Homogeneous isotropic turbulence with polymer additives

Prasad Perlekar

Department of Physics
Department of Mathematics and Computer Science
Eindhoven University of Technology
Eindhoven, The Netherlands.

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Work done in collaboration with

Homogeneous isotropic turbulence
Dhrubaditya Mitra (NORDITA, Stockholm)
Rahul Pandit (IISc, Bangalore)
Anupam Gupta (IISc, Bangalore)

Channel flow turbulence
Dhrubaditya Mitra (NORDITA, Stockholm)
Faranggis Bagheri (KTH, Stockholm)
Luca Brandt (KTH, Stockholm)

References

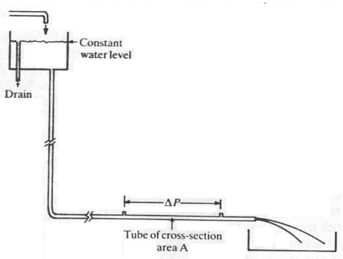
- Manifestations of Drag Reduction by polymer additives in Decaying, Homogeneous, Isotropic Turbulence, P. Perlekar, D. Mitra, and R. Pandit, Phys. Rev. Lett., **97**, 264501 (2006).
- Numerical studies of three dimensional turbulence with polymer additives and two dimensional turbulence in thin films, Prasad Perlekar, Ph.D. Thesis, Indian Institute of Science, Bangalore,
- Direct numerical simulations of statistically steady, homogeneous, isotropic fluid turbulence with polymer additives, P. Perlekar, D. Mitra, and R. Pandit, Phys. Rev. E., **82**, 066313 (2010).
- Statistics of polymer extensions in turbulent channel flow, F. Bagheri, D. Mitra, P. Perlekar, and L. Brandt, arXiv:1011.3766.

Outline

- Overview of turbulence with polymers.
- Modelling polymer solutions.
- Direct numerical simulations(DNS): Decaying and Forced Turbulence.
- Conclusions.

Drag Reduction

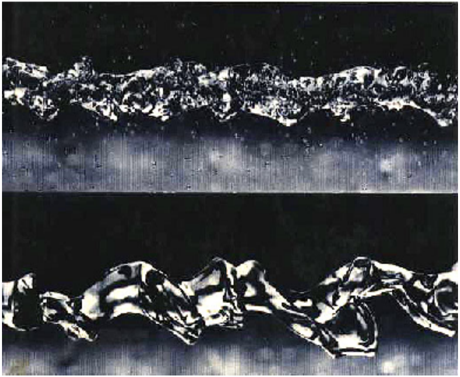
- Toms (1946): Monochlorobenzene with 0.25% (by weight) of polymethylmethacrylate



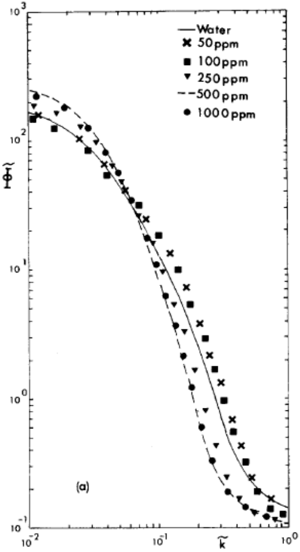
- Reduction in the pressure gradient across the pipe, on the addition of polymers, for the same volumetric flow rate
- Drag Reduction(in percentage) $DR \equiv \left(\frac{\Delta P_s - \Delta P_p}{\Delta P_s}\right) \times 100$

Reduction of small scale structures

- Turbulent jet of water with 50ppm polyethylene oxide at $Re \sim 225$
  [Turbulence structure in a water jet discharging in air, J.W. Hoyt and J.J. Taylor, Phys. Fluids, **20**, *S*253 (1977).]



