Spin Transport in Spin-Polarized Quantum Systems

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Exchange Hamiltonian

- In two particle systems, corresponds to the difference in singlet and triplet states
- J is the exchange integral that depends on the overlap of the single particle wavefunctions
- States where spins are associated with individual atoms are not stationary states …

$$H_{ex} = \sum_{i \neq j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$\frac{d}{dt}\vec{S}_i = \underbrace{\left[\frac{1}{\hbar} \sum_{i \neq j} J_{ij} \vec{S}_j \right]}_{\vec{\Omega}_i \text{ exchange field}} \times \vec{S}_i$$

» spin transport!

Quantum Fluids

- Liquid ^{4}He: spin 0 » Bose-Einstein statistics » superfluidity
- Liquid ^{3}He: spin 1/2 » Fermi-Dirac statistics » p-wave Cooper pairs; û**L** ∘ **S**ëû0.
- Normal ^{3}He described by Landau –Fermi liquid theory.

Leggett Equation (normal fluid)

- Spin-polarized 3He described by continuity eqn and a nonlinear eqn for spin current

$$\frac{d}{dt}\vec{M} = -\frac{d}{dz}\vec{J}_z + \vec{\Omega}_L(z)\times\vec{M}$$

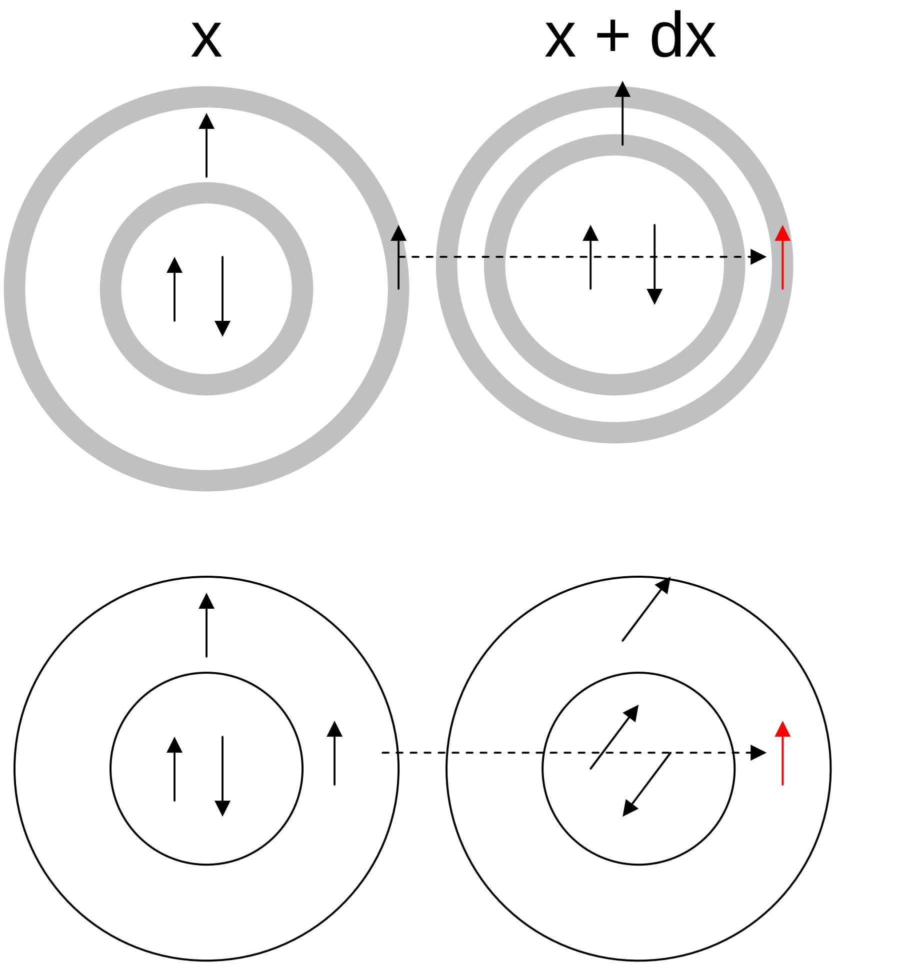
$$\vec{J}_z = -D_{\parallel}\hat{e}\partial_z M - D_{\perp}M\partial_z\hat{e} - \mu\vec{M}\times\vec{J}_z$$

