”the nature of turbulence"
KITP, may 2011

turbulence and the dynamo

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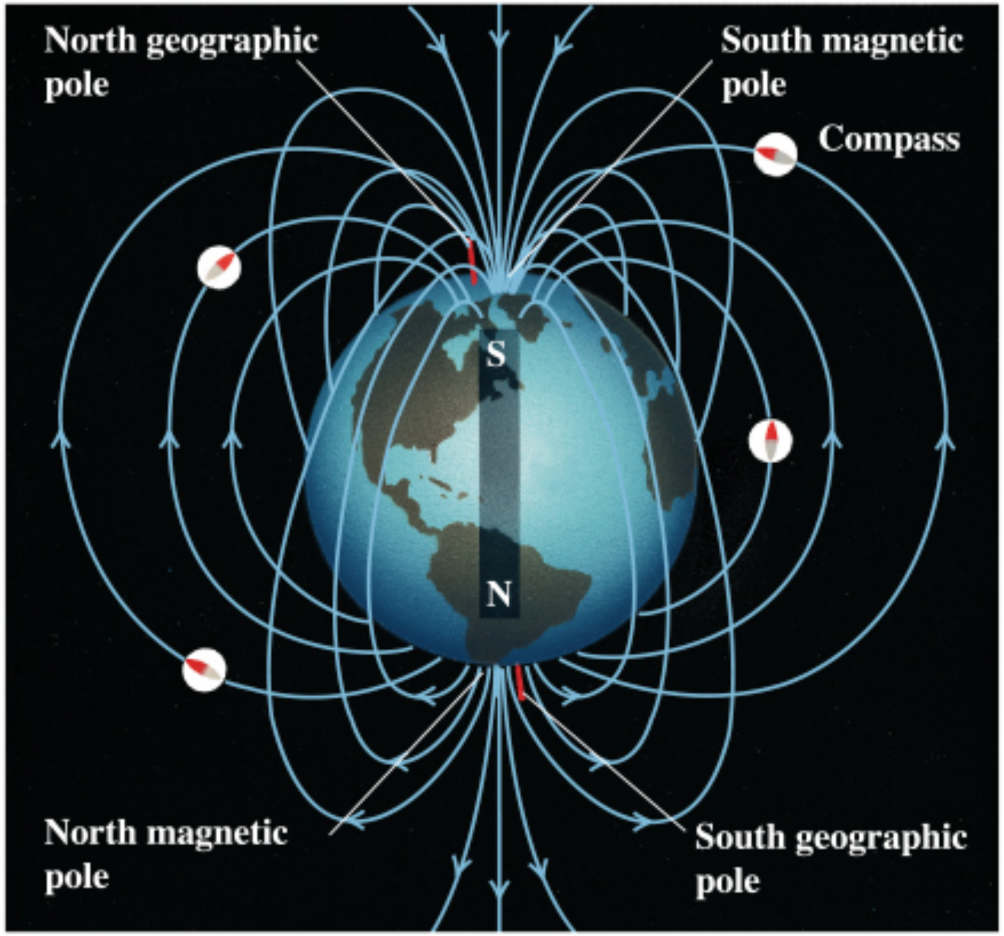
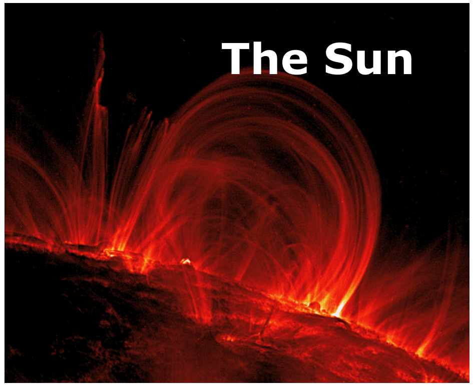
ENS DE LYON

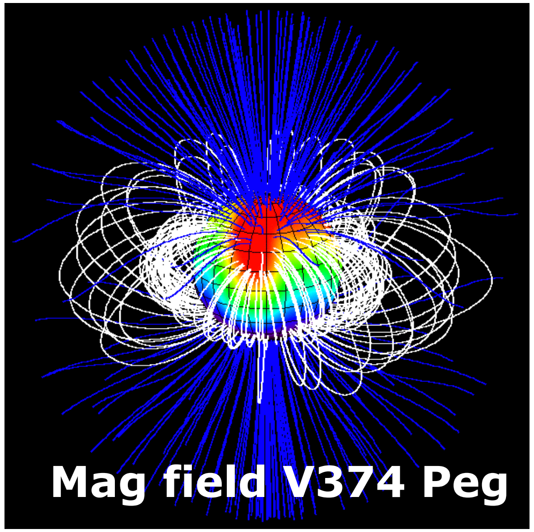
Agence Nationale de la Recherche ANR

cnrs dépasser les frontières

Focus on : Earth (liquid metals) & experiments







OUTLINE

1. Dynamos

Scalar, scalar gradient and B-equations
Is turbulence helpful or detrimental?
Experiments

2. **Mean field MHD**

easy and hard
Examples
Measurements

3. Turbulence in VKS

the $\alpha-\omega$ “Duddley & James”failed dynamo
the $\alpha-\omega$ ferroDynamo.
mechanism and low dimensional behavior
turbulence and noise.

4. Turbulent MHD

Saturation: large B-field experiment.

5. Next

Advection / diffusion / stretching equations

- passive scalar
$$\partial_t C + \mathbf{u} \cdot \nabla C = \kappa \Delta C$$

- gradient of a passive scalar
$$\mathbf{G} = \nabla C$$
$$\partial_t \mathbf{G} = -\nabla \cdot (\mathbf{u} \cdot \mathbf{G}) + \kappa \Delta G$$

- vorticity
$$\partial_t \omega = \nabla \times (\mathbf{u} \times \omega) + \nu \Delta \omega$$

- magnetic field
$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \Delta \mathbf{B}$$

Question : $\frac{d}{dt} \int_V d^3r \, x^2$

anti-dynamo theorems ....

Antidynamo theorem : Cowling 1934

Axisymmetric magnetic field:

$$\mathbf{B} = \nabla A_\phi \times \mathbf{e}_\phi + B_\phi \mathbf{e}_\phi$$

Ohm's law:

$$\lambda \nabla \times \mathbf{B} = \mathbf{u} \times \mathbf{B} - \nabla U - \partial_t \mathbf{A}$$

For a steady process, the C = r Aϕ component reads

$$u_r \partial_r C + u_z \partial_z C = \lambda \left(\partial_r^2 C + \partial_z^2 C - (1/r)\partial_r C \right)$$

However, dynamos with ***small*** deviations from axisymmetry are possible -- Bragiinsky 1976. Or small anostropy in the diffusivity – Lortz 1989.

Dynamo : stretch > Joule

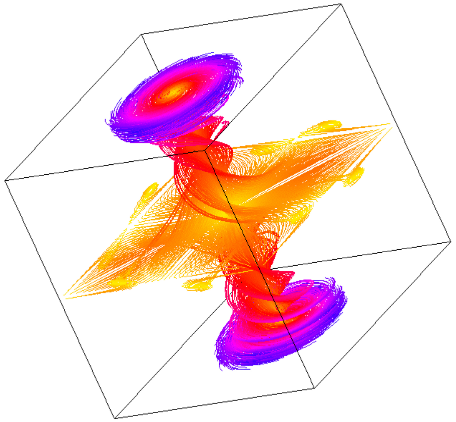
- Induction eqn. $$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \Delta \mathbf{B}$$
- Magnetic Reynolds number: $$Rm = \frac{UL}{\eta}$$
- Turbulence $$Re = \frac{UL}{\nu} = \frac{\eta}{\nu}\frac{UL}{\eta} = \frac{Rm}{Pm}$$
- Power $$Rm = \mu\sigma\left(\frac{PL}{\rho}\right)^{1/3}$$

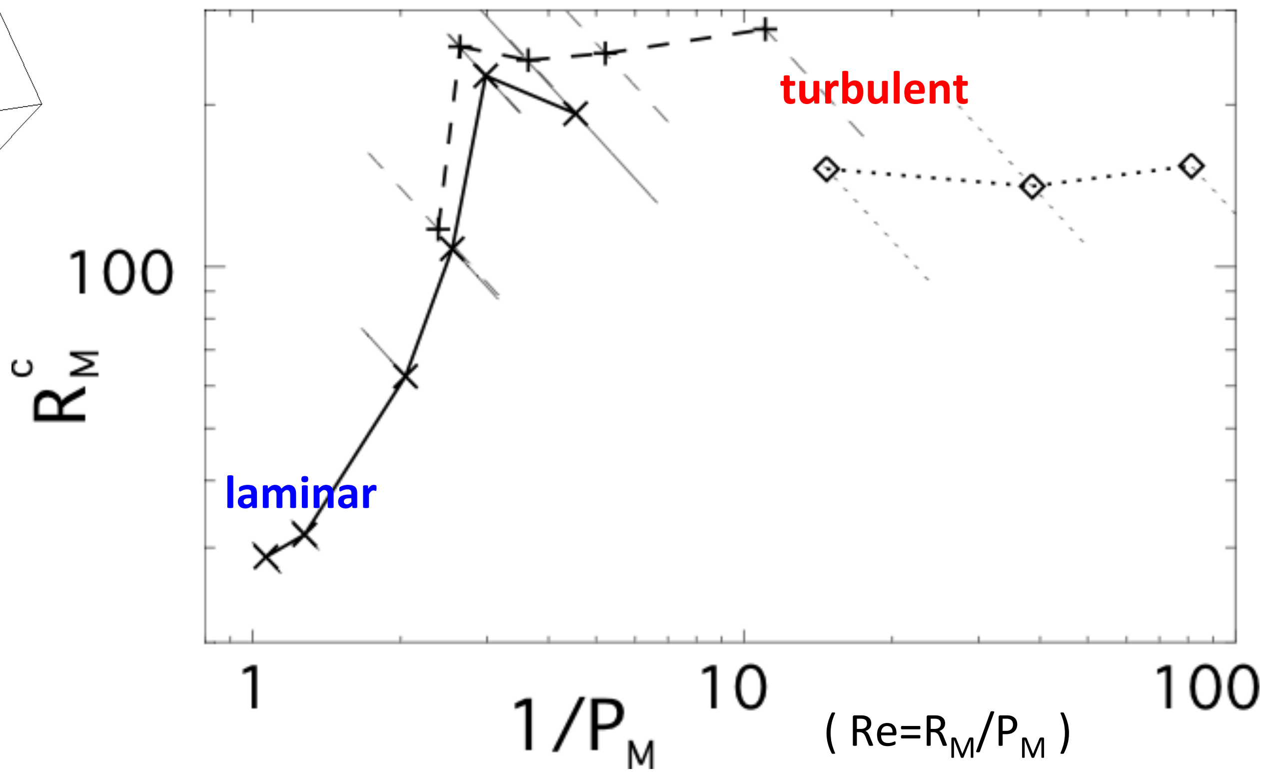
$Rm^c < Rm^{MAX}$, not $Rm^{MAX} > Rm^c$!!!!

Numerical study of the Taylor Green flow

Y. Ponty, P. Mininni et al., *PRL* **94** (2005)

$$\mathbf{F}_{\mathrm{TG}}(k_0) = 2F \begin{bmatrix} \sin(k_0\ x)\cos(k_0\ y)\cos(k_0\ z) \\ -\cos(k_0\ x)\sin(k_0\ y)\cos(k_0\ z) \\ 0 \end{bmatrix}$$



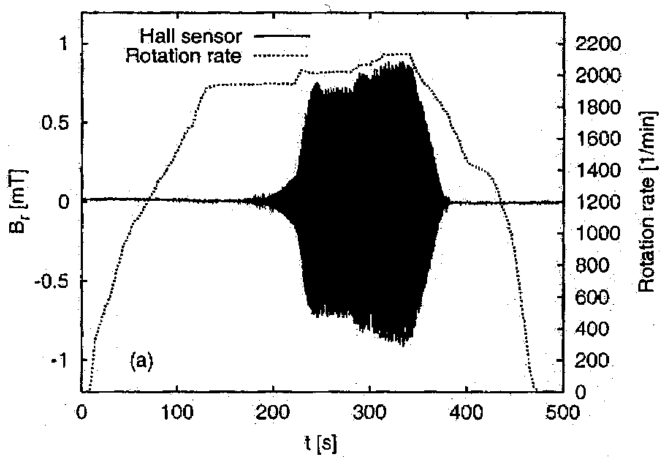


Exact solutions of the induction eqn

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \Delta \mathbf{B}$$

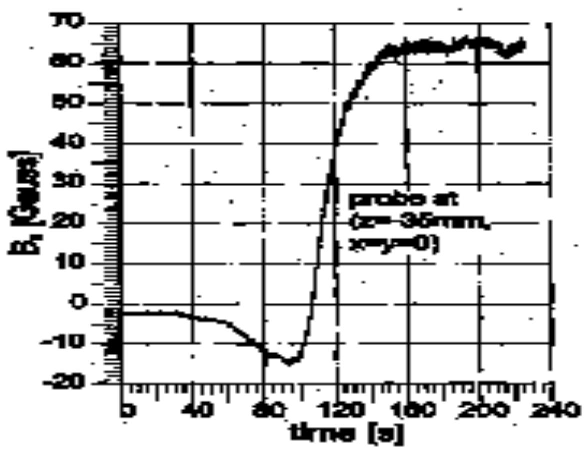
Ponomarenko – Riga dynamo
A. Gailitis et al., PRL (2001)

- Helical wave
- Threshold ≈ laminar prediction
- α-ω dynamo
- no further dynamics.