Energy spectra

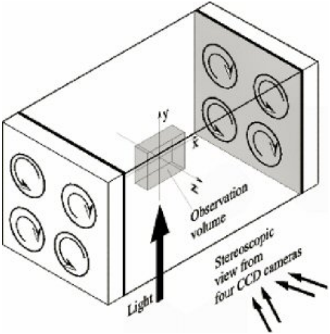


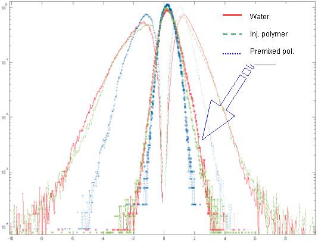
[Effect of polymer additives on the small-scale structure of grid-generated turbulence, W.D. McComb, J. Allan, and C.A. Greated, Phys. Fluids, **20**, 873 (1977).]

- Grid Reynolds number $Re_M = 7.6 \times 10^3$;
- For low polymer concentrations (50 and 100 ppm) there is no significant change in the energy spectrum; at somewhat higher concentrations(500 and 1000ppm) the spectra fall more steeply.

Eigenvalues of the strain tensor

[A. Liberzon, et al., Phys. Fluids, **17**, 031701 (2005).]

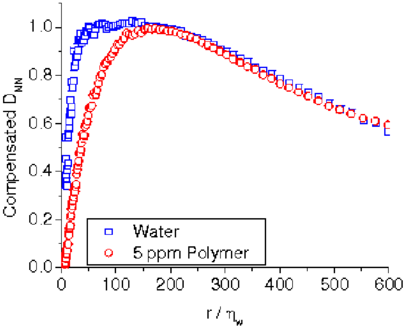




- Length:140mm, Width:120mm, Disk Dia.:40mm, Observation volume:$10 \times 10 \times 10 mm$, $Re_\lambda = 38$.
- Regions of large strains reduced on the addition of polymers.

Structure function: $S_2(r)$

N.T. Ouellette, H. Xu, and E. Bodenschatz, ICTR website, (2007).



- $c = 5ppm, Re_\lambda = 290, Wi = 3.5$,
- Small scale structures are modified on the addition of polymers.

Polymer Properties

Typical drag-reducing polymer:
Polyethylene oxide $N \times$[-CH_2-CH_2-O-]

- Degree of polymerization (N) $\simeq 10^4$
- Molecular weight $\simeq 4 \times 10^6$ amu
- Zimm relaxation time $\simeq 10^{-4}$s
- RMS end-to-end distance at maximal extension $\simeq 34\mu m$

Modelling polymer solutions

- Navier-Stokes(NS) with Polymer Additives:
  3D, unforced, incompressible, NS with additional stress because of polymers:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u} + \nabla \cdot \mathcal{T},$$

  where

  - $\mathbf{u}(\mathbf{x}, t)$: fluid velocity; point $\mathbf{x}$; time t;
  - ν: Kinematic viscosity of the fluid;
  - $\mathcal{T}$: polymer contribution to the fluid stress;

$\nabla \cdot \mathbf{u} = 0$ enforces incompressibility.

Modelling polymer solutions

- Finitely Extensible Nonlinear Elastic-Peterlin(FENE-P) model

$$\frac{\partial \mathcal{C}_{\alpha\beta}}{\partial t} + (u_\gamma \partial_\gamma)\mathcal{C}_{\alpha\beta} = (\partial_\gamma u_\alpha)\mathcal{C}_{\gamma\beta} + \mathcal{C}_{\alpha\gamma}(\partial_\gamma u_\beta) - \frac{1}{\mu}\mathcal{T}_{\alpha\beta}.$$

["Dynamics of polymeric liquids", Bird, *et al.*]

- $c = \mu/(\nu + \mu)$; $c = 0.1 \simeq 100 ppm$ of PEO
- $We = \tau_{poly}\sqrt{(\epsilon(t_m)/\nu)}$; t_m is the time corresponding to the peak in ϵ for $c = 0$

[Vaithianathan, *et al.*, JCP, **187**, 1 (2003).]

