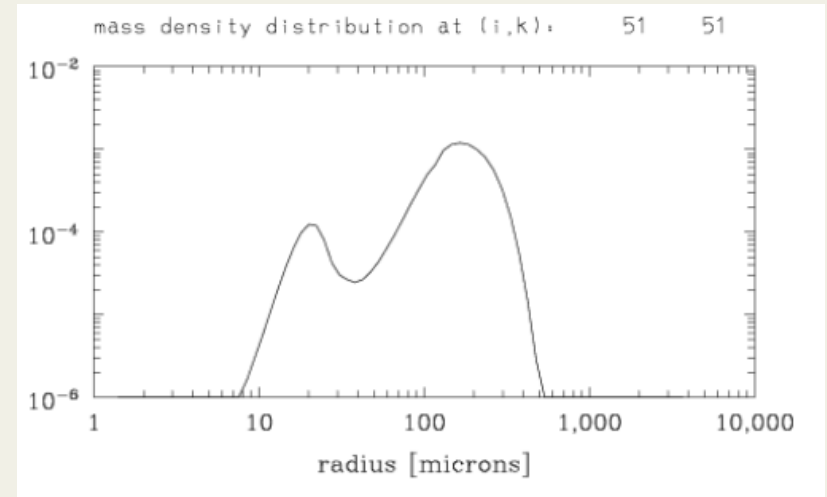
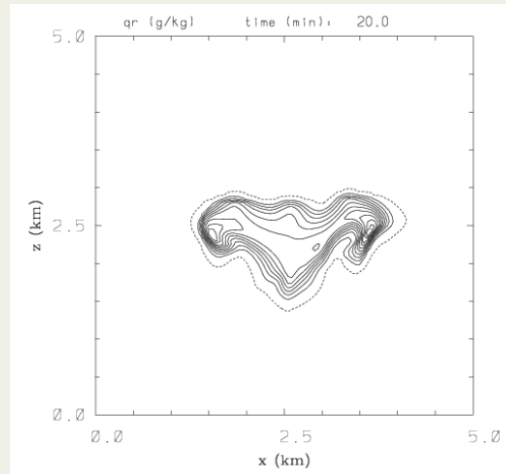
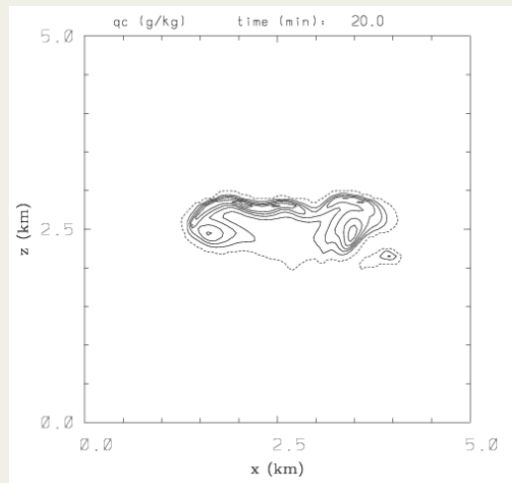


