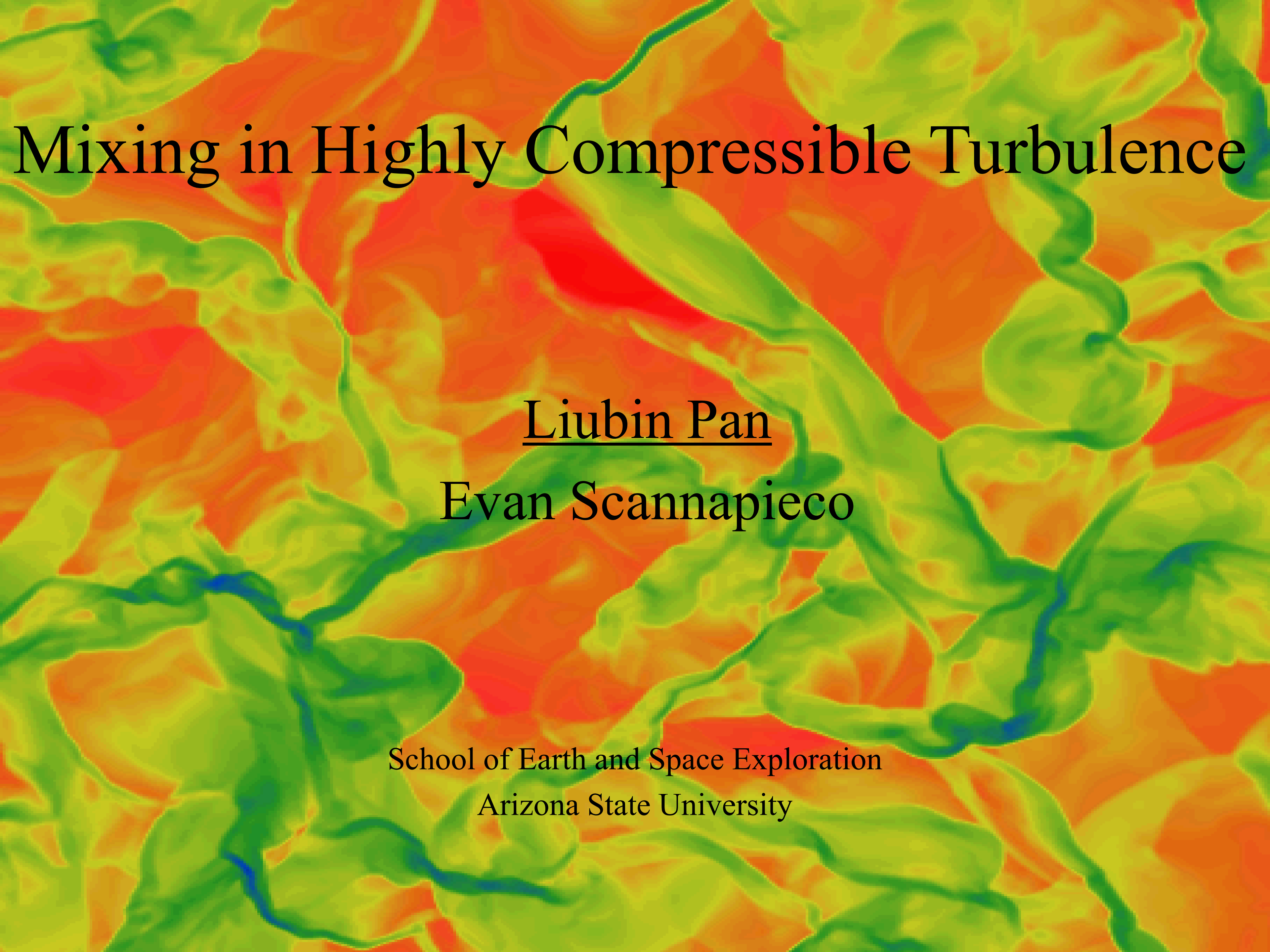

Mixing in Highly Compressible Turbulence

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Astrophysical motivation:

Heavy elements from supernova and stellar winds mix in the interstellar medium. The mixing efficiency controls the degree of chemical inhomogeneity, which can be observed.

The interstellar medium is turbulent and highly compressible. The rms Mach number in interstellar clouds can be as high as ~ 10.

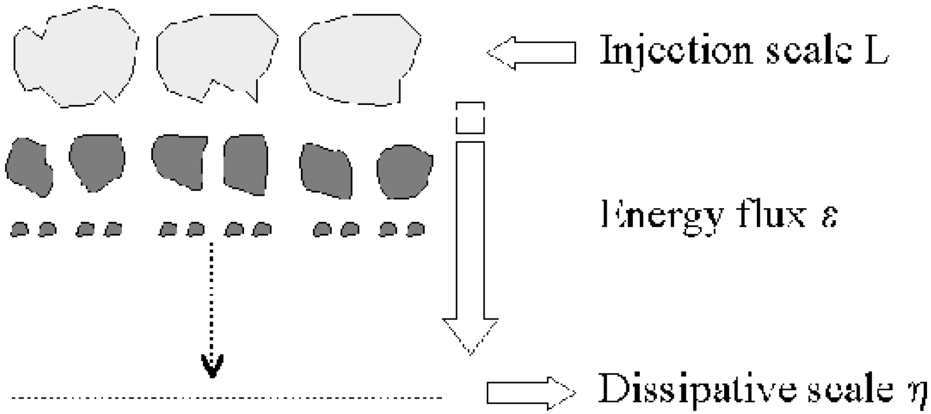
Questions:

