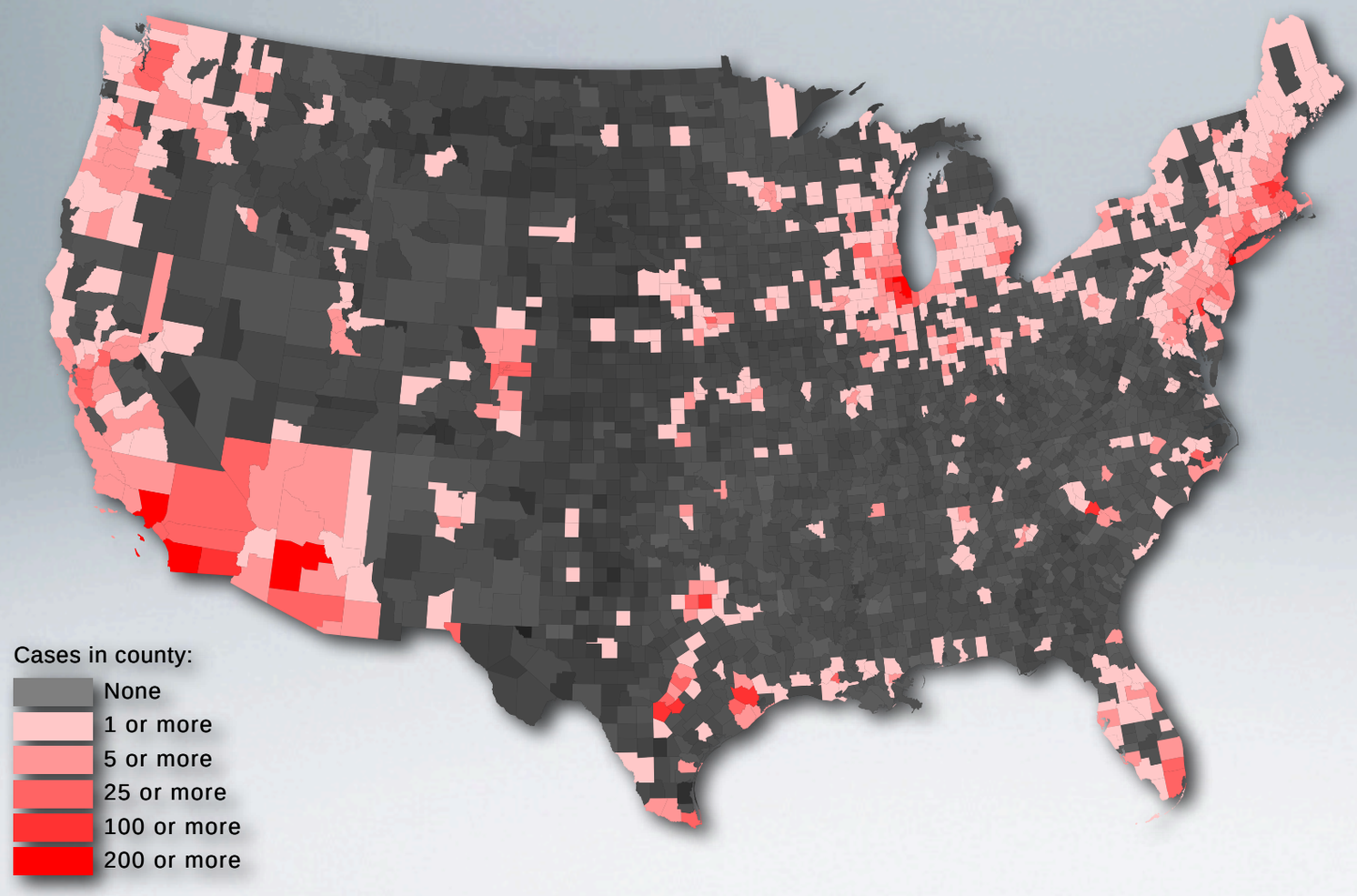
MODELLING DISEASE DYNAMICS ON A GLOBAL SCALE



Dirk Brockmann
Northwestern University

MODELLING DISEASE DYNAMICS ON A GLOBAL SCALE

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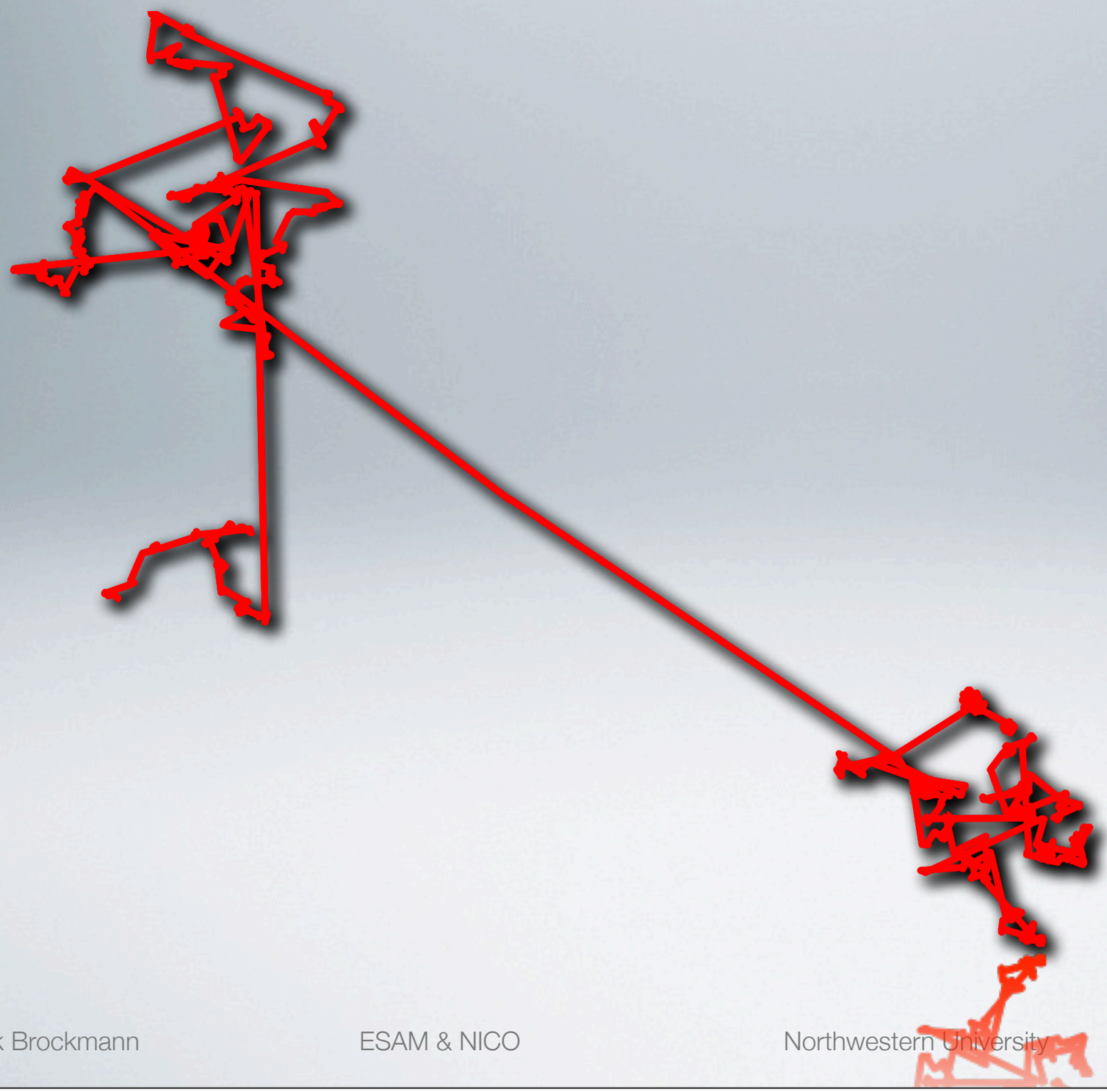

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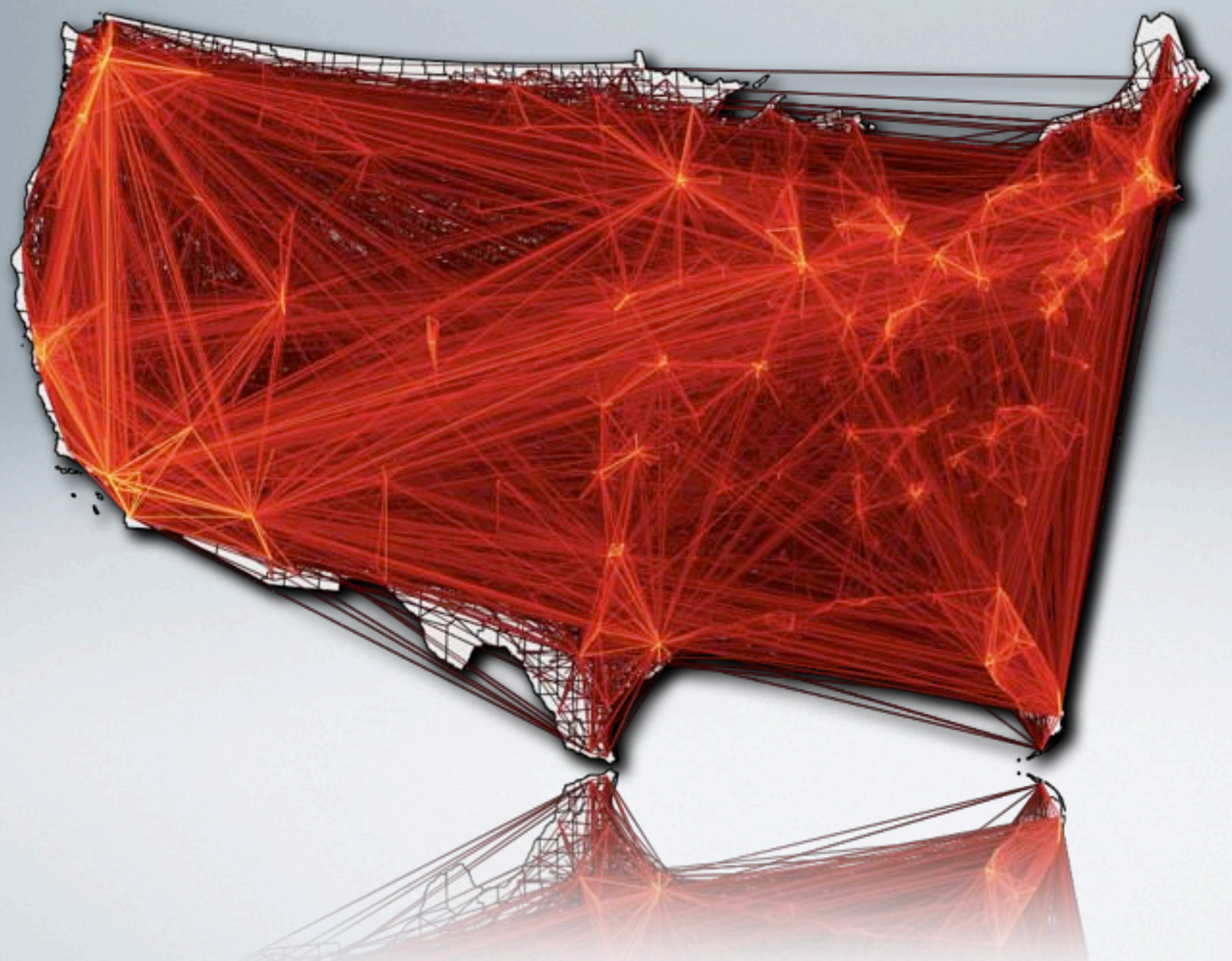

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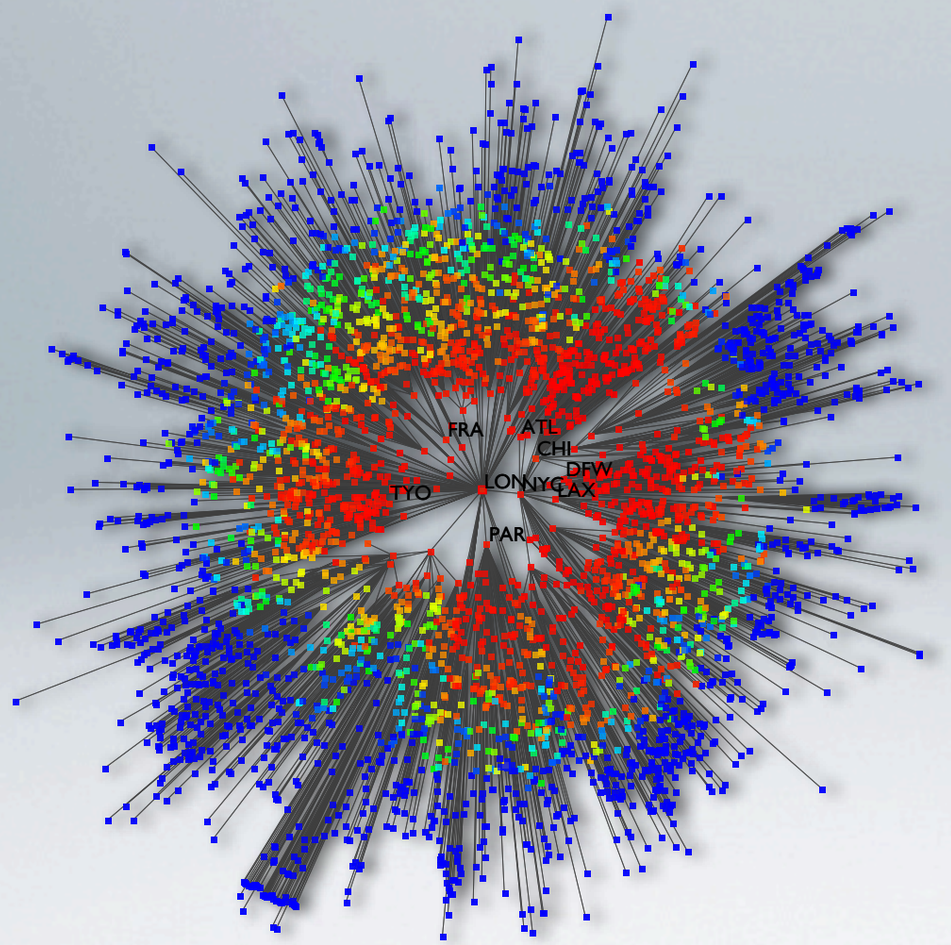
MODELLING DISEASE DYNAMICS ON A GLOBAL SCALE



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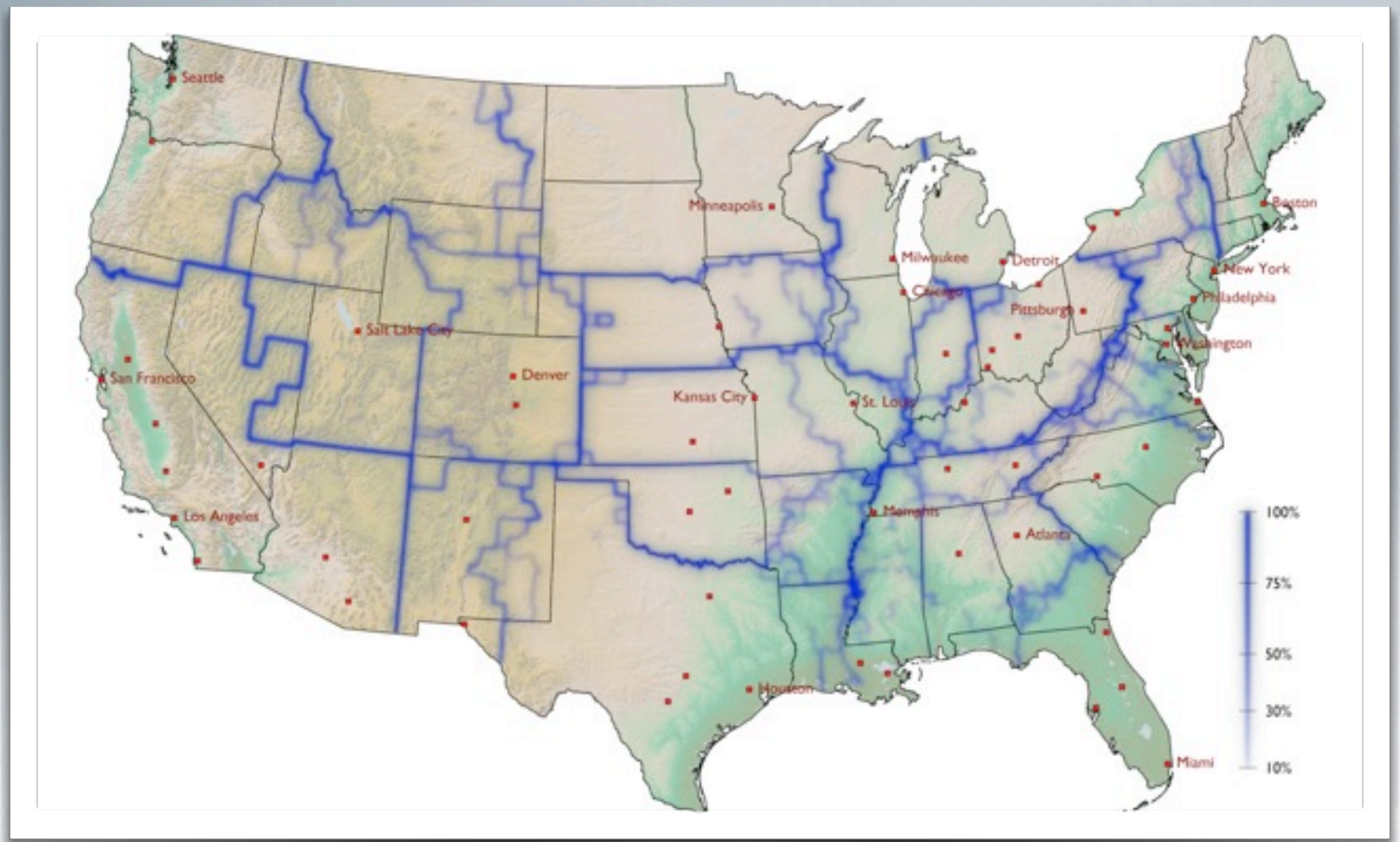
MODELLING DISEASE DYNAMICS ON A GLOBAL SCALE



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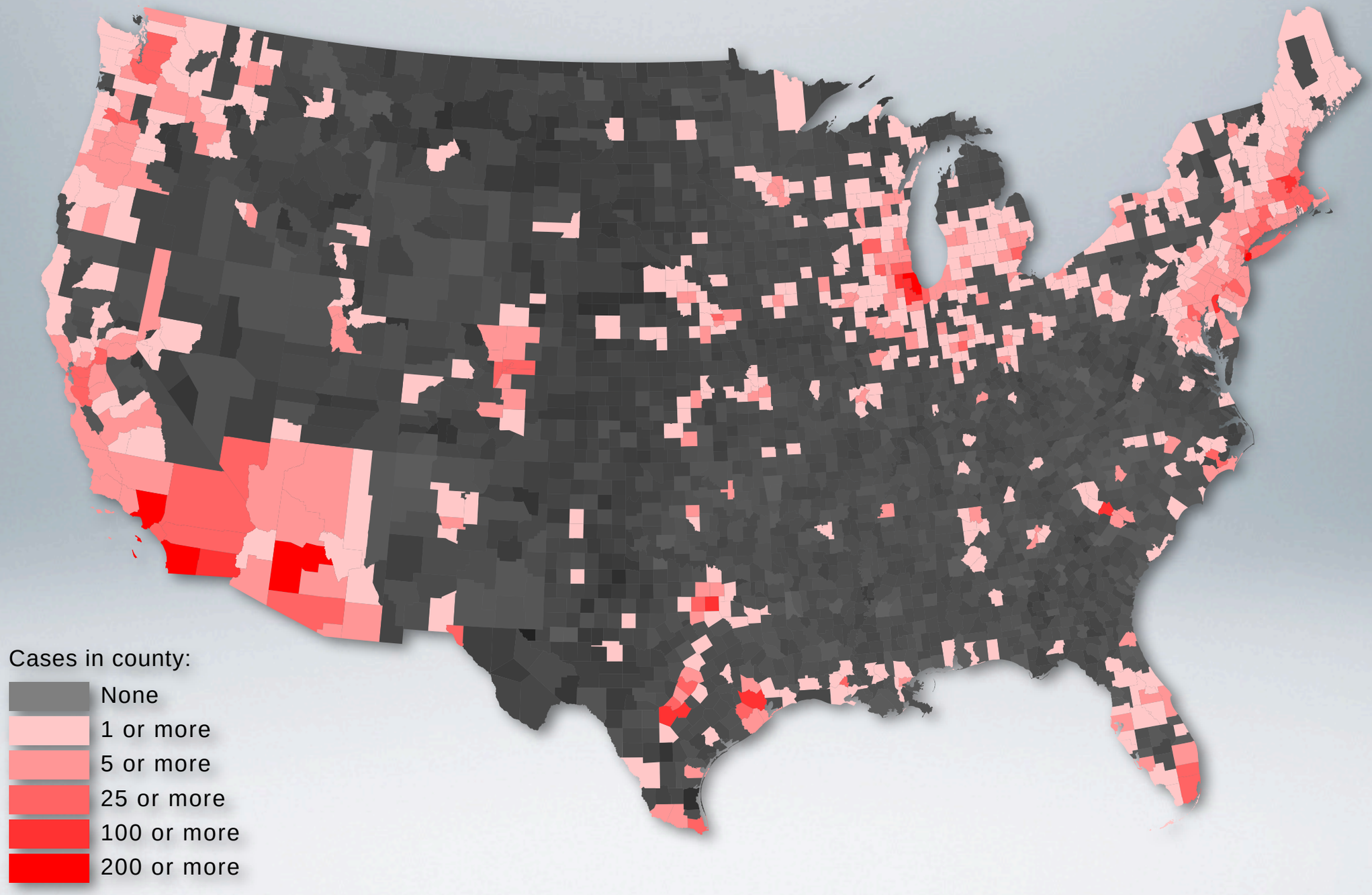
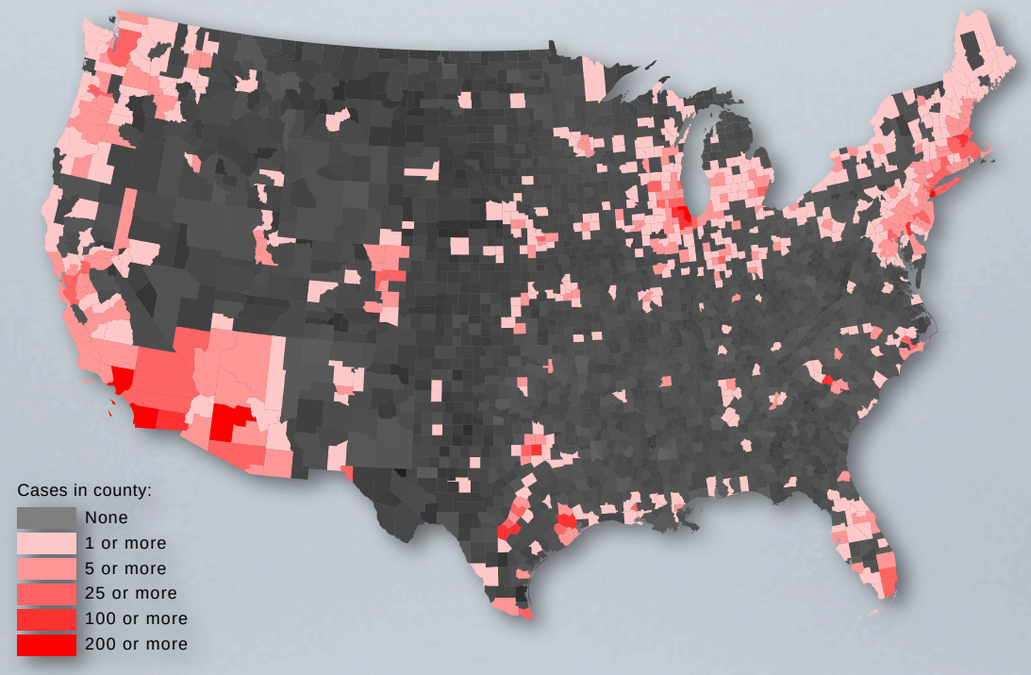
THE QUESTIONS



THE QUESTIONS

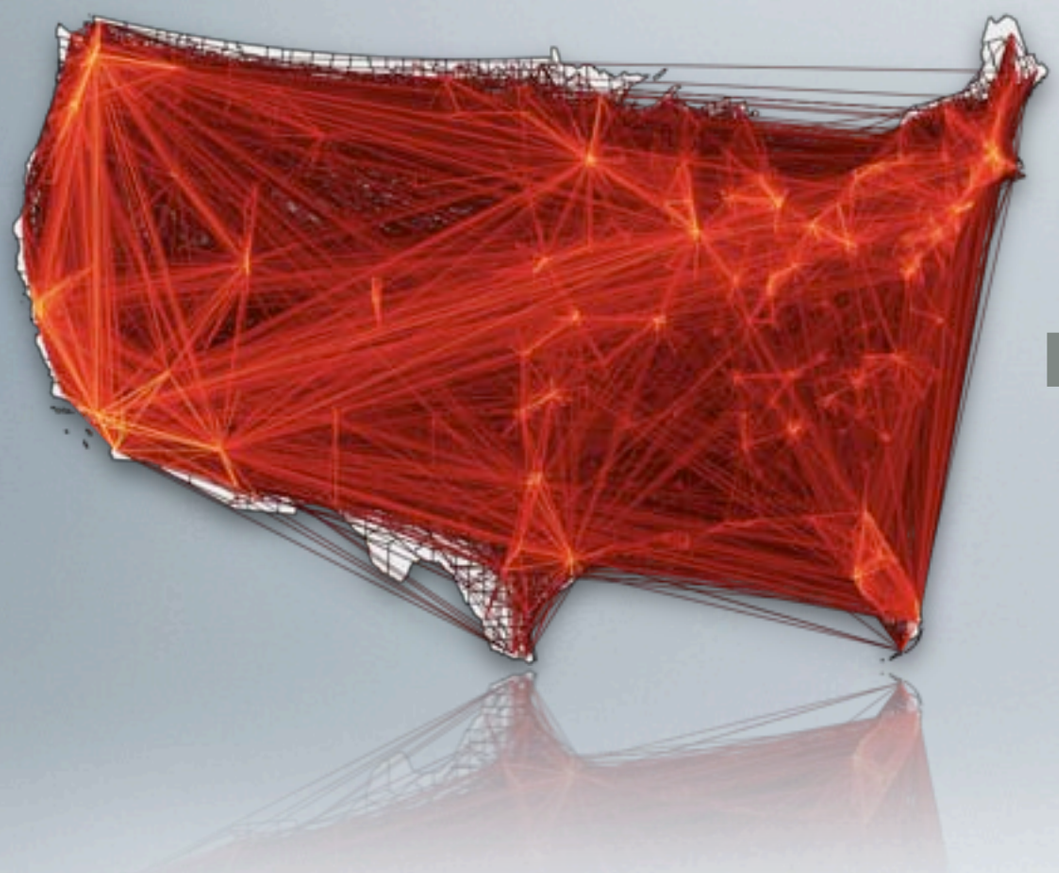


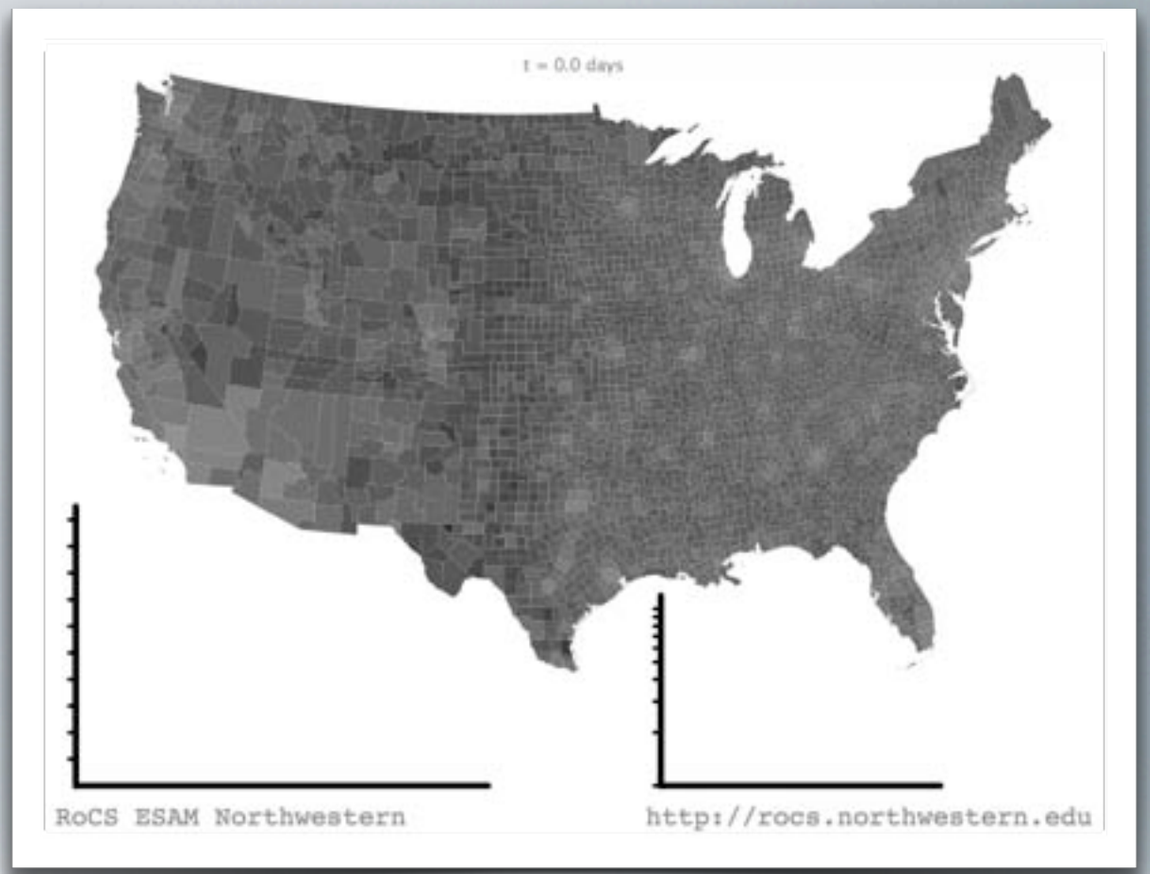
Where?

When?

How many?

MULTISCALE MOBILITY AND EPIDEMICS





what topological, structural and statistical features.....

...yield or induce dynamic phenomena?

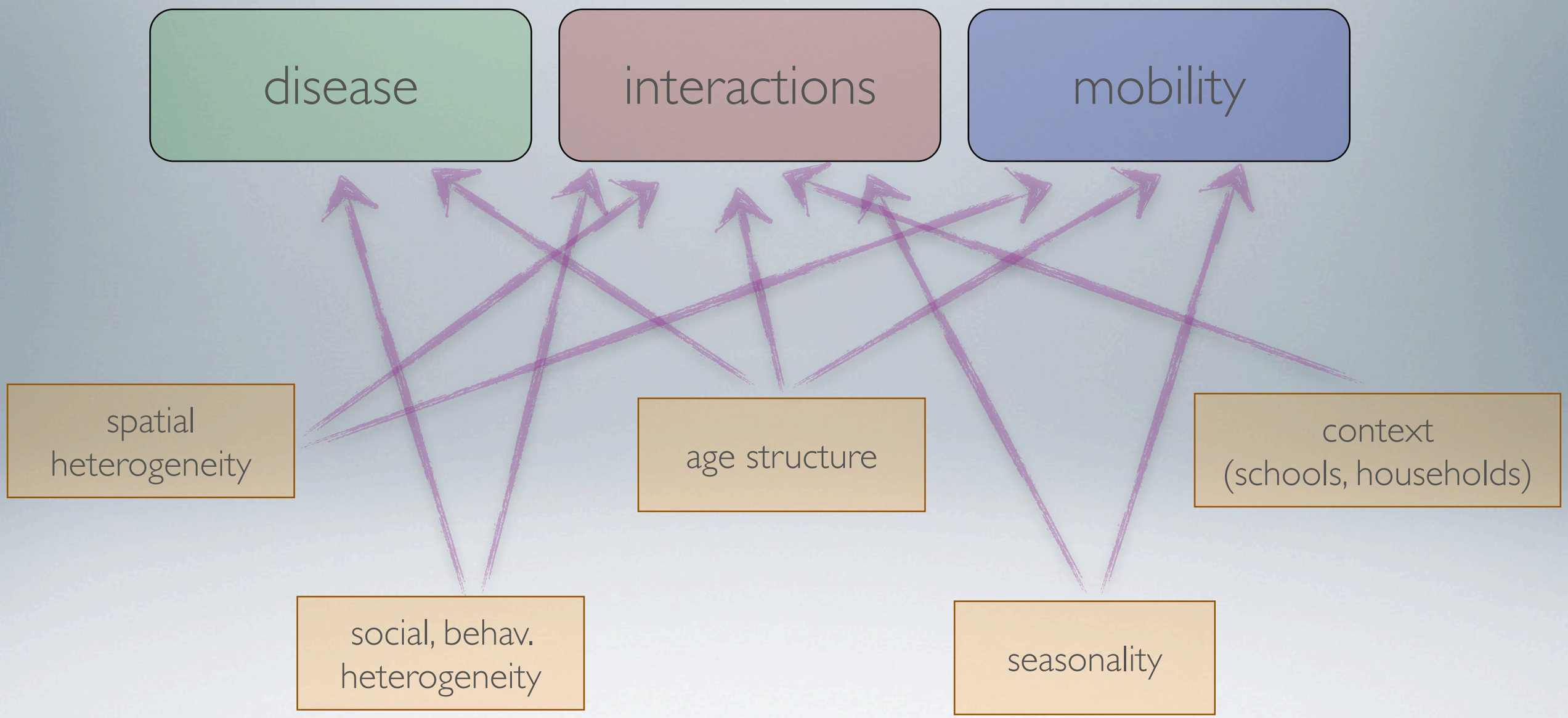
FACTORS

disease

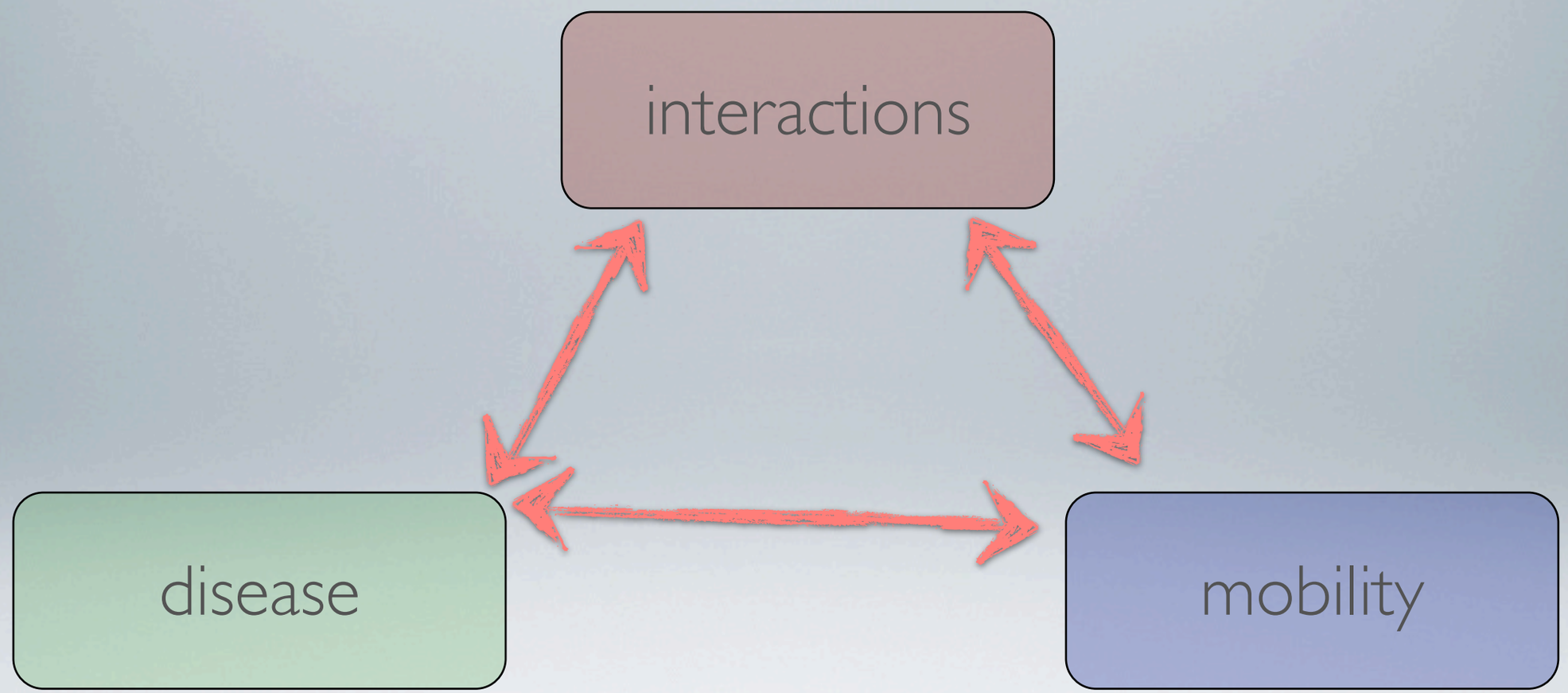
interactions

mobility

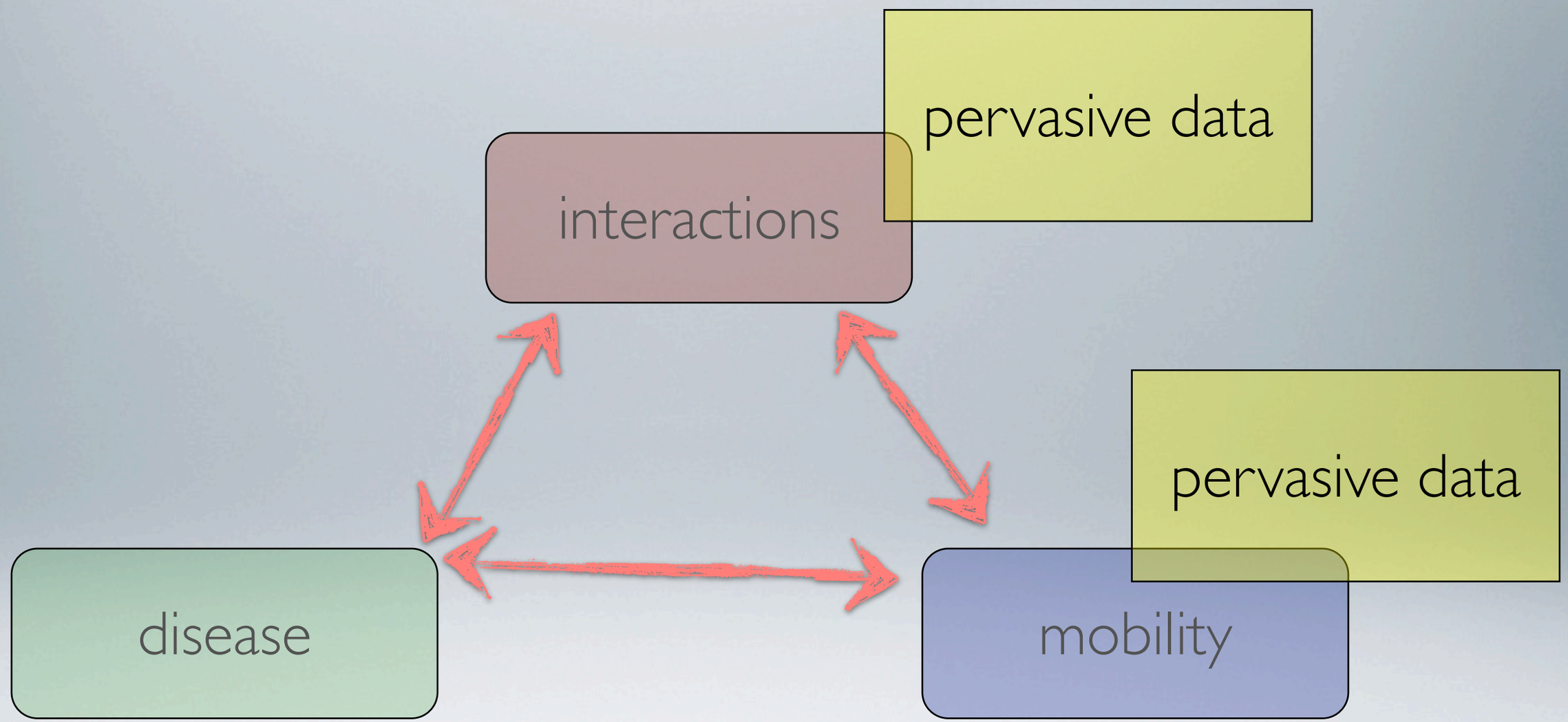
FACTORS


disease
interactions
mobility
spatial heterogeneity
age structure
context (schools, households)
social, behav. heterogeneity
seasonality

FACTORS



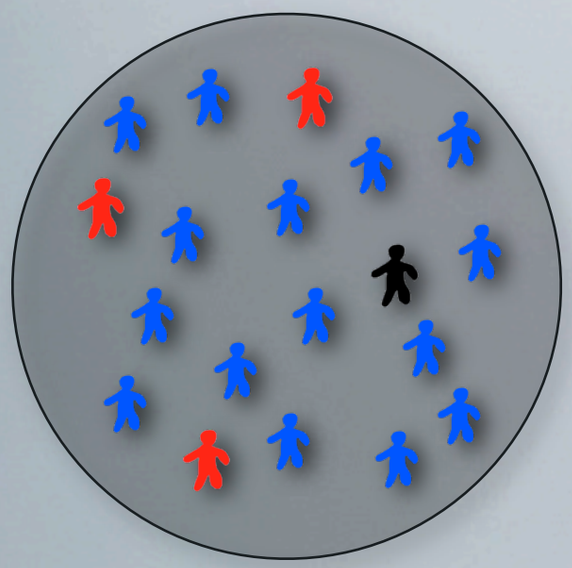
FACTORS


pervasive data
interactions
pervasive data
disease
mobility

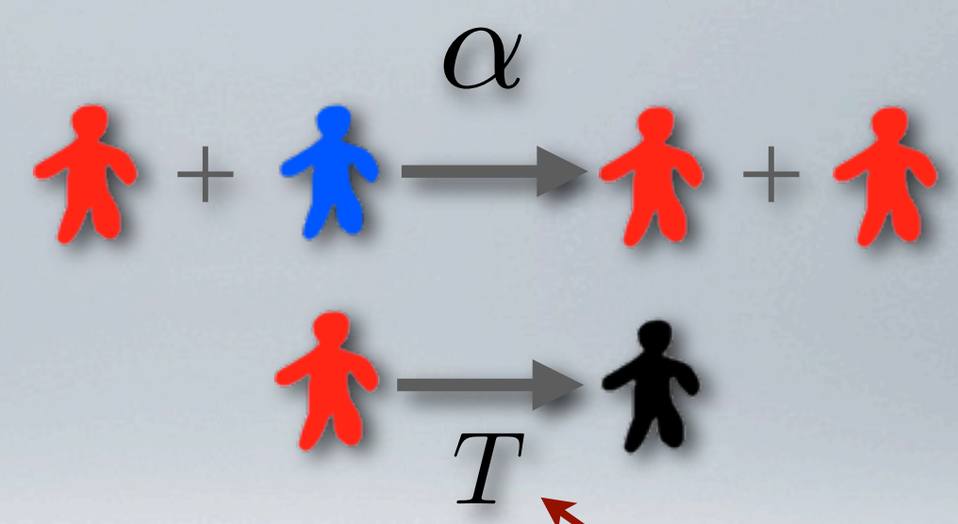
FACTORS

THE SIR MODEL

N ← Population Size



infection rate:
e.g. 2 effective contacts/week

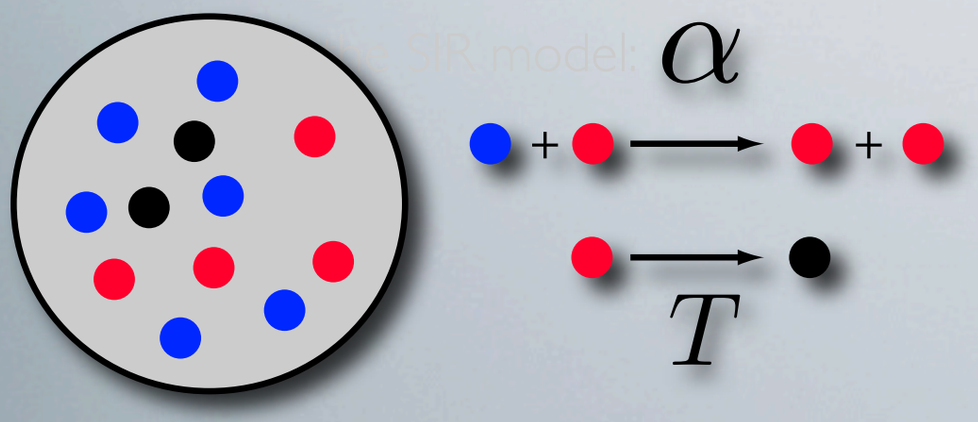


The basic reproduction number

$$\alpha \times T = R_0 > 1$$

infectious period
e.g. 4 days.