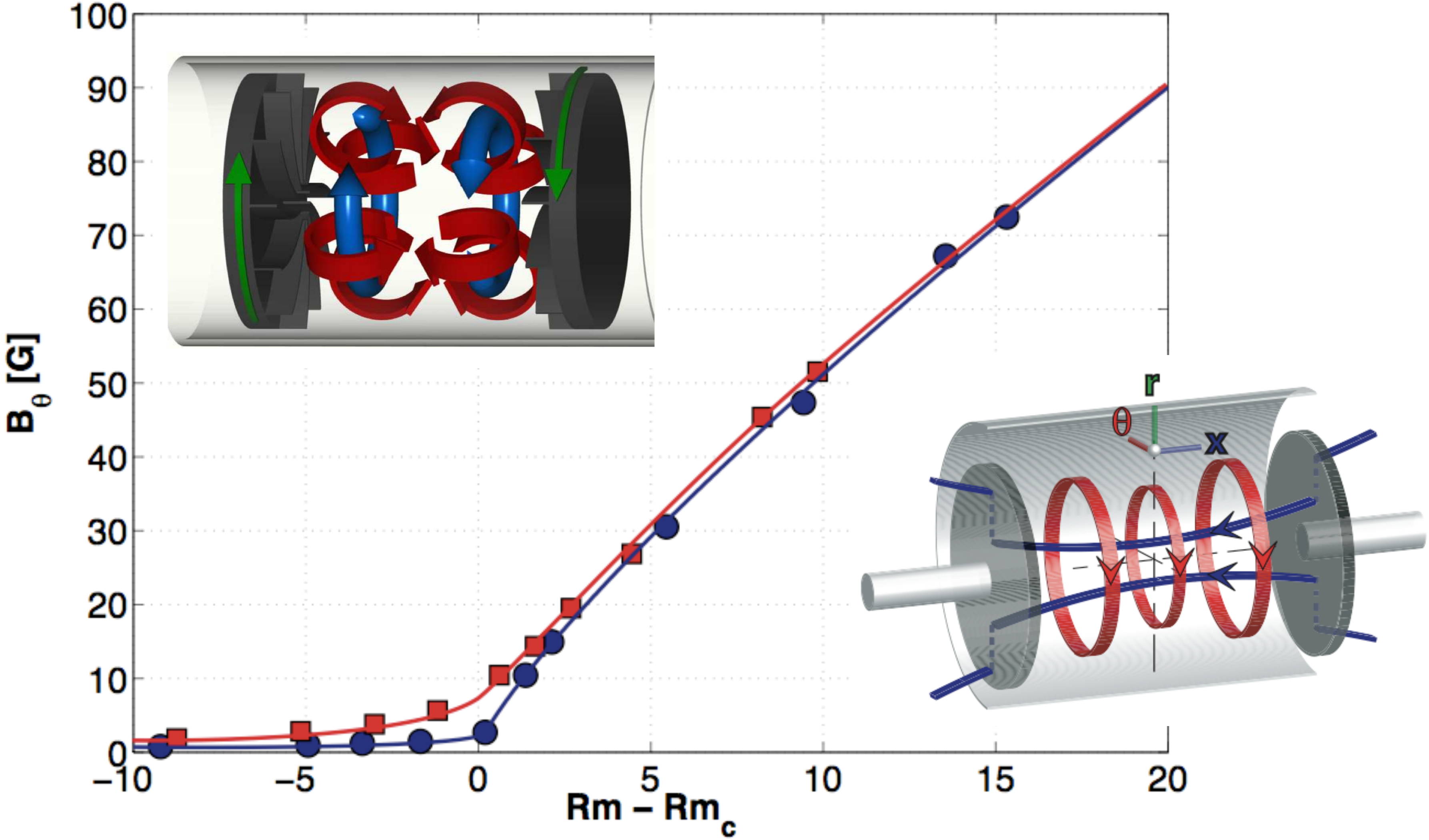


G.O. Roberts – Karlsruhe dynamo
Müller & Stieglitz, PoF (2001)

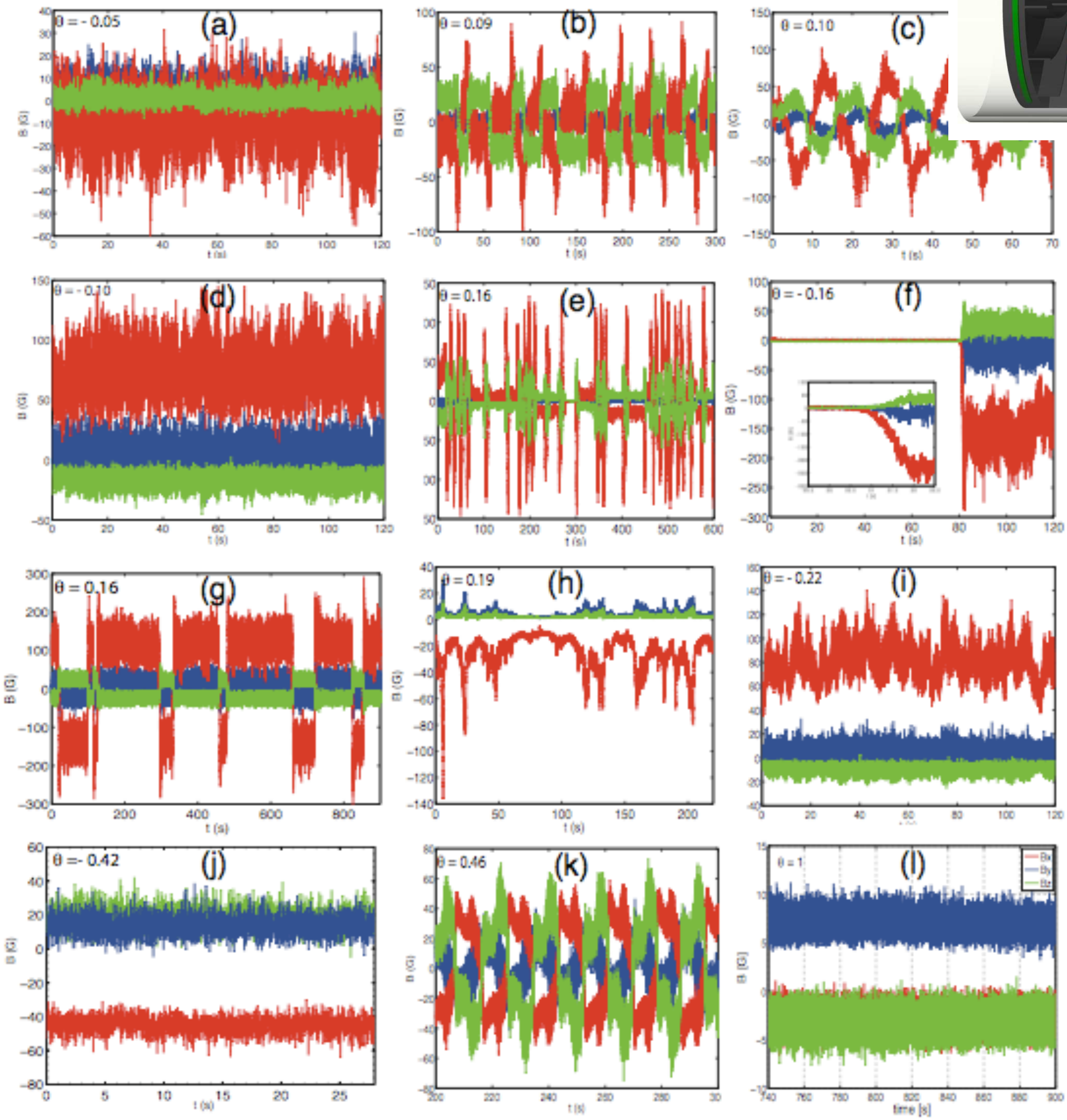
- Stationary dynamo
- Threshold ≈ laminar prediction
- α^2 dynamo
- no further dynamics.

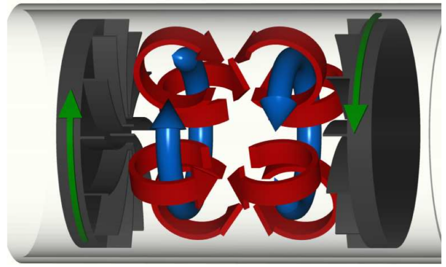


Monchaux et al., *PoF* **21** (2009)



Monchaux et al.,*PoF* **21** (2009)





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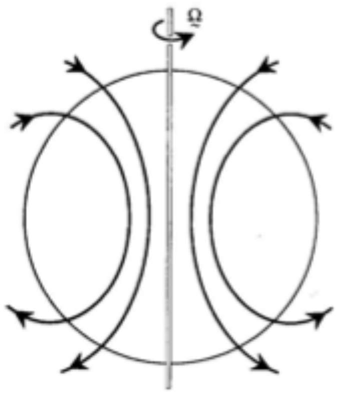
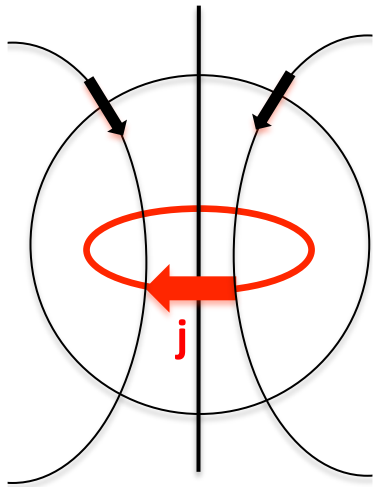
POL to TOR : the ω effect



TOR to POL : the α effect



$\langle u \times b \rangle = \alpha B$

j // B

Mean-field MHD 101

- Mean field MHD versus Prandtl mixing length

$$\tilde{B} = B + b$$

$$\tilde{u} = U + u = 0 + u$$

$$\langle induction \quad eqn \rangle \quad \Rightarrow \quad \partial_t B = \nabla \times \langle u \times b \rangle + \lambda \Delta B$$

$$\partial_t b = \nabla \times (u \times B + u \times b - \langle u \times b \rangle) + \lambda \Delta b$$

$$first \quad order \quad smoothing \quad \Rightarrow \quad u \times b \approx \langle u \times b \rangle$$

$$\partial_t b - \lambda \Delta b = \nabla \times (u \times B) \quad \Rightarrow \quad b = F_u(B)$$

$$\langle u \times b \rangle = \alpha B \ + \beta \nabla \times B$$

$$compute \quad \alpha \quad and \quad \beta \quad from \quad u(x)$$

$$enjoy \quad \partial_t B = \alpha \nabla \times B + (\lambda + \beta) \Delta B$$

Mean-field MHD 102

- calculation for homegenous isotropic (non mirror symmetric) turbulence

$$\tilde{B} = B + b$$
$$\tilde{u} = U + u = 0 + u$$
$$\langle induction\ \ eqn \rangle \ \Rightarrow\ \partial_t B = \nabla \times \langle u \times b \rangle + \lambda \Delta B$$
$$\partial_t b = \nabla \times (u \times B + u \times b - \langle u \times b \rangle) + \lambda \Delta b$$
$$first\ \ order\ \ smoothing\ \ \Rightarrow\ \ u \times b \approx \langle u \times b \rangle$$
$$\partial_t b - \lambda \Delta b = \nabla \times (u \times B)\ \ \Rightarrow\ \ b = F_u(B)$$
$$\langle u \times b \rangle = \alpha B + \beta \nabla \times B$$
$$compute\ \ \alpha\ \ and\ \ \beta\ \ from\ \ u(x)$$

$$\langle u_i(\mathbf{x}, t) u_j(\mathbf{x}', t') \rangle = R_{ij}(\mathbf{r}, \tau)$$

$$R_{ij}(\mathbf{r}, \tau) = \int\int \Phi_{ij}(\mathbf{k}, \omega)\, e^{i(\mathbf{k}.\mathbf{r} - \omega\tau)}\, d\mathbf{k}\, d\omega$$

$$E(k, \omega) = \tfrac{1}{2} \int_{S_k} \Phi_{ii}(\mathbf{k}, \omega)\, dS$$

$$F(k, \omega) = i \int_{S_k} \varepsilon_{ikl} k_k \Phi_{il}(\mathbf{k}, \omega)\, dS$$

$$\alpha \approx -\frac{1}{3\lambda} \int k^{-2} F(k)\, dk$$

$$\beta \approx \tfrac{2}{3} \lambda^{-1} \int_0^\infty k^{-2} E(k)\, dk$$

$$\alpha \sim u_0 R_M$$
$$\beta \sim u_0 \ell_0 R_M$$

The run-away problem

- Diffusivity and stretching increase at the same rate ?

$$\partial_t \mathbf{B} = \alpha \nabla \times \mathbf{B} + \beta \Delta \mathbf{B} \qquad \alpha \text{ and } \beta \text{ from } \langle u_i u_j' \rangle$$

$$\boxed{Rm_{\mathrm{eff}} = \frac{\alpha \ell}{\beta} \stackrel{?}{=} \mathrm{Const.} \quad < Rm^c}$$

- YES: Schekochihin et al., *APJ* **629** (2005)
- NO: Ponty et al., *PRL* **94** (2005)
- NO: Karlsruhe and Riga exp. : Rm^c(exp) = Rm^c (laminar)
  Denisov et al., *JETP Lett.*, **88** 2008
- MAYBE:?? Frick et al., PRL 2010

The run-away problem

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$$\partial_t \mathbf{B} = \alpha \nabla \times \mathbf{B} + \beta \Delta \mathbf{B} \qquad \alpha \text{ and } \beta \text{ from } \langle u_i u'_j \rangle$$

