Buoyancy Reversal at Cloud Boundaries

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Outline

Buoyancy Reversal and Latent Heat

Formulation of the Cloud-Top Mixing Layer

Turbulent Regime

Summary

Latent Heat and Clouds

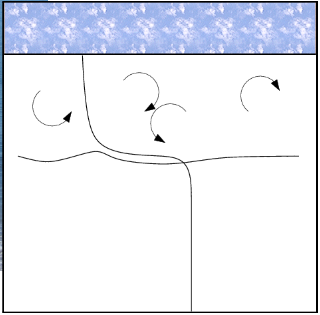
Condensation → warming → convection:

from T. Mauritsen (MPI-M), Slovakai July 2010

Evaporative Cooling

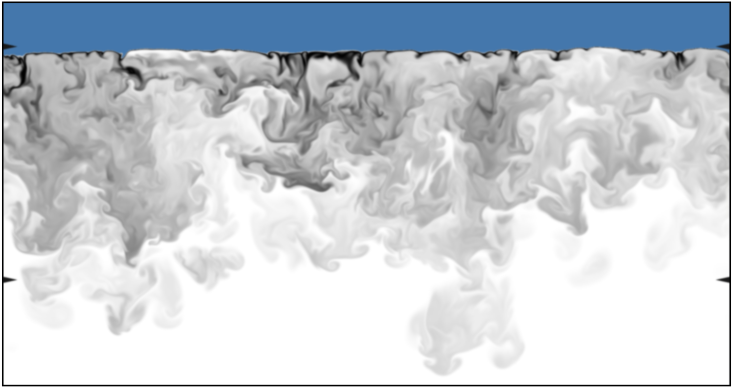
Evaporation at air/water interface $\rightarrow$ cooling $\rightarrow$ convection:

UCSB April 2011



Evaporative Cooling at Cloud Interface

Same if droplet suspension instead of pool of water:

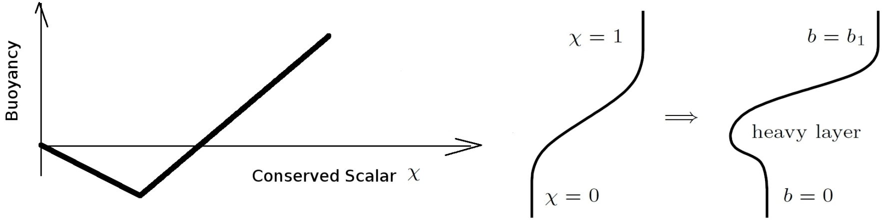


Latent heat effects on a small scale can also be important

Buoyancy Reversal

The buoyancy may not follow a linear relation wrt conserved scalars:

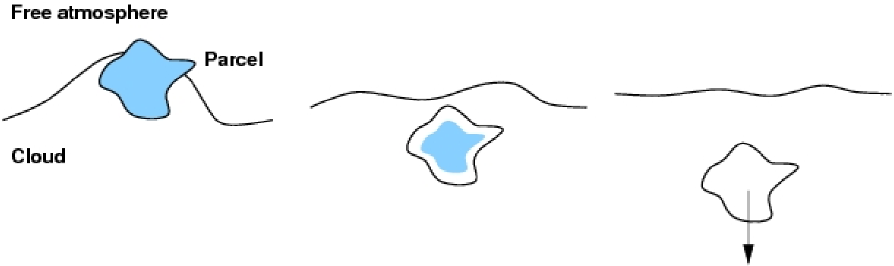
sign change $\Rightarrow$ buoyancy reversal



E.g. water has max. density at about 4°C, not at 0°C (Townsend, 1964)

Example: Stratocumulus Top

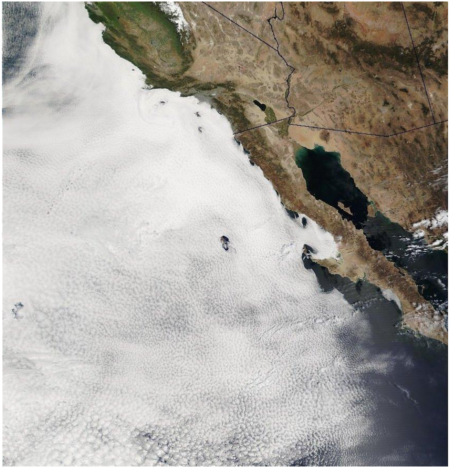
In addition, inversion (horizontal stably stratified density interface) at the cloud interface:



adapted from D. Randall, *J. Atmos. Sci.*, 1980

Cloud-Top Entrainment Instability? (Randall, 1980; Deardorff, 1980)