SIR DYNAMICS



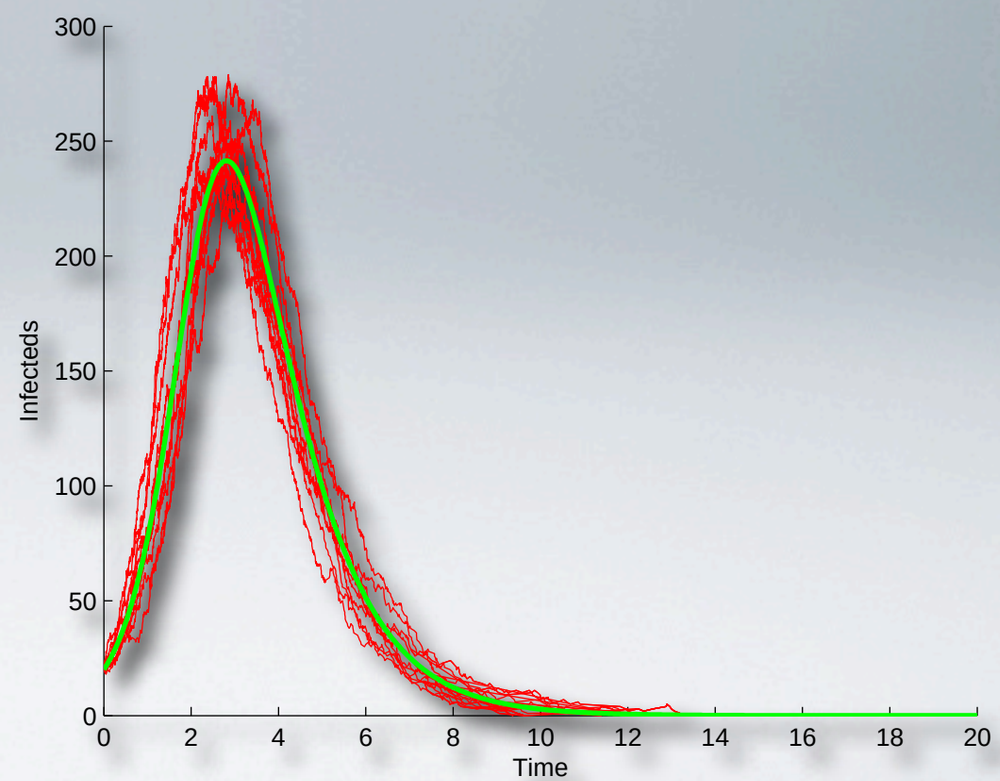
infected
susceptible
recovered/removed

mean field dynamics:

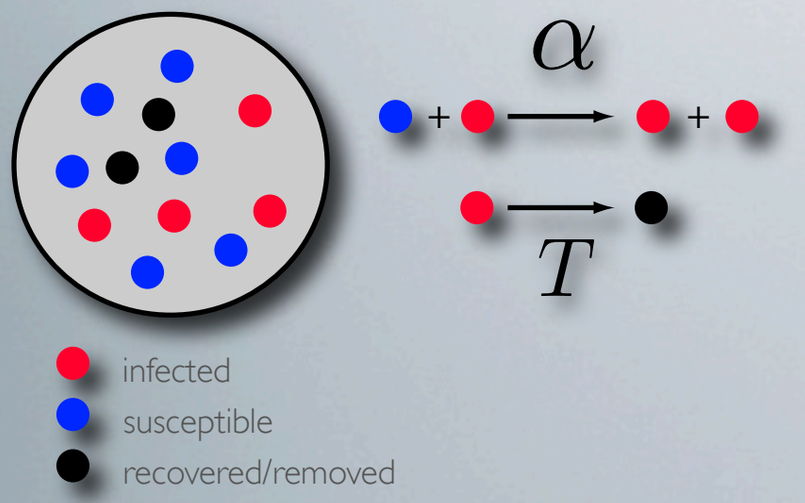
$$\begin{aligned} \partial_t S &= -R_0 I S \\ \partial_t I &= R_0 I S - I \\ R &= 1 - S - I \end{aligned}$$

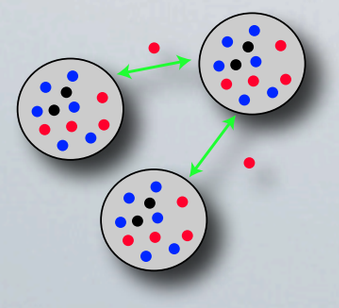
The basic reproduction number

$$\alpha \times T = R_0 > 1$$



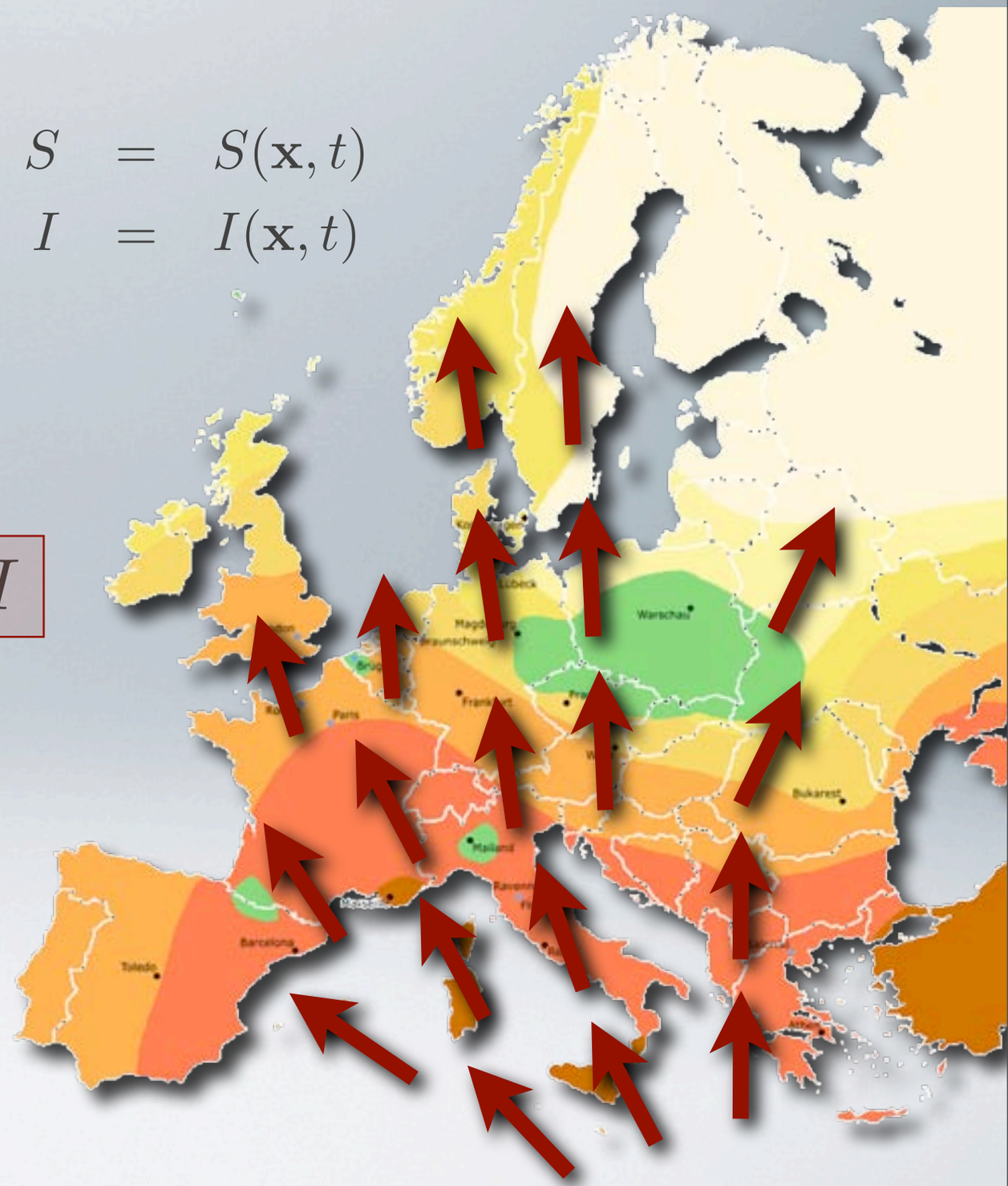
SPATIAL DYNAMICS





$$S = S(\mathbf{x}, t)$$
$$I = I(\mathbf{x}, t)$$

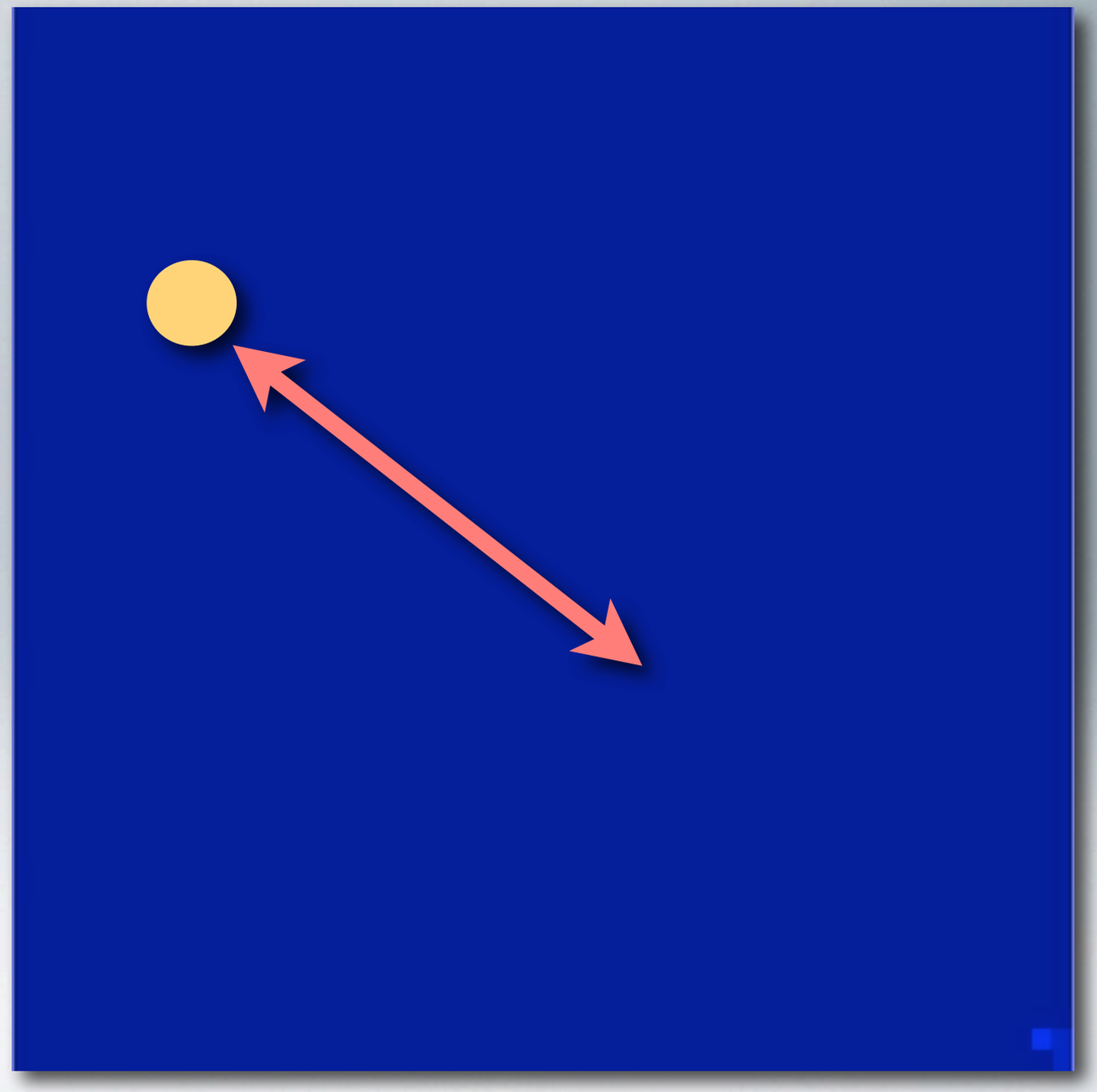
$$\partial_t S = -R_0 S I + D\partial_x^2 S$$
$$\partial_t I = R_0 S I - I + D\partial_x^2 I$$



WHEN?

WHEN?

$$T = \frac{d}{v}$$



FRONT VELOCITY IN FISHER TYPE SYSTEMS

$$u = u(\mathbf{x}, t)$$

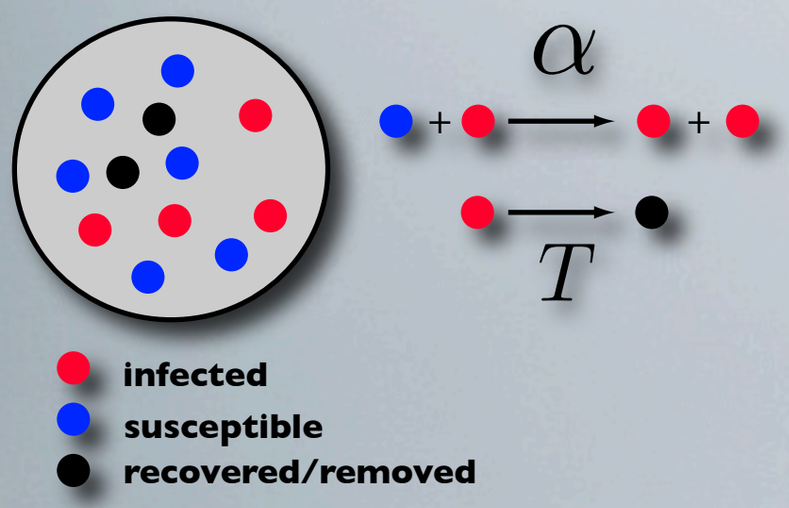
$$\partial_t u = \lambda u(1-u) + D\partial_x^2 u$$

$$\boxed{v \propto \sqrt{D\lambda}}$$

Noble, Nature (1974) vol. 250 (5469) pp. 726-728

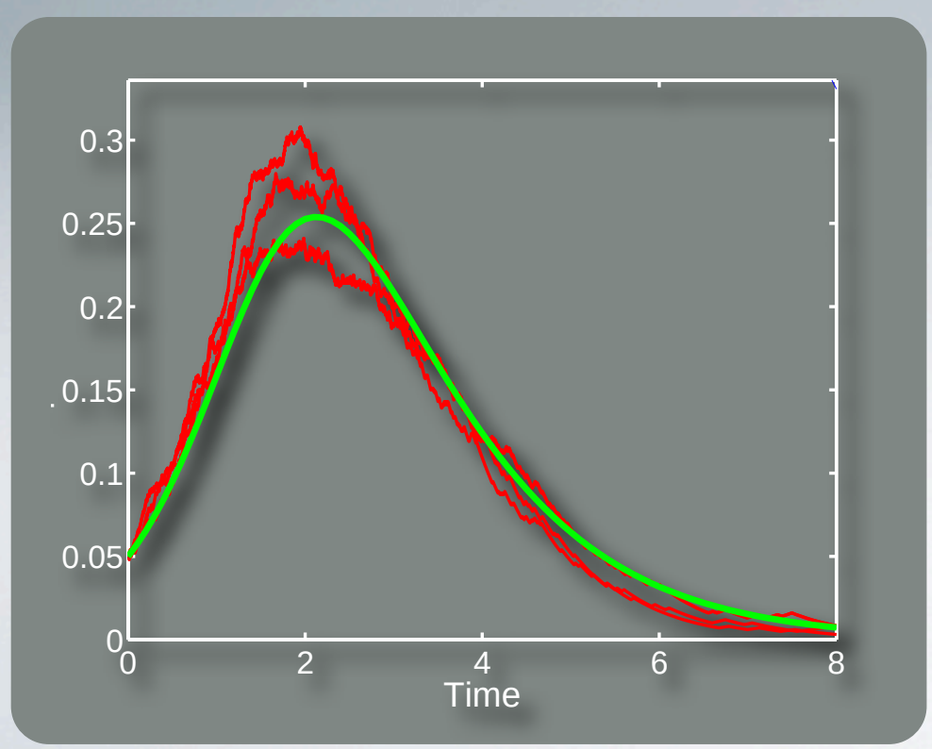
SINGLE POPULATION AND SPATIAL MODELS

SIR dynamics



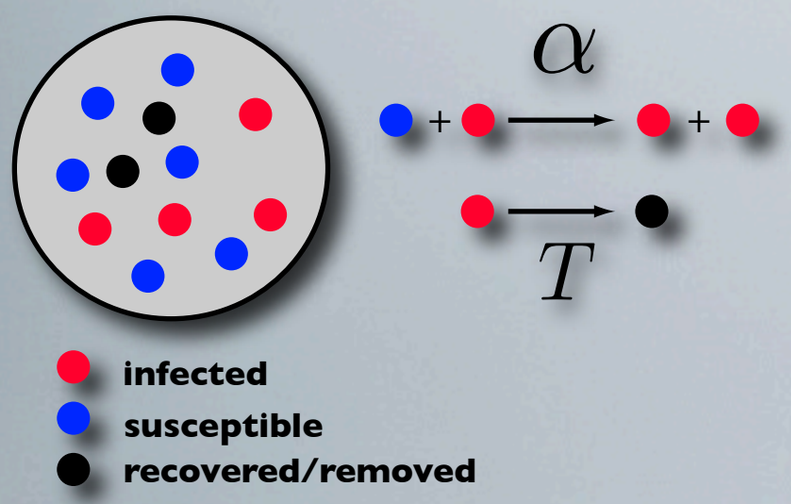
$$\partial_t I = \alpha I S/N - \beta I$$
$$\partial_t S = -\alpha I S/N$$

$$R_0 = \alpha/\beta$$



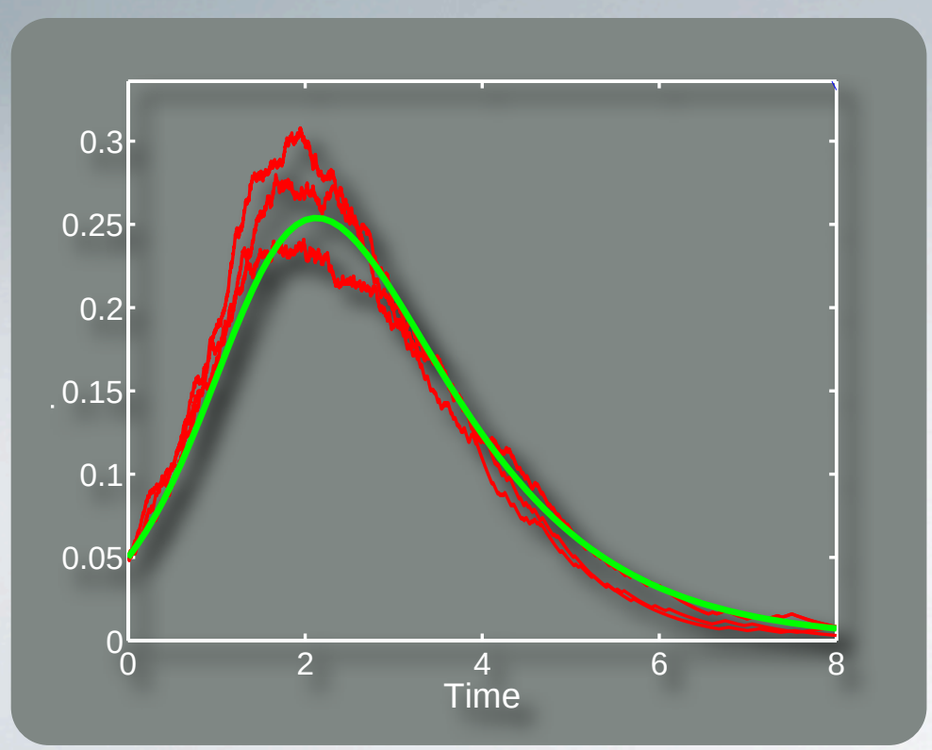
SINGLE POPULATION AND SPATIAL MODELS

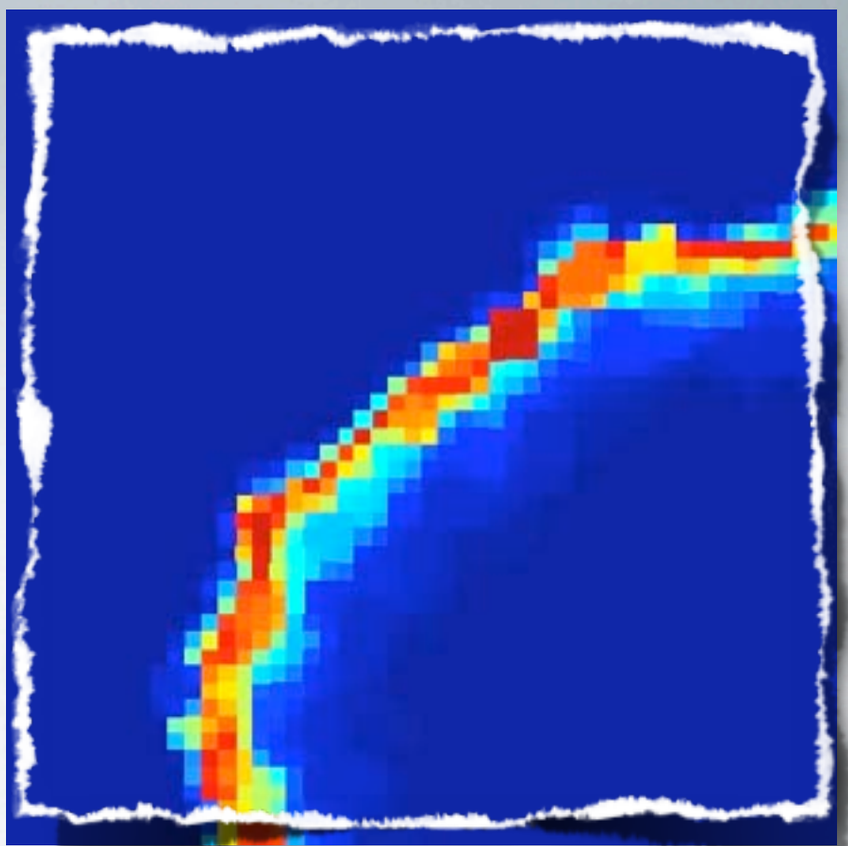
SIR dynamics

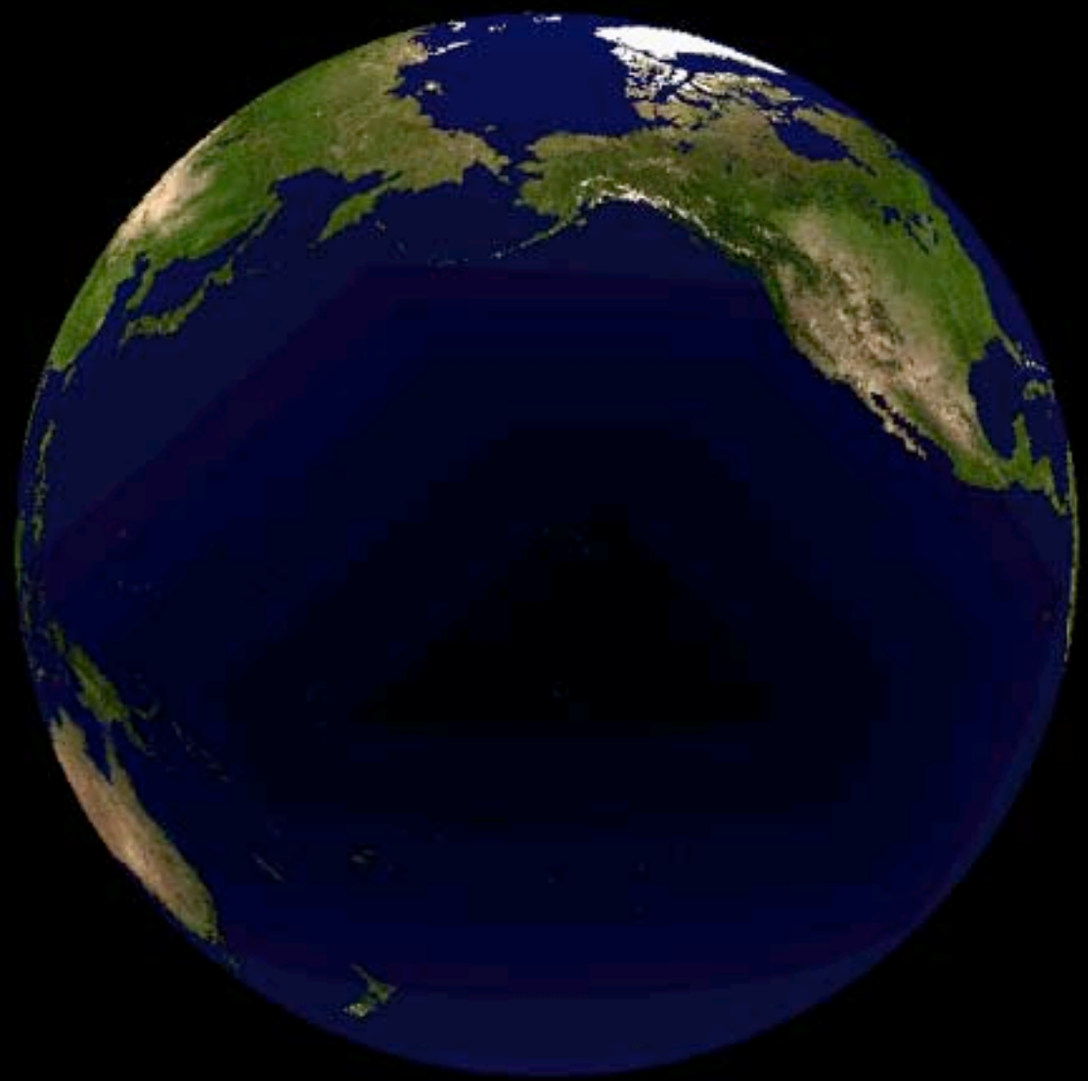


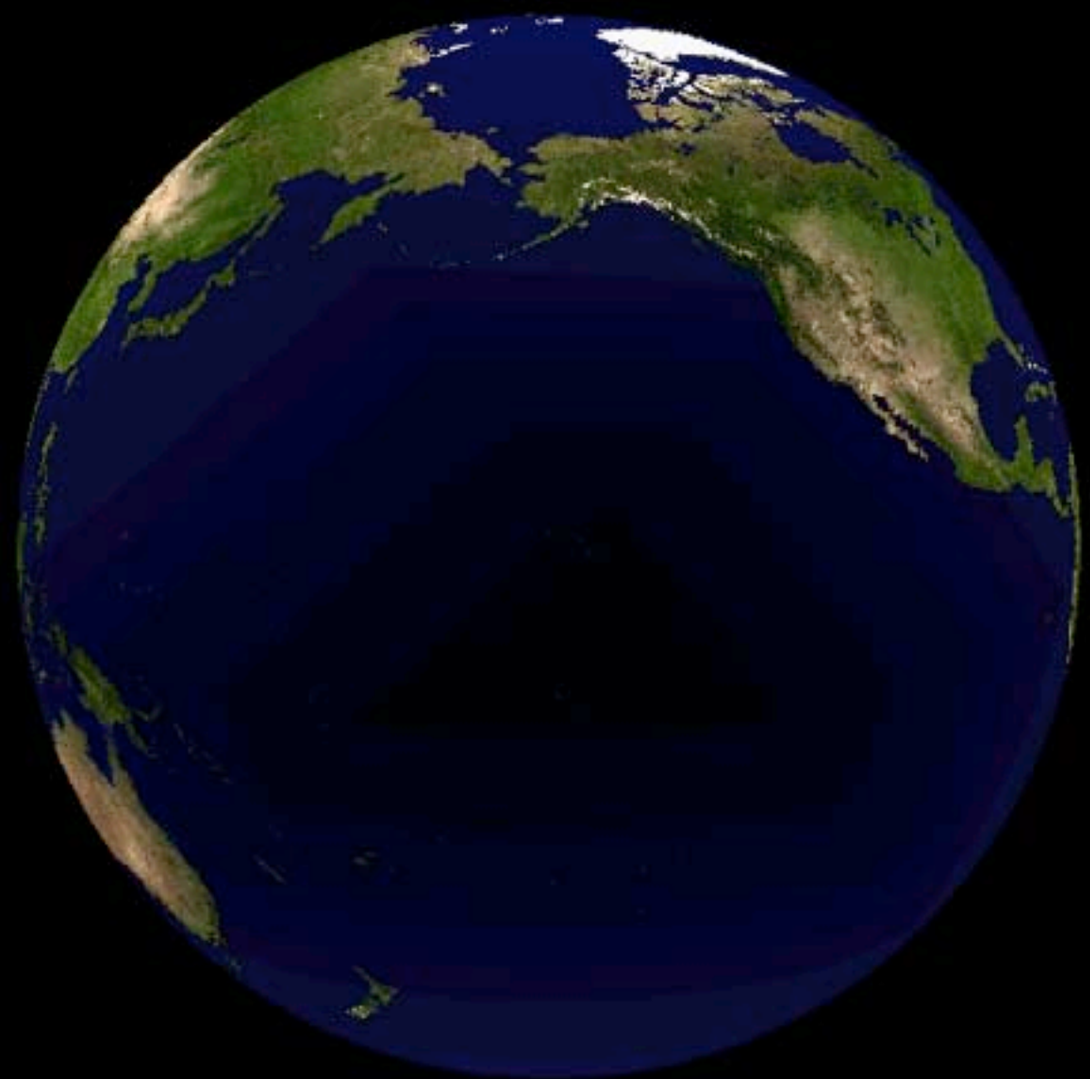
$$\partial_t I = \alpha I S/N - \beta I + D\partial_x^2 I$$
$$\partial_t S = -\alpha I S/N + D\partial_x^2 S$$

$$R_0 = \alpha/\beta \qquad v \propto \sqrt{(R_0 - 1)D}$$



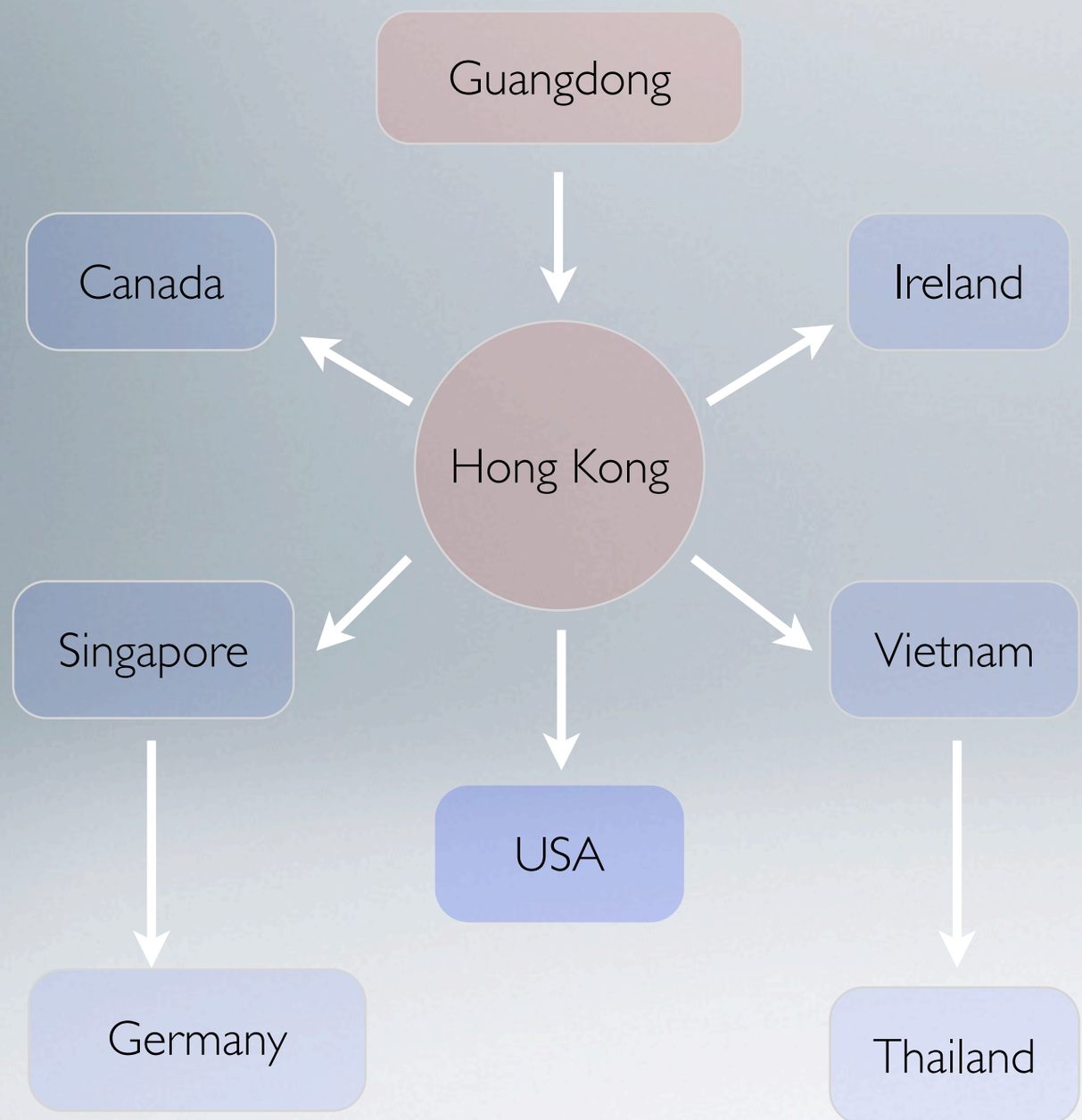




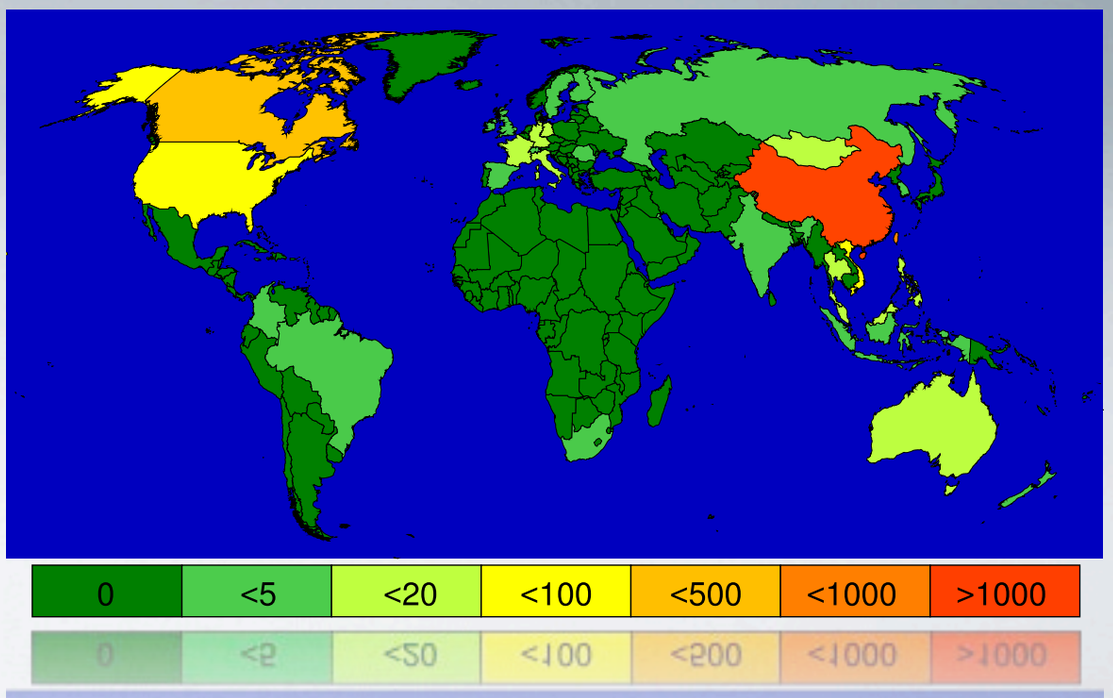


- 3 billion passengers/yr.
- 4095 airports
- 2 Mio. flights/week

THE „SEVERE ACUTE RESPIRATORY SYNDROME" (SARS)



- caused by coronavirus
- first outbreak: China
- February 2003 in Hong Kong
- worldwide spread in over 30 countries
- approx 8000 infections
- 10% die



MODEL FOR THE WORLDWIDE SPREAD OF AN

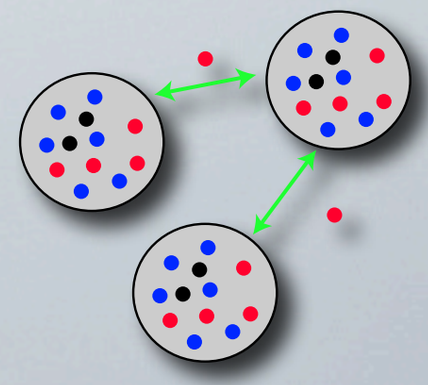
Hufnagel, L., Brockmann, D. & Geisel, T., *PNAS*, 2004

MODEL FOR THE WORLDWIDE SPREAD OF AN

Is it possible to describe (& and at some point predict) the global spread of human infectious diseases based on human air transportation?

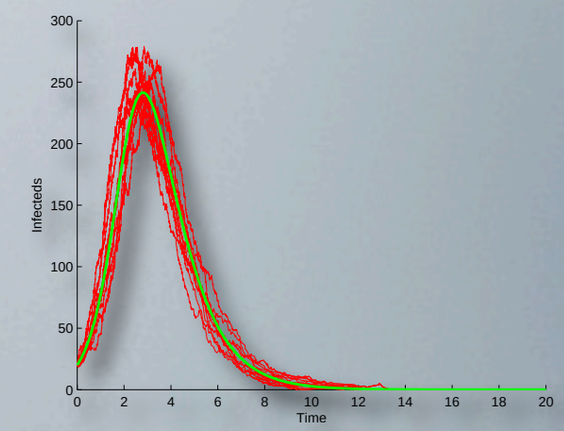
Hufnagel, L., Brockmann, D. & Geisel, T., *PNAS*, 2004

METAPOPULATION MODEL



+

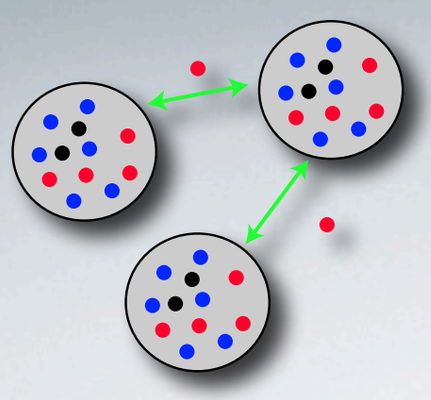
local SIR dynamics



$$\partial_t I_n = \alpha S_n I_n / N_n - \beta I_n + \sum_{m \neq n} (w_{nm} I_m - w_{mn} I_n)$$

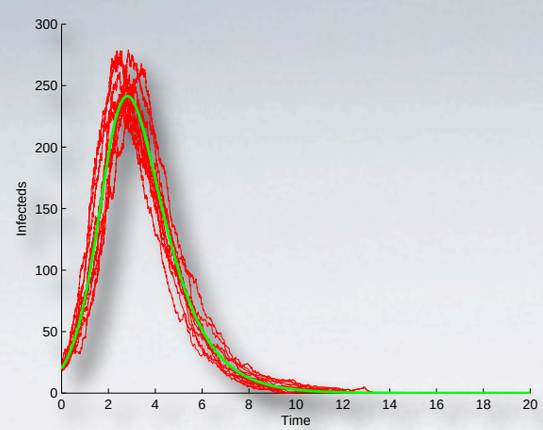
$$\partial_t S_n = -\alpha S_n I_n / N_n + \sum_{m \neq n} (w_{nm} S_m - w_{mn} S_n)$$

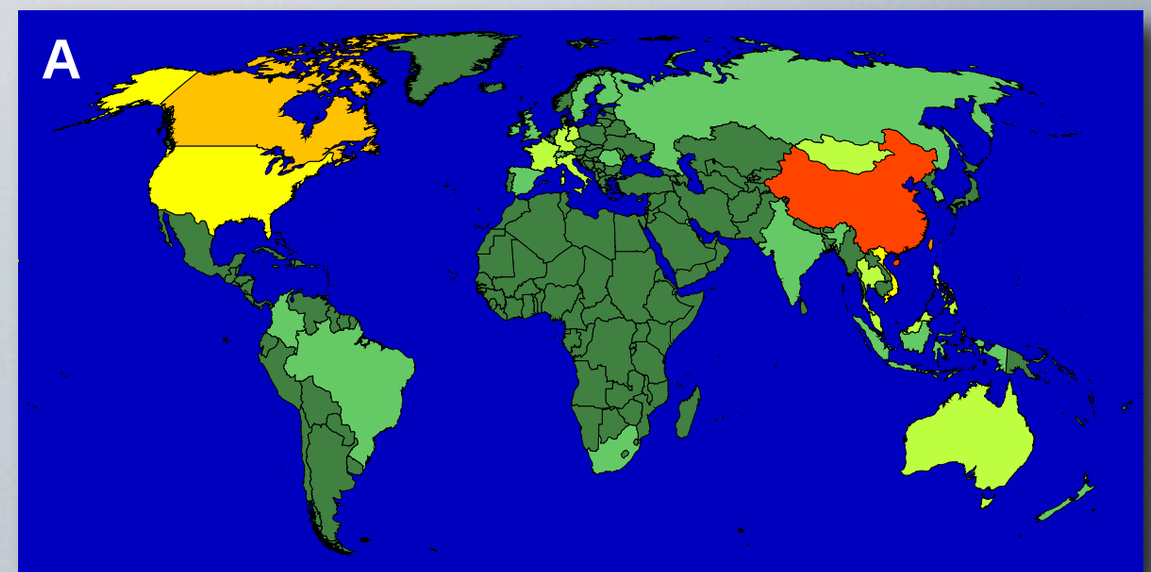
A MODEL FOR THE GLOBAL SPREAD OF SARS

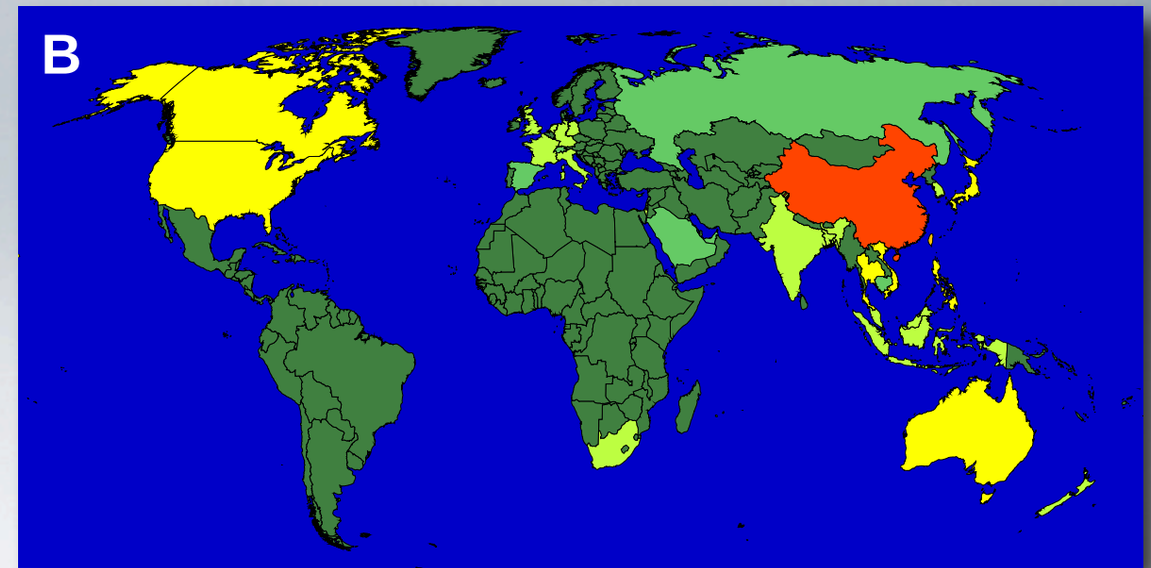


+

local SIR dynamics







0	<5	<20	<100	<500	<1000	>1000

A: WHO B: model

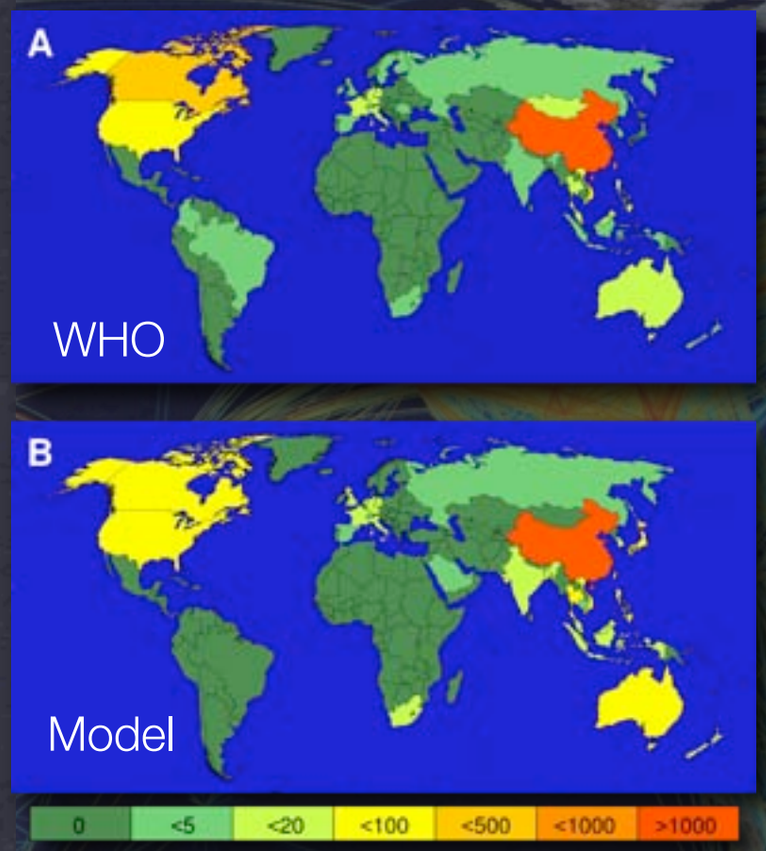
Hufnagel, L., Brockmann, D. & Geisel, T., *PNAS*, 2004

global air transportation in epidemic contexts

Dirk Brockmann ESAM & NICO Northwestern University http://rocs.northwestern.edu

global air transportation in epidemic contexts

SARS 2003



Hufnagel et al. Forecast and control of epidemics in a globalized world. PNAS (2004)

Ferguson et al. Strategies for containing an emerging influenza pandemic in Southeast Asia. Nature (2005)

Hollingsworth et al. Will travel restrictions control the international spread of pandemic influenza?. Nat Med (2006)

Colizza et al. The role of the airline transportation network in the prediction and predictability of global epidemics. PNAS (2006)

Ferguson et al. Strategies for mitigating an influenza pandemic. Nature (2006)

Colizza et al. Modeling the worldwide spread of pandemic influenza: Baseline case and containment interventions. Plos Med (2007)

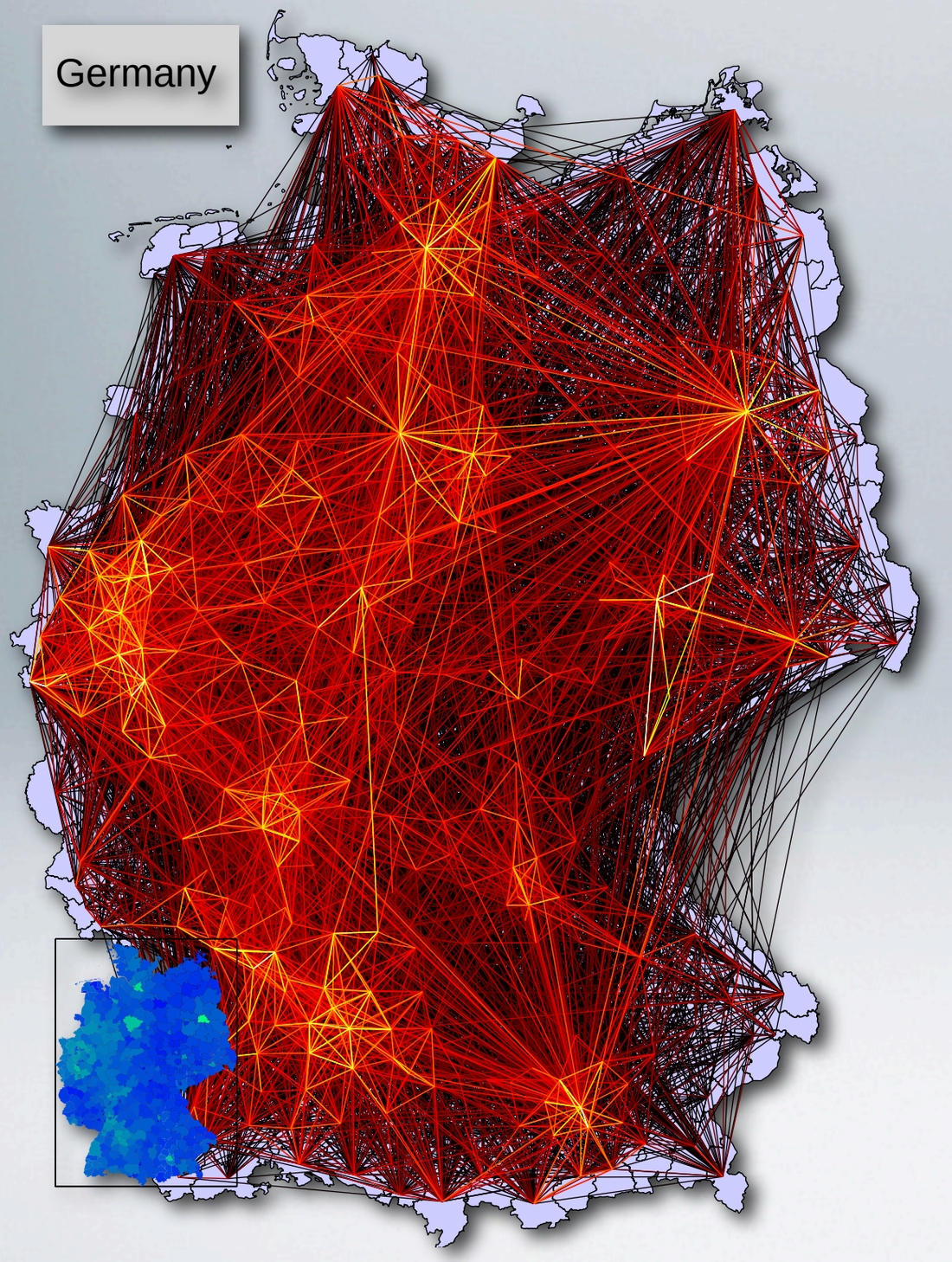
Colizza et al. Reaction-diffusion processes and metapopulation models in heterogeneous networks. Nat Phys (2007)

