The Wind-Turbine Array Boundary Layer

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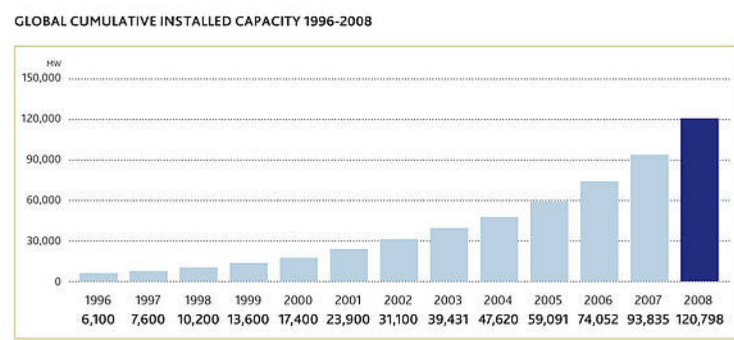
- Prof. Raúl B. Cal (now at Portland State Univ.) - exp
- Prof. Luciano Castillo (RPI) - exp
- José Lebrón-Torres (RPI) - exp
- Dr. Hyung-Suk Kang (JHU, now USNA) - exp

Funding: NSF CBET-0730922 (Energy for Sustainability)

Simulations: NCAR allocation (NSF)

Wind energy growth:
30% yearly growth, sustained



Renewables: low energy density

- solar, wind, wave energy
- need to cover "very, very big" areas
- wind: large wind-farms - on-land & off shore

Land-based HAWT

Shell's Rock River windfarm in Carbon County, Wyoming, USA
Source: http://www.the-eic.com/News/Archive/2005/May/Article503.htm

Horns Rev HAWT
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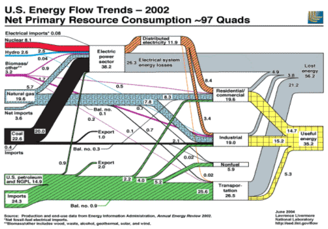
Some thoughts on how much we consume:

(i.e. "a few solar collectors or little wind-mills simply won't do")



- Consider 3 TW US power consumption

- $3 \times 10^{12} / 300 \times 10^{6}$ = 10 kW per person in US

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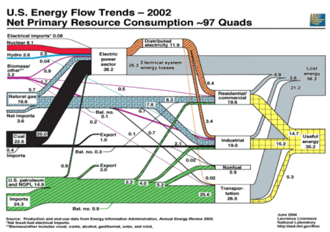
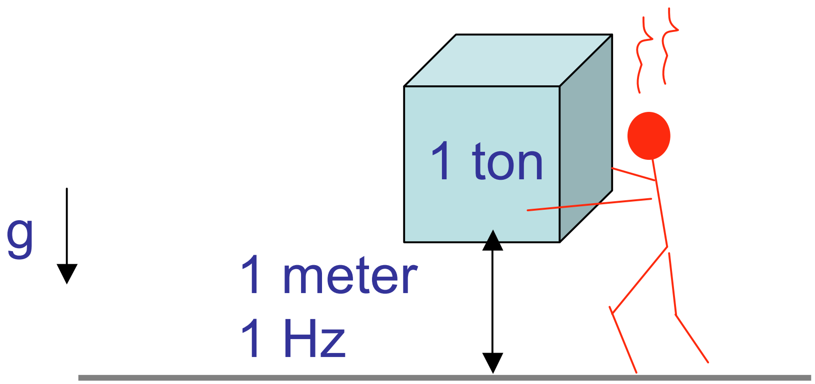
- Consider 3 TW US power consumption



- $3 \times 10^{12} / 300 \times 10^{6}$ = 10 kW per person in US

- That is the same as lifting 1 ton by 1 meter every second!!



Some thoughts on how much we consume:

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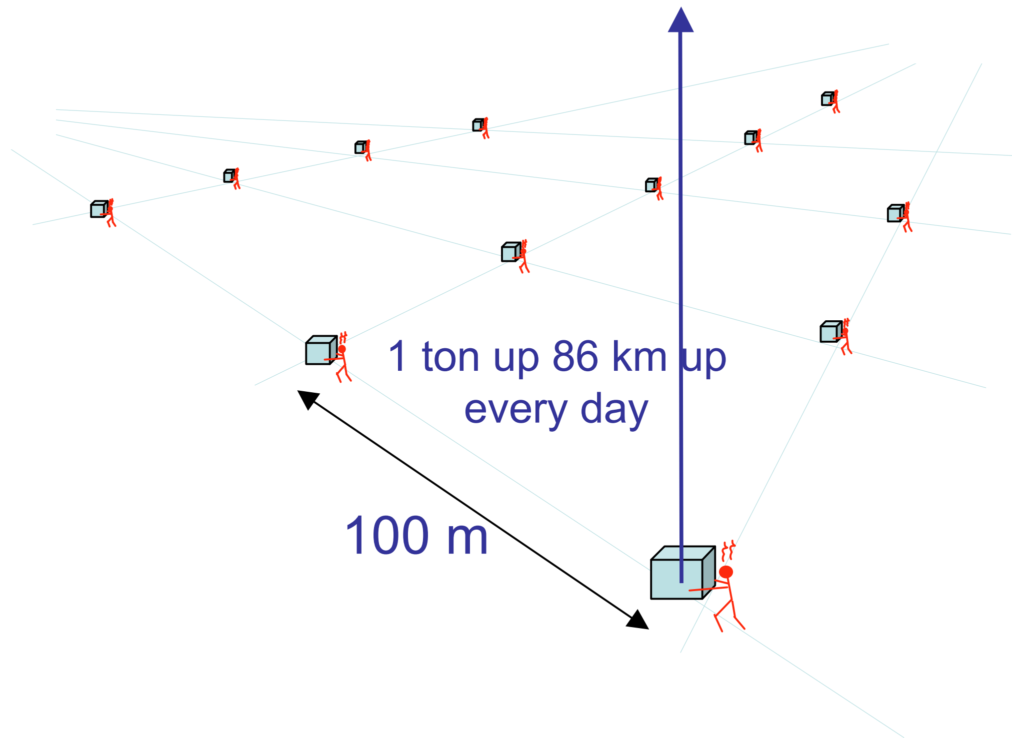
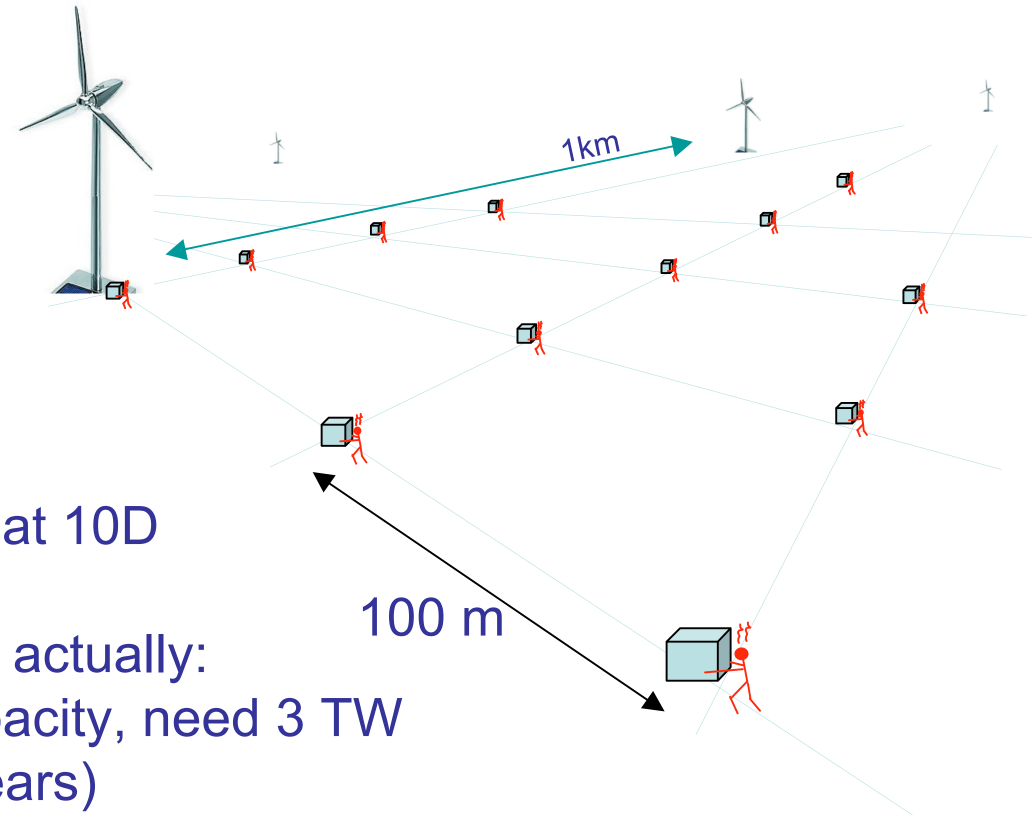
- Back to entire US (lower 48): 3.7 Million km^2
- $$\sqrt{\frac{3.7\times10^6\times10^6}{300\times10^6}}\approx 100\text{ m}$$



Some thoughts on how much we consume:

(i.e. “a few solar collectors or little wind-mills simply won’t do”)

- Back to entire US (lower 48): 3.7 Million km^2
- $$\sqrt{\frac{3.7\times10^6\times10^6}{300\times10^6}}\approx 100\text{ m}$$
- Need one 1MW WindTurbine for 100 people (100 x 10 kW)
- 1 WindTurbine every 1km …. at 10D
- 3 Million wind turbines (doable actually: now US: 6 GW, av. power capacity, need 3 TW factor 500 = 2^9 - 9x3 = 27 years)
- What can we say about land-atmosphere couplings in the presence of large wind farms?



The windturbine-array boundary layer (WTABL)

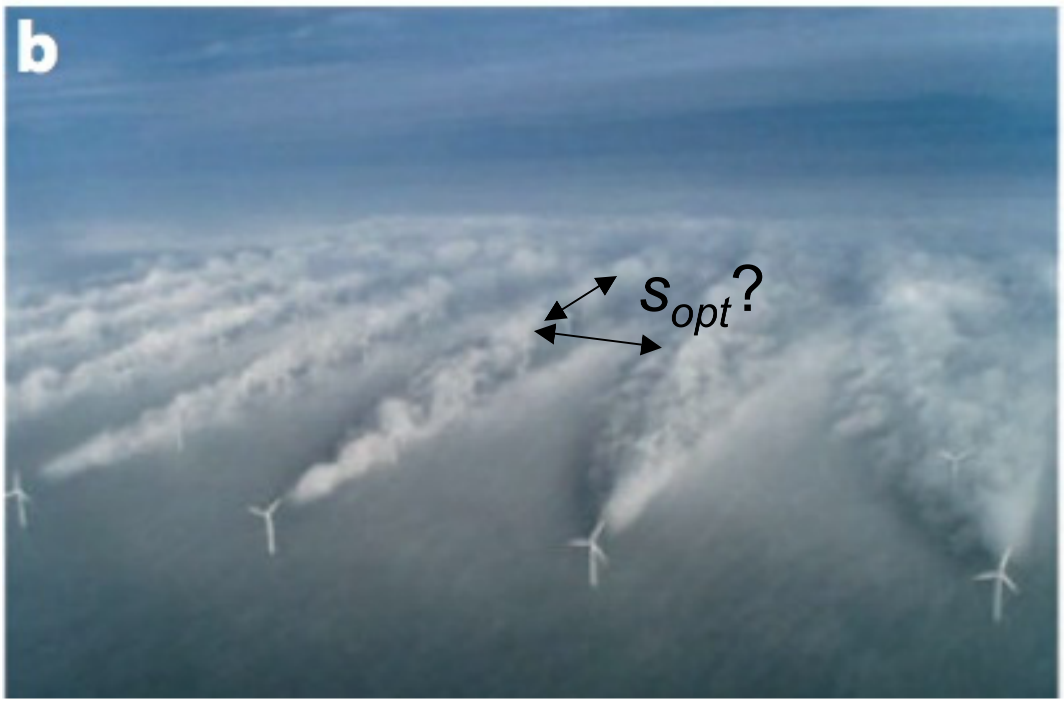


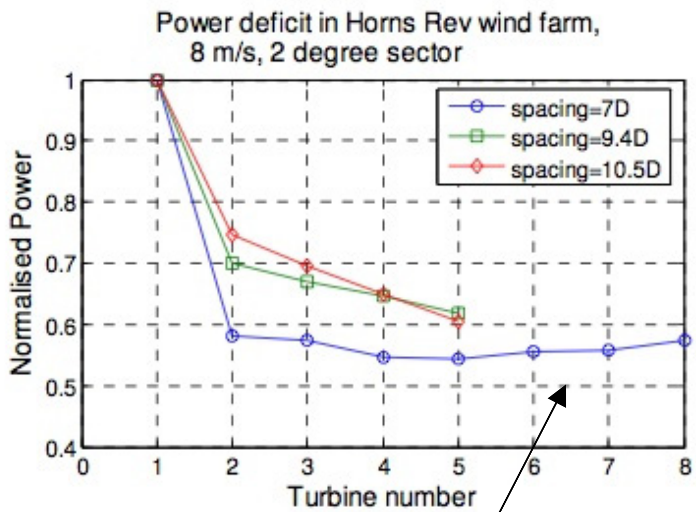
Photo by Uni-Fly A/S (Wind turbine maintenance company)

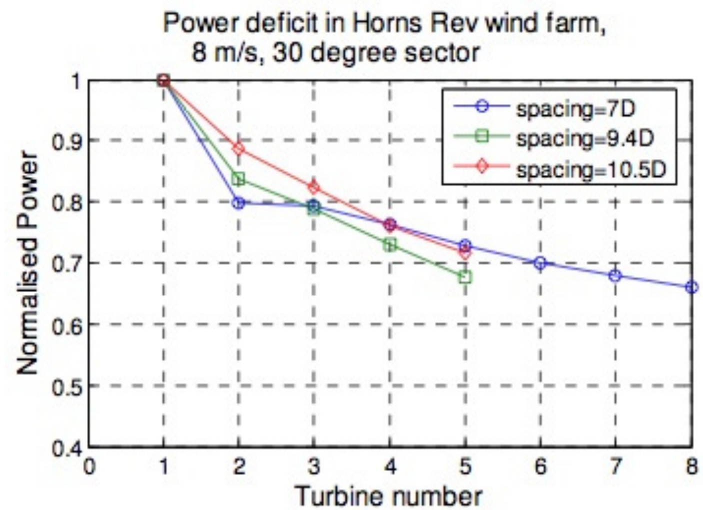
From J.N. Sørensen, Annual Rev. Fluid Mech. 2011:

Arrays are getting bigger: when L > 10 H (H: height of ABL),

approach "fully developed" **FD-WTABL**

Related problem: Wind farm power degradation



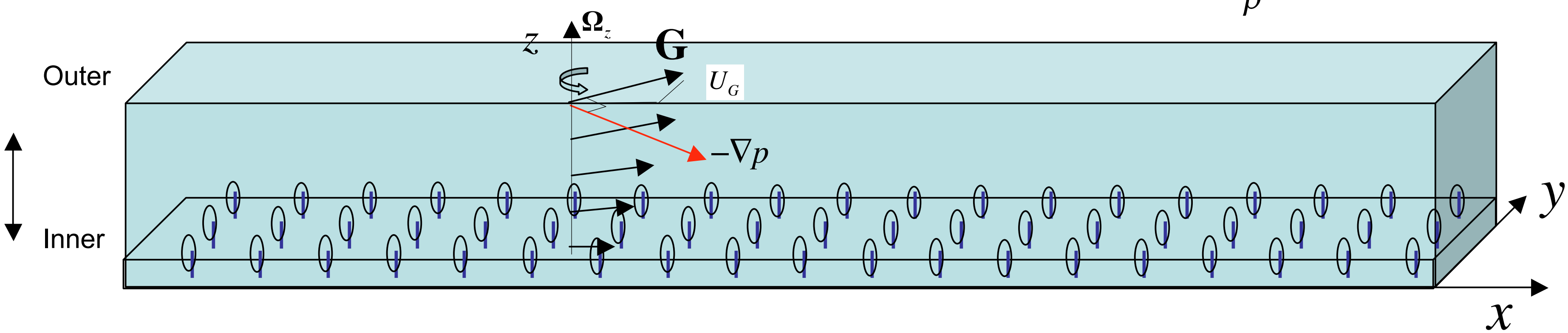


Modelling and measurements of wakes in large wind farms
Barthelemie, Rathmann, Frandsen, Hansen et al…
J. Physics Conf. Series 75 (2007), 012049

- asymptote ??
- how fast?
- is it really around 50%?
- mechanisms ?

The "fully developed" WTABL: Forcing by geostrophic wind

Above ABL (in mid-latitudes): geostrophic balance $2\boldsymbol{\Omega}\times\mathbf{G}-\frac{1}{\rho}\nabla P\approx 0$



Coupled through a stress $(u_*)^2$:

Outer length-scale: $H=\frac{u_*}{f}$ $f=2\Omega\sin\phi\approx 10^{-4}s^{-1}$ (mid-latitudes)

Inner length-scale: z_0

Inner-outer matching: $\frac{u(z)}{u_*}=\frac{1}{\kappa}\ln\left(\frac{z}{z_0}\right)$ $\frac{G}{u_*}=\sqrt{A^2+\left[\frac{1}{\kappa}\ln\left(\frac{u_*}{fz_0}\right)-C\right]^2}$

Given *G* and z_0 ----> find u_* and *H*

Example application of fully developed WTABL concepts and z_0: GCMs, mesoscale models, etc…

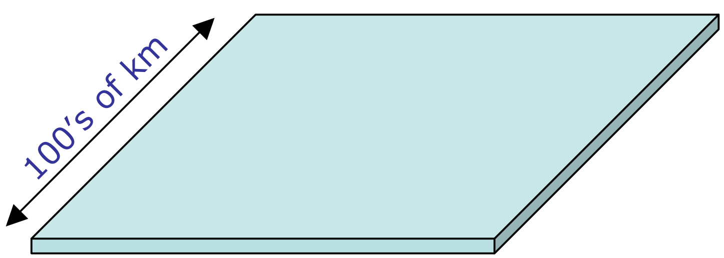
Keith et al. "The influence of large-scale wind power on climate" PNAS (2004)

Barrie & Kirk-Davidoff: "Weather response to management of large Wind turbine array", Atmos. Chem., 2010

Use z_0 ~ 0.8 m - using "Lettau's formula" (ad-hoc geometric arguments…)

Grid-spacings 100's of km, first vertical point ~ 80m

"horizontally averaged structure"



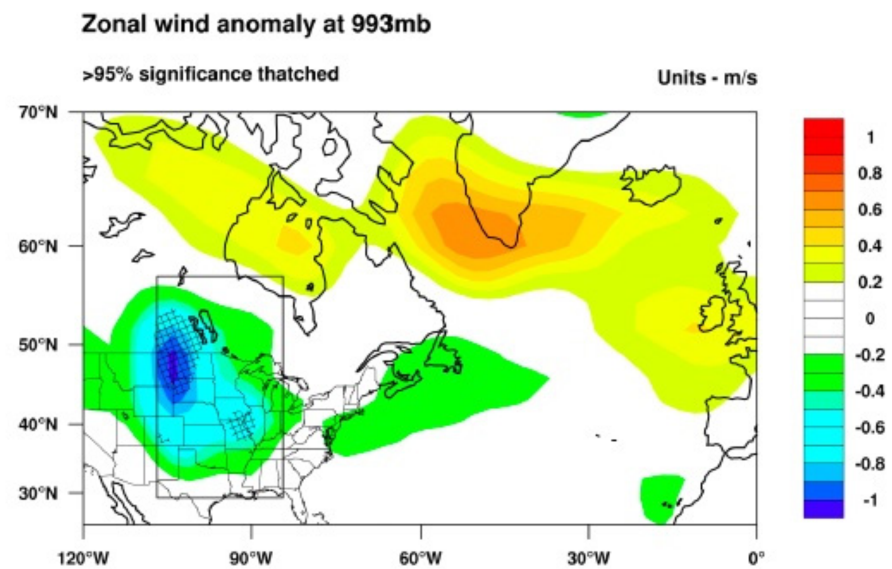
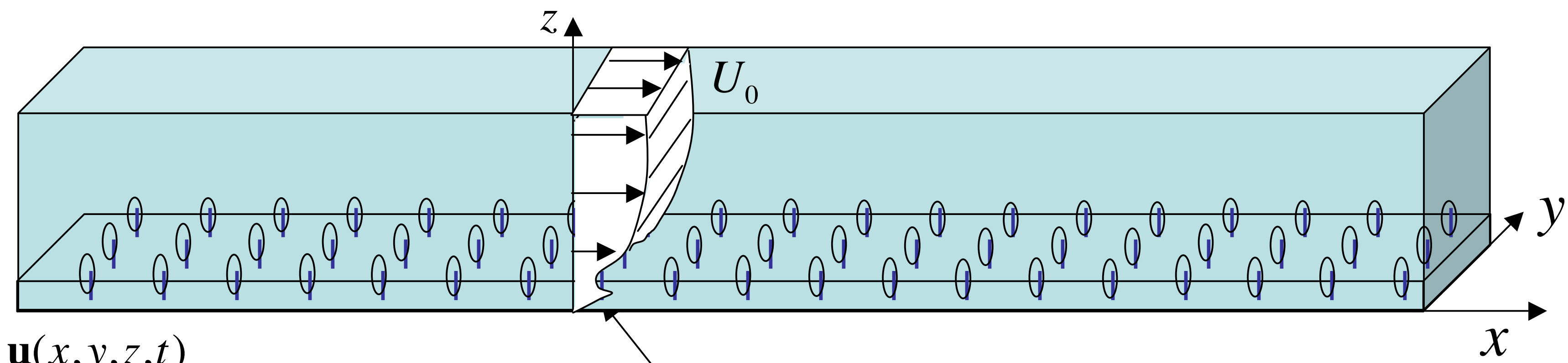


Fig. 1. 993 mbar zonal wind anomaly. The mean difference in the eastward wind in the lowest model level between the control and perturbed model runs highlights regions of atmospheric modification. Regions where significance exceeds 95%, as determined by a Student's t-test, are thatched. The wind farm is located within the rectangular box over the central United States and central Canada. Areas of the wind farm located over water are masked out during the model runs.