Stratocumulus-Topped Boundary Layer



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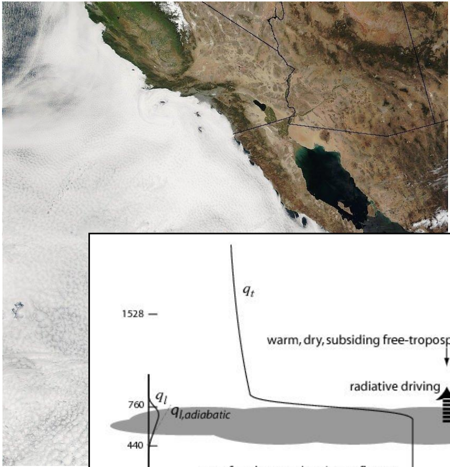
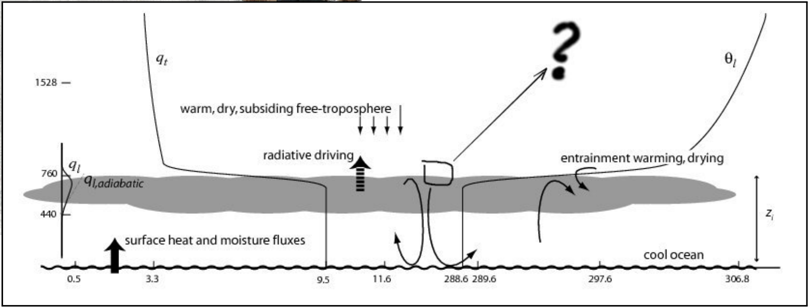
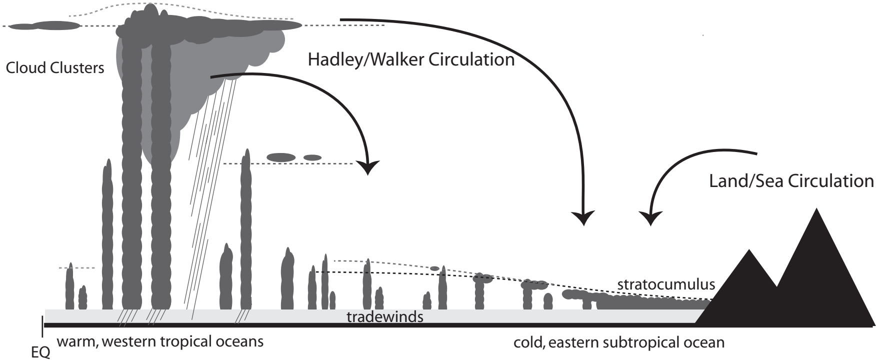




from B. Stevens, *Annu. Rev. Earth Planet. Sci.*, 2005

Key Piece of the Climate Puzzle



from B. Stevens, *Annu. Rev. Earth Planet. Sci.*, 2005

Previous Work

- Lilly (1968)
- Caughey et al. (1982), Gerber et al. (2005), Haman et al. (2007)
- Deardorff (1980), Moeng et al. (2005), Yamaguchi and Randall (2008), Kurowski et al. (2009)

Stevens (2002):

"Such differences [in entrainment parametrizations] imply steady-state boundary layers that can differ by as much as a factor of two in climatologically important properties such as vertically integrated liquid water and boundary layer depth"

Previous Work

...on small-scale buoyancy reversal

Siems et al. (1990), Shy and Breidenthal (1990), Siems et al. (1992)

- Tank experiments with liquid mixtures. Mechanically driven ICs.
- Definition of the problem in terms of $D = -b_s/b_1$ and χ_s.
- Sims. Almost laminar behavior for $D \simeq 0.04$ (real conditions).
- Small reversal ($D \ll 1$) cannot explain cloud break-up.

Wunsch (2003)

- Stochastic models.
- Confirms previous results.
- Points to possible relevance of diffusion at cloud interface.

What is really going on at the interface?

Outline

Buoyancy Reversal and Latent Heat

Formulation of the Cloud-Top Mixing Layer

Turbulent Regime

Summary

The mixture(mixing) fraction χ

Cloud: disperse and dilute two-phase flow.

Albrecht et al. (1985)

- Introduced in the analysis of field data at the cloud top based on the mixing of a cloud parcel with an environmental parcel.
- Ratio of the mass of the latter to the total (*).

Bretherton (1987)

- Linear mixing properties $a = a_0(1 - \chi) + a_1\chi$.
- Discussed the equilibrium thermodynamics.

No direct connection to governing equations.

Two-Fluid Formulation

Drew (1983), Zhang & Prosperetti (1994)

Local average to define a continuous field from the disperse and dilute liquid phase:

$$
\begin{aligned}
\tilde{\rho} &= \phi_g\tilde{\rho}_g + \phi_l\tilde{\rho}_l = \phi_d\tilde{\rho}_d + \phi_v\tilde{\rho}_v + \phi_l\tilde{\rho}_l \\
1 &= q_g + q_l \qquad\quad = q_d + q_v + q_l \\
\tilde{\mathbf{v}} &= q_g\tilde{\mathbf{v}}_g + q_l\tilde{\mathbf{v}}_l = q_d\tilde{\mathbf{v}}_d + q_v\tilde{\mathbf{v}}_v + q_l\tilde{\mathbf{v}}_l \\
\tilde{e} &= q_g\tilde{e}_g + q_l\tilde{e}_l \;\; = q_d\tilde{e}_d + q_v\tilde{e}_v + q_l\tilde{e}_l
\end{aligned}
\qquad \mathbf{V}^D = \tilde{\mathbf{v}}_l - \tilde{\mathbf{v}}_g
$$

Conditions:

$$\eta^3 \gg d^3/\phi_l \gg d^3$$

$\phi_l \simeq 10^{-6}$ ✓ $\quad d \simeq 10\ \mu\text{m}$ ✓ $\quad n_l = \phi_l/(\pi d^3/6) \simeq 100\ \text{cm}^{-3}$ ✗