Direct Numerical Simulations

Direct Numerical Simulation(DNS)

Solve NS and FENE-P numerically

$$\begin{aligned}\frac{\partial u_\alpha}{\partial t} + (u_\gamma \partial_\gamma) u_\alpha &= -\partial_\alpha p + \nu \partial_{\gamma\gamma} u_\alpha + \partial_\gamma \mathcal{T}_{\alpha\gamma}, \\ \partial_\gamma u_\gamma &= 0,\end{aligned}$$

$$\frac{\partial \mathcal{C}_{\alpha\beta}}{\partial t} + (u_\gamma \partial_\gamma) \mathcal{C}_{\alpha\beta} = (\partial_\gamma u_\alpha) \mathcal{C}_{\gamma\beta} + \mathcal{C}_{\alpha\gamma} (\partial_\gamma u_\beta) - \frac{1}{\mu} \mathcal{T}_{\alpha\beta}.$$

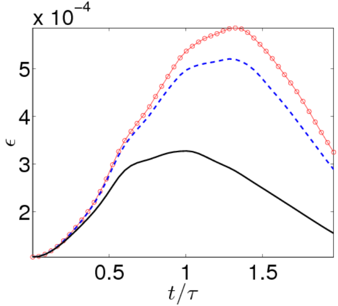
Decaying Turbulence

Results: Initial Condition

- Start from an initial energy spectrum with energy concentrated in the first few Fourier modes and the polymers unstretched
- Monitor the decay of the energy dissipation rate and the energy spectrum for the fluid with and without polymer additives.

Energy Dissipation Rate

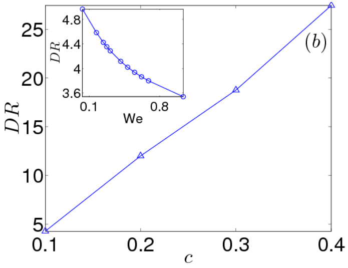
$N = 256$, $\nu = 10^{-3}$, $\tau_{poly} = 1$



- The energy dissipation rate $\epsilon(t)$ as a function of time t for different values of c.
- The peak in $\epsilon(t)$ decreases as c increases.

Dissipation Reduction(DR)

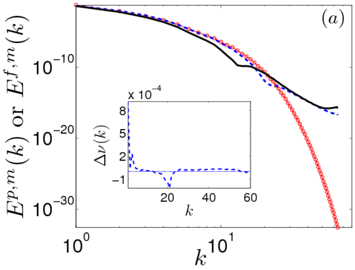
$N = 96$, $\nu = 10^{-2}$



- Natural definition of dissipation-reduction $\%DR = \left(\frac{\epsilon^{f,m} - \epsilon^{p,m}}{\epsilon^{f,m}}\right) \times 100;$
- f and p stand, respectively, for the fluid without and with polymers.
- An increase in c enhances the dissipation reduction DR (cf., earlier shell-model study).
- DR decreases marginally with an increase in We.

Fluid energy spectrum

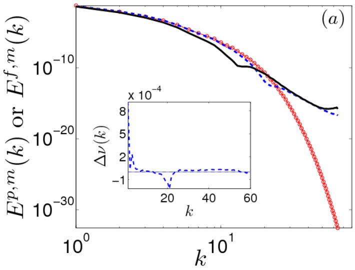
$N = 192$, $\nu = 10^{-2}$, $\tau_{poly} = 1$



- $E_f(k) = \sum_{k-1/2<k'<k+1/2} |\mathbf{u}(k')|^2$ at t_m for polymer concentrations $c = 0$(o-), $c = 0.1$(–) and $c = 0.4$(-).
- Energy spectrum at cascade completion changes significantly for large Fourier modes.
- This had not been resolved by earlier, high-Re simulations!

Scale-dependent viscosity

$N = 192$, $\nu = 10^{-2}$, $\tau_{poly} = 1$



- The change in the spectra and ϵ can be understood in terms of an additional, effective, scale-dependent viscosity $\Delta\nu(k) \equiv -\mu \sum_{k-1/2<k'\leq k+1/2} \mathbf{u}_{\mathbf{k}'} \cdot (\nabla \cdot \mathcal{J})_{-\mathbf{k}'} / [\tau_{poly} k'^2 E^{p,m}(k')]$.
- Since $\Delta\nu$ becomes negative, polymers pump energy into the fluid around $k \simeq 10$.