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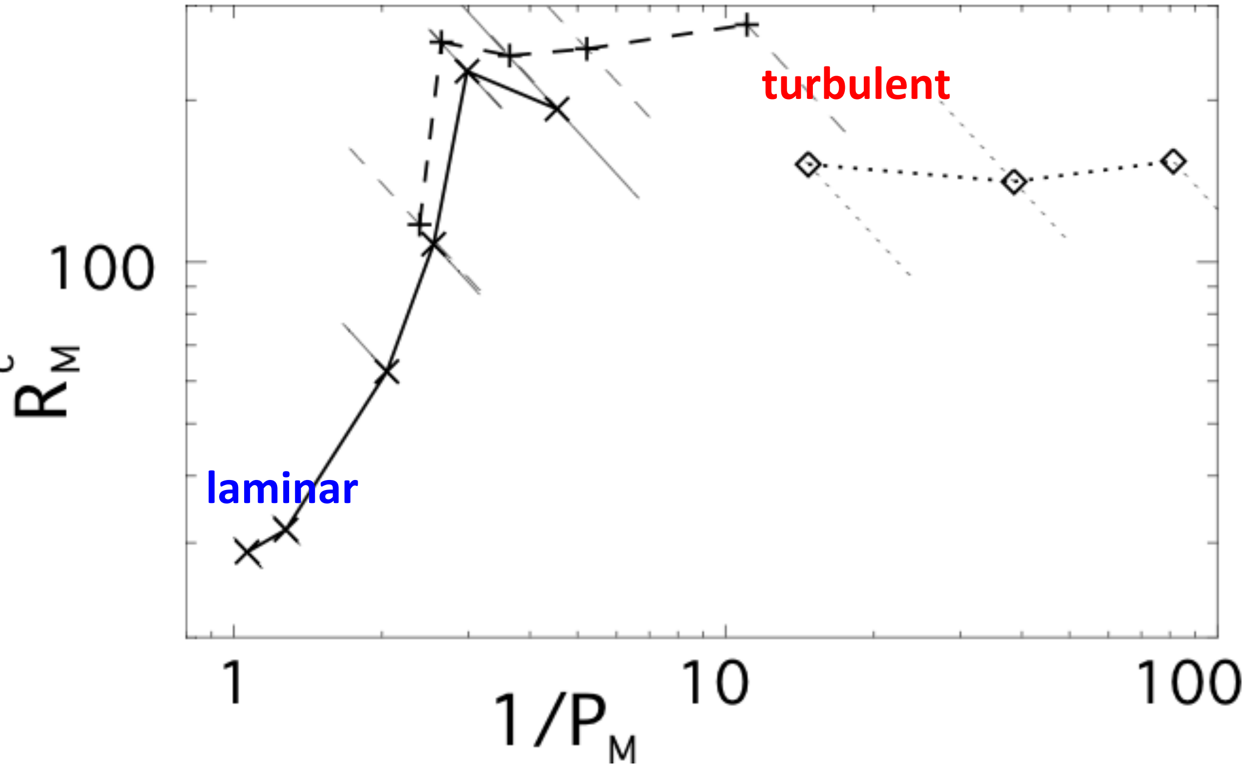
- YES: Schekochihin
- NO: Ponty et al., F
- NO: Karlsruhe and Denisov et al.
- MAYBE:?? Frick et al



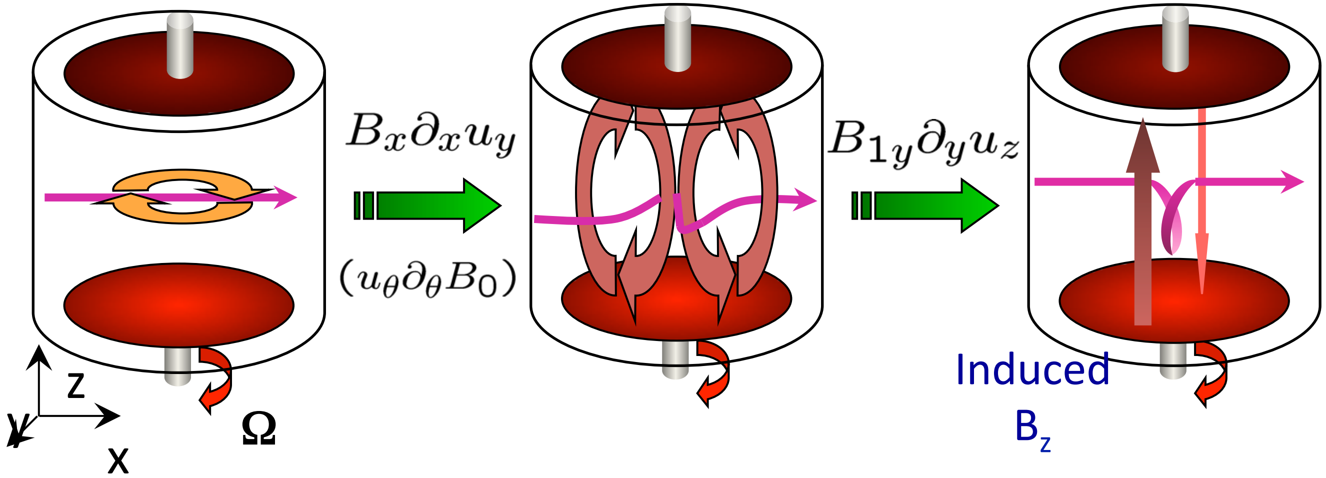
Mean-field MHD 715

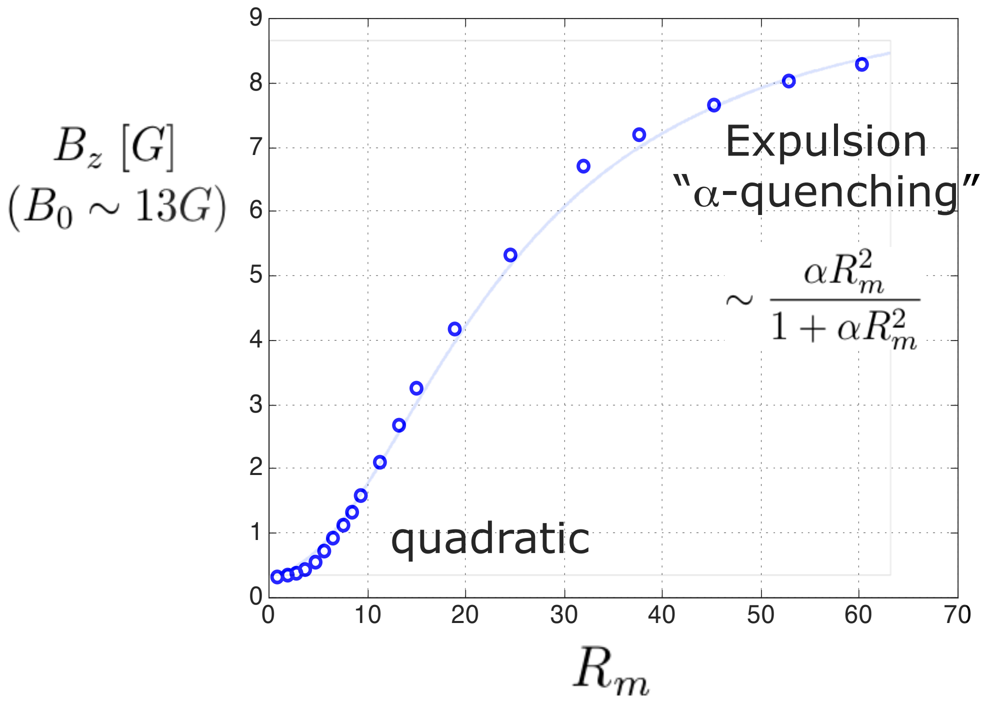
- Now, do it right … Rädler and Stepanov

$$
\begin{aligned}
\boldsymbol{\mathcal{E}} = &- \alpha_1^{(\Omega)}(\mathbf{g}\cdot\boldsymbol{\Omega})\overline{\mathbf{B}} - \alpha_2^{(\Omega)}((\boldsymbol{\Omega}\cdot\overline{\mathbf{B}})\mathbf{g} + (\mathbf{g}\cdot\overline{\mathbf{B}})\boldsymbol{\Omega}) \\
&- \alpha_1^{(W)}(\mathbf{g}\cdot\mathbf{W})\overline{\mathbf{B}} - \alpha_2^{(W)}((\mathbf{W}\cdot\overline{\mathbf{B}})\mathbf{g} + (\mathbf{g}\cdot\overline{\mathbf{B}})\mathbf{W}) \\
&- \alpha^{(D)}\hat{\alpha}(\mathbf{g},\mathbf{D})\circ\overline{\mathbf{B}} \\
&- (\gamma^{(0)}\mathbf{g} + \gamma^{(\Omega)}\mathbf{g}\times\boldsymbol{\Omega} + \gamma^{(W)}\mathbf{g}\times\mathbf{W} + \gamma^{(D)}\mathbf{g}\circ\mathbf{D}) \\
&\times\overline{\mathbf{B}} - \beta^{(0)}\boldsymbol{\nabla}\times\overline{\mathbf{B}} - \beta^{(D)}\mathbf{D}\circ(\boldsymbol{\nabla}\times\overline{\mathbf{B}}) \\
&- (\delta^{(\Omega)}\boldsymbol{\Omega} + \delta^{(W)}\mathbf{W})\times(\boldsymbol{\nabla}\times\overline{\mathbf{B}}) \\
&- (\kappa^{(\Omega)}\boldsymbol{\Omega} + \kappa^{(W)}\mathbf{W})\circ(\boldsymbol{\nabla}\overline{\mathbf{B}})^{(s)} - \kappa^{(D)}\hat{\boldsymbol{\kappa}}(\mathbf{D})\circ(\boldsymbol{\nabla}\overline{\mathbf{B}})^{(s)}
\end{aligned}
$$

K.-H. Rädler and R. Stepanov, Phys. Rev. E 73, 056311 (2006)

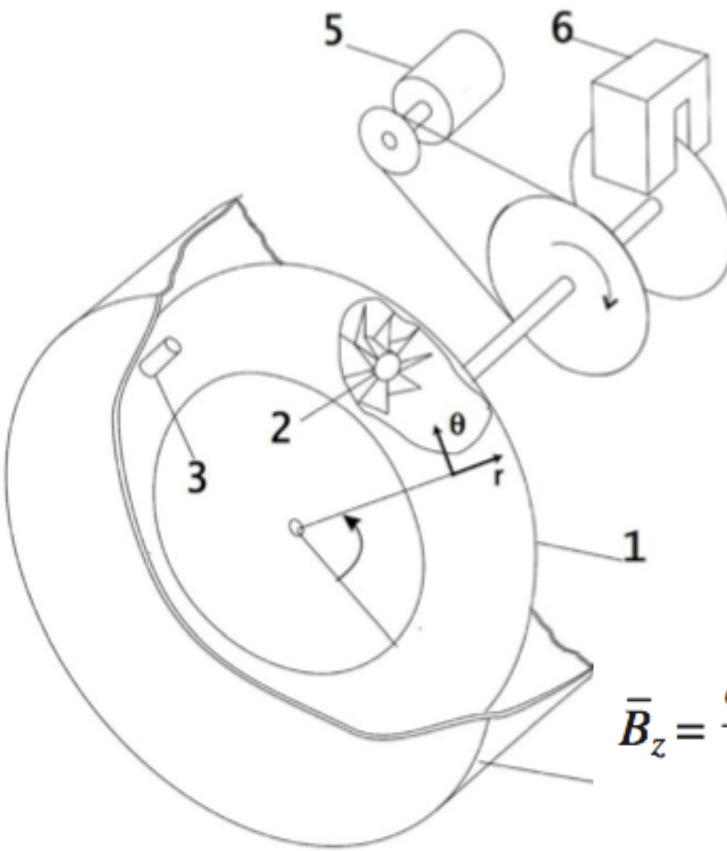
Induction : helicity and alpha effect

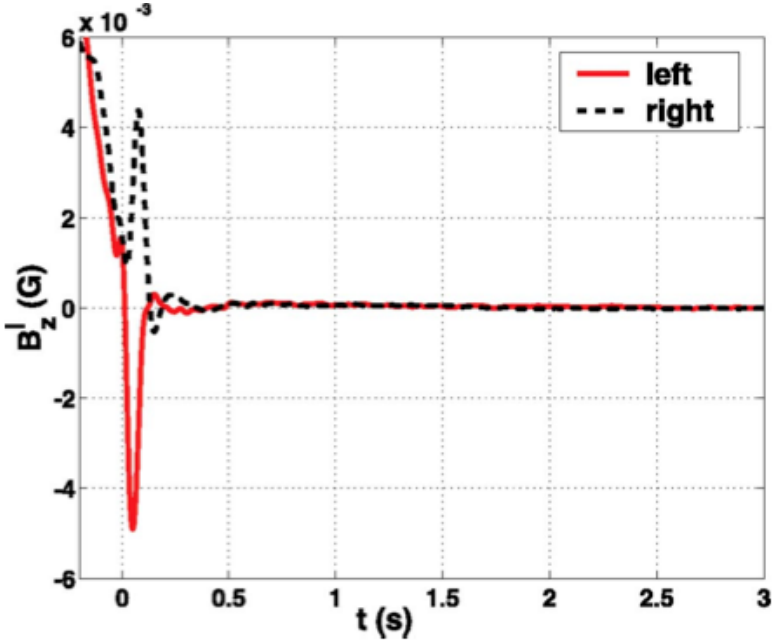




Perm Torus experiment : alpha

Stepanov et al.,*PRE* **73** (2006)





$$\bar{B}_z = \frac{\alpha \pi r_0^2}{2\lambda R} B_0 \Rightarrow [\alpha] \approx \frac{2\lambda R}{\pi r_0^2} \frac{\bar{B}_z}{B_0} \sim (3 \pm 0.3) \times 10^{-3}\ \text{m/s}.$$