Mean Drift Velocity

Droplet dynamics at low Reynolds ($Re_d \simeq 10^{-2}$) (Shaw, 2003; Maxey and Riley, 1983)

$$\frac{d\mathbf{v}}{dt} = -\frac{\mathbf{w}}{t_d} + \left(1 - \frac{\rho_g}{\rho_l}\right)\mathbf{g} + \frac{\rho_g}{\rho_l}\dot{\mathbf{u}} - \frac{1}{2}\frac{\rho_g}{\rho_l}\dot{\mathbf{w}} - \left(\frac{9}{2\pi t_d}\frac{\rho_g}{\rho_d}\right)^{1/2}\int_0^t \frac{\dot{\mathbf{w}}(t')}{\sqrt{t-t'}}dt'$$

where $\mathbf{w} = \mathbf{v} - \mathbf{u}$, $\rho_g/\rho_l \simeq 10^{-3}$ and $t_d = \rho_l d^2/(18\mu_g) \simeq 0.3 \times 10^{-3}$ s.

Local average: $\mathbf{w} \to \mathbf{V}^D$ (Druzhinin and Elghobashi, 1998)

Local Thermodynamic Equilibrium

Mashayek (1998), Shaw (2003)

Conditions:

Mechanical	Small	$St = t_d/t_\eta$
Thermal	Small	$St\,Pr\,(c_{p,l}/c_{p,g})$ (low Mach)
Phase	Large	$Da = t_\eta/t_s$

$St \simeq 0.01$ ✓ $Da \simeq 0.01$ ✗

$St \ll 1 \Rightarrow \mathbf{V}^D = \mathbf{g}t_d \simeq 3$ mm/s.

Settling parameter $V^D/v_\eta \simeq 0.3$; leading order terms $(q_l/q_v)(V^D/v_\eta)$ in transport equation for q_t and thus e are then small.

Liquid-Phase Diffusion

The standard formulation includes a diffusion term of the form

$$\rho q_l(\tilde{\mathbf{v}}_l - \tilde{\mathbf{v}}) = -\rho\kappa_l \nabla q_l$$

Brownian motion $\kappa_l = (k_B T)/(3\pi\mu_g d) \simeq 10^{-12}$ m^2/s ✗

Eqns. for total mixture $\{\rho, q_t = q_l + q_v, q_l, \mathbf{v}, e\}$. Fluxes:

$$\mathbf{j}_t = -\rho\kappa_v \nabla q_t - (\rho\kappa_l - \rho\kappa_v)(q_d/q_g)\nabla q_l$$
$$\mathbf{j}_q = -\rho\kappa_v \nabla h - (\lambda/c_p - \rho\kappa_v)c_p \nabla T - (\rho\kappa_l - \rho\kappa_v)(h_l - h_g)\nabla q_l$$

Limits: $\kappa_l = \kappa_v$ for $d/\eta \to 0$, and $\kappa_l \to 0$ for real conditions.

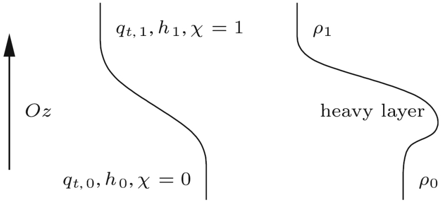
Mixture Fraction χ

1. Neglecting differential diffusion effects $Sc_l/Pr = Sc/Pr = 1$
2. Low Mach; domain $\ll$ scale height.

$$\partial/\partial t\,(\rho q_t) + \nabla\cdot(\rho q_t \mathbf{v}) = \nabla\cdot(\rho\kappa\nabla q_t)$$
$$\partial/\partial t\,(\rho h) + \nabla\cdot(\rho h \mathbf{v}) = \nabla\cdot(\rho\kappa\nabla h)$$

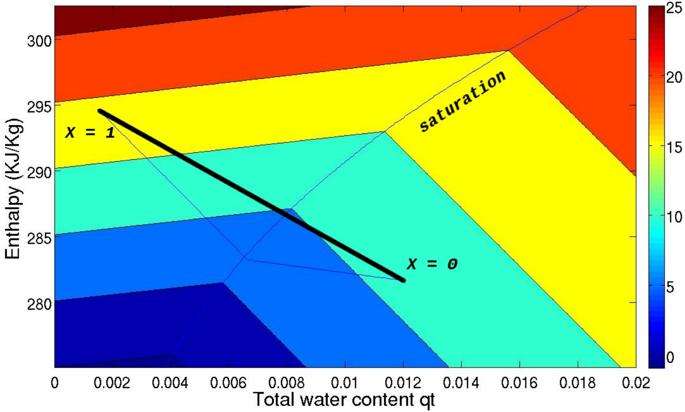
$$\frac{q_t - q_{t,0}}{q_{t,1} - q_{t,0}} - \frac{h - h_0}{h_1 - h_0} \longrightarrow 0$$

$$\mathbf{\frac{q_t - q_{t,0}}{q_{t,1} - q_{t,0}} = \frac{h - h_0}{h_1 - h_0} = \chi}$$



Mixing Line χ

Mixing at fixed pressure level (isobaric mixing). Equilibrium.