The "fully developed" WTABL:

What is the structure of this specific type of boundary layer?



$\mathbf{u}(x,y,z,t)$
DNS

$\tilde{\mathbf{u}}(x,y,z,t)$
LES

$\bar{\mathbf{u}}(x,y,z)$
RANS

$$U(z) = \langle \bar{u}(x,y,z) \rangle_{xy}$$

horizontal (canopy) average

What is the "averaged" velocity distribution? $U(z) = \langle \bar{u}(x,y,z) \rangle_{xy}$

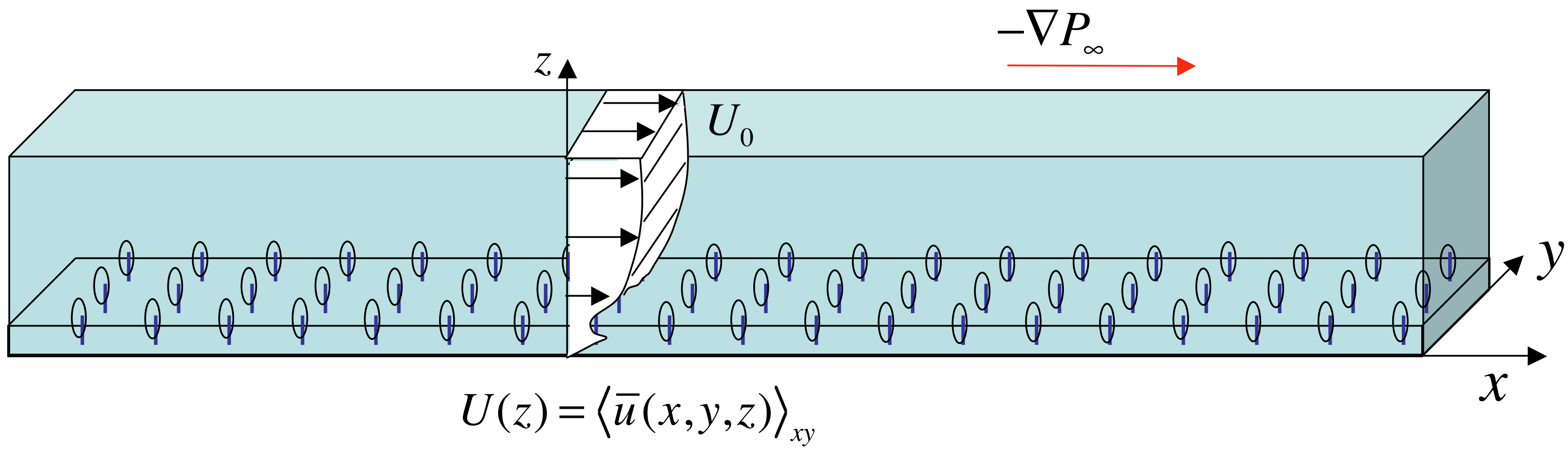
Is there a "universal" WTABL profile?

What are profiles of shear stresses? $\tau_{xz}(z) = -\langle \overline{u'w'} \rangle_{xy}$

Fluxes? TKE flux profiles?

The "fully developed" WTABL:

assume pressure-gradient forcing ("wind farm in a channel")



- Momentum theory: Reynolds Eq. + horizontal average + fully dev.

$$0=-\frac{1}{\rho}\frac{dp_\infty}{dx}+\frac{d}{dz}\left(-\langle\overline{u'w'}\rangle_{xy}-\langle\overline{u}''\overline{w}''\rangle_{xy}\right)+\langle f_x\rangle_{xy}$$

thrust force due to WT

$$\overline{u}''=\overline{u}-\langle\overline{u}\rangle_{xy}$$

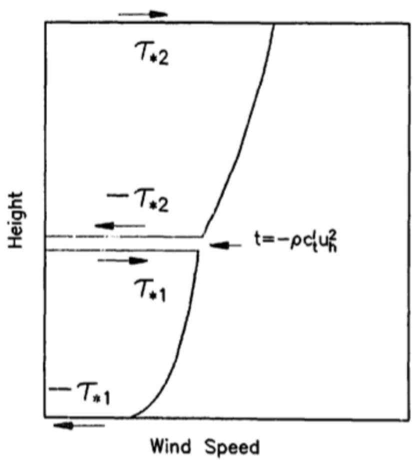
Horizontal average
of turbulent Reynolds shear stress

We must include "correlations"
between mean velocity deviations
from their spatial mean
(Raupach et al. Appl Mech Rev **44**, 1991,
Finnigan, Annu Rev Fluid Mech **32**, 2000)

The fully developed WTABL: momentum theory

Horizontally averaged variables

Sten Frandsen,
J. Wind Eng & Ind
Appl **39**, 1992):



$$0 = -\frac{1}{\rho}\frac{dp_\infty}{dx} + \frac{d}{dz}\left(-\left\langle \overline{u'w'} \right\rangle_{xy} - \left\langle \bar{u}''\bar{w}'' \right\rangle_{xy}\right) + \left\langle f_x \right\rangle_{xy}$$

Integrate in z-direction:

If top of WT canopy still

falls in the "surface layer", where $\frac{dp}{dx} z \approx 0$

and if wakes have "diffused" so that $\left\langle \bar{u}''\bar{w}'' \right\rangle_{xy} \approx 0$

$$-\left\langle \overline{u'w'} \right\rangle_{xy}(z_{top}) \approx -\left\langle \overline{u'w'} \right\rangle_{xy}(z_{bottom}) + \frac{1}{2} C_T \frac{A_{disk}}{A_{xy}} U_R^2$$

$$s_x = \frac{L_x}{D}$$

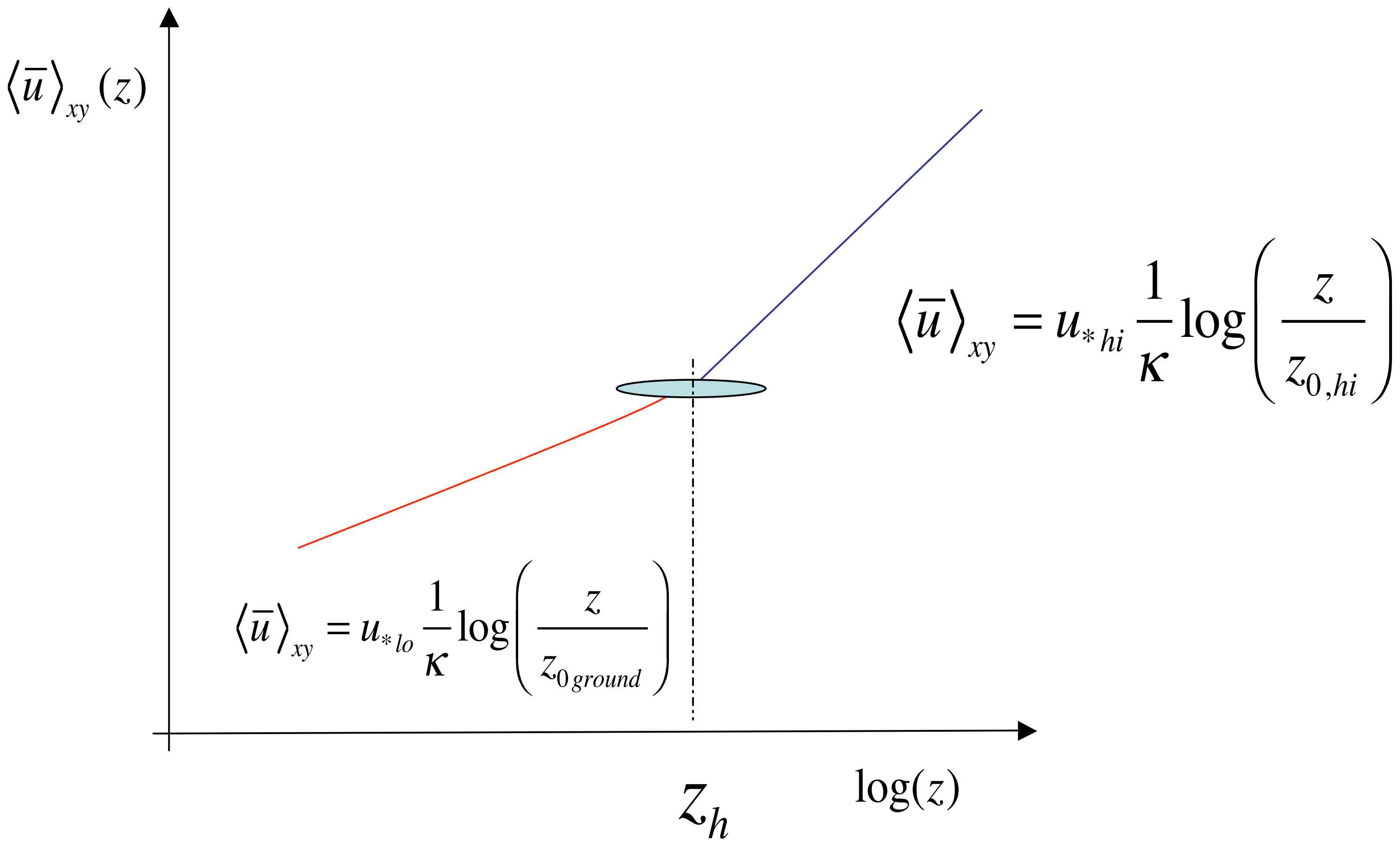
$$s_y = \frac{L_y}{D}$$

$$u_{*\,hi}^2 \approx u_{*\,lo}^2 + \frac{1}{2} C_T \frac{\pi}{4 s_x s_y} U_R^2$$

The fully developed WTABL: momentum theory

$$u_{*hi}^2 \approx u_{*lo}^2 + \frac{1}{2} C_T \frac{\pi}{4 s_x s_y} U_R^2$$

Frandsen 1992: postulated the existence of 2 log laws



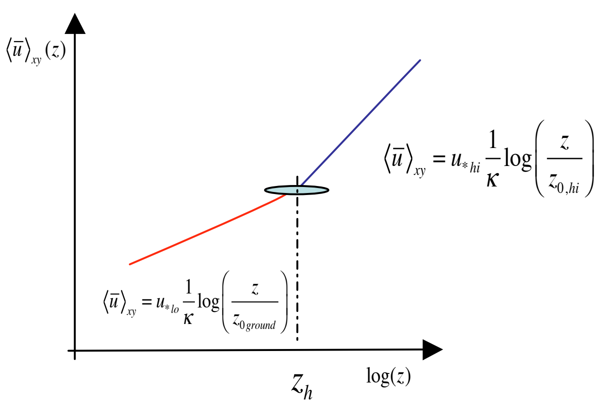
The fully developed WTABL: momentum theory

S. Frandsen 1992, Frandsen et al. 2006:

Knowns: $u_{*hi},\ z_{0,ground},\ C_T,\ s_x,\ s_y$

3 unknowns: $z_{0,hi},\ U_R,\ u_{*lo}$

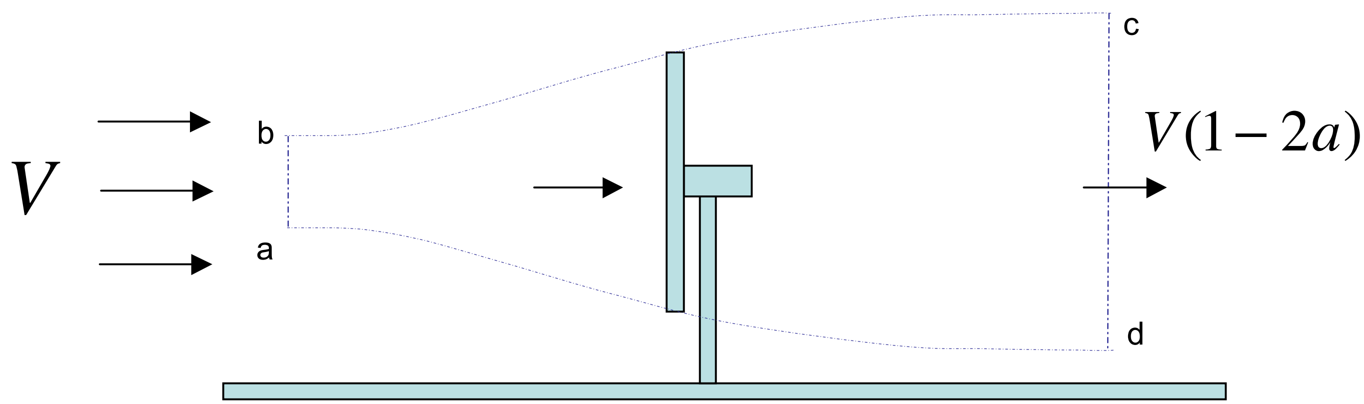
$$\begin{cases} u_{*hi}^2 \approx u_{*lo}^2 + \dfrac{1}{2} C_T \dfrac{\pi}{4 s_x s_y} U_R^2 \\ U_R = u_{*hi} \dfrac{1}{\kappa} \log\left(\dfrac{z_h}{z_{0,hi}}\right) \\ u_{*hi} \dfrac{1}{\kappa} \log\left(\dfrac{z_h}{z_{0,hi}}\right) = u_{*lo} \dfrac{1}{\kappa} \log\left(\dfrac{z_h}{z_{0,ground}}\right) \end{cases}$$



Solve for effective roughness:

$$z_{0,hi} = z_h \exp\left(-\kappa \left[\frac{\pi C_T}{8 s_x s_y} + \left(\frac{\kappa}{\ln(z_h / z_{0,ground})} \right) \right]^{-1/2} \right)$$

Another important question: Fluxes of kinetic energy



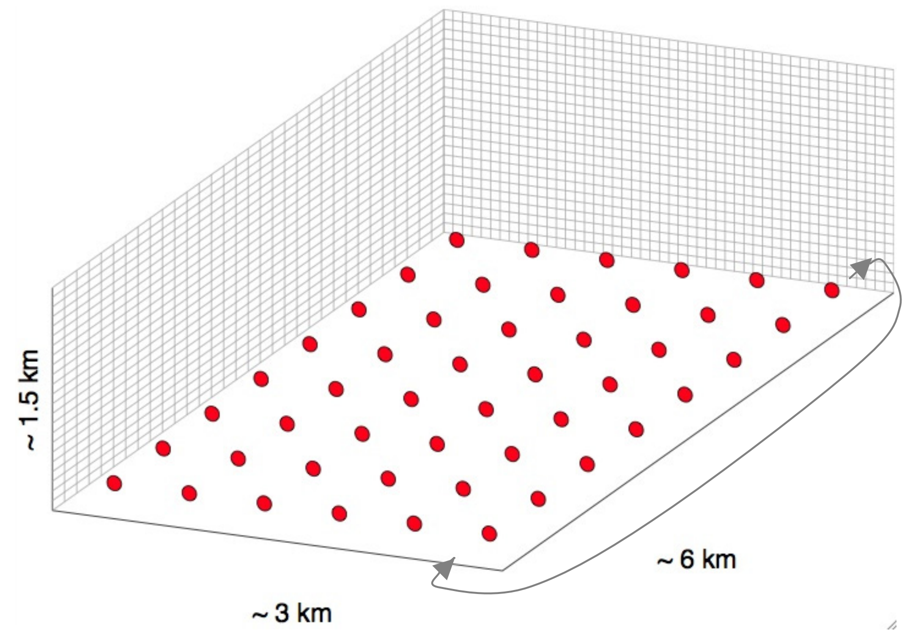
For **single** wind turbine, extracted power =
difference in front and back fluxes of kinetic energy

Betz limit, etc…

Simulations setup:

- LES code: horizontal pseudo-spectral (periodic B.C.), vertical: centered 2nd order FD (Moeng 1984, Albertson & Parlange 1999, Porté-Agel et al. 2000, Bou-Zeid et al. 2005)

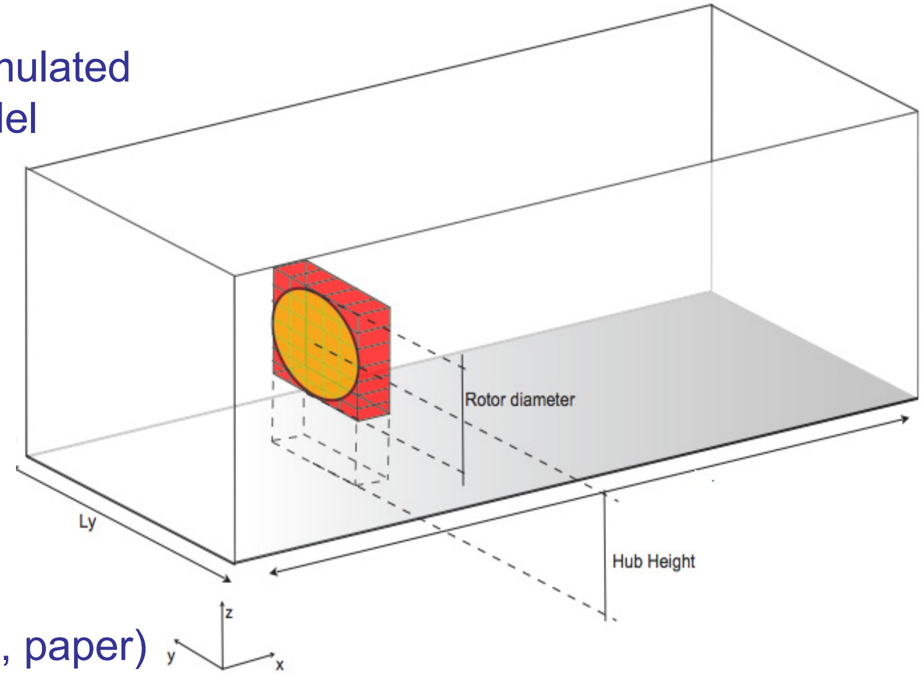
$$H = 1000 - 1500m, \quad L_x = \pi H - 2\pi H, \quad L_y = \pi H$$
$$(N_x \times N_y \times N_z) = 128 \times 128 \times 128$$



- Horizontal periodic boundary conditions (only good for FD-WTABL)
- Top surface: zero stress, zero w
- Bottom surface B.C.: Zero w + Wall stress: Standard wall function relating wall stress to first grid-point velocity
- Scale-dependent dynamic Lagrangian mode (*no* adjustable parameters)
- More details: Calaf, Meneveau & Meyers, "Large eddy simulation study of fully developed wind-turbine array boundary layers" Phys. Fluids. **22** (2010) 015110

Actuator disk modeling of turbines in LES

Jimenez et al., J. Phys. Conf. Ser. **75** (2007) simulated single turbine in LES using dynamic Smag. model