*with turbulence – Ayala kernel with  $100 \text{ cm}^2 \text{ s}^{-3}$*



## 2D simulation of a small precipitating cloud: $t=20$ min

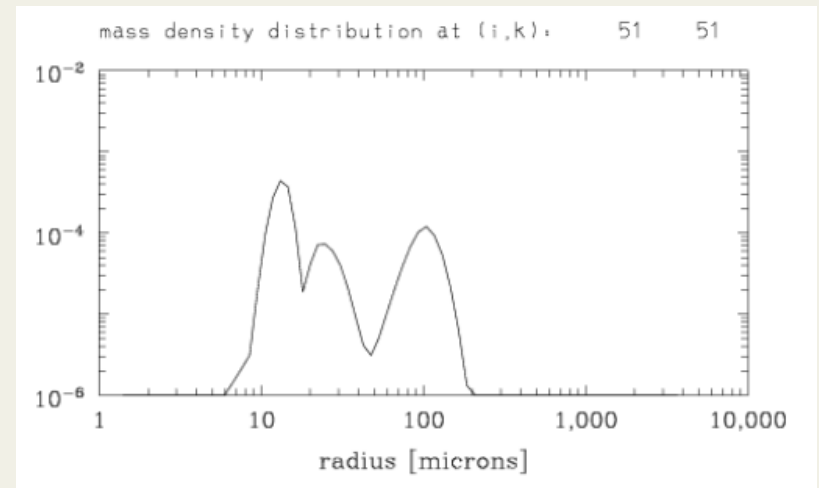
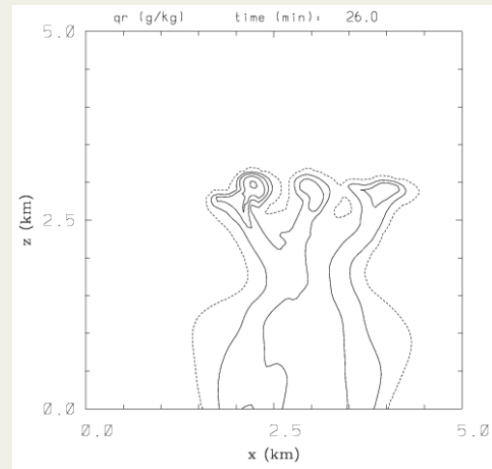
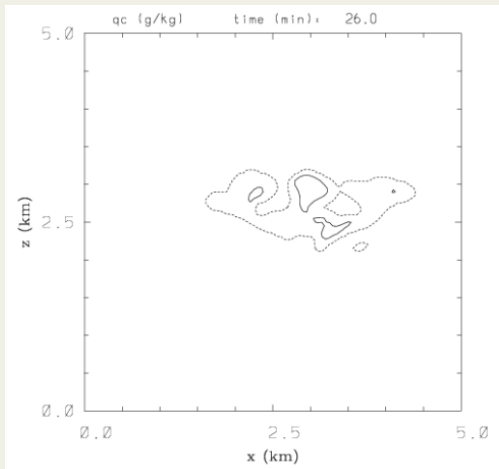
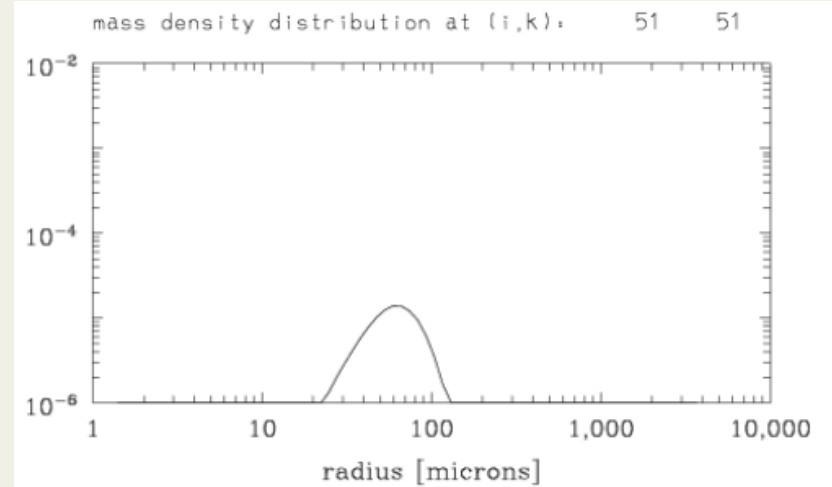
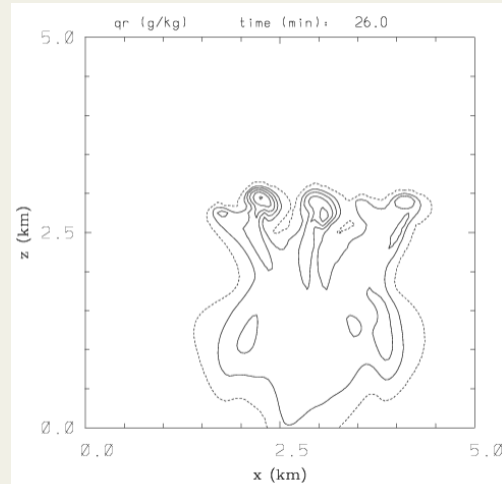
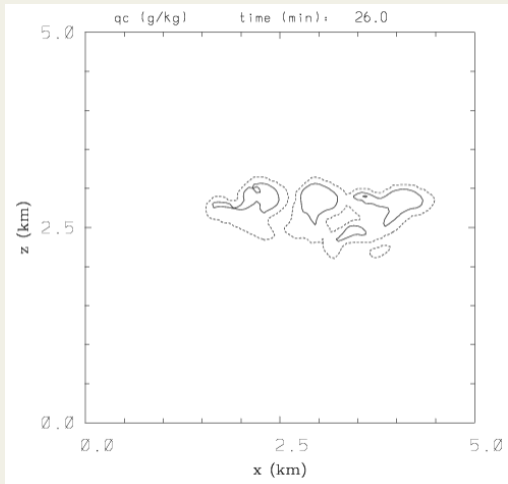
*no turbulence*