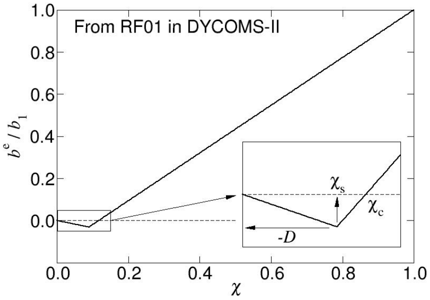


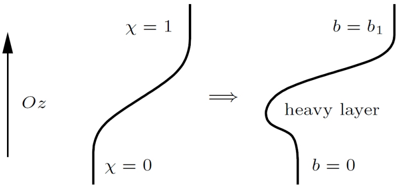
from Alberto de Lózar

Governing Equations

Boussinesq + Mixture Fraction χ + Non-Linear Eqn. State (Bretherton, 1987)

$$\begin{aligned}
\partial_t u_k &= -\partial_i(u_k u_i) - \partial_k p + \nu\partial_i\partial_i u_k + b\delta_{k3}\,,\ \partial_i u_i = 0 \\
\partial_t \chi &= -\partial_i(\chi u_i) + \kappa\partial_i\partial_i\chi \\
b &= b^e(\chi;\, b_1,\, b_s,\, \chi_s)
\end{aligned}$$

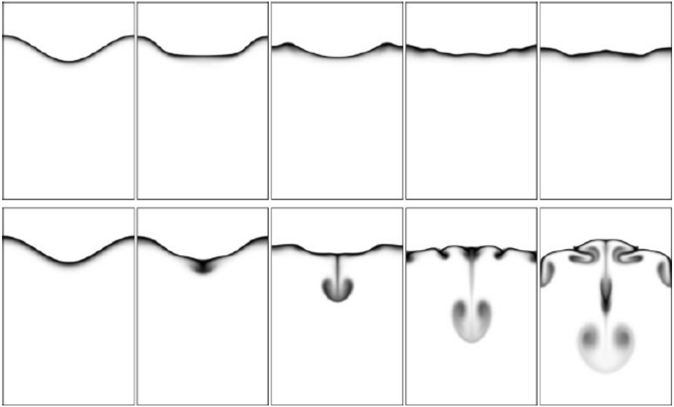




Parameter space $\{\nu,\, \kappa,\, b_1,\, b_s,\, \chi_s\} \rightarrow \{Pr = 1,\, D = -b_s/b_1,\, \chi_s\}$

Buoyancy Reversal Instability

Two time scales associated w/ buoyancies b_1, $b_s \rightarrow D = -b_s/b_1$.



from JPM et al., *Q. J. R. Meteorol. Soc.*, 2009

Outline

Cases

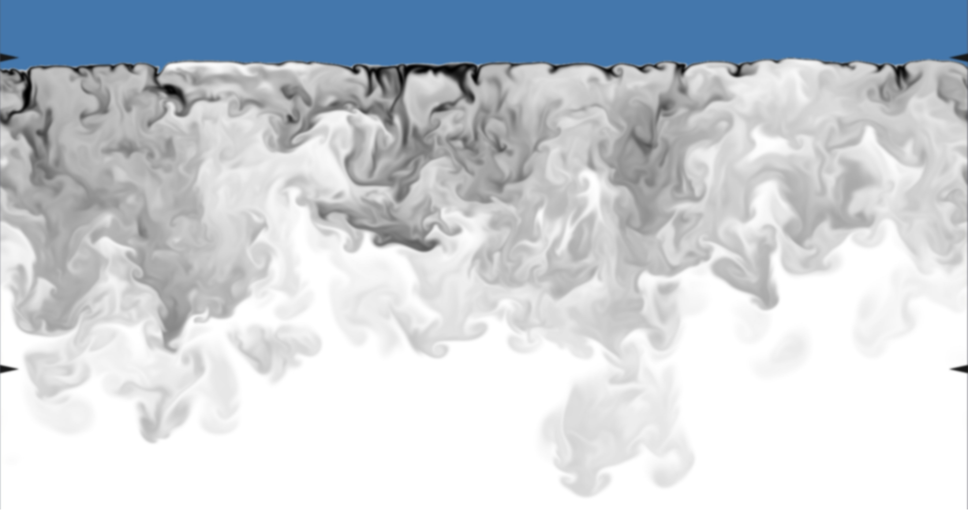
	ΔT_l (°C)	Δq_t (g kg^{-1})	$q_{l,c}$ (g kg^{-1})	D	χ_s	χ_c	f_1	f_2
A11	9.7	−7.5	0.5	0.031	0.09	0.117	1.33	0.48
A21	8.5	−8.4	0.5	0.062	0.09	0.143	1.37	0.53
A12	13	−8.2	1.2	0.031	0.18	0.205	1.40	0.54