1. What is the mixing timescale in supersonic turbulence?
   How do compressible modes affect the mixing efficiency?

2. What do passive scalar structures look like in supersonic turbulence?

3. Is there an inverse scalar cascade in highly compressible turbulent flows?

Cascade Picture for Energy Dissipation and Turbulent Mixing



Velocity

Scalar

In incompressible turbulence:

Kolmogorov's similarity hypothesis:

$$\delta\upsilon(l) \sim \bar{\varepsilon}^{1/3} l^{1/3} \text{ and } E(k) \sim \bar{\varepsilon}^{2/3} k^{-5/3}$$

Energy dissipation timescale $\tau_v \sim L / U$.

Classic mixing theory (Obukohov--Corrsin) :

$$\delta C(l)^2 \sim \bar{\varepsilon}_C \frac{l}{\delta\upsilon(l)}$$

Mixing timescale τ_c also ~ L/U, about $\tau_v/2$.

In supersonic turbulence:

Structure function and spectrum steepens with the Mach number (e.g, Kritsuk et al. 2007)

Energy decays at a timescale $\tau_v \sim L / U$ for all Mach numbers (Stone et al. 1998, Mac Low 1999, Padoan & Norlund 1999, Lemaster and Stone 2009)

Does the scalar structure function and spectrum flatten with Mach number?

What is the mixing timescale as a function of the Mach number?

Hydrodynamic and Concentration Equations

Hydro:

$$\partial_t \rho + \partial_i (\rho \upsilon_i) = 0$$

$$\partial_t \upsilon_i + \upsilon_j \partial_j \upsilon_i = -\frac{\partial_i p}{\rho} + \frac{\partial_j \sigma_{ij}}{\rho} + f_i$$

viscous stress tensor $\sigma_{ij} = \rho \nu (\partial_i \upsilon_j + \partial_j \upsilon_i - \frac{2}{3} \delta_{ij} \partial_k \upsilon_k)$

isothermal equation of state: $p = \rho C_s^2$

Concentration:

$$\partial_t C + \upsilon_i \partial_i C = \frac{1}{\rho} \partial_i (\rho \kappa \partial_i C) + S$$

C is the *ratio* of the tracer density to the flow density

Simulations

We use the FLASH code with modified "Stir unit" for flow and scalar driving.
Simulations are performed in a periodic box of unit size with 512^3 computation cells.

Turbulent flows:

Flows driven and maintained by a solenoidal external force at large scales, generated in Fourier space with wave number in the range $2\pi \leq k \leq 6\pi$.

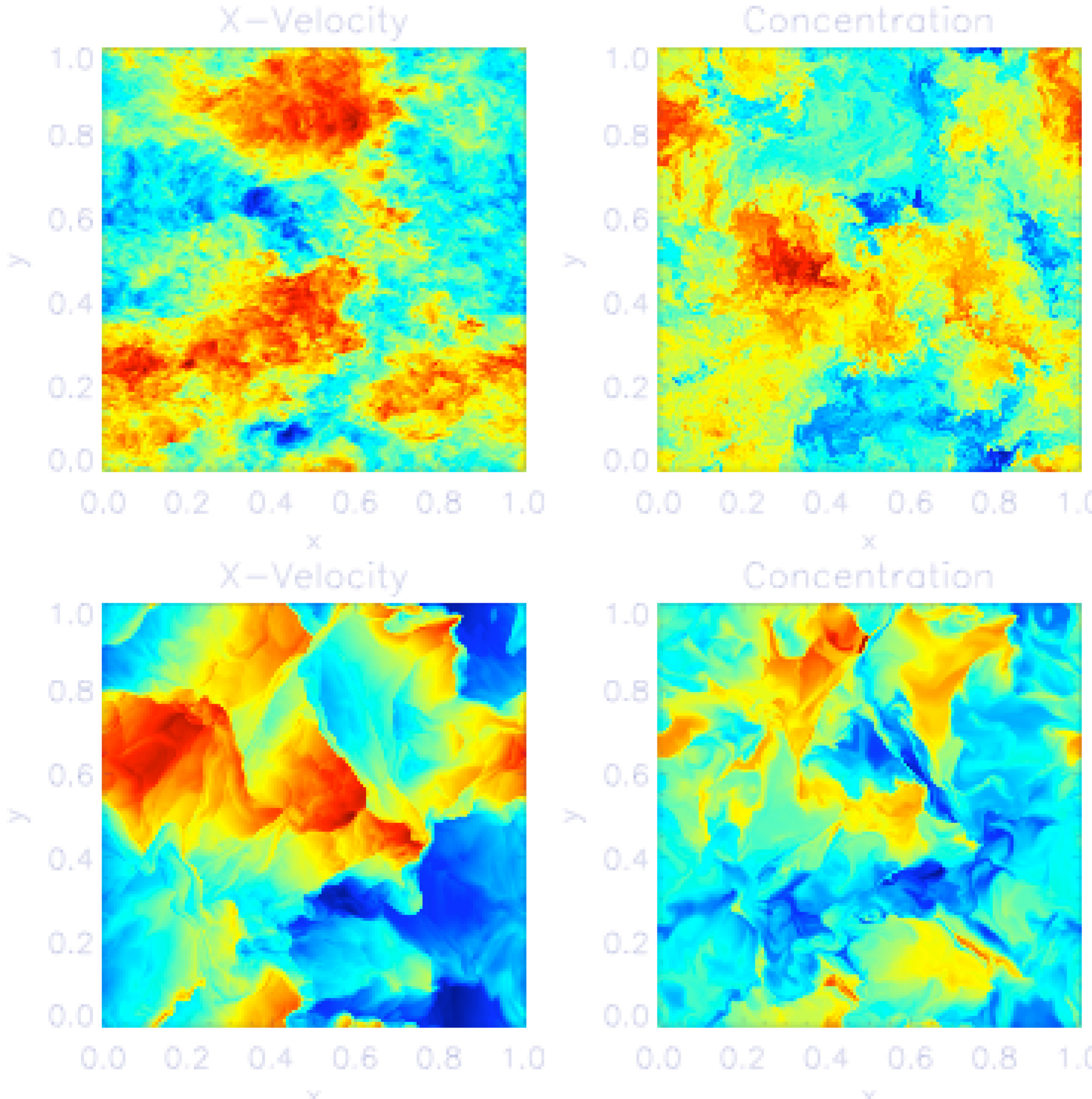
External force set to be Gaussian with exponential temporal correlation; Each mode in the wave number range given the same driving power; Amplitude of the force adjusted to obtain 6 Mach numbers in the range $0.9 \leq M \leq 6.1$.