$$D_{\parallel,\perp} \sim v_F^2\tau_{\parallel,\perp} \qquad \mu M = \Omega_{ex}\tau_{\perp}$$

$$m^+ = Ae^{iqz-i\omega t} \qquad \omega_q = \frac{v_F^2}{\Omega_{ex}}q^2 + i\frac{D_{\perp}}{(\mu M)^2}q^2$$

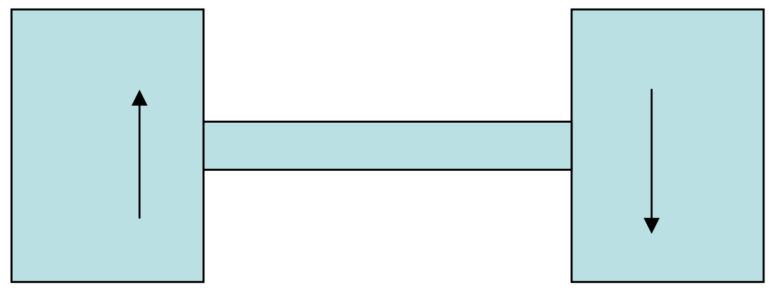
Anisotropic Spin Diffusion

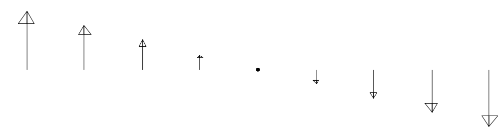
- Longitudinal spin transport (gradient in polarization) collisions forbidden except at Fermi surface w£ $1/T^2$
- Transverse case (gradient in direction) scattering possible between Fermi spheres w£ P^2



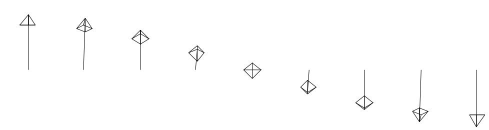
Longitudinal spin diffusion

- Measured $D_{||} \propto 1/T^2$
- Large spin current is unstable (Castaing instability)
- A domain wall forms and spin transport occurs via transverse spin diffusion