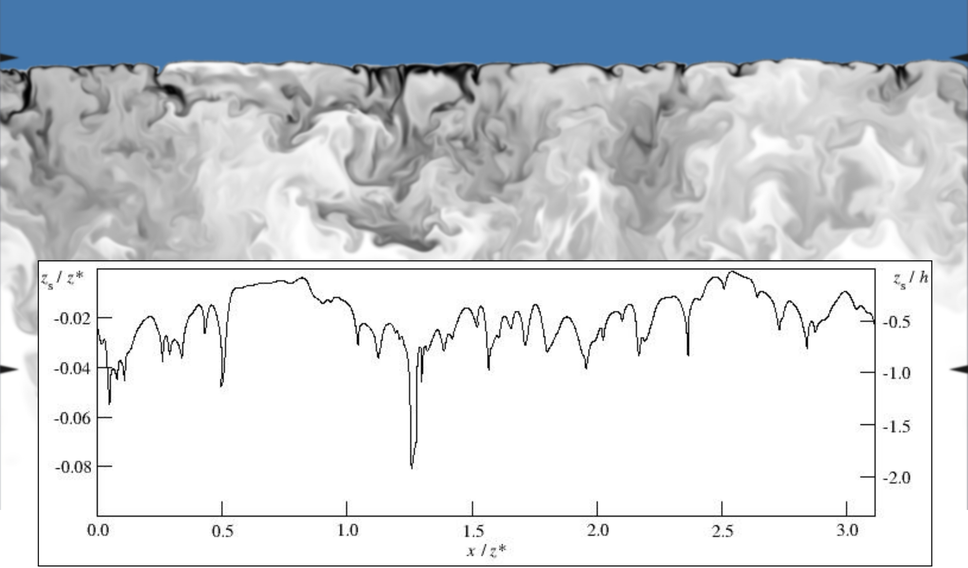
TABLE 1. Simulation series. Reference case A11 taken from field measurements of nocturnal marine stratocumulus (Stevens *et al.* 2003*a*): ΔT_l, jump across the inversion in liquid-water static energy temperature; Δq_t, jump in total-water content; $q_{l,c}$, cloud liquid-water content. Cases A21 and A12 are derived to investigate the effects of the buoyancy-reversal parameters D and χ_s independently. The cross-over mixture fraction χ_c is defined in (2.4). The last two columns contain the prefactors of the upward mean entrainment velocity (cf. (5.6)) and the rate of broadening of the convection layer (cf. (6.12)).

from JPM, *J. Fluid Mech.*, 2010; JPM et al. *New J. Phys.*, 2010

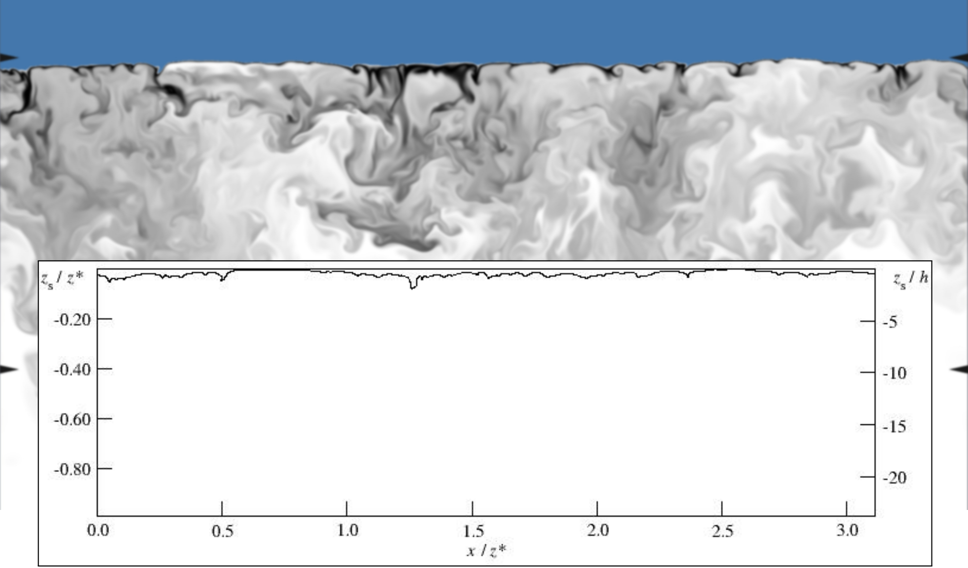
Grid size $2048 \times 2048 \times 1536$.

DNS run on IBM Power 6 at DKRZ.

