THE AUSTRALIAN NATIONAL UNIVERSITY

Turbulence and condensate in fluid layers

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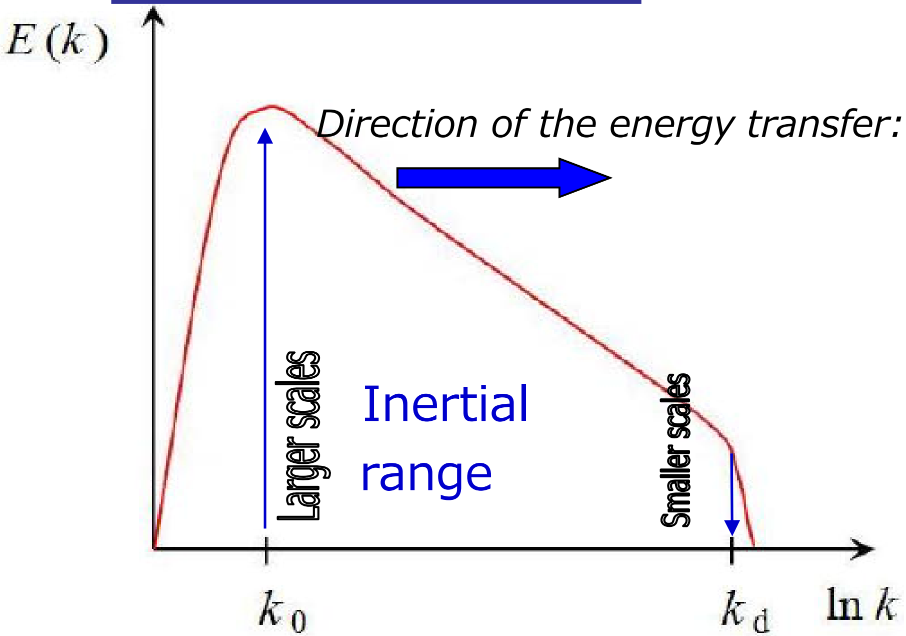
Kolmogorov theory (1941)

$$S_3(l) = \left\langle \delta v_L^3 (r, l) \right\rangle = -\frac{4}{5}\varepsilon l$$

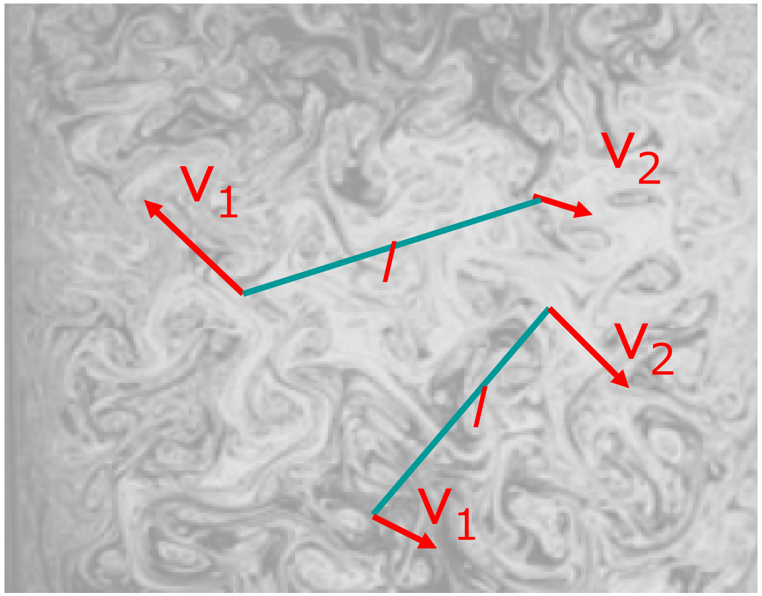
Kolmogorov spectrum:

$$E(k) = C\varepsilon^{2/3} k^{-5/3}$$



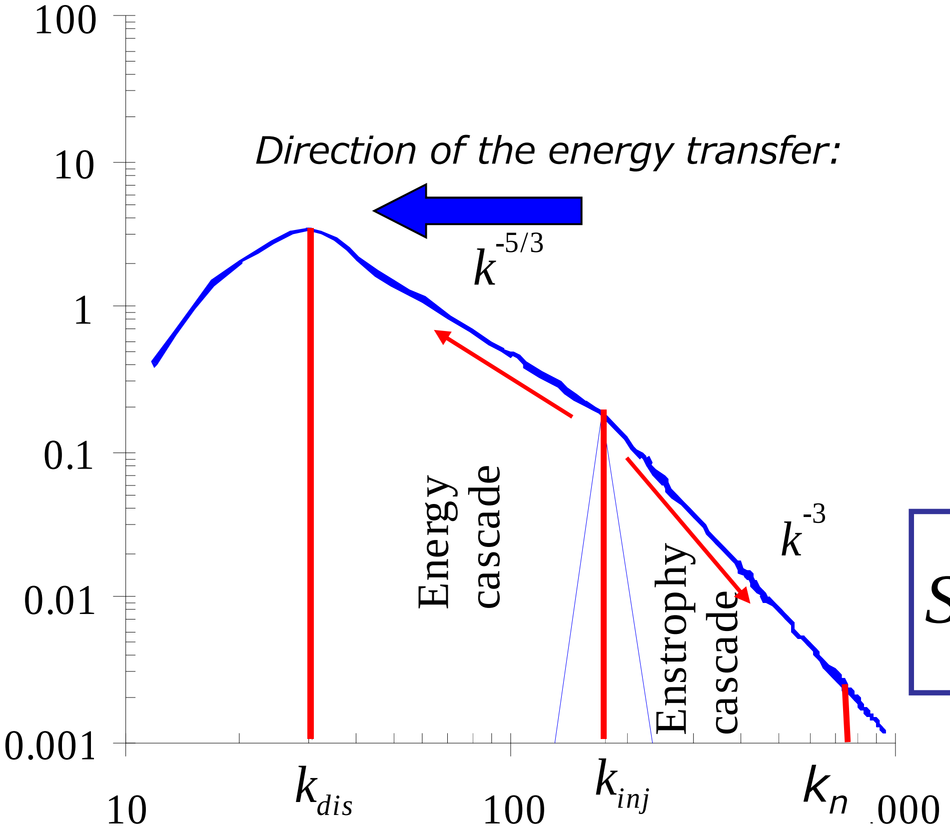
Structure functions = statistical moments of velocity increments δv_L across a distance l:

$$S_n(l) = \left\langle \delta v_L^n (\mathbf{r}, l) \right\rangle = \left\langle \left(v_L(\mathbf{r}) - v_L(\mathbf{r} + l) \right)^n \right\rangle$$



Negative S_3 ⇔ energy cascades from large to small scales

Inverse energy cascade in 2D turbulence



Robert Kraichnan (1967)

vorticity: $\boldsymbol{\omega} = \nabla \times \mathbf{v}$

enstrophy $\Omega = \frac{1}{2}\int_V |\boldsymbol{\omega}|^2 dV$

Kolmogorov theorem in 2D:

$$S_{3L}(r) = \left\langle \delta V_L^3(r) \right\rangle = \frac{3}{2}\varepsilon r > 0$$

In a 2D flow both energy and enstrophy are conserved!

$$E(k) = C\varepsilon^{2/3}k^{-5/3}$$

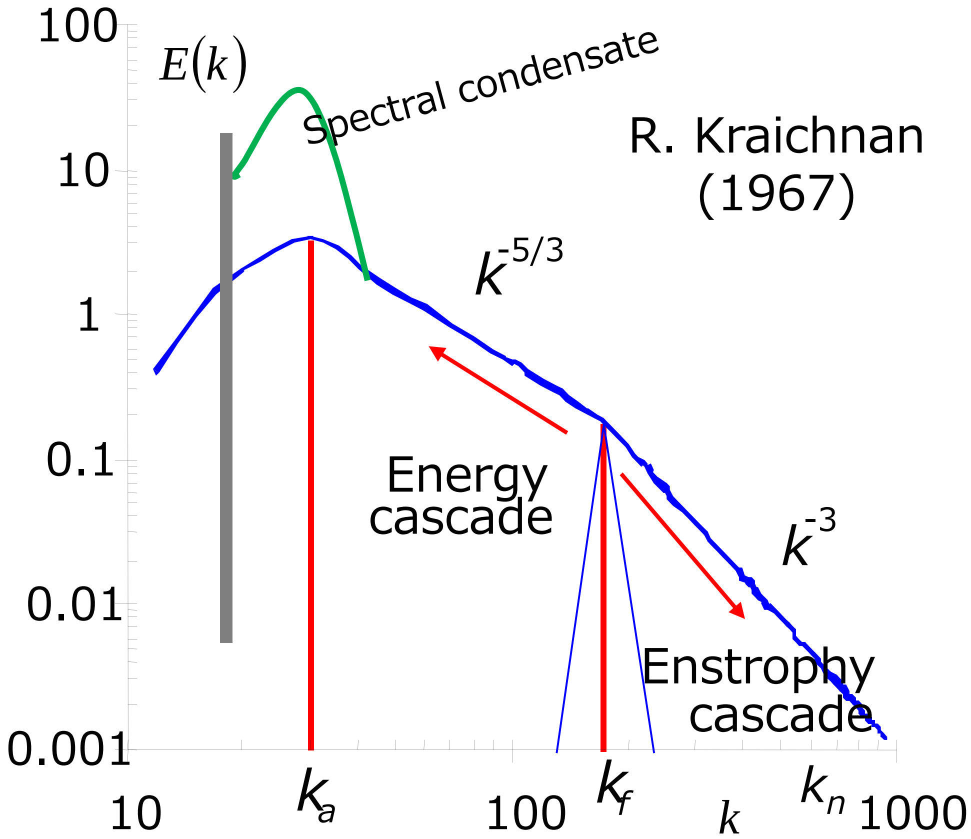
Energy cascade: inverse cascade

$$E(k) = C_\omega \varepsilon_\omega^{2/3} k^{-3}$$

Enstrophy cascade: forward cascade

Turbulence formation and spectral condensate

Idealized homogeneous isotropic 2D turbulence



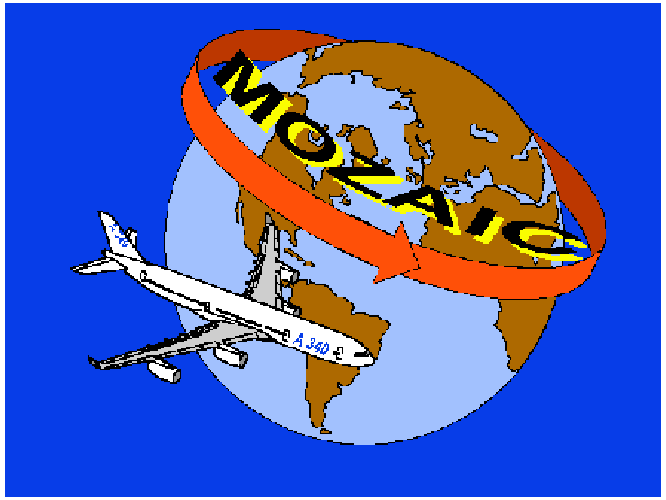
Application of turbulence theory to atmosphere