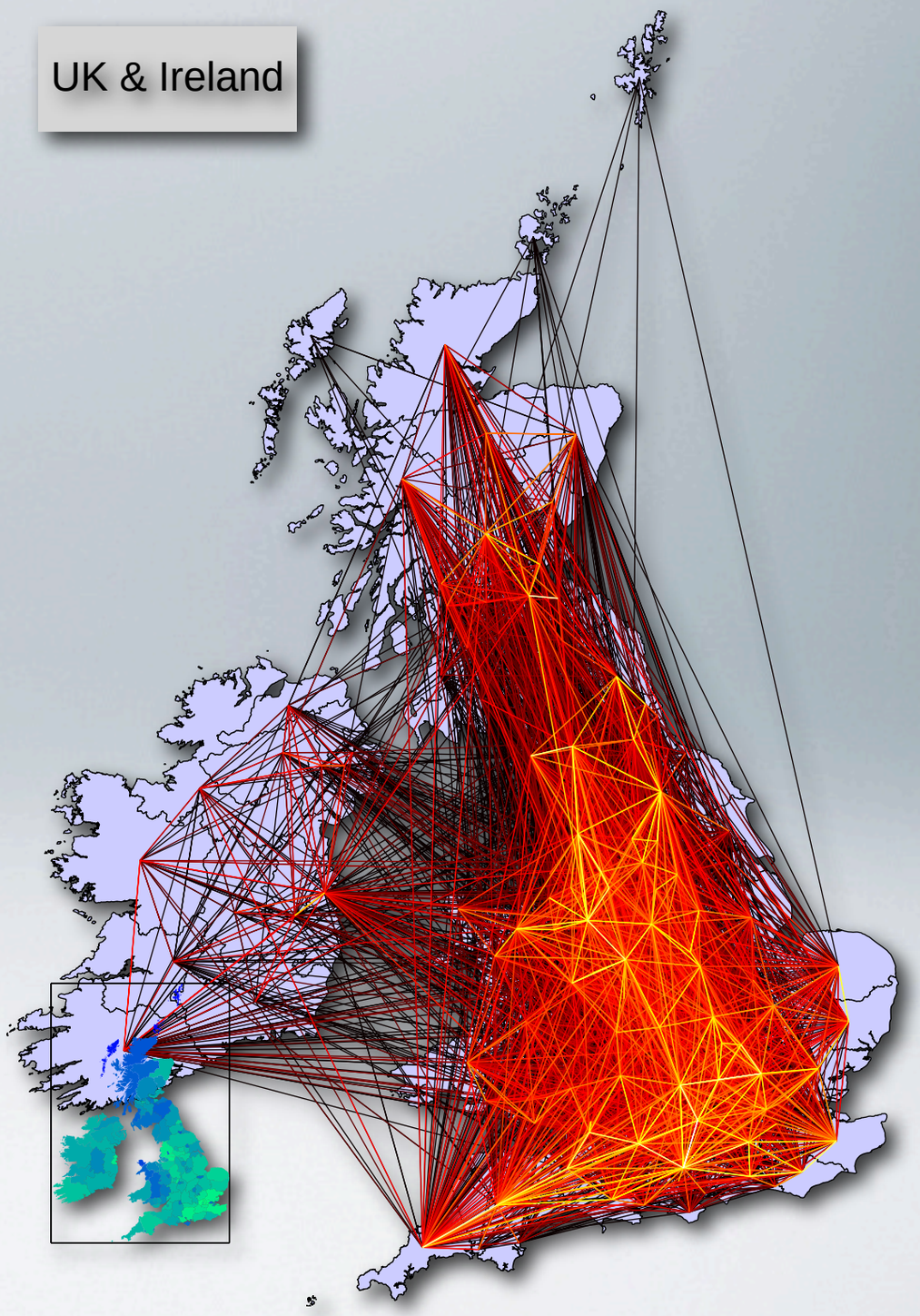
Colizza et al. Invasion threshold in heterogeneous metapopulation networks. Phys Rev Lett (2007) vol. 99 (14)

Riley. Large-scale spatial-transmission models of infectious disease. Science (2007)

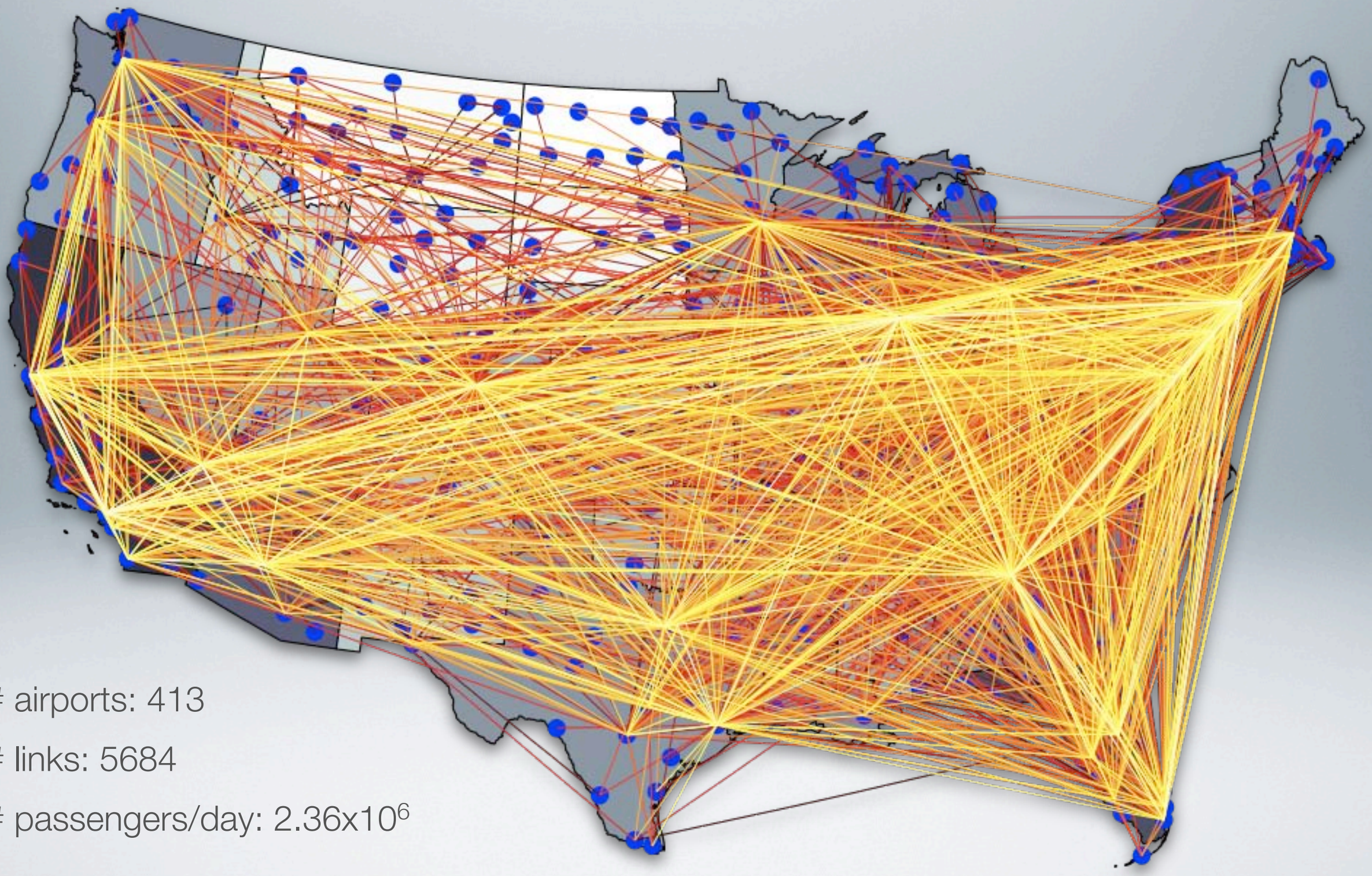
Colizza et al. Epidemic modeling in metapopulation systems with heterogeneous coupling pattern: Theory and simulations. J Theor Biol (2008)

MULTI-SCALE MOBILITY





US AIR TRANSPORTATION

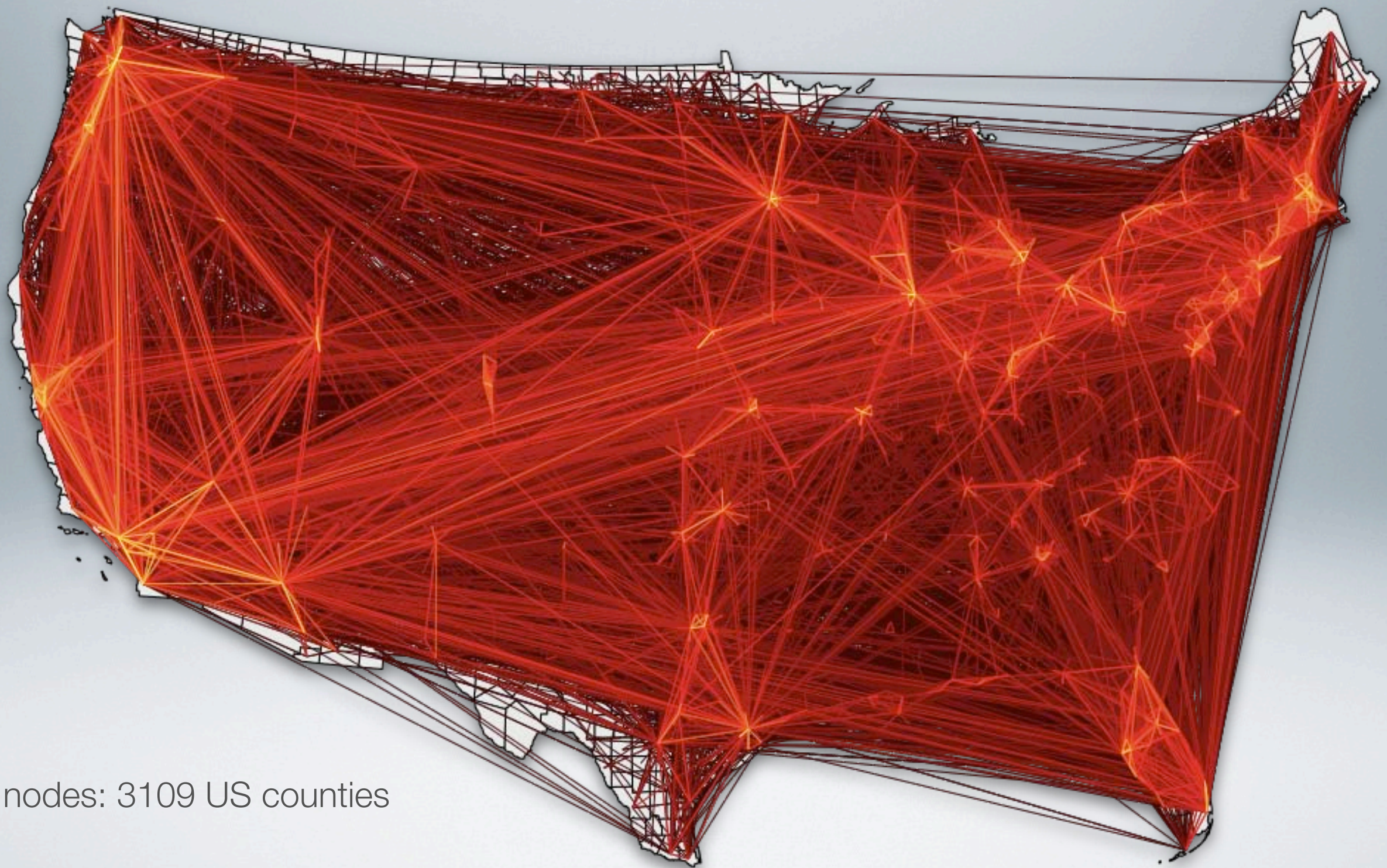


airports: 413

links: 5684

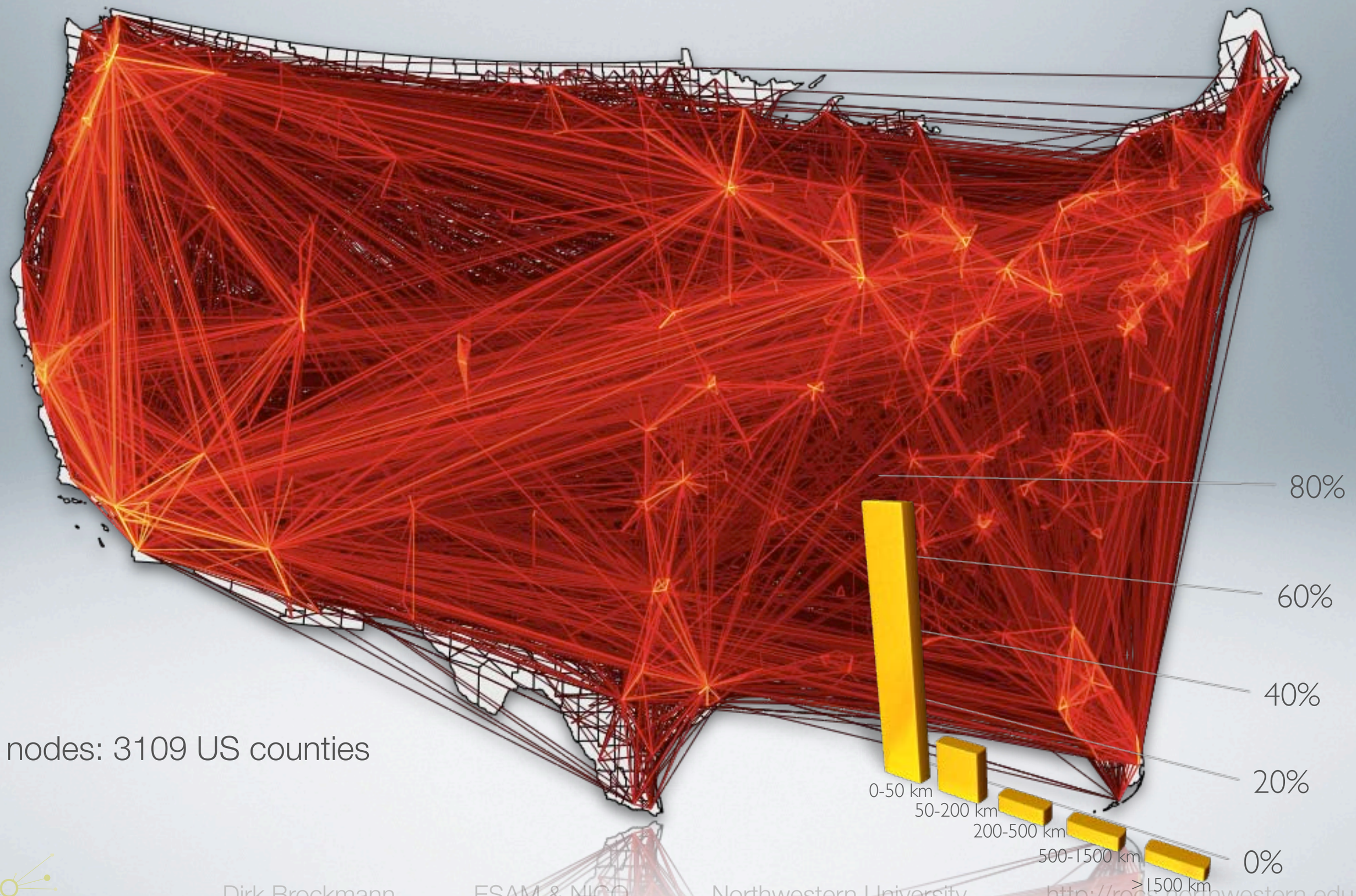
passengers/day: 2.36×10^6

MULTI SCALE HUMAN MOBILITY



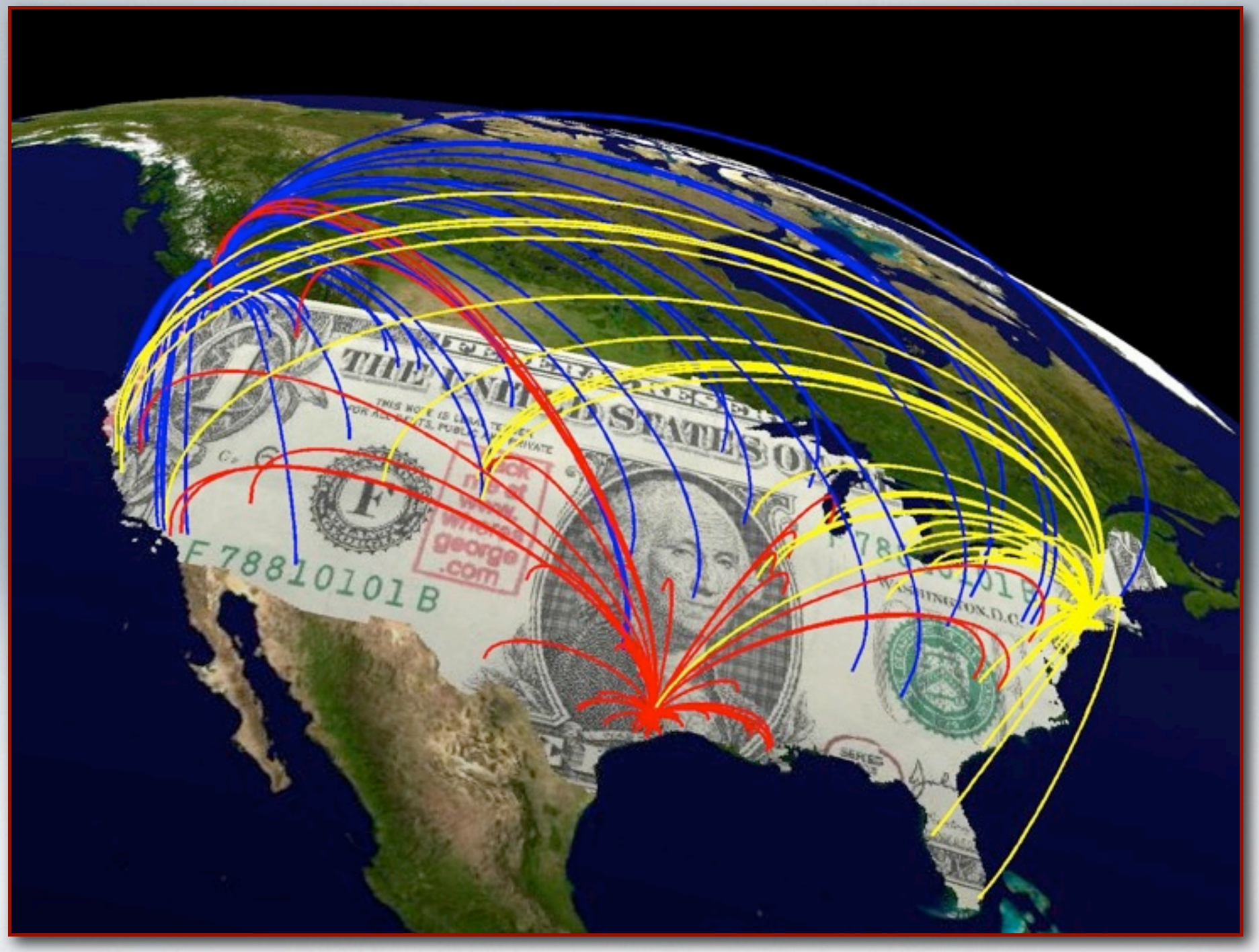
nodes: 3109 US counties

MULTI SCALE HUMAN MOBILITY



nodes: 3109 US counties

IDEA : „FOLLOW THE MONEY“



IDEA : „FOLLOW THE MONEY“

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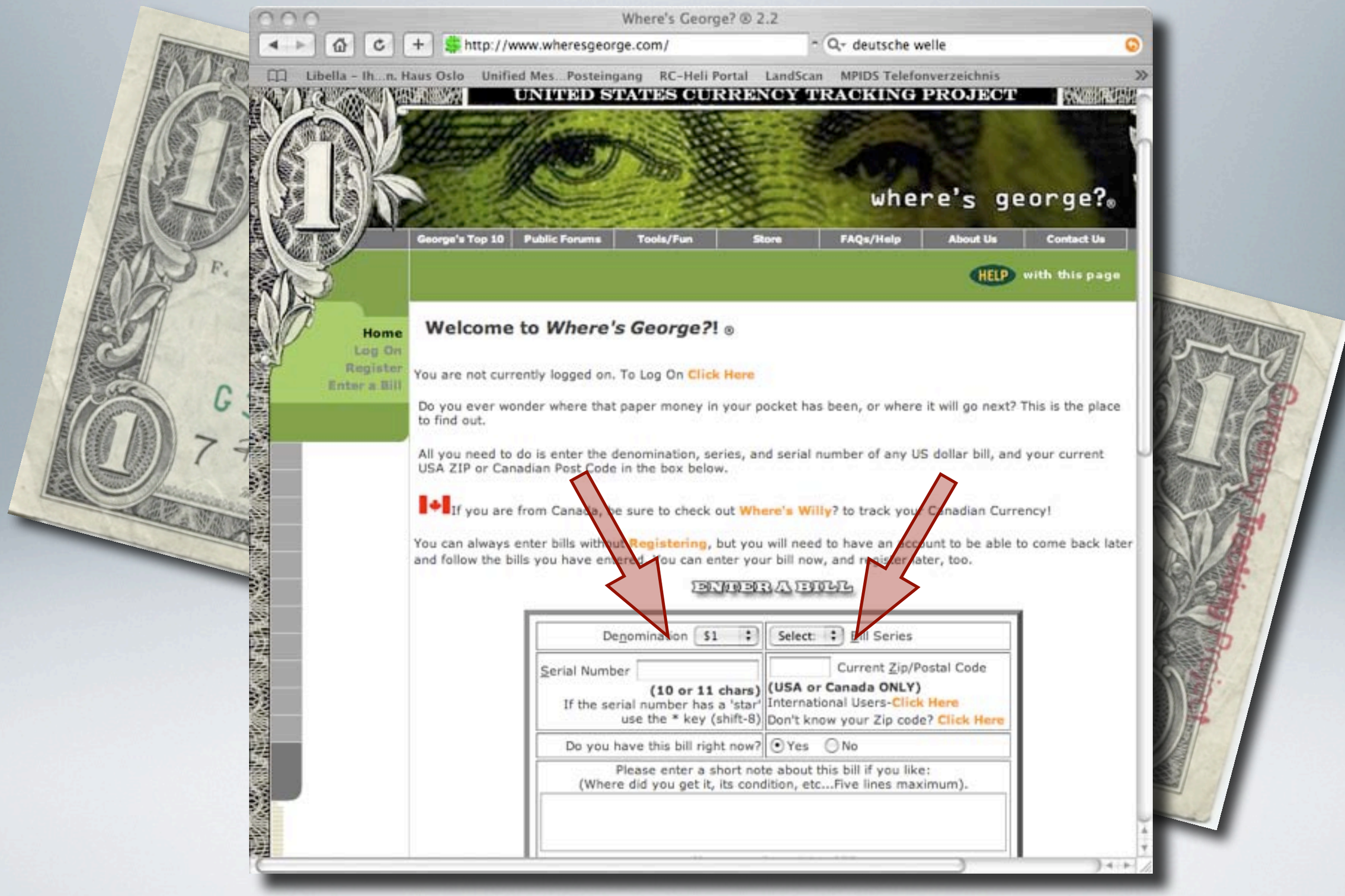


HOW WHERESGEORGE WORKS

HOW WHERESGEORGE WORKS

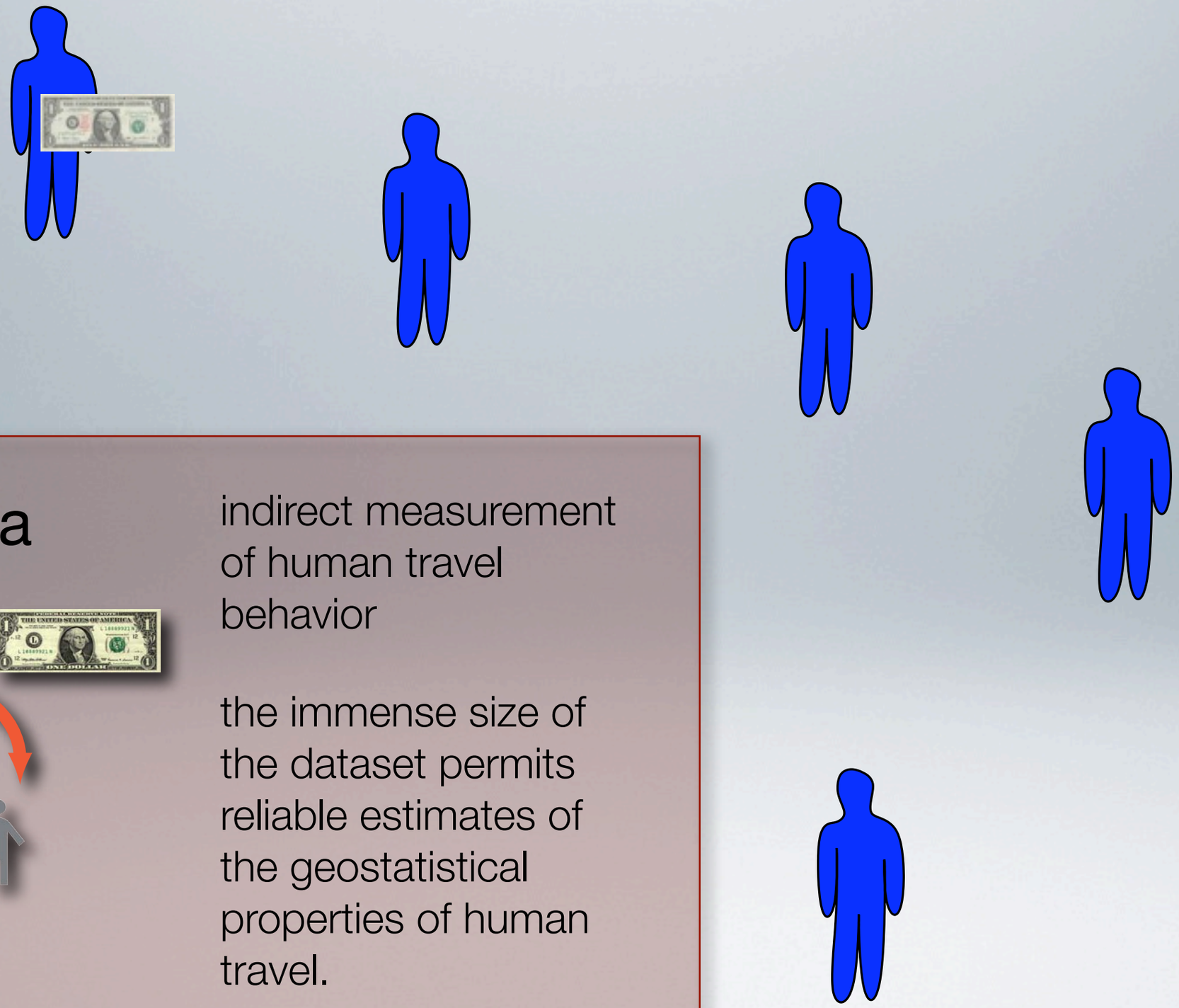
1. bills are registered
2. bills enter circulation
3. bills travel
4. bills are reported
5. bills reenter circulation
6. ...

HOW WHERESGEORGE WORKS

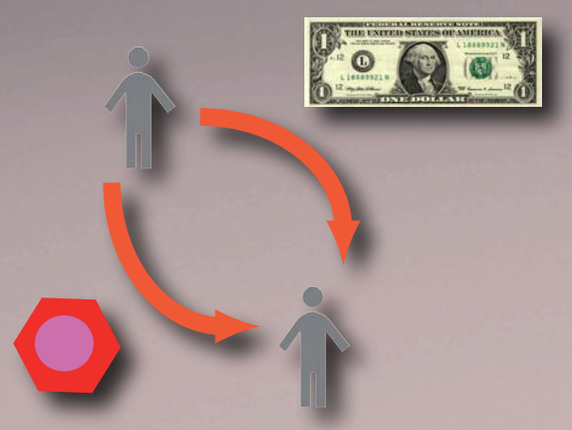
1. bills are registered
2. bills enter circulation
3. bills travel
4. bills are reported
5. bills reenter circulation
6. ...

- initiated in 1998 by Hank Eskin
- high spatial and temporal resolution
- more than 100 million bills are registered
- more than 10 million bills with hits

HUMAN MOBILITY AND MONEY DISPERSAL



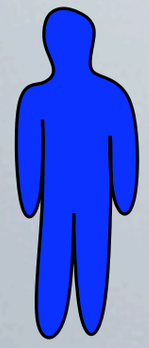
the idea

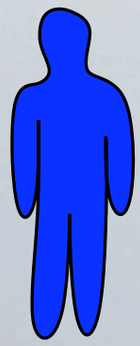


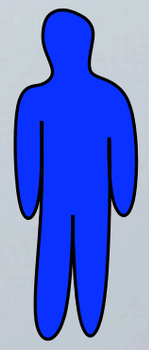
indirect measurement of human travel behavior

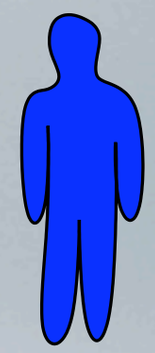
the immense size of the dataset permits reliable estimates of the geostatistical properties of human travel.

HUMAN MOBILITY AND MONEY DISPERSAL

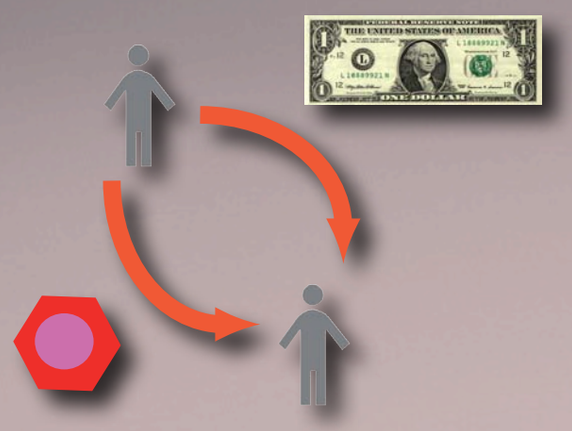








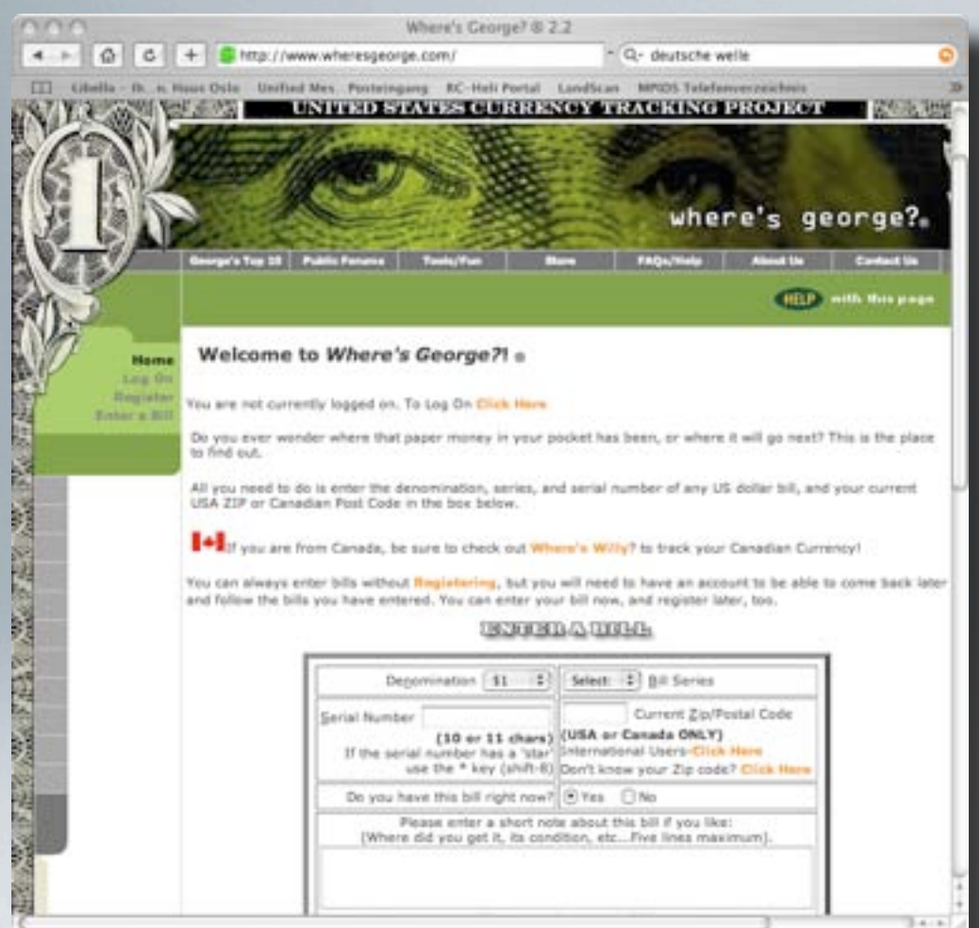
the idea



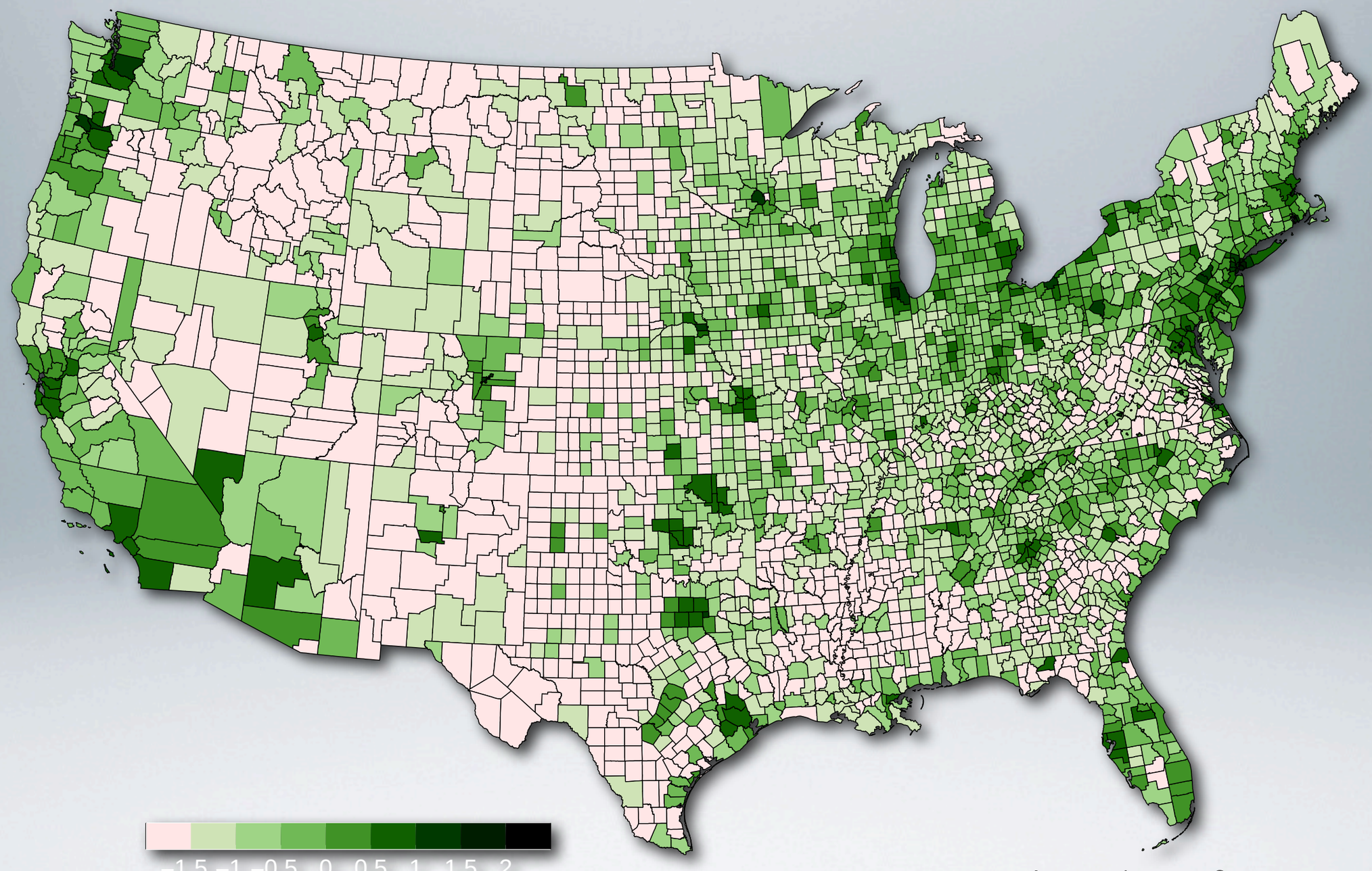
indirect measurement of human travel behavior

the immense size of the dataset permits reliable estimates of the geostatistical properties of human travel.

www.wheresgeorge.com www.eurobilltracker.com

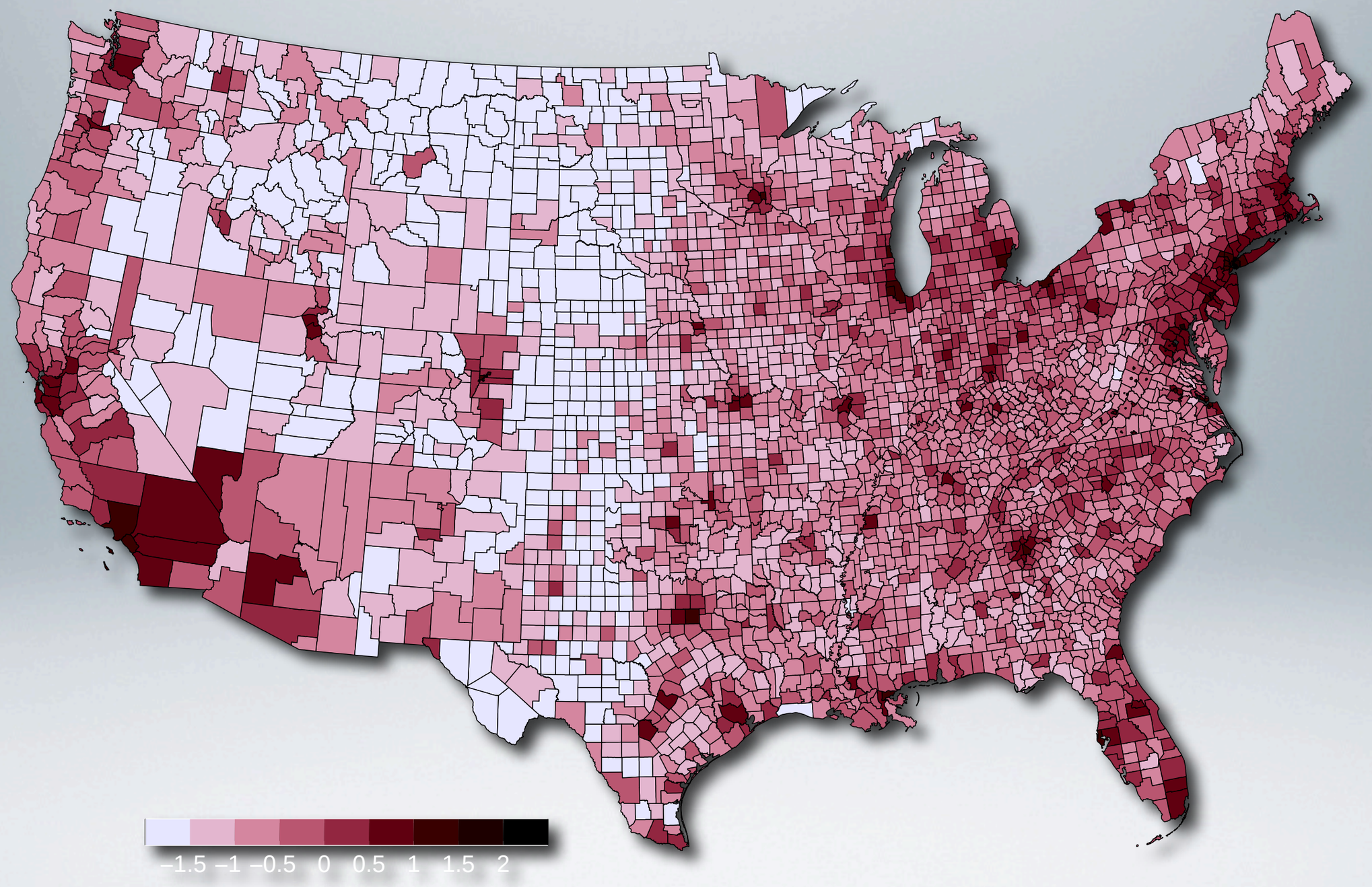


REPORT DENSITY



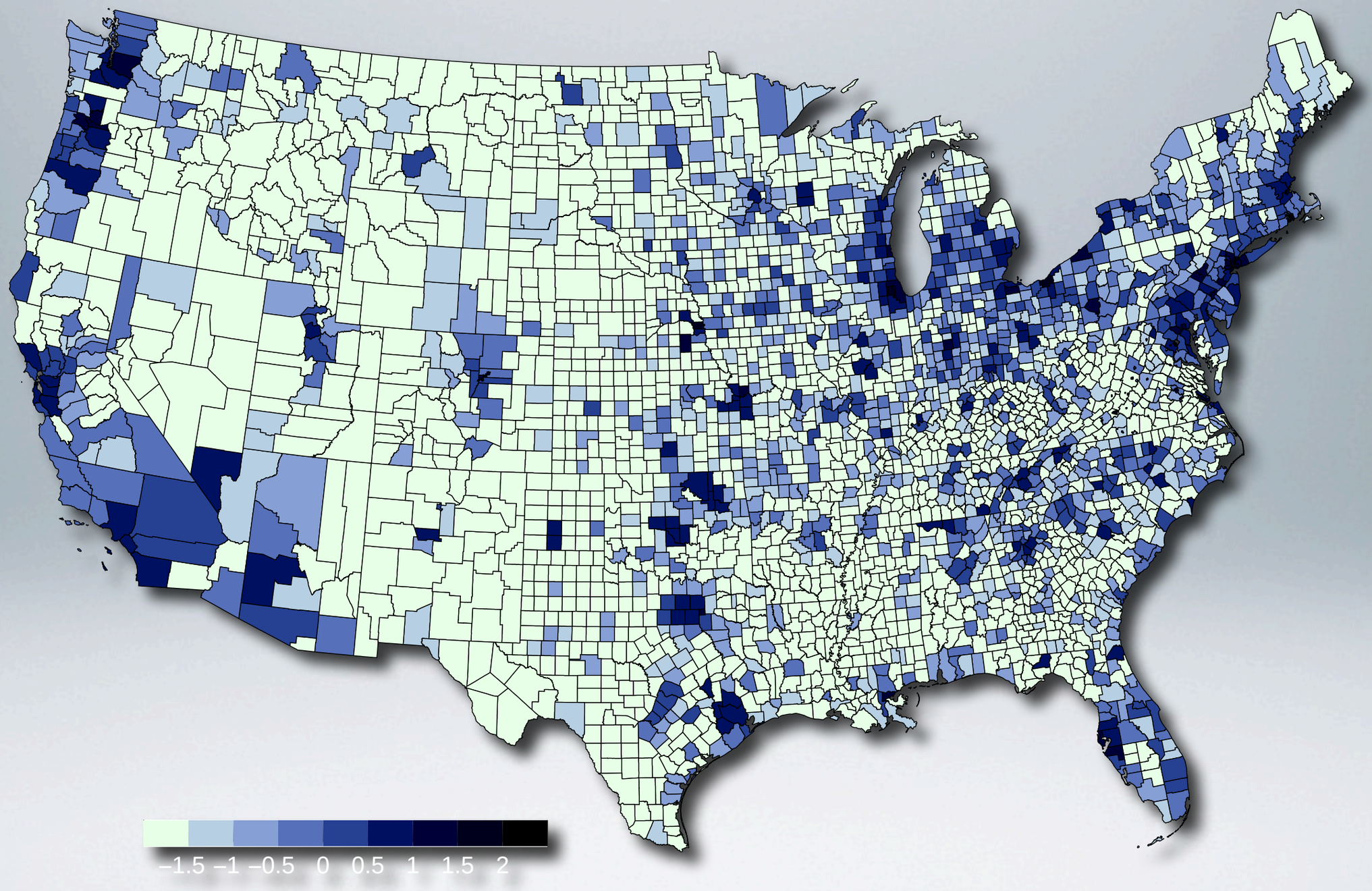
total number of reports: 1,033,095

POPULATION DENSITY



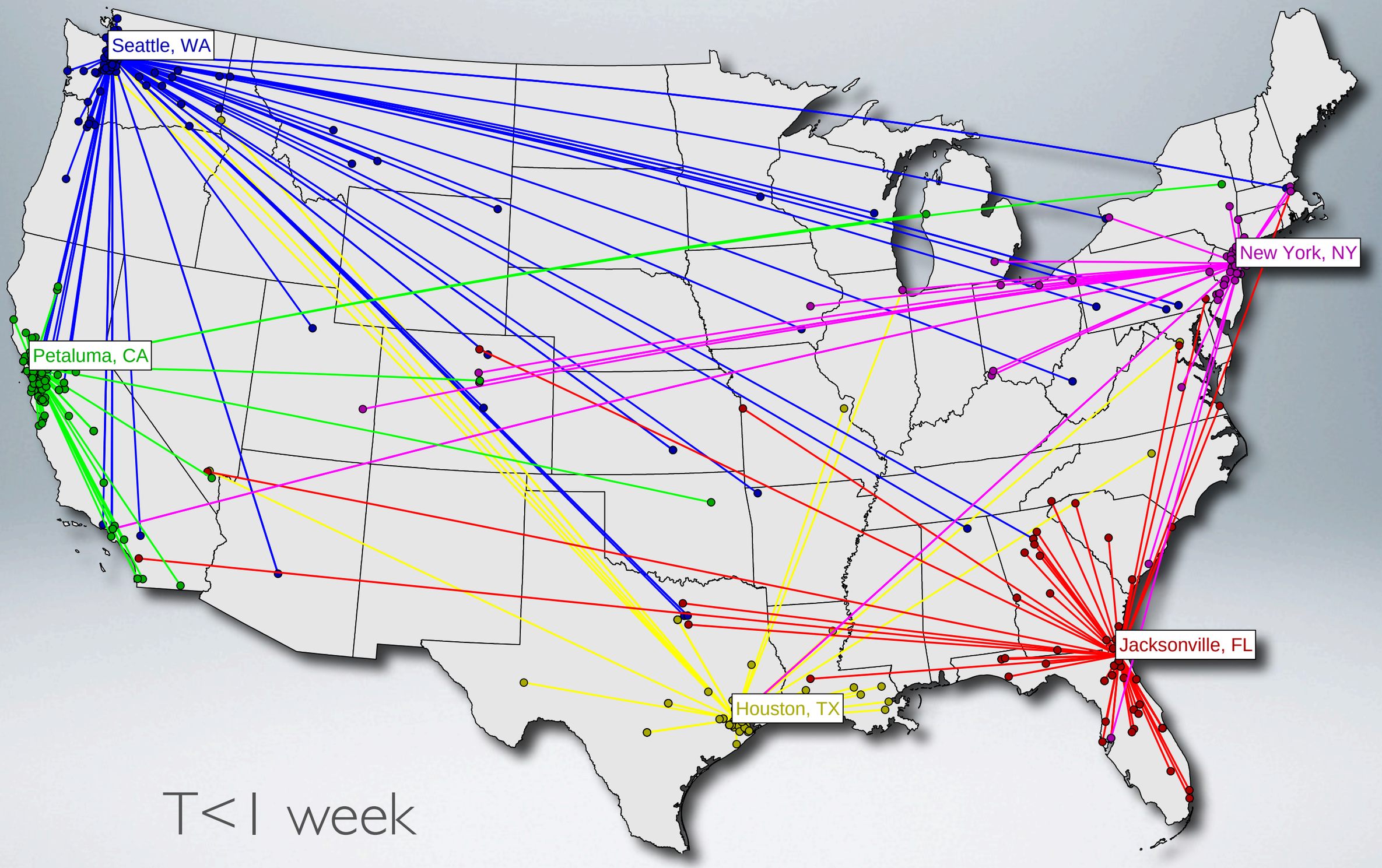
total population: 280 Mio.