$J_{||} = D_{||} M/L$

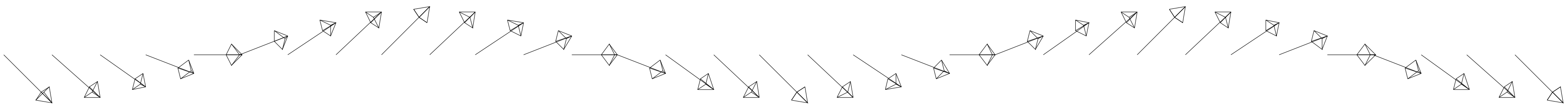


$J_{\perp} = D_{\perp} M\pi/\mu ML$

180º twist

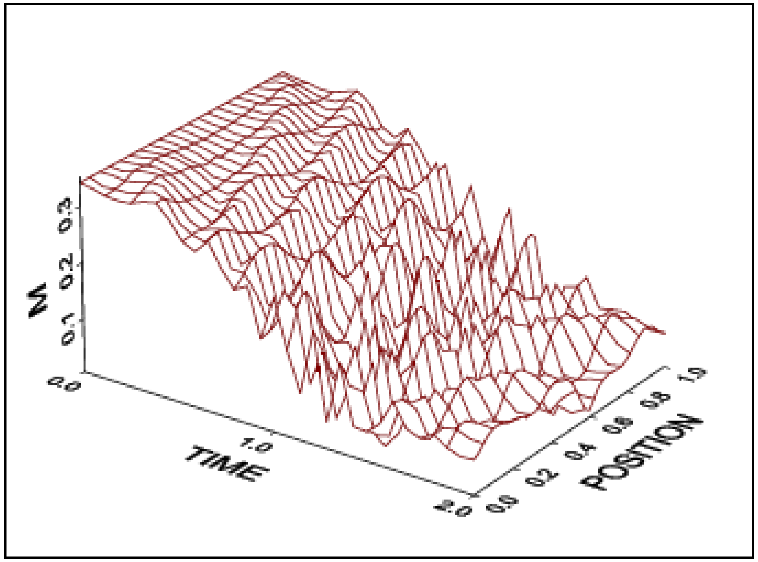
Spin-Wave Instabilities

- $D_\perp$ is measured in NMR spin-wave and spin-echo experiments in helical spin wave configurations



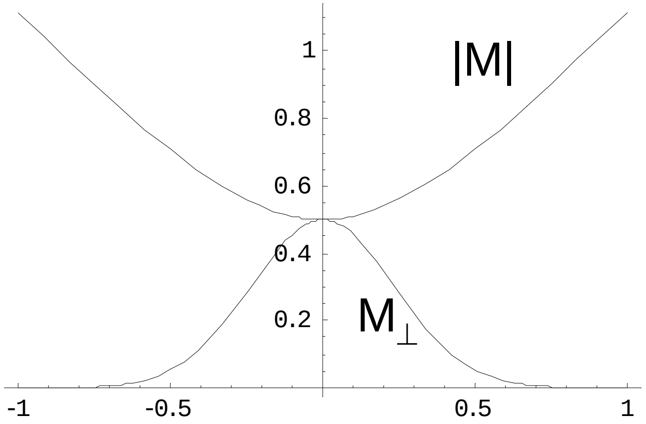
•Castaing instabilities occur here too – can mimic effects of zero-temperature attenuation in NMR experiments

•In fact solitons and cnoidal waves occur in the nonlinear dynamics



Other phenomena

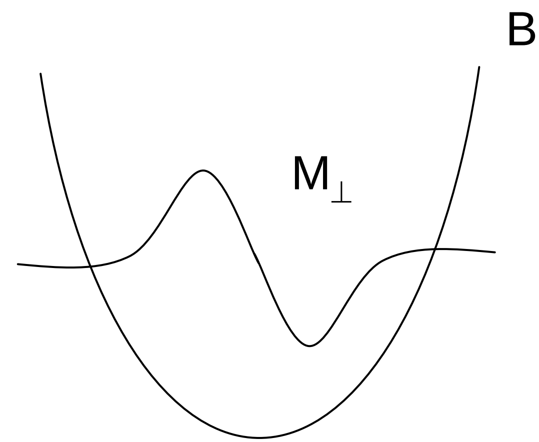
In 3.8% 3He-4He mixtures exchange effects cancel and spin transport is by anisotropic spin diffusion only. In this case transverse magnetization can be trapped in the minimum of |M| by longitudinal diffusion!