They used fixed reference (undisturbed) velocity:

$$f_{Tx} = -\frac{1}{2} C_T U_{ref}^2 \frac{\delta A_{yz}}{\delta V}, \quad C_T = 0.75$$

Here we use disk-averaged and time-averaged velocity, but local at the disk (see Meyers & Meneveau 2010, 48th AIAA conf., paper)

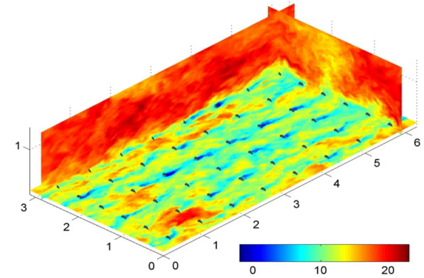
$$f_{Tx} = -\frac{1}{2} C_T \left(\frac{1}{1-a} \overline{U} \right)^2 \frac{\delta A_{yz}}{\delta V} = -\frac{1}{2} C_T' \overline{U}^2 \frac{\delta A_{yz}}{\delta V}$$

$$C_T = 0.75 \Rightarrow a \approx 0.25 \rightarrow C_T' = 1.33$$

Also, use first-order relax process to time-average:

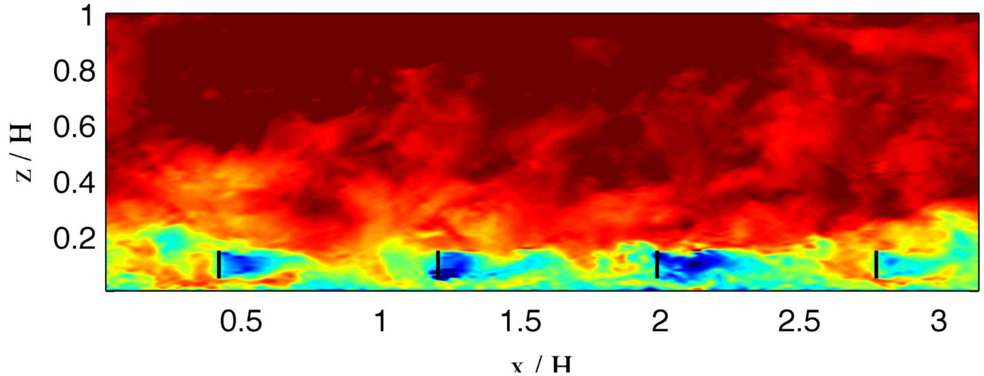
$$\overline{U}(t) = (1-\varepsilon)\overline{U}(t-dt) + \varepsilon U_{disk}(t)$$

Simulations results:

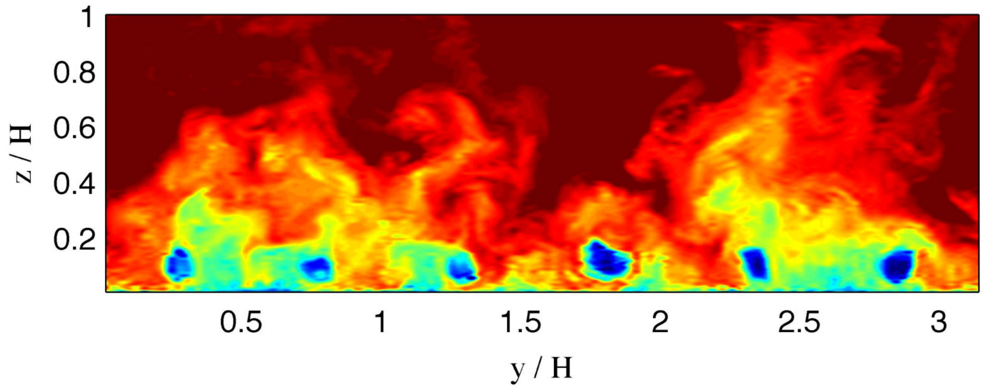


Instantaneous stream-wise velocity contours:

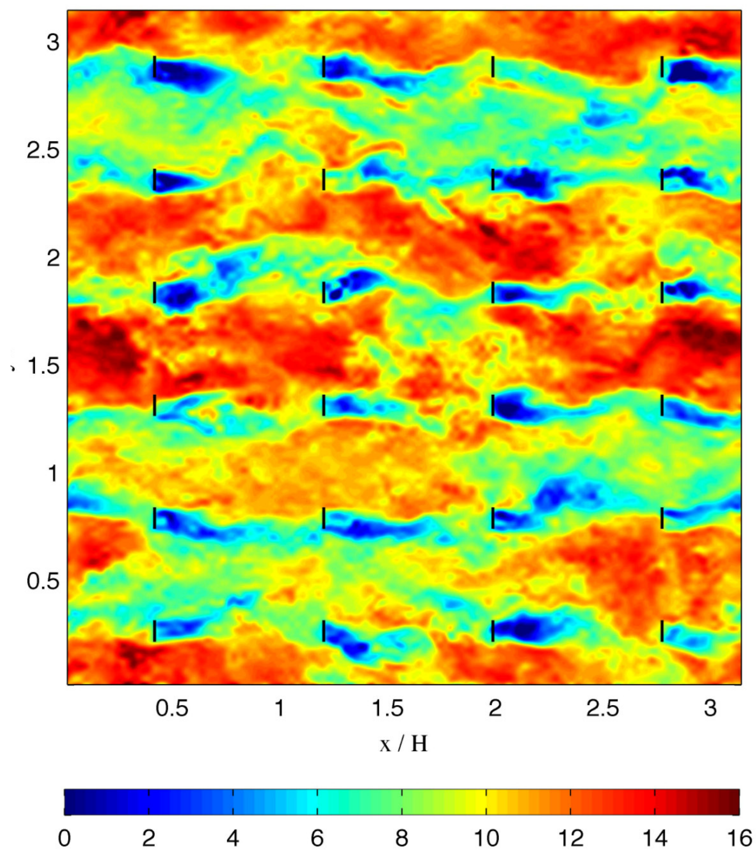
side-view



front-view

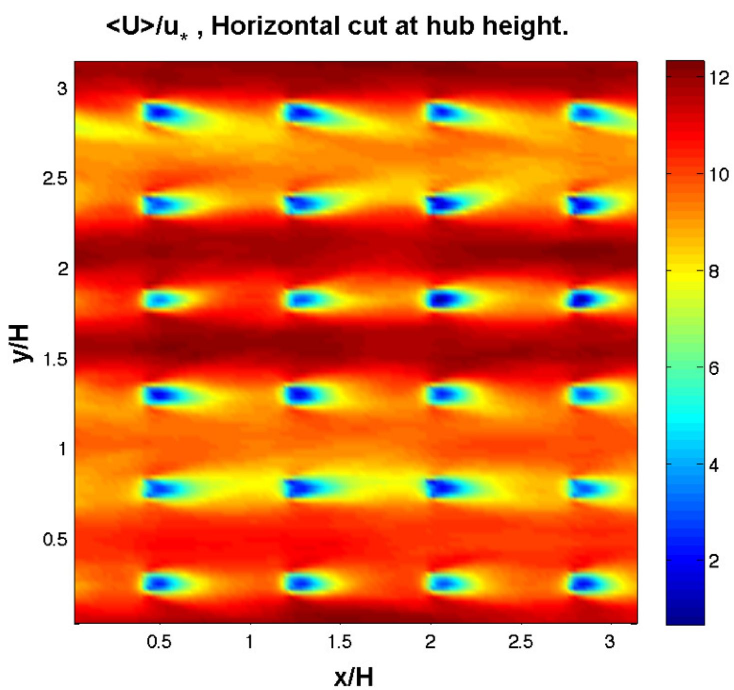


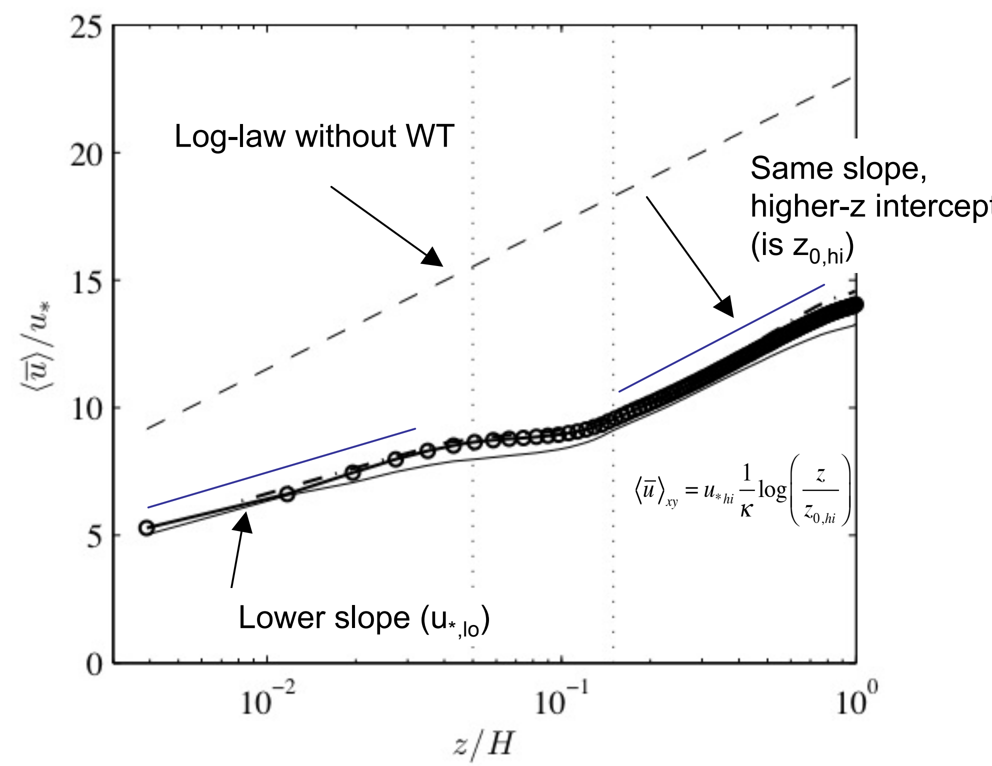
top-view



Simulations results: horizontally averaged velocity profile *U(z)*

Mean velocity profile: $U(z) = \langle \overline{u} \rangle_{xy}$





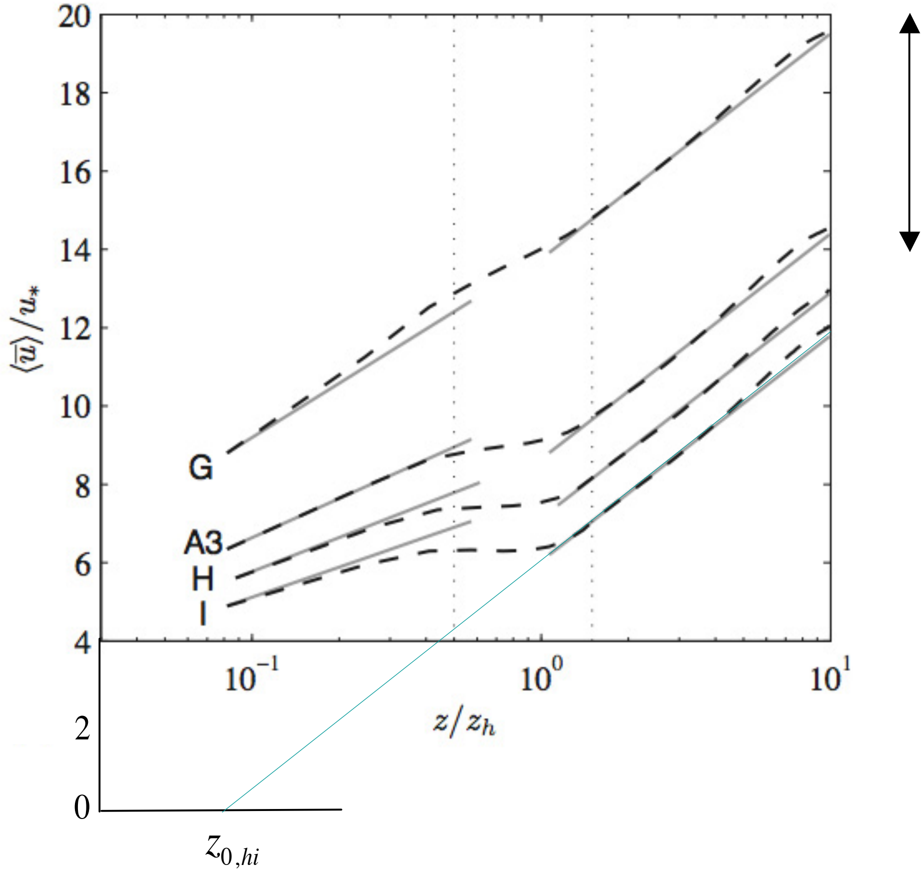
Observation:
Two log-laws (as assumed in Frandsen, 1992)

Suite of LES cases:

TABLE I. Summarizing parameters of the various LES cases. Between brackets is indicated which code is used: "L" refers to the KULeuven code and "J" refers to the JHU-LES code.

	s_x/s_y	s_x	$4s_xs_y/\pi$	N_t	$L_x \times L_y \times H$	$N_x \times N_y \times N_z$	$z_{0,\mathrm{lo}}$	C_T'	c_{ft}'
A1 (L)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-4}	1.33	0.025
A2 (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-4}	1.33	0.025
A3 (L)	1.5	7.85	52.36	8×6	$2\pi\times\pi\times1$	$128\times192\times61$	10^{-4}	1.33	0.025
A4 (L)	1.5	7.85	52.36	8×6	$2\pi\times\pi\times1.5$	$128\times192\times92$	10^{-4}	1.33	0.025
B (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-4}	2.00	0.038
C (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-4}	0.60	0.012
D (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-3}	1.33	0.025
E (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-5}	1.33	0.025
F (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-6}	1.33	0.025
G (L)	1.5	15.7	209.4	4×3	$2\pi\times\pi\times1$	$128\times192\times61$	10^{-4}	1.33	0.0064
H (L)	1.5	6.28	33.51	10×8	$2\pi\times1.07\pi\times1$	$128\times192\times57$	10^{-4}	1.33	0.040
I (L)	1.5	5.24	23.27	12×9	$2\pi\times\pi\times1$	$128\times192\times61$	10^{-4}	1.33	0.057
J (L)	2	9.07	52.36	7×7	$2.02\pi\times1.01\pi\times1$	$128\times192\times61$	10^{-4}	1.33	0.025
K (L)	1	6.41	52.36	10×5	$2.04\pi\times1.02\pi\times1$	$128\times192\times60$	10^{-4}	1.33	0.025

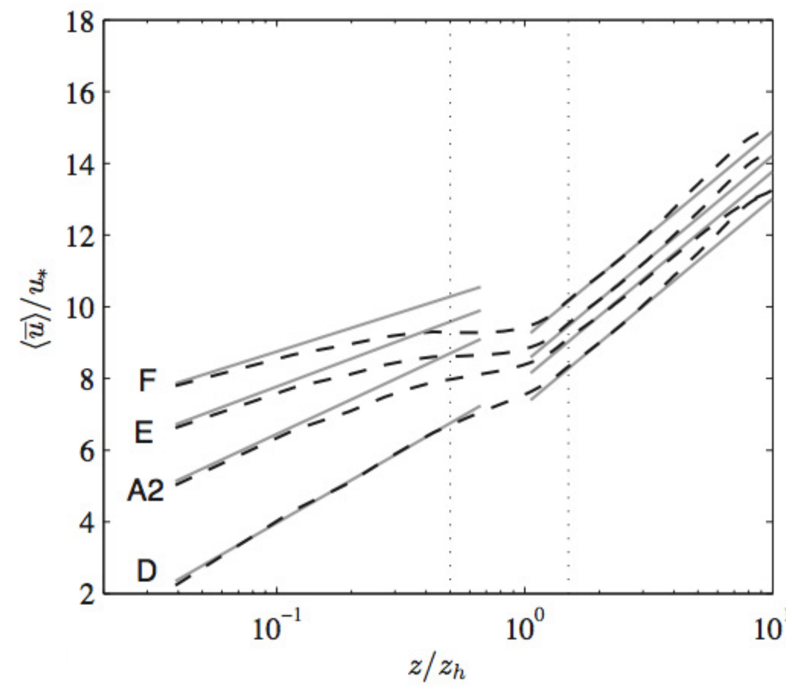
Suite of LES cases:



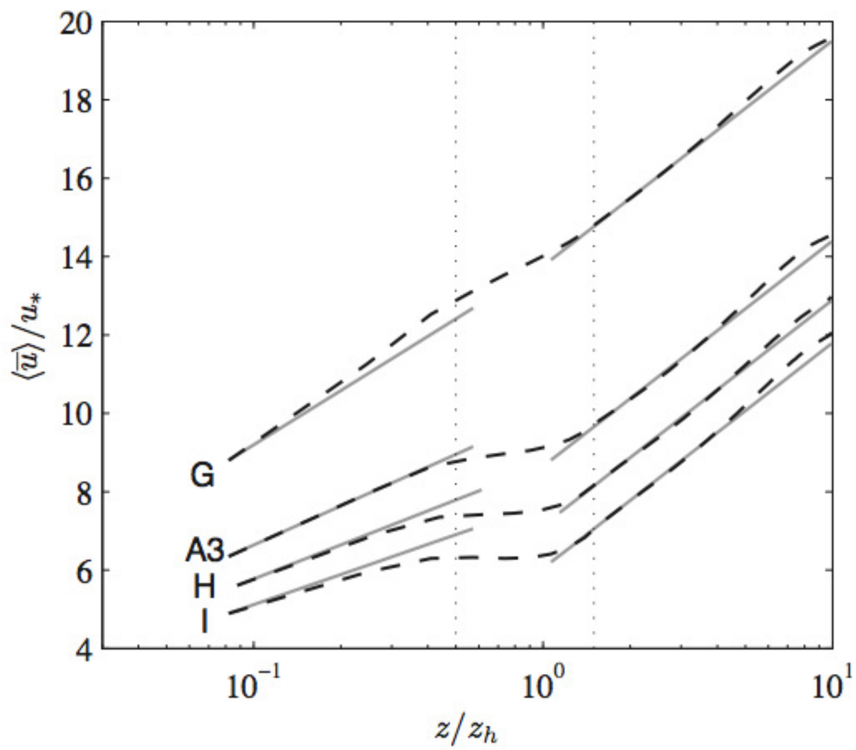
measure $z_{0,hi}$ from intercept

$$\langle \overline{u} \rangle_{xy} = u_{*hi} \frac{1}{\kappa} \log\left(\frac{z}{z_{0,hi}} \right)$$

(essentially the “Clauser plot” method)



"Wake upgrade" to Frandsen's model:



$$\left(\kappa u_* z_h + \nu_w\right)\frac{\partial \langle \overline{u} \rangle}{\partial z} = u_*^2$$

In wake, reduced slope:

$$\frac{\partial \langle \overline{u} \rangle}{\partial z} = \frac{1}{\kappa u_* z_h + \nu_w} u_*^2$$