INITIAL ENTRY DENSITY



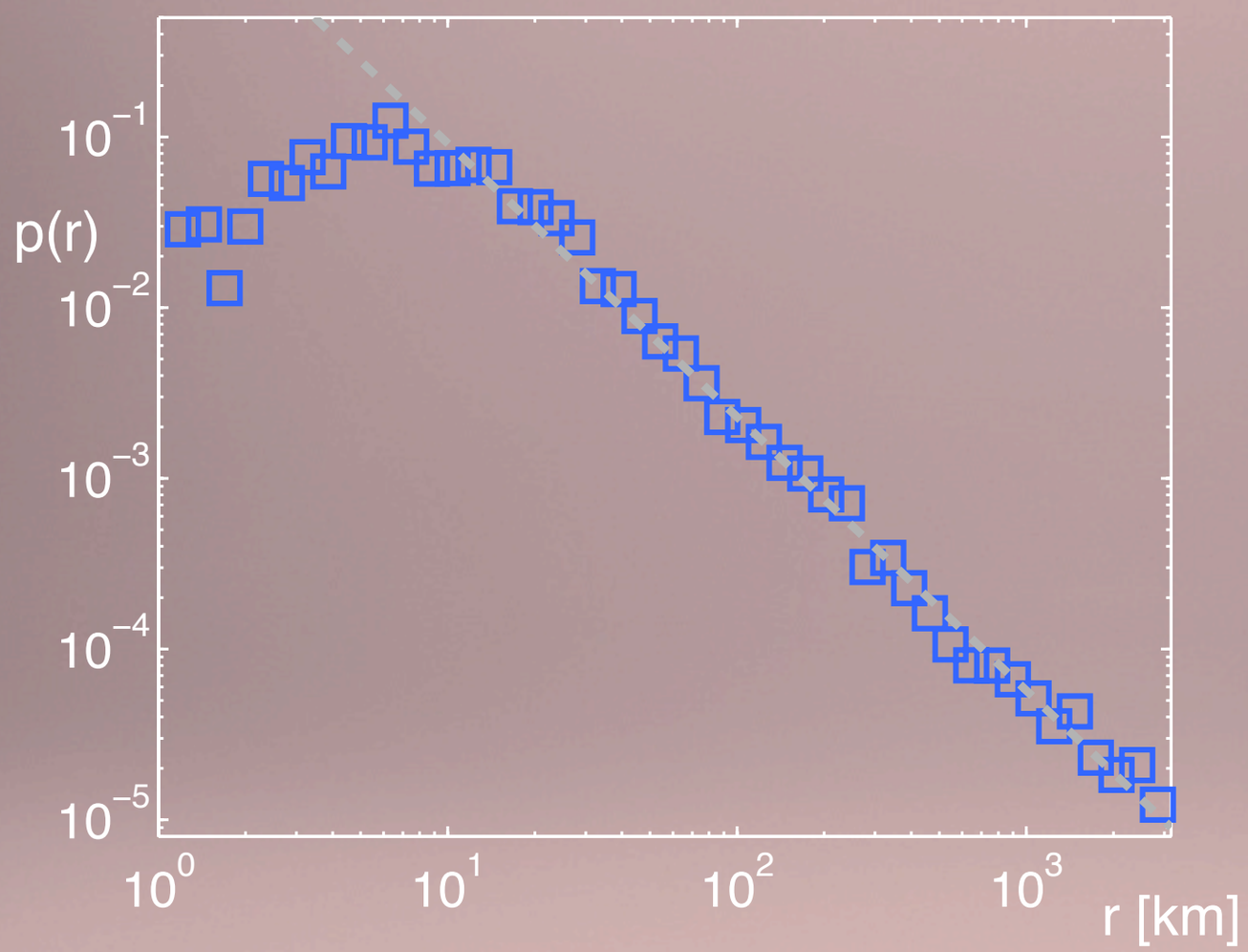
total number of initial entries: 408,321

DYNAMICS AND SHORT TIME TRAJECTORIES



T<1 week

SHORT TIME TRAJECTORY STATISTICS

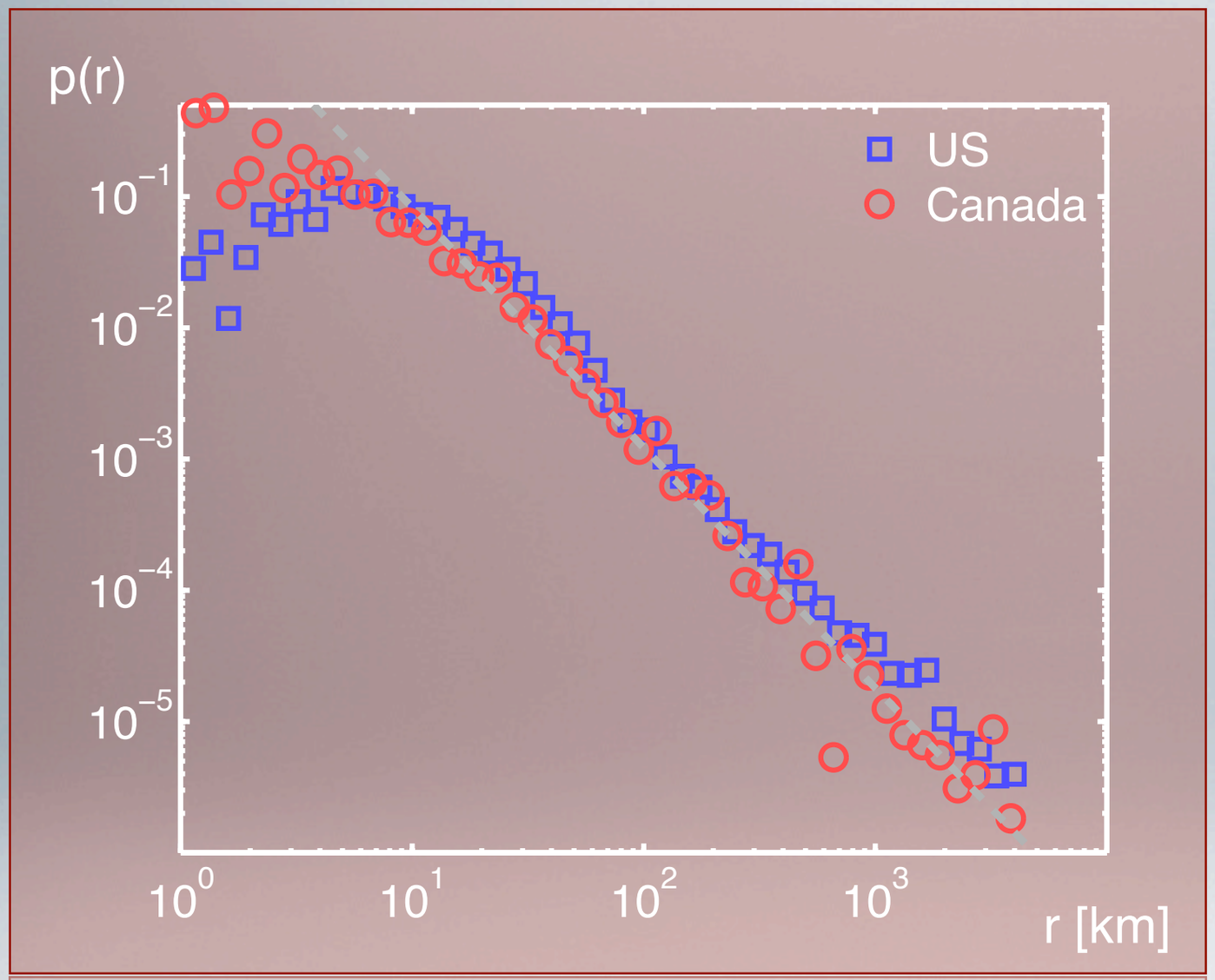


$$p(r) \sim \frac{1}{r^{1+\beta}} \qquad \beta \approx 0.6$$

- power law (20-3500 km)
- travel lacks a scale
- trajectories of bills are reminiscent of lévy flights
- dispersal is superdiffusive

Brockmann,D., Hufnagel, L. & Geisel, T., Nature 2006

SHORT TIME TRAJECTORY STATISTICS



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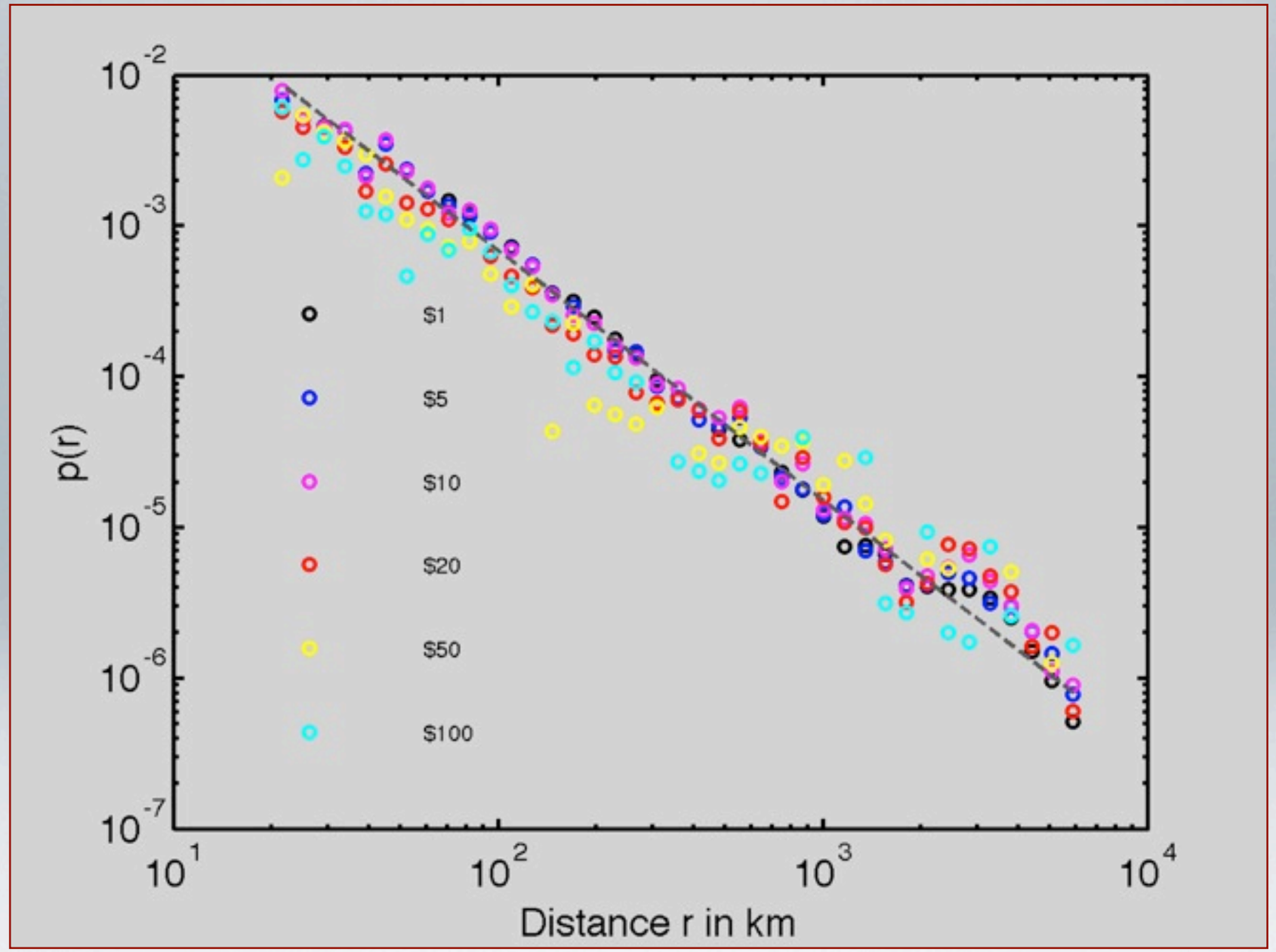
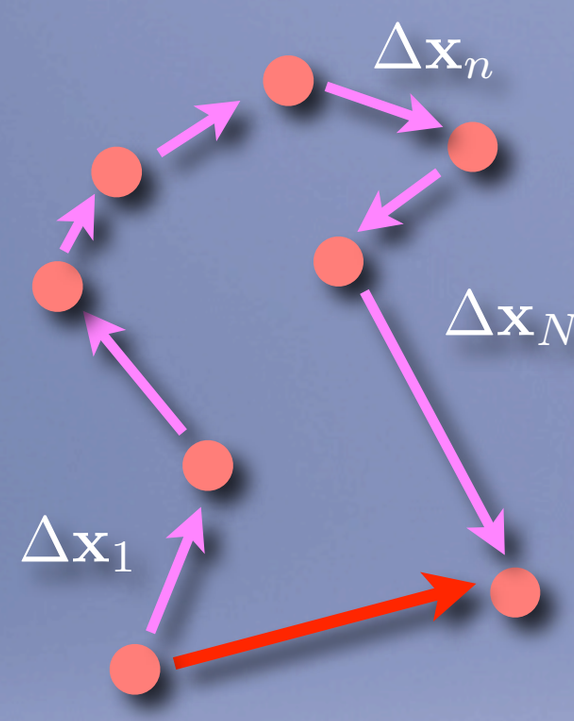
DENOMINATION ANALYSIS



RANDOM WALKS

2-d random walk



position after N steps

$$\mathbf{x}_N = \sum_{n=1}^{N} \Delta\mathbf{x}_n$$

$$p(\Delta\mathbf{x}) = p(|\Delta\mathbf{x}|) \quad \langle \Delta\mathbf{x}^2 \rangle = \sigma^2$$

$$p(\mathbf{x}, N) \sim N^{-1} G\left(\mathbf{x}/N^{1/2}\right)$$

$$|\mathbf{x}_N| \sim N^{1/2}$$

gaussian

RANDOM WALKS

2-d random walk

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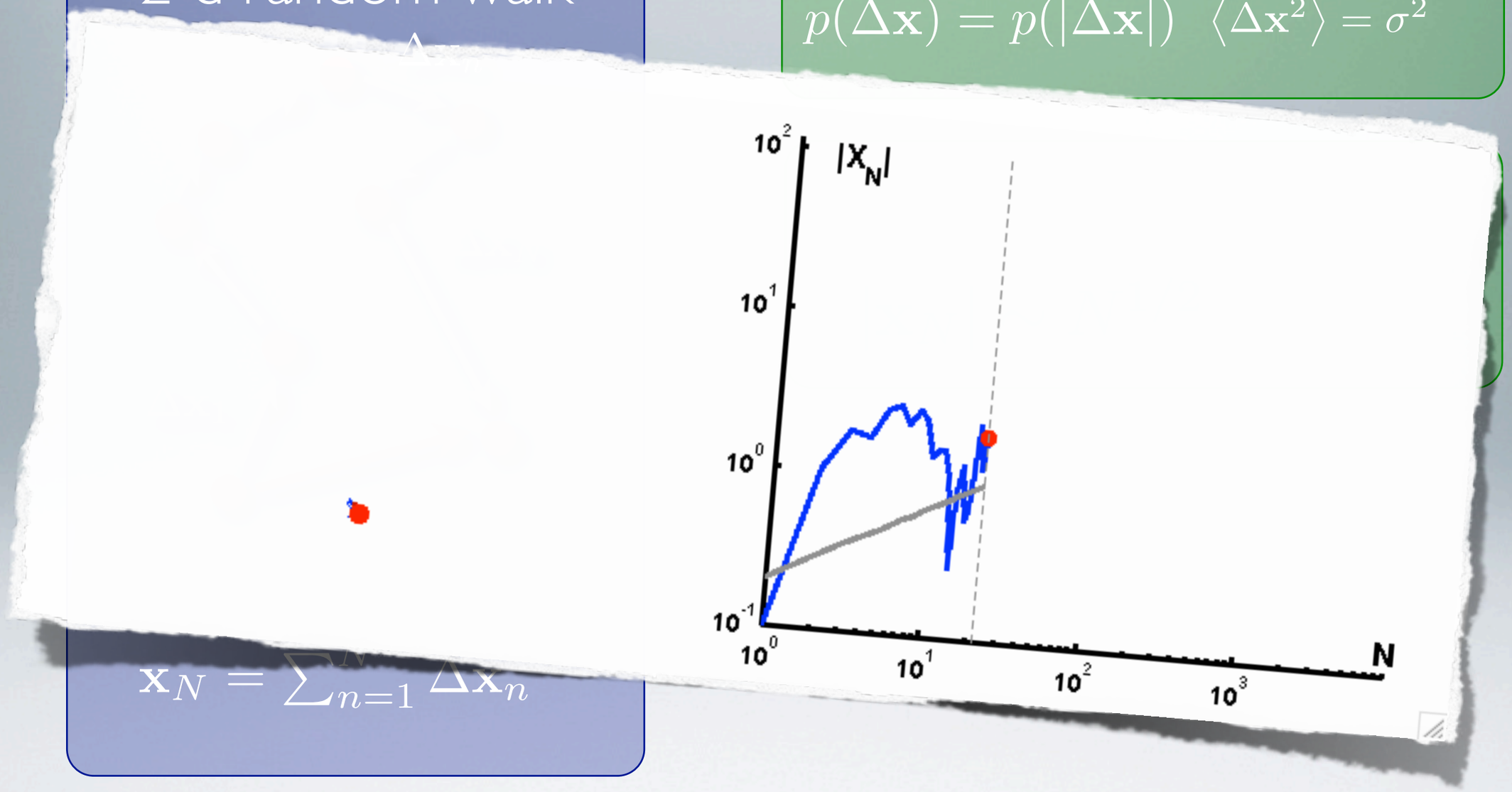
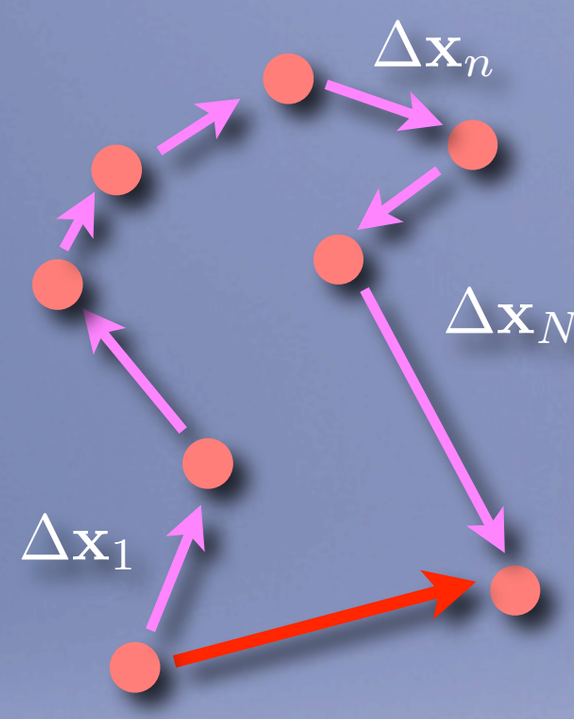


LEVY FLIGHTS

2-d random walk



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$$p(\Delta\mathbf{x}) = \frac{1}{|\Delta\mathbf{x}|^{D+\beta}} \qquad \beta < 2$$

$$p(\mathbf{x}, N) \sim N^{-2/\beta} L_\beta\left(\mathbf{x}/N^{1/\beta}\right)$$

$$|\mathbf{x}_N| \sim N^{1/\beta}$$

lévy stable law

LEVY FLIGHTS

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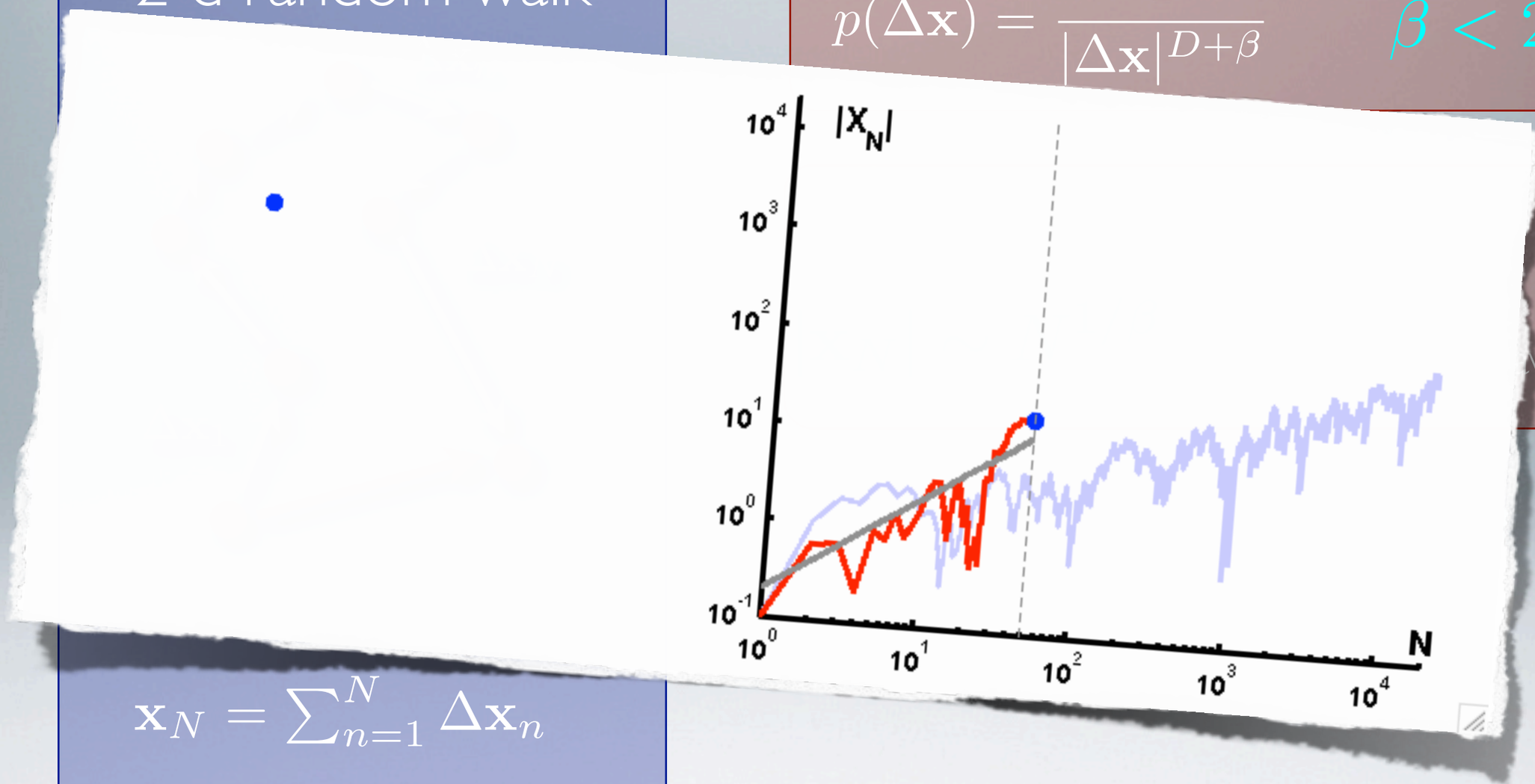
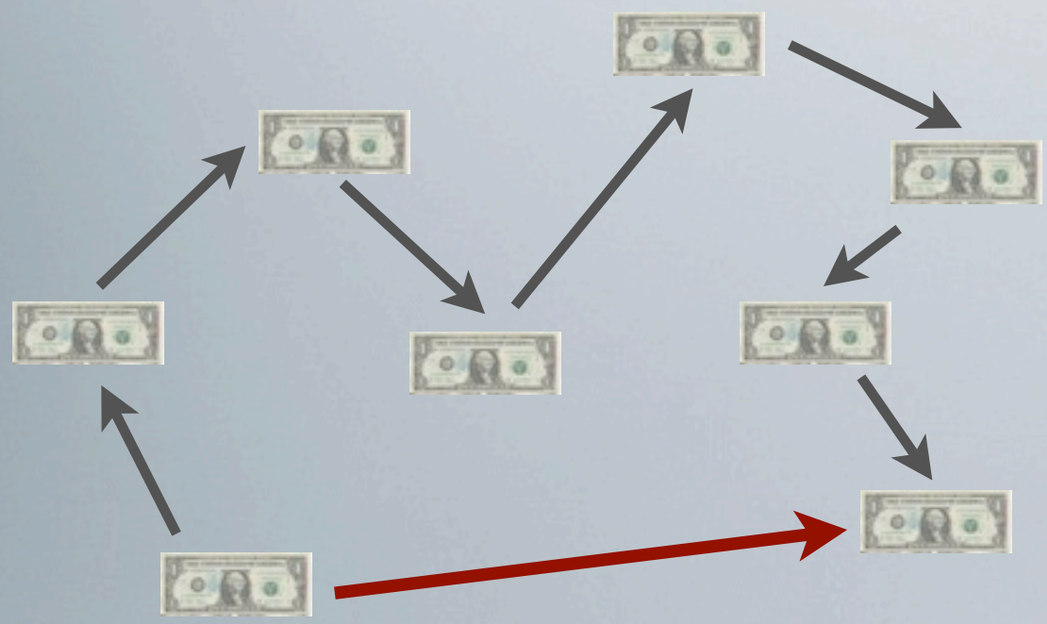


$$\mathbf{x}_N = \sum_{n=1}^{N} \Delta\mathbf{x}_n$$

IMPLICATIONS FOR THE GEOMETRY OF MONEY CIRCULATION



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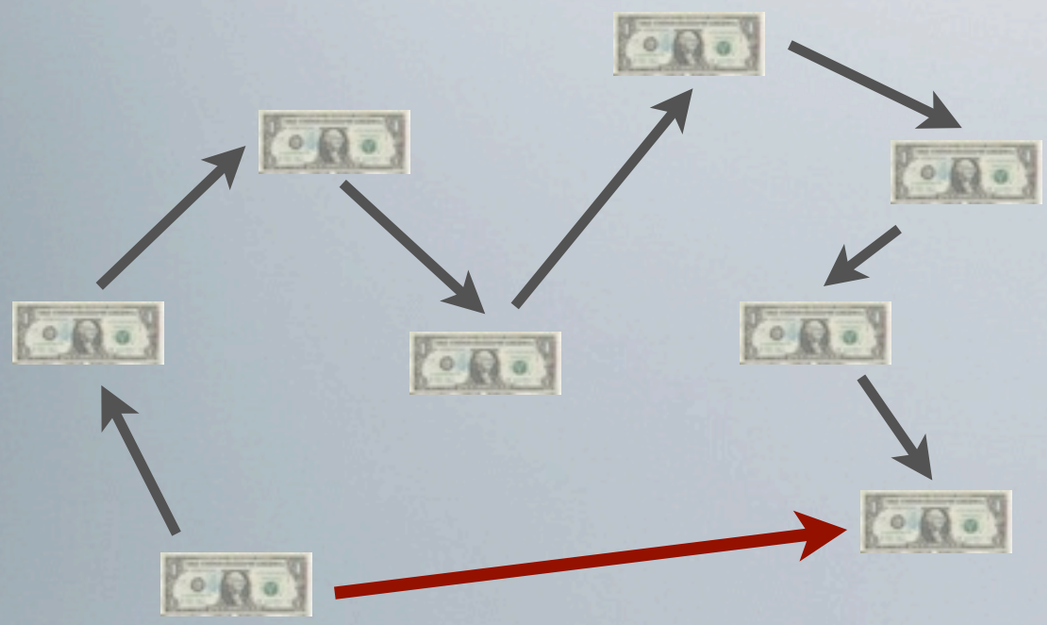

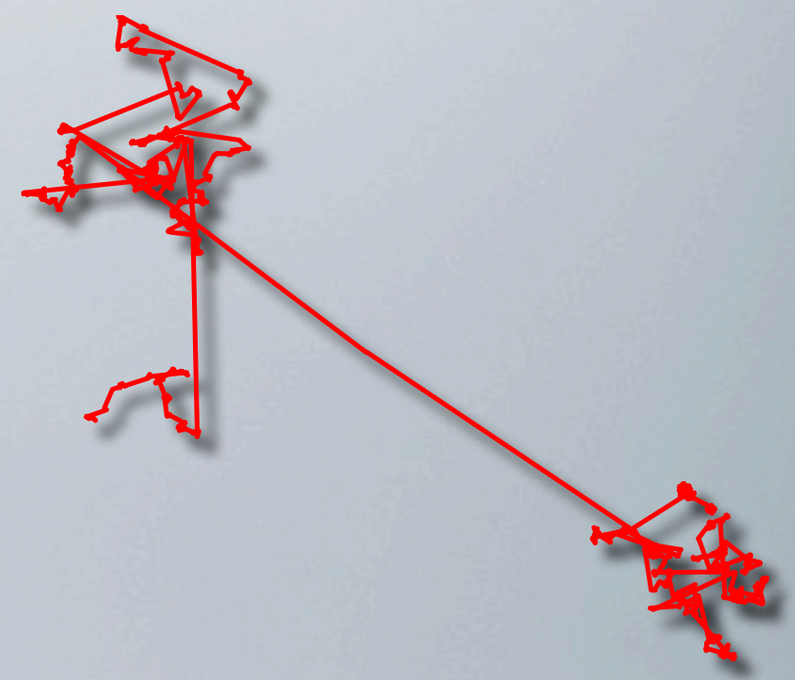

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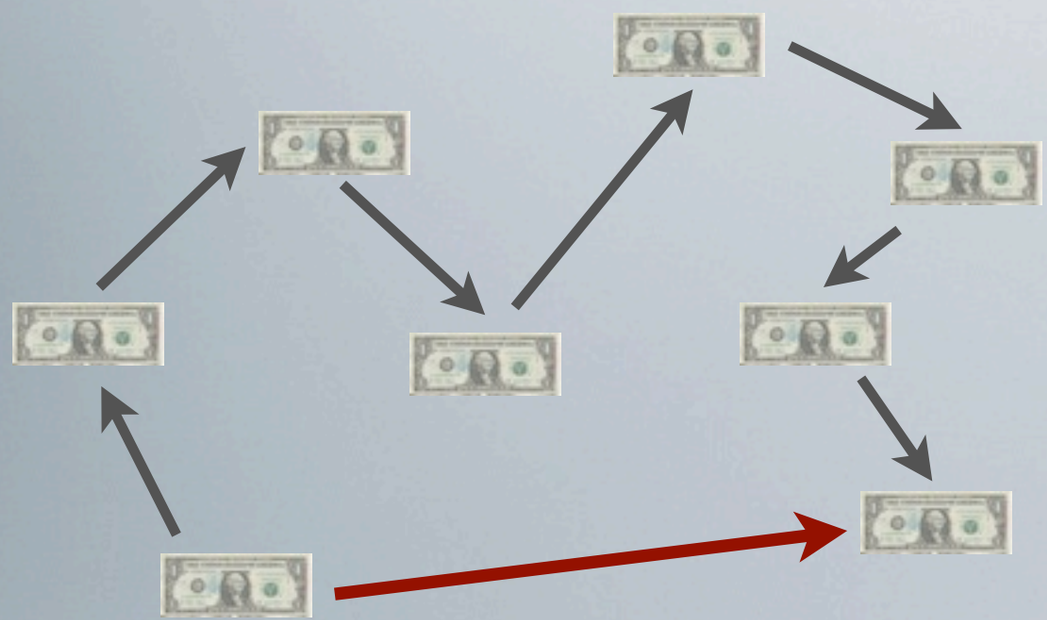
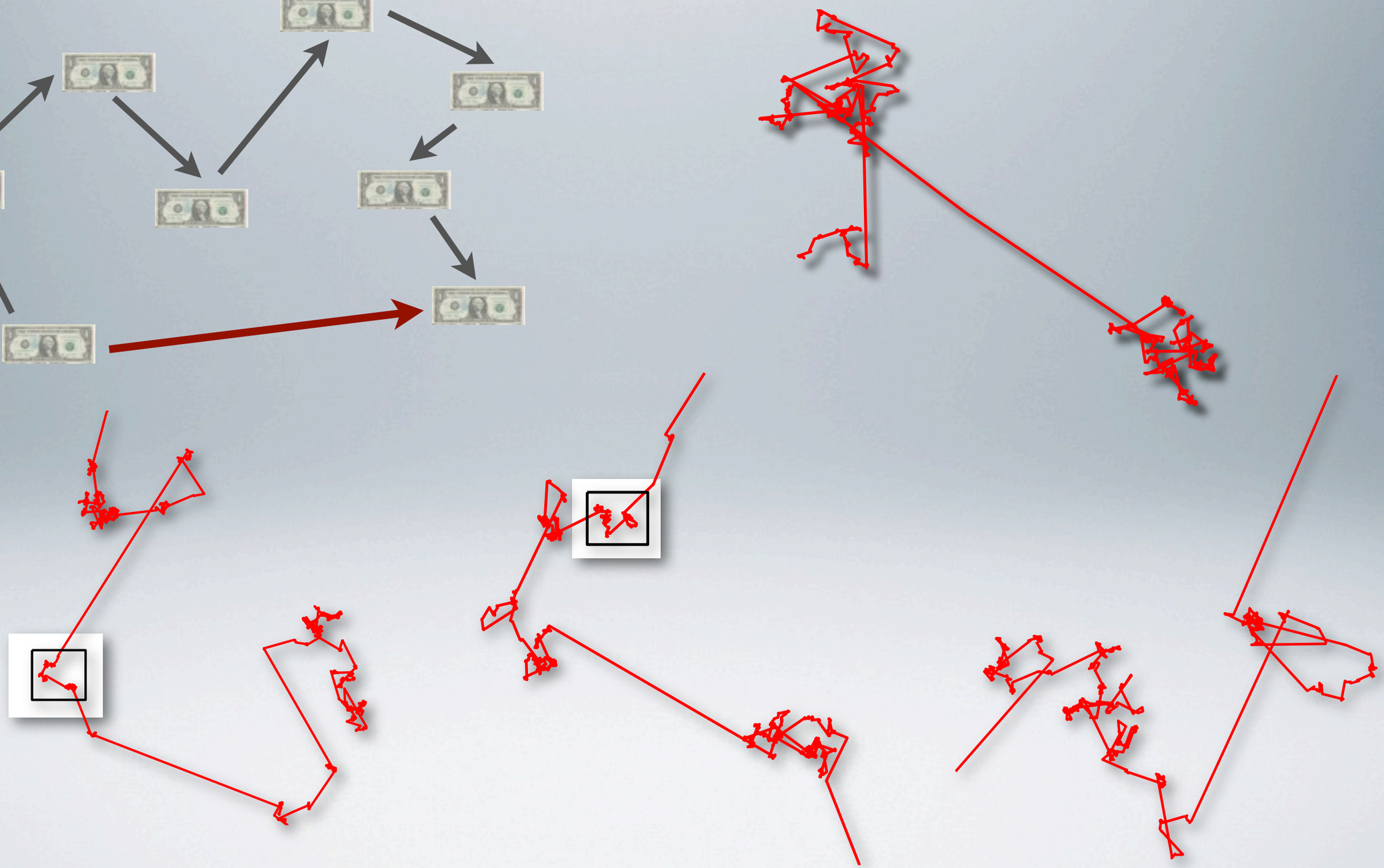
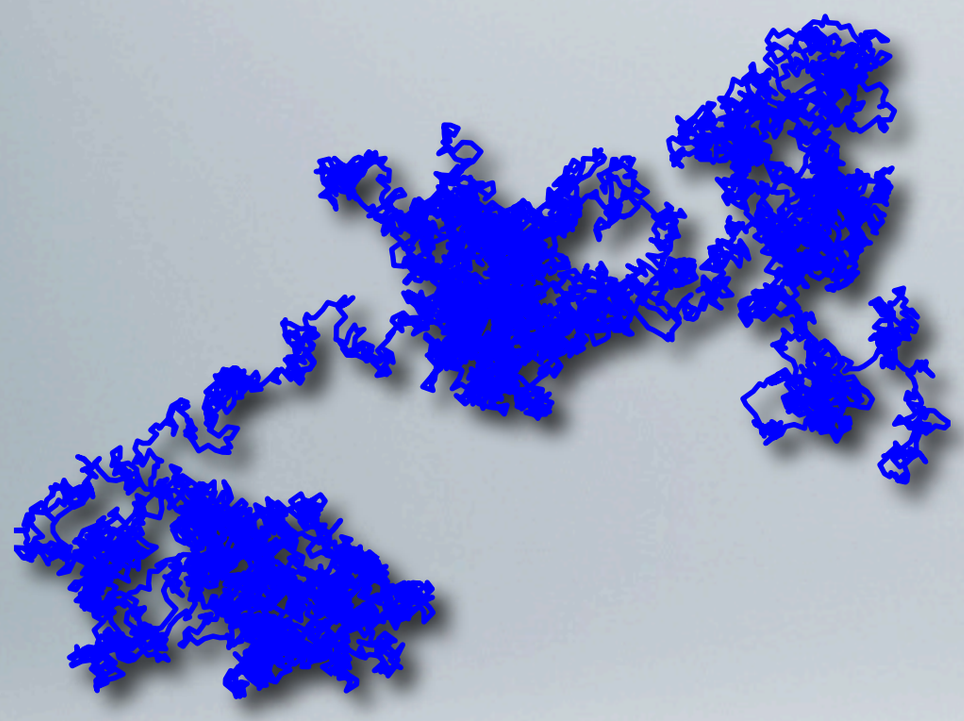


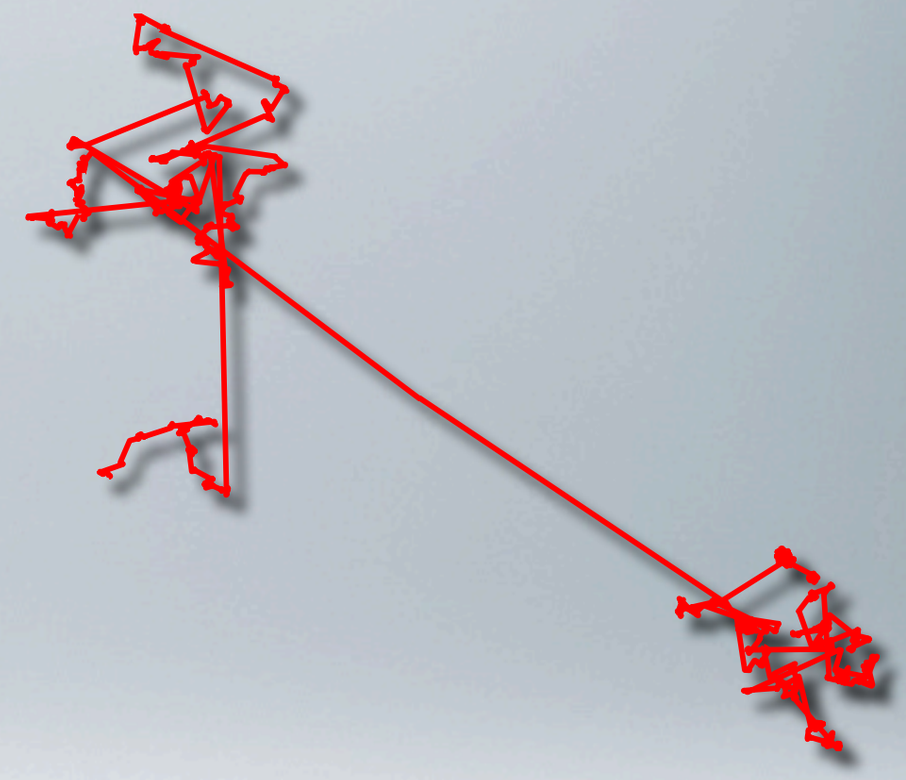


LÉVY FLIGHTS AND FRACTIONAL DISPERSAL

ordinary diffusion



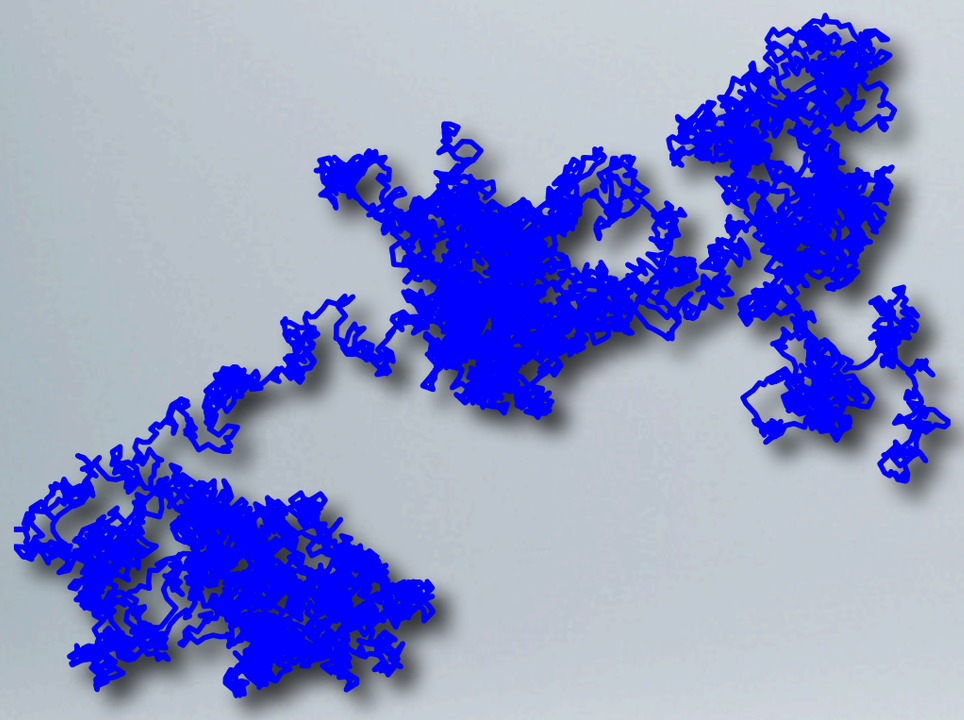
lévy stable process



$$\partial_t p = D\nabla^2 p$$

LÉVY FLIGHTS AND FRACTIONAL DISPERSAL

ordinary diffusion



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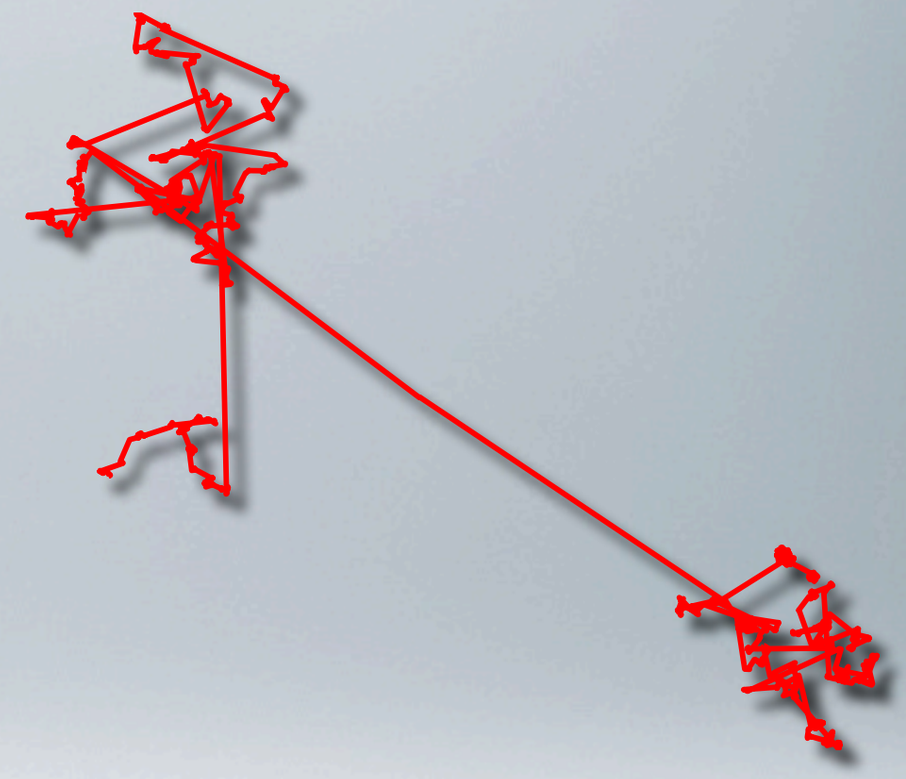


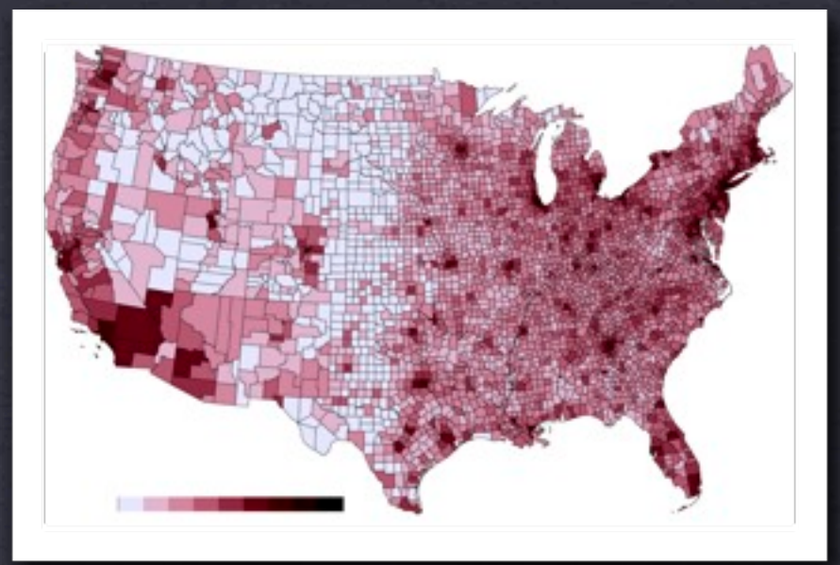
$$\partial_t p = D\nabla^2 p$$

$$\partial_t p = D_\beta |\nabla|^\beta p$$

fractional Laplacian

lévy flights and heterogeneities

$n(\mathbf{x}) \neq \text{const}$



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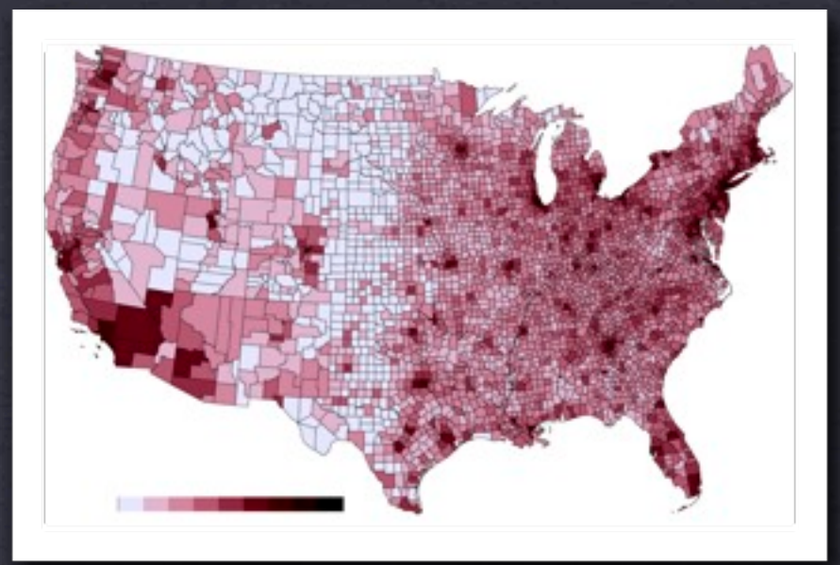