*with turbulence – Ayala kernel with  $100 \text{ cm}^2 \text{ s}^{-3}$*

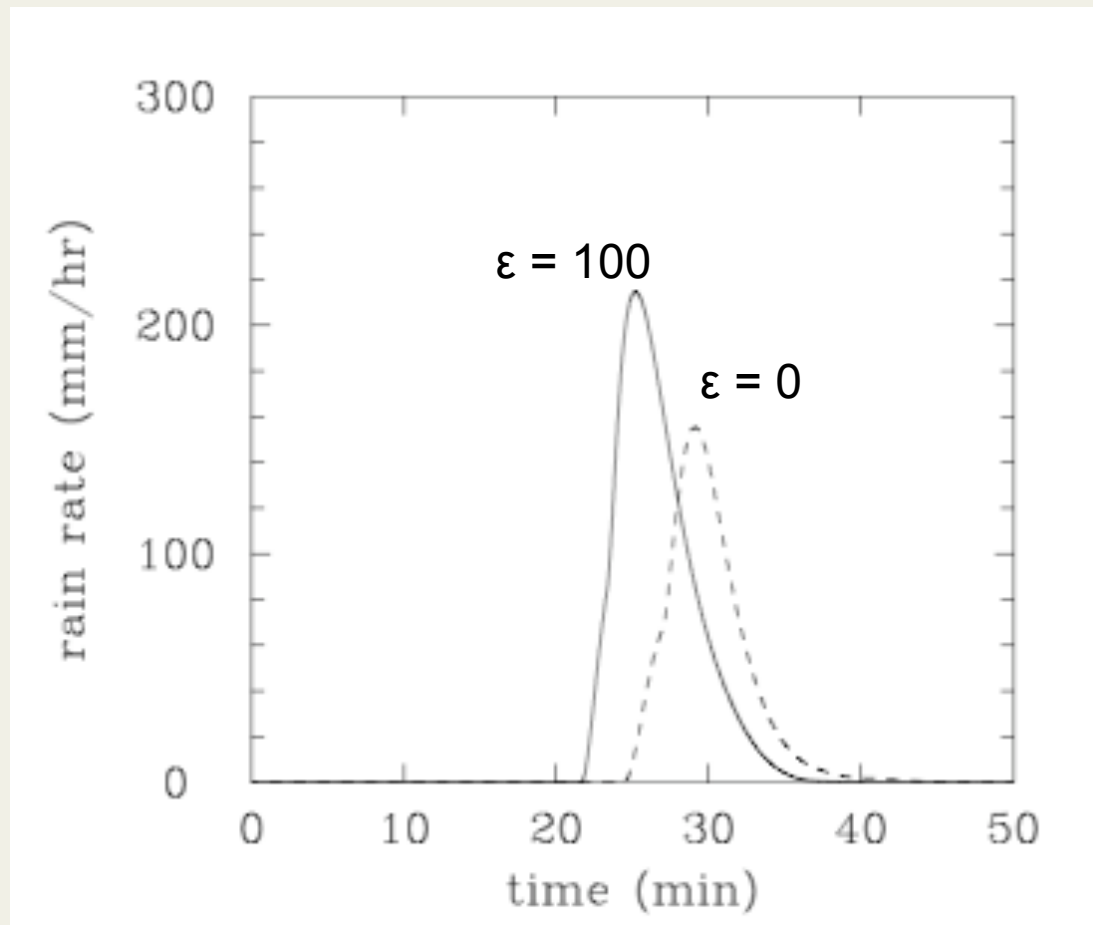
## 2D simulation of a small precipitating cloud: $t=26$ min