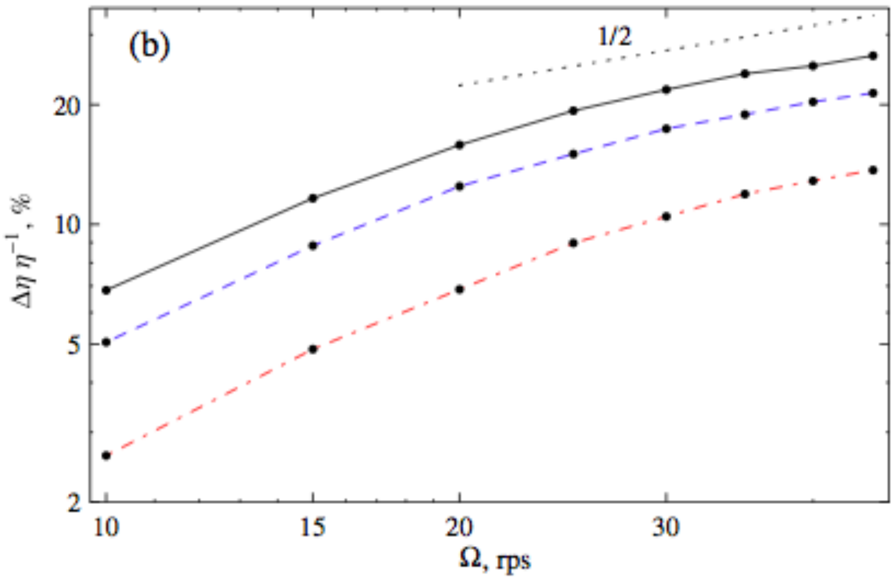
$$\alpha_{max} \sim 10\ \text{cm/s}$$

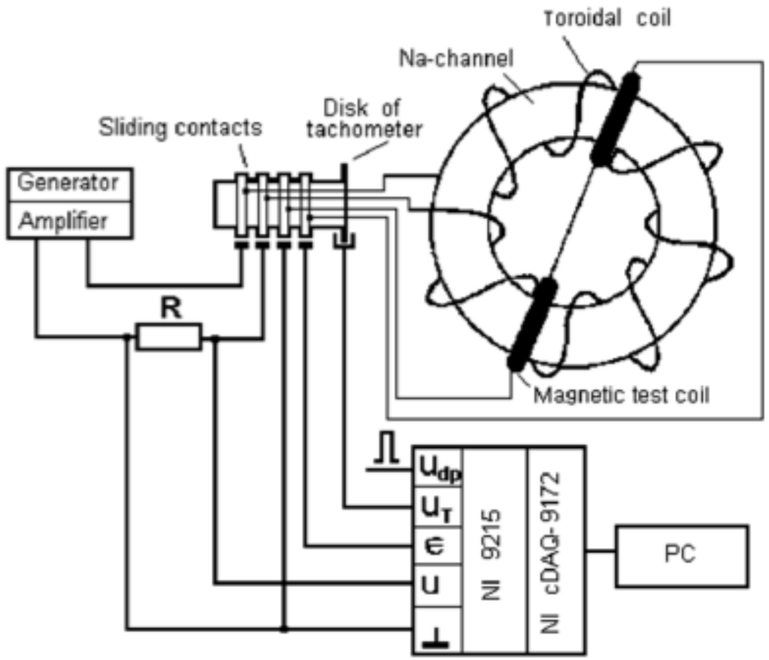
$$\alpha/u \sim 10^{-2\ -3}$$

$$\frac{\alpha(\text{Na})}{\alpha(\text{Ga})} \sim \frac{L(\text{Na})}{L(\text{Ga})} \frac{\text{R}_\text{m}(\text{Na})}{\text{R}_\text{m}(\text{Ga})}$$

Perm Torus experiment : beta

Frick et al.,*PRL* **105** (2010)





$$\lambda \sim \lambda_{\rm mol}(1 + 0.25) \;\; \text{at} \;\; R_M \sim 50$$

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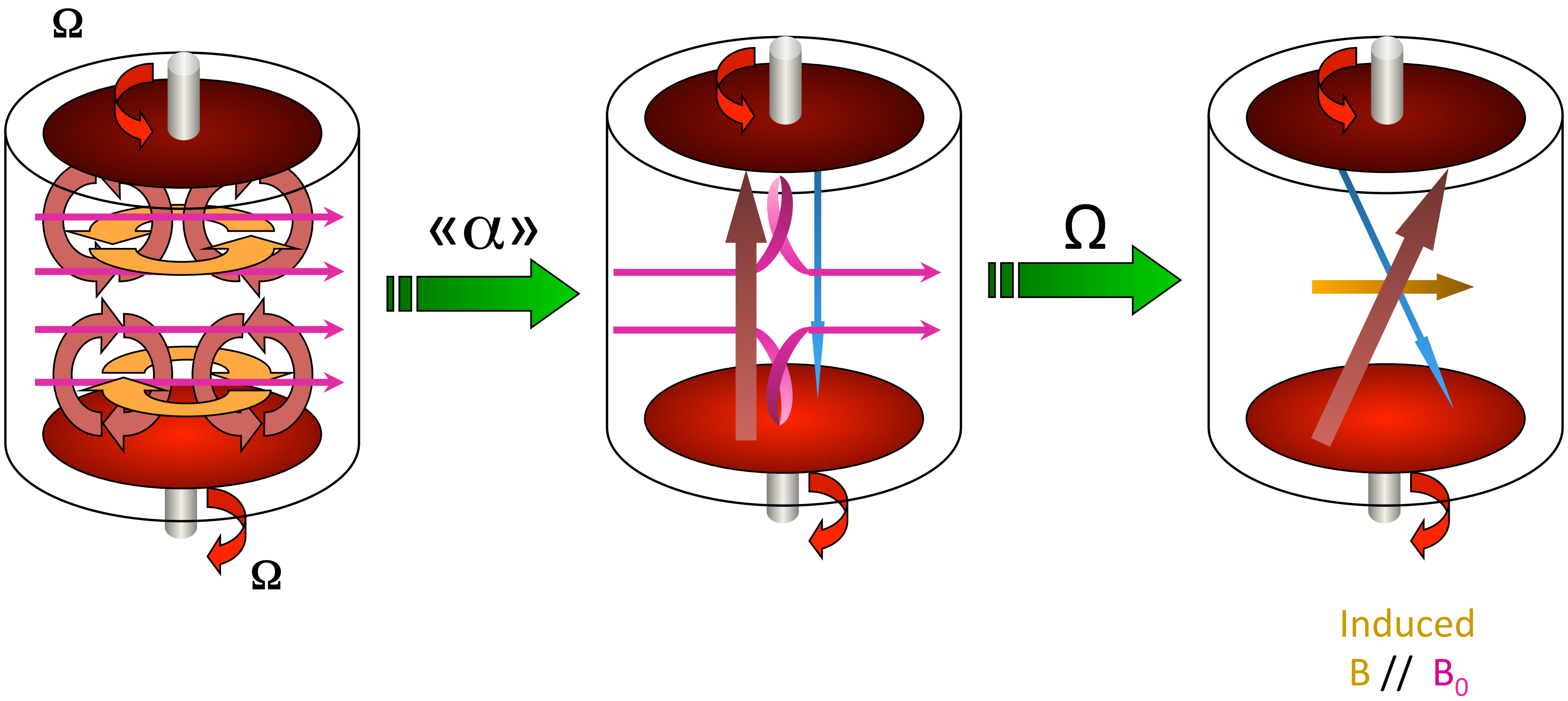
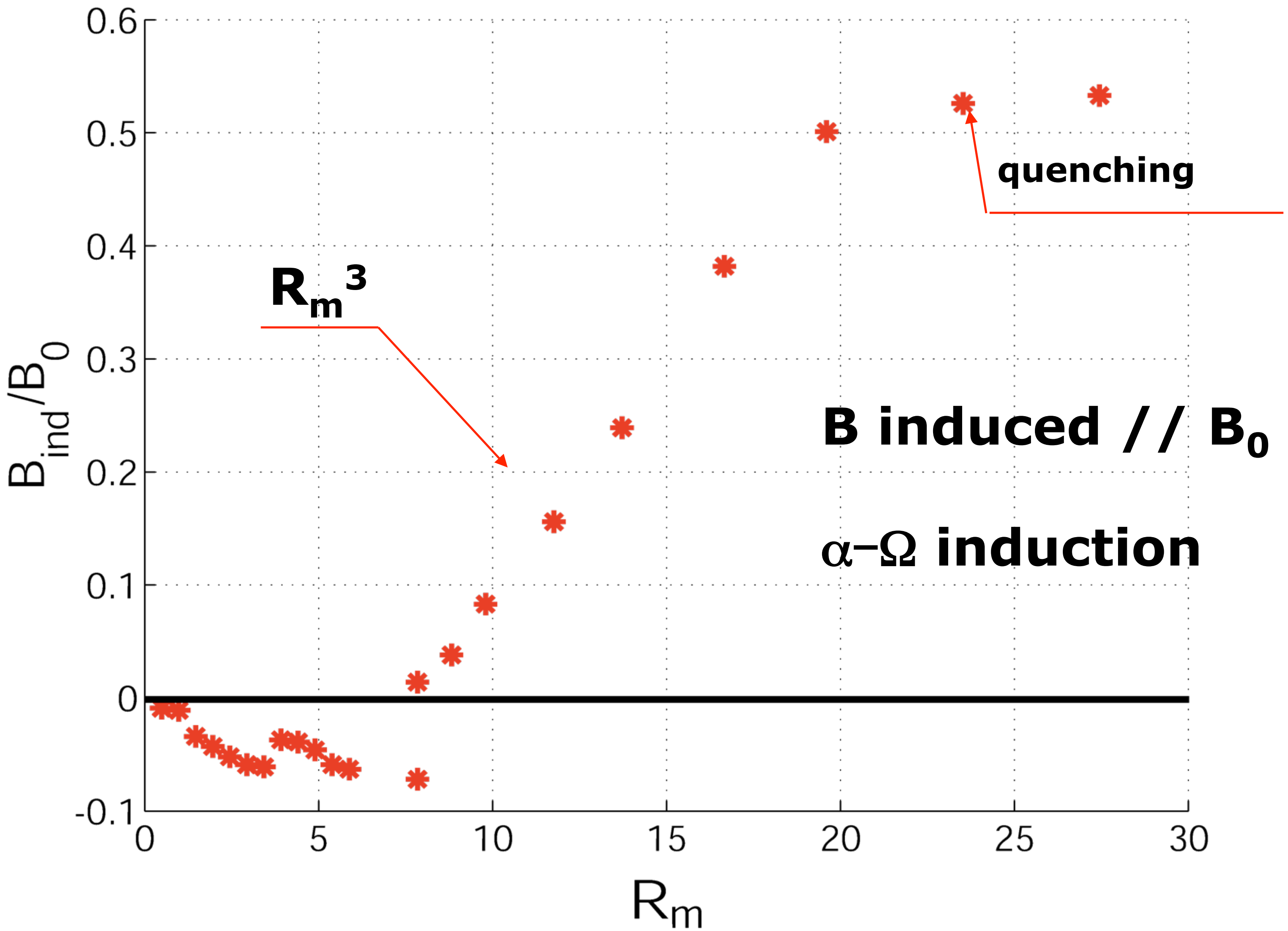
VKS, as a Duddley and James s2t2 dynamo ?



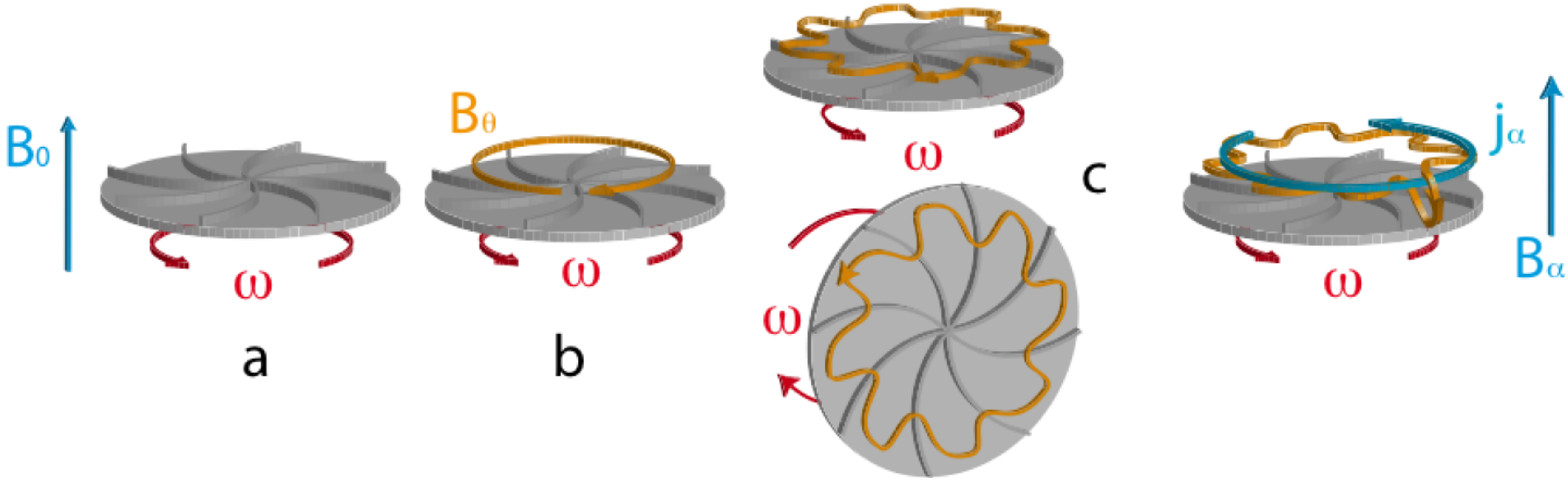
α-ω loop-back mechanism

VKS1 measurment
(M. Bourgoin PhD thesis)