GRAVITY MODEL

$$w(\mathbf{x}|\mathbf{y}) \propto \frac{n(\mathbf{x})^{\xi} n(\mathbf{y})^{\xi-1}}{|\mathbf{x}-\mathbf{y}|^{D+\beta}}$$

$$0 < \xi \leq 1$$

lévy flights and heterogeneities

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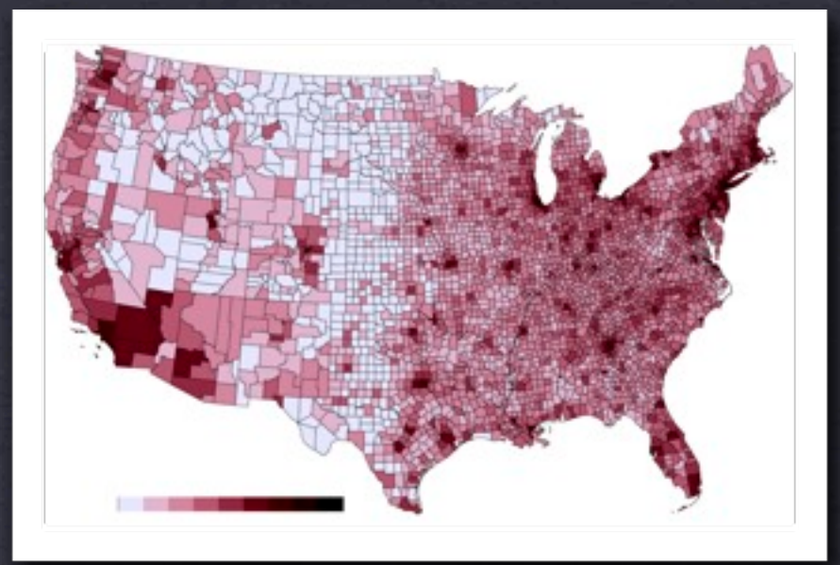

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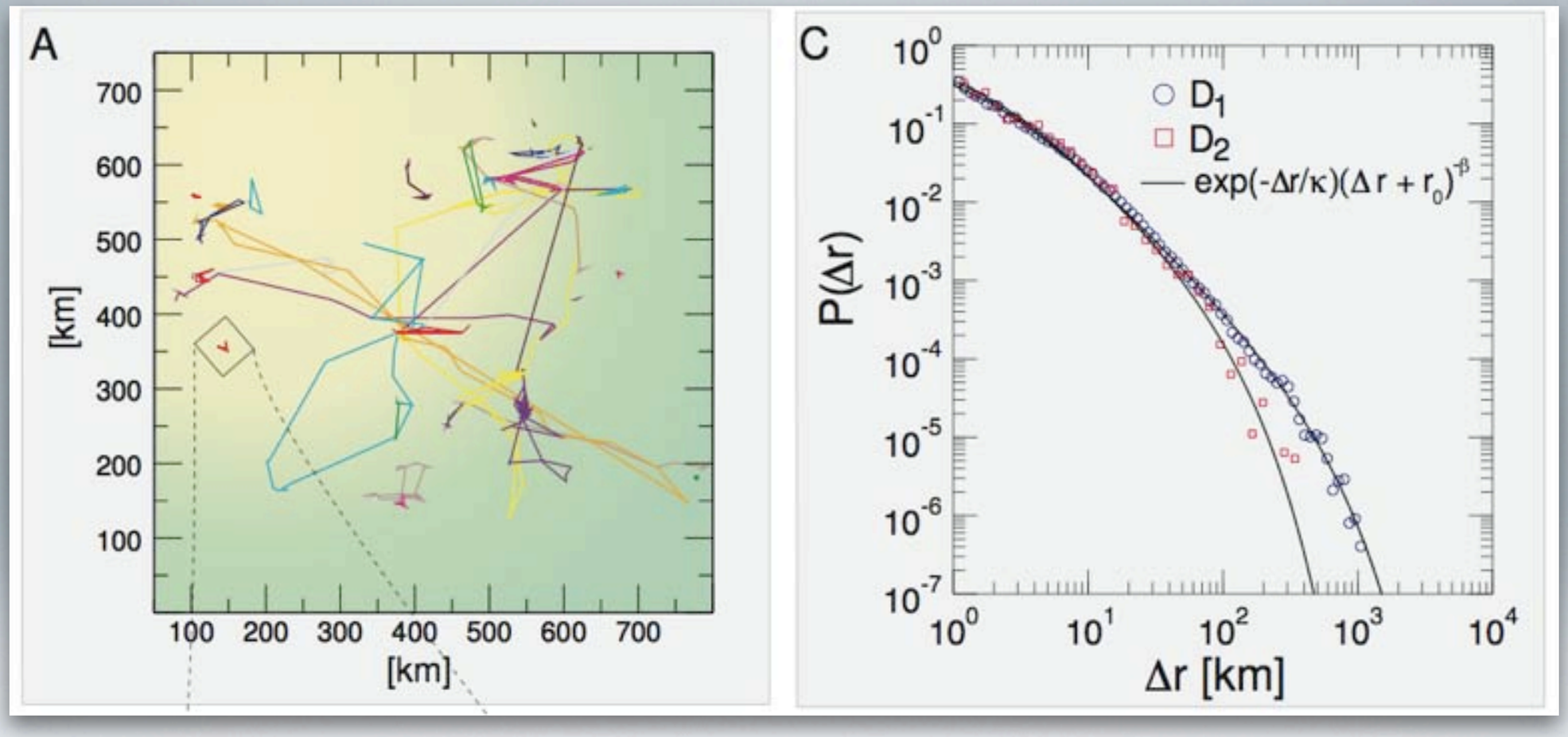
$$0 < \xi \leq 1$$

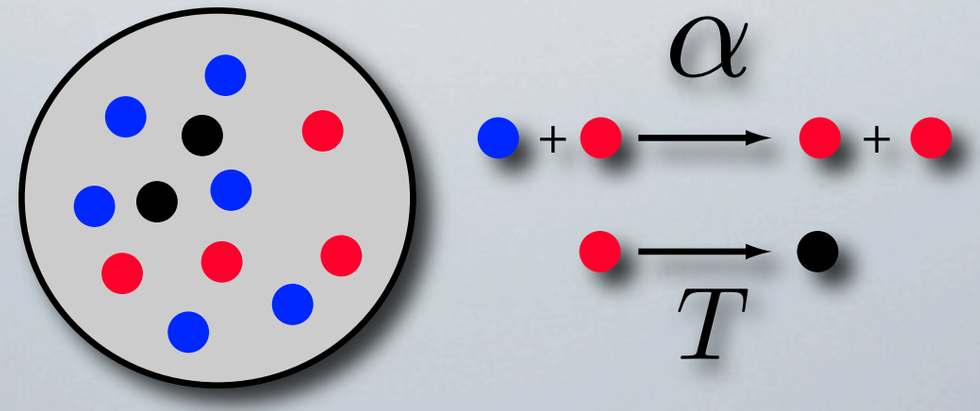
$$\partial_t p(\mathbf{x},t) = n(\mathbf{x})\Delta^{\beta/2} p(\mathbf{x},t) - p(\mathbf{x},t)\Delta^{\beta/2} n(\mathbf{x})$$

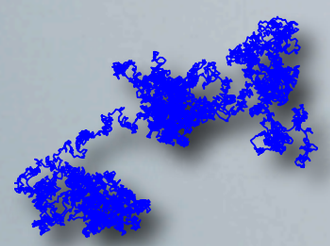
INDIVIDUAL MOBILITY PATTERS



from Gonzalez et al. Nature (2008)

GENERIC PATTERNS

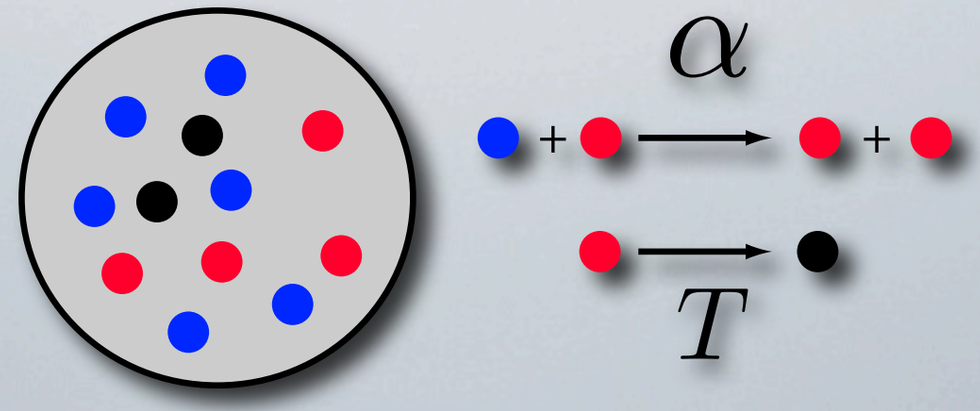




infected
susceptible
recovered/removed

reaction-diffusion

GENERIC PATTERNS



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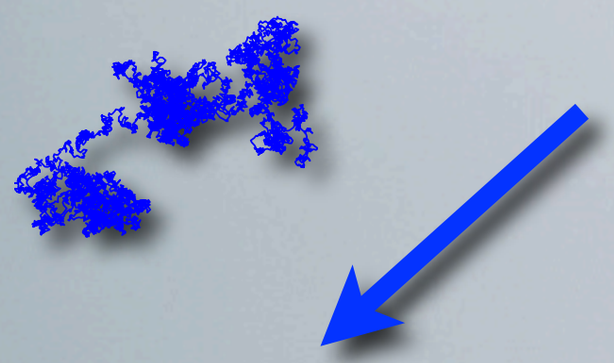
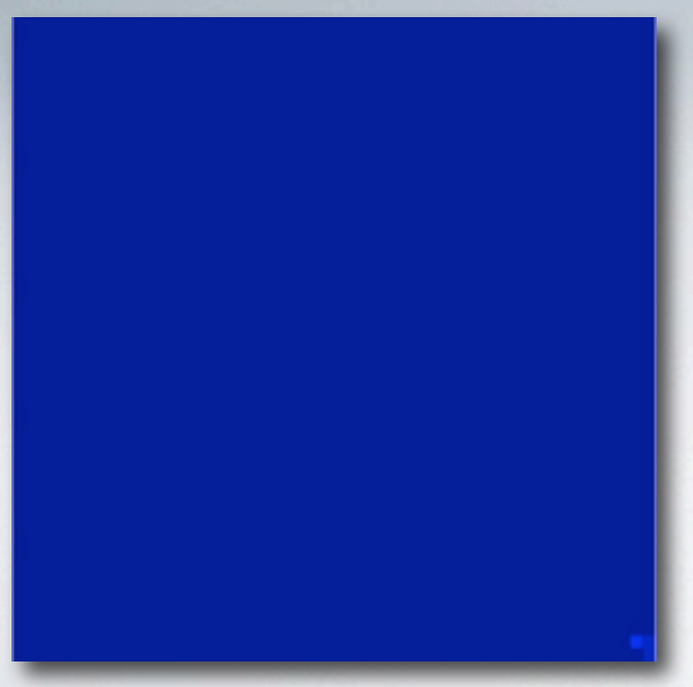


reaction-diffusion



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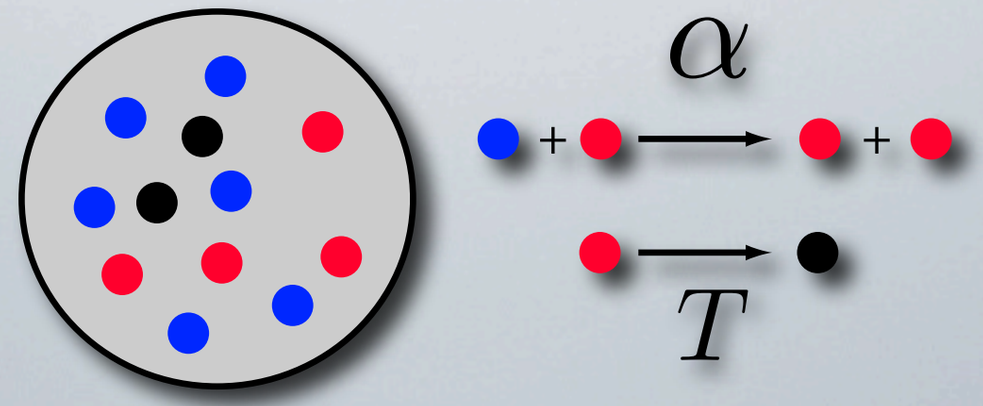

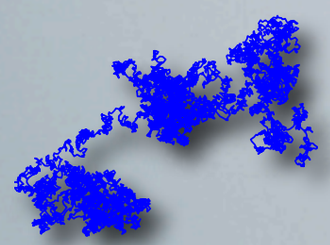

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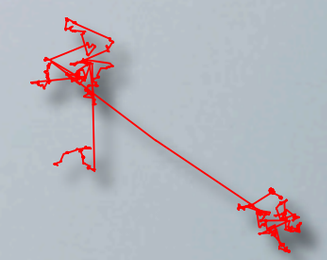
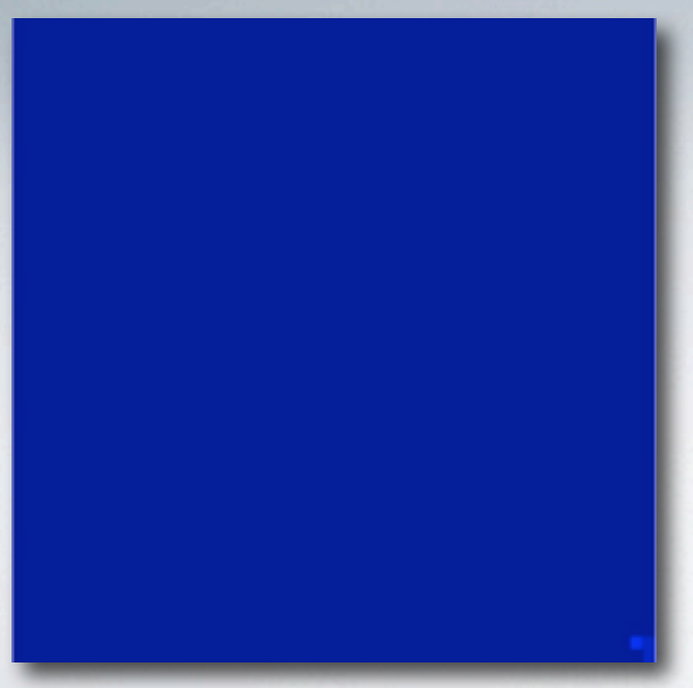
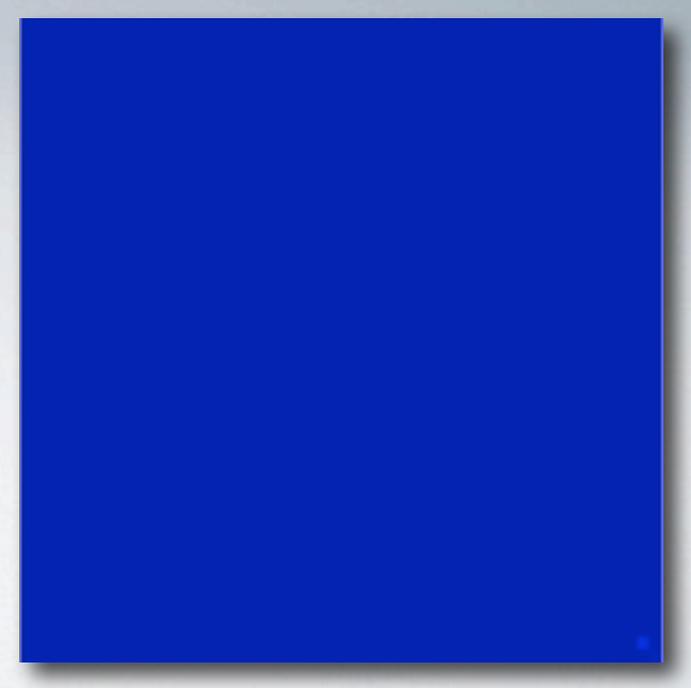


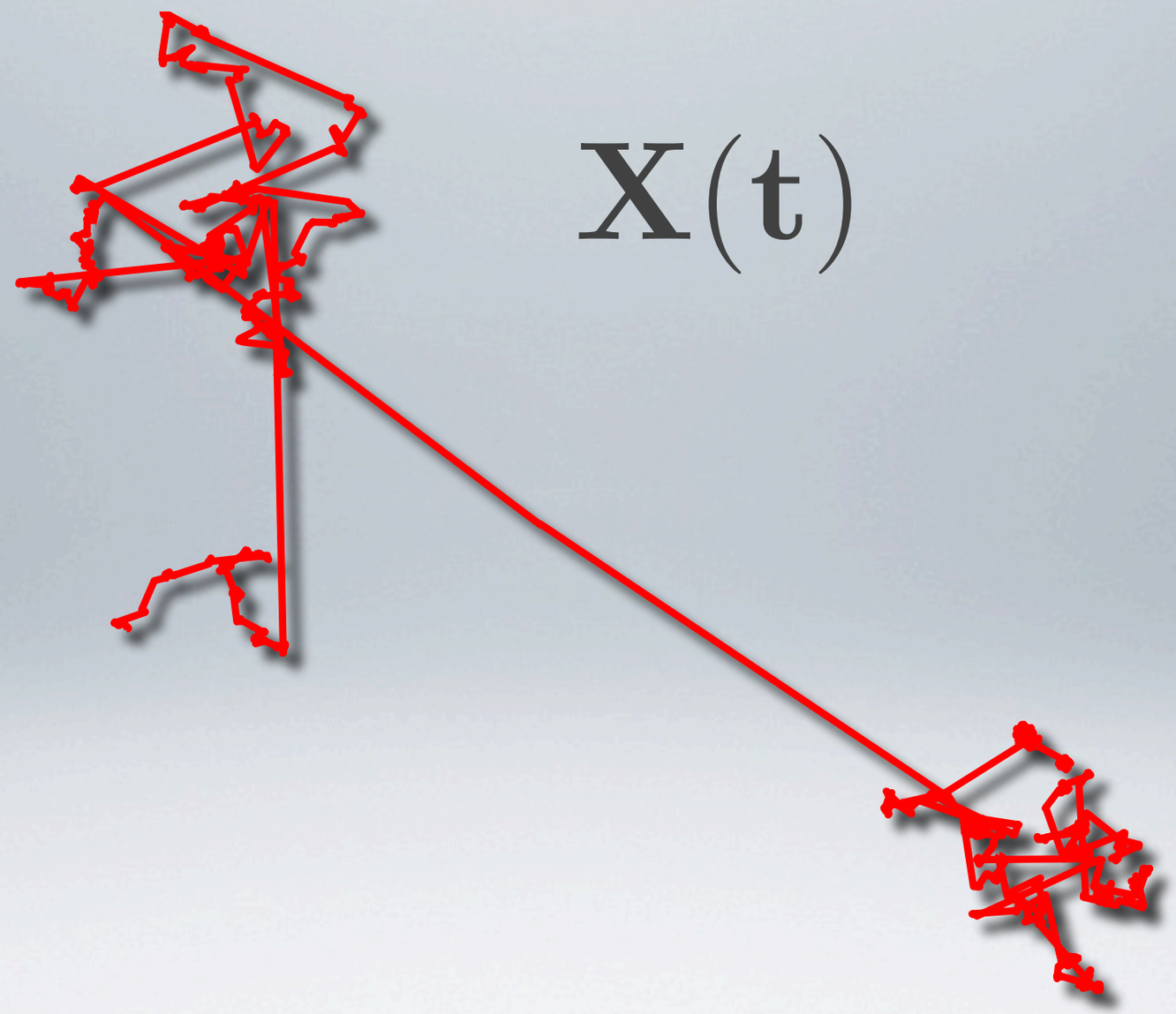
reaction-diffusion



reaction-superdiffusion



PATH AND NETWORKS



$$\mathbf{X(t)}$$

THE FLUX OF DOLLARS

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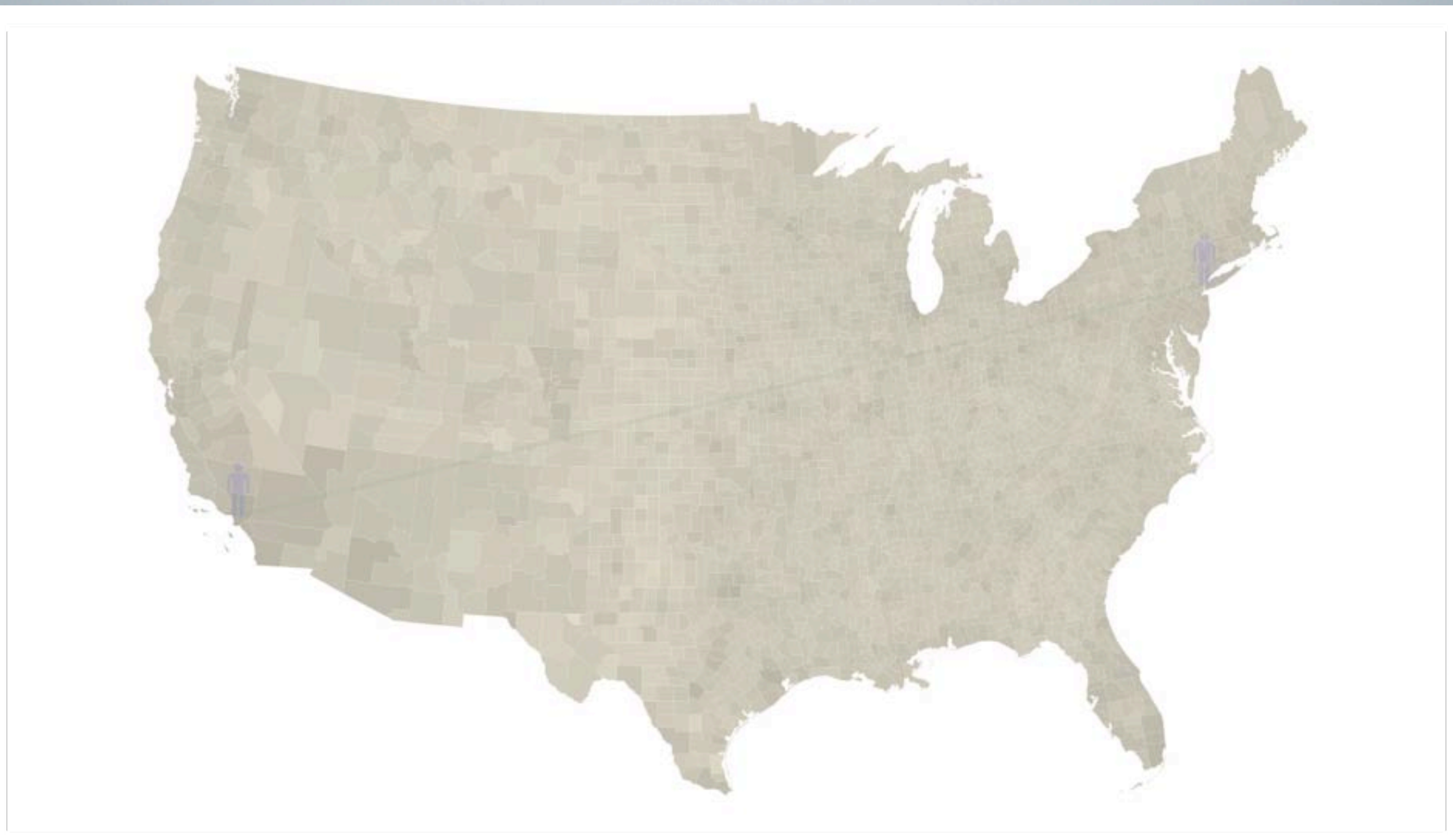
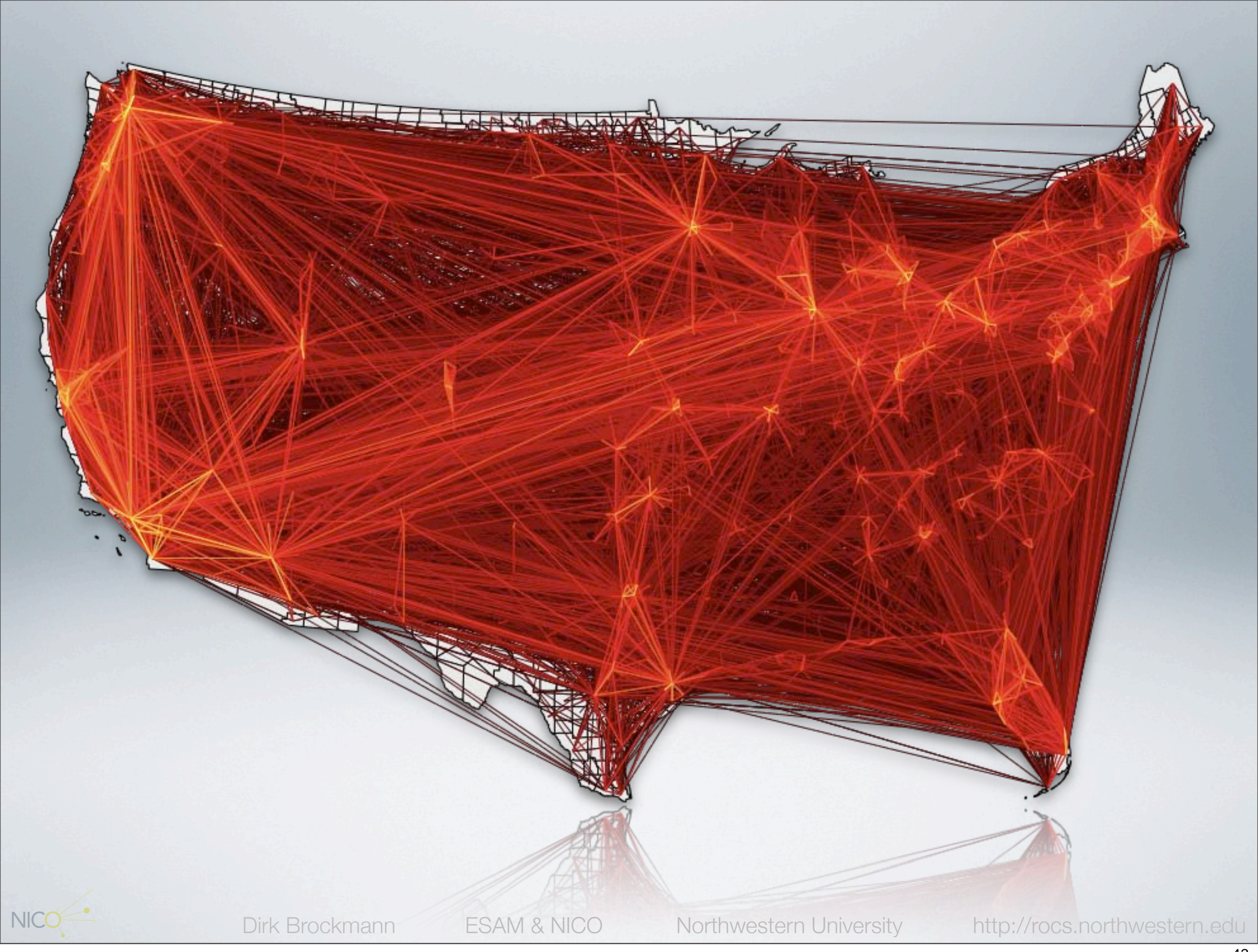
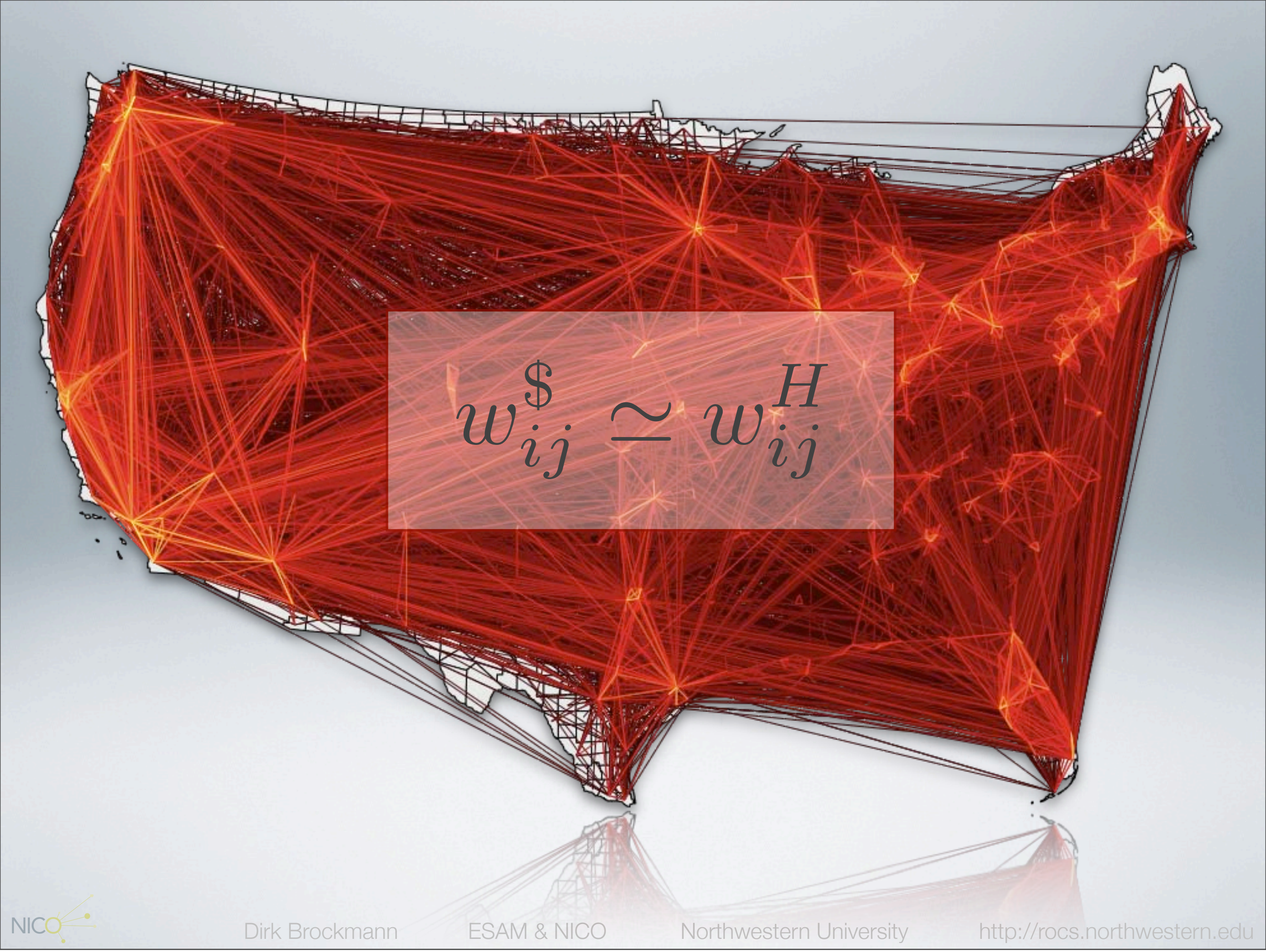
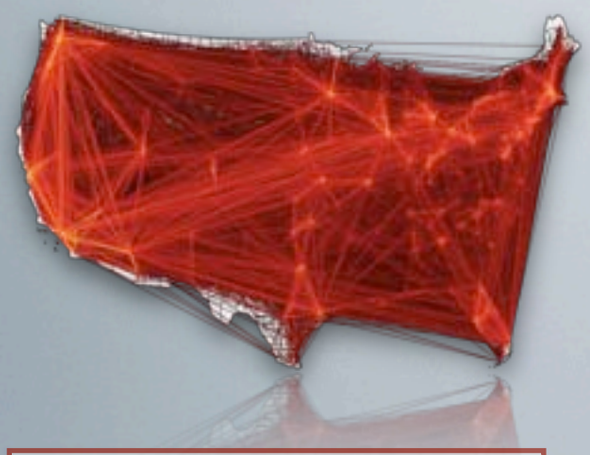





$w_{ij}^{\$} \simeq w_{ij}^{H}$

FLUX COMPARISON

FLUX COMPARISON



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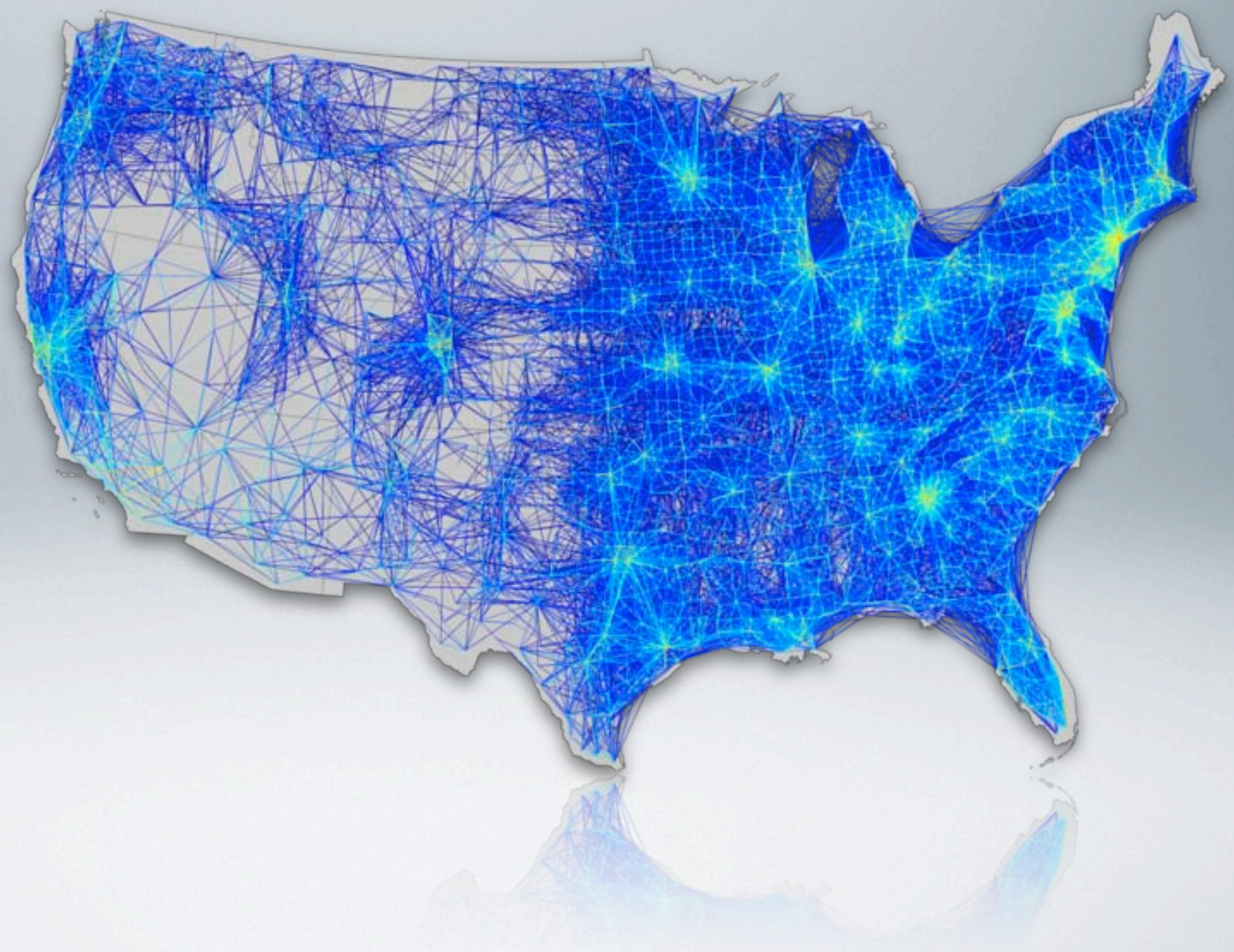
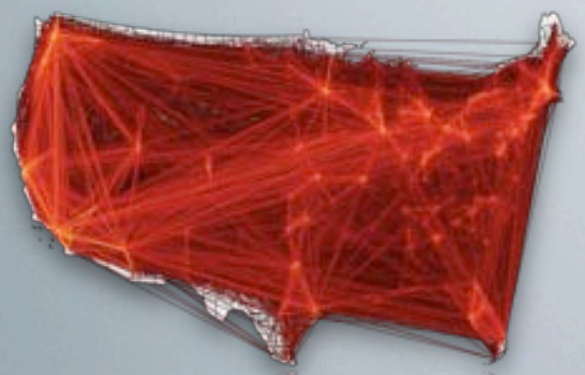


FLUX COMPARISON



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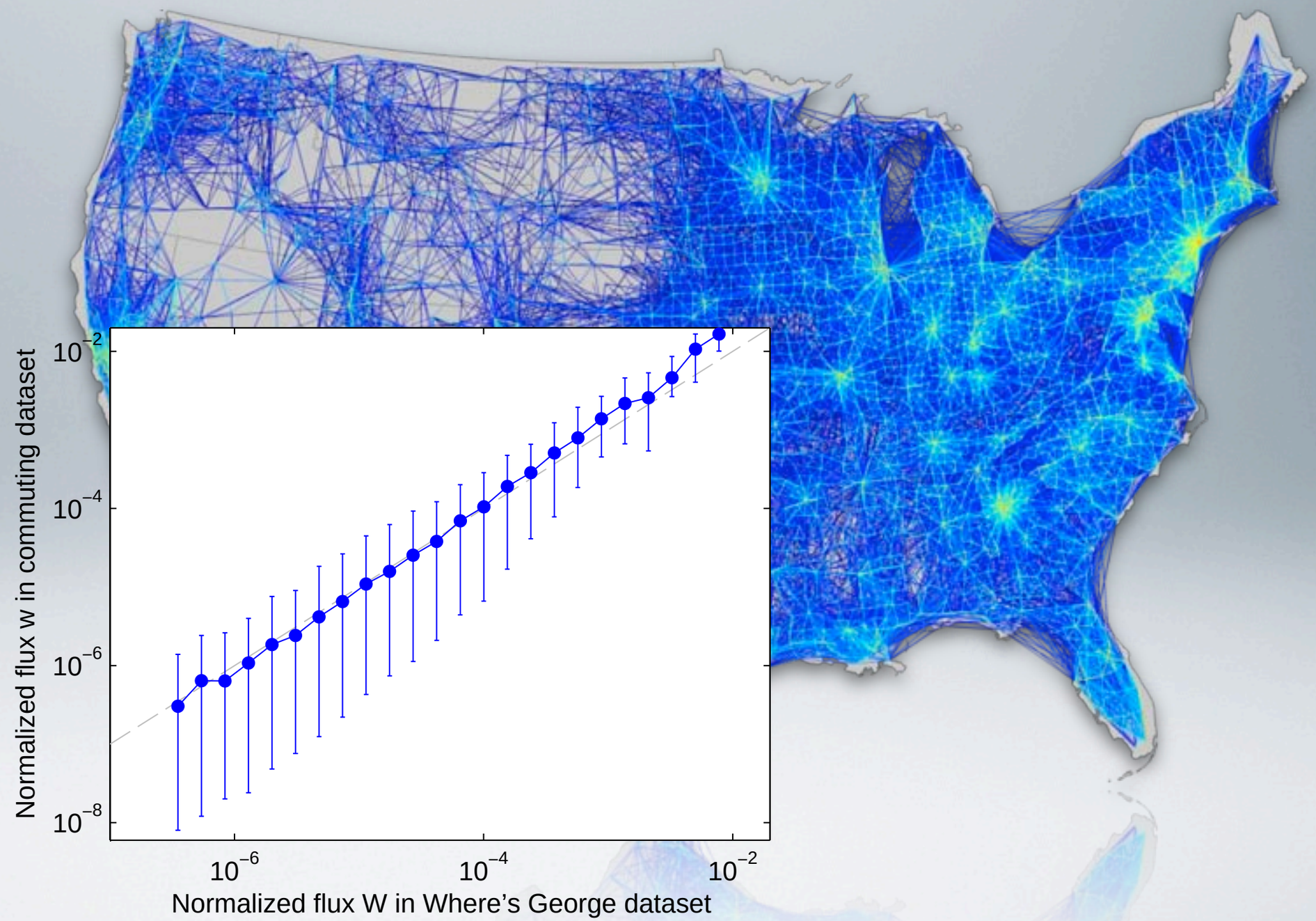
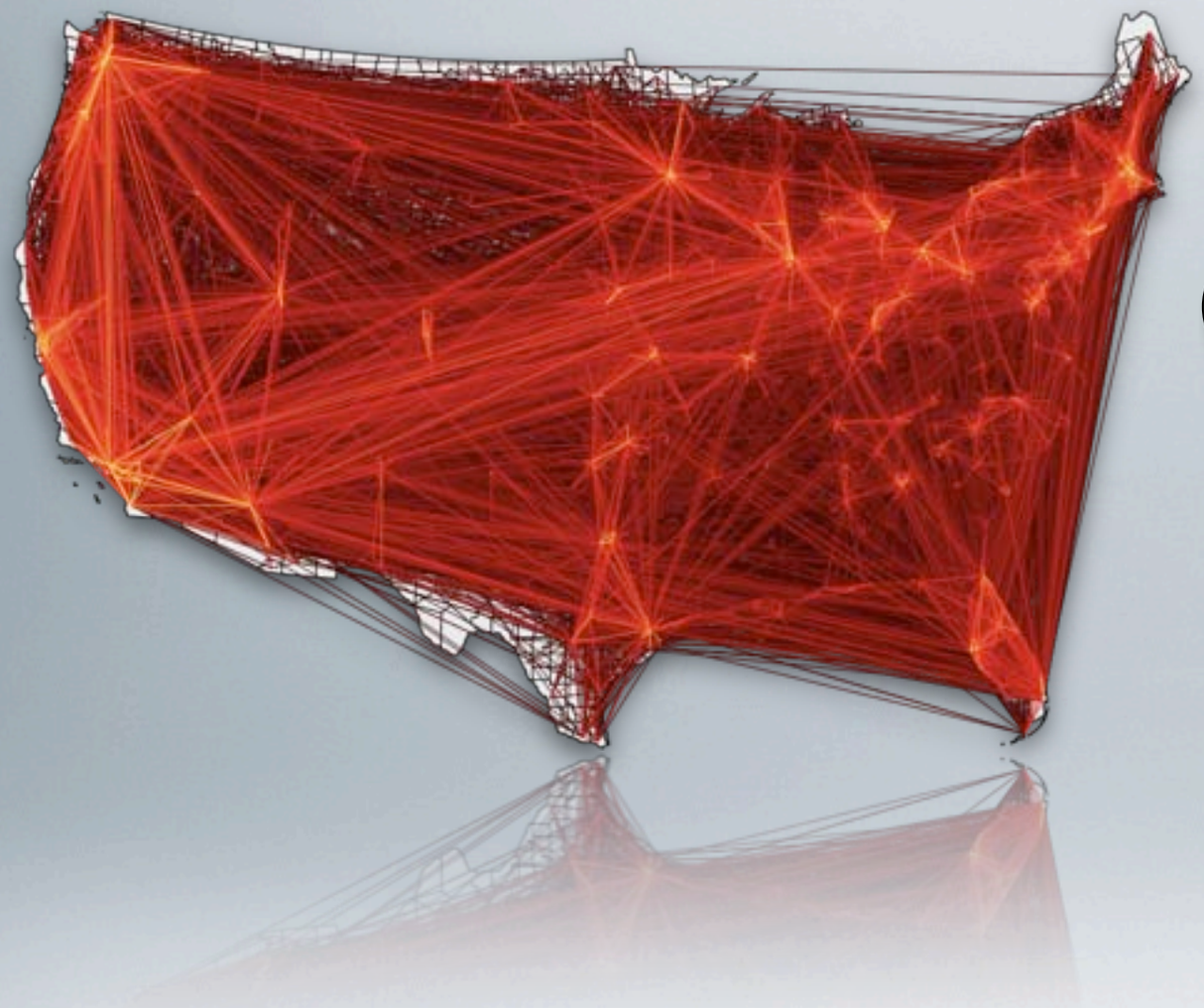
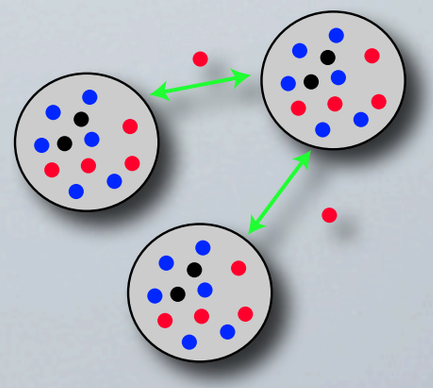


A MODEL FOR THE SPREAD OF H1N1 IN THE US





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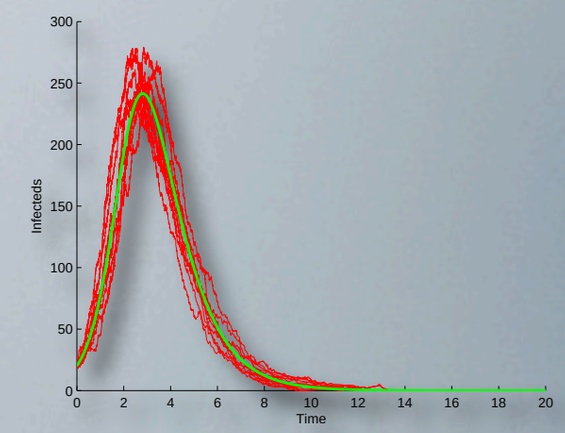

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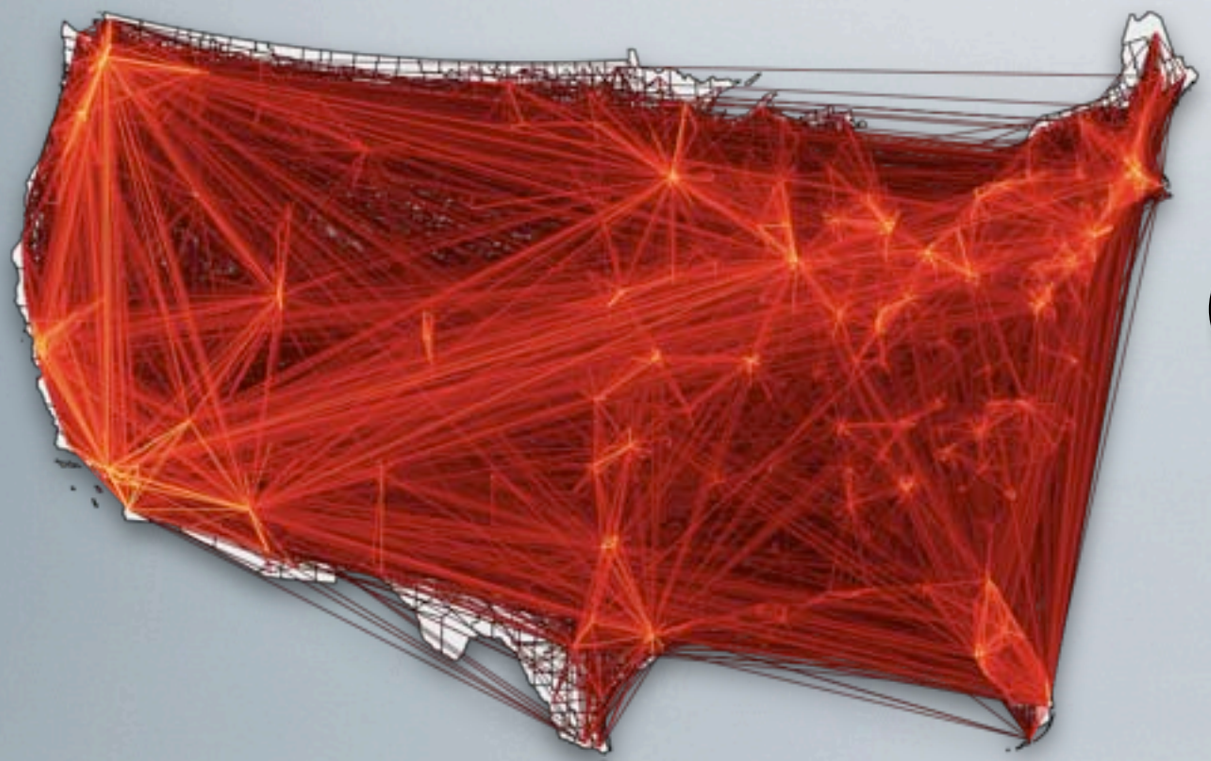