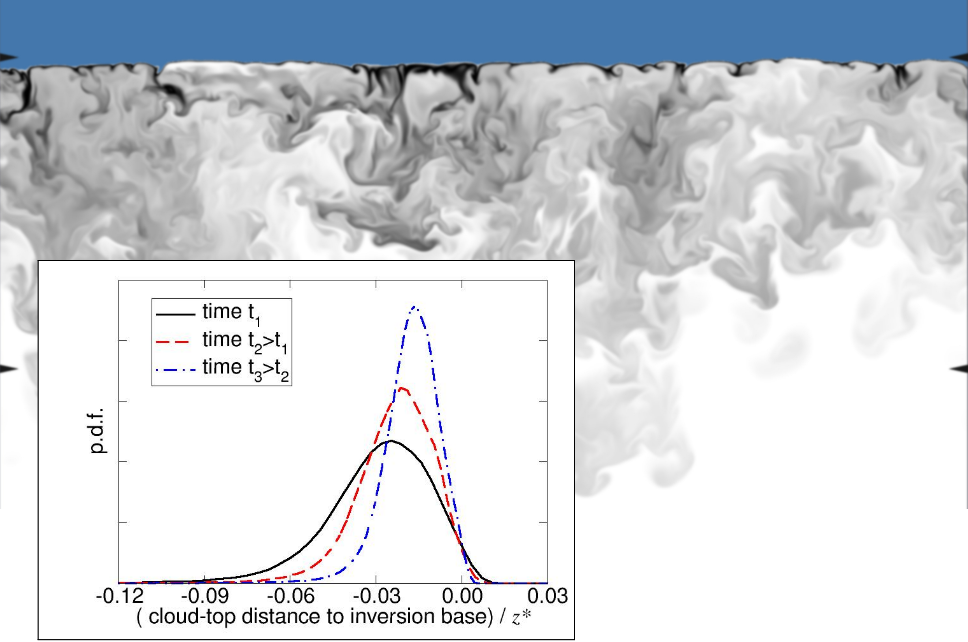


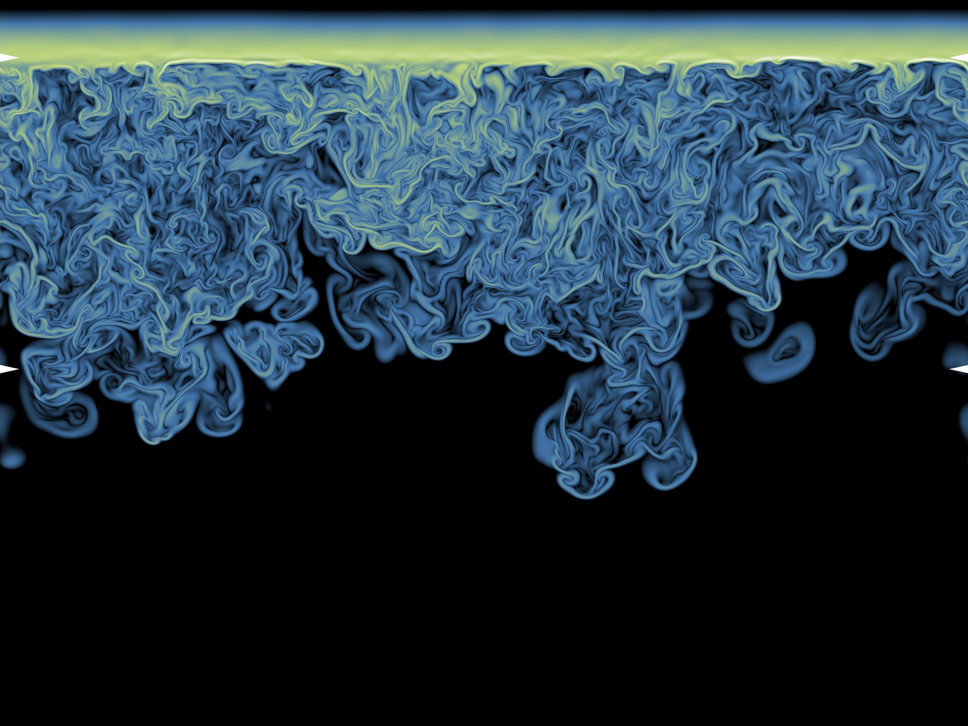


Instantaneous cloud-top $\{\mathbf{x} : \chi(\mathbf{x}, t) = \chi_s\}$

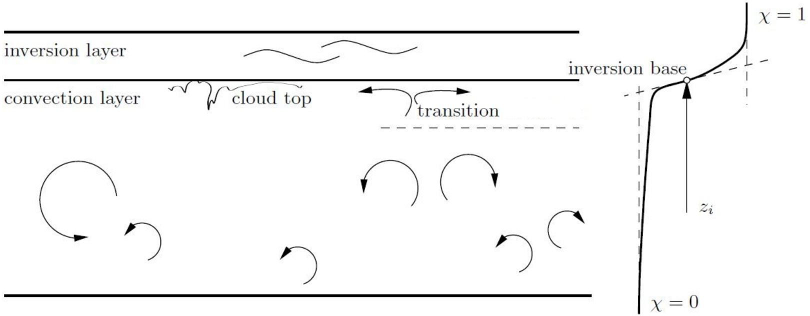


Instantaneous cloud-top $\{\mathbf{x} : \chi(\mathbf{x}, t) = \chi_s\}$


time t_1
time $t_2 > t_1$
time $t_3 > t_2$
p.d.f.
-0.12
-0.09
-0.06
-0.03
0.00
0.03
(cloud-top distance to inversion base) / z^*



Layered Vertical Structure

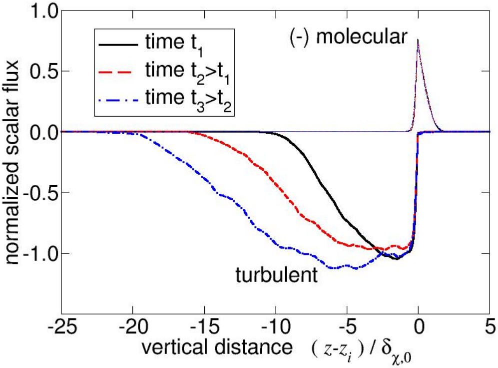


Height z_i of maximum stratification separates two diff. regions:

1. **Inversion layer** on top, dominated by molecular transport
2. **Convection layer** below, dominated by turbulent transport

Inversion Layer

$$\frac{d}{dt}\left[\int_{z_i}^{\infty}(1-\langle\chi\rangle)\,dz\right]+\langle w'\chi'\rangle(z_i,t)=\kappa\frac{\partial\langle\chi\rangle}{\partial z}(z_i,t)-(1-\chi_i)\frac{dz_i}{dt}$$



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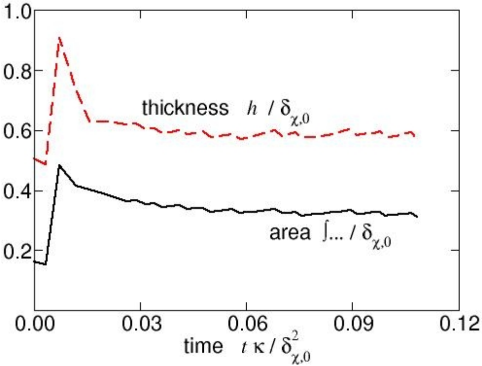