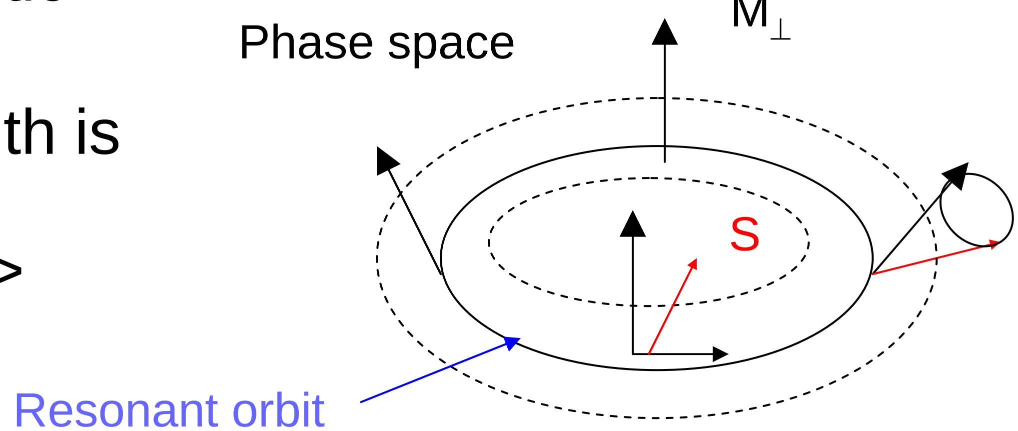


- Spin-wave damping in ferromagnetic Fermi liquids
- Castaing instabilities and anisotropic pseudo spin diffusion in trapped alkali gases (Rb-87)

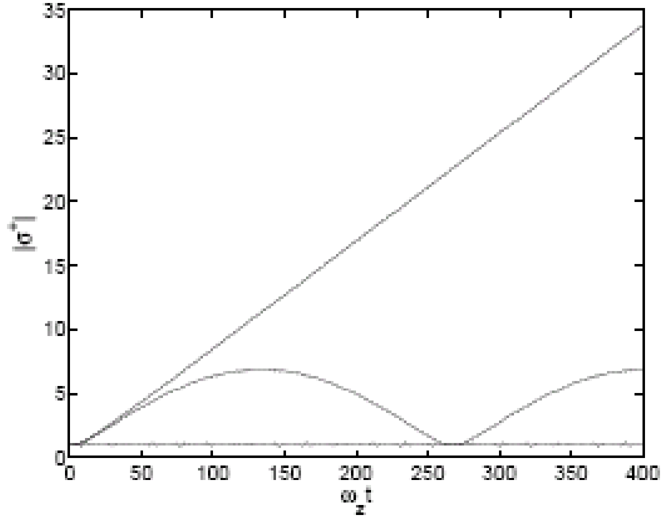
Landau damping in Trapped Boltzmann Gases

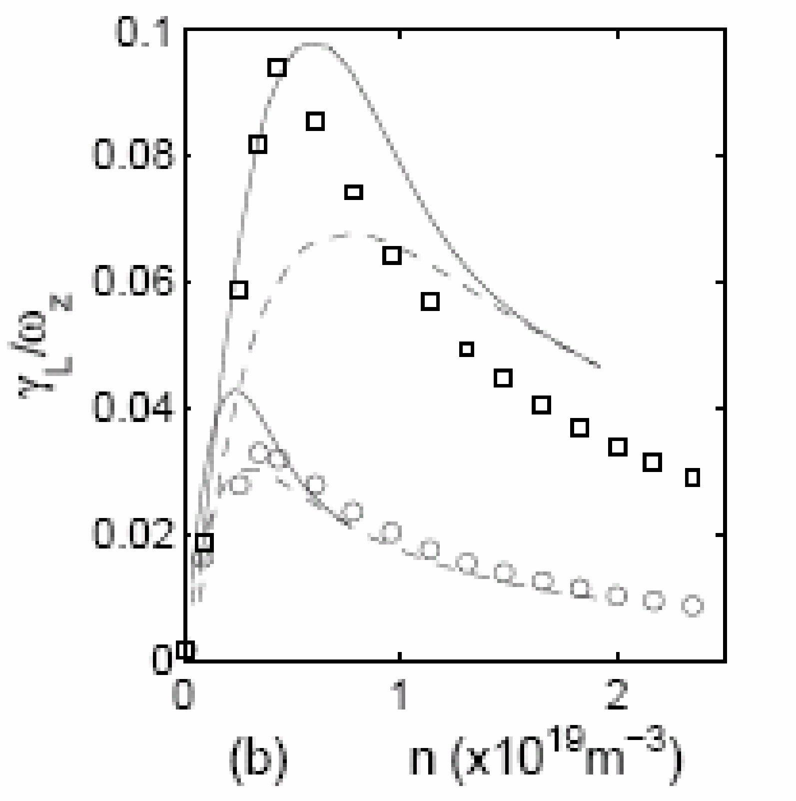
- Two Hyperfine levels in Rb-87 form pseudo spin-1/2 system
- At temperatures above BEC, the system supports harmonically trapped paramagnetic spin waves
- deBroglie wavelength is longer than s-wave scattering length => exchange