Structure Functions

Order-p equal-time, longitudinal velocity structure function.

$$\begin{aligned} \mathcal{S}_p(r) &\equiv \langle \delta u(r,t)^p \rangle, \\ \delta u_{||}(r,t) &\equiv [\vec{u}(\vec{x}+\vec{r},t) - \vec{u}(\vec{x},t)] \cdot (\vec{r}/r). \end{aligned}$$

Second order structure function $S_2(r)$

Experiments(Ouellette et al.)

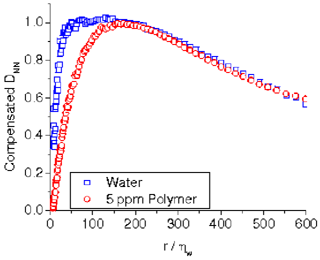


Figure: $c = 5ppm$, $Re_\lambda = 290$, and $We = 3.5$

Our DNS

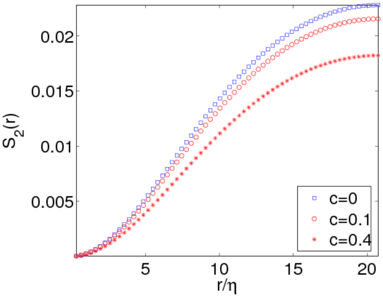
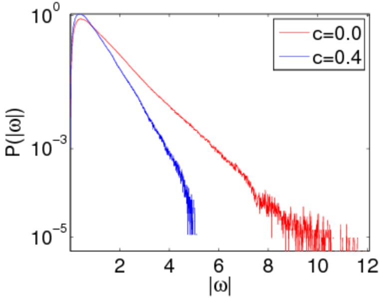


Figure: $N = 128$, $\nu = 0.01$, and $\tau_P = 1.5$

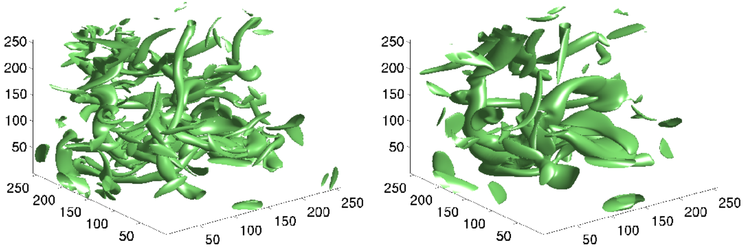
PDF of $|\omega|$



- Probability distribution of the modulus of the vorticity($P(|\omega|)$) at cascade completion(c=0, c=0.4).
- Addition of polymers leads to a decrease in the regions of large vorticity.

Isosurfaces of $|\omega|$

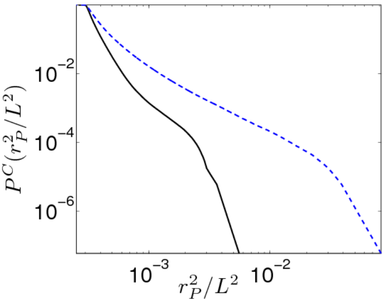
$N = 256$, $\nu = 10^{-3}$, $\tau_{poly} = 1$



- Iso-$|\omega|$ surfaces for $|\omega| = \langle|\omega|\rangle + 2\sigma$ for $c = 0$(left) and $c = 0.4$(right) at t_m.
- Small-scale structures are suppressed on the addition of polymers.

Stretching of Polymers:Cumulative distribution(CDF)

$N = 256$, $\nu = 10^{-3}$, $\tau_{poly} = 1$



- $c = 0.1$(dashed line), $c = 0.4$(line).
- An increase in c leads to a decrease in the polymer extension.
- A decrease in ν leads to turbulent flows and large polymer extensions.