Interpretation of atmospheric wind measurements:

Nastrom-Gage spectrum (1983)

Energy flux measurements (2001)

Atmospheric sampling programs

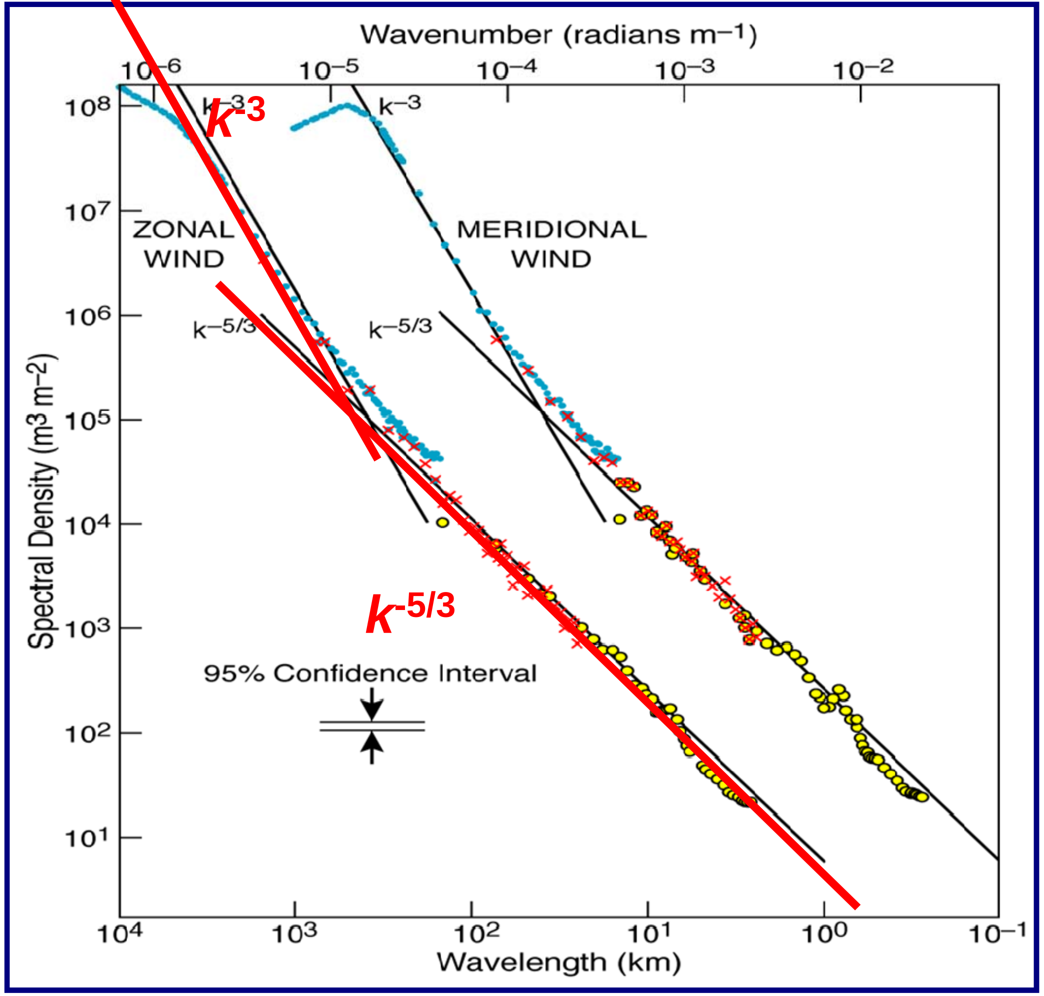


- NASA Global Atmospheric Sampling Program:
  - 1972-1979
- Measurement of Ozone and Water Vapor by Airbus In-Service Aircraft (MOZAIC) program
  - 1994-2008
- Civil Aircraft for the Regular Investigation of the atmosphere Based on an Instrument Container (CARIBIC)
  - Started 1997
- Integration of routine Aircraft measurements into a Global Observing System (IAGOS project)
  - Started 2006

Turbulent wind spectra: a 30 year long puzzle

Nastrom-Gage spectrum (1983)

Wind spectra analyzed based on data from 6900 aircraft flights



k^{-3} and $k^{-5/3}$ ranges are present but in the reversed order compared to the Kraichnan theory

$$E(k) = C_k \varepsilon^{2/3} k^{-5/3} \quad \text{at } k < k_f$$

$$E(k) = C_\omega \varepsilon_\omega^{2/3} k^{-3} \quad \text{at } k > k_f$$

What is the origin of

k^{-3} and $k^{-5/3}$ ranges in atmosphere?

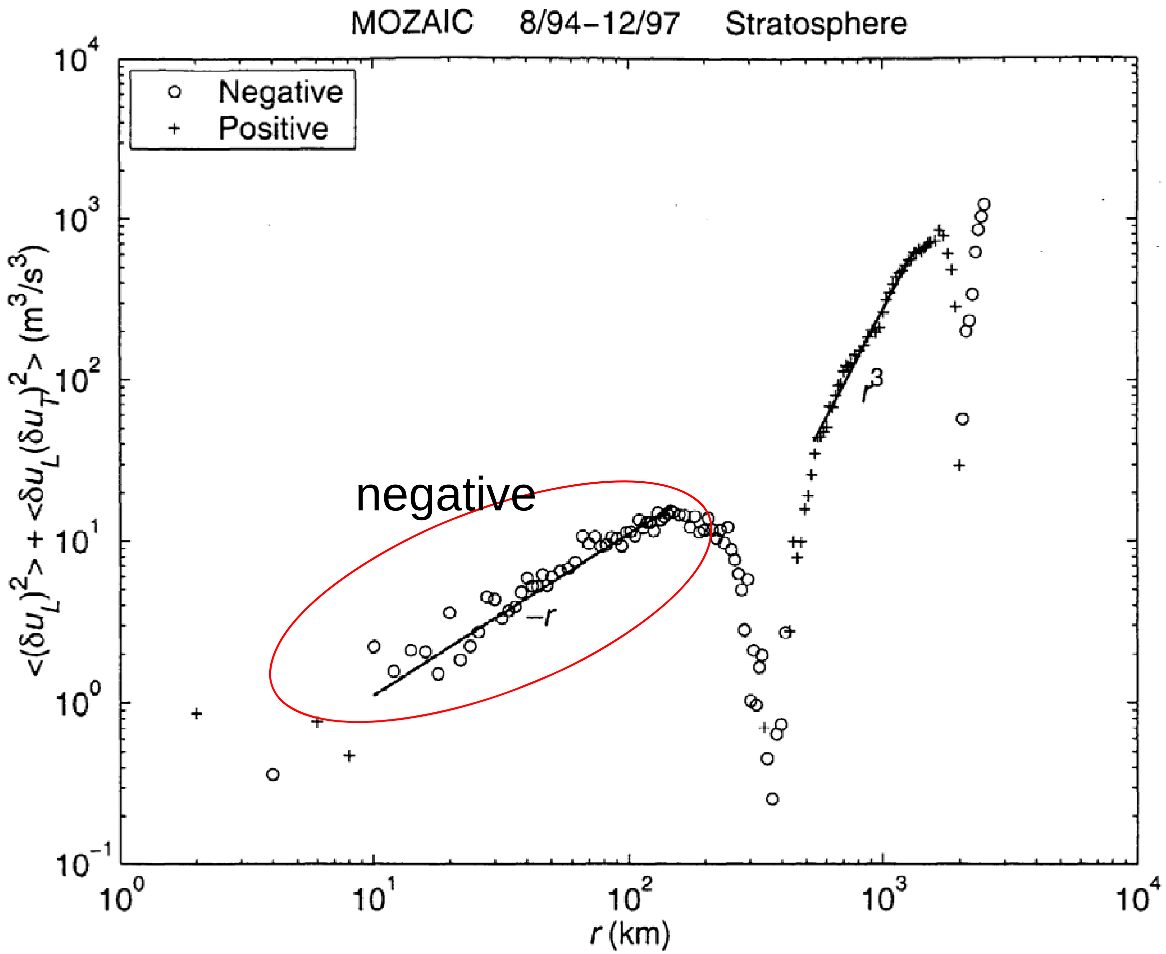
Meso-scale $k^{-5/3}$ range can be due to

- *3D (downscale) direct energy cascade,*
- *2D inverse (upscale) cascade*

Large-scale k^{-3} range can be due to

- *direct enstrophy cascade (large-scale forcing)*
- *spectral condensation*

Energy flux measurement in the atmosphere



Circles and crosses indicate negative and positive values

[Cho, Lindborg, J. Geophys. Res. (2001)]

Problems in 2D turbulence theory application

Laboratory experiments in thin fluid layers:

- “Intrinsic” three-dimensionality of fluid layers

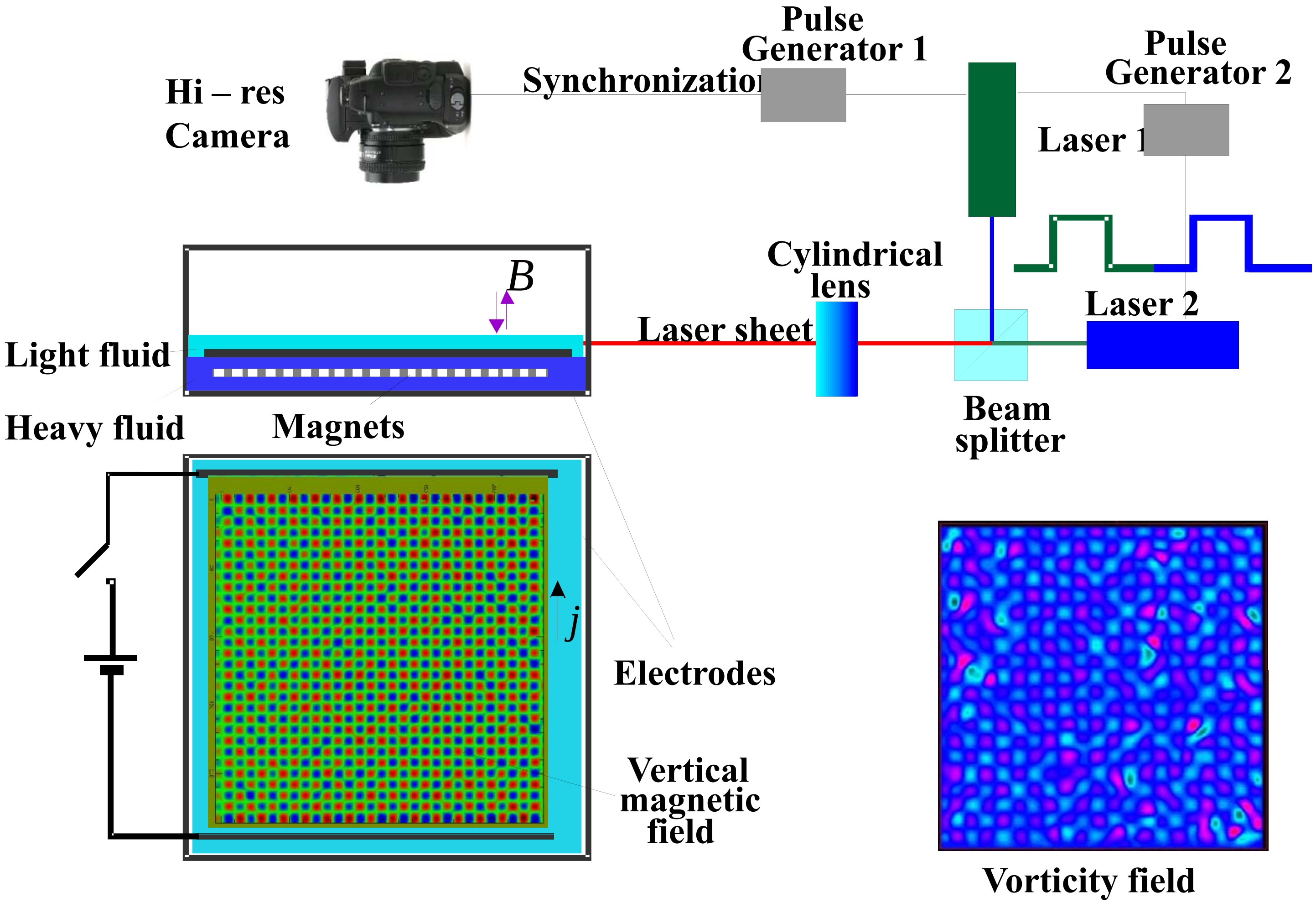
[e.g. Akkermans et al. 2008-2010]