VKS dynamo generation mechanism

G. Verhille et al., *NJP* **12** (2010)



- **LARGE ω-effect enhaced by iron**
- **WEAK α-effect, several possibilities**

- m=0 is promoted by ferro impellers
- the neutral mode is NOT the one predicted for μ_r=1

Generation mechanism and ferromagnetic impellers

A. Giesecke et al., *PRL* **104** (2010)

Kinematic approach,
MND flow
μ_r variations implemented in rotaing disk
uniform alpha

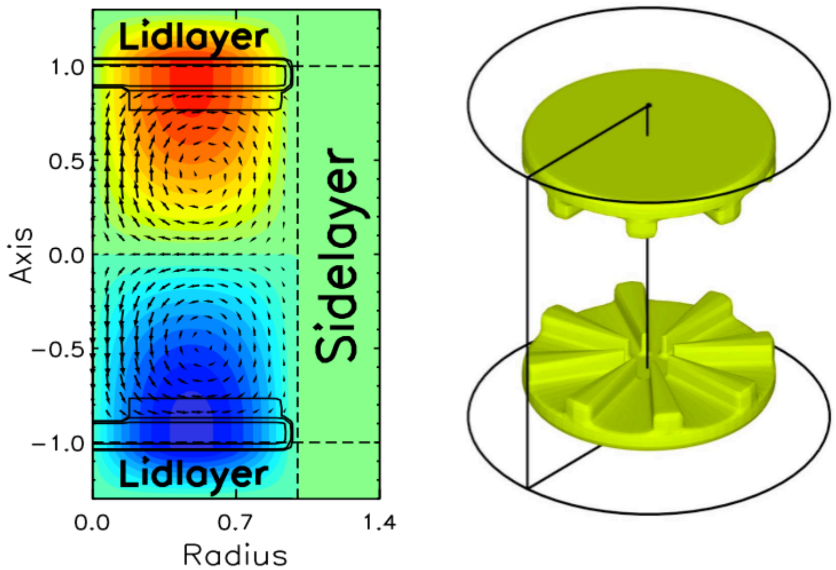
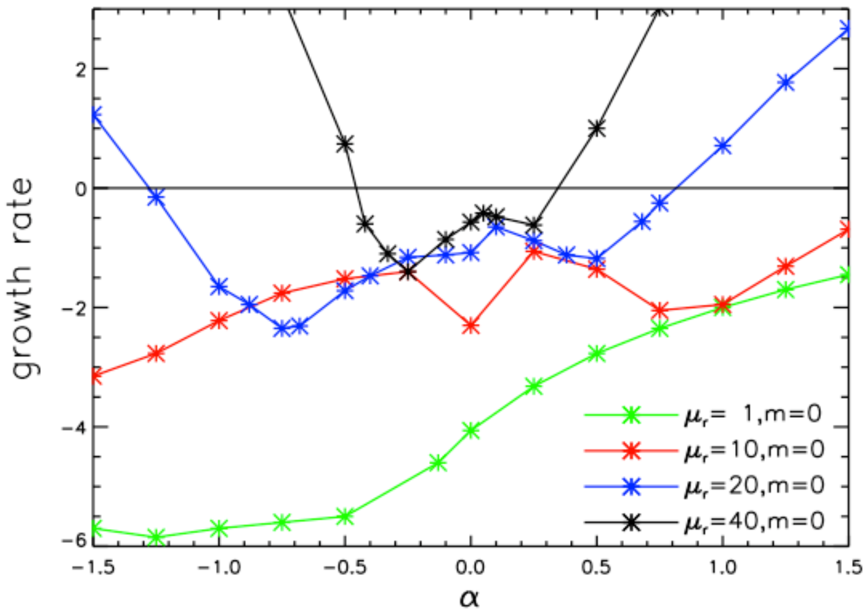
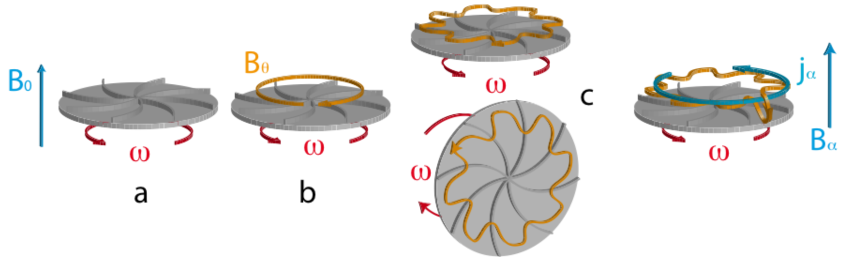


FIG. 1: Left panel: Structure of the prescribed axisymmetric velocity field. The color coded pattern represents the azimuthal velocity and the arrows show the poloidal velocity field. The black solid lines represent the shape of the impeller system (disk and blades). Right panel: assumed permeability distribution; isolevel at $\mu_r = 35$ where the peak value is given by $\mu_r = 40$. In the fluid region μ_r is equal to unity.



- $\alpha \approx 0.15$ m/s for $\mu_r = 40$
- $\alpha \approx 15$ m/s for $\mu_r = 1$

LARGE ω, WEAK α



α−ω dynamo

Threshold corresponds to $Rm^{\alpha}.Rm^{\omega} \approx 1$

$$Rm^{\alpha} = \frac{\alpha h}{\eta} \sim 0.2\alpha$$

$$Rm^{\omega} = K\frac{Uh}{\eta} \sim 0.4K$$

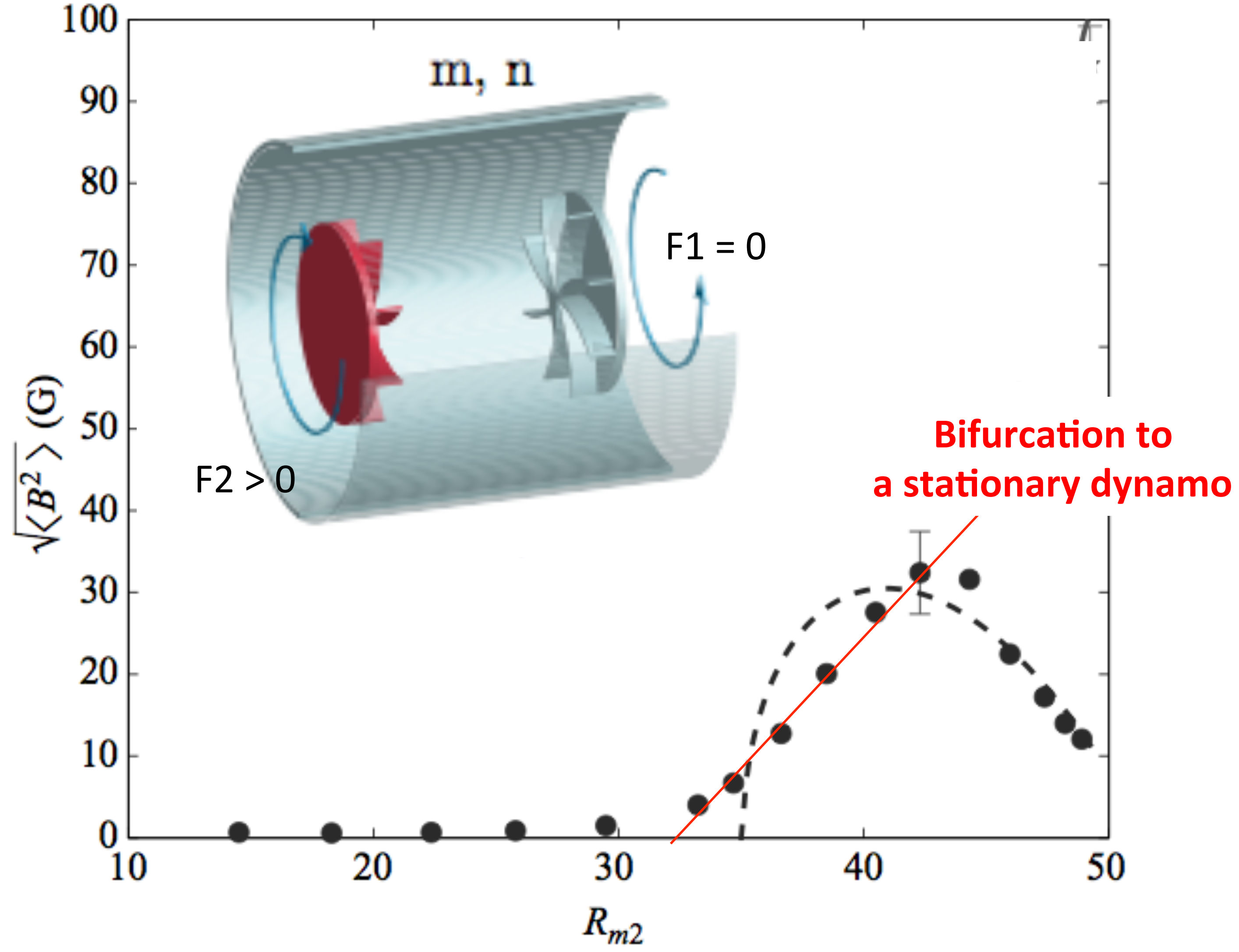
$$Rm^{\alpha} \cdot Rm^{\omega} \sim 0.1\,\alpha K$$

α≈0.1, Ga measurements
Stepanov et al.,*PRE* **73** (2006)

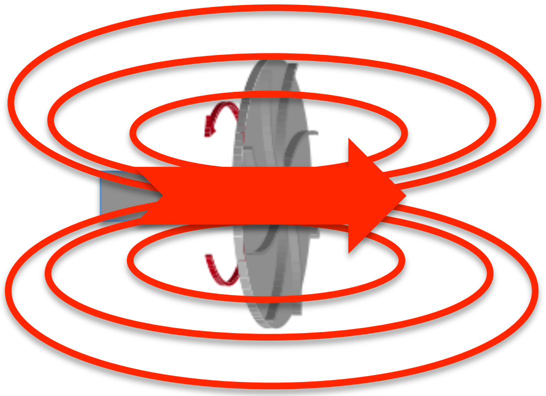
K: iron enhancement,
K≈100, Ga measurements.
G. Verhille et al., *NJP* **12** (2010)