Summary of Results: Decaying turbulence

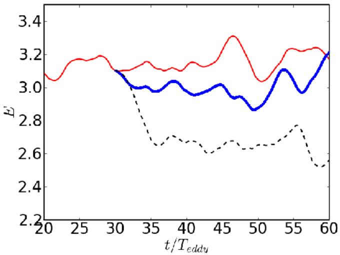
- Polymer additives lead to a decrease in small-scale structures.
- Polymers decrease the energy of the turbulent fluid at intermediate length scales and increase it at small scales.
- Dissipation reduction is the analogue in homogeneous, isotropic turbulence of drag-reduction in wall-bounded turbulence.
- An effective scale-dependent viscosity leads to a natural explanation of our results.
- This points toward an increase in the effective viscosity, but one that is scale-dependent.
- Ref: Manifestations of Drag Reduction by polymer additives in Decaying, Homogeneous, Isotropic Turbulence, P. Perlekar, D. Mitra, and R. Pandit, Phys. Rev. Lett., **97**, 264501 (2006).

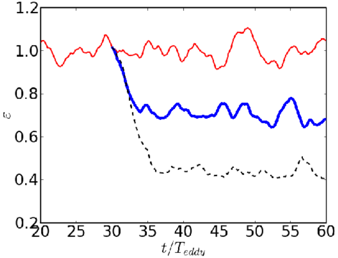
Forced Turbulence

- Deterministic forcing of M.A. Taylor, S. Kurien, and G. Eyink, Phys. Rev. E, **68**, 26310, (2003).

Time evolution of E and ϵ

$N = 256$, $Re_\lambda \simeq 80$, $c = 0.1$

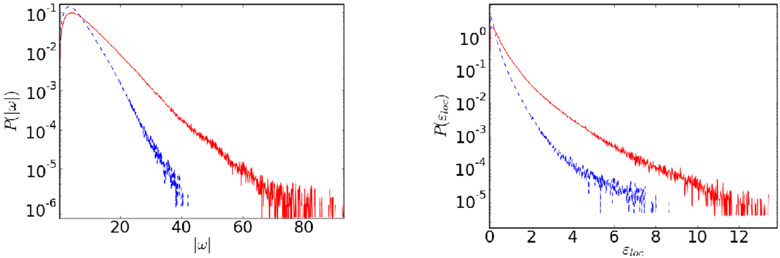




- Time averaged E decreases with an increase in We
- Time averaged ϵ decreases with an increase in We
- $We = 3.5$ (blue circles), $We = 7.1$ (black dashed line), NS (red)

PDF of $|\omega|$ and ϵ_{loc}

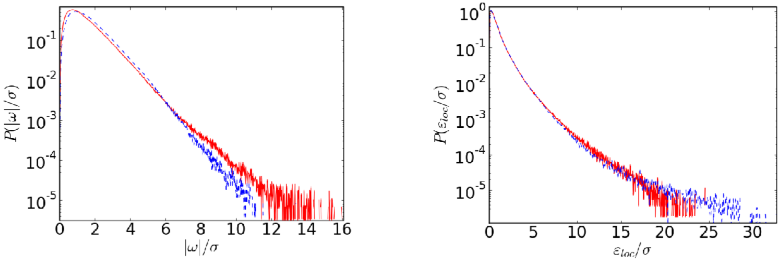
$N = 256,\ Re_\lambda \simeq 80,\ c = 0.1$



- $\omega \equiv |\sqrt{\sum_{i,j} \omega_{ij}\omega_{ij}}|,\ \epsilon_{loc} = \nu s^2 \equiv \sum_{i,j} s_{ij}s_{ij},$
  $s = (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)/2,\ \omega = \nabla \times \mathbf{u}$
- Regions of large strain and vorticity decrease on the addition of polymers

PDF of $|\omega|$ and ϵ_{loc}

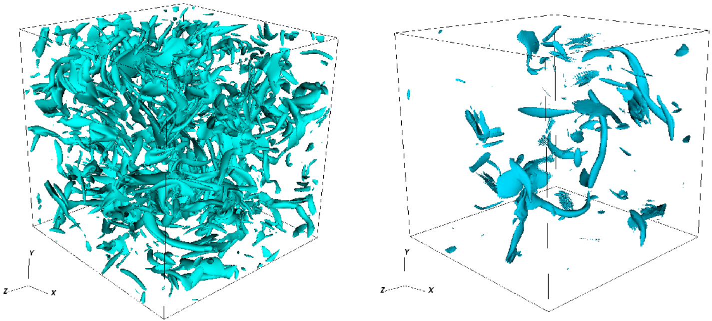
$N = 256,\ Re_\lambda \simeq 80,\ c = 0.1$