- Finite dissipation => no inertial interval

[e.g. Lindborg, 2008]

- Isotropy assumption – broken in the presence of anisotropic coherent flows, e.g condensate?

Experiments on turbulence in thin layers



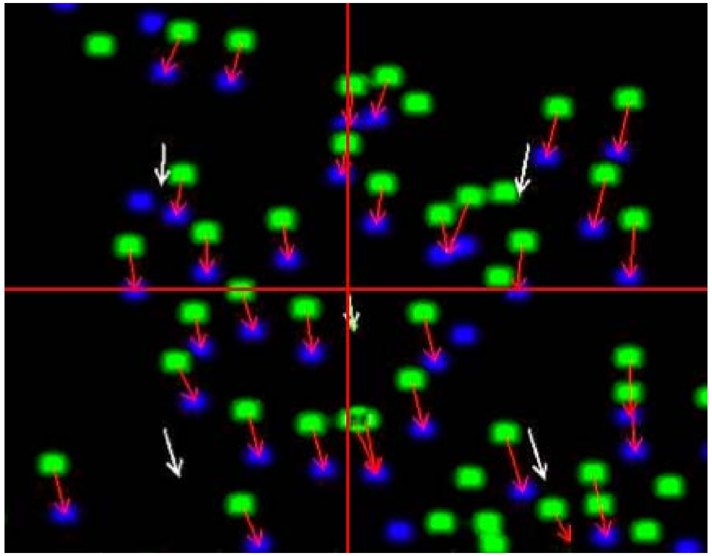
Configurations:

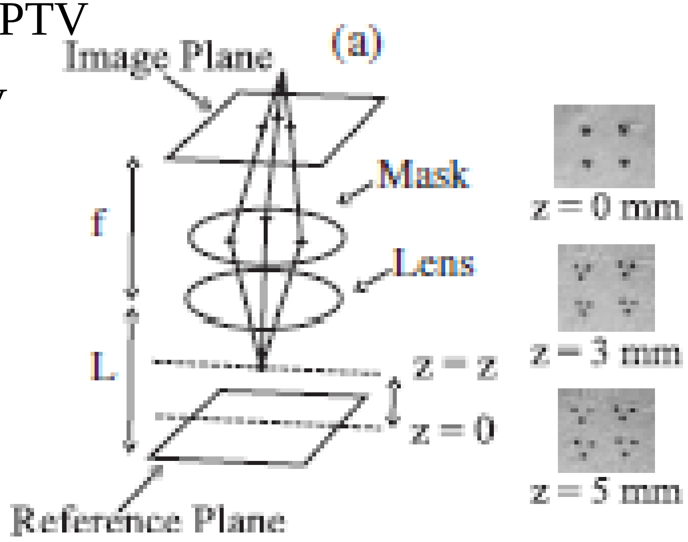
- Single layer: electrolyte
- Double layer: heavier non-conducting fluid
- Various forcing scale: 8mm, 10mm, 25mm

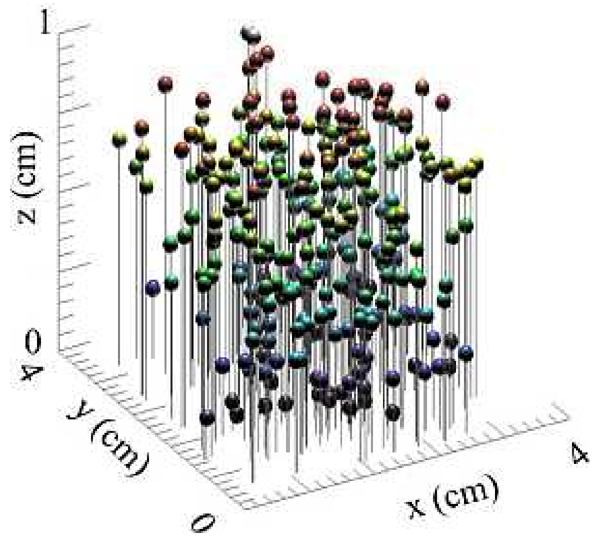
Visualization: vertical and horizontal streaks

Measurement techniques:

- PIV
- Two-color PIV/PTV
- Defocusing PIV





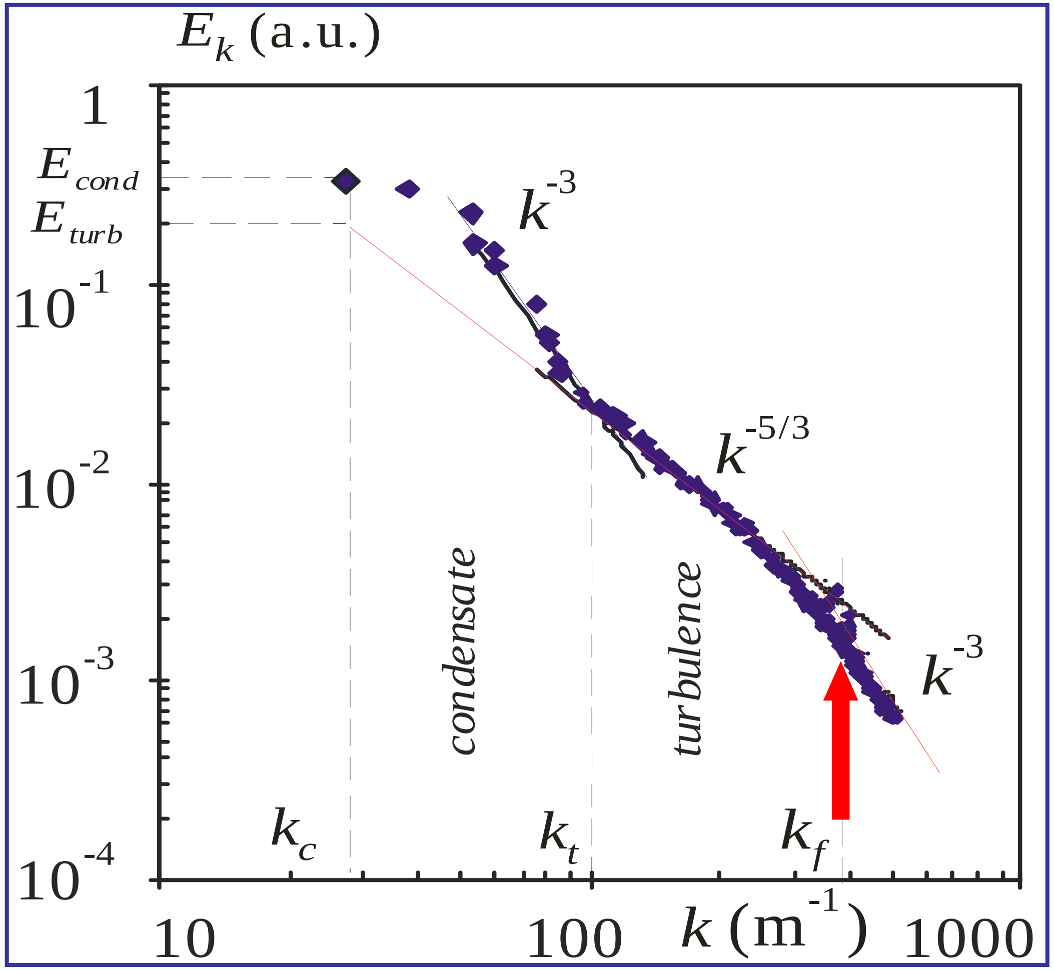


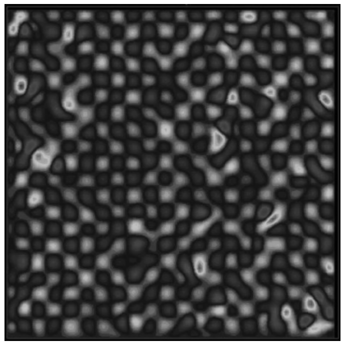
Quasi-2D experiments:

- Double layers
- Thin layer

Turbulence formation and spectral condensate

Spectrum of condensed turbulence





[Xia, et al, Phys. Fluids, **21,** 125101 (2009);
Chertkov, et al, Phys. Rev. Lett. **99**, 084501 (2007)]

Vortex-turbulence interaction

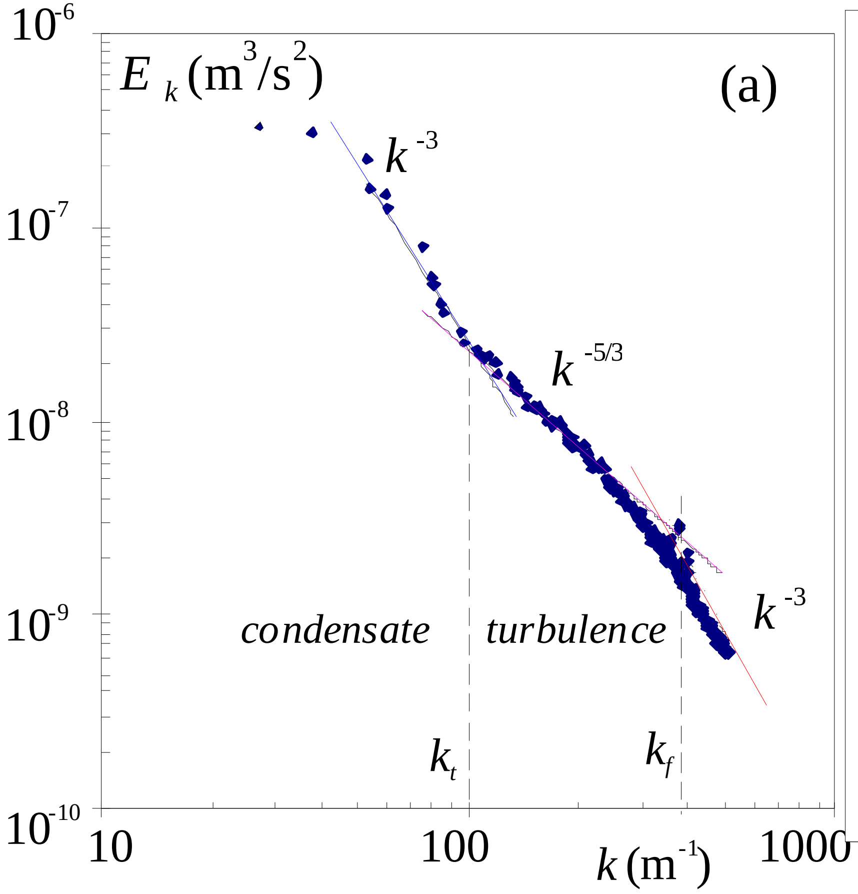
- Vortex suppresses turbulence (shear etc.)