Passive scalars:

Same driving scheme used for the scalar source term representing new sources of pollutants at large scales. Three independent scalars evolved in each flow to achieve accurate statistical measurements.

M = 0.9

M = 6.1


X−Velocity
Concentration
1.0
0.8
0.6
0.4
0.2
0.0
y
0.0 0.2 0.4 0.6 0.8 1.0
x
X−Velocity
Concentration
y
x

Solenoidal to Compressible Ratio as a Function of M

M	$\langle\bar{\rho}(\nabla\cdot\mathbf{v})^2\rangle/\langle\bar{\rho}(\nabla\times\mathbf{v})^2\rangle$	$\langle(\nabla\cdot\mathbf{v})^2\rangle/\langle(\nabla\times\mathbf{v})^2\rangle$	f_{comp}	$f_{comp}(k/2\pi \geqslant 4)$
0.9	0.06	0.06	0.05	0.07
1.4	0.19	0.20	0.10	0.16
2.1	0.41	0.49	0.15	0.25
3.0	0.52	0.75	0.17	0.30
4.6	0.55	0.88	0.18	0.33
6.1	0.54	0.86	0.19	0.32

Pan & Scannapieco (2010)

1. The divergence to vorticity ratio increases with M for M < 3, and appears to saturate at M>3. This indicates that compressible modes contributed more to energy dissipation with increasing M for M<3.

2. The fraction of kinetic energy contained in compressible modes first increases with M and then saturates at 1/3 for M>3, corresponding to energy equipartition between the two modes.

Mixing Timescale and Energy Dissipation Timescale

Concentration variance: $$\partial_t\langle\tilde{\rho}C^2\rangle+\partial_i\langle\tilde{\rho}C^2\upsilon_i\rangle=-2\langle\tilde{\rho}\kappa(\partial_i C)^2\rangle+2\langle\tilde{\rho}SC\rangle$$

Kinetic energy:

$$\partial_t\langle\frac{1}{2}\tilde{\rho}\upsilon^2\rangle+\partial_i\langle\frac{1}{2}\tilde{\rho}\upsilon^2\upsilon_i\rangle=\frac{\langle p\partial_i\upsilon_i\rangle}{\bar{\rho}}-\frac{1}{2}\langle\tilde{\rho}\nu(\partial_i\upsilon_j+\partial_j\upsilon_i-\frac{2}{3}\delta_{ij}\partial_k\upsilon_k)^2\rangle+\langle\tilde{\rho}f_i\upsilon_i\rangle$$

Mixing timescale:

$$\tau_m=\langle\tilde{\rho}C^2\rangle/2\langle\tilde{\rho}\kappa(\partial_i C)^2\rangle$$

$$=\langle\tilde{\rho}C^2\rangle/2\langle\tilde{\rho}SC\rangle \qquad \text{at steady state}$$

Timescale for kinetic energy loss

$$\tau_\upsilon=\langle\frac{1}{2}\tilde{\rho}\upsilon^2\rangle/\left[-\frac{\langle p\partial_i\upsilon_i\rangle}{\bar{\rho}}+\frac{1}{2}\langle\tilde{\rho}\nu(\partial_i\upsilon_j+\partial_j\upsilon_i-\frac{2}{3}\delta_{ij}\partial_k\upsilon_k)^2\rangle\right]$$