Dominant balance

$$w_e=\frac{dz_i}{dt}\simeq\frac{\kappa}{h}$$

Constant thickness

$$h=\frac{1-\chi_i}{\frac{\partial\langle\chi\rangle}{\partial z}(z_i,t)}$$

$\Rightarrow$ **Constant entrainment rate**

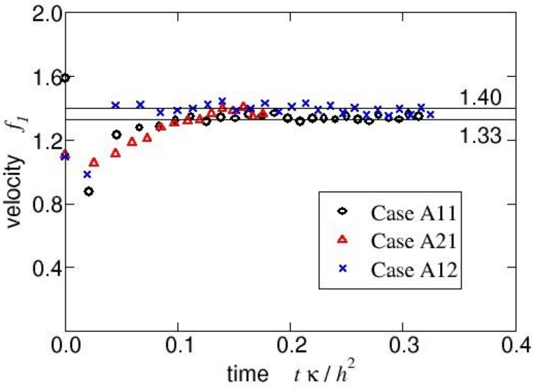
Mean Entrainment Rate

Marginally stable thermal boundary layer $\chi_c h$:

$$\frac{(\chi_c h)/(\kappa/h)}{\nu/(\chi_c h|b_s|)} \simeq 10^3 \qquad \Rightarrow \qquad h \simeq (10/\chi_c^{2/3})(\kappa^2/|b_s|)^{1/3}$$

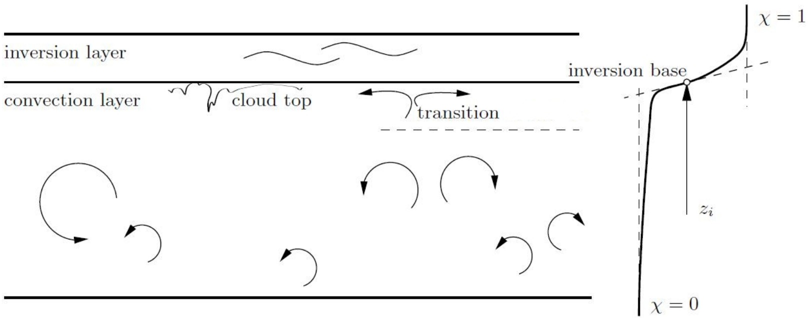
$$f_1 = \frac{w_e}{(\chi_c^{2/3}/10)(\kappa|b_s|)^{1/3}}$$

$$f_1 = f_1(D, \chi_s)$$



Turbulent Regime

Convection Layer



Convection Layer

$$\frac{d}{dt}\int_{-\infty}^{z_i}\langle\chi\rangle dz = \kappa\frac{\partial\langle\chi\rangle}{\partial z}(z_i,t) - \langle w'\chi'\rangle(z_i,t) + \chi_i w_e \simeq w_e$$

Then, w_e determines also scaling inside the turbulent zone.

From functional dependence between b and χ, **reference buoyancy flux**

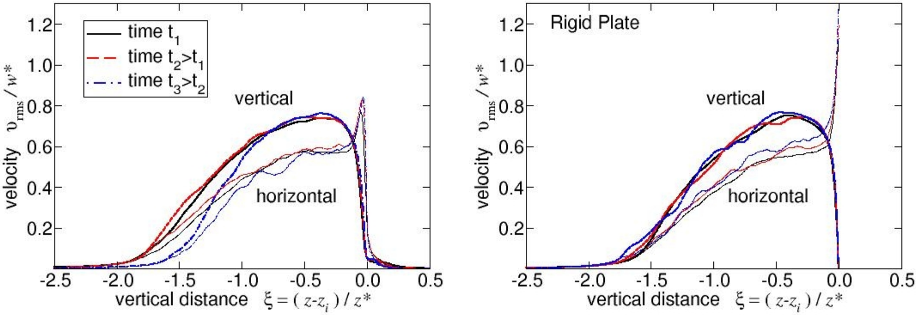
$$B_s = w_e|b_s|/\chi_s = (0.1 f_1 \chi_c^{2/3}/\chi_s)(\kappa b_s^4)^{1/3}$$

and then **convection scales**

$$z^* = (1/B_s)\int B dz \qquad , \qquad w^* = (z^* B_s)^{1/3}$$

characterize (some statistics of) the turbulent region (Deardorff, 1980)

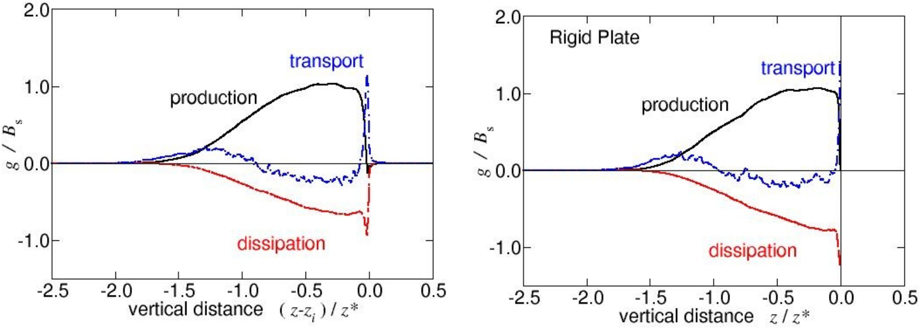
Turbulent Velocity and Vorticity Fluctuations



- Self-preservation: $f(z,t) \to \phi(\xi = (z - z_i(t))/z^*(t))$.
- Anisotropy.
- Inhomogeneity: Entrainment/Bulk/Transition/Inversion.