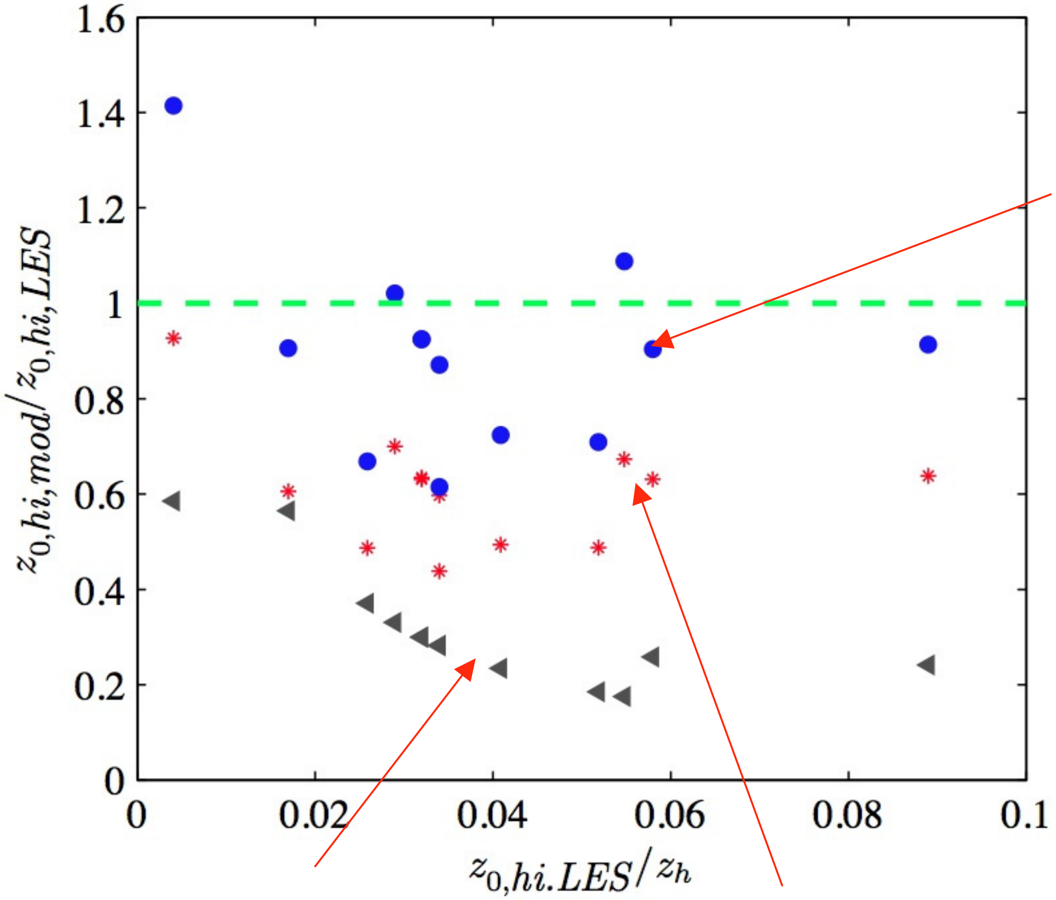
$$\nu_w = \sqrt{\tfrac{1}{2} c_{ft}}\, \langle \overline{u} \rangle D$$

$$\nu_w^* = \frac{\sqrt{\tfrac{1}{2} c_{ft}}\, \langle \overline{u}(z_h) \rangle D}{\kappa u_* z_h} \approx 28\sqrt{\tfrac{1}{2} c_{ft}}$$

$$z_{0,hi} = z_h \left(1 + \frac{D}{2z_h}\right)^{\beta} \exp\left(- \left[\frac{\pi C_T}{8\kappa^2 s_x s_y} + \left(\ln\left[\frac{z_h}{z_{0,ground}} \left(1 - \frac{D}{2z_h}\right)^{\beta} \right] \right)^{-2} \right]^{-1/2} \right)$$

where $\beta = \dfrac{28\sqrt{\tfrac{1}{2} c_{ft}}}{1 + 28\sqrt{\tfrac{1}{2} c_{ft}}}$,

Comparison of LES results with models:



Circles: improved Frandsen model
Calaf, Meneveau & Meyers,
(Phys. Fluids 2010, **22**)

$$z_{0,hi} = z_h \left(1+\frac{D}{2z_h}\right)^{\beta} \exp\left(-\left[\frac{\pi C_T}{8\kappa^2 s_x s_y} + \left(\ln\left[\frac{z_h}{z_{0,ground}}\left(1-\frac{D}{2z_h}\right)^{\beta}\right]\right)^{-2}\right]^{-1/2}\right)$$

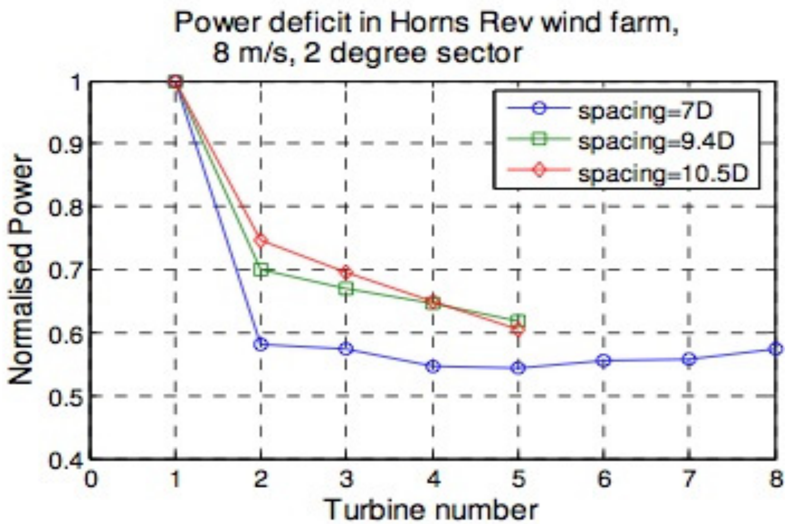
where $\beta = \frac{\nu_w^*}{1+\nu_w^*}$, and $\nu_w^* = \frac{\nu_T}{\kappa u_* z_h}$, eddy viscosity due to wake

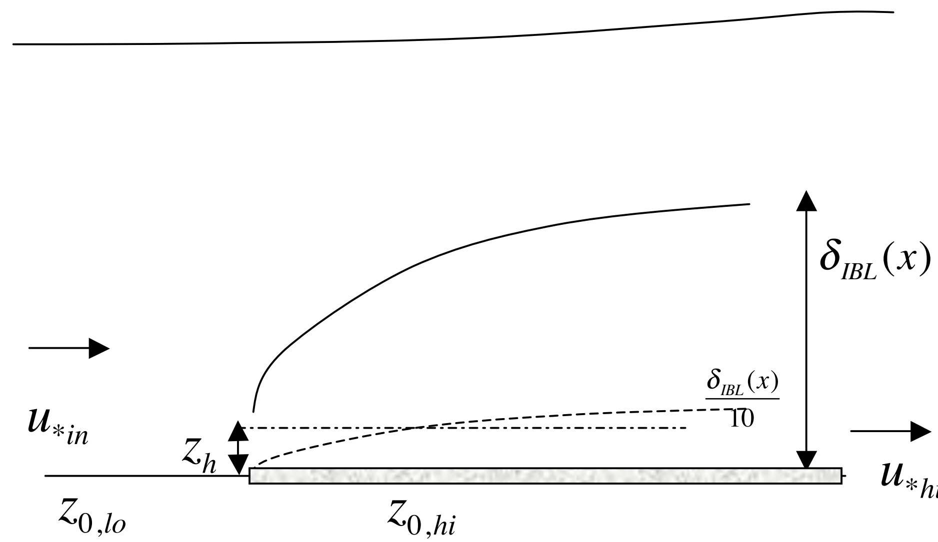
Triangles: Lettau formula

Asterisks: Frandsen et al. (2006) formula

$$z_{0,hi} = z_h \exp\left(-\kappa\left[\frac{\pi C_T}{8 s_x s_y} + \left(\frac{\kappa}{\ln(z_h / z_{0,ground})}\right)^2\right]^{-1/2}\right)$$

Impact on evaluation of power degradation: internal boundary layers





Composite profile
(Chamorro & Porté-Agel, BLM 2009)

$$\overline{u}(z_h) = [1-\lambda(x)]\frac{u_{*,hi}}{\kappa}\ln\left(\frac{z_h}{z_{0,hi}}\right) + \lambda(x)\frac{u_{*,in}}{\kappa}\ln\left(\frac{z_h}{z_{0,lo}}\right)$$

Set λ=0 when $z_h = 0.1\,\delta_{IBL}$

$$\lambda(x) = \frac{z_h - \alpha\,\delta_{IBL}}{(1-\alpha)\delta_{IBL}}$$

$$\delta_{IBL} = z_h + z_{0,hi}\left(\frac{x}{z_{0,hi}}\right)^{0.8}$$

Impact on evaluation of power degradation:

$D = 80m$

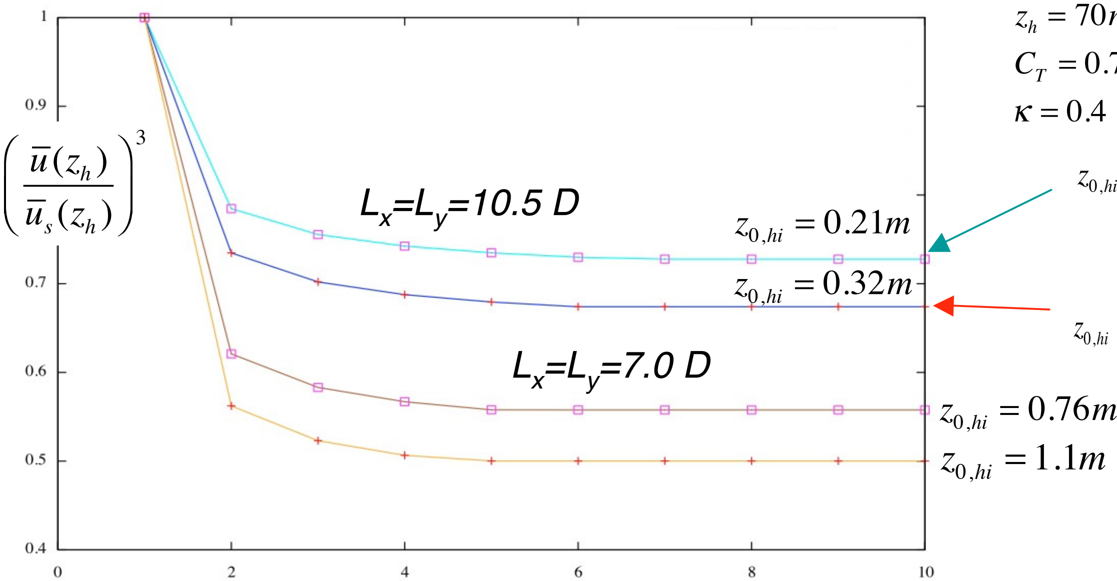
$z_h = 70m$

$C_T = 0.7$

$\kappa = 0.4$

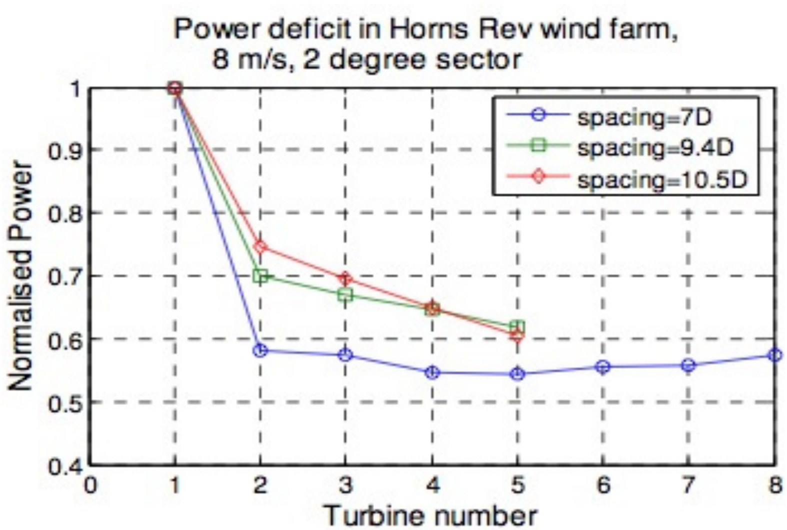
Vestas V80

$$\frac{P}{P_0} = \left(\frac{\overline{u}(z_h)}{\overline{u}_s(z_h)}\right)^3$$



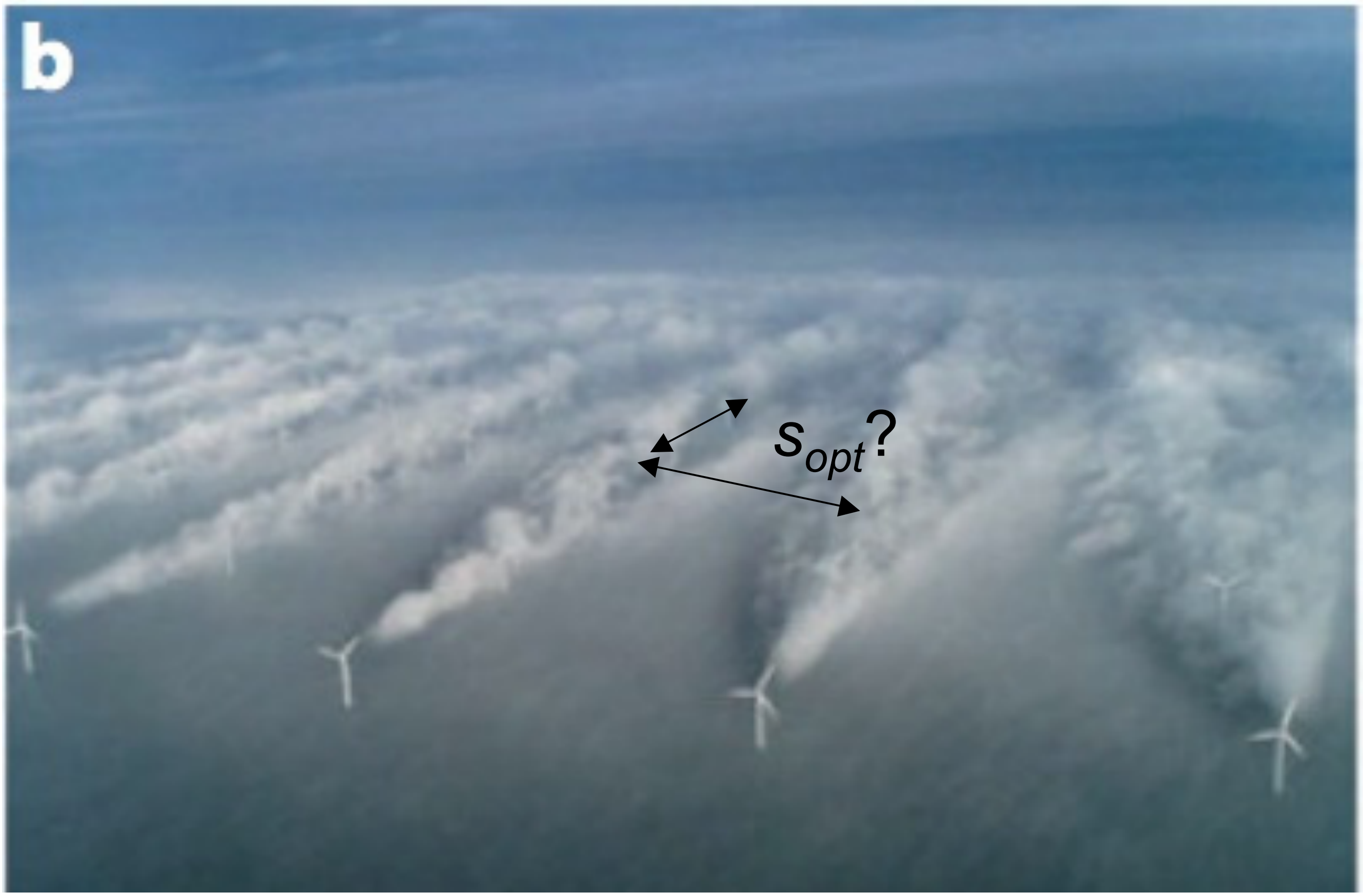
$$z_{0,hi} = z_h \exp\left(-\kappa\left[\frac{\pi C_T}{8 s_x s_y} + \left(\frac{\kappa}{\ln(z_h / z_{0,ground})}\right)^2\right]^{-1/2}\right)$$

$$z_{0,hi} = z_h \left(1 + \frac{D}{2z_h}\right)^{\beta} \exp\left(-\left[\frac{\pi C_T}{8\kappa^2 s_x s_y} + \left(\ln\left[\frac{z_h}{z_{0,ground}}\left(1 - \frac{D}{2z_h}\right)^{\beta}\right]\right)^{-2}\right]^{-1/2}\right)$$



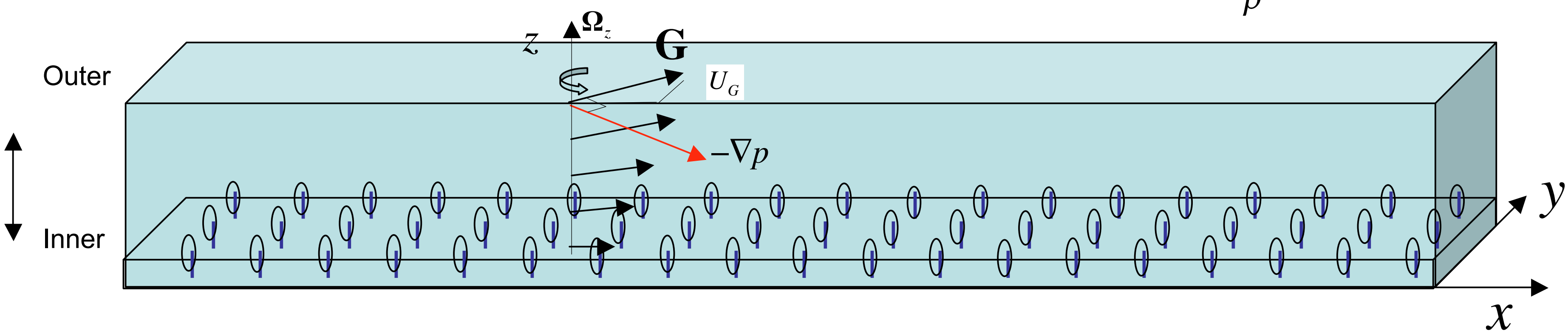
$$\frac{\overline{u}(z_h)}{\overline{u}_s(z_h)} = \lambda(x) + [1 - \lambda(x)]\,\frac{\ln\left(\frac{z_h}{z_{0,hi}}\right)}{\ln\left(\frac{z_h}{z_{0,lo}}\right)}\,\frac{\ln\left[Ro_h\left(\frac{z_h}{z_{0,lo}}\right)\right] - C_*}{\ln\left[Ro_h\left(\frac{z_h}{z_{0,hi}}\right)\right] - C_*}.$$

What is the most optimal spacing s_{opt} of wind turbines in the fully developed WTABL?