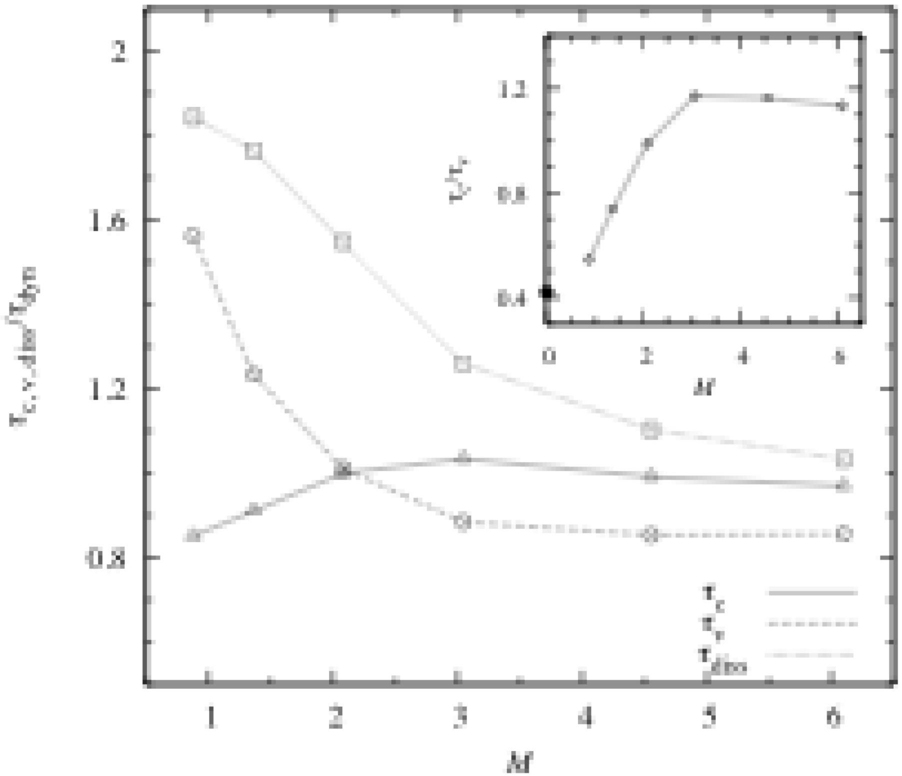
$$=\langle\frac{1}{2}\tilde{\rho}\upsilon^2\rangle/\langle\tilde{\rho}f_i\upsilon_i\rangle \qquad \text{at steady state}$$

Timescale by viscous dissipation only

$$\tau_{diss}=\langle\frac{1}{2}\tilde{\rho}\upsilon^2\rangle/\left[\frac{1}{2}\langle\tilde{\rho}\nu(\partial_i\upsilon_j+\partial_j\upsilon_i-\frac{2}{3}\delta_{ij}\partial_k\upsilon_k)^2\rangle\right]$$

$$=\langle\frac{1}{2}\tilde{\rho}\upsilon^2\rangle/\left[\frac{\langle p\partial_i\upsilon_i\rangle}{\bar{\rho}}+\langle\tilde{\rho}f_i\upsilon_i\rangle\right] \qquad \text{at steady state}$$

Mixing and Energy Dissipation Timescales



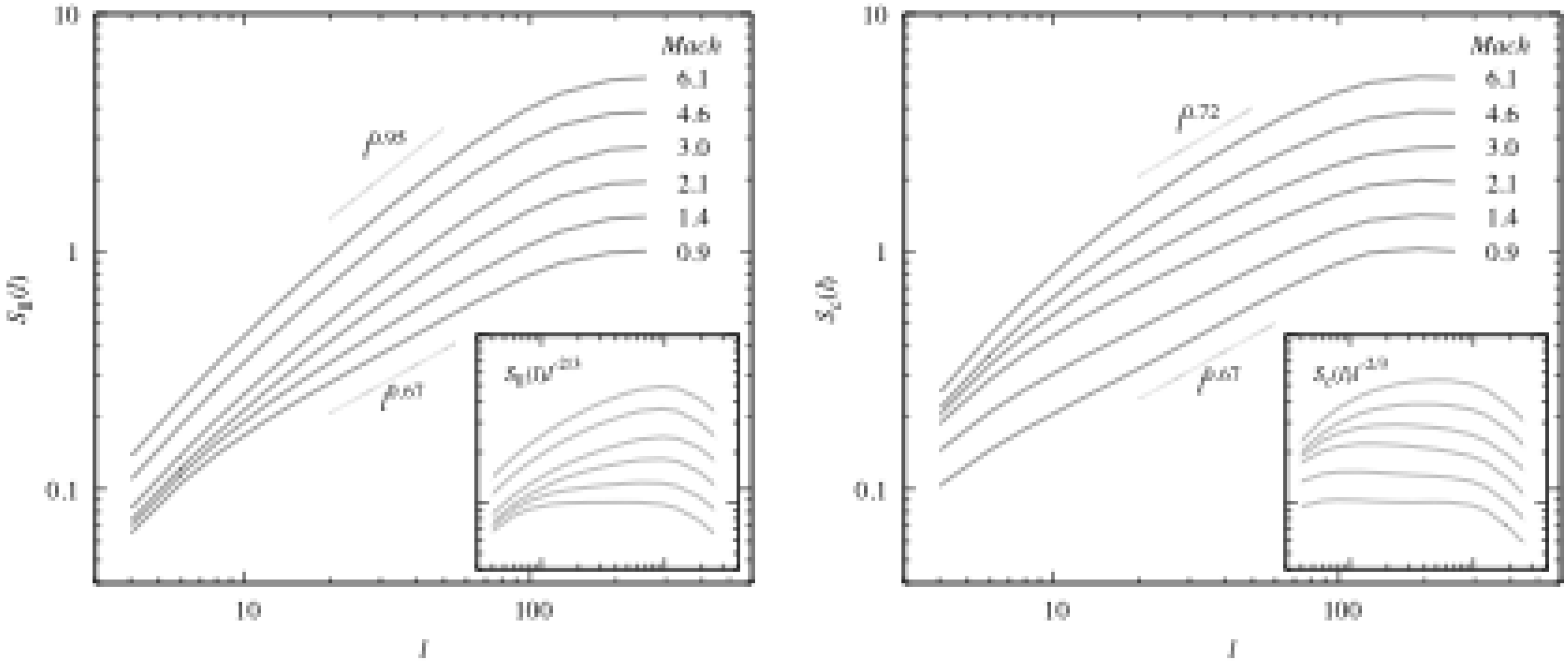
Pan & Scannapieco (2010)