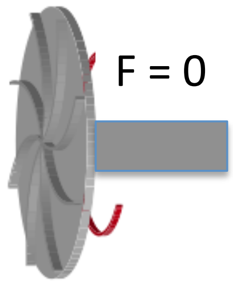
1Disk – dynamo generation

M. Berhanu et al., *JFM* **641** (2009)

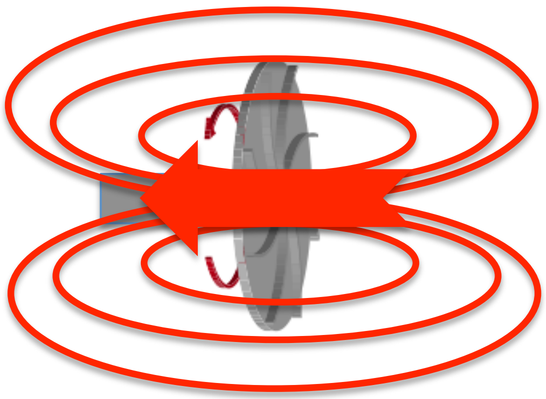


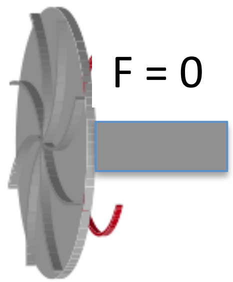
VKS / 1 Disk dynamo



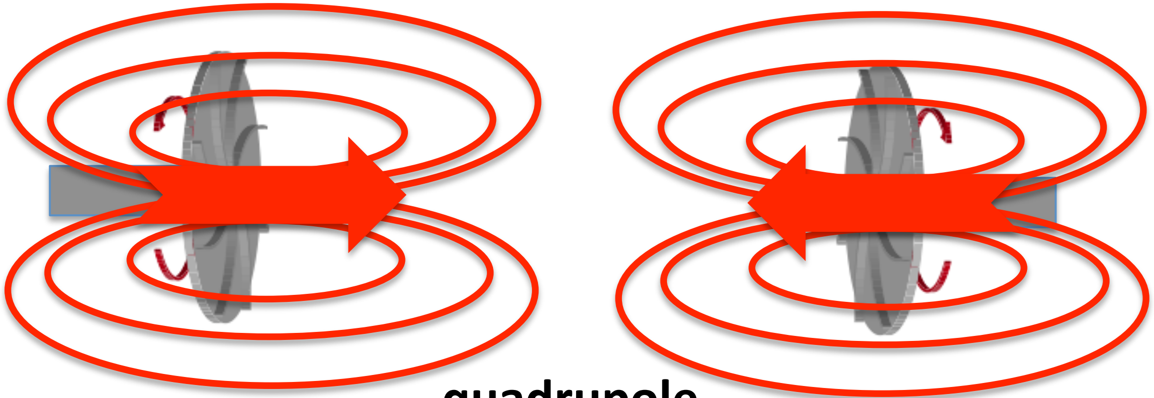


B and –B are
equally possible

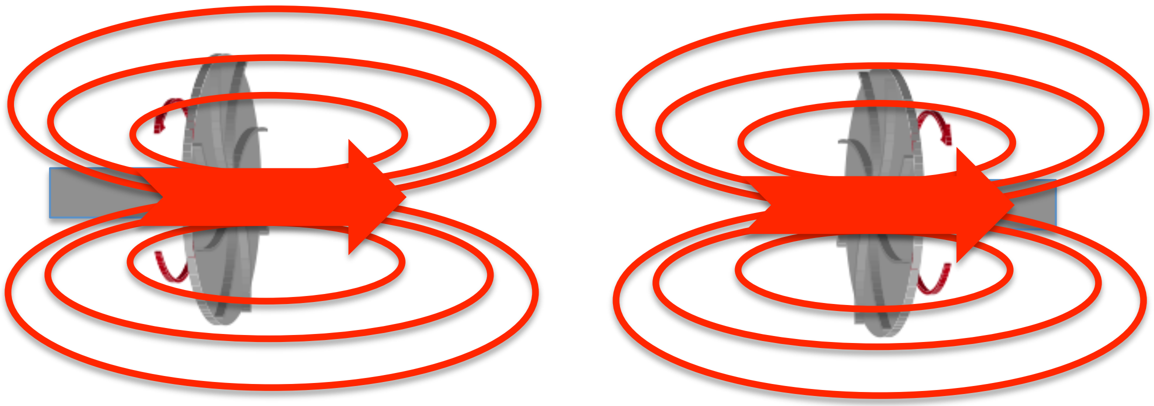




VKS / 2 disks : dynamo modes



quadrupole

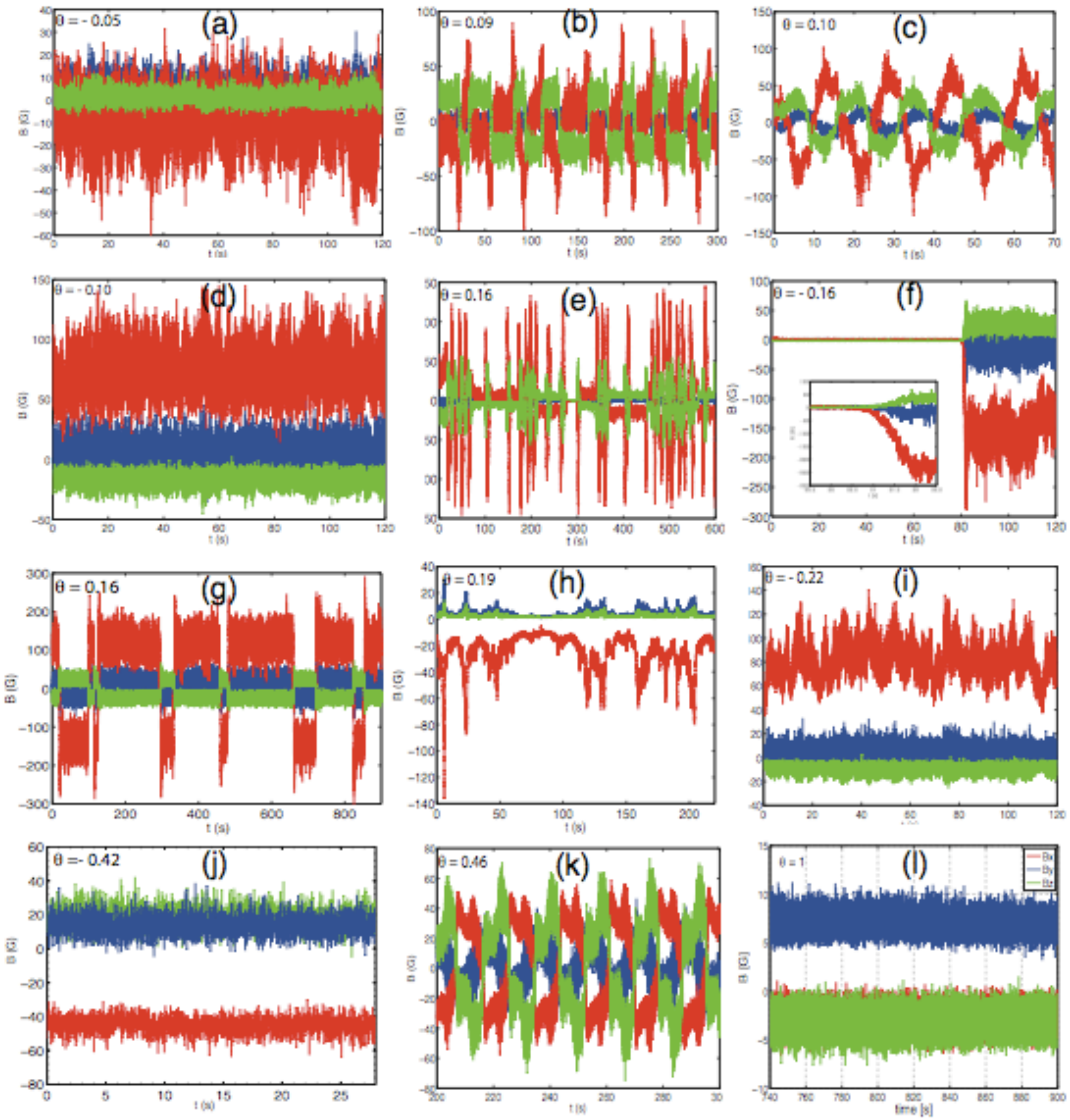


dipole

$$\partial t\ D = f_{NL}(D,Q)$$
$$\partial t\ Q = g_{NL}(D, Q)$$

➔ **Dynamics !**

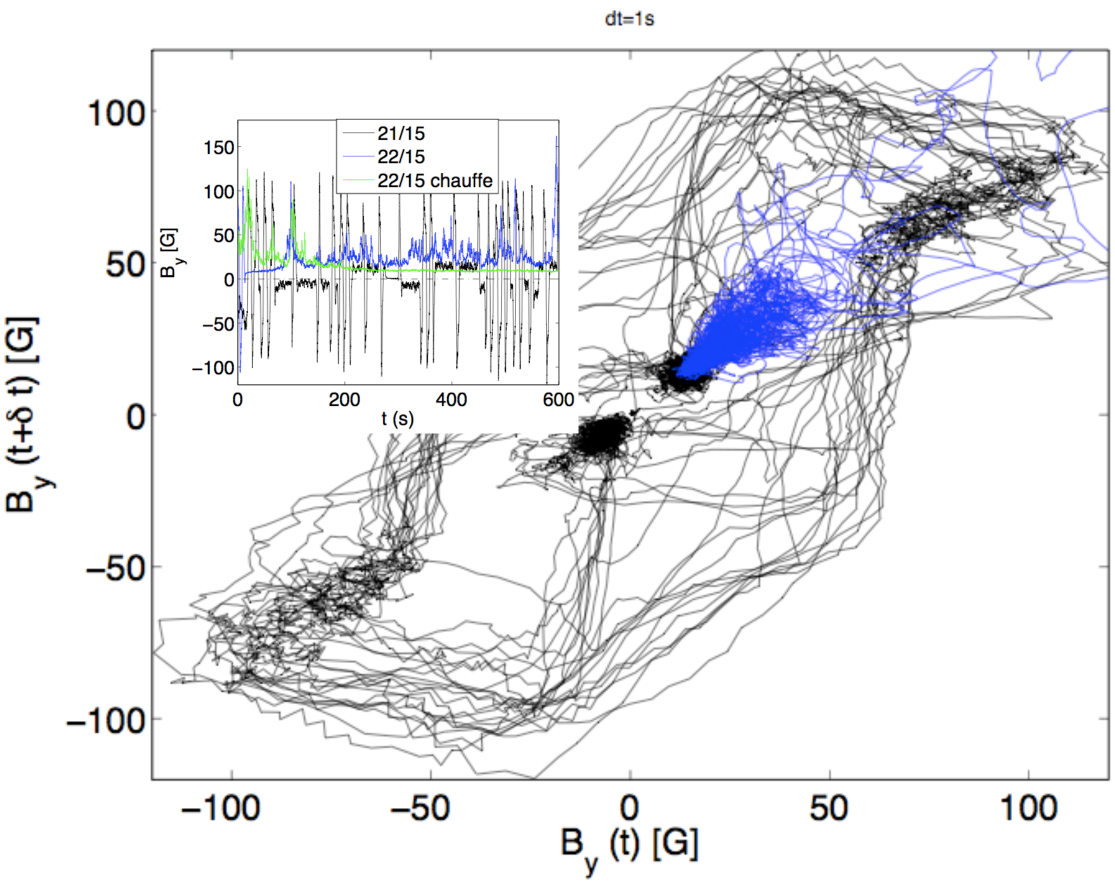
Monchaux et al., *PoF* **21** (2009)



Low dimensional behavior

R. Monchaux et al., *Phys. Rev. Lett.* **101**, 074502 (2008)

F. Petrelis & S. Fauve, *J Phys-CM* **20** (2008)

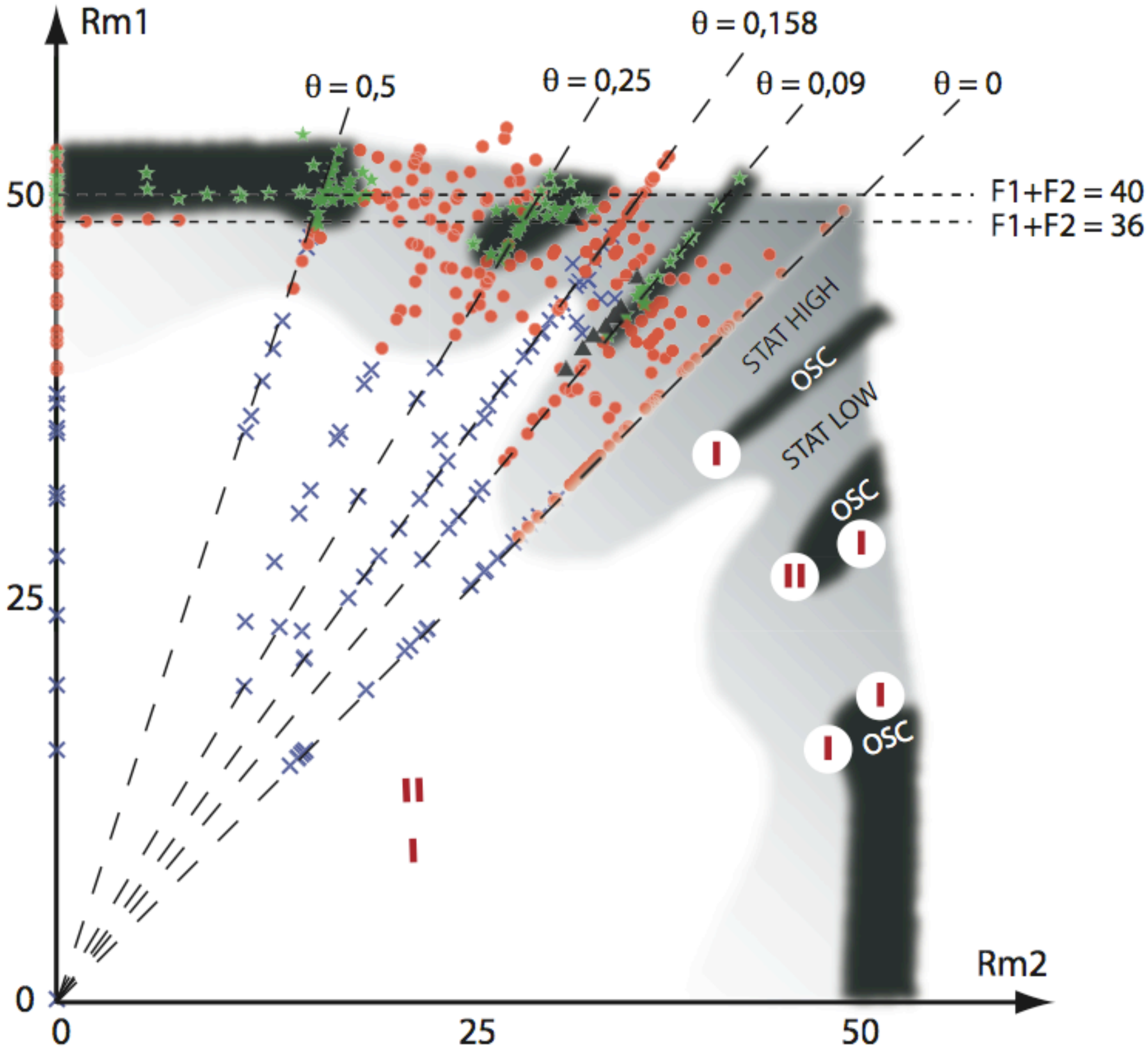


$$A = D + iQ = R\exp(i(\theta + \theta_0))$$

$$\dot{A} = \mu A + \nu\bar{A} + \beta_1 A^3 + \beta_2 A^2\bar{A} + \beta_3 A\bar{A}^2 + \beta_4\bar{A}^3$$