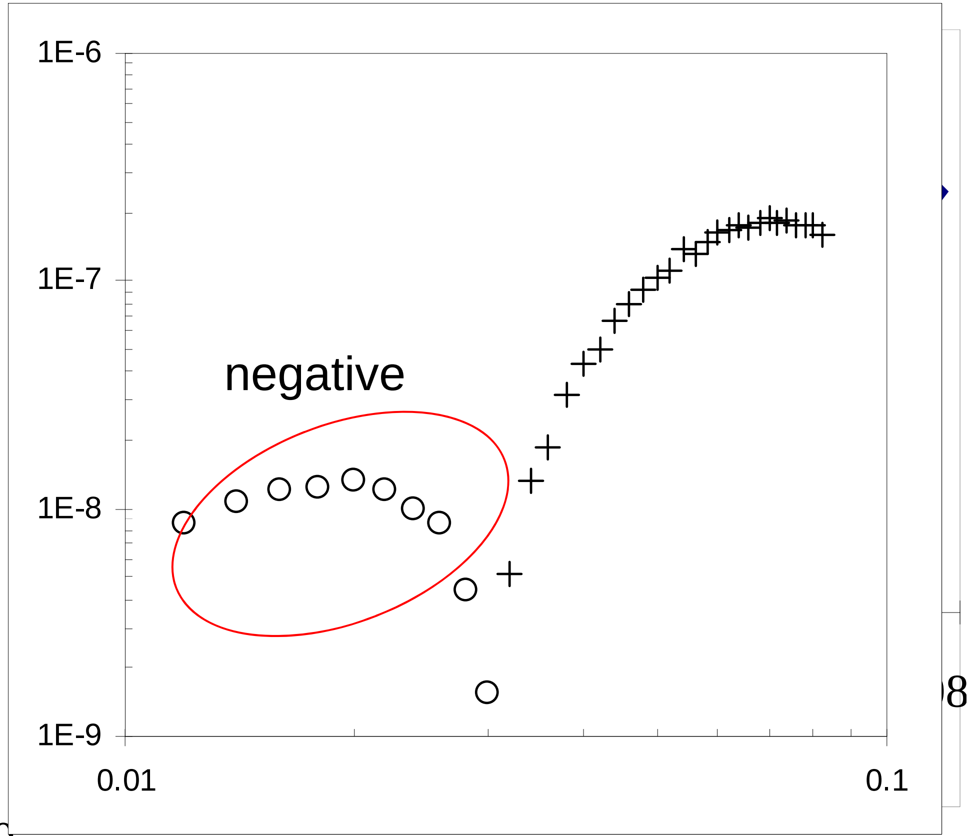
  [Shats, et al, Phys. Rev. Lett. (2007)]

- How the energy is distributed in the system

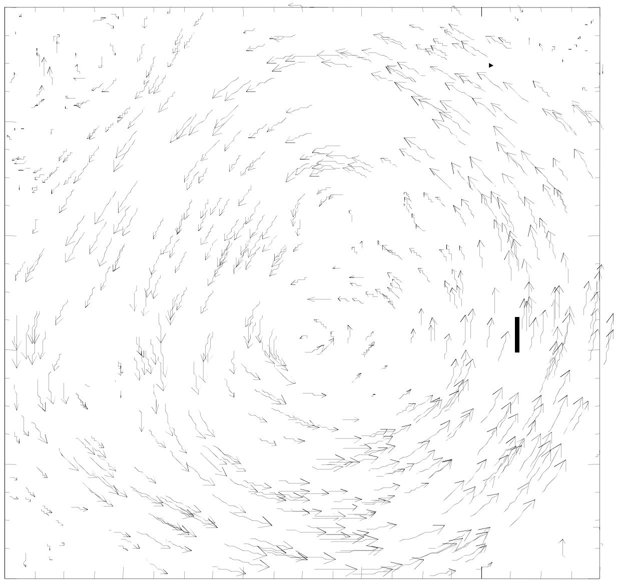
  [Xia, et al, Phys. Rev. Lett. (2008); Xia et al, Phys. Fluids, (2009)]

Condensate: experimental results





Turbulence inside the vortex



Coherent structure =
mean flow =
time average

$$\delta V = \delta\overline{V} + \delta\tilde{V}$$

$$\left\langle \delta V^2 \right\rangle = \left\langle \delta\overline{V}^2 + \cancelto{0}{2\delta\overline{V}\,\delta\tilde{V}} + \delta\tilde{V}^2 \right\rangle$$

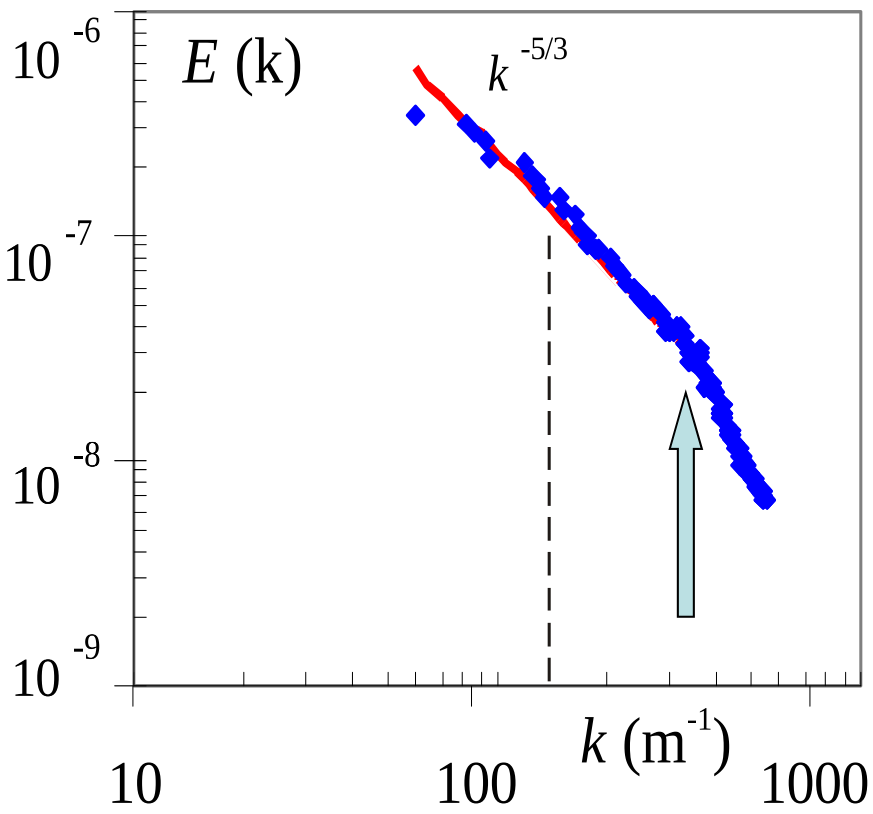
$$\left\langle \delta V^3 \right\rangle = \left\langle \delta\overline{V}^3 - \cancelto{0}{3\delta\overline{V}^2\delta\tilde{V}} + 3\delta\overline{V}\,\delta\tilde{V}^2 - \delta\tilde{V}^3 \right\rangle$$

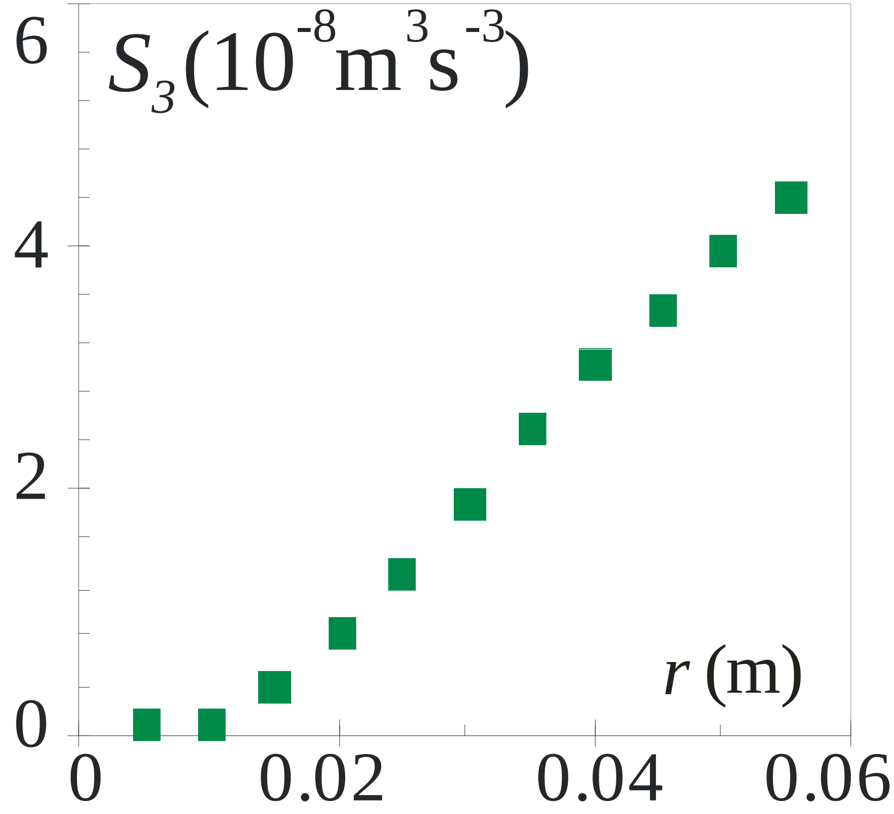
Mean flow modifies velocity moments

Mean subtraction recovers isotropic turbulence!