- Energy dissipation rate increases with M ---compressible modes, essentially shocks, provide an additional faster channel for energy dissipation.
- Mixing rate decreases with M --- compressible modes are less efficient at producing scalar structures at small scales than solenoidal modes, and hence less efficient at enhancing mixing.

Velocity shocks squeeze the scalar field and amplify the scalar gradients. However, the amplification factor is finite and no scalar shocks are produced.

The ratio of energy dissipation timescale to the mixing timescale increases to 1.2 at M=3 and then saturates.

2nd Order Velocity and Scalar Structure Functions



1. The velocity structure function steepens from the Kolmogorov slope of 2/3 at M = 0.9 continuously to 0.95 at M = 6.1, primarily due to increasing frequency and intensity of shocks with M.

2. The slope of the scalar structure function first decreases from 2/3 at M=0.9 to 0.6 at M=2.1, consistent with the prediction of the classic theory of Obukohov and Corrsin . However, at M >3, the slope starts to increase, due to the effect of strong compressible modes on scalar structures at large Mach numbers.

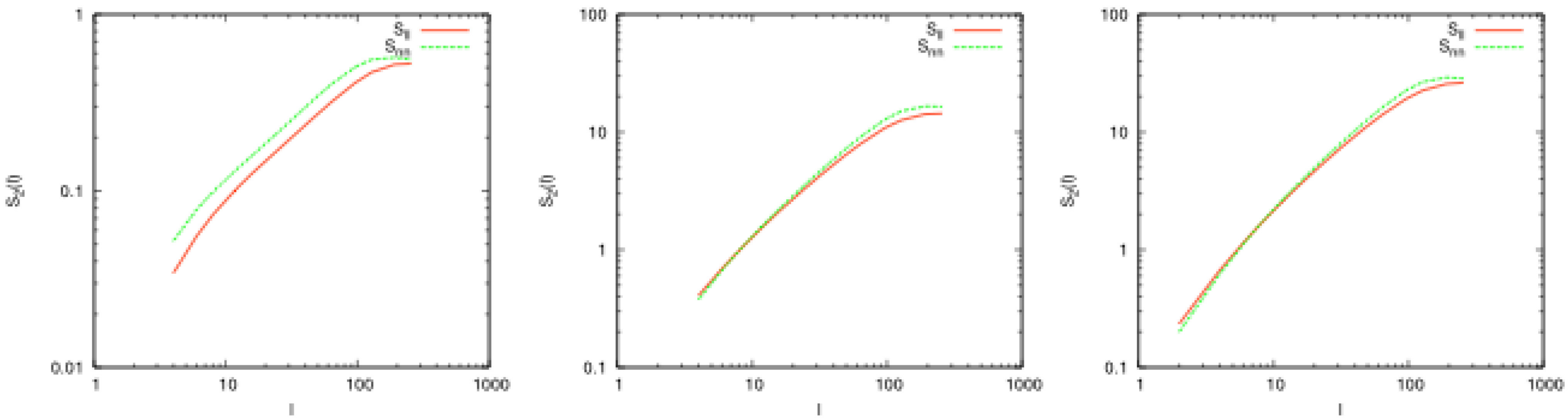
Inverse Scalar Cascade?

Theoretical models predicted an inverse scalar cascade in highly compressible flows Chertkov et al. (1997, 1998) , Gawedzki and Vergassola (2000), Falkovich, Gawedzki and Vergassola (2001). There exists a critical compressibility above which the scalar energy cascades to large scales.