Parameter space

M. Berhanu et al., *Euro. Phys. J. B* (2010)

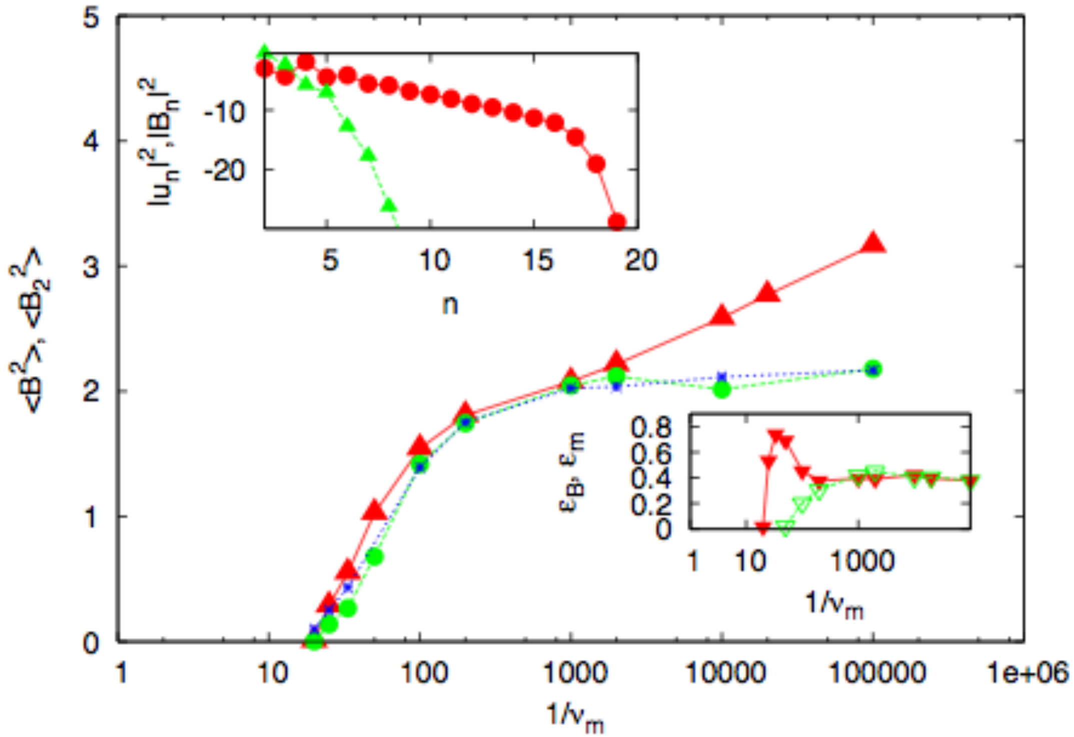


Shell model: deterministic dynamics + "turbulent" fluctuations

R. Benzi, JFP, *PRL* **105** (2010)

$$\frac{du_n}{dt} = \frac{i}{3}(\Phi_n(u,u) - \Phi_n(B,B)) - \nu k_n^2 u_n + f_n \ ,$$
$$\frac{dB_n}{dt} = \frac{i}{3}(\Phi_n(u,B) - \Phi_n(B,u)) - \nu_m k_n^2 B_n \ ,$$

$$\frac{dB_2}{dt} = F_2(u,B) - M_2(B_2) - \nu_m k_2^2 B_2$$

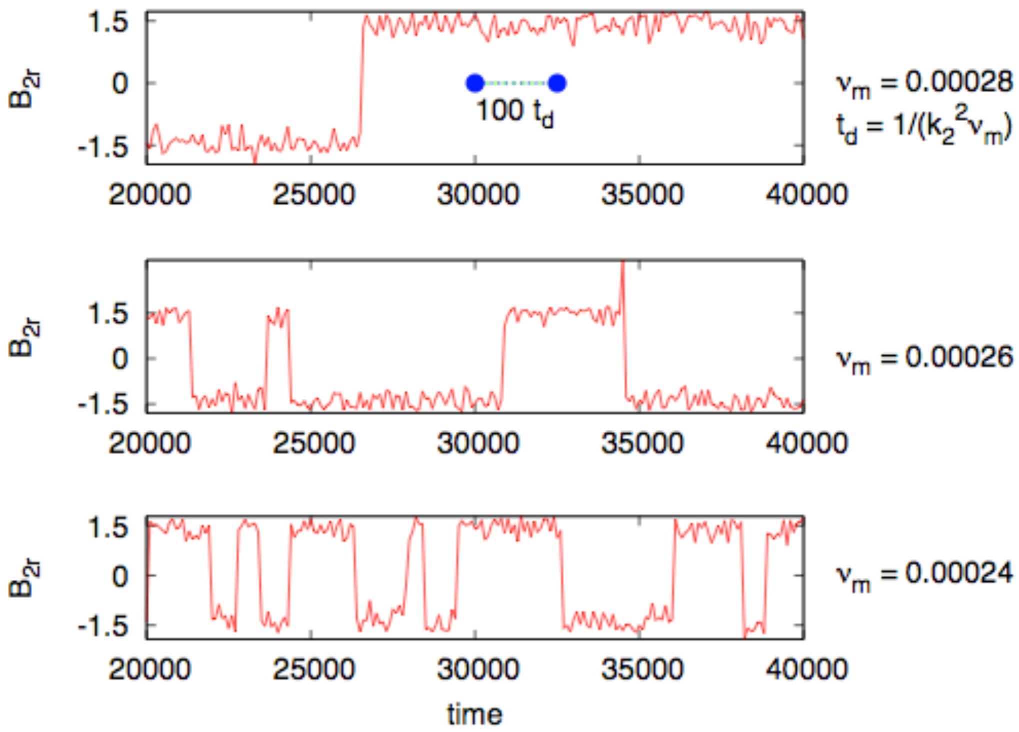


Shell model: deterministic dynamics + “turbulent” fluctuations

R. Benzi, JFP, *PRL* **105** (2010)

$$\frac{du_n}{dt} = \frac{i}{3}(\Phi_n(u,u) - \Phi_n(B,B)) - \nu k_n^2 u_n + f_n \ ,$$
$$\frac{dB_n}{dt} = \frac{i}{3}(\Phi_n(u,B) - \Phi_n(B,u)) - \nu_m k_n^2 B_n \ ,$$

$$\frac{dB_2}{dt} = F_2(u,B) - M_2(B_2) - \nu_m k_2^2 B_2$$

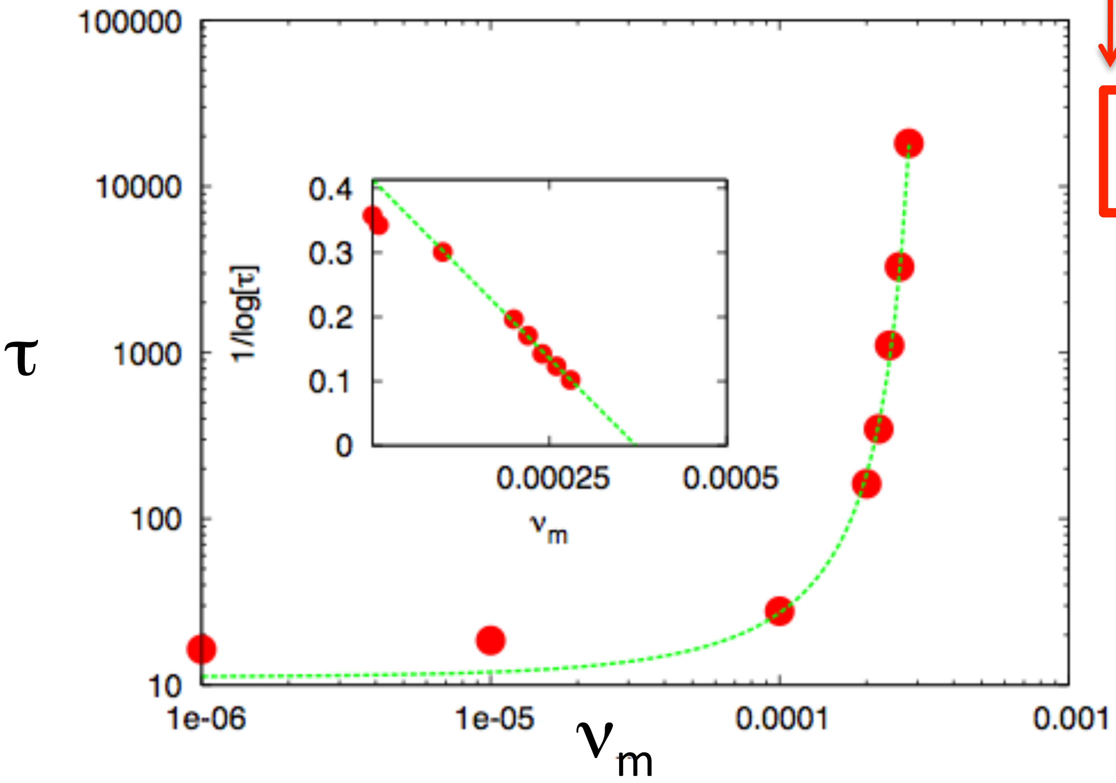


Shell model: deterministic dynamics + "turbulent" fluctuations

$$\frac{dB_2}{dt} = F_2(u,B) - M_2(B_2) - \nu_m k_2^2 B_2$$

$$F_2(u,B) = \beta B_2 + \phi'$$

$$\frac{dB_2}{dt} = \beta B_2 - a_m B_2^3 + \phi' \qquad \tau \sim \exp\left(\frac{\beta^2}{a_m \sigma}\right) = \exp\left(\frac{A}{\sigma}\right)$$



governed by variance of "noise"

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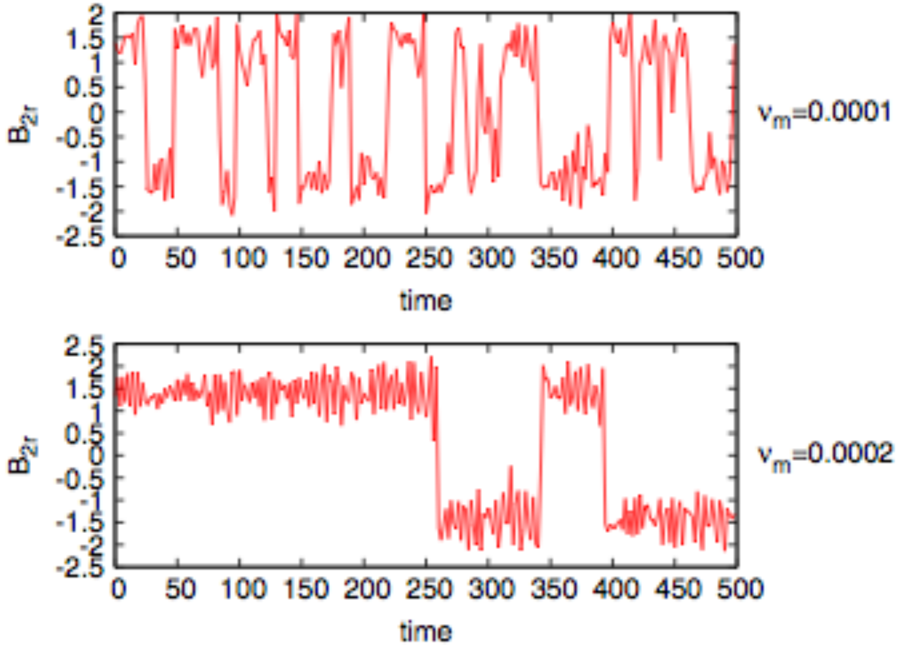


Time-scale separation !

OUTLINE

1. **Dynamos**
   Scalar, scalar gradient and B-equations
   Is turbulence helpful or detrimental?
   Experiments

2. **Mean field MHD**
   easy and hard
   Examples
   Measurements

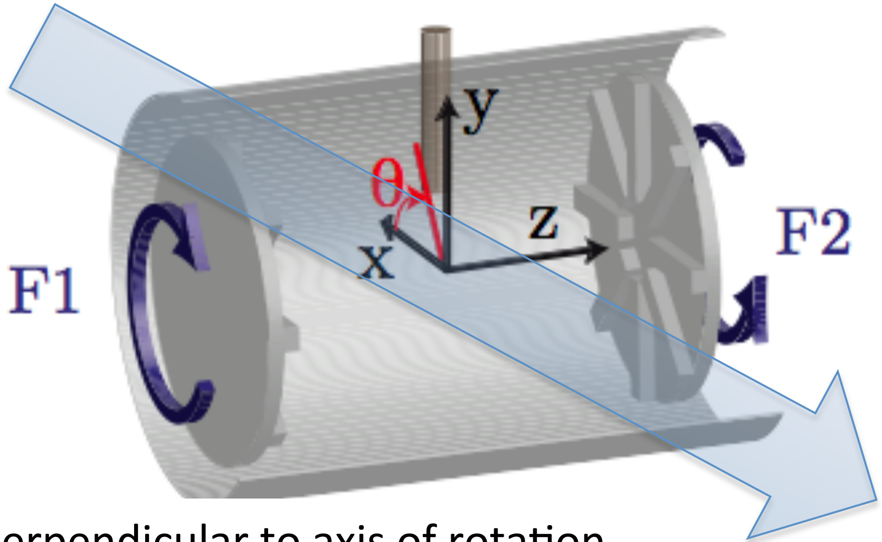
3. **Turbulence in VKS**
   the $\alpha-\omega$ "Duddley & James" failed dynamo
   the $\alpha-\omega$ ferroDynamo.
   mechanism and low dimensional behavior
   turbulence and noise.

4. **Turbulent MHD**
   Saturation: large B-field experiment.