*no turbulence*

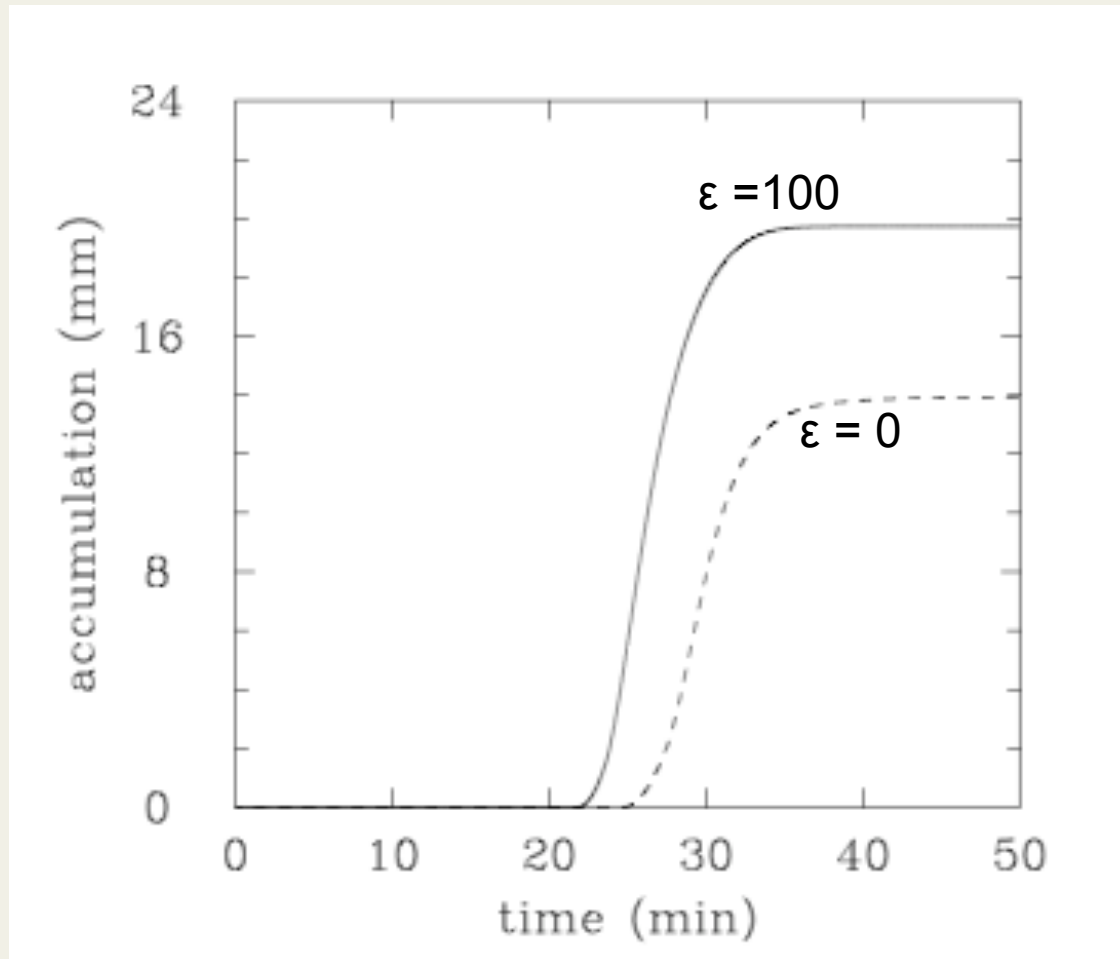


*with turbulence – Ayala kernel with  $100 \text{ cm}^2 \text{ s}^{-3}$*

Time evolution of the surface precipitation intensity: turbulent collisions lead to earlier rain at the ground and higher peak intensity...



...but also to more rain at the surface. This implies higher precipitation efficiency!



## Summary:

Small-scale turbulence seems to have an insignificant effect on diffusional growth of cloud droplets.

Turbulence seems to play a significant role when entrainment and mixing is considered through “large-eddy hopping”, local heterogeneity of mixing, and in-cloud activation.

Small-scale turbulence appears to have a significant effect on collisional growth. Not only rain tends to form earlier in a single cloud, but also turbulent clouds seem to rain more. More realistic numerical studies are needed to quantify this aspect.