Compressibility: $$P = (\xi + (C-1)(d-1+\xi))/(\xi(d+C-1))$$

C: ratio of longitudinal to transverse structure functions

M=0.9 M=4.5 M=6.1

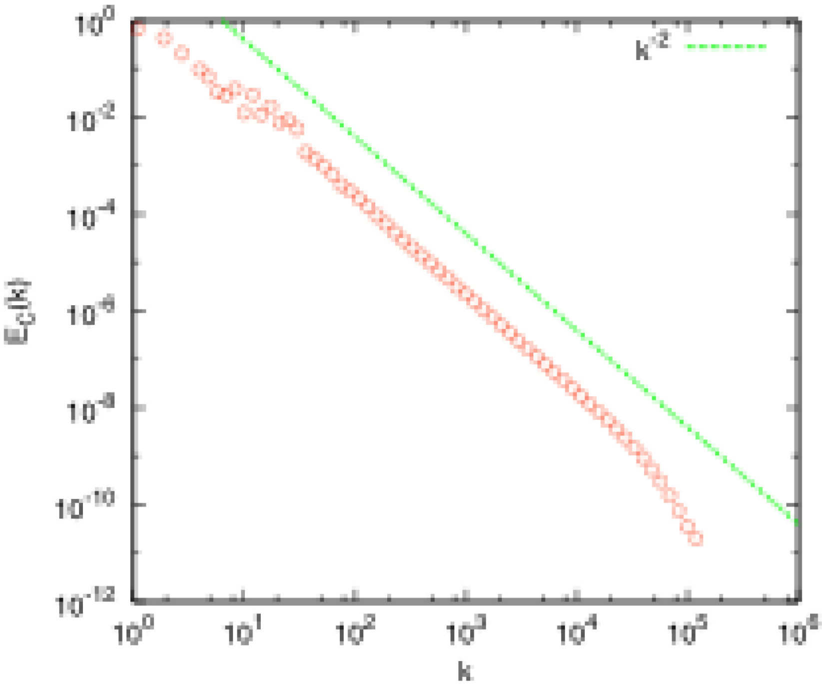


In our 3D flows, due to the equipartition between solenoidal modes and potential modes, the compressibility does not exceed 1/3, in the weak compressible limit.

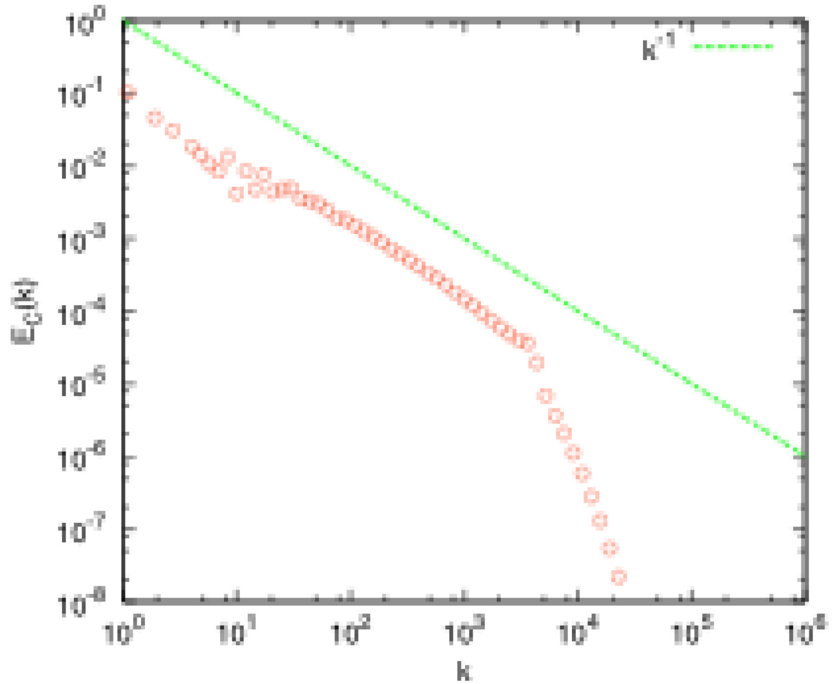
In 2D supersonic flows, the compressibility is smaller than ½ at all Mach numbers, and no inverse cascade was found either.

Inverse Scalar Cascade in 1D flows

1D flows are fully compressible, and an inverse scalar cascade is indeed found.