The "fully developed" WTABL: Forcing by geostrophic wind

Above ABL (in mid-latitudes): geostrophic balance $2\mathbf{\Omega}\times\mathbf{G}-\frac{1}{\rho}\nabla P\approx 0$



Coupled through a stress $(u_*)^2$:

Outer length-scale: $L=\frac{u_*}{f}$ $\quad f=2\Omega\sin\phi\approx 10^{-4}s^{-1}$ (mid-latitudes)

Inner length-scale: z_0

Inner-outer matching: $\frac{u(z)}{u_*}=\frac{1}{\kappa}\ln\left(\frac{z}{z_0}\right)$ $\quad \frac{G}{u_*}=\sqrt{A^2+\left[\frac{1}{\kappa}\ln\left(\frac{u_*}{fz_0}\right)-C\right]^2}$

Given *G* and z_0 ----> find $u_{*,hi}$ and *H*

Using the roughness model for array optimization - find s-opt:

Driving forces is geostrophic wind G (assuming large but not regional-scale WT, i.e. assume wind farm does not affect *G*)

$$P^{+} = \frac{P}{\frac{\rho}{2}(s_x s_y D^2)G^3} = \frac{\pi C_T'}{4 s_x s_y}\left(\frac{U_d}{G}\right)^3 = \frac{\pi C_T'}{4 s_x s_y}\left(\frac{u_{*,hi}}{G}\right)^3\left(\frac{U_d}{u_{*,hi}}\right)^3$$

$$\frac{U_d}{u_{*hi}} = \sqrt{\frac{1-\frac{u_{*\,hi}^2}{u_{*\,lo}^2}}{\frac{1}{2}C_T\frac{\pi}{4 s_x s_y}}}$$

Classical ABL relationship
(Tennekes & Lumley, 1972) - *C=4.5, A=11.25*

$$\frac{G}{u_{*,hi}} = \sqrt{A^2 + \left[\frac{1}{\kappa}\ln\left(\frac{u_{*,hi}}{G}\frac{z_h}{z_{0,hi}}Ro_h\right) - C\right]^2}$$

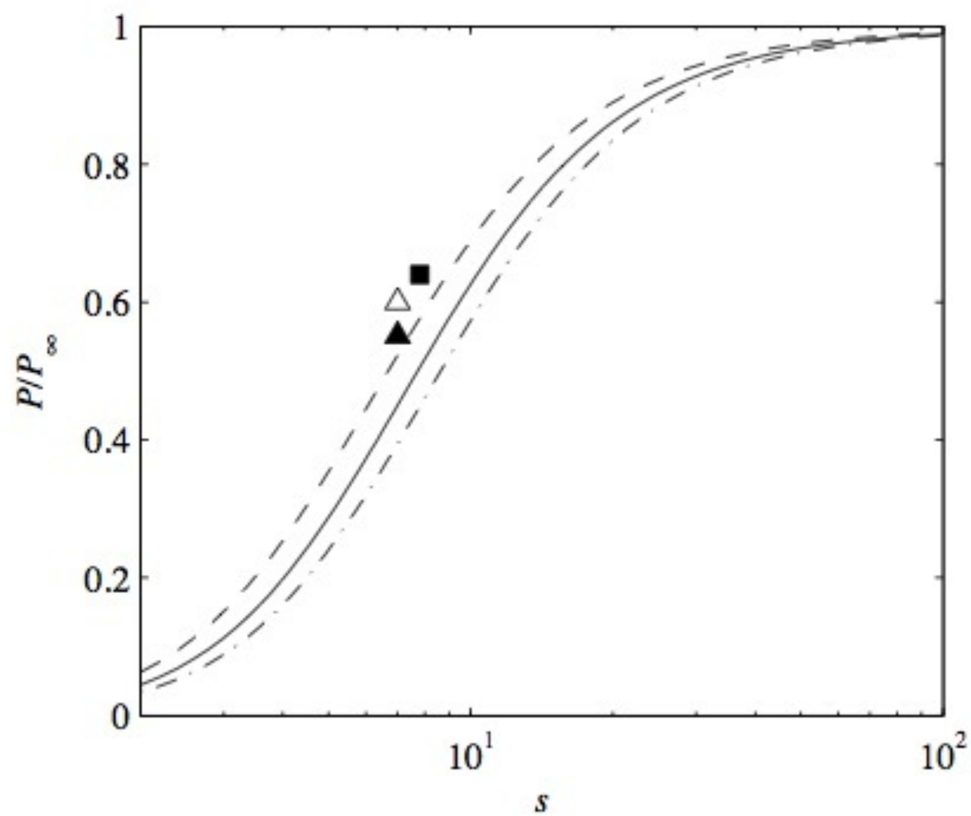
$$Ro_h = \frac{G}{f z_h} \approx 2{,}000$$

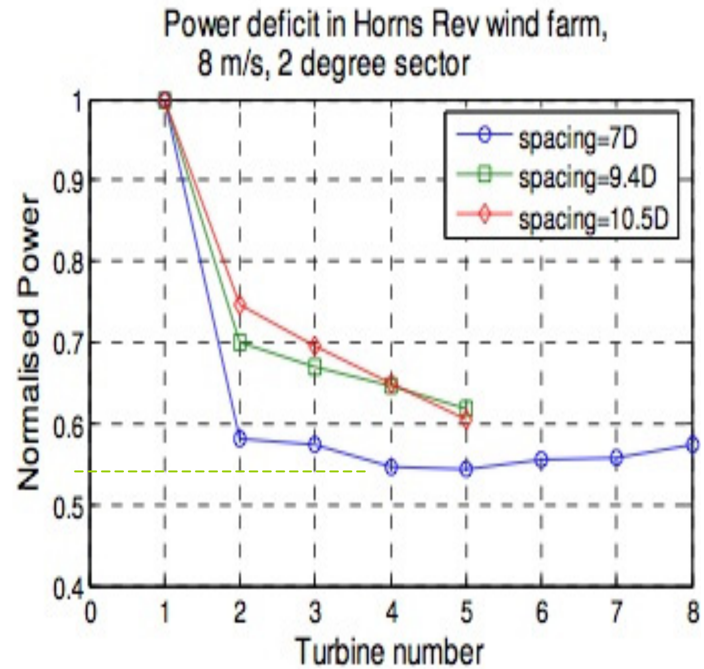
typical hub-height Rossby number

$$\frac{z_{0,hi}}{z_h} = \left(1+\frac{D}{2z_h}\right)^{\beta}\exp\left(-\left[\frac{\pi C_T}{8\kappa^2 s_x s_y} + \left(\ln\left[\frac{z_h}{z_{0,ground}}\left(1-\frac{D}{2z_h}\right)^{\beta}\right]\right)^{-2}\right]^{-1/2}\right)$$

Using the roughness model for array optimization - find s-opt:

For given s, $z_{0,lo}$, D, z_h, C_T evaluate P
divide by P_∞ of single WT ($z_{0,hi} = z_{0,lo}$ case)





From: Barthelmie et al.
J. of Phys. Conf. (2007)

Using the roughness model for array optimization - find s-opt:

"Cost optimization":

consider total $Cost = Cost_{land}$ [$/m²] x S + $Cost_{turb}$ [$]

Define dimensionless ratio:

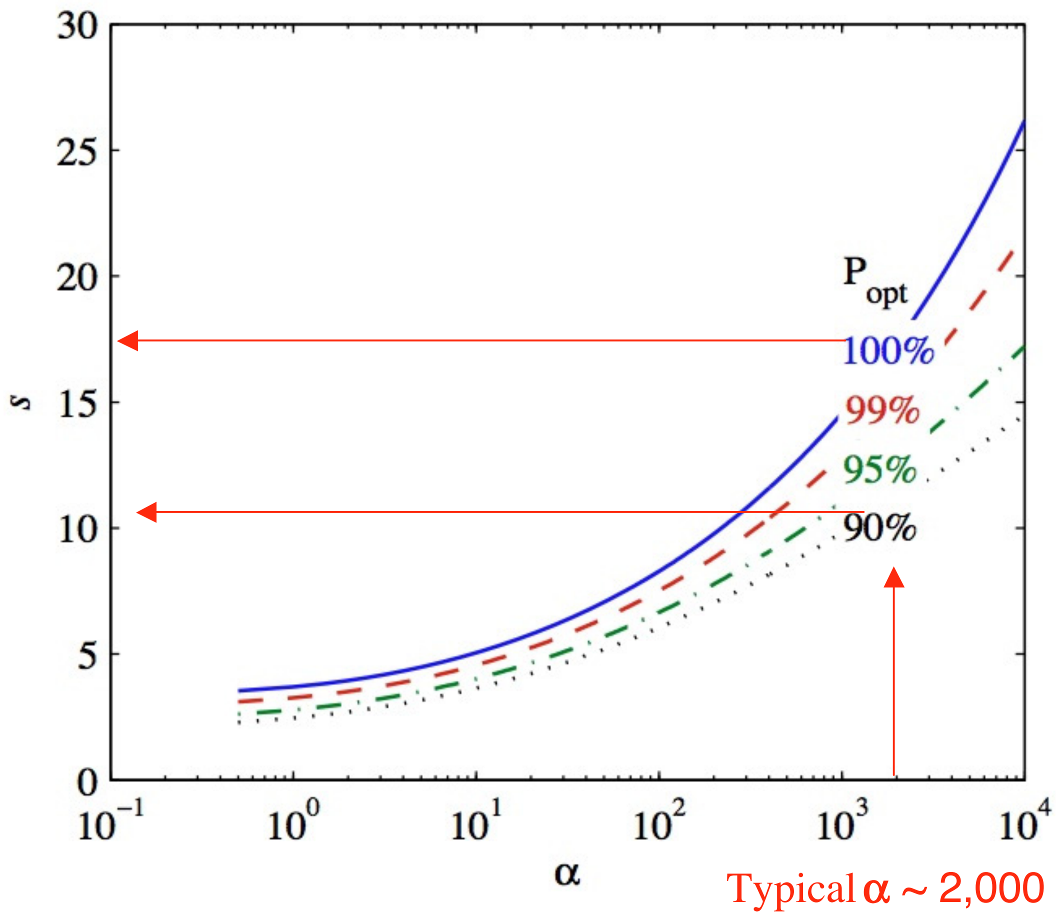
$$\alpha = \frac{Cost_{turb} / \left(\frac{\pi}{4} D^2\right)}{Cost_{land}}$$

Power per unit cost:

$$P^* = \frac{P}{Cost_{turb} / (s_x s_y D^2) + Cost_{land}} \propto \frac{C_T'}{4 s_x s_y / \pi + \alpha} \left(\frac{u_{*,hi}}{G} \right)^3 \left(\frac{U_d}{u_{*,hi}} \right)^3$$

(region II)

Using the roughness model for array optimization - find s-opt:

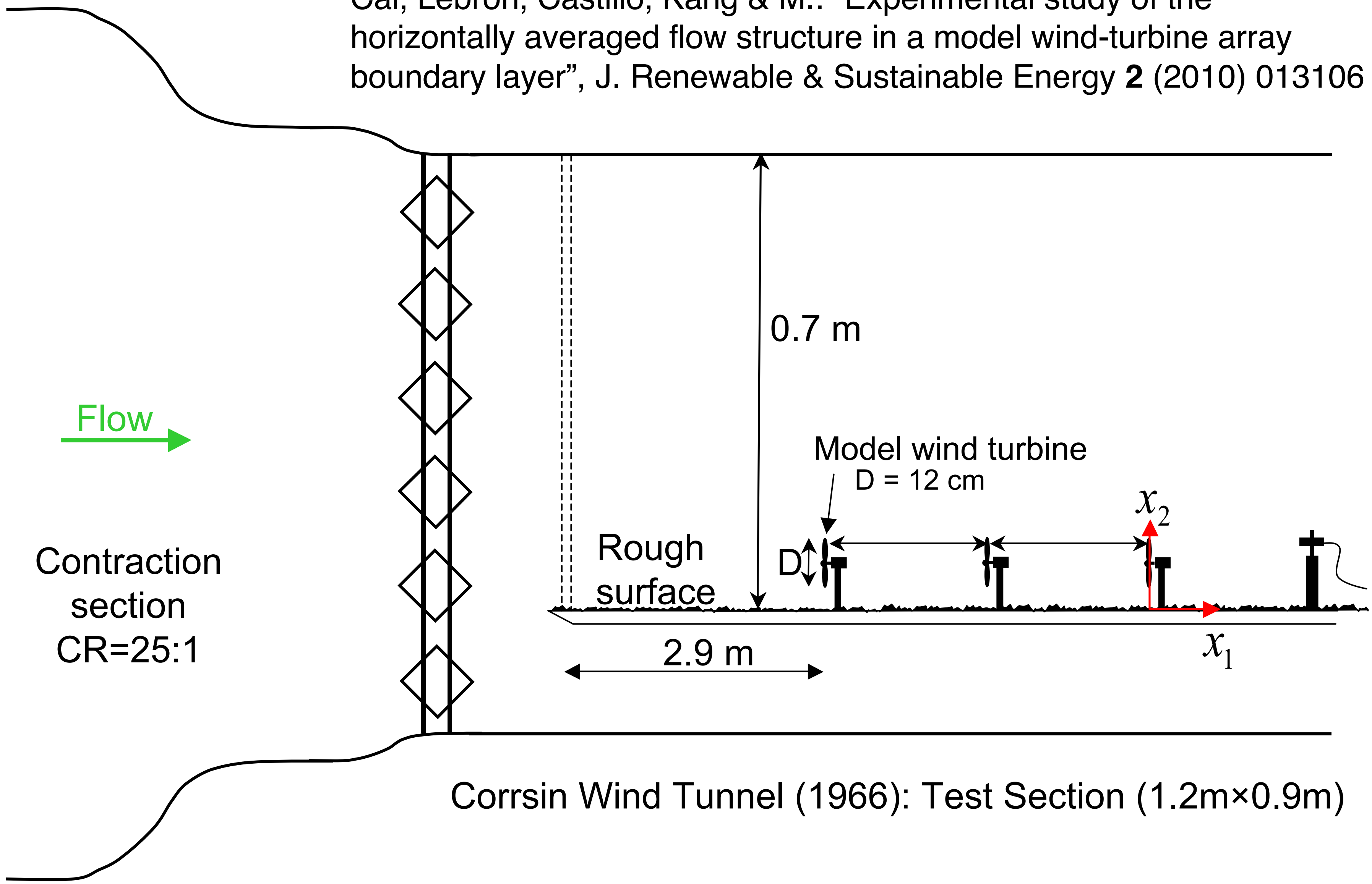


At common s ~ 7D, 10-20% suboptimal - use 15 D instead

Meyers & Meneveau, 2011
(in press, Wind Energy)

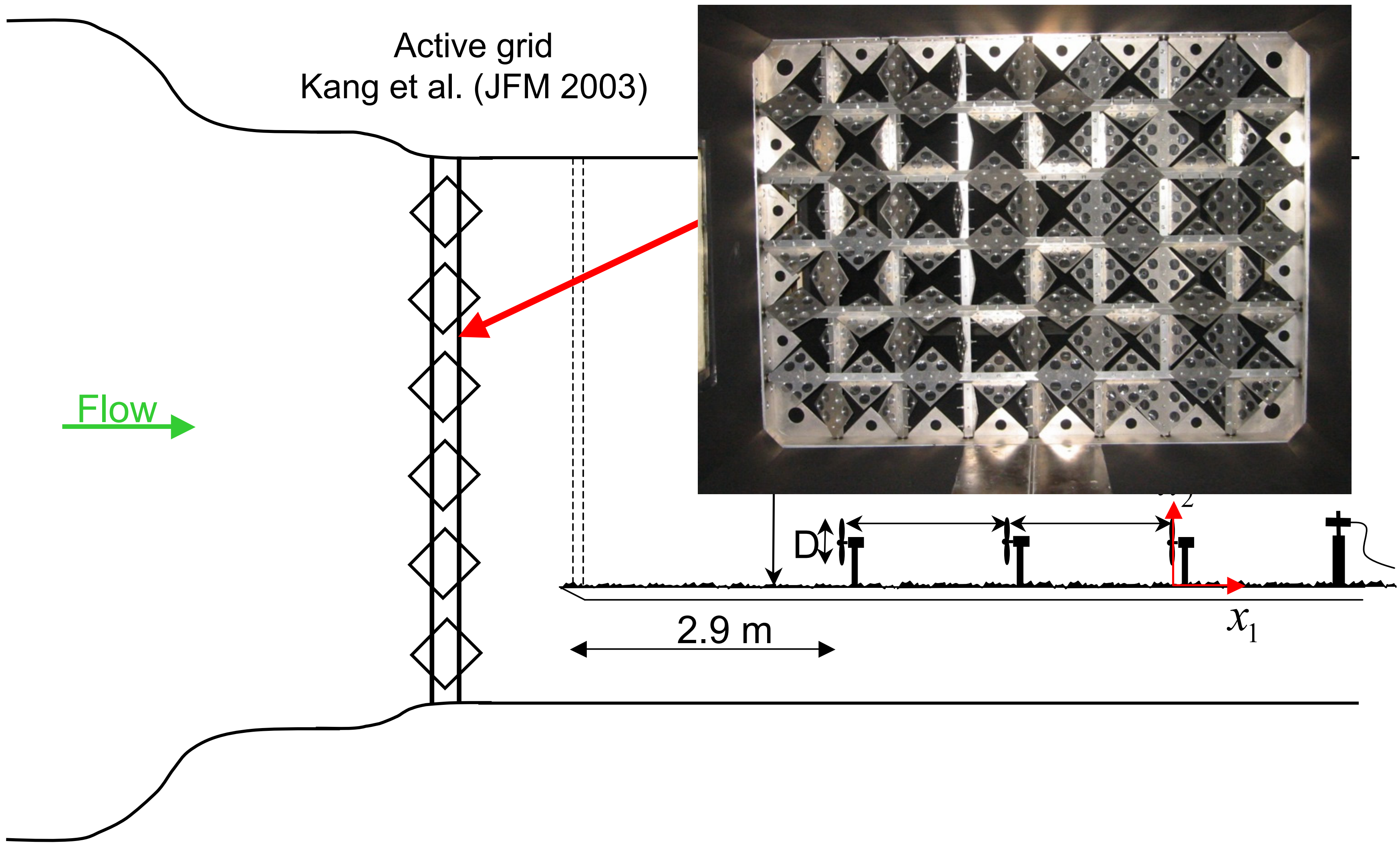
Wind-tunnel measurements: mechanics of vertical KE entrainment??

Cal, Lebrón, Castillo, Kang & M.: "Experimental study of the horizontally averaged flow structure in a model wind-turbine array boundary layer", J. Renewable & Sustainable Energy **2** (2010) 013106

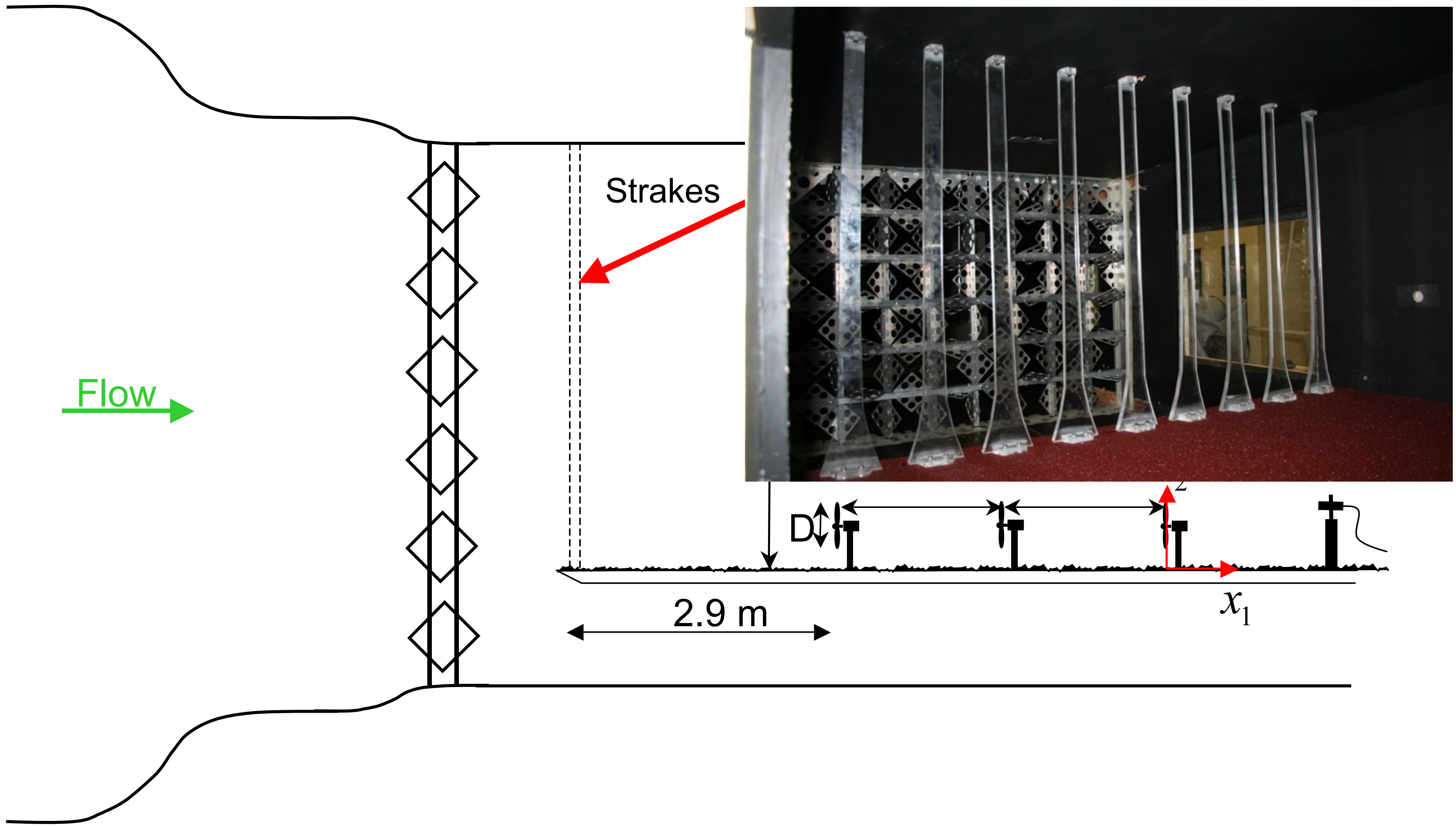


Corrsin Wind Tunnel (1966): Test Section (1.2m×0.9m)

Wind-tunnel measurements: mechanics of vertical KE entrainment??