1. Compute time-average velocity field (N=400): $\overline{V}(x,y) = 1/N \sum_{n=1}^{N} V(x,y,t_n)$

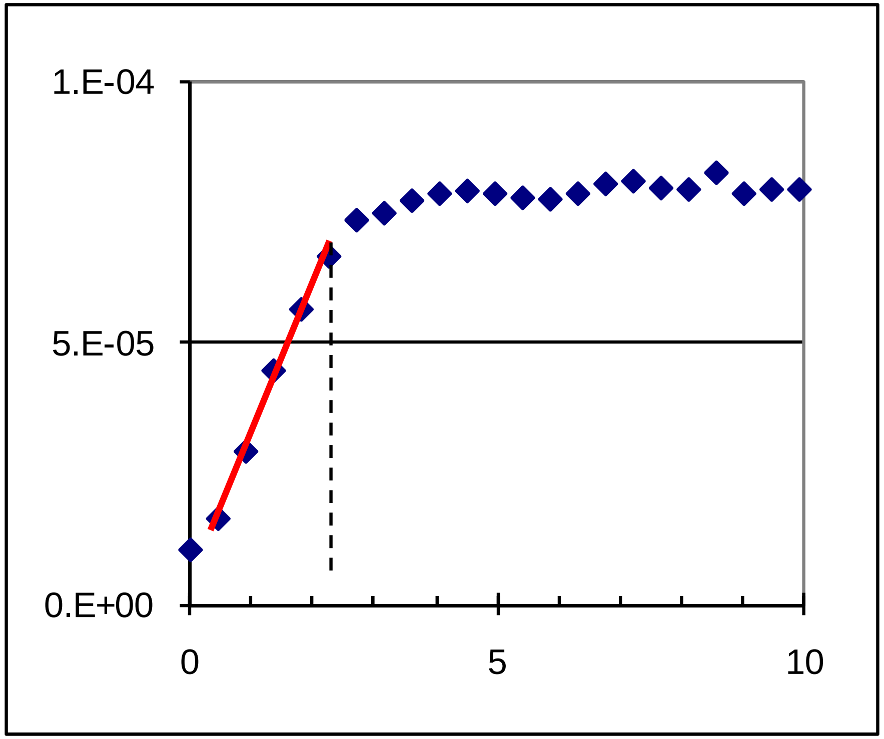
2. Subtract $\overline{V}(x,y)$ from instantaneous velocity fields

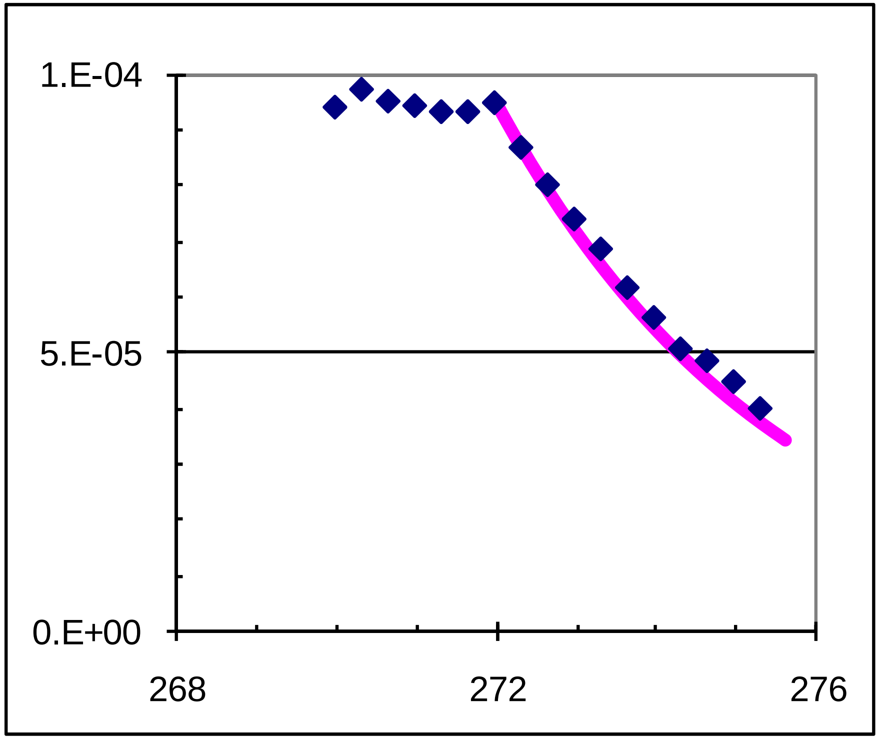




[Xia, et al, Physical Review Letters **101**, 194504 (2008)]

Energy balance



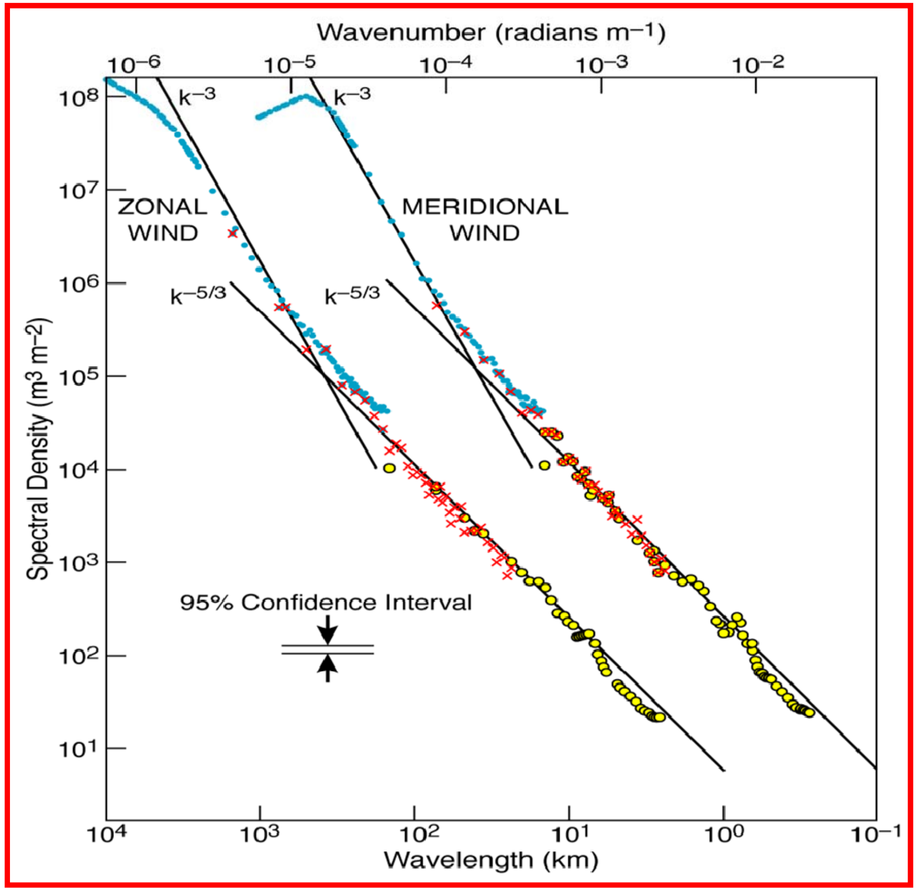


$$\varepsilon_{in} = \frac{dE}{dt}\Big|_{t=0} \qquad \frac{dE}{dt} = \varepsilon - \alpha E \qquad E_t = E_{t_0} e^{-\alpha(t-t_0)}$$

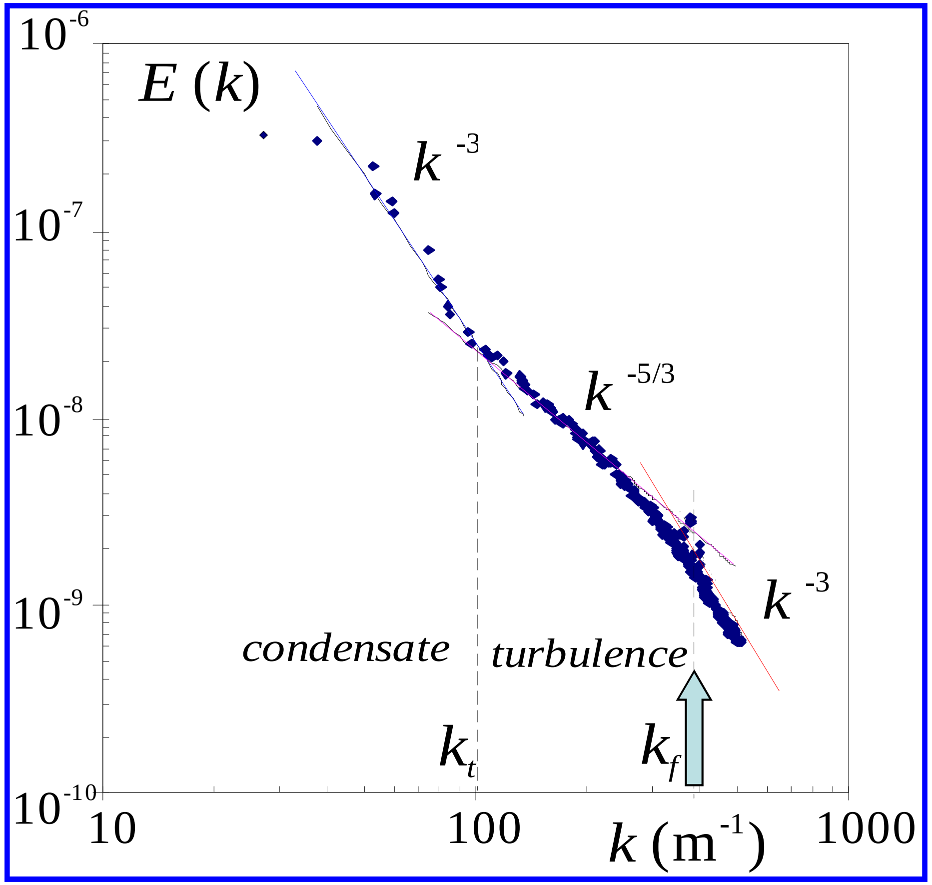
Energy flux derived from S_{3l} in agreement with that derived from the energy balance