Energy spectrum

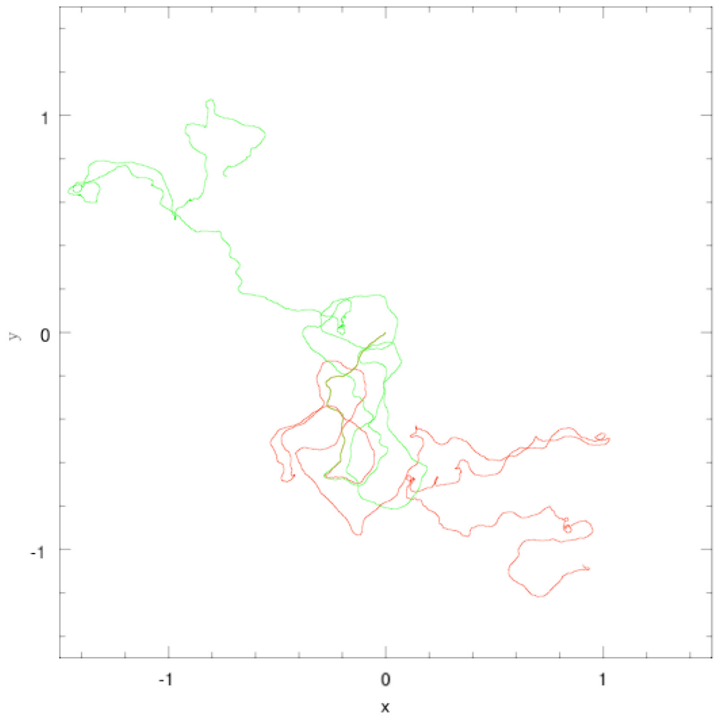


Scalar spectrum

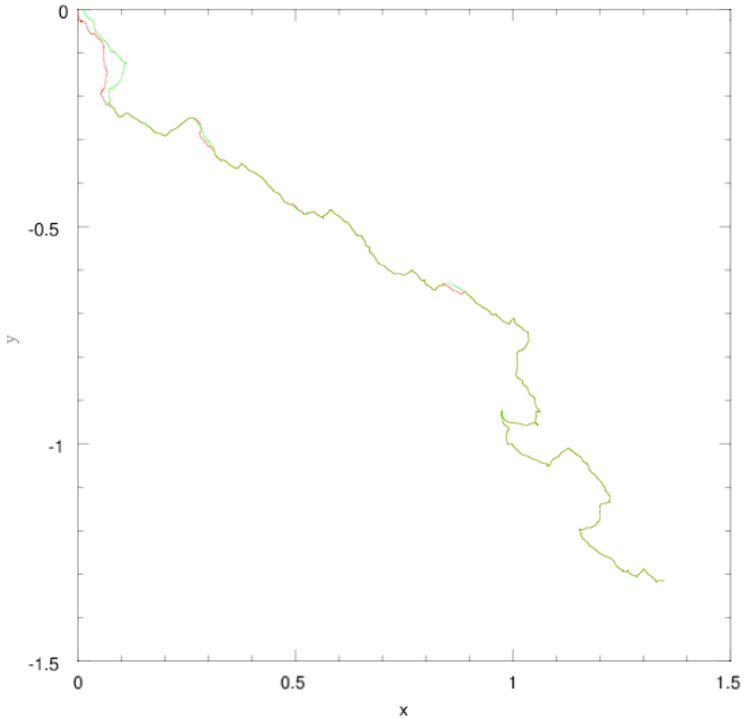
We also found inverse scalar cascade 1D decaying Burgers turbulence.

Particle pair dispersion in a 2D synthetic flow

Inverse scalar cascade corresponds to sticking behavior of particle pairs.



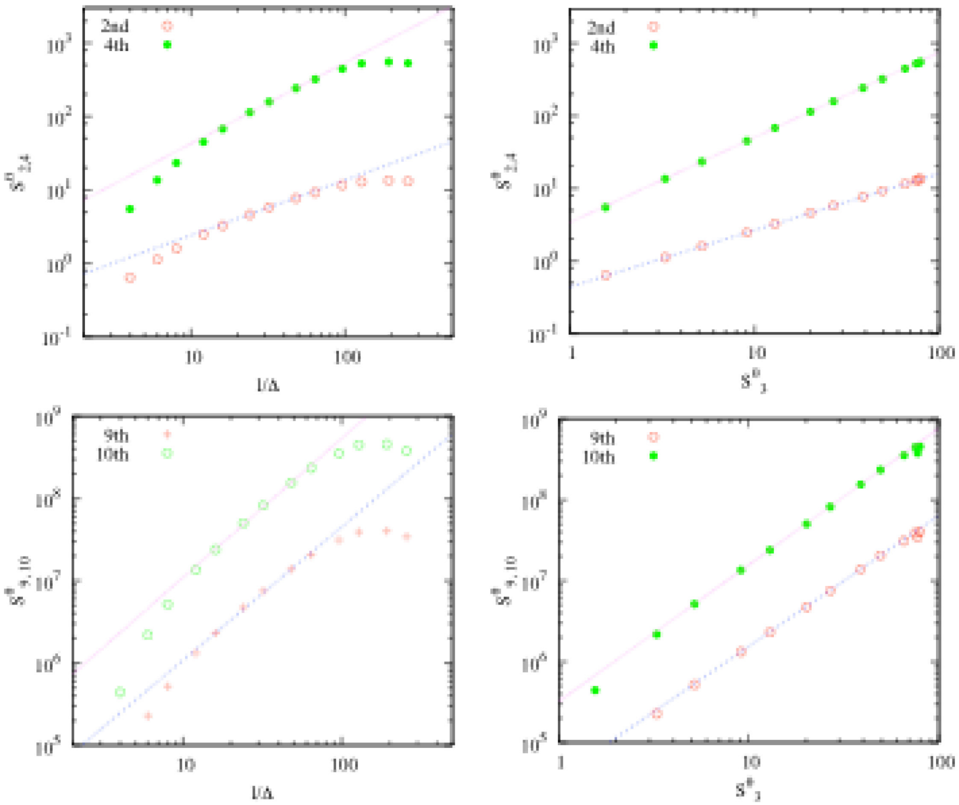
Incompressible



Irrotational

Inverse scalar cascade can happen at high enough compressibility in synthetic flows.