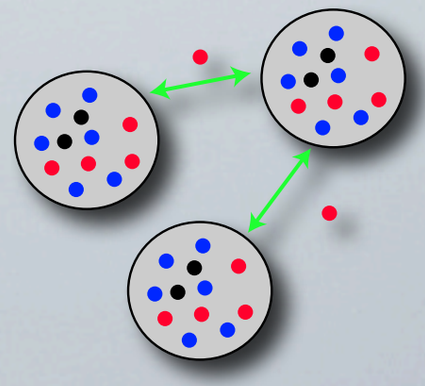

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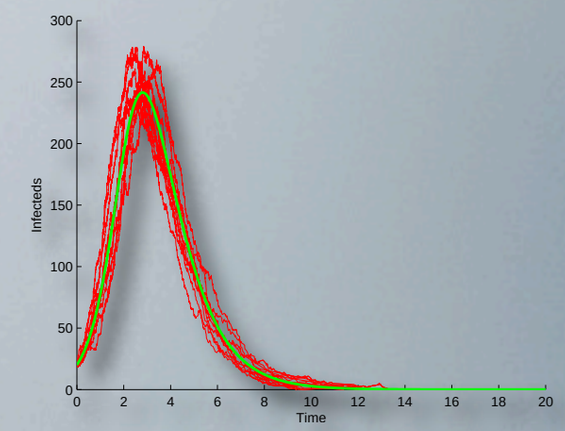
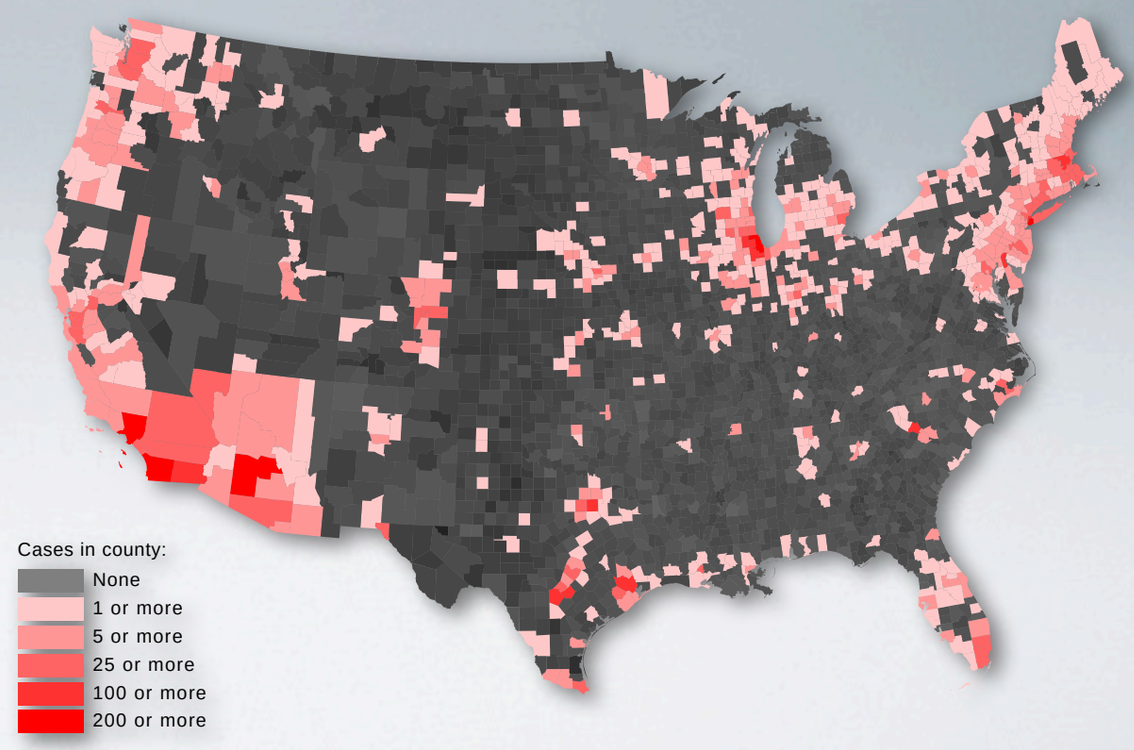
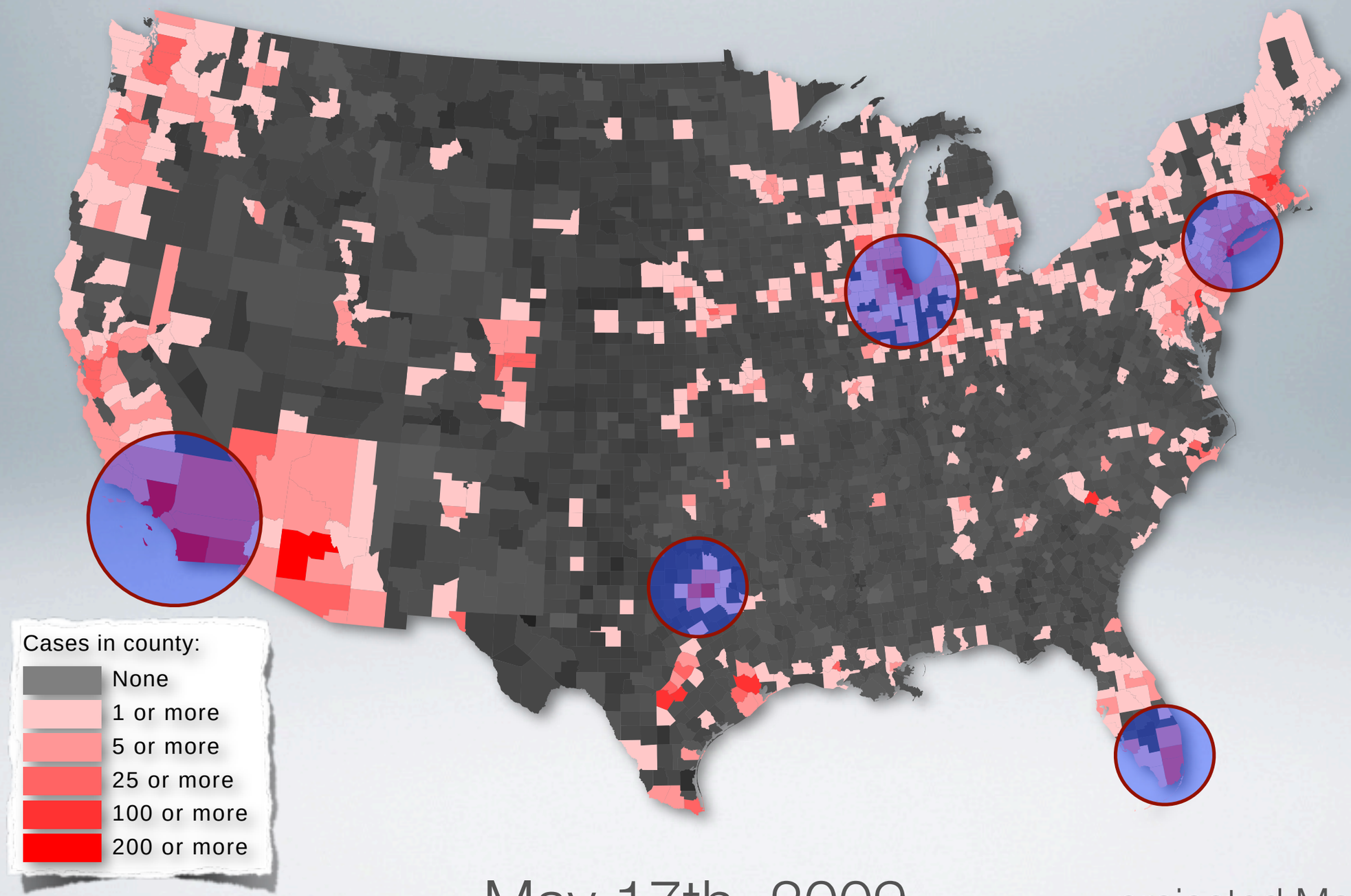




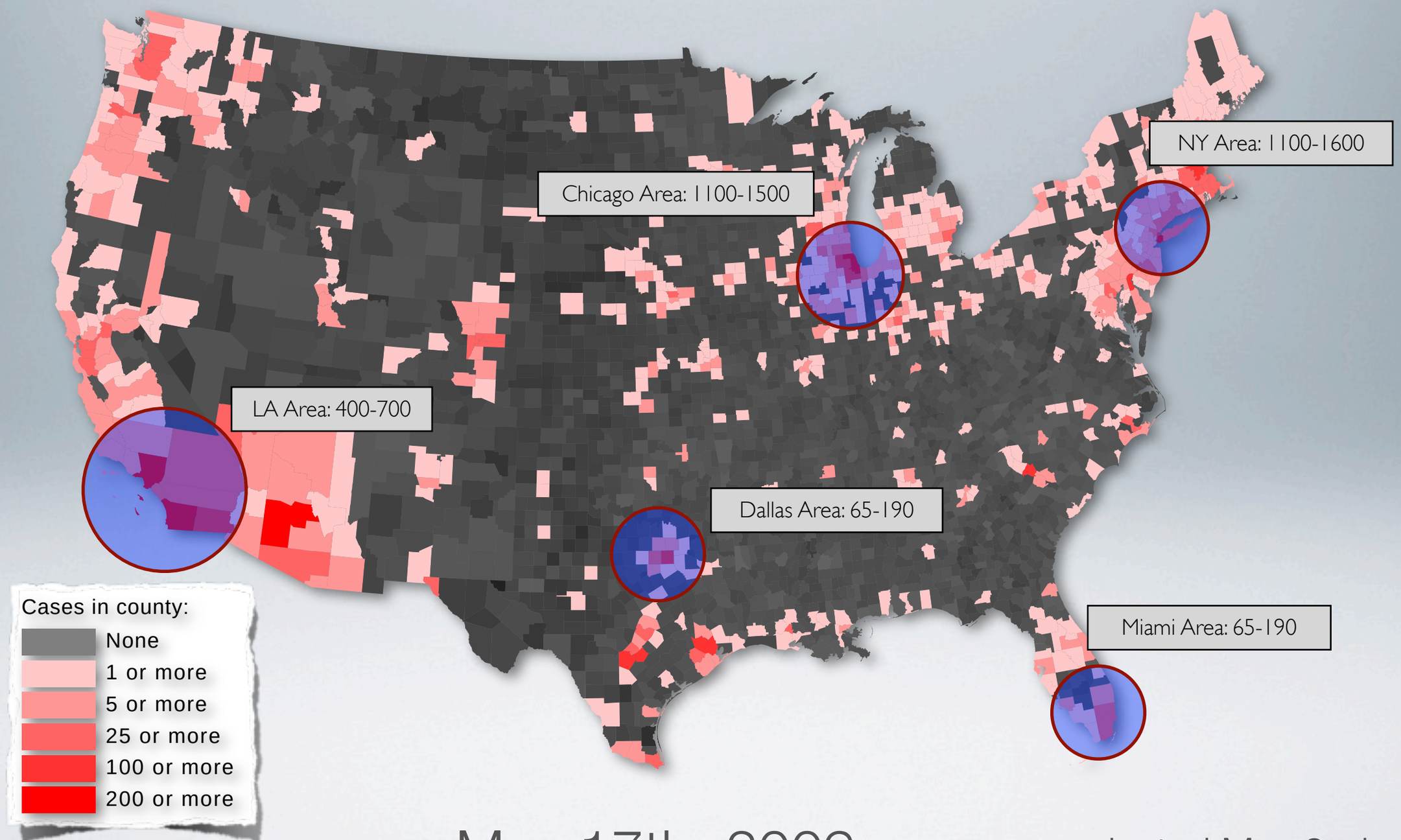
COMPUTATIONAL MODELS
H1N1 PROJECTIONS, US 2009



May 17th, 2009

projected May 2nd

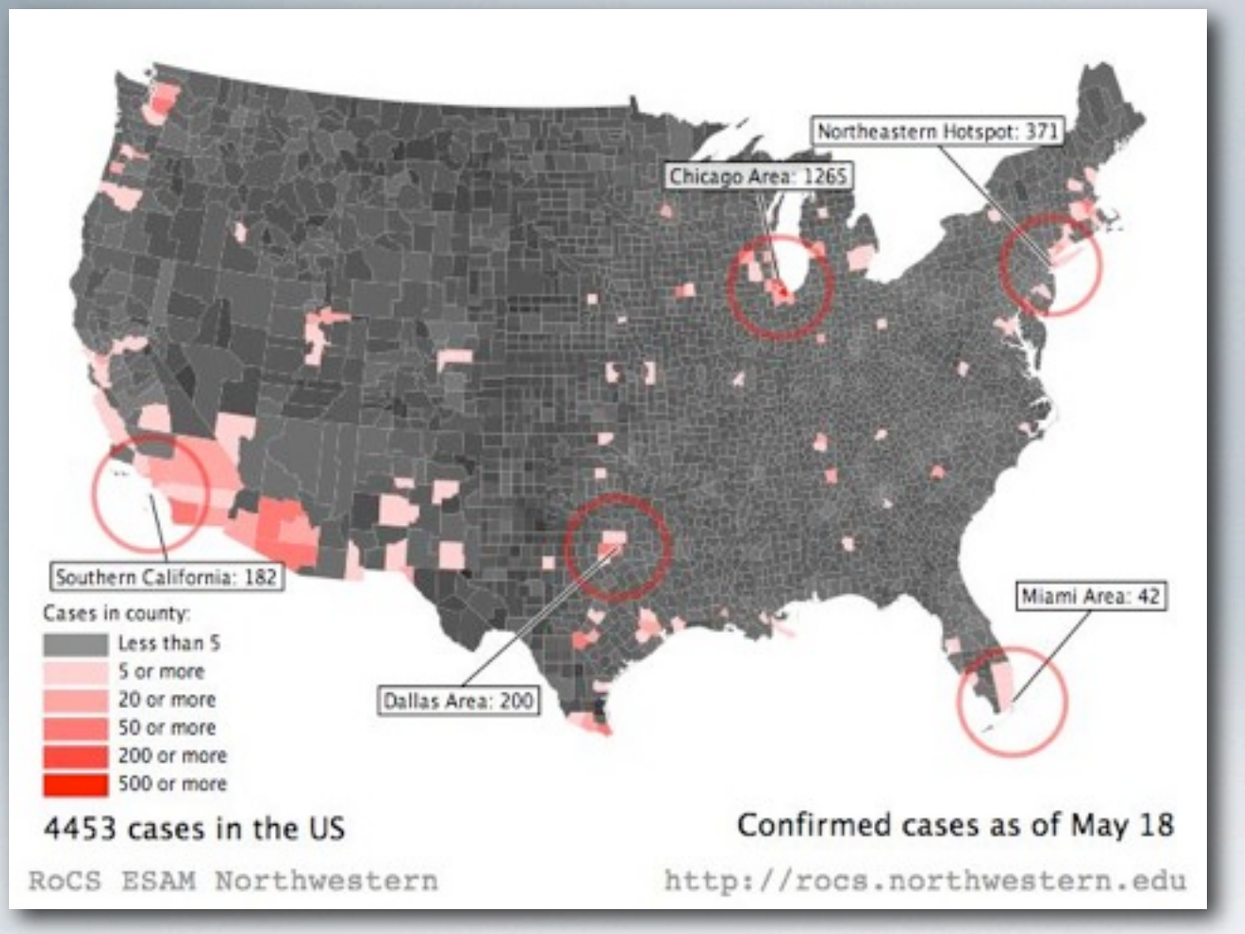
COMPUTATIONAL MODELS
H1N1 PROJECTIONS, US 2009



May 17th, 2009

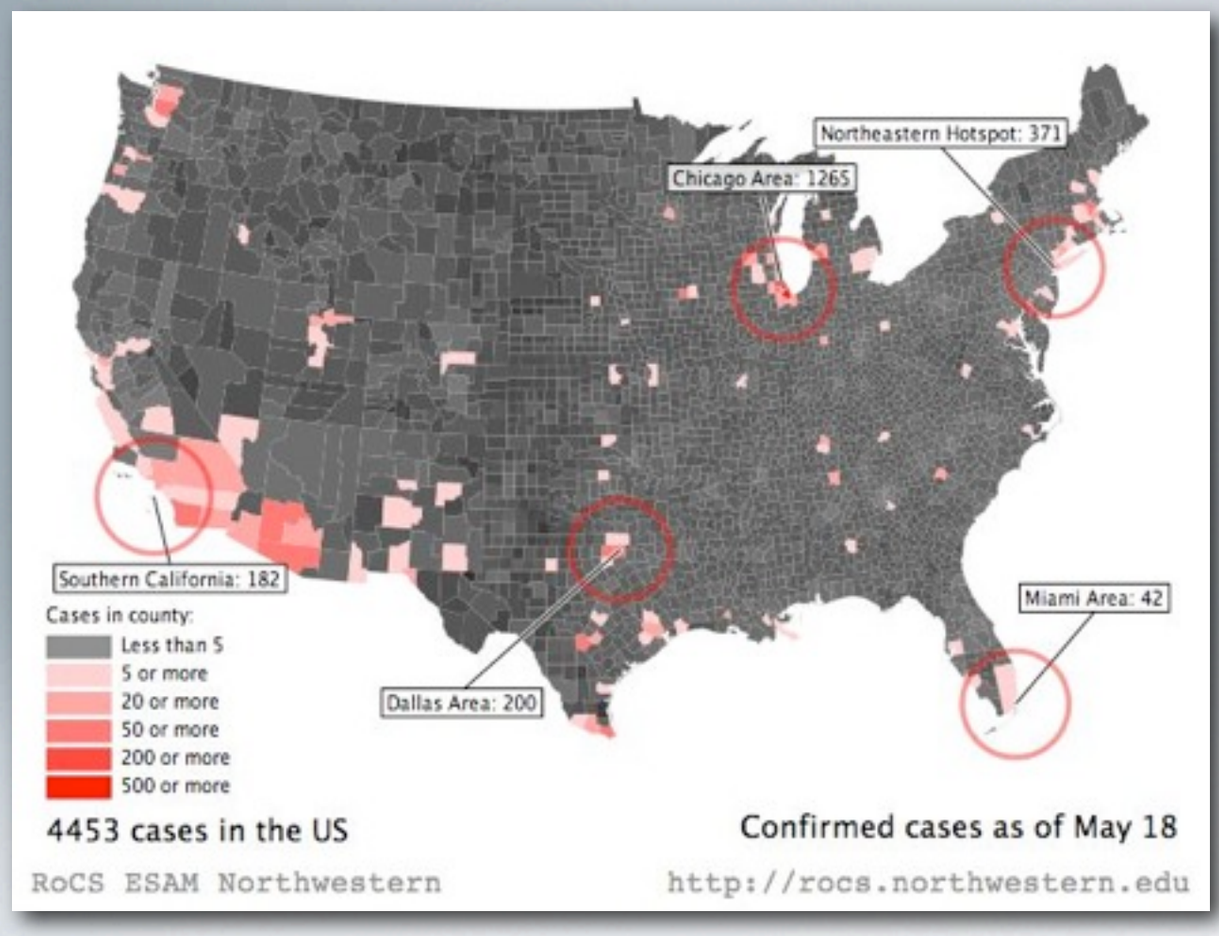
projected May 2nd

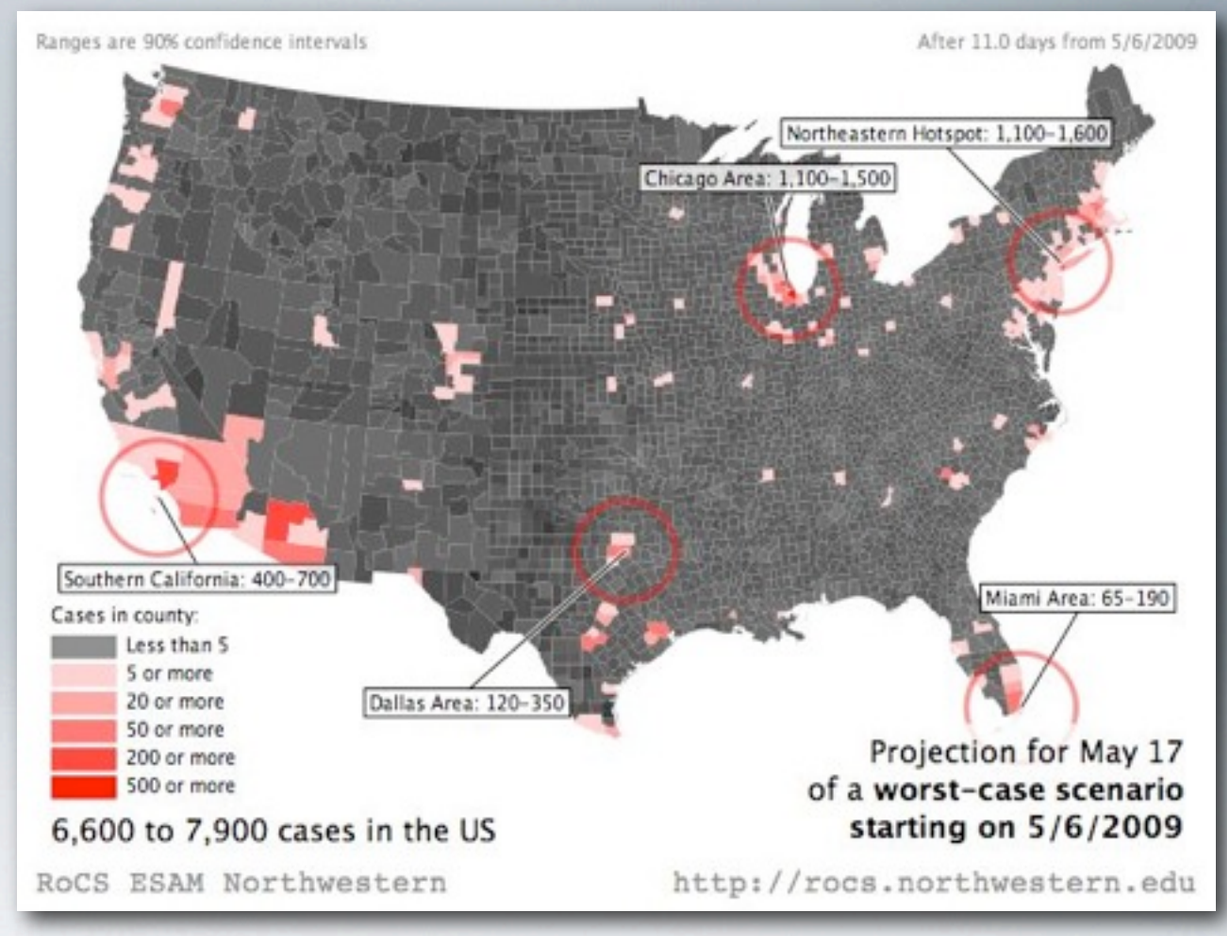
H1N1 PANDEMIC 2009



http://rocs.northwestern.edu

H1N1 PANDEMIC 2009





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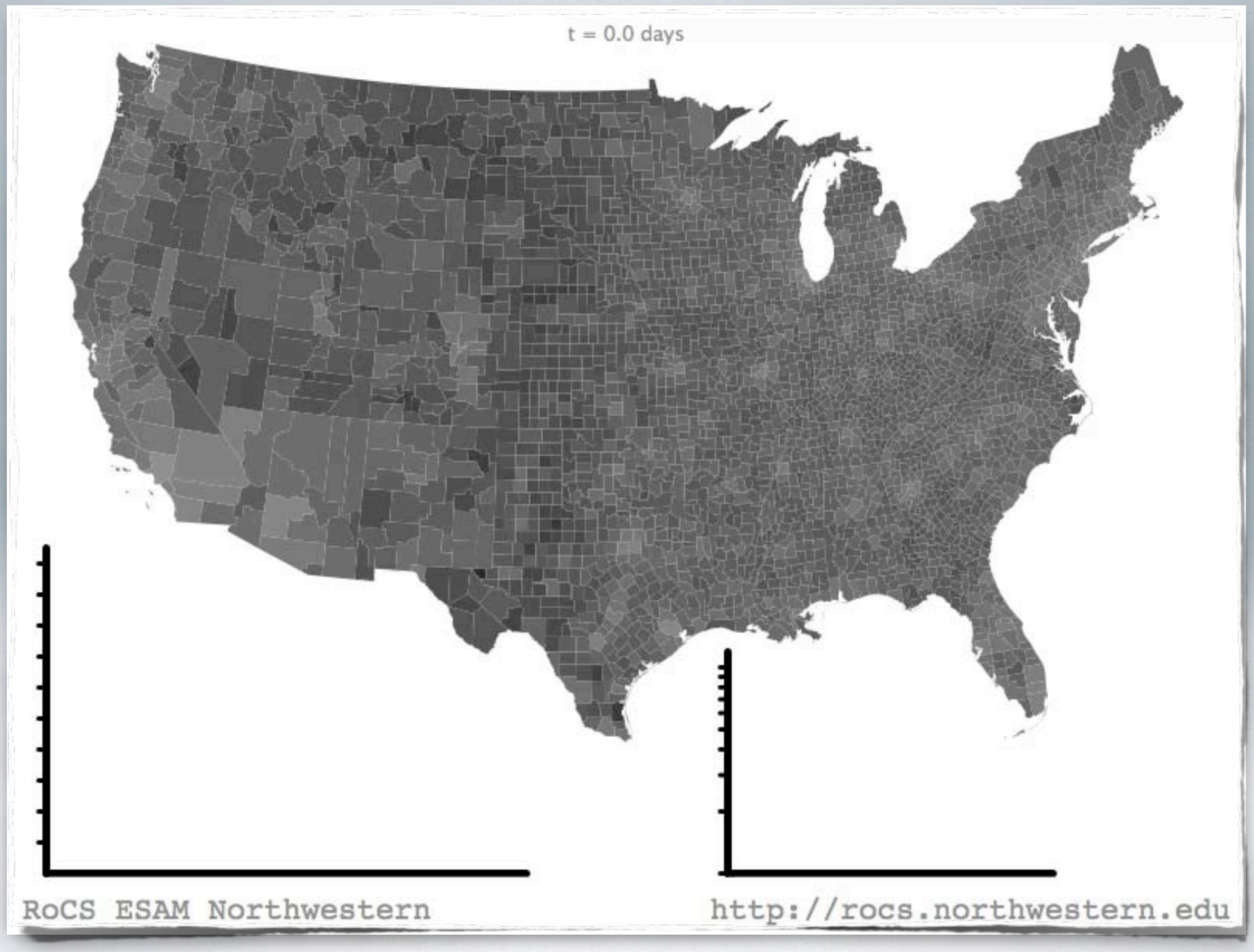
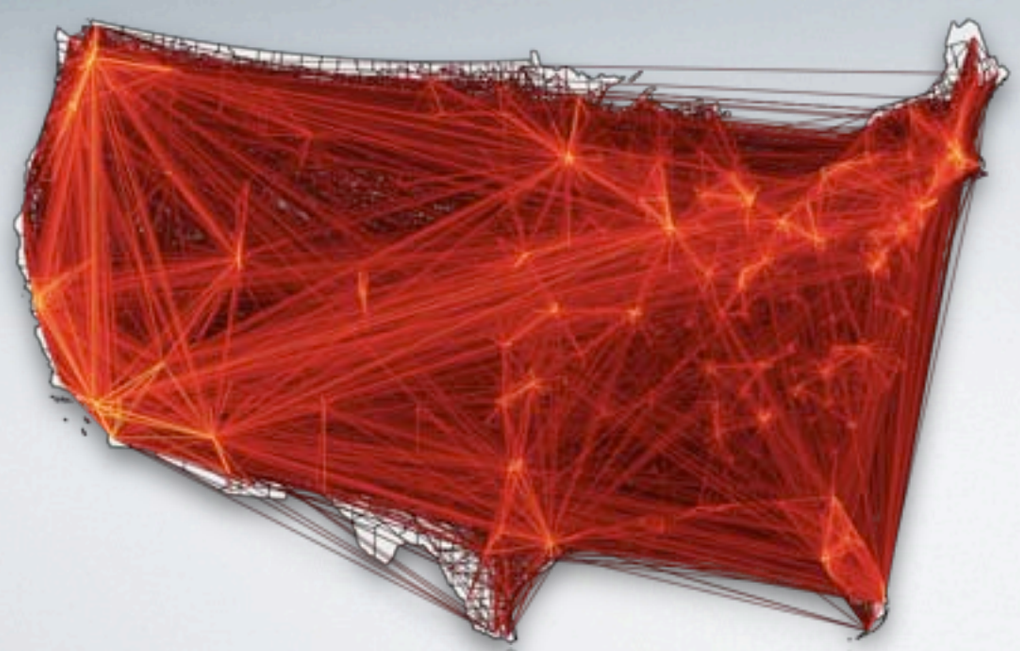
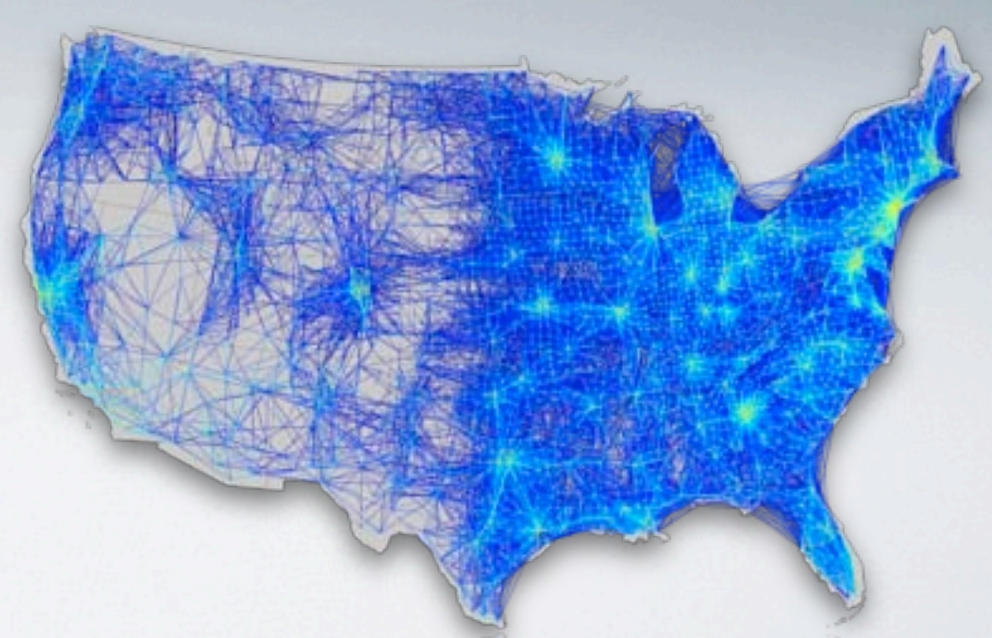
PANDEMIC SCENARIO

PANDEMIC SCENARIO

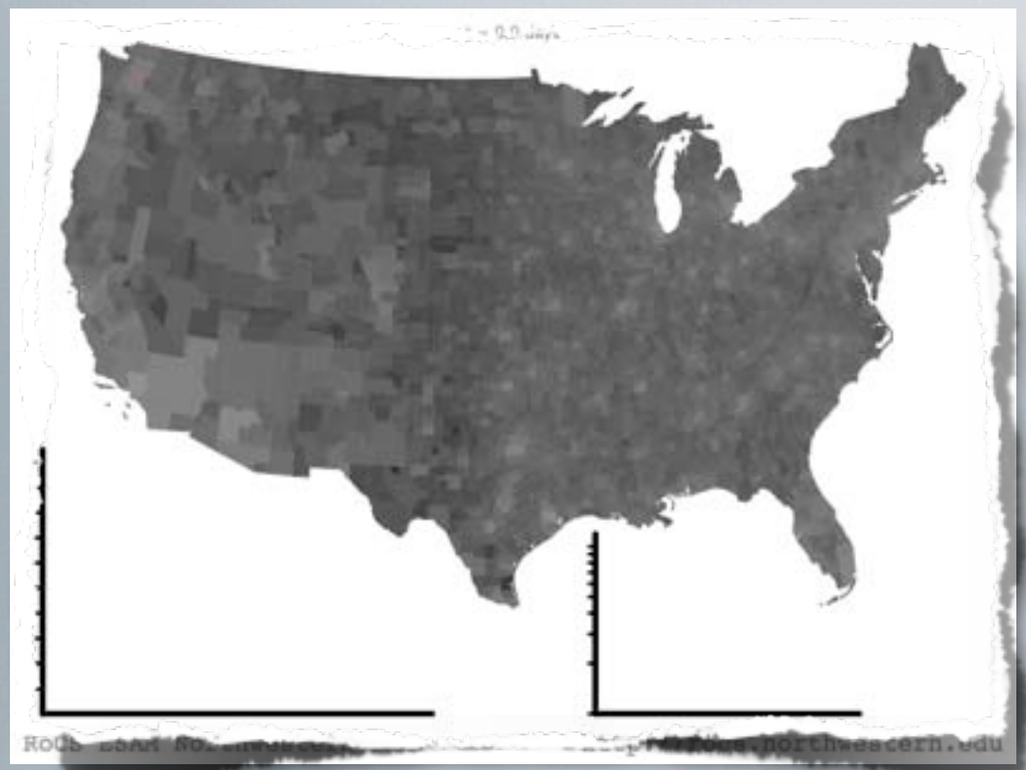


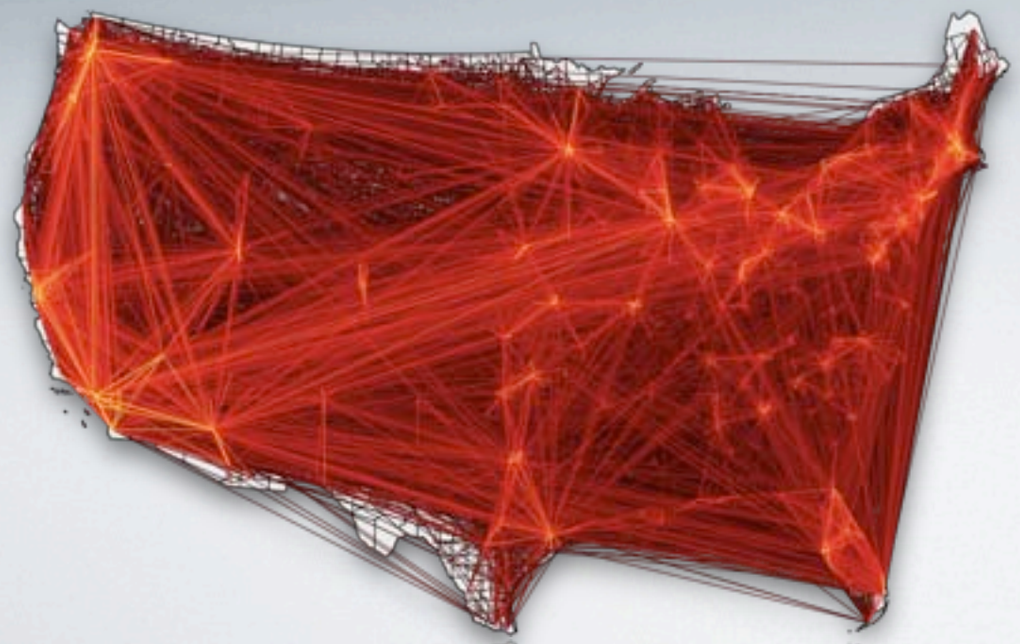
THE IMPACT OF LONG RANGE TRAFFIC

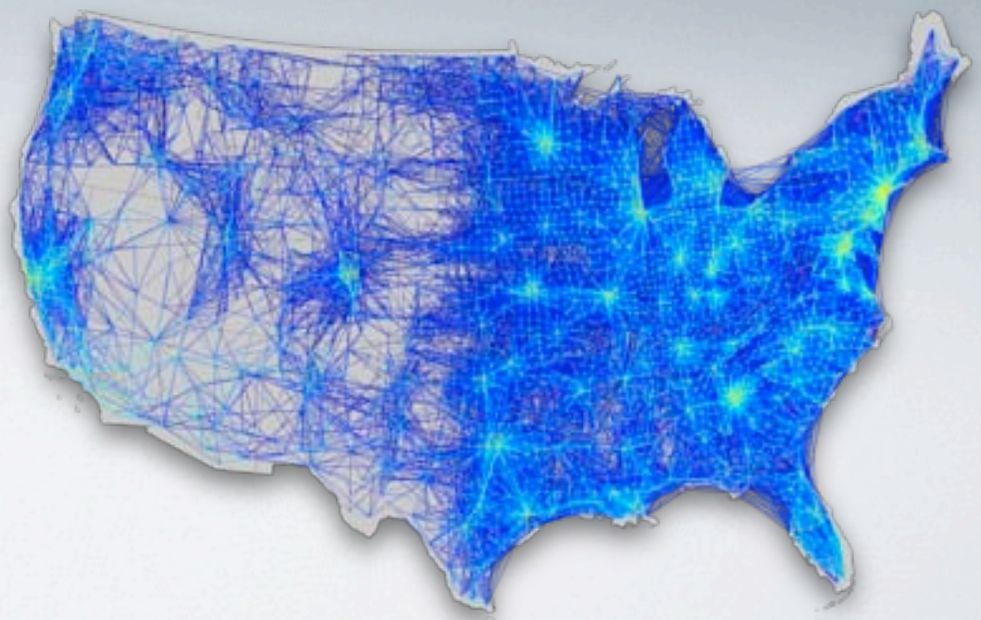




THE IMPACT OF LONG RANGE TRAFFIC







QUESTION

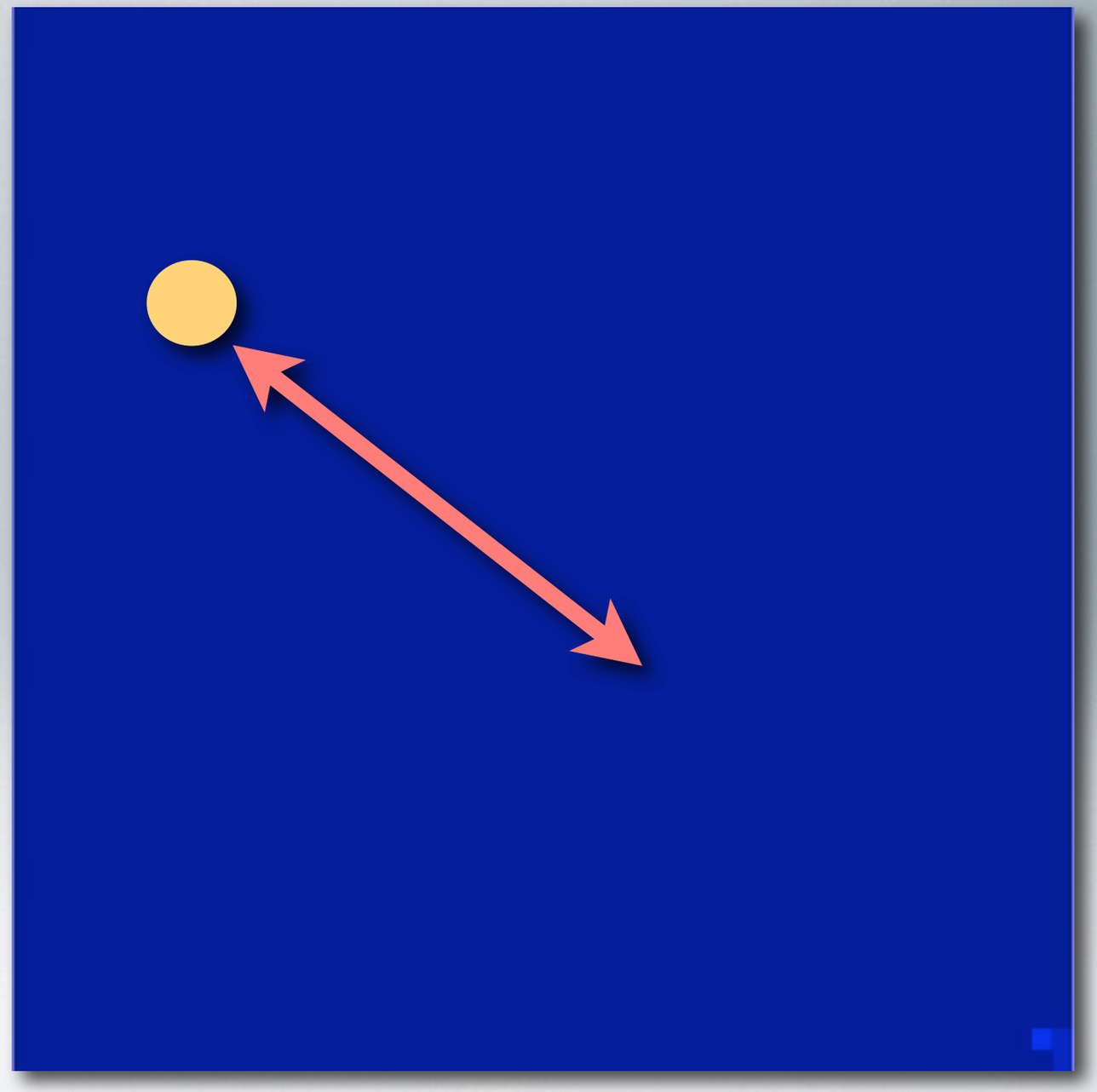
can we find simplify the picture?

can we compute effective “front” velocities?

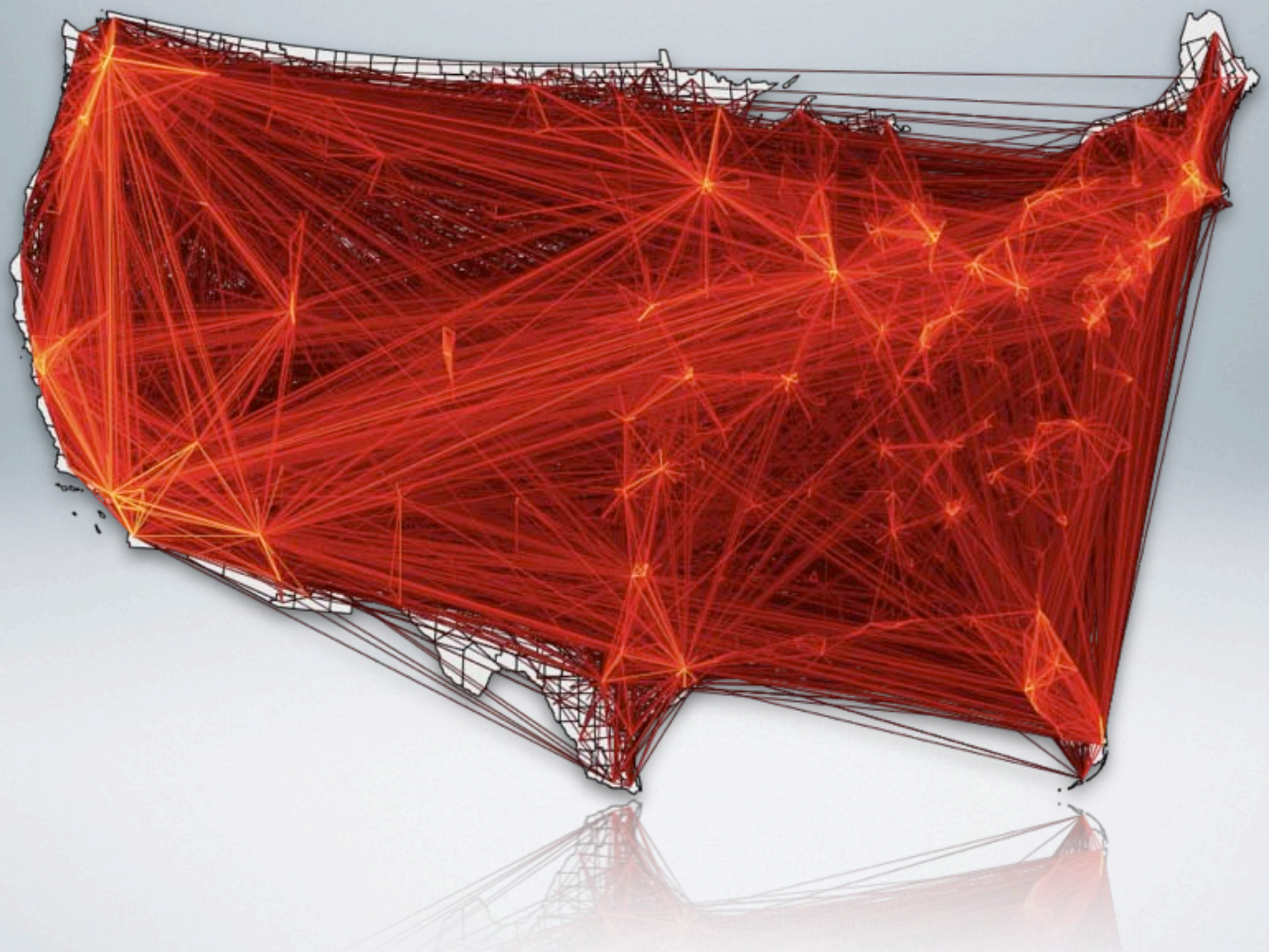
WHEN?

WHEN?