Atmospheric and laboratory data

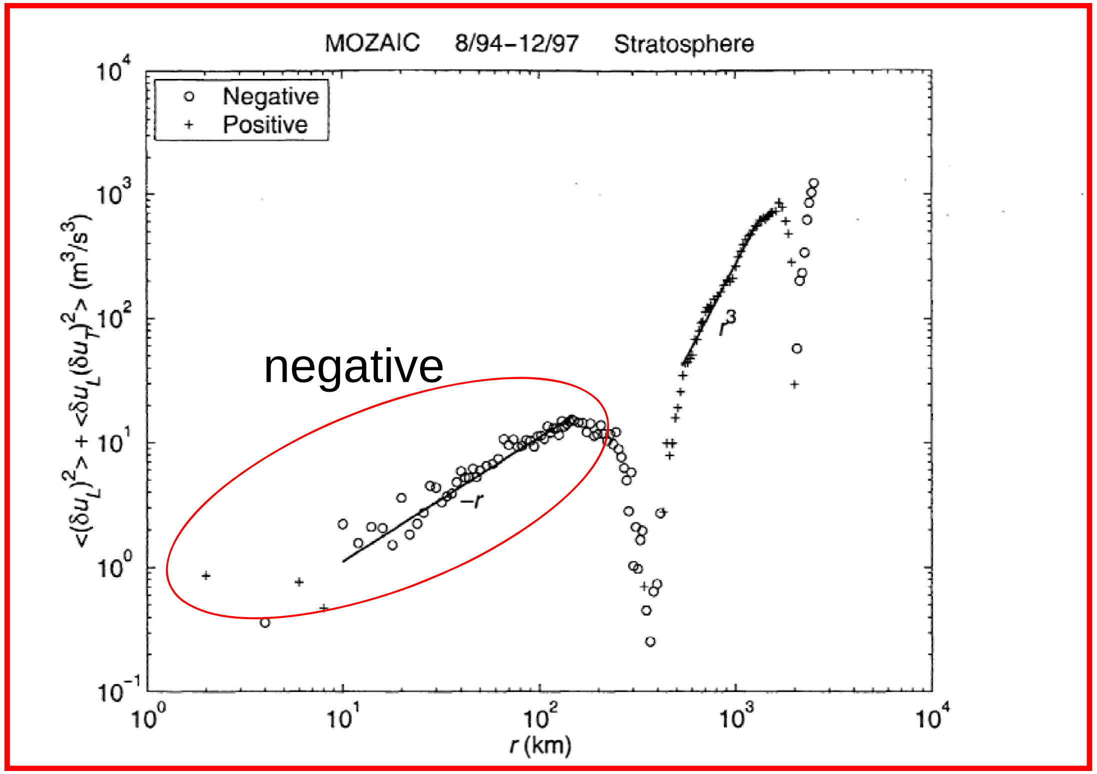
Atmospheric spectrum



Experimental spectrum of spectrally condensed turbulence

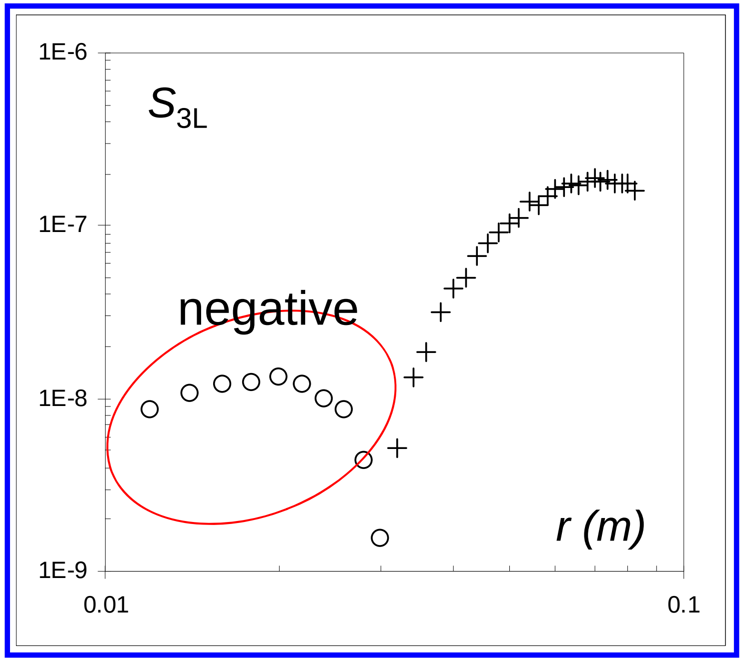


Atmospheric and laboratory data



Mean shear flows present in the atmosphere must affect velocity moments, similarly to laboratory experiments

Laboratory experiment



How thin a layer should be for turbulence to remain 2D?

What is the effect of 3D motions?

Split energy cascade in fluid layers

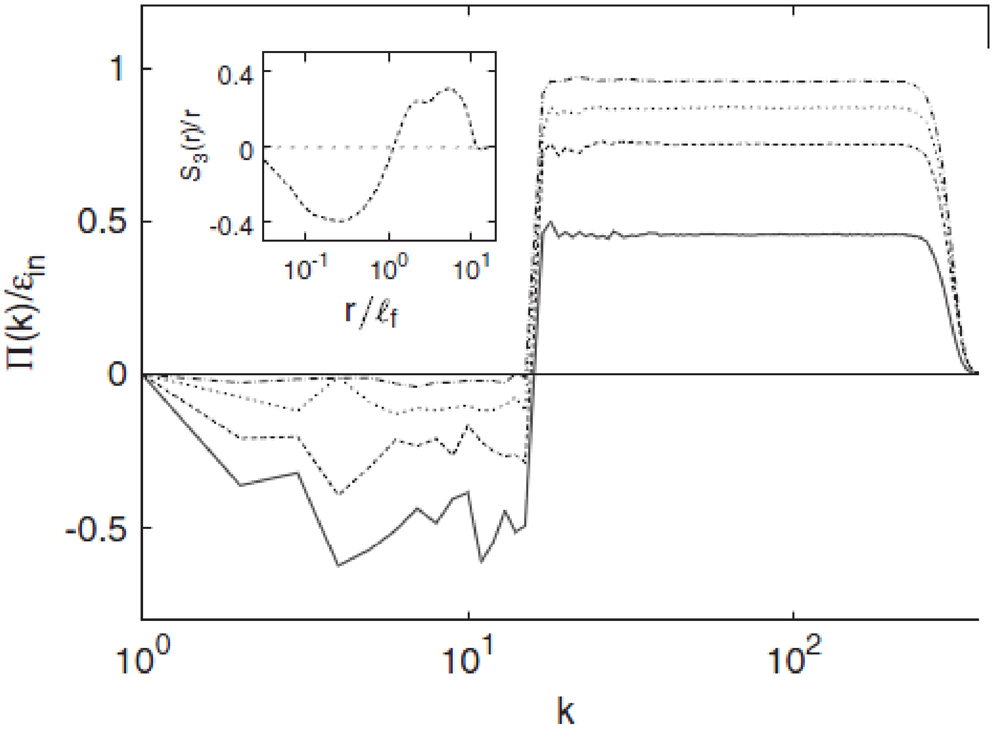
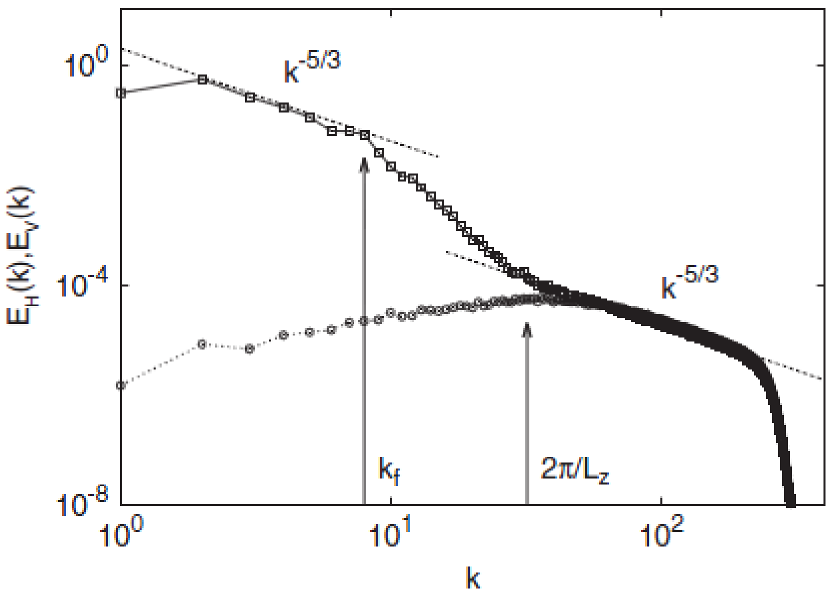


FIG. 2. Spectral flux of kinetic energy for various aspect ratio $L_z/\ell_f = 1/8, 1/4, 3/8, 1/2$ (from bottom to top). Simulation parameters as in Fig. 1. The inset reports the third order structure function of the velocity, $S_3(r)$, for $L_z/\ell_f = 1/4$.

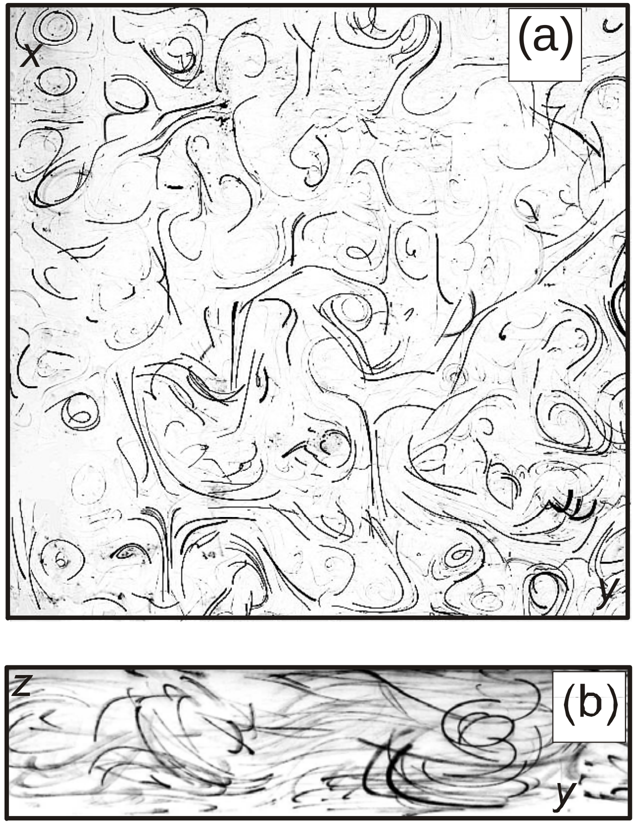
[Celani, et al., Phys. Rev. Let. 2010]

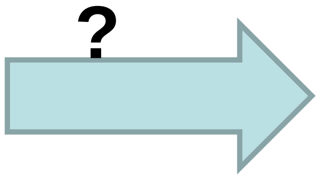


- Split of the energy cascade
- Coexistence of 2D and 3D turbulence

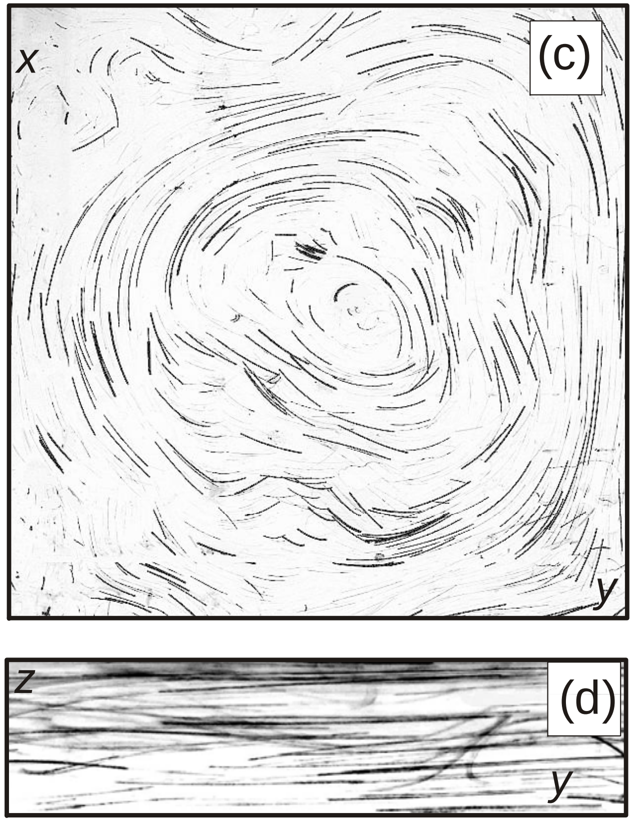
Condensate in thick fluid layers ?