- $\omega \equiv |\sqrt{\sum_{i,j} \omega_{ij}\omega_{ij}}|$, $\epsilon_{loc} = \nu s^2 \equiv \sum_{i,j} s_{ij}s_{ij}$,
  $s = (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)/2$, $\omega = \nabla \times \mathbf{u}$
- Regions of large strain and vorticity decrease on the addition of polymers

Isosurfaces of $|\omega|$

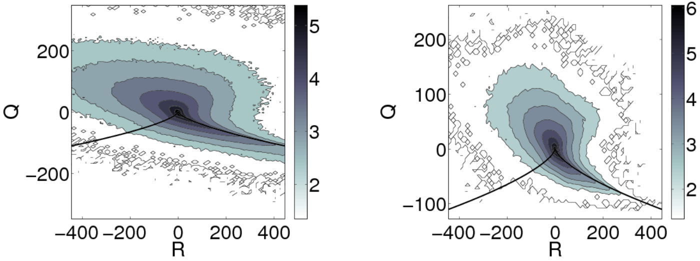
$N = 256,\ Re_\lambda \simeq 80,\ c = 0.1$



- Iso-$|\omega|$ surfaces for $|\omega| = \langle|\omega|\rangle + 2\sigma$ for $c = 0$(left) and $c = 0.1, We = 7.1$(right).
- Small-scale structures are suppressed on the addition of polymers.

QR plots

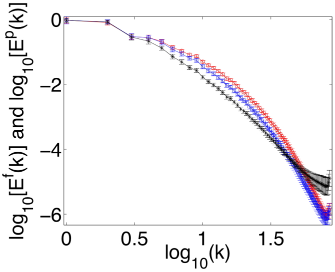
$N = 256,\ Re_\lambda \simeq 23,\ We = 7.1,\ c = 0.1$

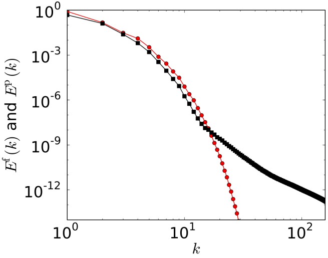


- Left: NS; Right: Polymer ($c = 0.1,\ We = 7.1$)

Energy spectrum

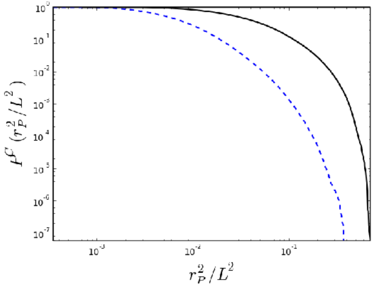
(Left) $N = 256$, $Re_\lambda = 80$
(Right) $N = 512$, $Re_\lambda = 20$





Polymer extensions

$N = 256,\ Re_\lambda \simeq 23,\ c = 0.1$



- $We = 7.1,\ c = 0.1$(line); $We = 3.5,\ c = 0.1$(dashed line);
- Polymer extensions larger in comparison to decaying turbulence
- At fixed c, polymer extension increases with an increase in We

Conclusions

- Our simulations show that the addition of polymers to flows that display homogeneous isotropic turbulence leads to dissipation reduction in both decaying and statistically steady turbulence; this dissipation reduction is the analogue of drag reduction in wall-bounded flows.
- Our numerical results agree with the experimental results of (a) Liberzon, et al., op. cit. and (b) Ouellette et al., op. cit.
- Polymers decrease the energy of the turbulent fluid at intermediate length scales and increase it at small scales; a scale-dependent viscosity provides a natural means of understanding our results.