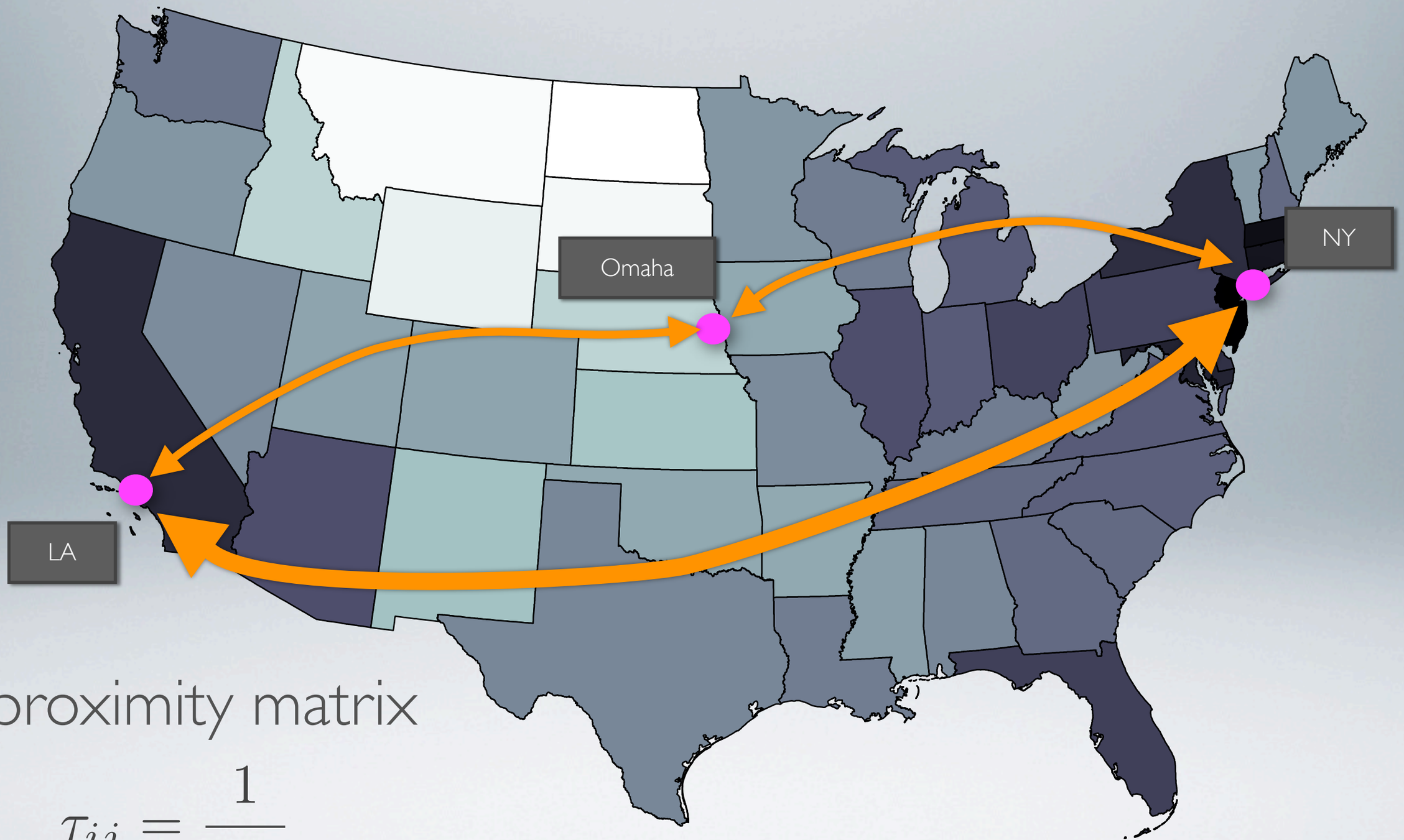
$$T = \frac{d}{v}$$



PROXIMITY, PATHS AND TREES



EFFECTIVE PROXIMITY



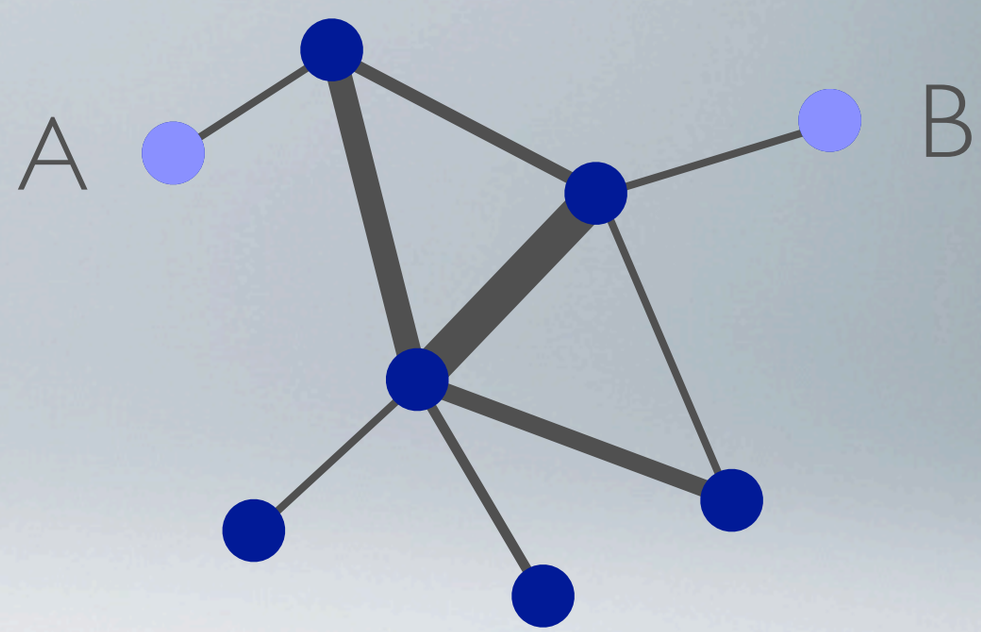
proximity matrix

$$\tau_{ij} = \frac{1}{w_{ij}}$$

SHORTEST PATHS AND EFFECTIVE DISTANCE

proximity matrix

$$\tau_{ij} = \frac{1}{w_{ij}}$$



weighted shortest path distance

$$d_{ij} = \sum_{\{k,l\} \in P^{\star}_{ij}} \tau_{kl}$$

SHORTEST PATHS AND EFFECTIVE DISTANCE

proximity matrix

$$\tau_{ij} = \frac{1}{w_{ij}}$$

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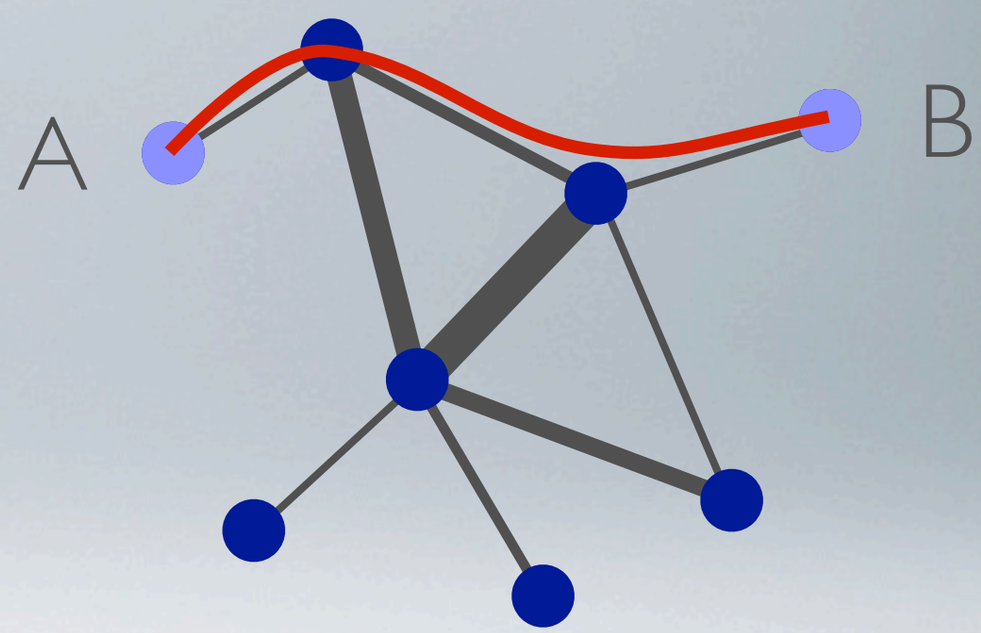
through topology



SHORTEST PATHS AND EFFECTIVE DISTANCE

proximity matrix

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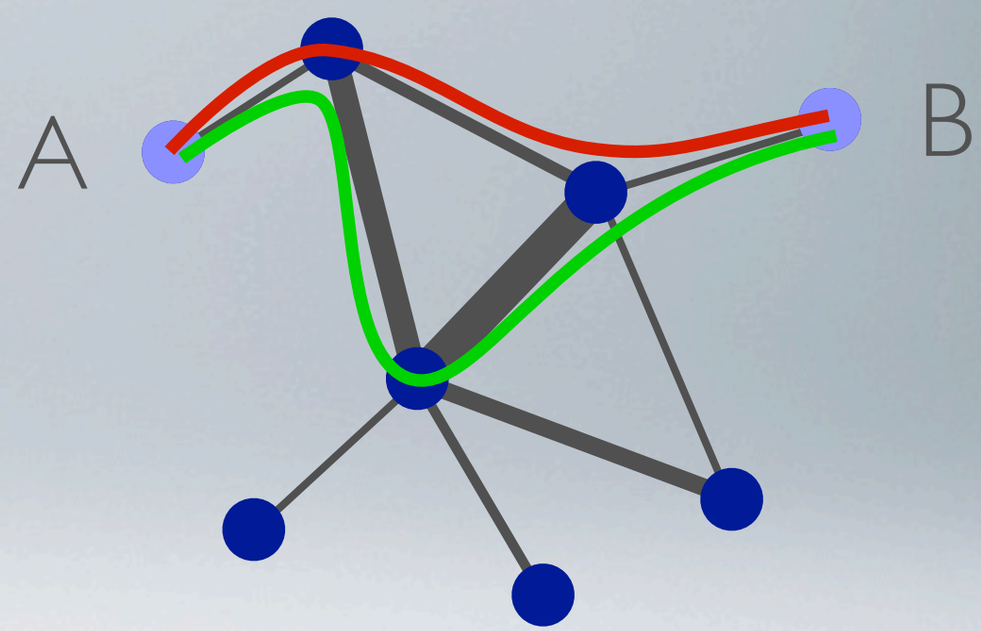
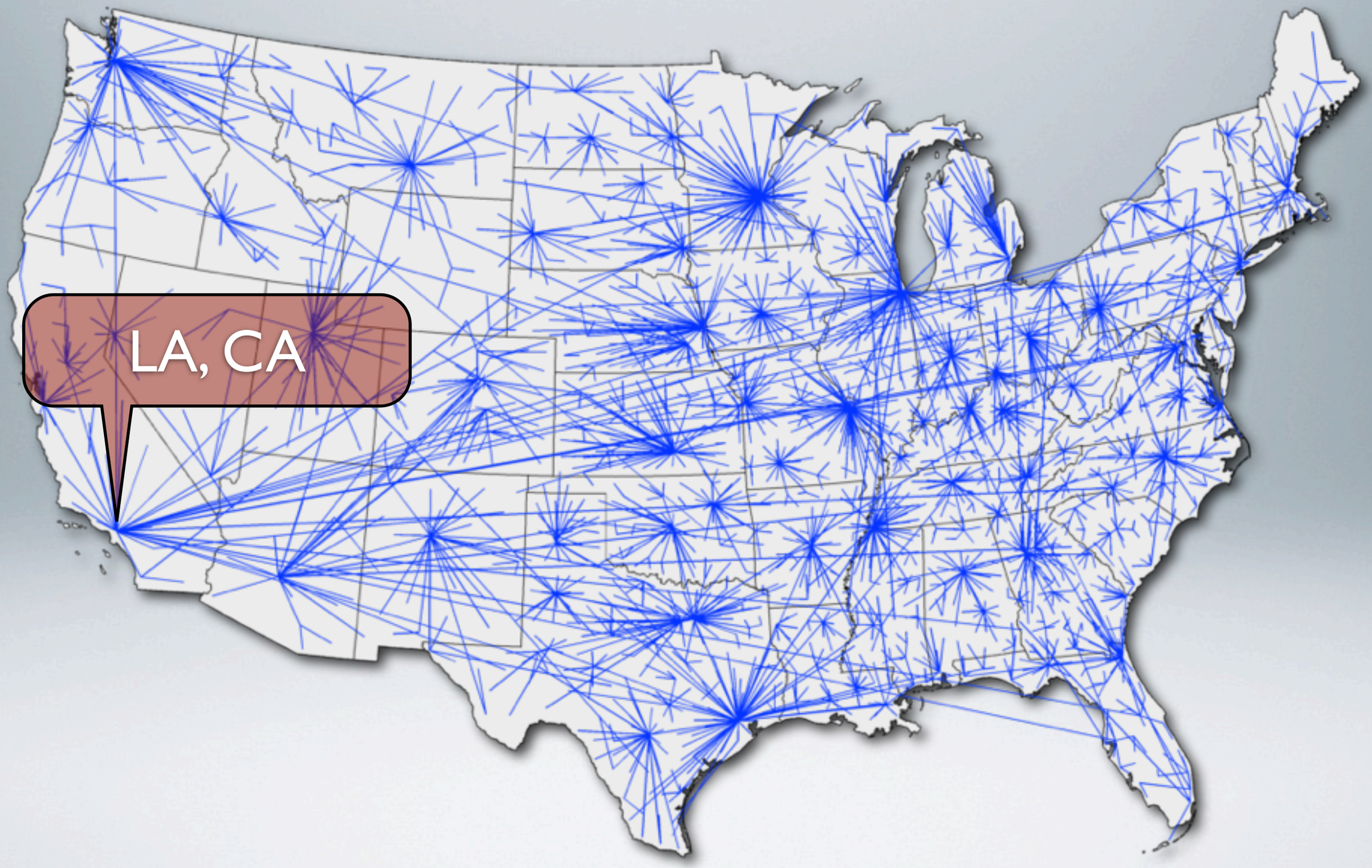


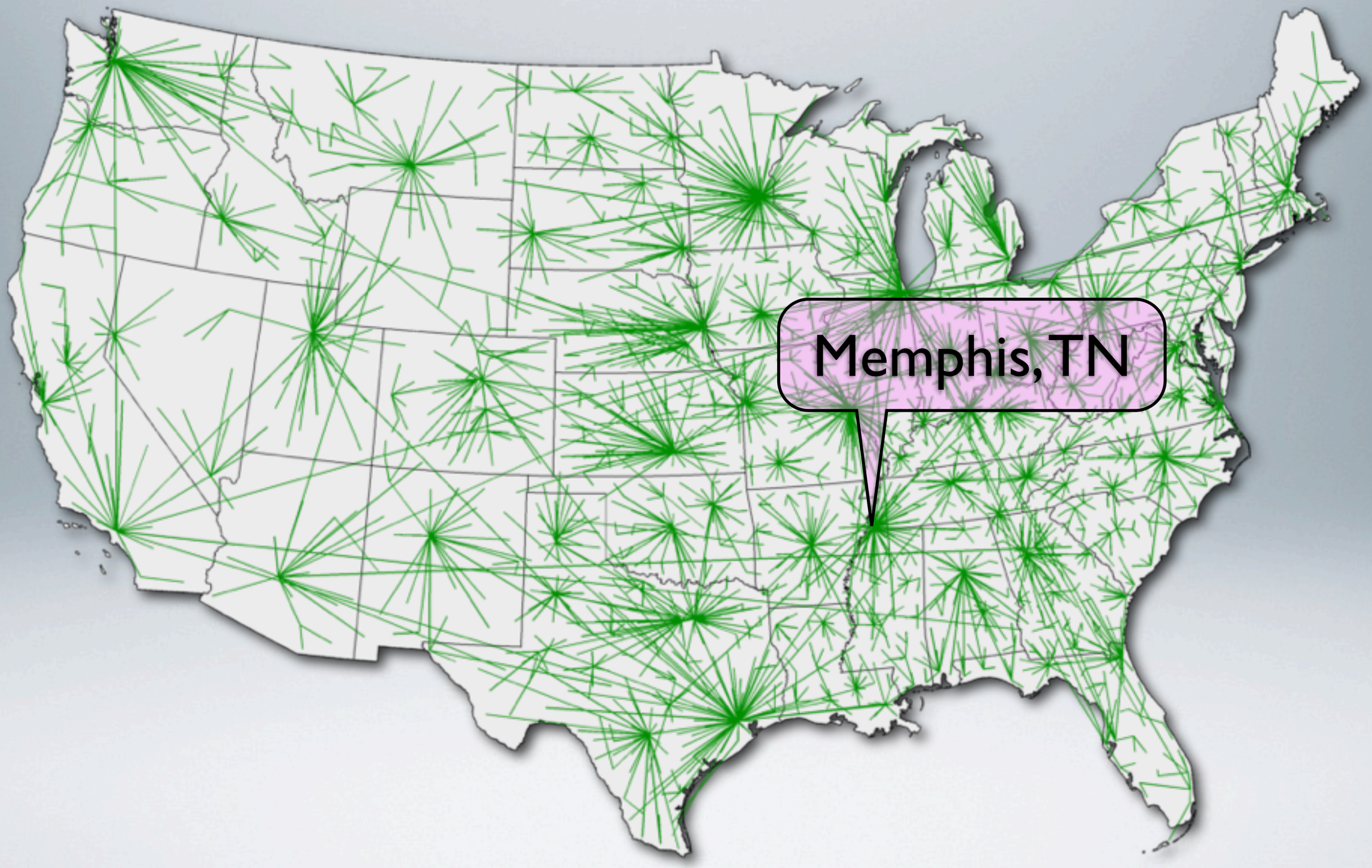
through weighted network

SHORTEST PATH TREES EXAMPLES

SHORTEST PATH TREES EXAMPLES

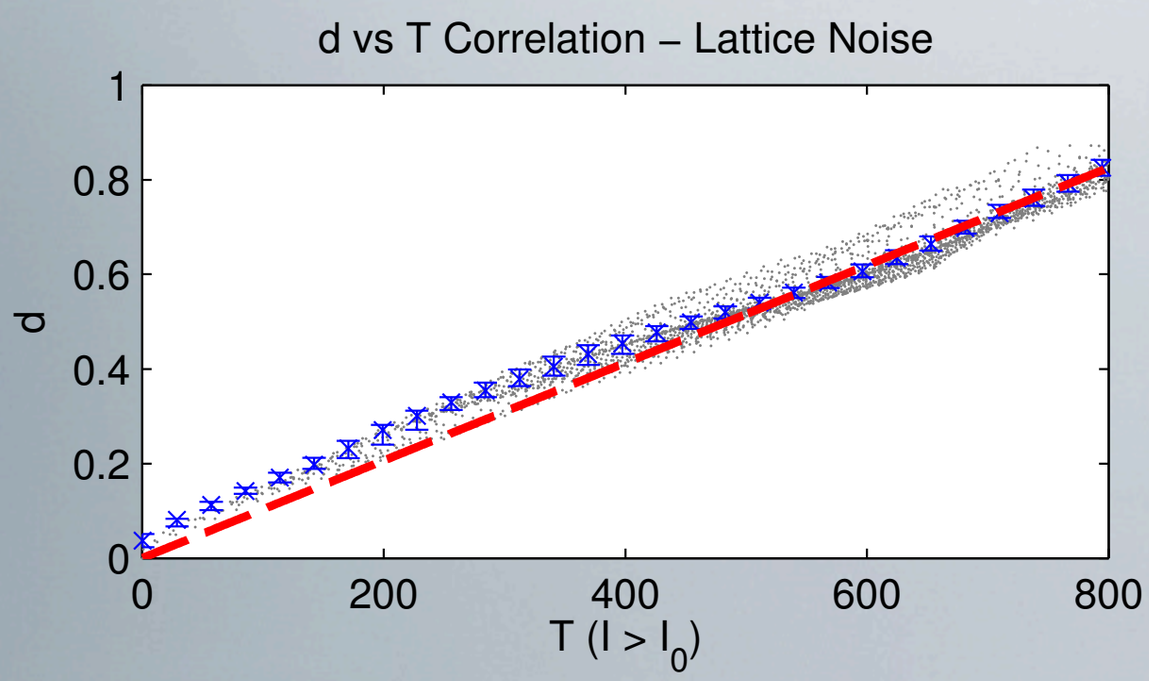


SHORTEST PATH TREES EXAMPLES



SPATO

EFFECTIVE VELOCITIES

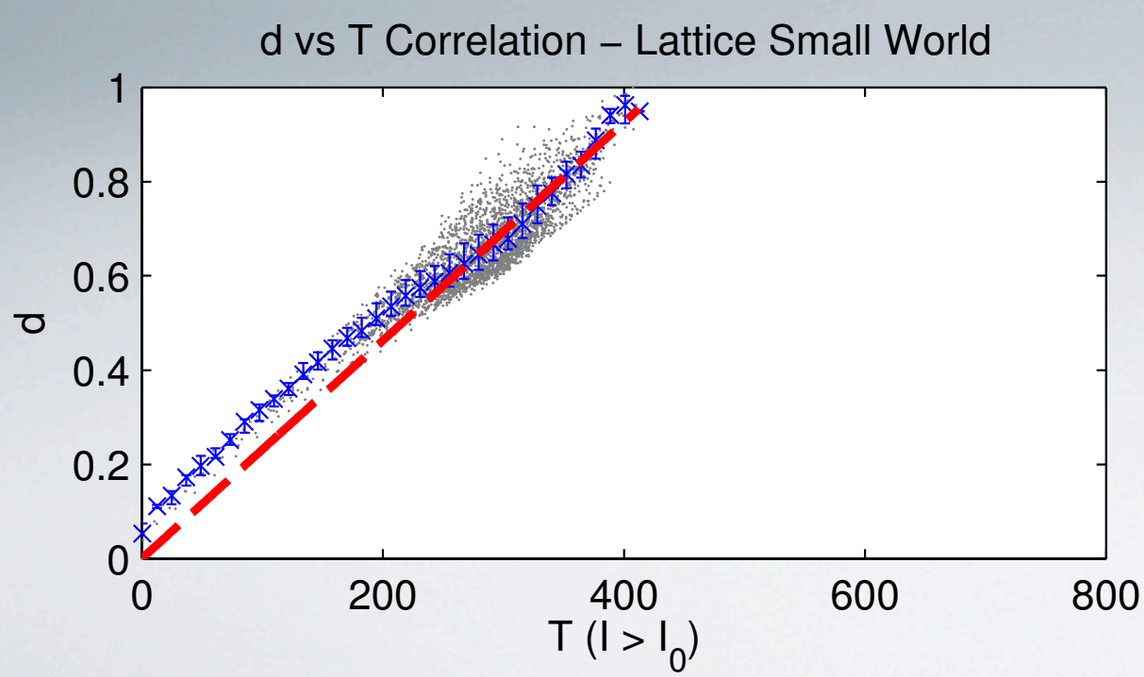


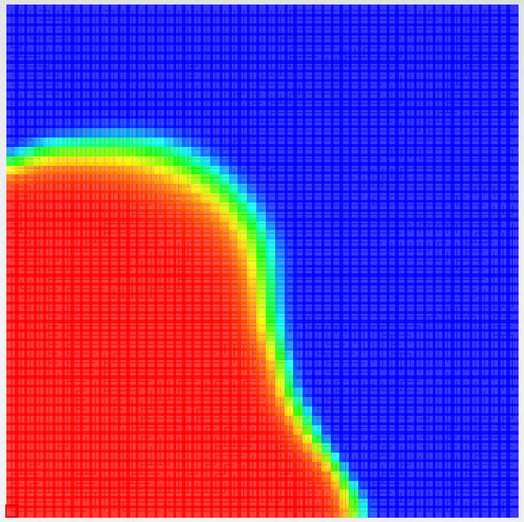
arrival time: $T(I > I_0)$

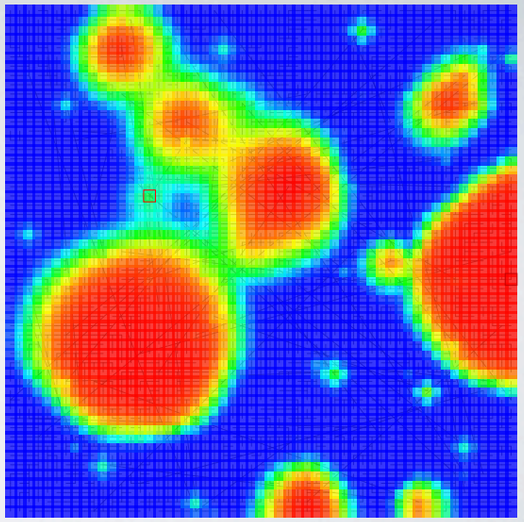
shortest path distance: d

effective velocity

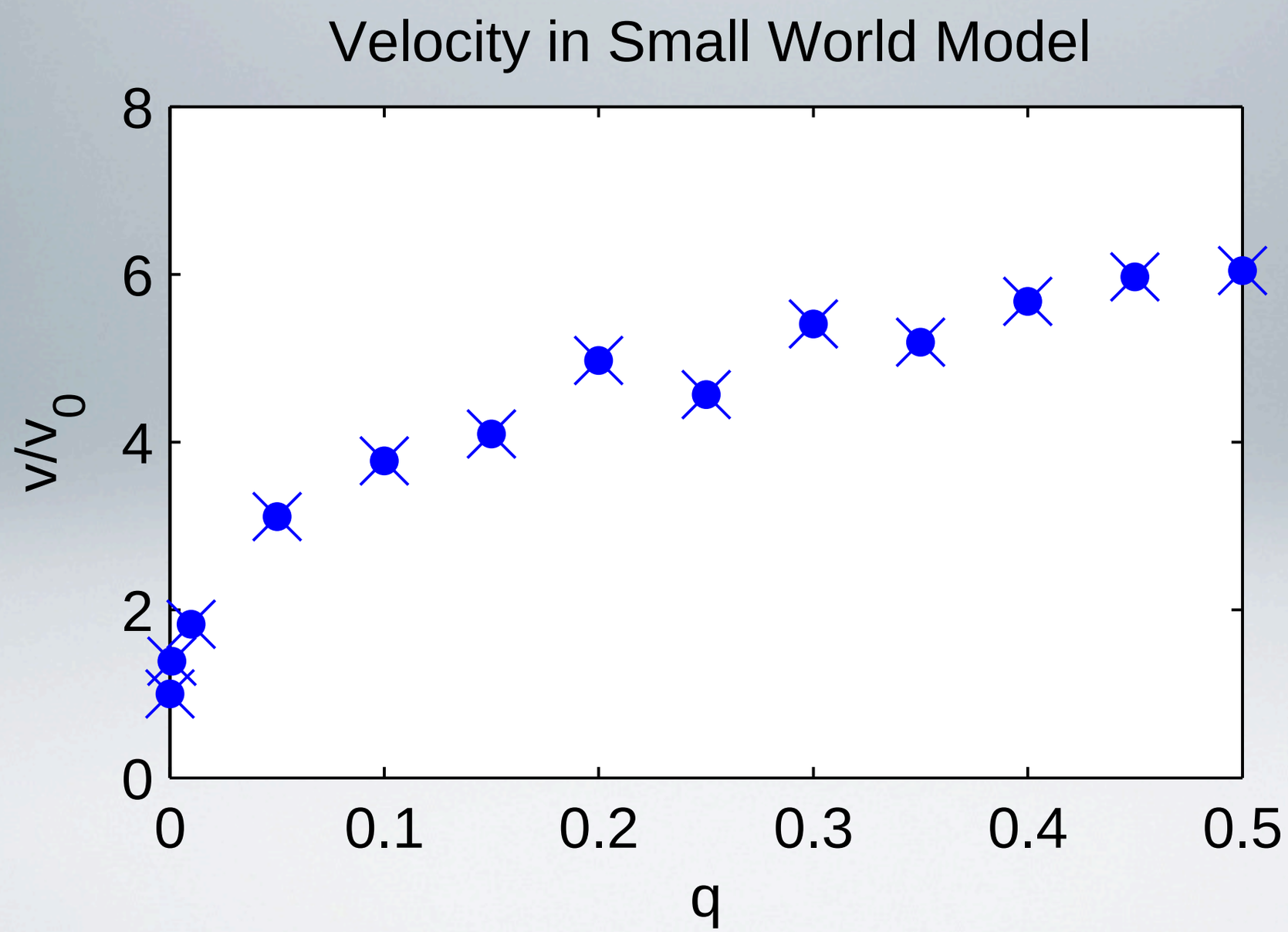
$$v = d/T$$







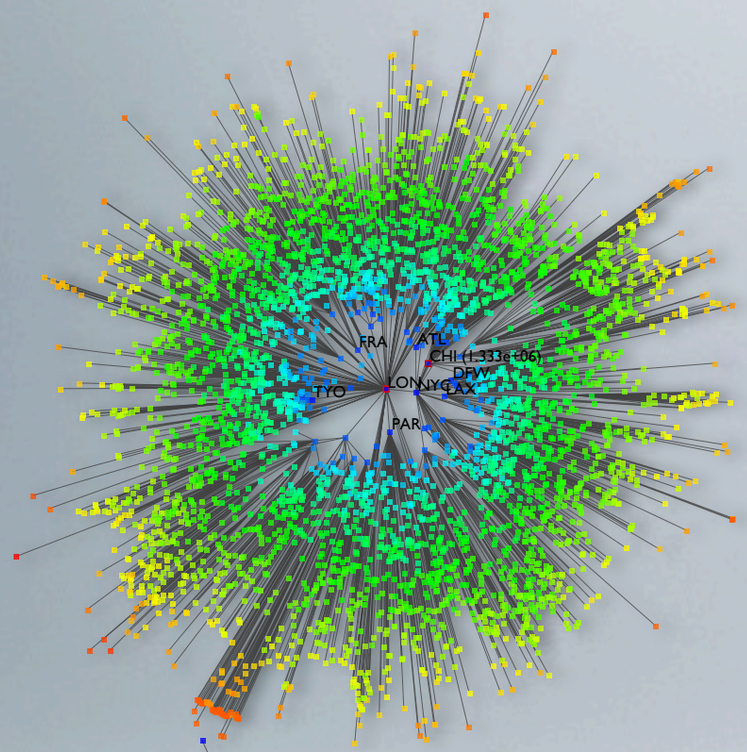
EFFECTIVE VELOCITY AS A FUNCTION OF LONG RANGE LINKS



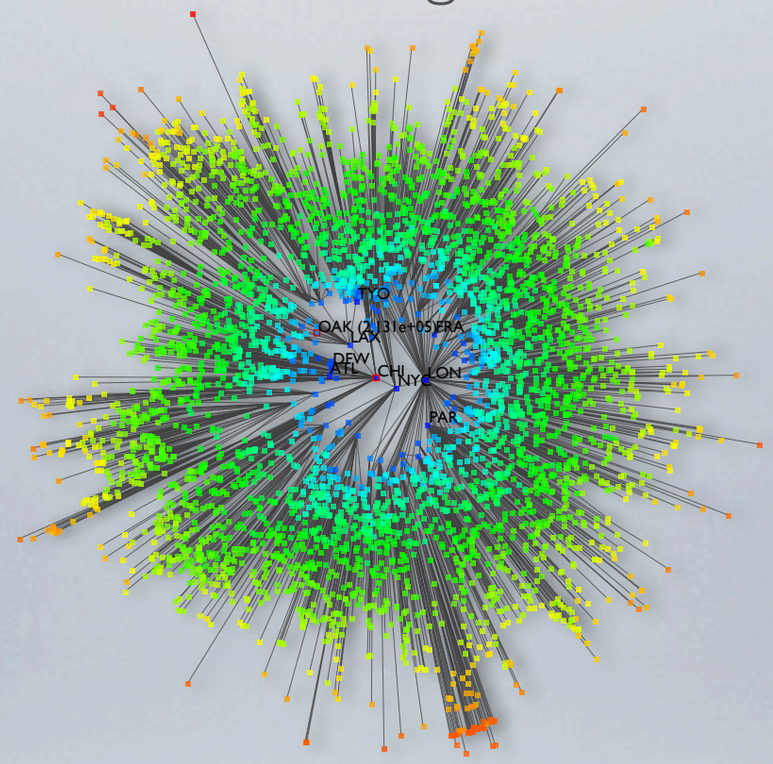
SPATO

WORLDWIDE AIR TRANSPORTATION NETWORK

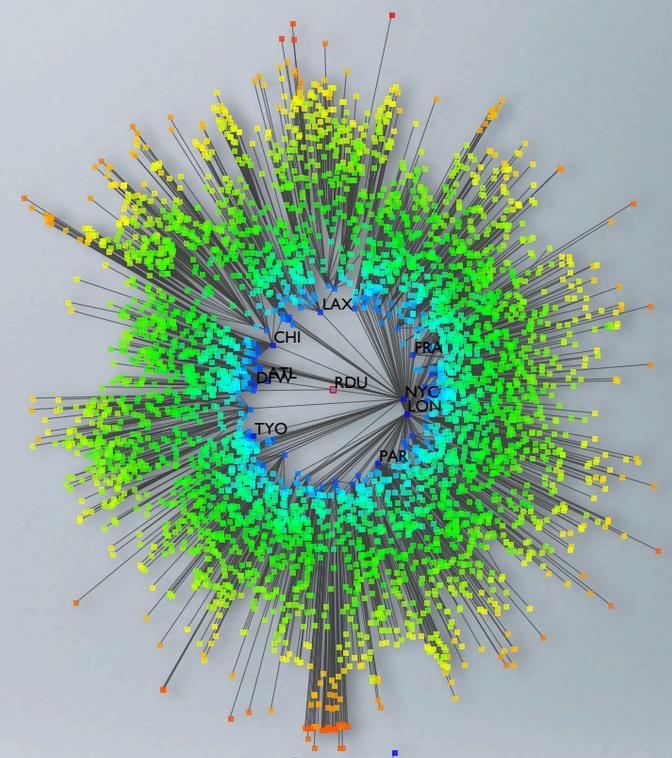
London

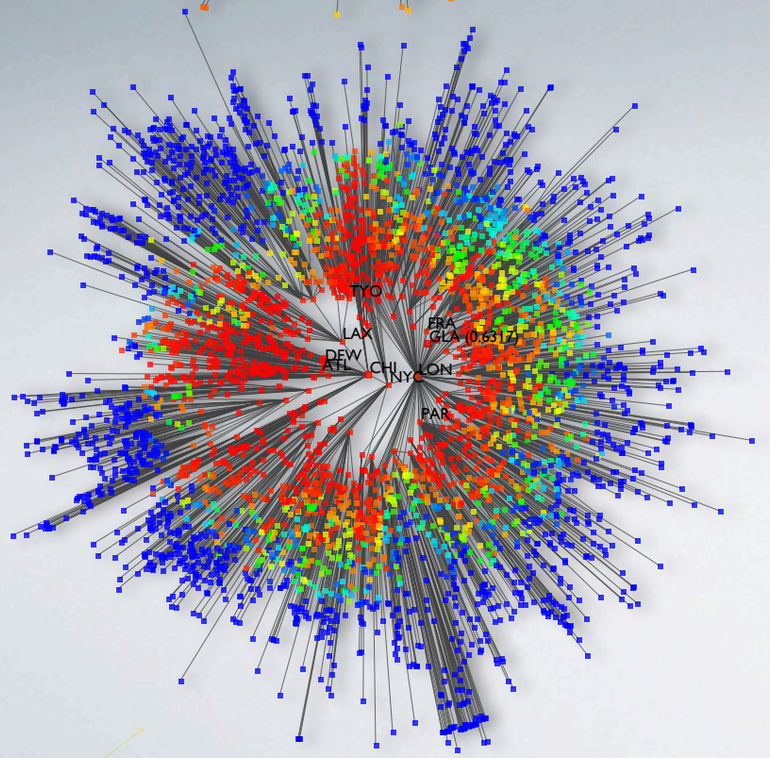


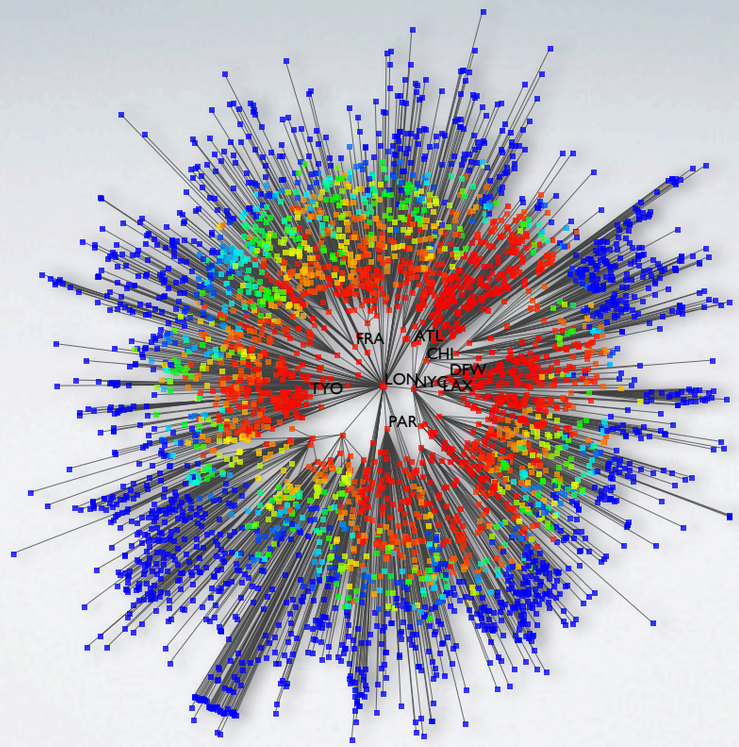
Chicago

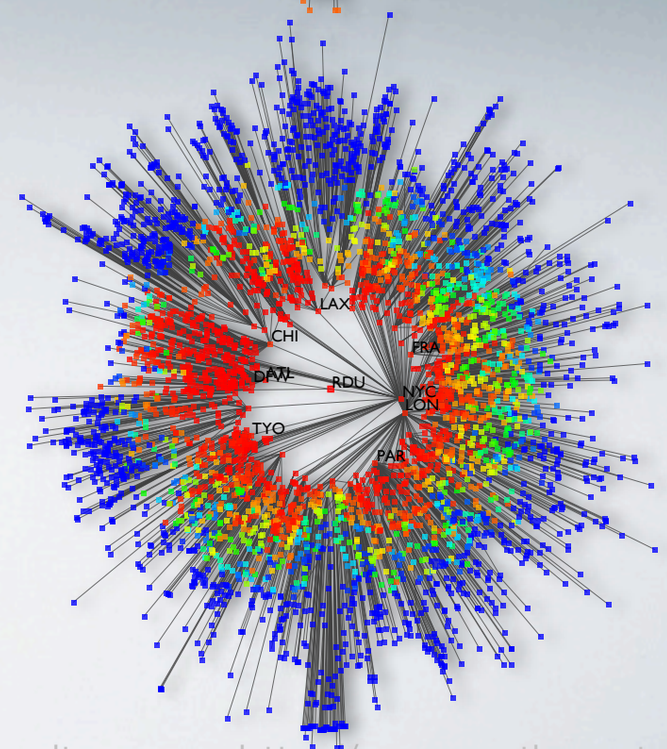


durham, NC

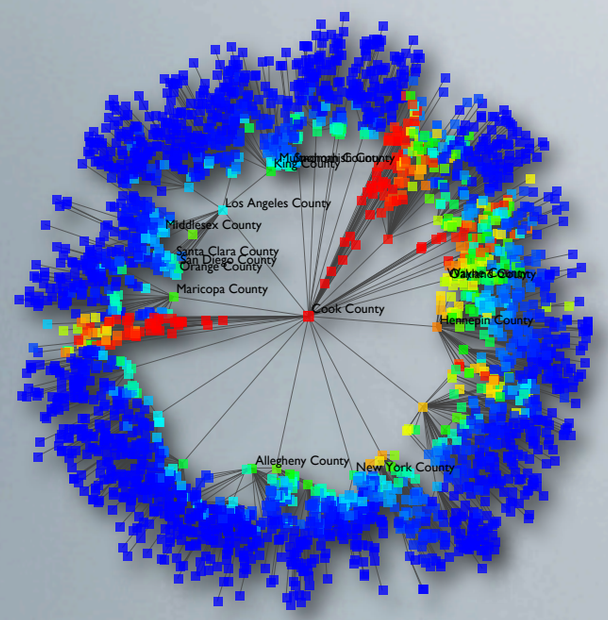


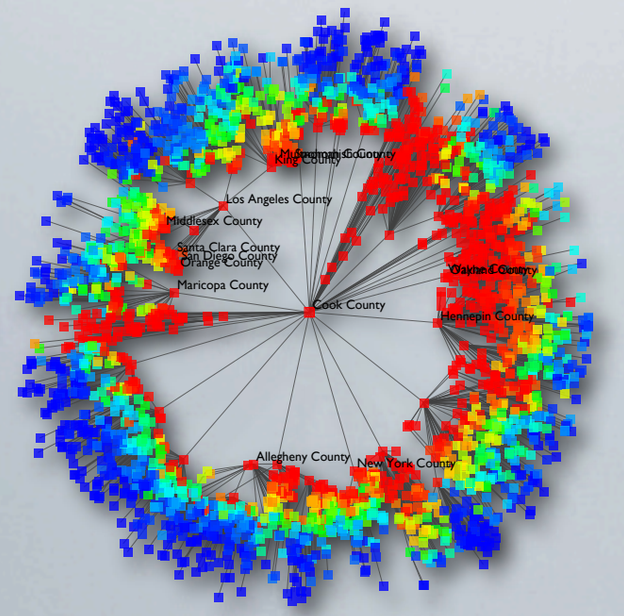


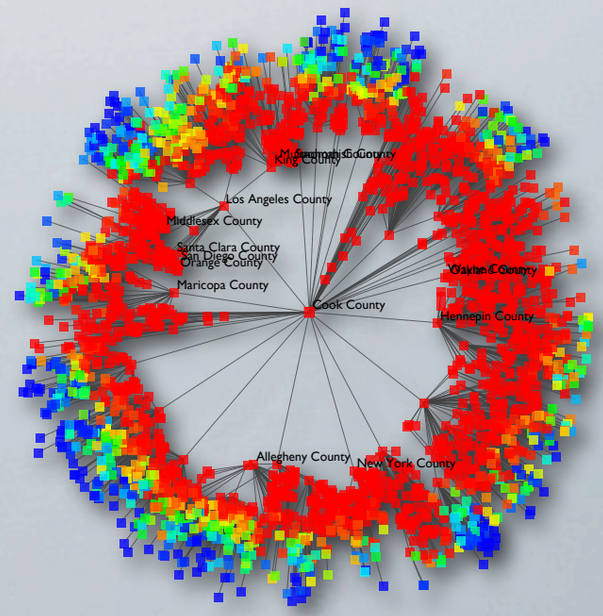


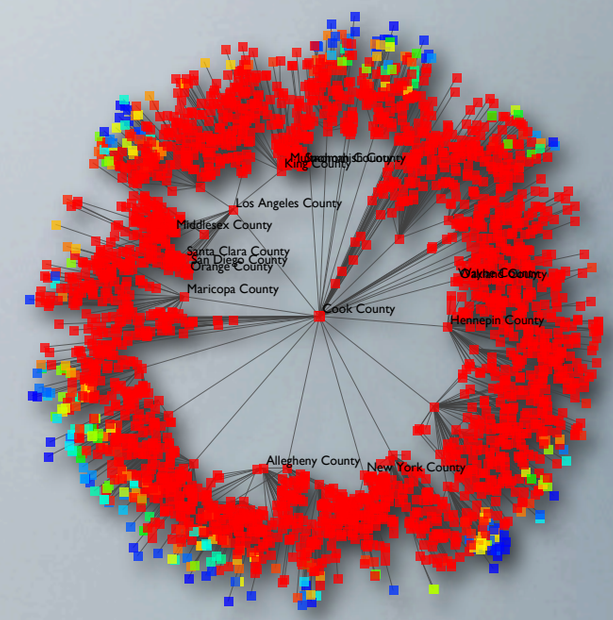


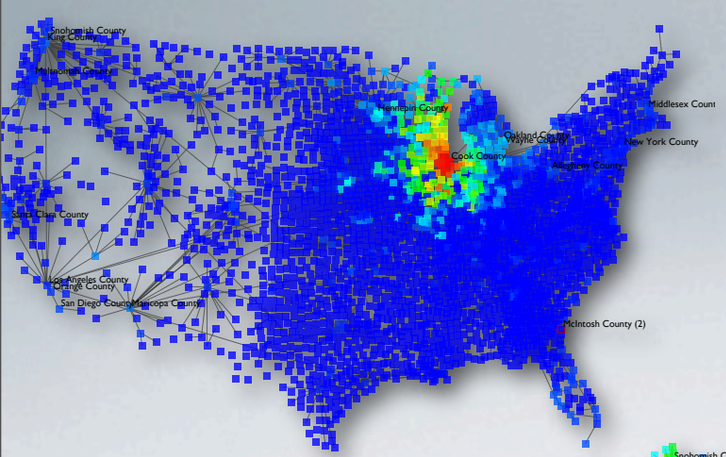
US MULTISCALE MOBILITY

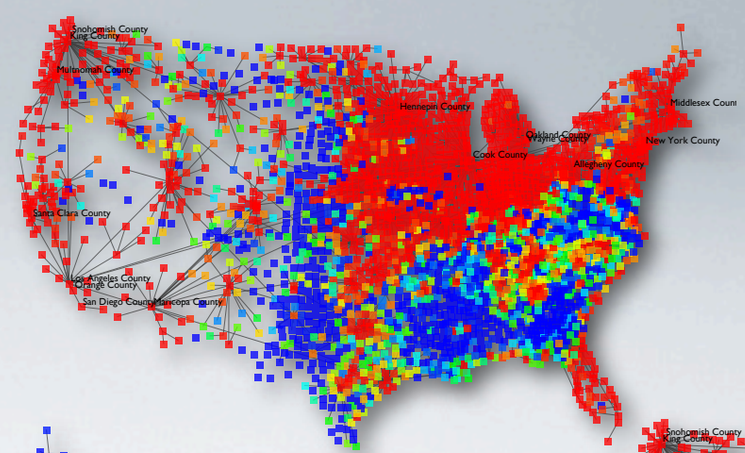


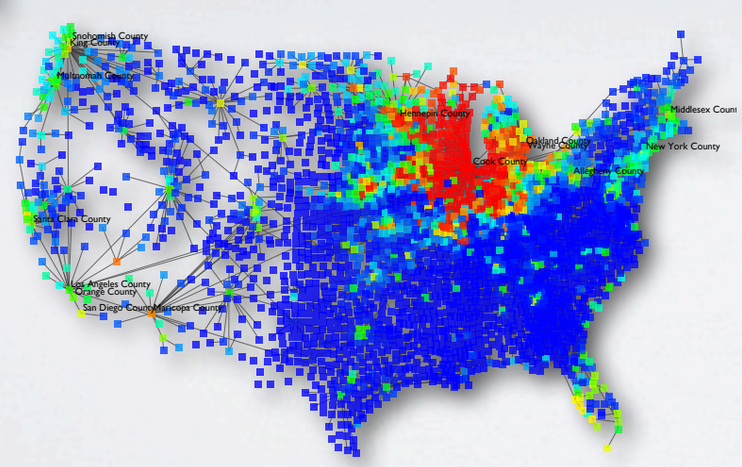


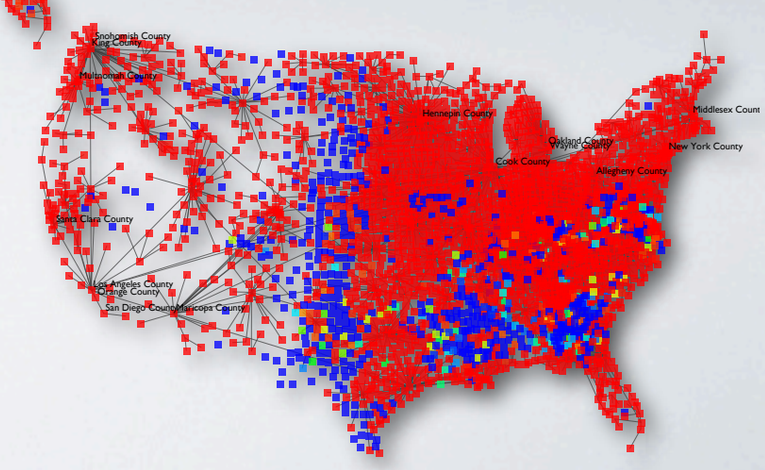






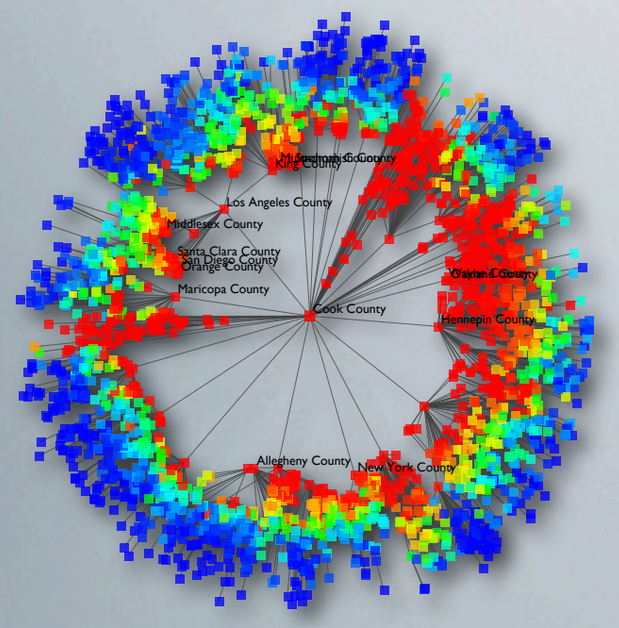






US MULTISCALE MOBILITY

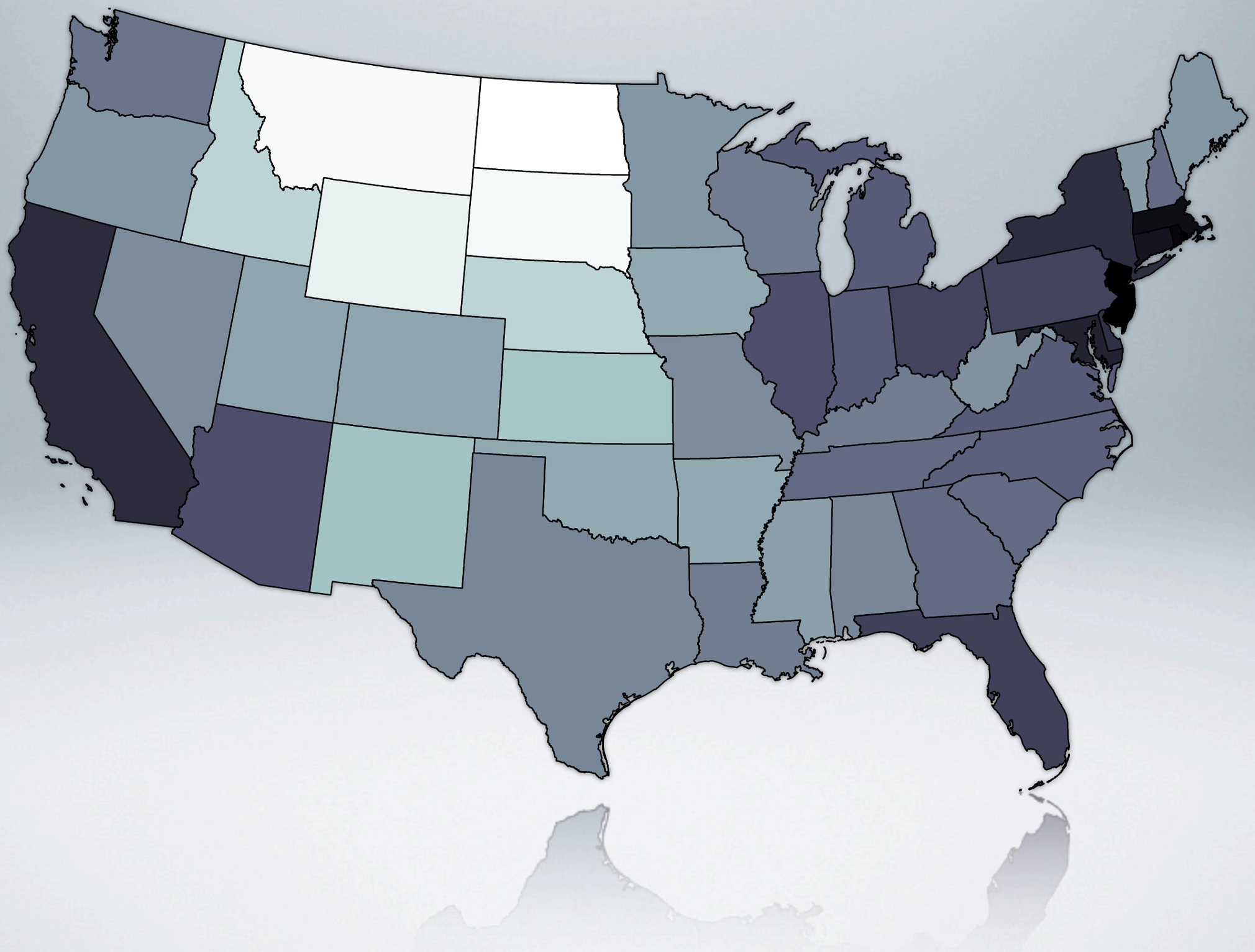
CONCLUSIONS



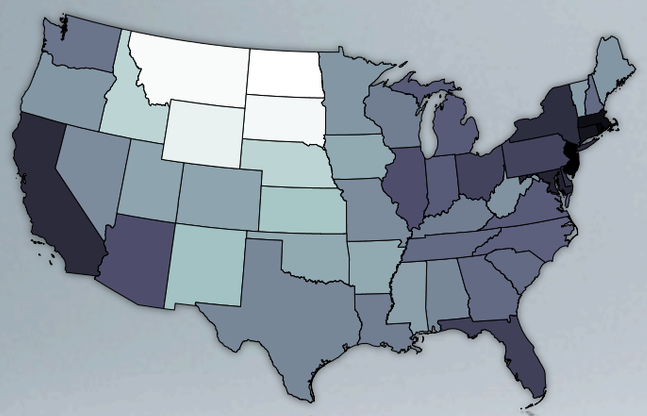
- Shortest paths and shortest path tree representations simplify the dynamics
- effective propagation speeds based on effective distance
- long links along imply constant velocity
- the combination of scale free traffic and weight heterogeneity yield exponential spread