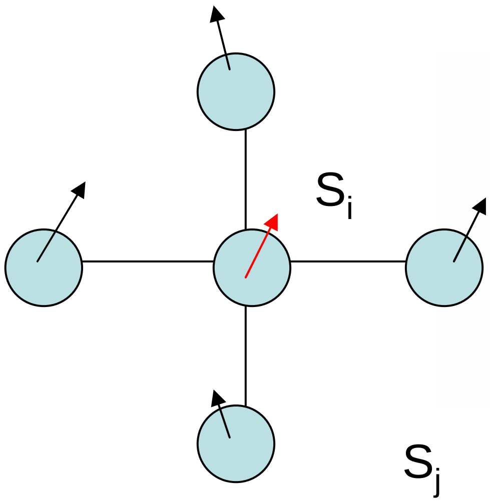


Landau damping simulations





Heisenberg Paramagnets

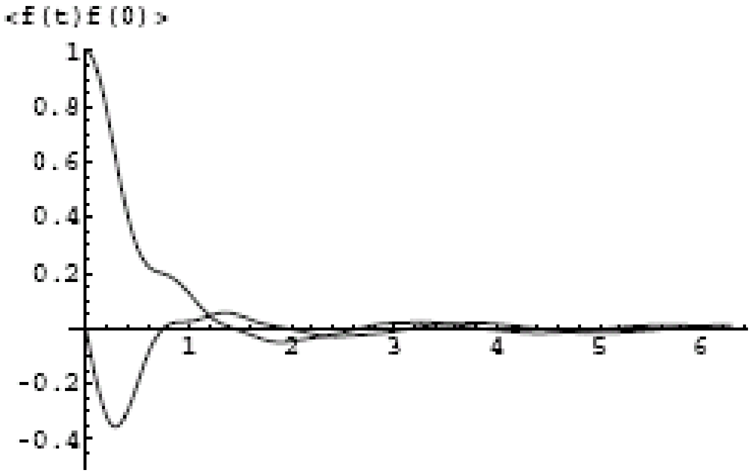


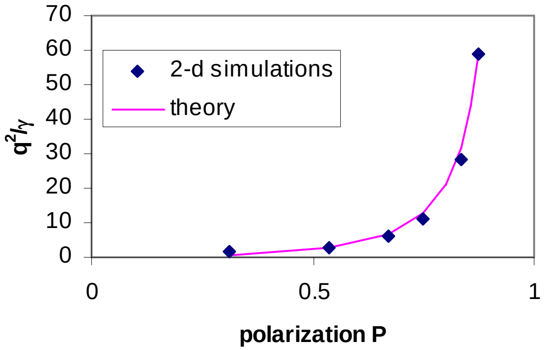
$$\frac{d}{dt} S_q^+(t) = \omega_q S_q^+(t)$$
$$- i\Omega \sum_{q_1} \sum_{q_2} S_{q-q1}^+(t) S_{q1-q2}^+(t) S_{q2}^-(t) \left[\cos(q - q_1^z)a - \cos q_1^z a\right]$$

$$\frac{d}{dt} S_q^+ = i\omega_q S_q^+ - \gamma S_q^+ + f_q^+(t)$$

Langevin approach

- At high polarization, system acts like a dilute gas of magnons
- The hydrodynamic modes act like Brownian particles.
- Interactions (the “random force”) of the hydrodynamic modes with fluctuations have a very short correlation time.
- Damping rate is found from the total “impulse” of the random force and the fluctuation-dissipation theorem.





$$\gamma_q = \frac{\left\langle S_q^+ S_q^- \right\rangle}{2\int_0^\infty \left\langle f(t) f(0) \right\rangle dt}$$

Future Work for $P \to 1$

- S=1/2 case
- Short wavelength (neutron scattering)