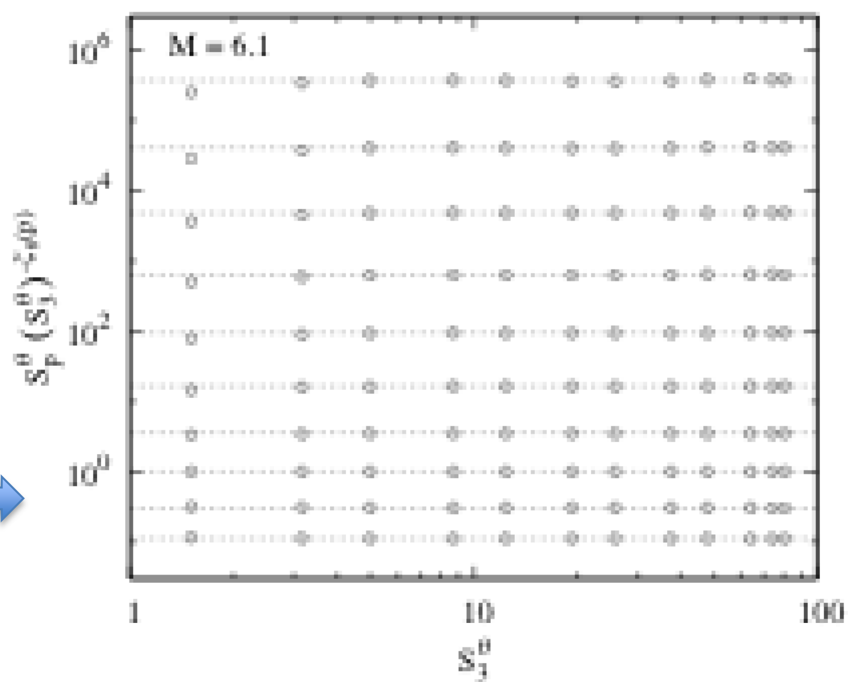
High Order Scalar Structure Functions



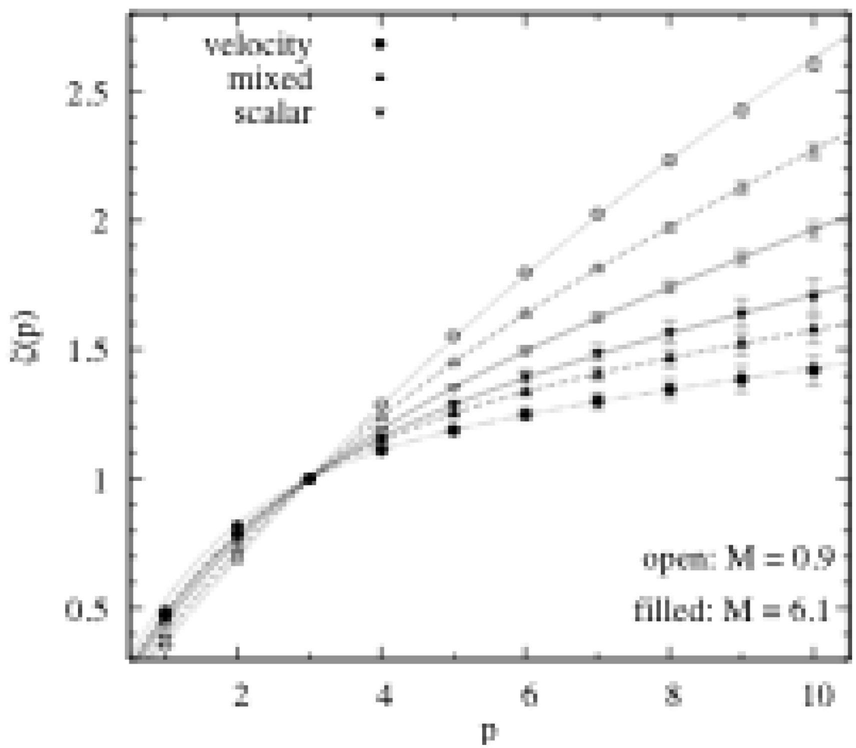
Extended Self-similarity gives broad power-law scale ranges

Compensated structure functions

Pan & Scannapieco (2011)



High Order Velocity and Scalar Structure Functions



Velocity: $\left\langle |\delta v(r)|^p \right\rangle \propto r^{\zeta_v(p)}$

Mixed: $\left\langle |\delta v(r)\delta C(r)^2|^{p/3} \right\rangle \propto r^{\zeta_m(p)}$

Scalar: $\left\langle |\delta C(r)|^p \right\rangle \propto r^{\zeta_c(p)}$

Pan & Scannapieco (2011)

Velocity structures become much more intermittent with increasing M.
But for scalars, the degree of intermittency changes only slightly.

The intermittency model by She and Leveque (1994) well fit
velocity and scalar structures in supersonic turbulence.

Intermittent Velocity and Scalar Structures

Fit parameters for the She-Leveque model

M	$\xi_v(3)$	β_v	d_v	$\xi_m(3)$	β_m	d_m	$\xi_\theta(3)$	β_θ	d_θ
0.9	0.98	0.88	1.0	0.96	0.75	1.9	0.87	0.64	2.2
1.4	1.07	0.85	1.0	0.96	0.77	1.8	0.82	0.63	2.3
2.1	1.18	0.77	1.2	0.96	0.78	1.5	0.78	0.65	2.2
3.0	1.22	0.63	1.7	1.01	0.72	1.7	0.82	0.62	2.2
4.6	1.22	0.54	1.8	1.03	0.64	1.8	0.91	0.62	2.1
6.1	1.22	0.52	1.8	1.07	0.61	1.8	0.96	0.61	2.0

The strongest velocity structures change from filamentary to sheet-like as the flow changes
from transonic to highly supersonic (Padoan et al. 2004).

The most intense scalar structures are sheet-like at all Mach numbers.

Conclusions

1. The mixing timescale is similar to the timescale for the kinetic energy dissipation at all Mach numbers, suggesting a scalar cascade similar to that of the kinetic energy.

2. The mixing timescale increases with the Mach number for M <3, and is essentially constant at larger M. Compressible modes are less efficient at enhancing mixing, and solenoidal modes are the primary “mixer” at all Mach numbers.

3. The second-order scalar structure function obeys a 2/3 scaling in subsonic flows. As the Mach number increases, the scalar structure function first flattens, and then becomes steeper when M>3.

4. Inverse scalar cascade was found in 1D simulations but not in 2D and 3D. The critical compressibility was not reached in 2D or 3D.

5. The high order scalar structure functions are described by the She-Leveque model. The most intense scalar structures are sheet-like at all Mach numbers.

Energy and Scalar Power Spectra