t=5 s





t=20 s



Thick fluid layer

(double layer) →

3D turbulence →

Residual inverse

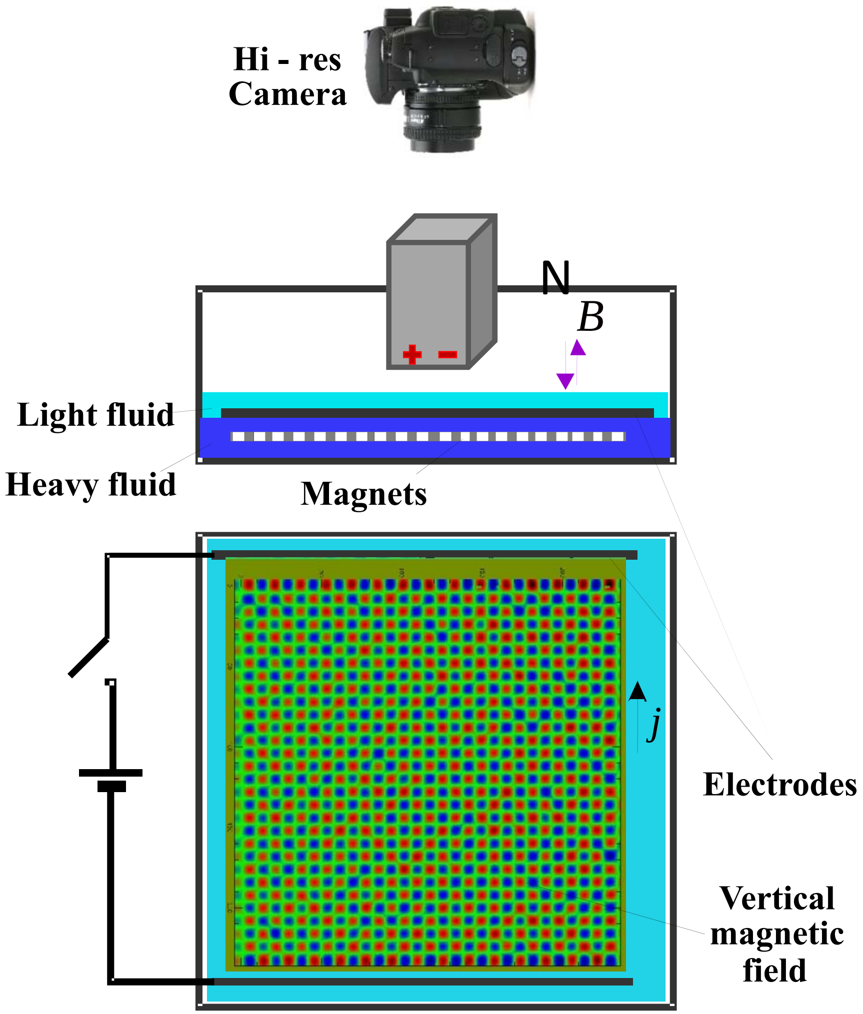
energy cascade →

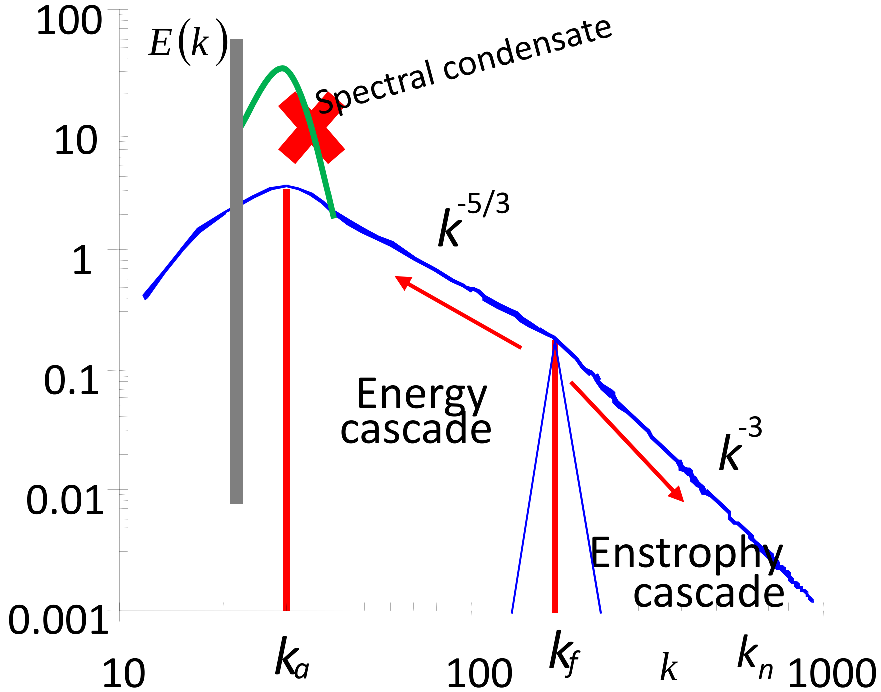
Condensation →

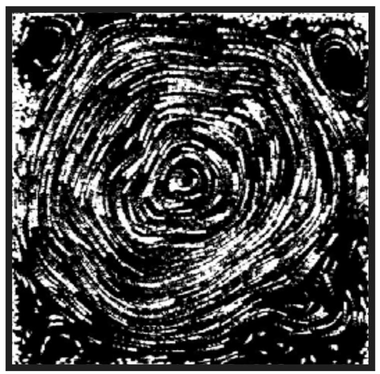
Planarity of the flow

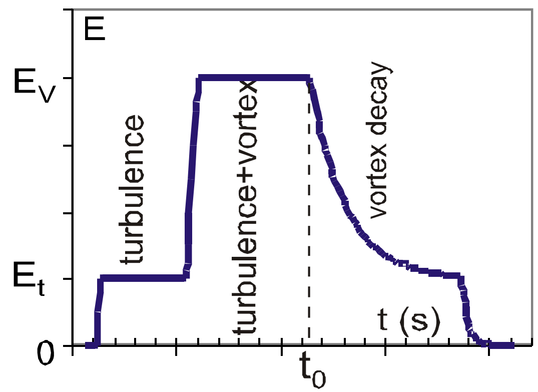
Shear flow makes (initially 3D) turbulence two dimensional