5. **Next**

Magnetic field and vorticity interactions

G. Verhille, preprint (2011)



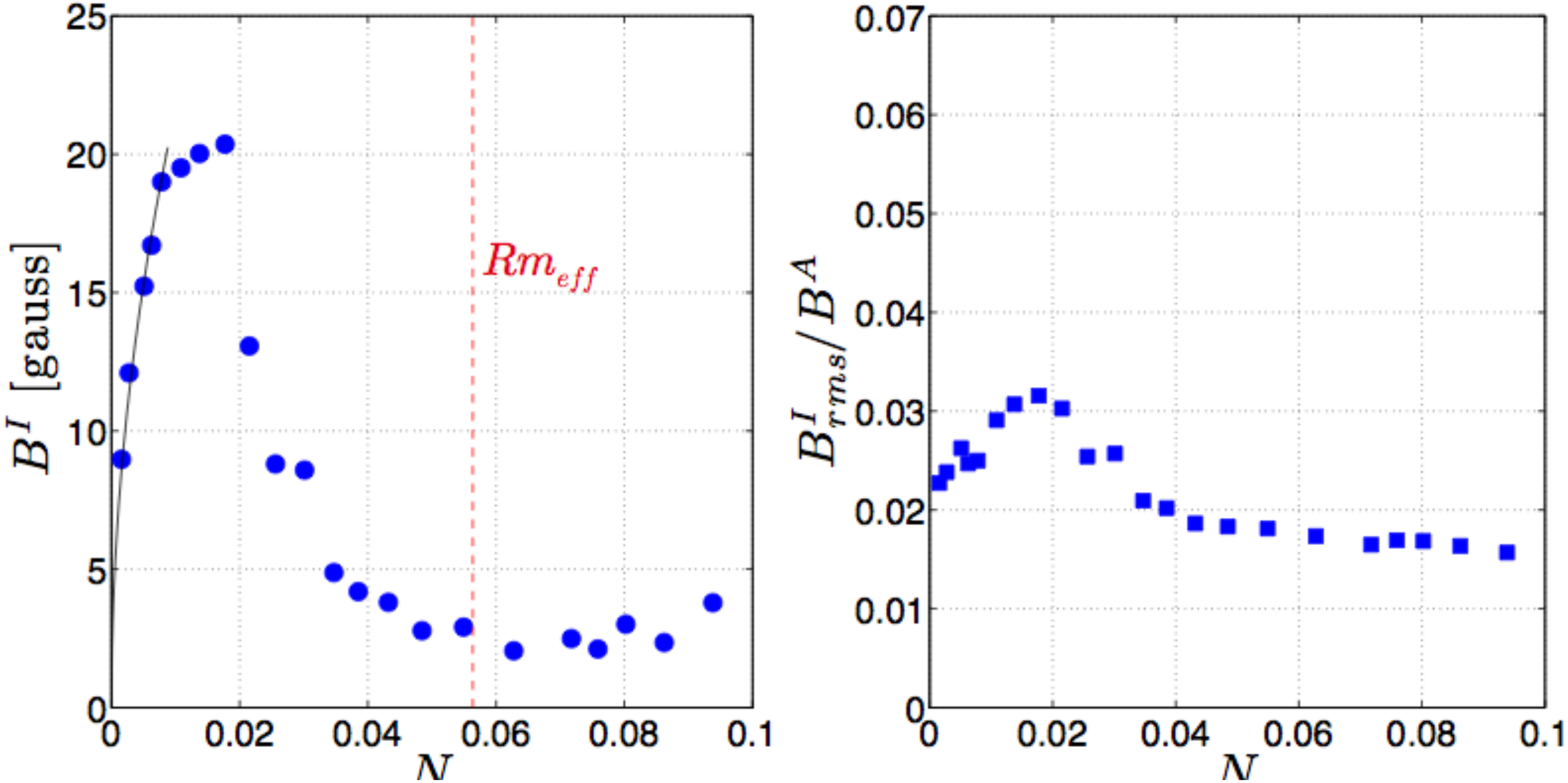
$\mathbf{B}^A$ perpendicular to axis of rotation

$\mathbf{B}^A$ in [0 – 1500] G

N in [0 – 0.5]

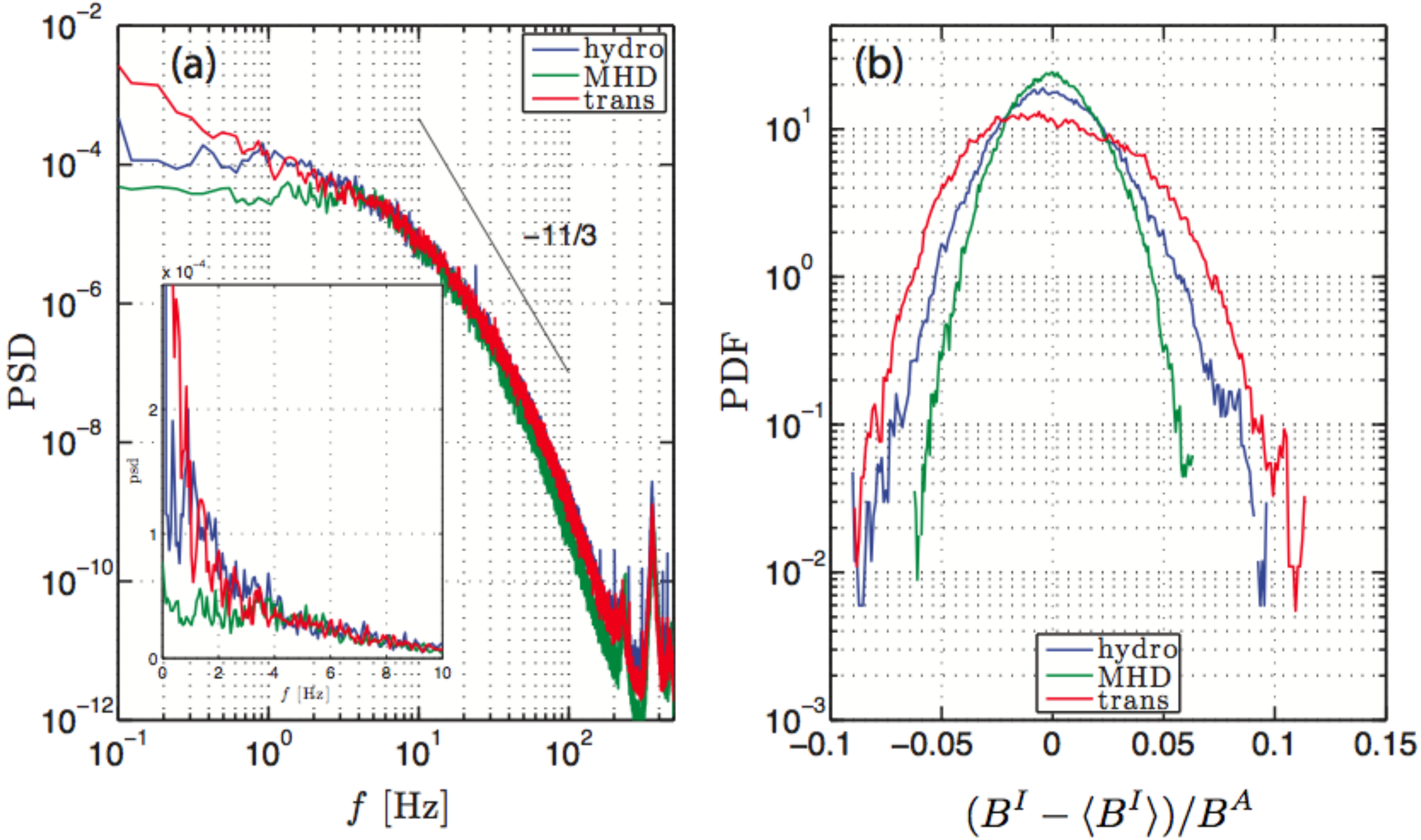
Magnetic field and vorticity interactions

G. Verhille, preprint (2011)

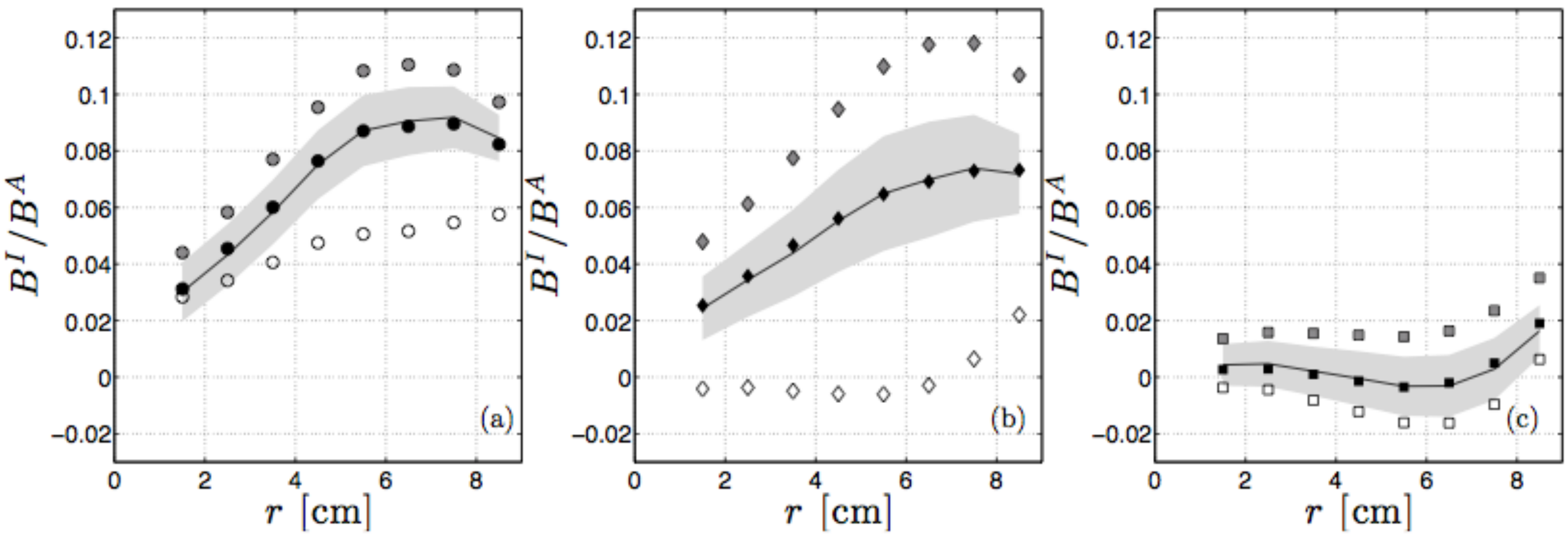


$$N = \frac{\sigma R {B^A}^2}{\rho u_{\mathrm{rms}}} = \mu\sigma R u_{\mathrm{rms}} \cdot \frac{{B^A}^2}{\rho\mu u_{\mathrm{rms}}^2} = Rm \cdot \left(\frac{v_A}{u_{\mathrm{rms}}}\right)^2$$

Magnetic field and vorticity interactions



Magnetic field and vorticity interactions



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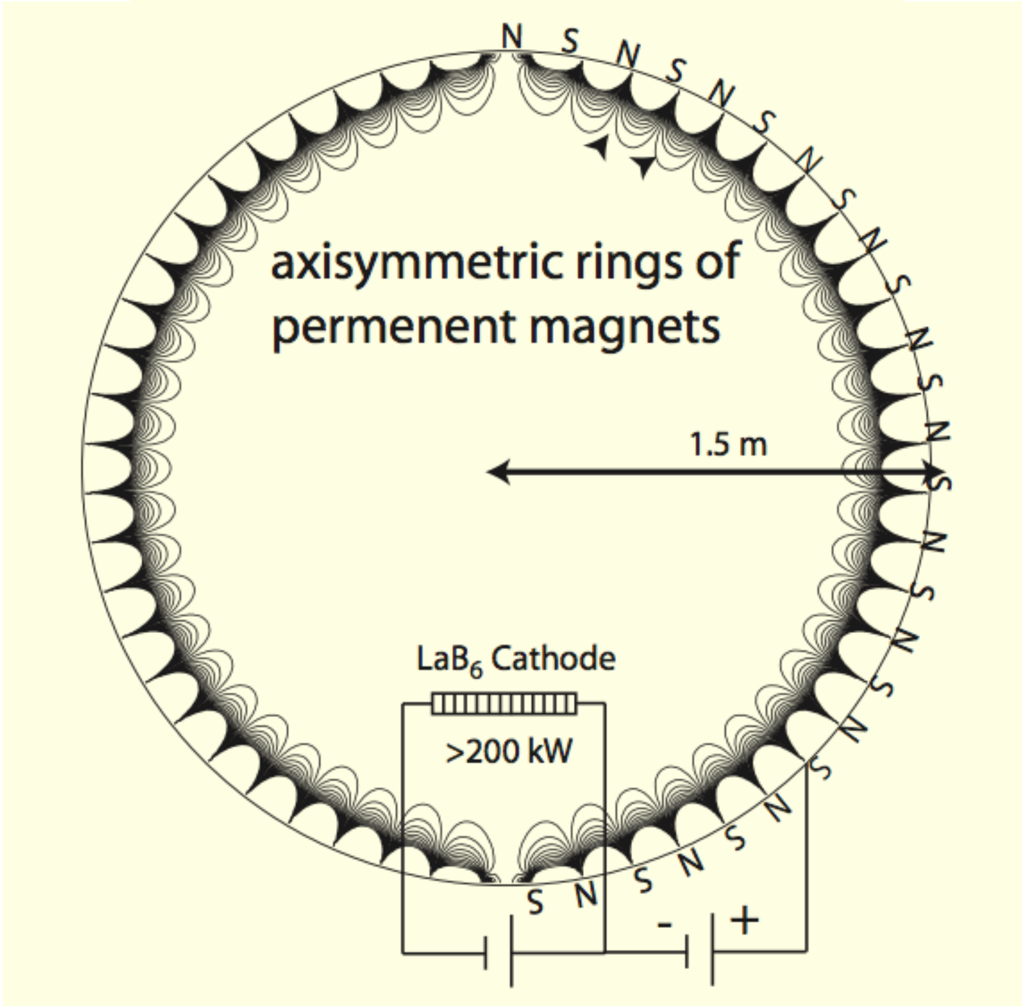
Quantity	Symbol	Value	Unit
Plasma radius	a	1.1	m
Number density	n	10^{17}–10^{19}	m^{-3}
Ion temperature	T_i	0.5–4	eV
Electron temperature	T_e	2–10	eV
Peak speed	U_{max}	0–20	km s^{-1}
Ion species	H, He, Ar	1, 4, 40	amu
Pulse length	τ_{pulse}	5	s
Plasma beta	β	10^4	
Resistive time	τ_η	50	ms
Magnetic Reynolds number	Rm_{max}	∼1000–2000	
Reynolds number	Re	2.4×10^1 to 3.8×10^6	
Magnetic Prandtl number	Pm	3.0×10^{-4} to 5.6×10^1	

$$Pm = 0.18 \frac{T_e^{3/2} T_i^{5/2}}{\mu^2 n} \qquad Rm = 1.5 \frac{T_e^{3/2} U a}{Z}$$

Wisconsin plasma dynamo

Cary Forest

Rm_{max}	> 1000
Re	24—3.8×10^{6}
Pm	3×10^{-4}—56



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VKS

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