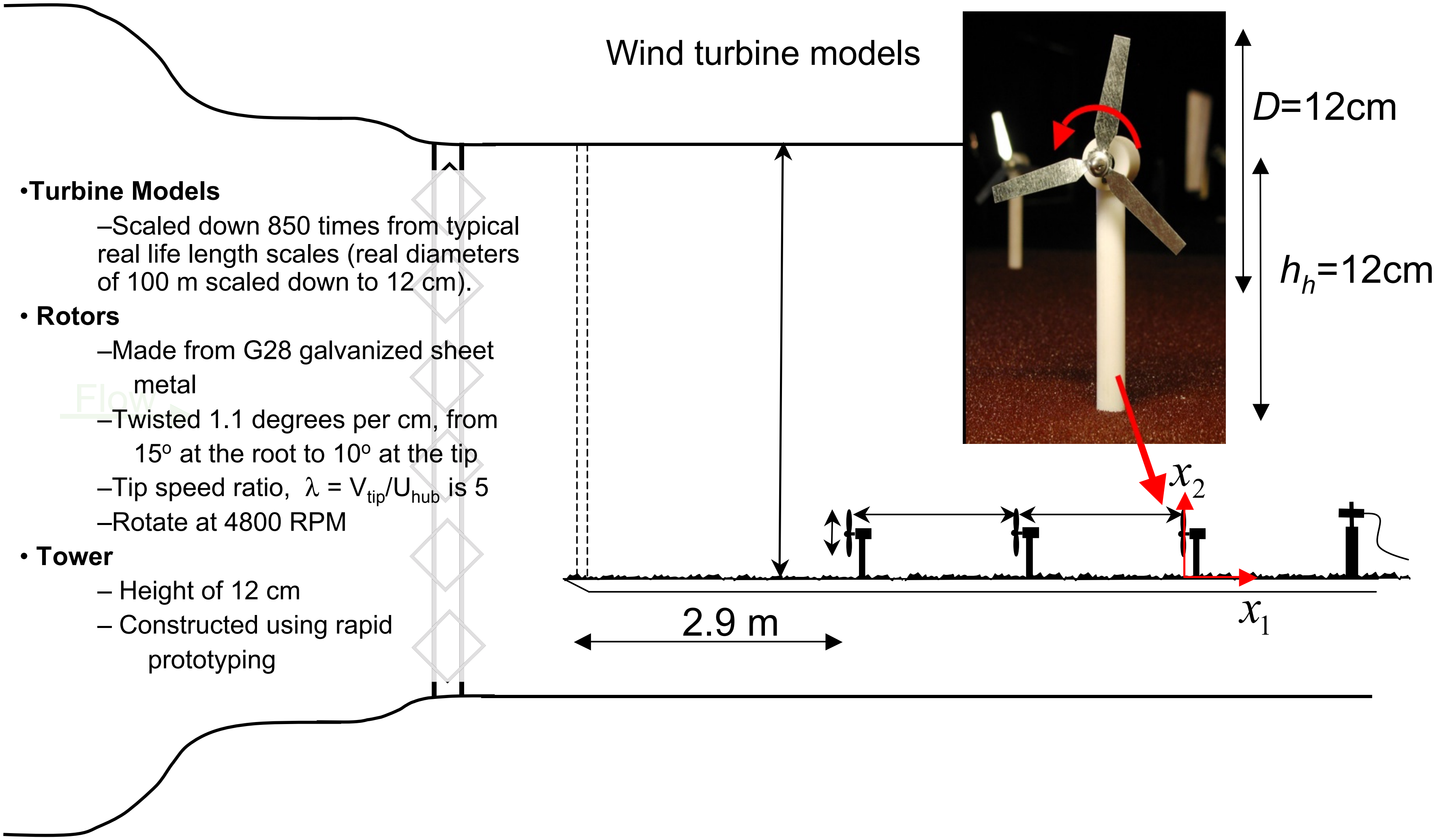


Wind-tunnel measurements

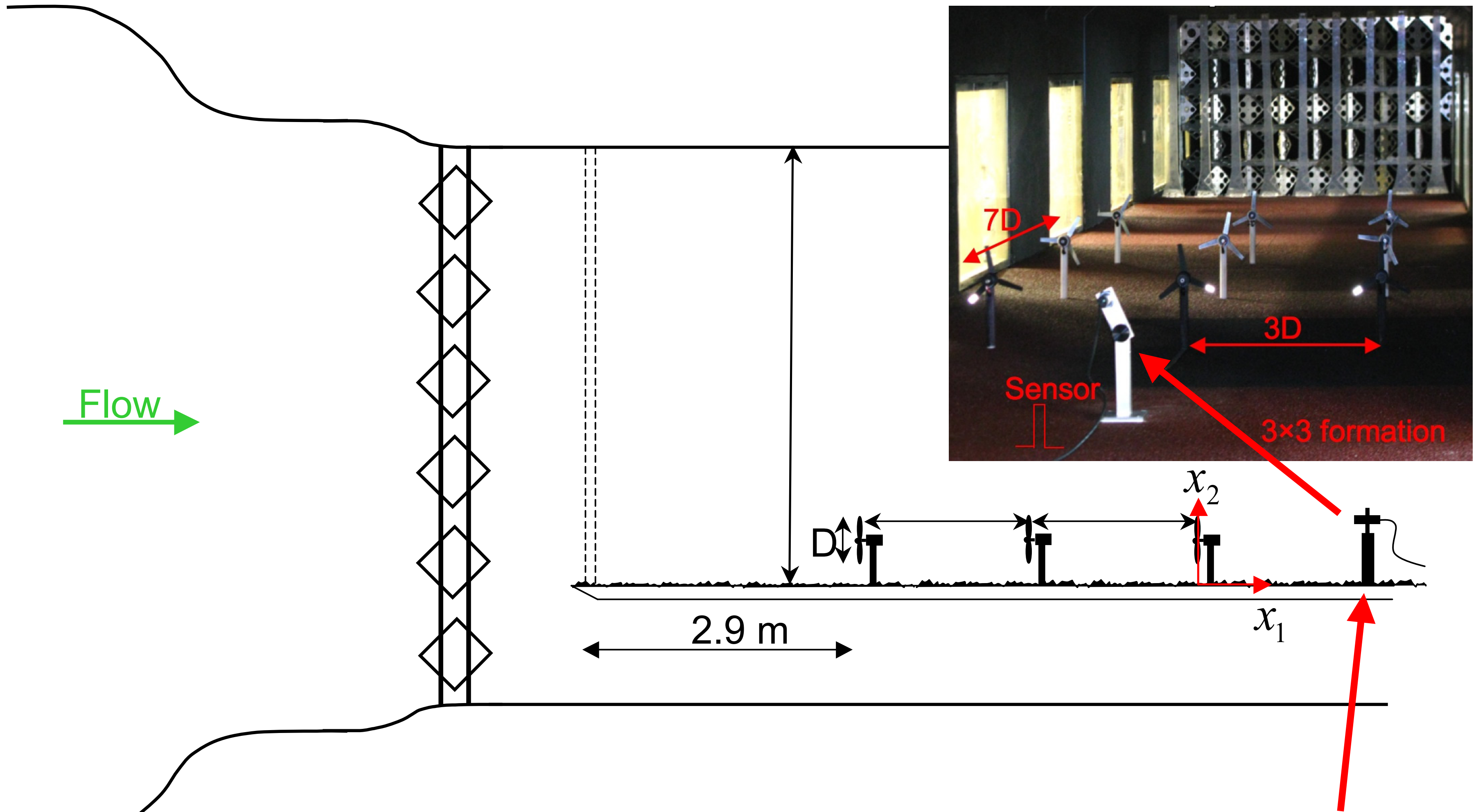


Wind-tunnel measurements

- **Turbine Models**
  - –Scaled down 850 times from typical real life length scales (real diameters of 100 m scaled down to 12 cm).
- **Rotors**
  - –Made from G28 galvanized sheet metal
  - –Twisted 1.1 degrees per cm, from 15º at the root to 10º at the tip
  - –Tip speed ratio, $\lambda = V_{tip}/U_{hub}$ is 5
  - –Rotate at 4800 RPM
- **Tower**
  - – Height of 12 cm
  - – Constructed using rapid prototyping



Wind-tunnel measurements

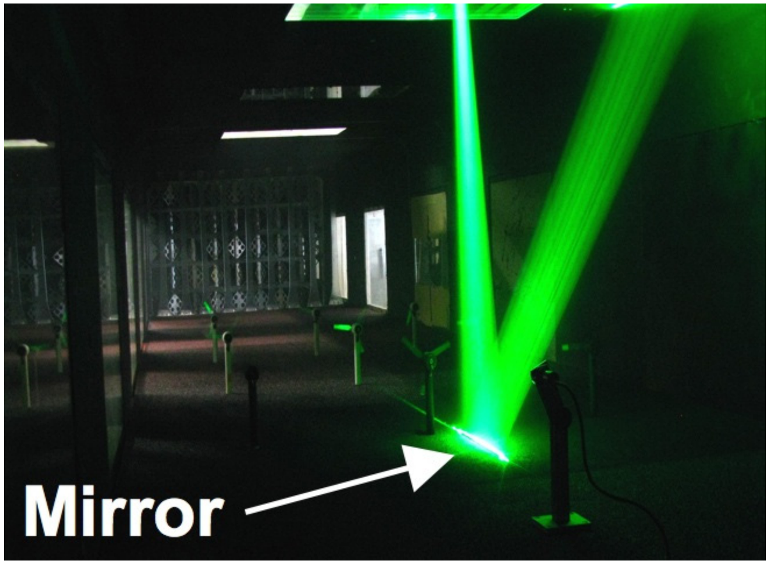


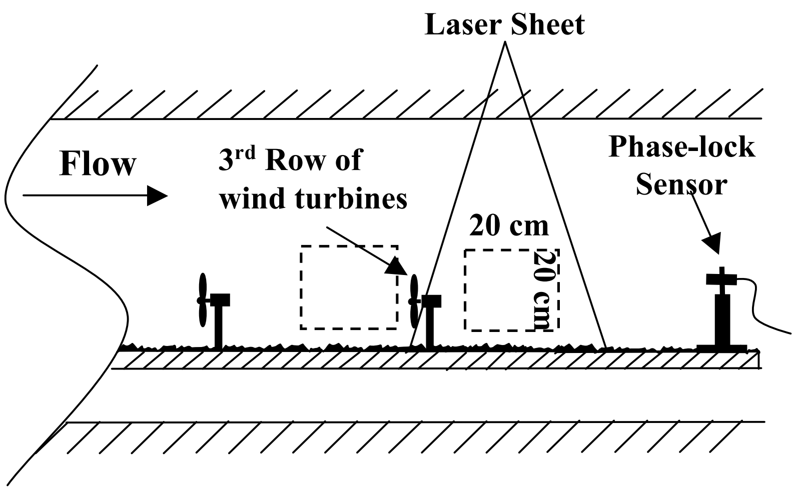
optical sensor
for phase-lock and
Ω rpm measurements

Stereo-PIV system

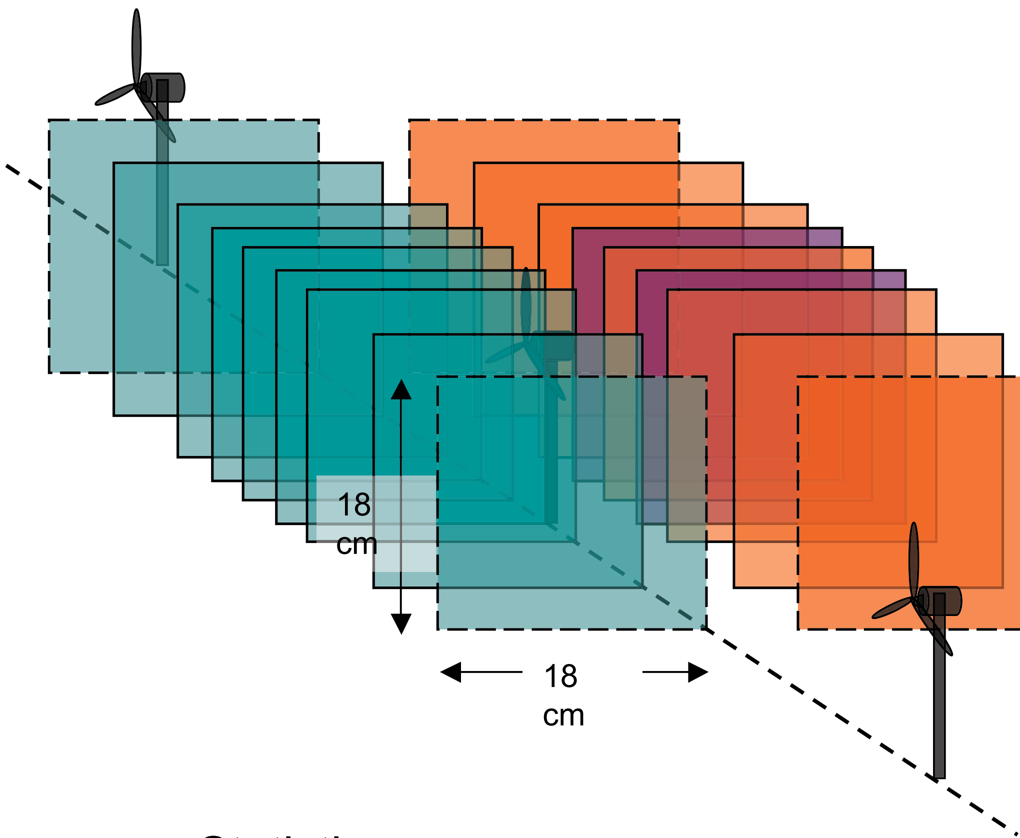
TSI System with:

- Double pulse Nd:YAG laser(120 mJ/pulse)
  - Laser sheet thickness of 1.2 mm
  - Time between pulses of 50 ms
  - Optical sensor external trigger for phase lock measurements
- Two high resolution cross/auto correlation digital CCD cameras with
  - a frame rate of 16 frames/sec.
  - Interrogation area of 20 cm by 20 cm

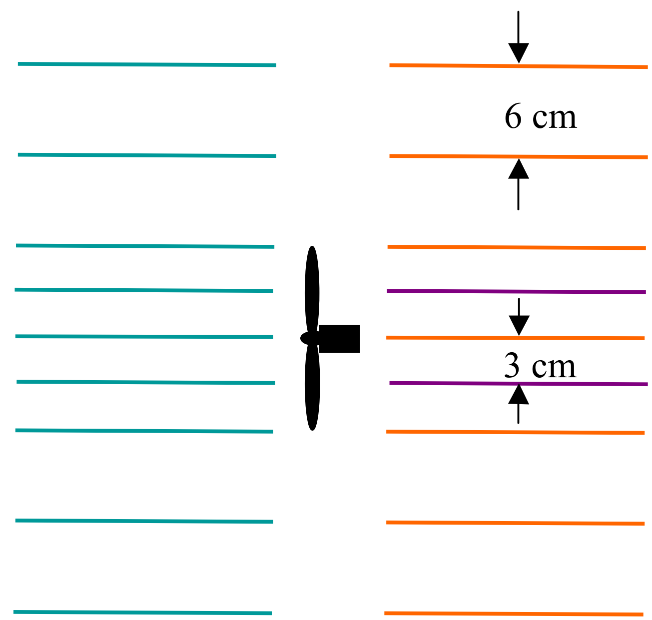




PIV data planes:



Top view:

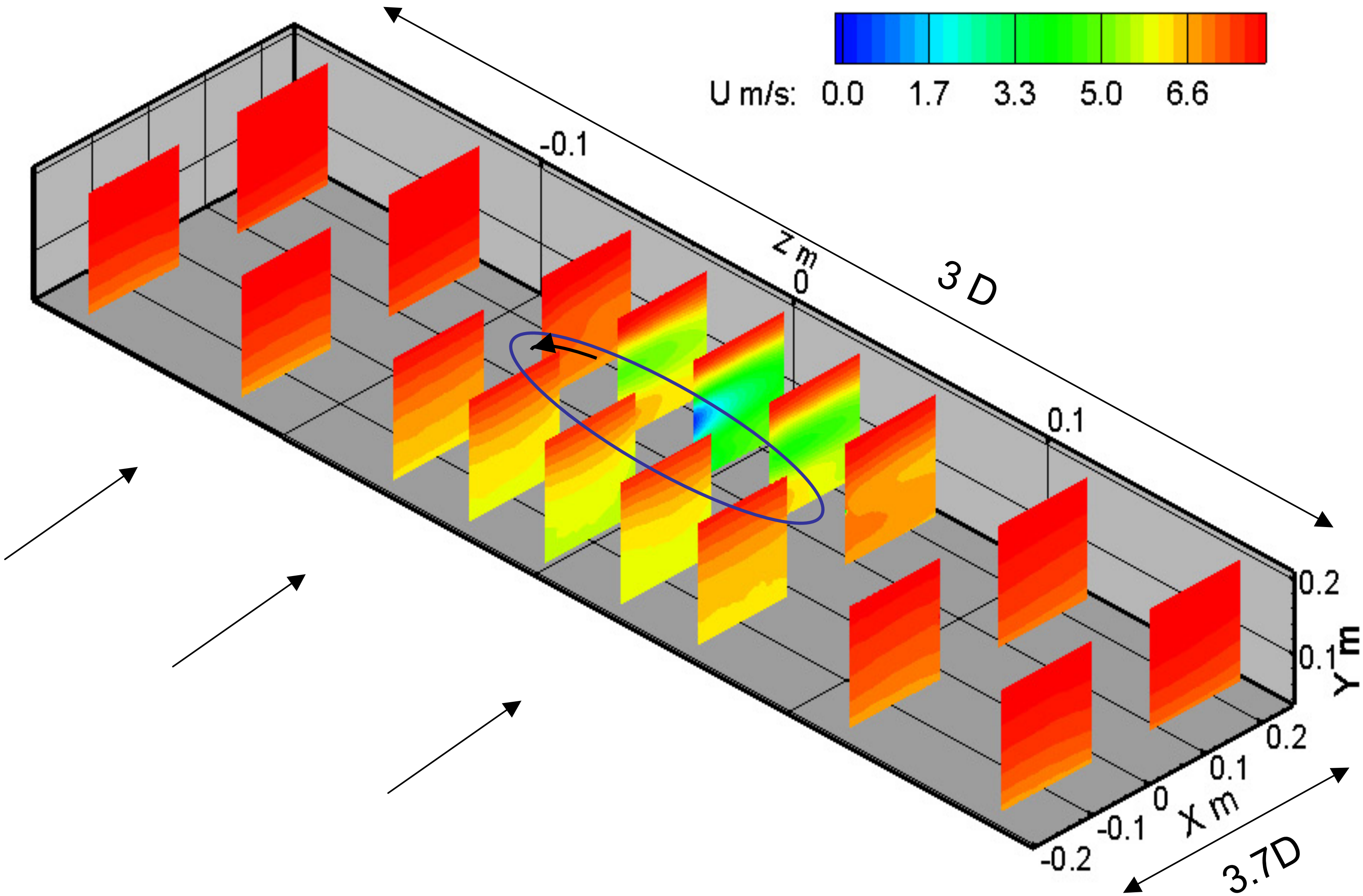


Statistics:

2000 vector maps for each front plane
12000 samples each back plane (6 phase-locked cases)