Externally imposed flow in a single layer

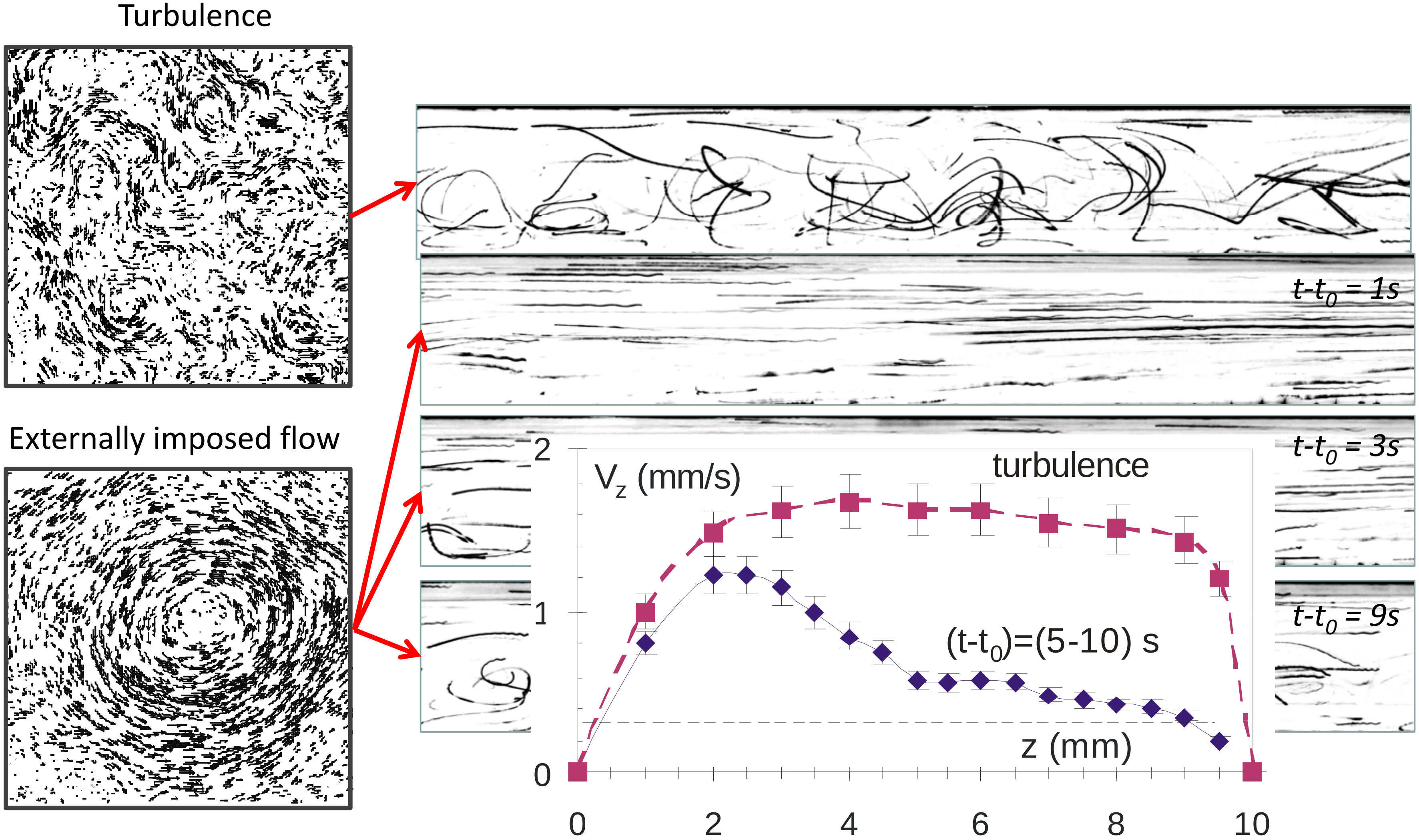






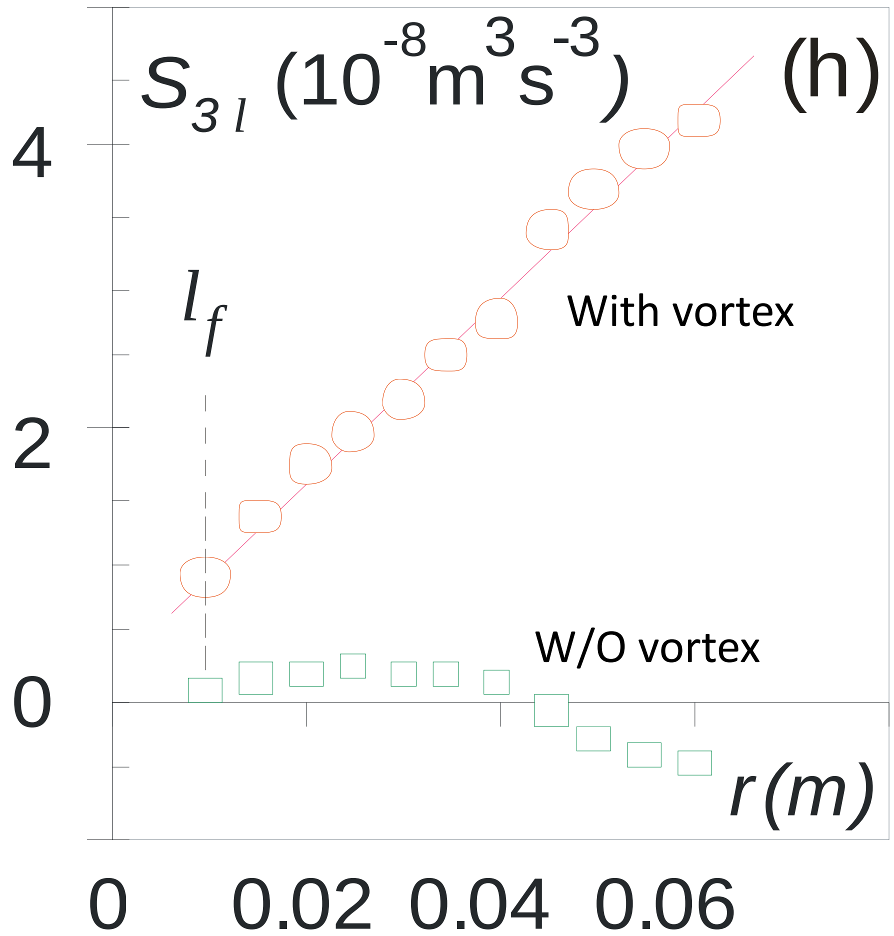


Turbulence with imposed flow



Large-scale flow suppresses 3D eddies

Inverse energy transfer



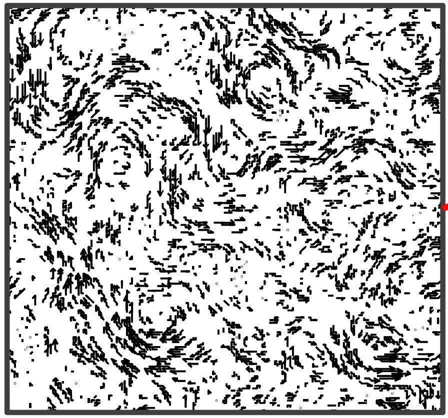
This may be important for:
Atmospheric turbulence:
Decay of large vortices:
e.g. cyclone

[Xia, Byrne, Falkovich & Shats, *Nature Physics*, doi:10.1038/nphys1910 (2011)]

Shear suppression of vertical eddies by large vortex
→ inverse energy cascade, as in 2D turbulence

Turbulent damping

Turbulence



3D eddies affect damping through eddy viscosity:

$$\left\langle \tilde{V}_{x,y}\tilde{V}_z \right\rangle = -K\left(\frac{\partial V_{x,y}}{\partial z}\right)$$

$$K \approx \langle V_z\rangle\langle V_{x,y}\rangle\left(\frac{\partial V_{x,y}}{\partial z}\right)^{-1}$$

$$\boxed{\alpha = \frac{(\nu + K)\pi^2}{2h^2}}$$

Quasi-2D model of turbulent damping:

$$\partial V_{x,y} / \partial t = \nu\partial^2 V_{x,y} / \partial z^2$$

$$V_{x,y}(z=0,t) = 0$$

$$\partial V_{x,y}(z=h,t) / \partial z = 0$$

$$V(z,t) = \sin\left(\frac{\pi z}{2h}\right)e^{-t\alpha_L/2}$$

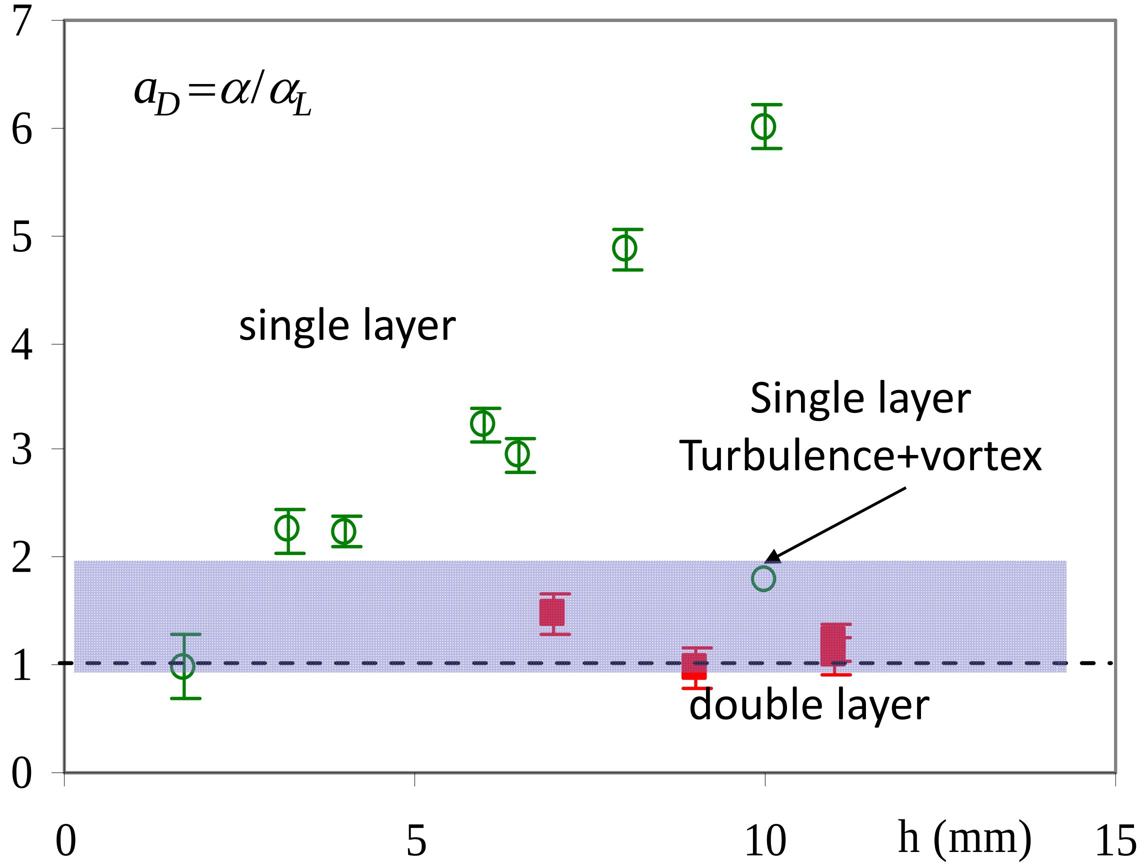
$$\boxed{\alpha_L = \frac{\nu\pi^2}{2h^2}}$$

[F.V. Dolzhanskii, et al, JFM, 241, 705 (1992).]

Turbulent damping in fluid layers

- Turbulent damping (with eddy viscosity) agrees with the measured damping
- Measure of dimensionality:

$$a_D = \alpha / \alpha_L$$



Realization of 2D flow:

→ very thin layers (large damping)

→ with large scale imposed flow

→ double layer configuration (condensate)

[Shats, Byrne, Xia, Phys. Rev. Lett., **105**, 264501 (2010)]

Summary

In thin fluid layers (double layer configuration):

- System size vortex develops in a finite size 2D turbulent system
- Subtraction of mean vortex recovers the energy flux in the underlying turbulence.
- We reproduced main features (spectrum and third-order structure function) of the atmospheric turbulence in laboratory

In thick fluid layers

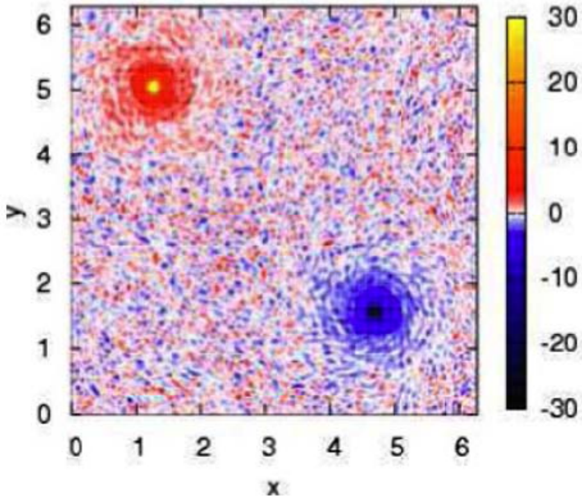
- 3D motions ➔Eddy viscosity
- Measure of flow dimensionality: $a_D = \alpha / \alpha_L$
- Planarization of the flow through shear suppression of vertical eddies in 3D turbulence restores 2D cascade

Open questions:

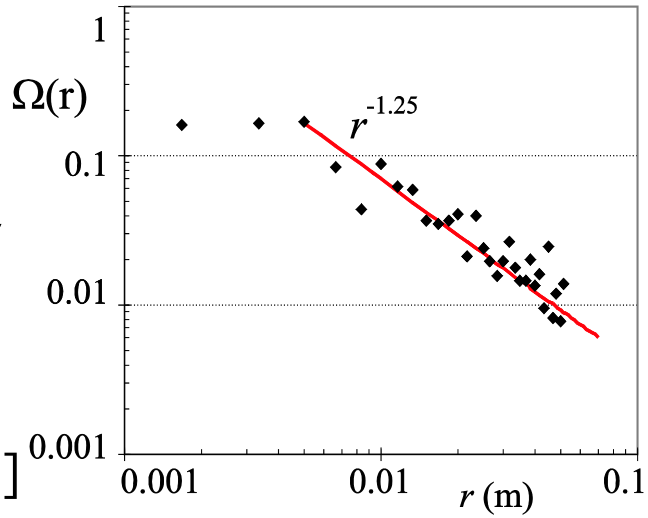
- Symmetry breaking and generation of non-zero vorticity
  - Numerics: dipole
  - Experiments:

    monopole (small box)

    several structures



- Universality of the shape of the spectral condensate:
  - Numerics & Theory: [Chertkov et al, PRL, 2007; Chertkov et al, PRE, 2010]
  - Experimental results at low damping, small boundary box: [Xia et al, PoF, 2009]



Open questions:

- Isotropy

For isotropic turbulence:

$$S_{3T} = (r/3)d\left[S_{3L}(l)\right]/dr \qquad S_{3L} = 3S_{3T}$$

[Yakhot, Phys. Rev. E **60**, 5544 (1999)]