Budget of Turbulent Kinetic Energy

$$\frac{\partial q^2/2}{\partial t} = -\frac{\partial T}{\partial z} + B - \varepsilon \; ,$$

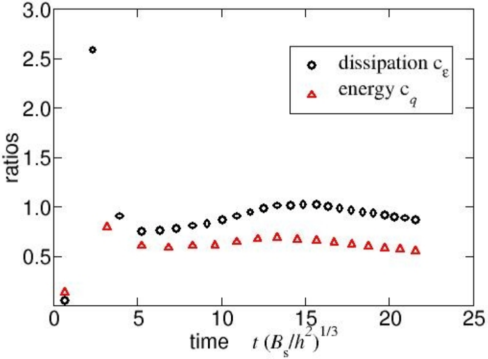


Temporal Evolution of Convection Scales

Integrating transport equation for $q^2/2$,

$$\frac{d}{dt}(c_q z^* w^{*2}) = 2(1 - c_\varepsilon) w^{*3}$$

$$c_\varepsilon = \frac{1}{w^{*3}} \int \varepsilon \, dz \;, \qquad c_q = \frac{1}{w^{*2} z*} \int q^2 \, dz$$



Constant $c_q(D, \chi_s)$

Constant $c_\varepsilon(D, \chi_s)$

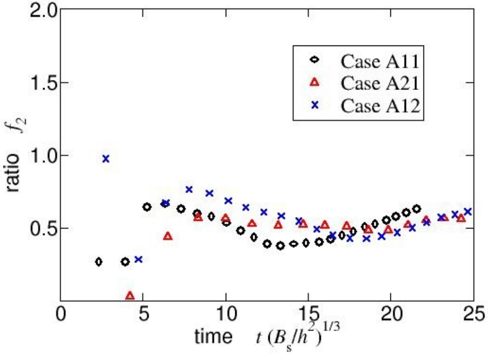
With def. $w^{*3} = B_s z^*$, closed system for $w^*(t)$ and $z^*(t)$

Temporal Evolution of Convection Scales

Solution

$$z^*(t) = z^*(t_1)\left[1 + (2f_2/3)\frac{t - t_1}{[z^*(t_1)^2/B_s]^{1/3}}\right]^{3/2}$$

$$dz^*/dt = f_2\, w^*$$



Scalings:

$z^* \propto t^{3/2}$
$w^* \propto t^{1/2}$
$b^* \propto t^{-1/2}$

Time scale $(z^{*2}/B_s)^{1/3}$

Further Discussion

	z^*/h	$\eta/\Delta x$	z^*/λ_z	λ_z/η	u'/w^*	w'/w^*	Re_t	Re_λ	Re^*	Ri^*	Ra^*
A11	24	1.2	19	28	0.84	0.74	1800	220	4800	590	0.4×10^9
A21	39	0.9	26	31	0.86	0.78	2400	250	8000	293	1.1×10^9
A12	39	1.2	19	28	0.90	0.76	1600	200	4800	716	0.5×10^9

TABLE 2. Length-scale ratios, turbulence intensities and derived quantities at the final time t_2. Reynolds numbers $Re_t = (q^2/2)^2/(\varepsilon\nu)$, $Re_\lambda = w'\lambda_z/\nu$ and $Re^* = z^* w^*/\nu$; convection Richardson number $Ri^* = b_1 z^*/w^{*2}$; Rayleigh number $Ra^* = z^{*3}|b_s|/(\kappa\nu)$; Nusselt number $Nu^* = w_e z^*/\kappa = z^*/h$. Maximum values are used for the mean turbulent dissipation rate ε and the turbulence intensities.

- Unsteady free convection; $Nu^*(t)/(Ra^*(t))^{1/3} = 0.1 f_1 \chi_c^{2/3}$ const.
- Turbulent mixing across a density interface; $Ri^*(t)$ increasing.
- Stratocumulus.

Stratocumulus

Turbulence does not break the cloud top, but enhance mixing up to a linear entrainment rate.

Some numbers:

$w_e \simeq$ 0.16 mm/s; $h \simeq$ 0.1 m; $B_s \simeq 10^{-5}$ m^2/s^3; $z^* \simeq$ 2.5 m; $w^* \simeq$ 30 mm/s.

From previous growth rates, 100 m reached in about 45 min. Then, $w^* \simeq$ 0.1 m/s; still $\ll$ measurements of 1 m/s (radiative forcing).

Structure:

Vertical interface displacement $\delta = w^{*2}/b_1$ small and $Ri^* = z^*/\delta \propto t^{1/2}$. Internal Richardson number $Ri_{(I)} = h/\delta \propto t^{-1}$ decreases, but order 1 only after $z^* = 300$ m.

Outline

Summary

- Latent heat can also play a role in small scales.
- Buoyancy reversal, in the absence of mean shear, depends on molecular transfer properties.
- Evaporative cooling effects are one order of magnitude smaller than radiative cooling effects.

- Models for two-phase flows in clouds need to be improved.
- External flows: Unsteady convection and entrainment.
- Boundary layers adjacent to density interfaces; interaction with gravity waves.