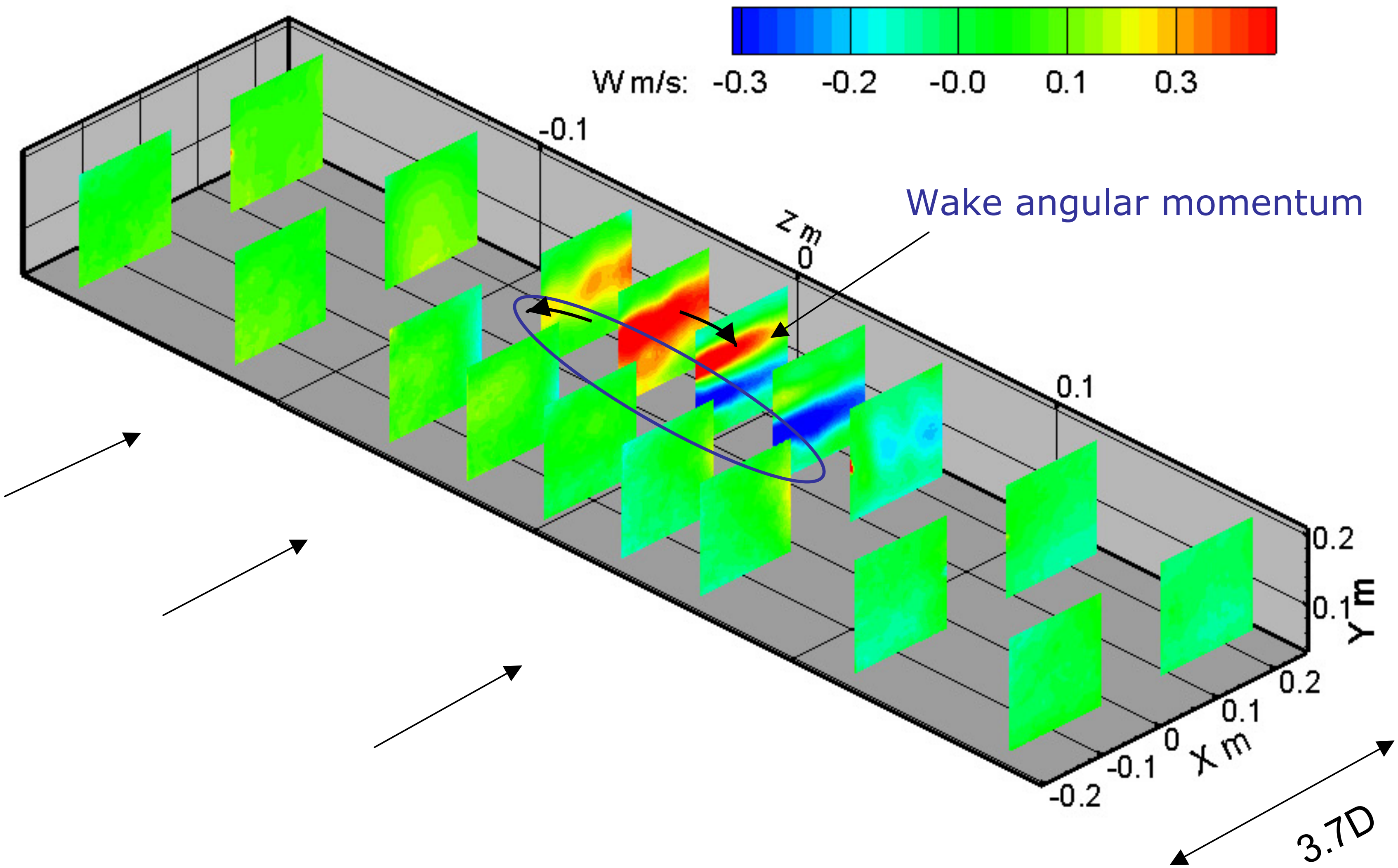
Velocity maps:

Mean streamwise velocity



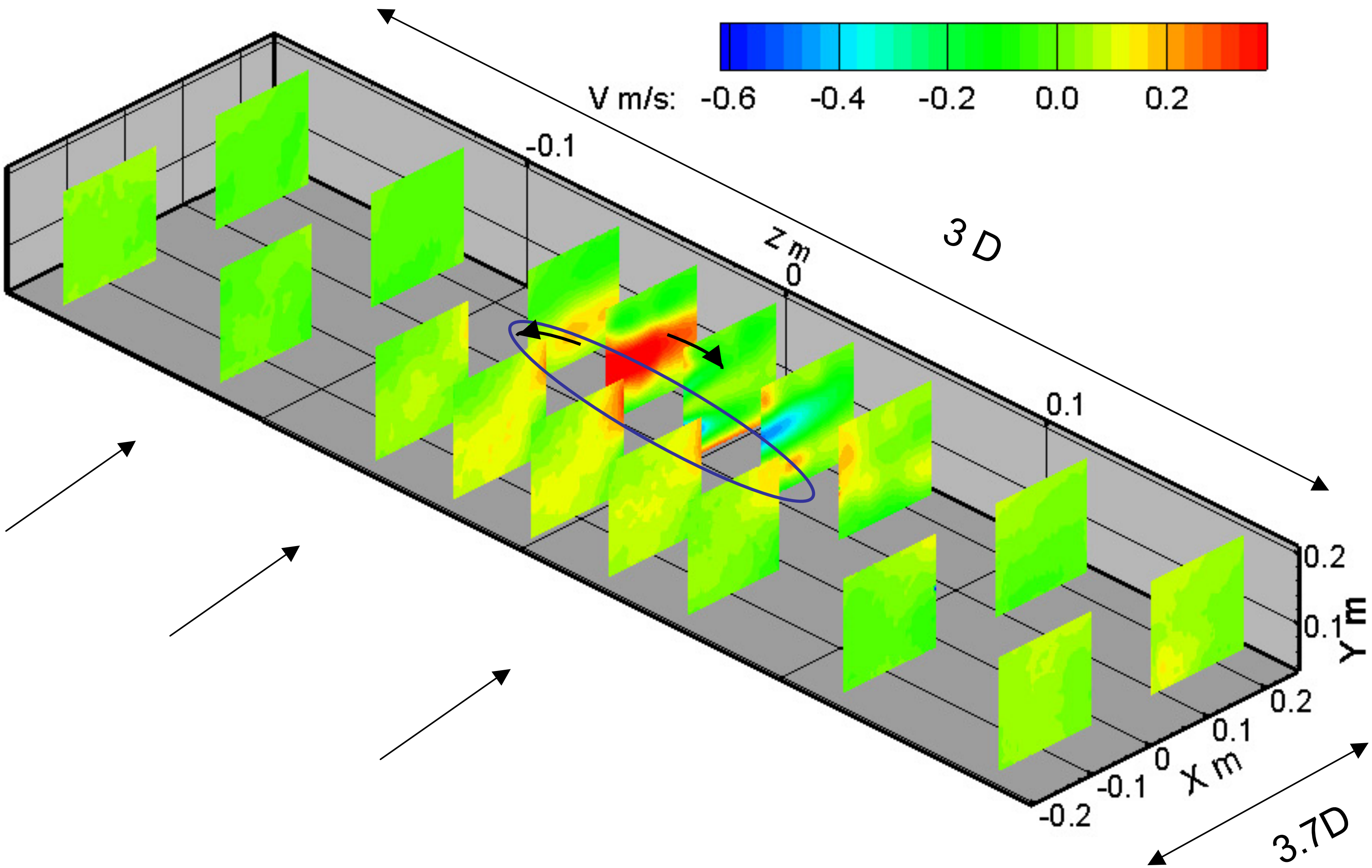
Velocity maps:

Mean transverse velocity



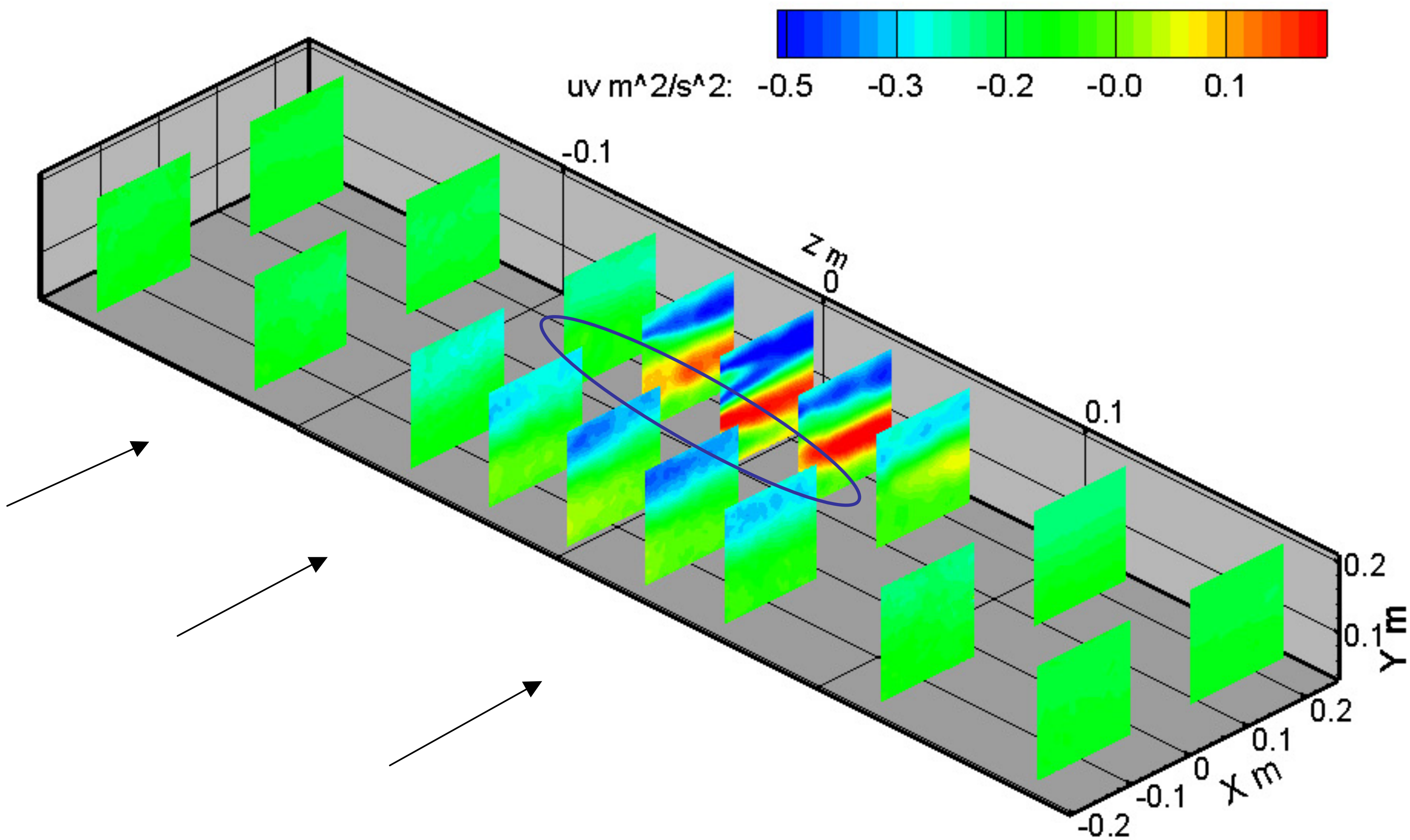
Velocity maps:

Mean vertical velocity



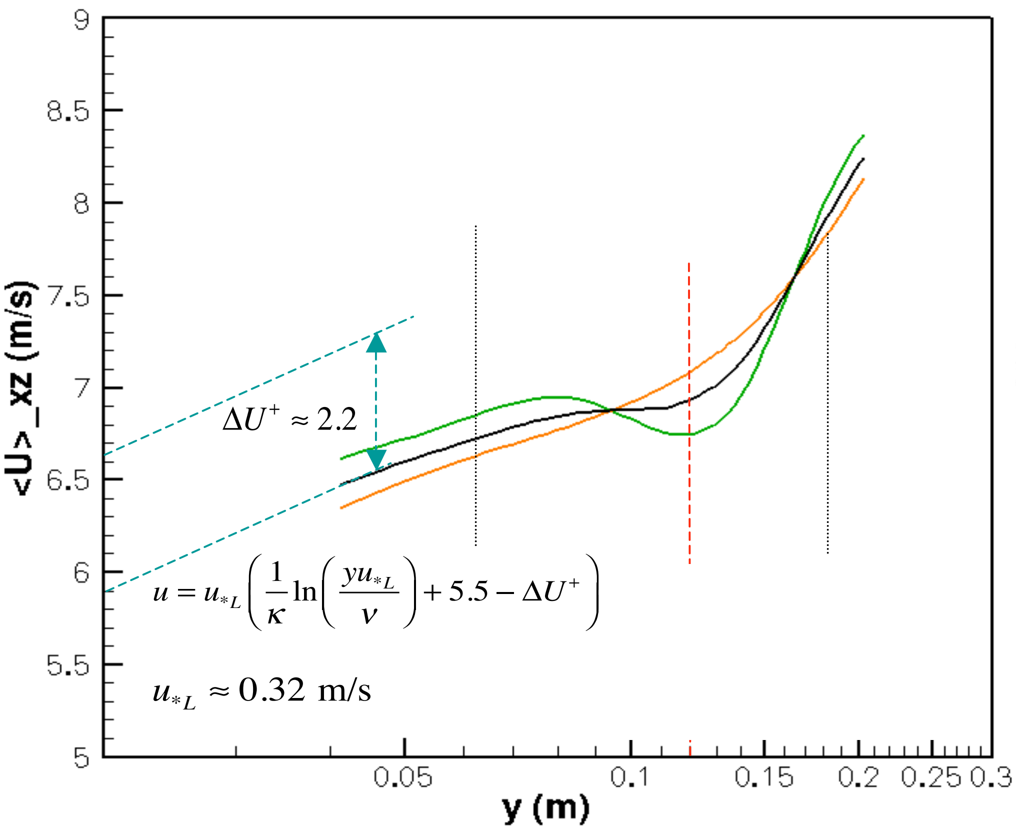
Velocity maps:

(negative) Reynolds shear stress

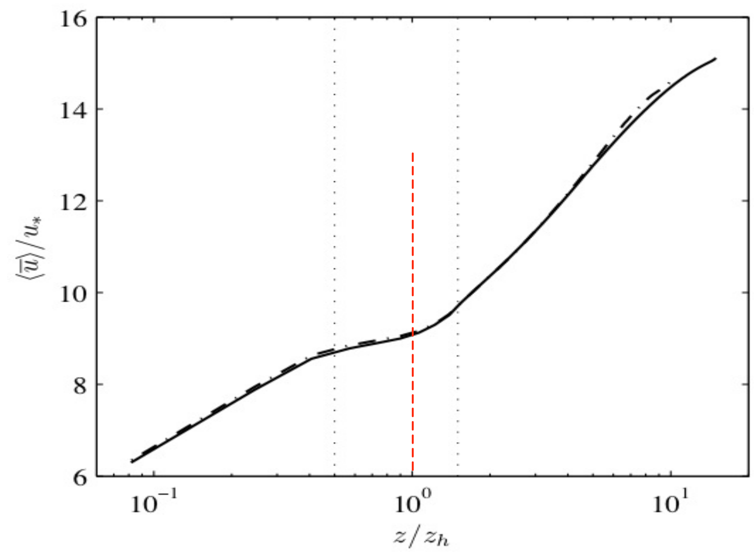


Horizontally (canopy) averaged profiles:

experiment



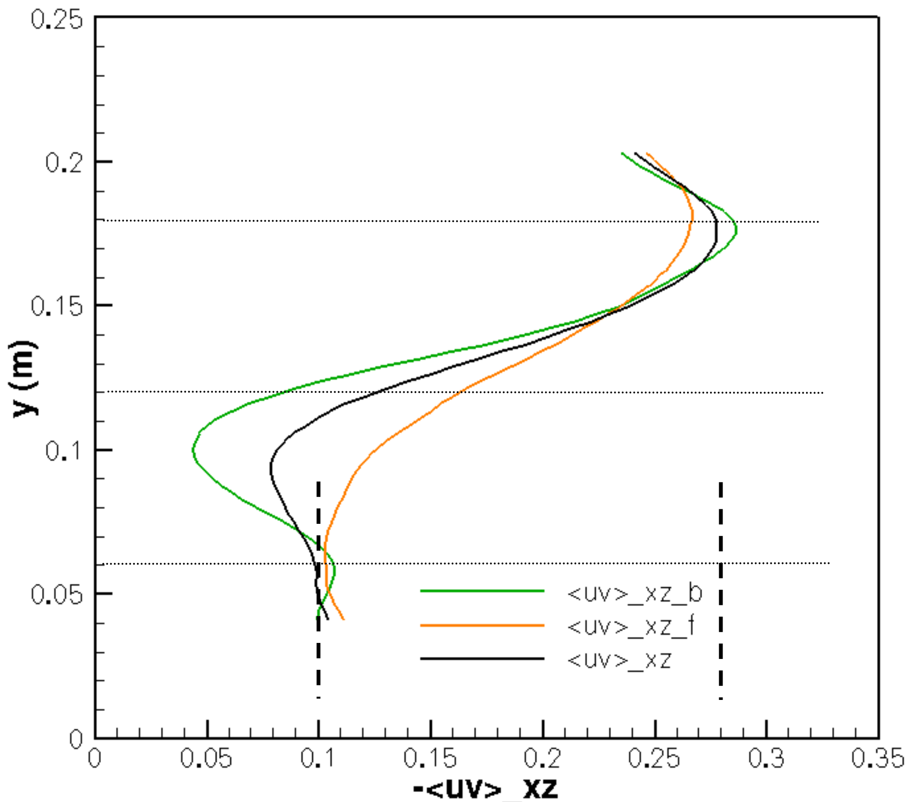
LES



Horizontally averaged Reynolds shear stress profiles:

Wind tunnel measurements

Cal et al.: J. Renewable
And Sustainable Energy **2**, 2010

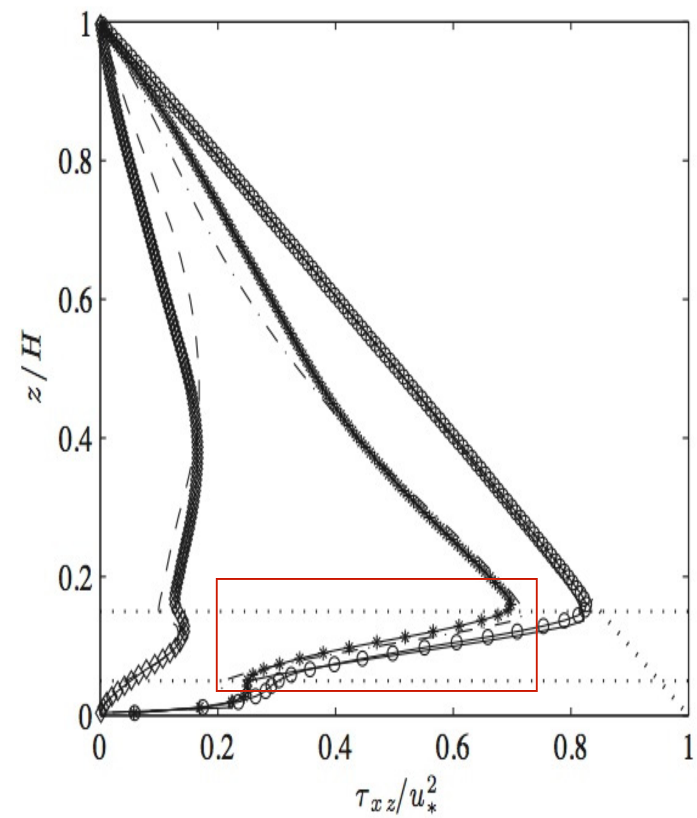


$u_{*L} \approx \sqrt{0.1} \approx 0.32$ m/s

$u_{*H} \approx \sqrt{0.28} \approx 0.53$ m/s

LES

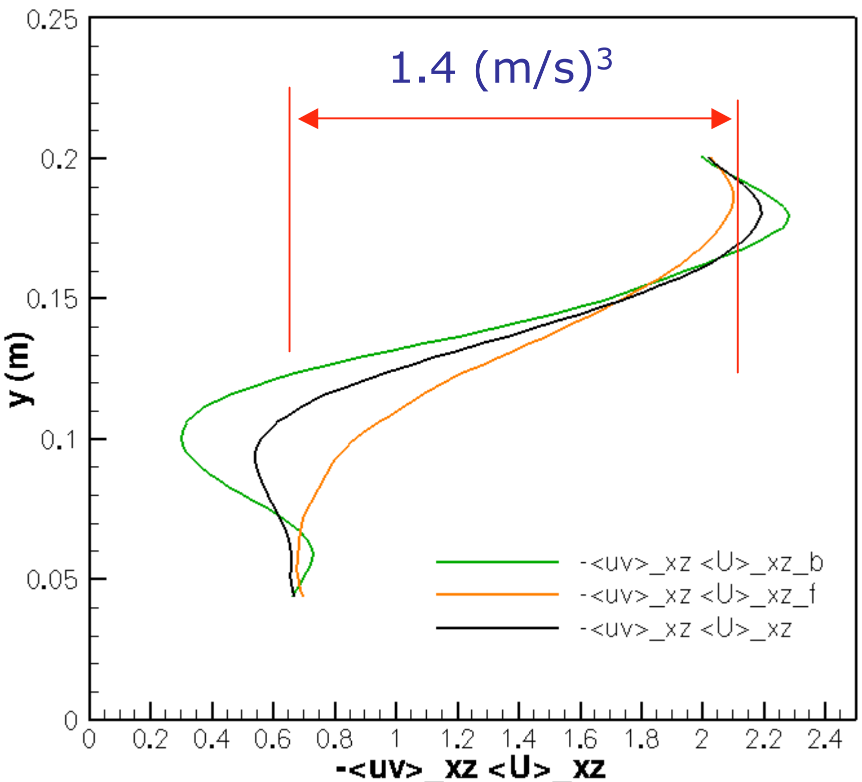
Calaf et al.: Phys. Fluids **22**, 2010



Horizontally averaged profiles - kinetic energy terms:

$$\frac{d\frac{1}{2}\langle u\rangle_{xz}^2}{dt} = -\varepsilon_{turb} - \varepsilon_{canop} - \frac{d}{dy}\left(\langle\overline{u'v'}\rangle_{xz}\langle u\rangle_{xz} + \langle\overline{u}''\overline{v}''\rangle_{xz}\langle u\rangle_{xz}\right) - \langle u\rangle_{xz}\frac{1}{\rho}\frac{dp_\infty}{dx} - P_T(y)$$

found to be negligible here



$$\langle\overline{u'v'}\rangle_{xz}\langle u\rangle_{xz}(y_{top}) - \langle\overline{u'v'}\rangle_{xz}\langle u\rangle_{xz}(y_{bottom}) \approx 1.4 \quad \frac{W/m^2}{(kg/m^3)}$$

$$\Downarrow$$

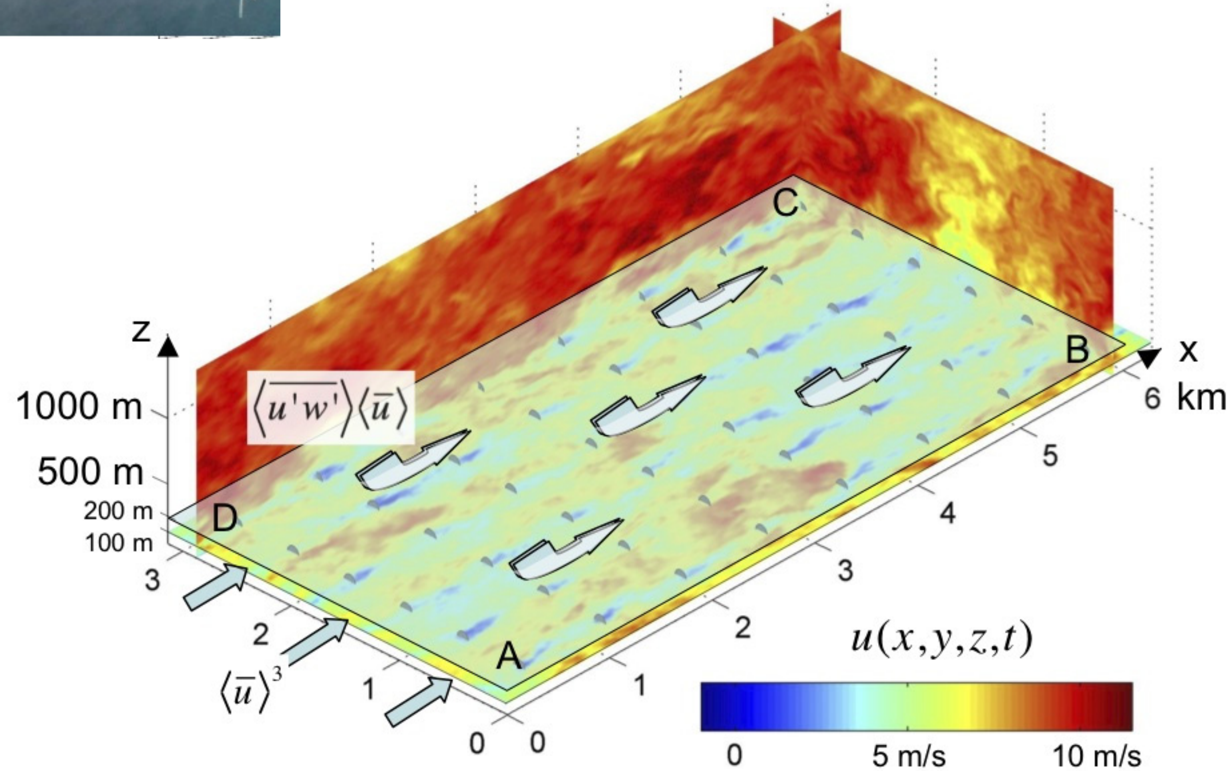
$$P^{turb-flux} = 1.4\rho A$$

$$P^{turb-flux} = 1.4 \times 1.2 \times (3 \times 0.12)(7 \times 0.12)$$

$$P^{turb-flux} = 0.51\ W$$

Analysis consistent with view that kinetic energy extracted by turbine (0.34W) is delivered vertically by turbulence fluxes (0.51W) (rest goes into dissipation, etc…)

to scale:

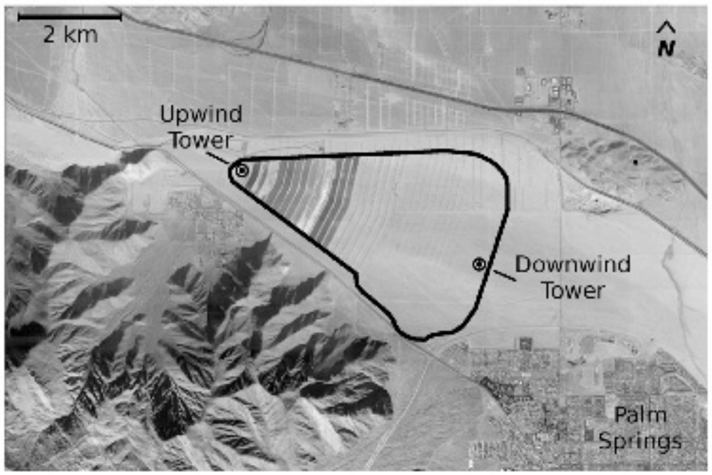


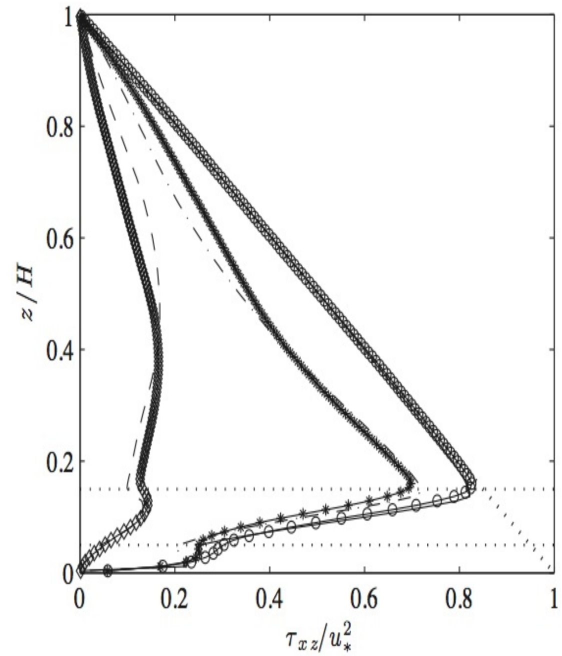
z
1000 m
500 m
200 m
100 m
$\langle \overline{u'w'} \rangle \langle \bar{u} \rangle$
C
B
D
A
x
6 km
5
4
3
2
1
0
3
2
1
0
$\langle \bar{u} \rangle^3$
$u(x,y,z,t)$
0
5 m/s
10 m/s

Effects of large wind farms on scalar fluxes: Heat and moisture

Observations: increased fluxes

(evaporation, drying, ??)
Baidya-Roy & Traiteur PNAS 2010
in San Gorgonio wind farm (CA)





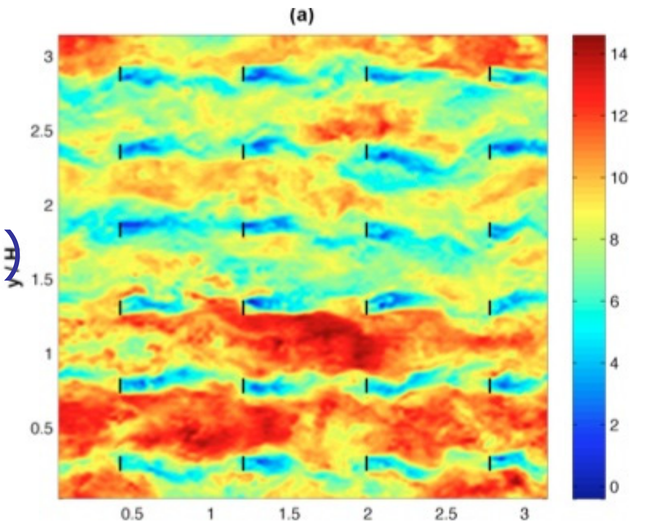
But: Farm increases turbulence in wakes and $u_{*,hi}$ is increased, but $u_{*,lo}$ is DECREASED. Net effect?

First step: Passive scalar LES

(no Boussinesq term in momentum equations)

(M. Calaf, Parlange & M, in preparation)

Velocity (hub-height)



Temperature

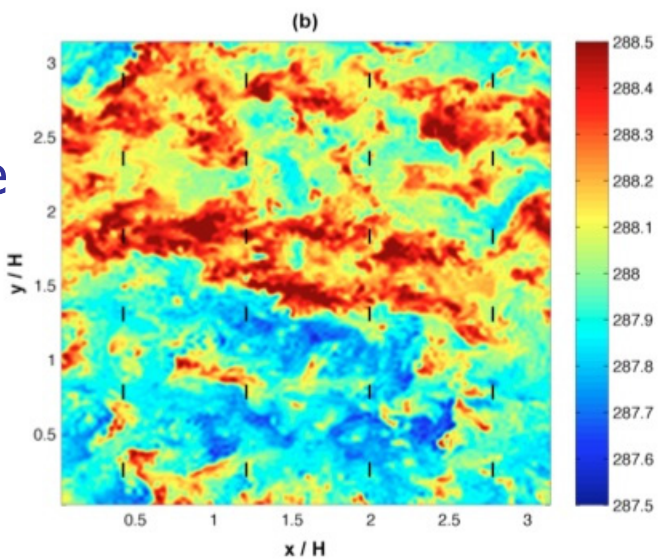
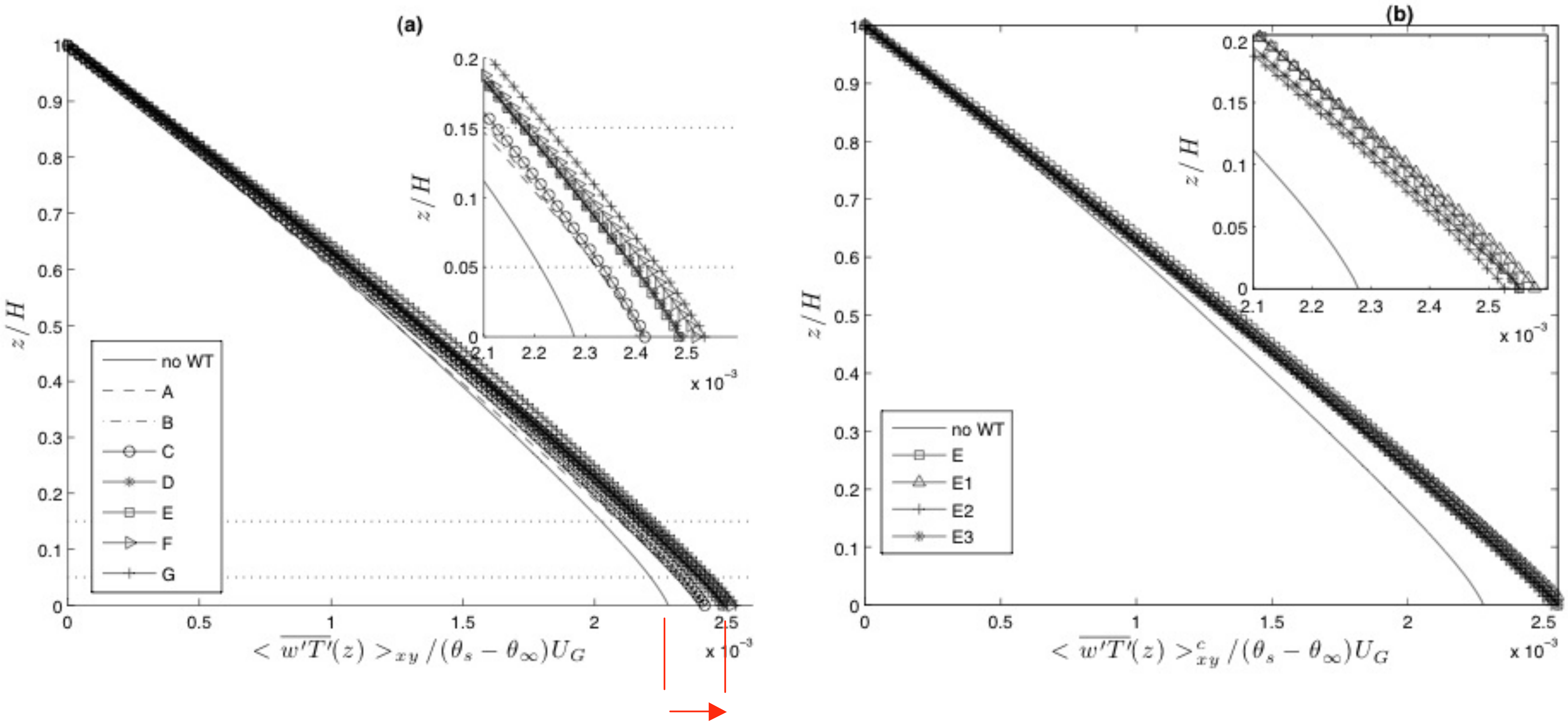


TABLE I: Table summarizing parameters of the various LES cases.

	s_x	s_y	$4s_xs_y/\pi$	N_t	C_T	C'_T	c_{ft}	c'_{ft}
A	7.85	$s_x/1.5$	52.3	4×6	0.45	0.6	0.009	0.011
B	7.85	$s_x/1.5$	52.3	4×6	0.52	0.7	0.01	0.013
C	7.85	$s_x/1.5$	52.3	4×6	0.6	0.88	0.012	0.017
D	7.85	$s_x/1.5$	52.3	4×6	0.68	1.13	0.013	0.022
E	7.85	$s_x/1.5$	52.3	4×6	0.75	1.33	0.014	0.025
F	7.85	$s_x/1.5$	52.3	4×6	0.82	1.63	0.016	0.031
G	7.85	$s_x/1.5$	52.3	4×6	0.88	2	0.017	0.038
$E1$	7.85/2	7.85/1.5	26.15	8×6	0.75	1.33	0.029	0.051
$E2$	7.85	$s_x/3$	26.15	4×12	0.75	1.33	0.029	0.051
$E3$	7.85/2	$s_x/1.5$	13.1	8×12	0.75	1.33	0.057	0.1

Horizontally averaged scalar flux from LES



10-15% increase, not strongly dependent on loading

Horizontally averaged scalar balance: constant flux

$$q_H^{WT} = \begin{cases} \frac{u_{*,lo}\kappa z}{Pr_T^{WT}} \frac{d[\theta_s - \tilde{\theta(z)}]}{dz} & (z_{0,s} < \text{z} < z_h\text{-D}/2) \\ \frac{(u_{*,lo}\kappa z + \sqrt{c_{ft}/2}\langle\tilde{u}(z_h)\rangle D)}{Pr_T^{WT}} \frac{d[\theta_s - \tilde{\theta(z)}]}{dz} & (z_h - D/2 < \text{z} < z_h) \\ \frac{(u_{*,hi}\kappa z + \sqrt{c_{ft}/2}\langle\tilde{u}(z_h)\rangle D)}{Pr_T^{WT}} \frac{d[\theta_s - \tilde{\theta(z)}]}{dz} & (z_h < \text{z} < z_h + D/2) \\ \frac{u_{*,hi}\kappa z}{Pr_T^{WT}} \frac{d[\theta_s - \tilde{\theta(z)}]}{dz} & (z_h + D/2 < \text{z} < H) \end{cases}$$

Horizontally averaged scalar balance: constant flux

For imposed geostrophic wind,
ratio of scalar flux with and without wind farm

$$\frac{q_H^{WT}}{q_H^0} = \frac{u_{*,hi}}{u_*}\frac{Pr_T^0}{Pr_T^{WT}}\left\{\frac{ln\left(\frac{u_{*,hi}}{fz_{0,s}}\right) - kC + \frac{u_{*,hi}}{u_{*,lo}}ln\left[\frac{z_h}{z_{0,s}}\left(1-\frac{D}{2z_h}\right)^{\beta}\right] - ln\left[\frac{z_h}{z_{0,s}}\left(1+\frac{D}{2z_h}\right)^{\beta}\right]}{ln\left(\frac{u_*}{fz_{0,s}}\right) - kC}\right\}$$

Term 1: increase due to increased turbulence in wake

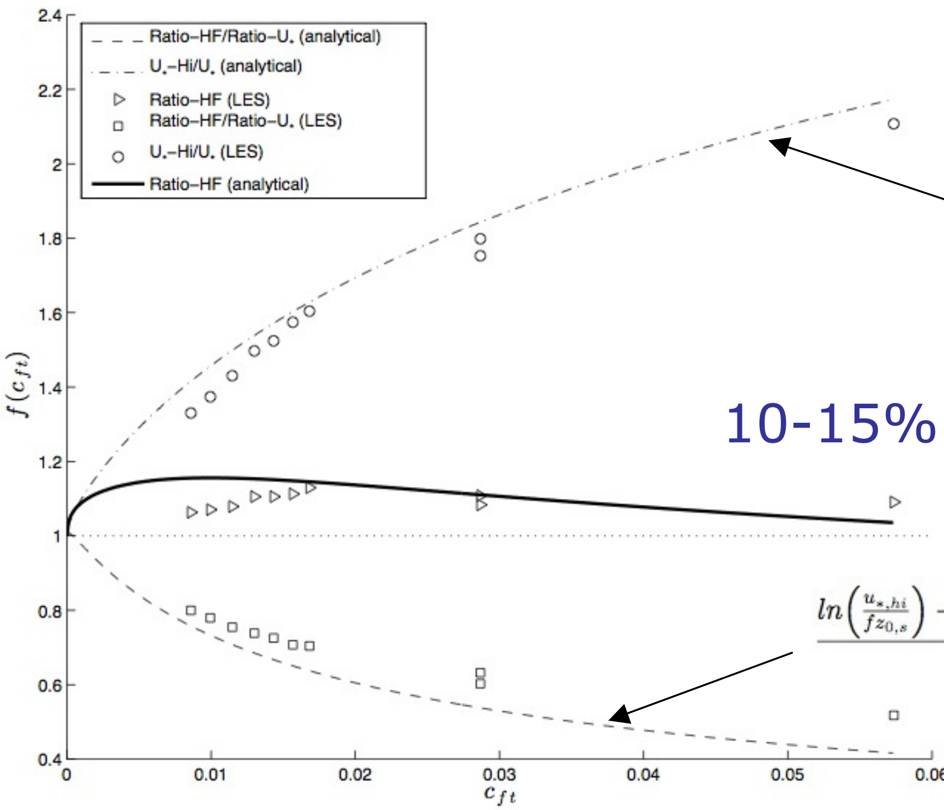
Term 2: decrease due to "dead water region" below WT

$$\frac{u_{*,hi}}{u_*} = \left[1 - \frac{\ln\left(\frac{z_{0,hi}}{z_{0,lo}}\right)}{\ln\left(\frac{U_G}{fz_{0,lo}}\right) - C_*}\right]^{-1}$$

LES measured and model terms as function of loading

(neutral stratification)

For imposed geostrophic wind,
ratio of scalar flux with and without wind farm (symbols=LES)



$$\frac{u_{*,hi}}{u_*} = \left[1 - \frac{\ln\left(\frac{z_{0,hi}}{z_{0,lo}}\right)}{\ln\left(\frac{U_G}{f z_{0,lo}}\right) - C_*} \right]^{-1}$$

10-15% increase now “understood”

$$\frac{ln\left(\frac{u_{*,hi}}{f z_{0,s}}\right) - kC + \frac{u_{*,hi}}{u_{*,lo}} ln\left[\frac{z_h}{z_{0,s}}\left(1 - \frac{D}{2z_h}\right)^{\beta}\right] - ln\left[\frac{z_h}{z_{0,s}}\left(1 + \frac{D}{2z_h}\right)^{\beta}\right]}{ln\left(\frac{u_*}{f z_{0,s}}\right) - kC}$$

Questions ?