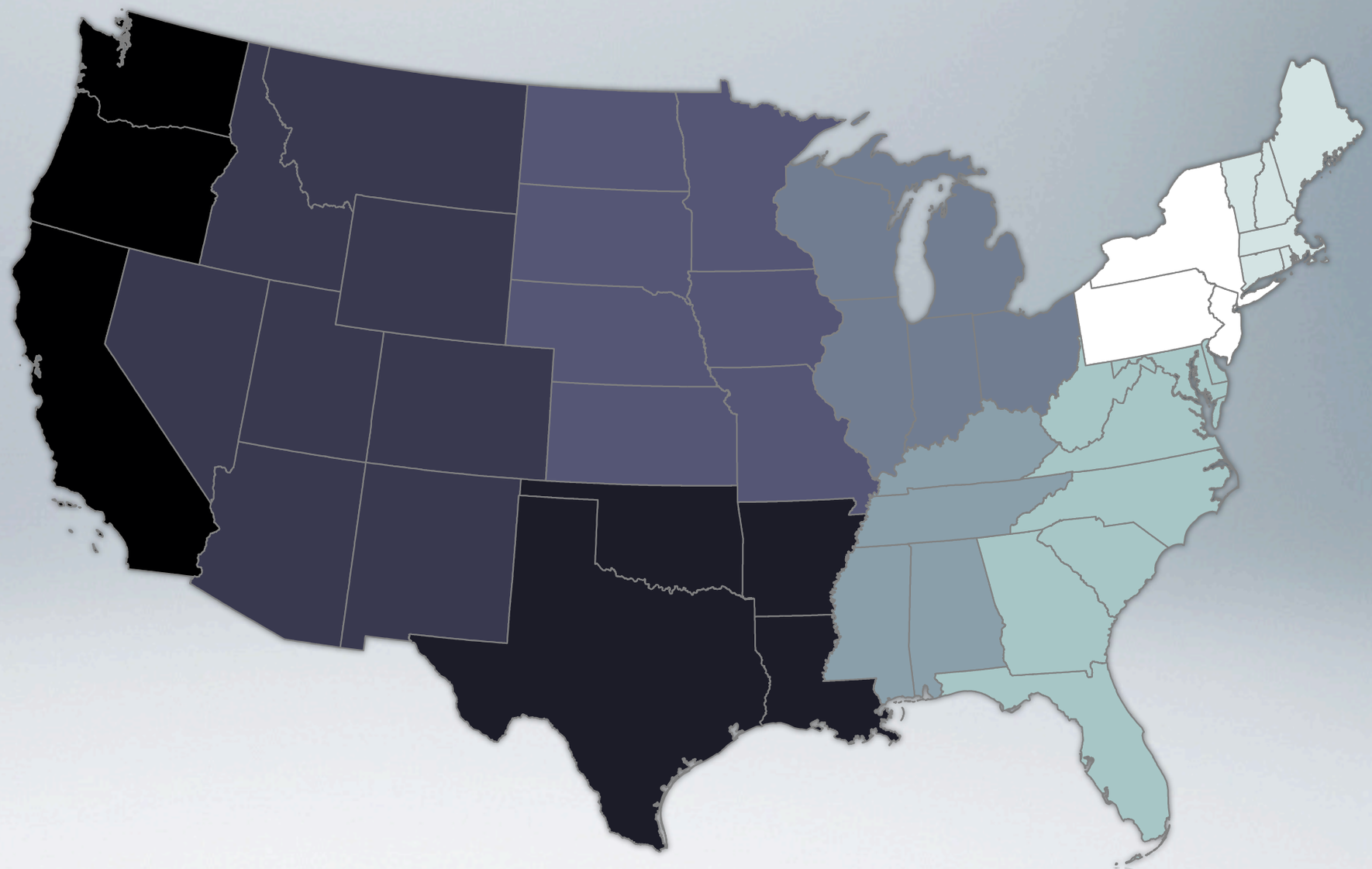
THE BORDERS IN HUMAN MOBILITY

GEOGRAPHICAL SUBDIVISIONS

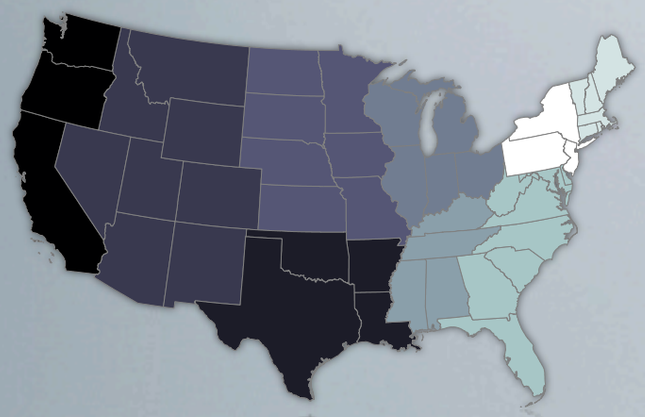


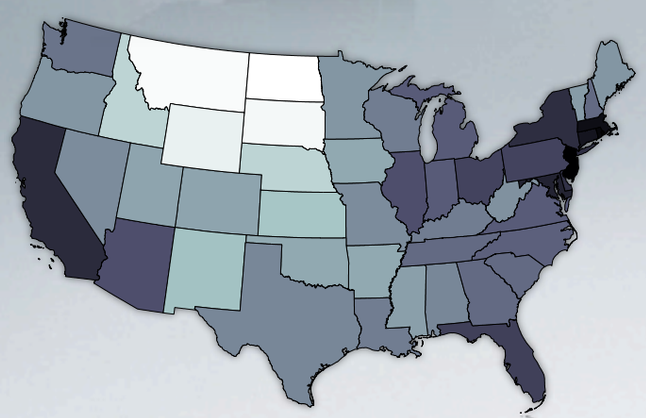
GEOGRAPHICAL SUBDIVISIONS

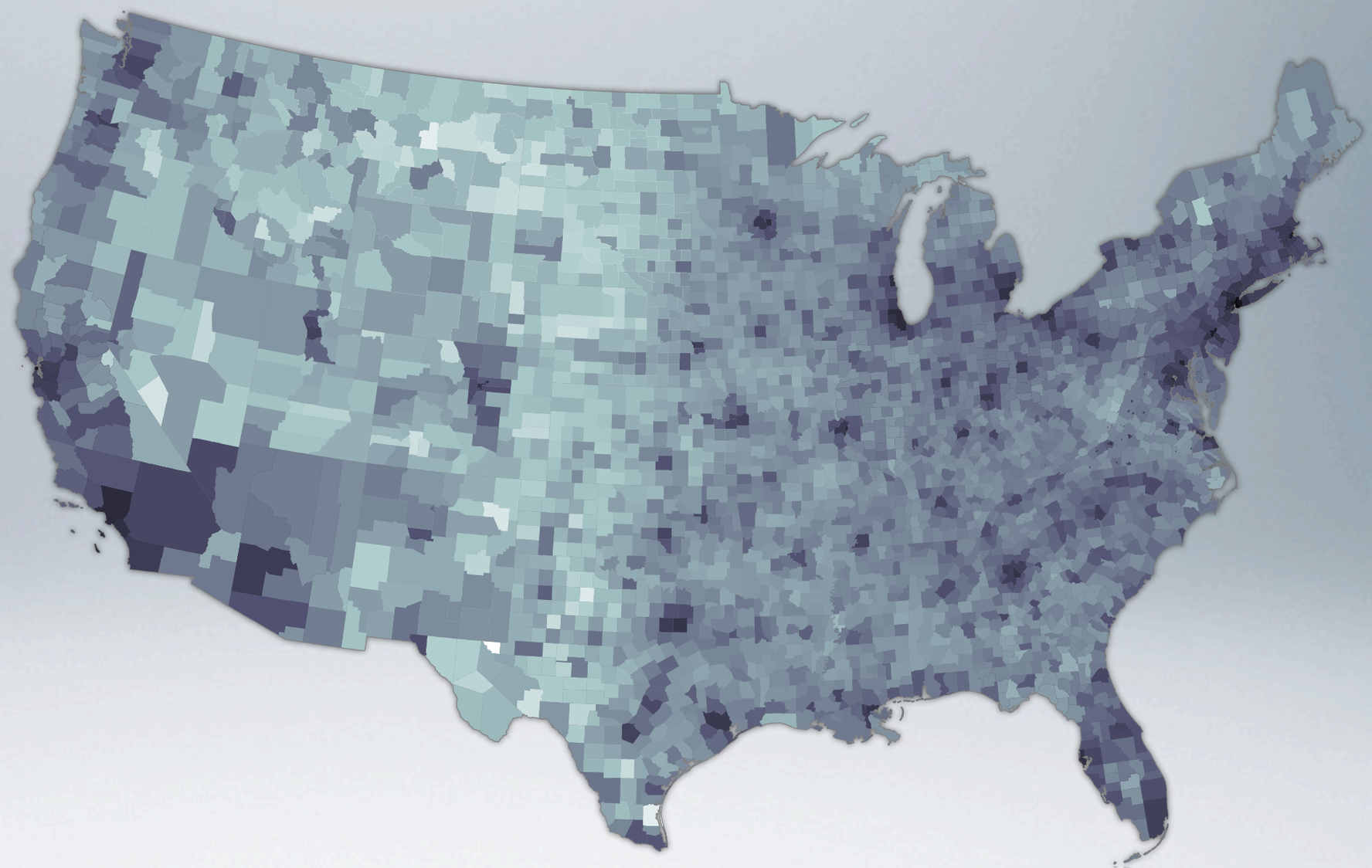




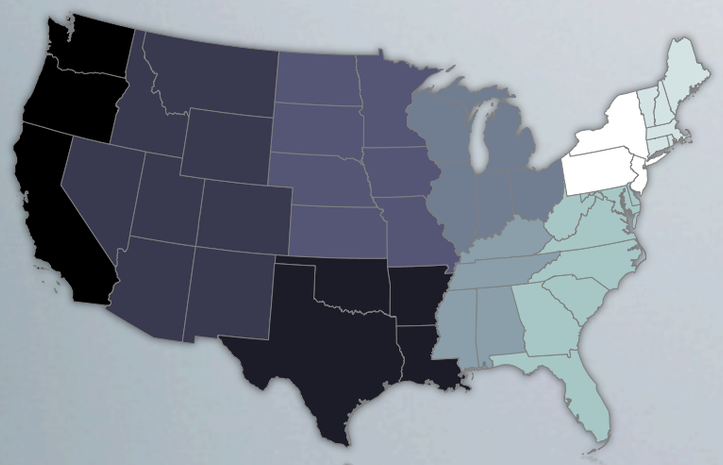
GEOGRAPHICAL SUBDIVISIONS

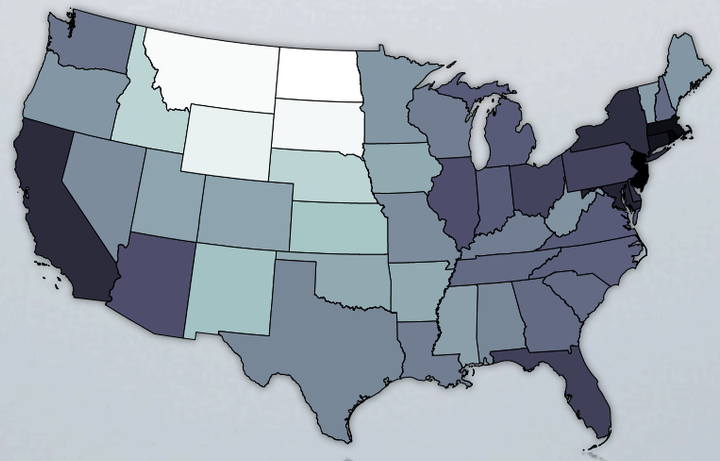


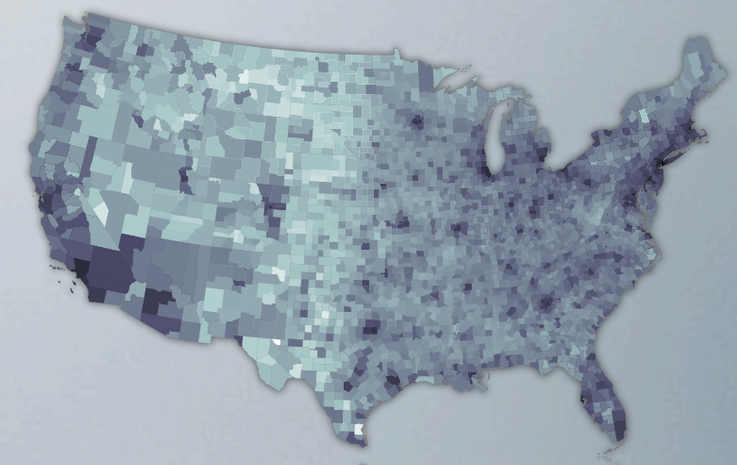




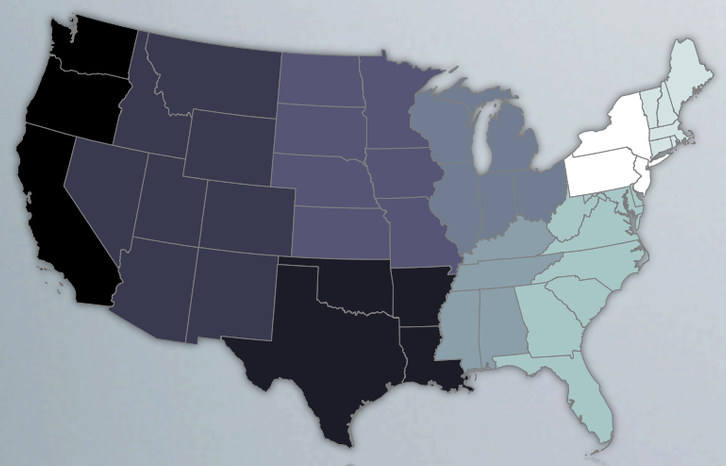
GEOGRAPHICAL SUBDIVISIONS

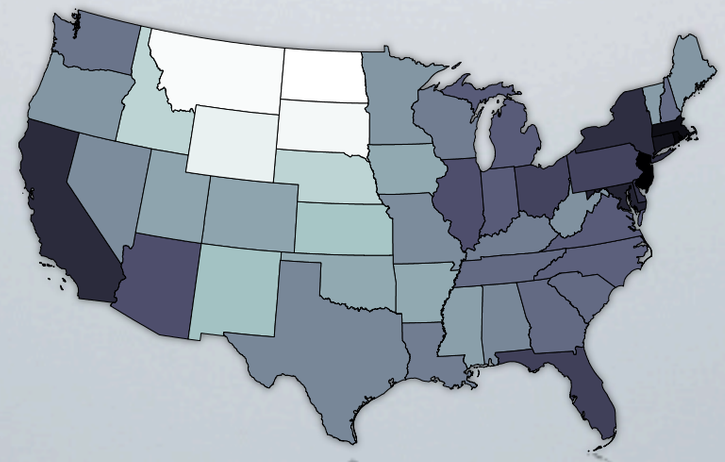


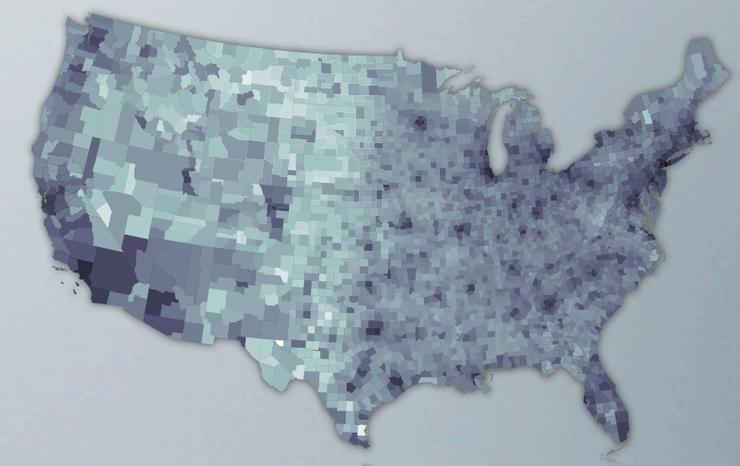




GEOGRAPHICAL SUBDIVISIONS

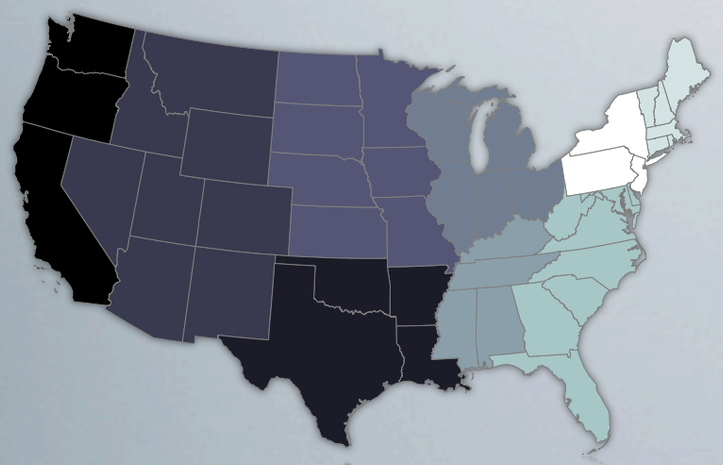


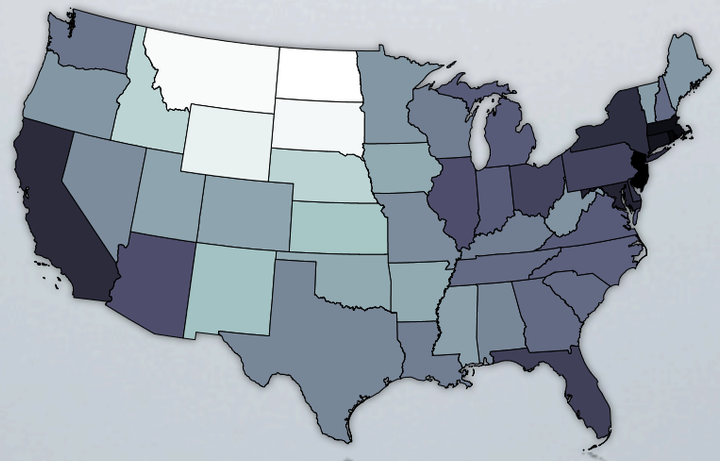


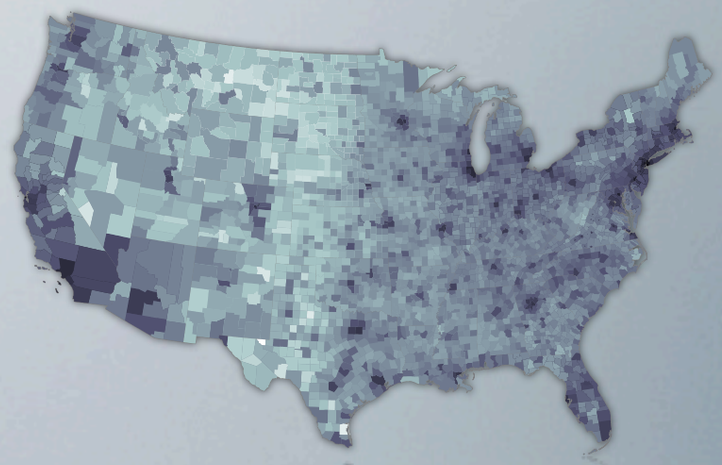


hierarchical structure
spatially compact
historically evolved
geographic determinants

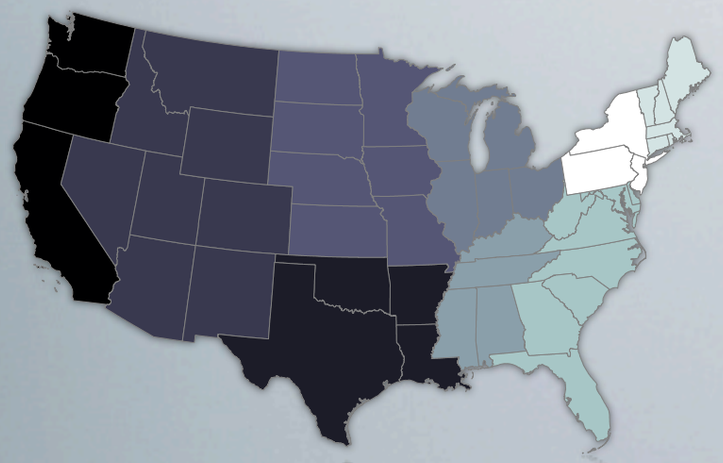
GEOGRAPHICAL SUBDIVISIONS

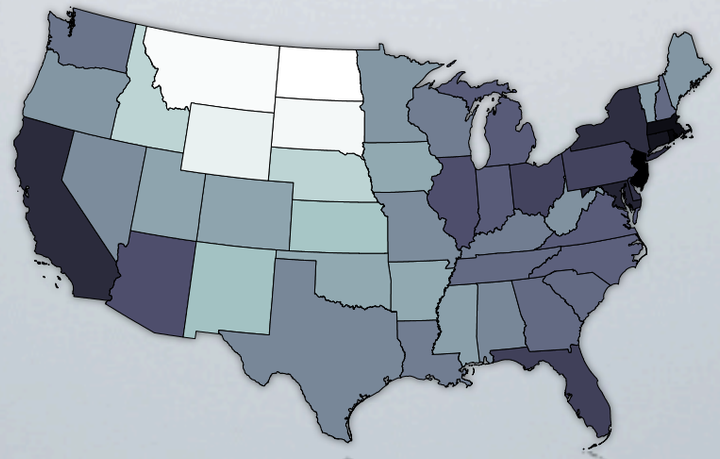


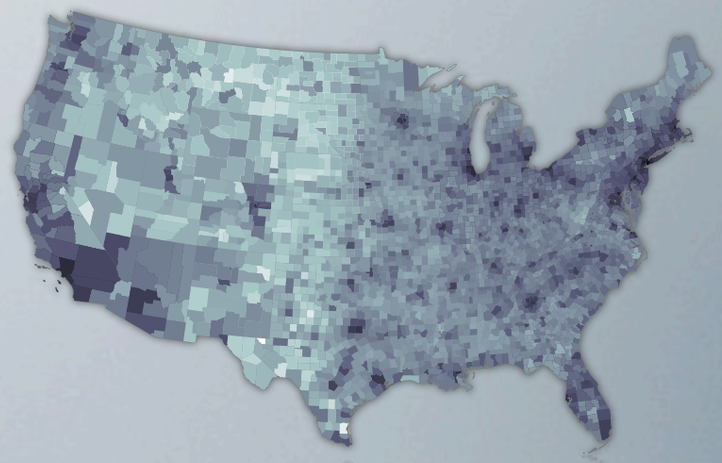




GEOGRAPHICAL SUBDIVISIONS

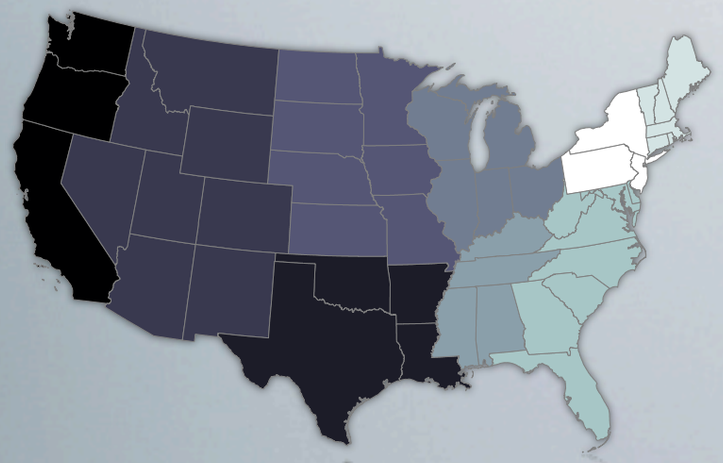


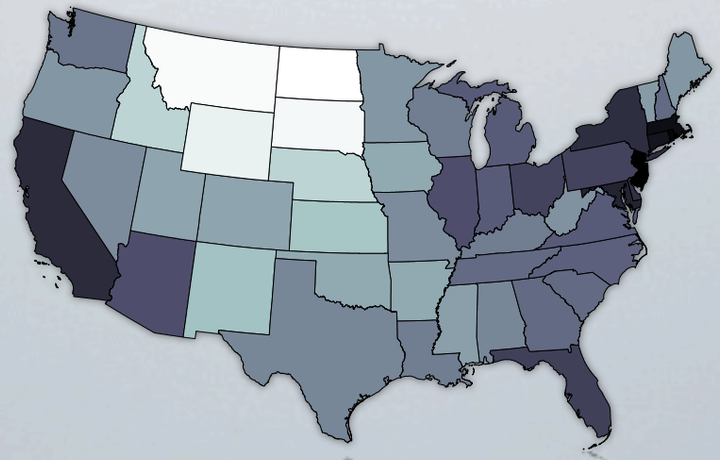


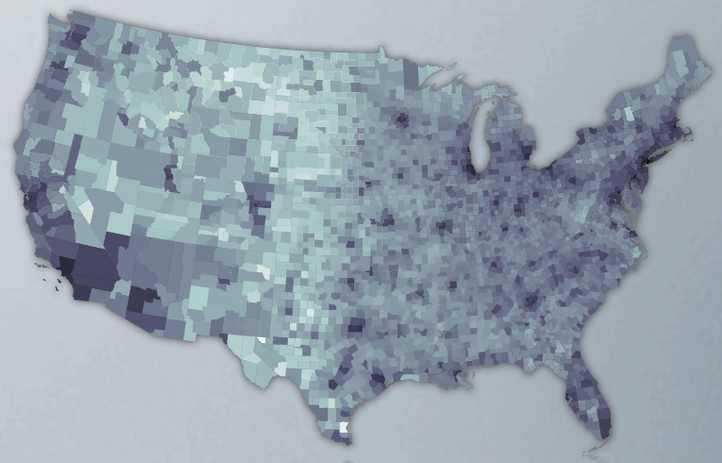


- What are the **effective** subdivisions of the United States?
- Where are **effective** borders?
- Are **effective** subdivisions spatially compact?

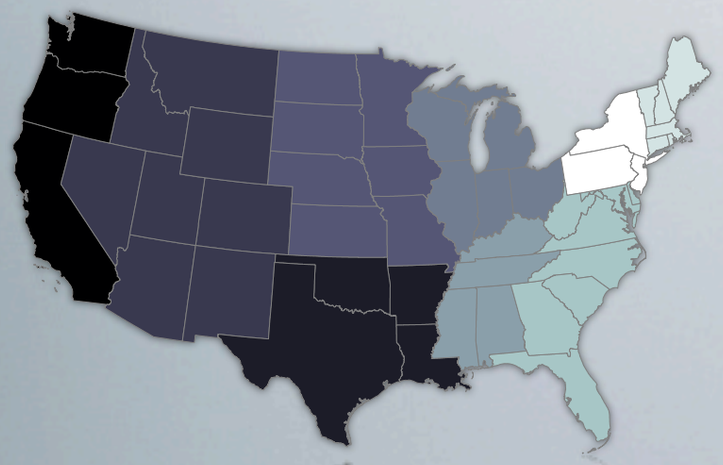
GEOGRAPHICAL SUBDIVISIONS

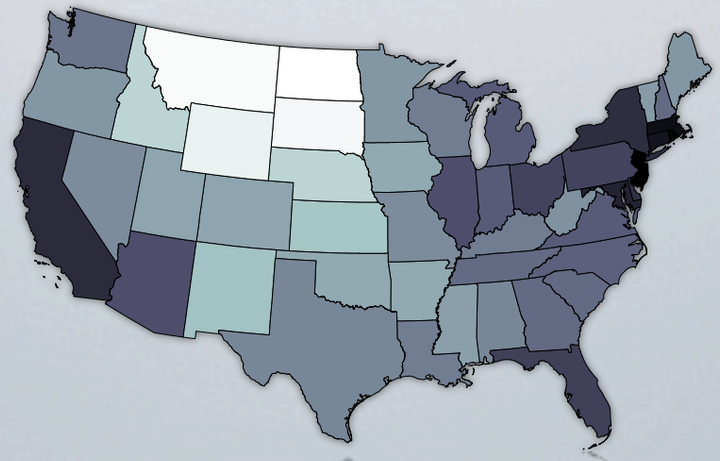


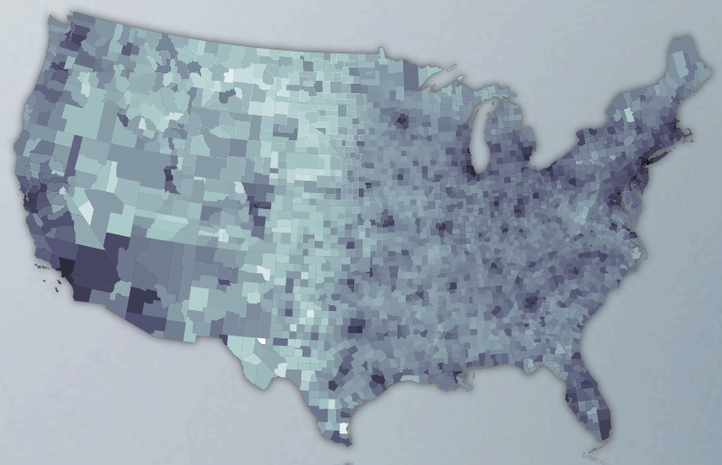


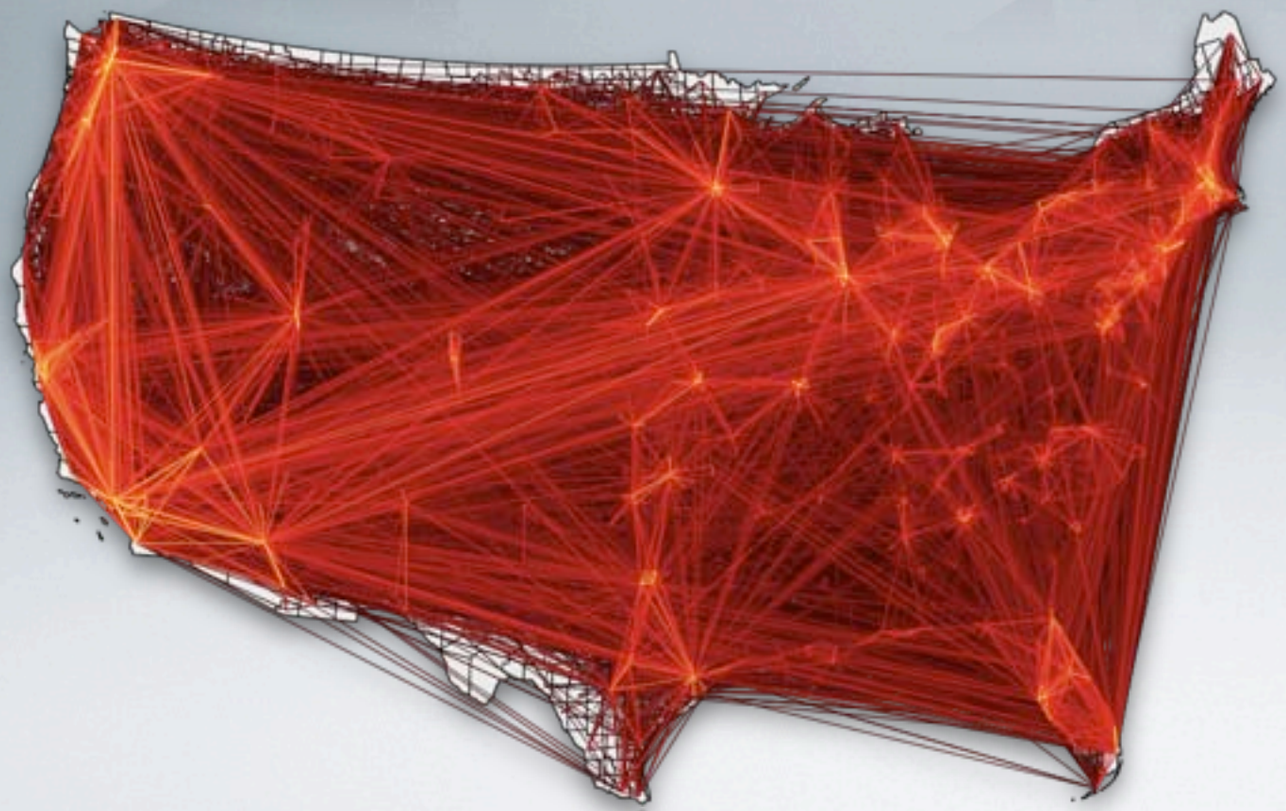


GEOGRAPHICAL SUBDIVISIONS





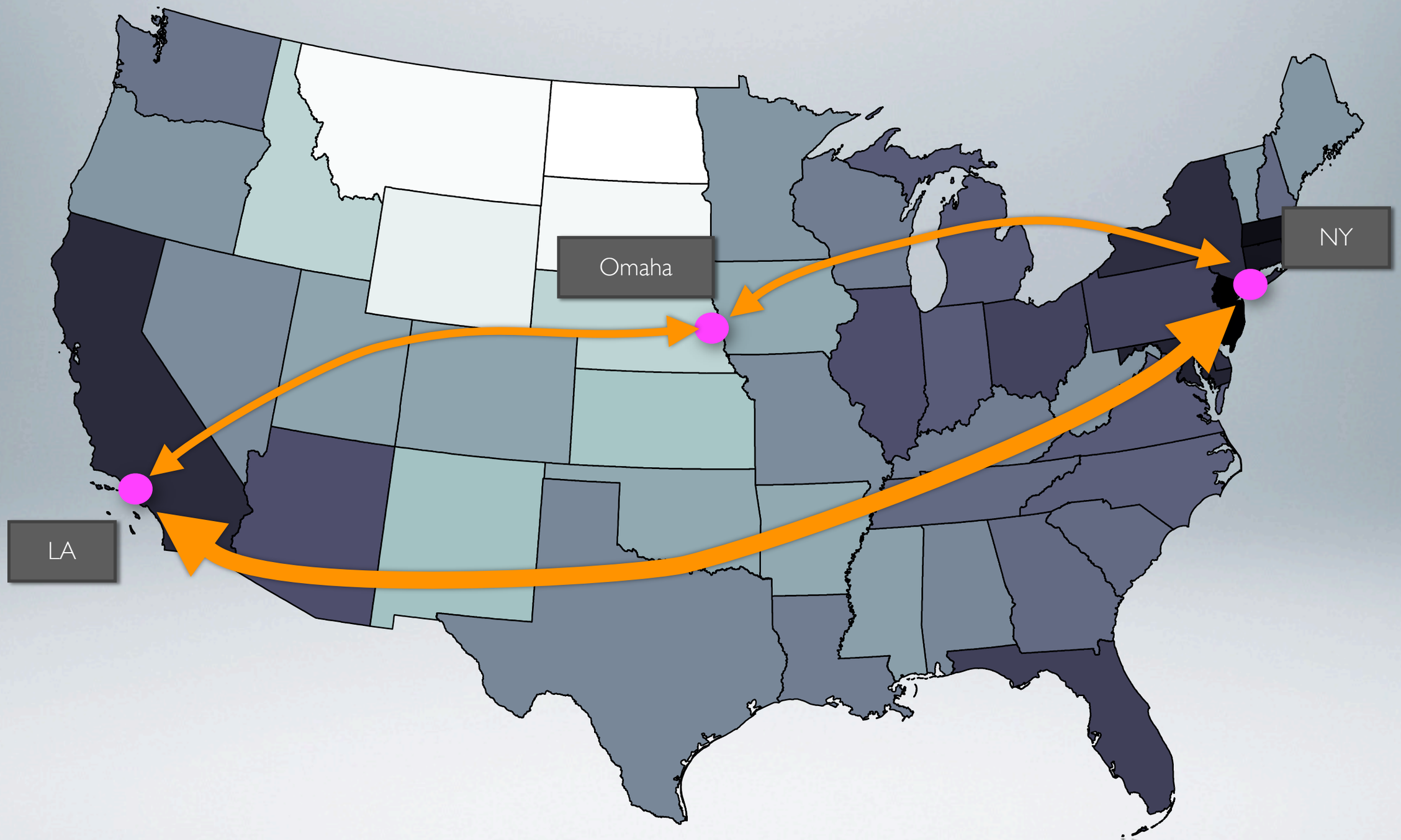




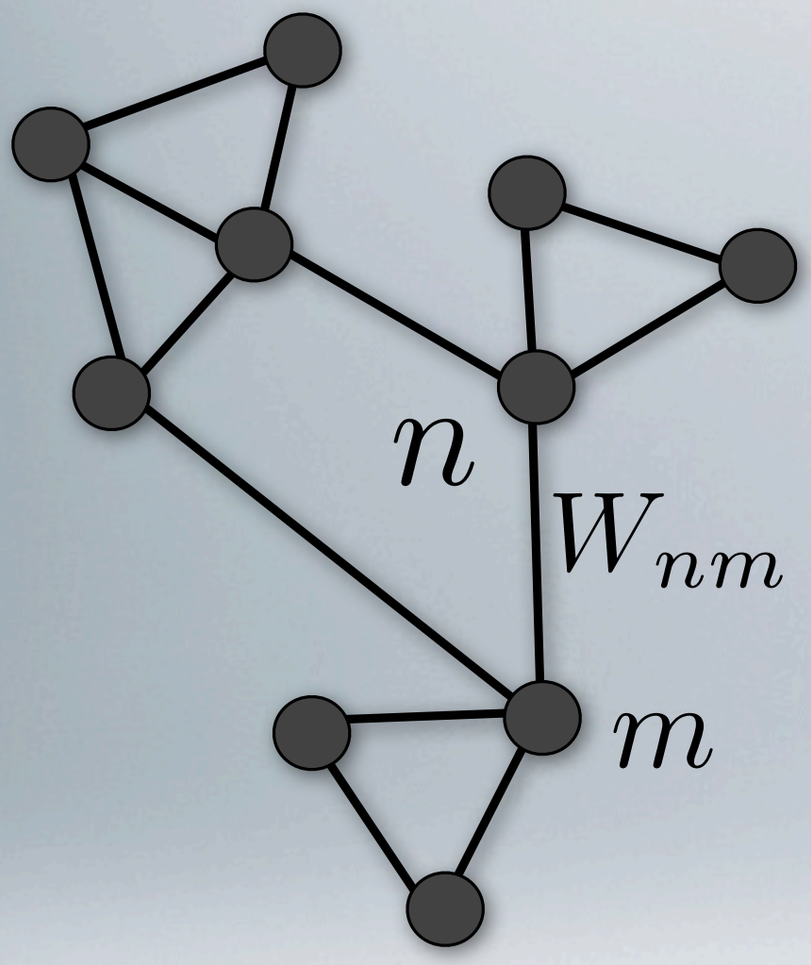
effective:

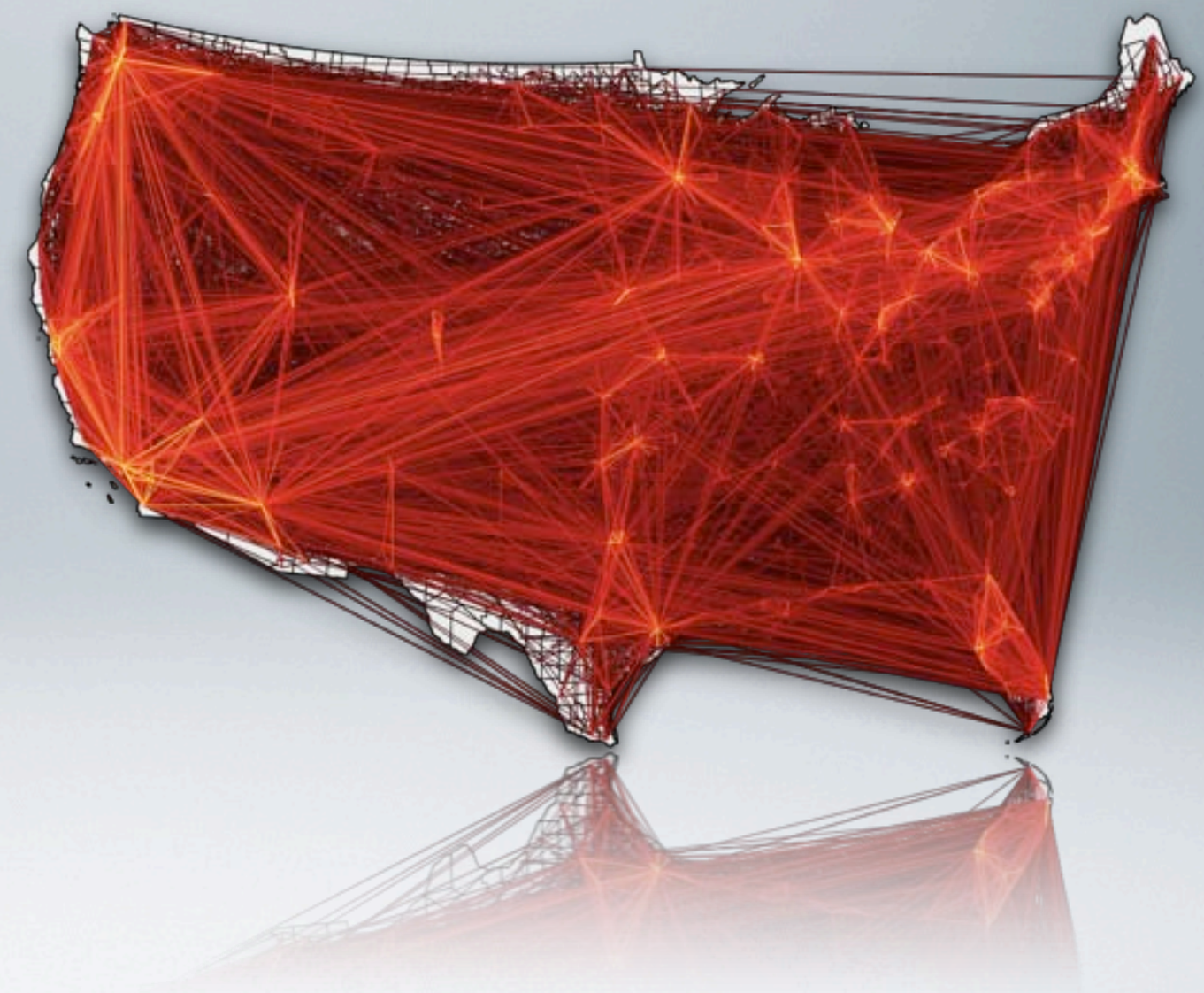
based on mobility

EFFECTIVE DISTANCE

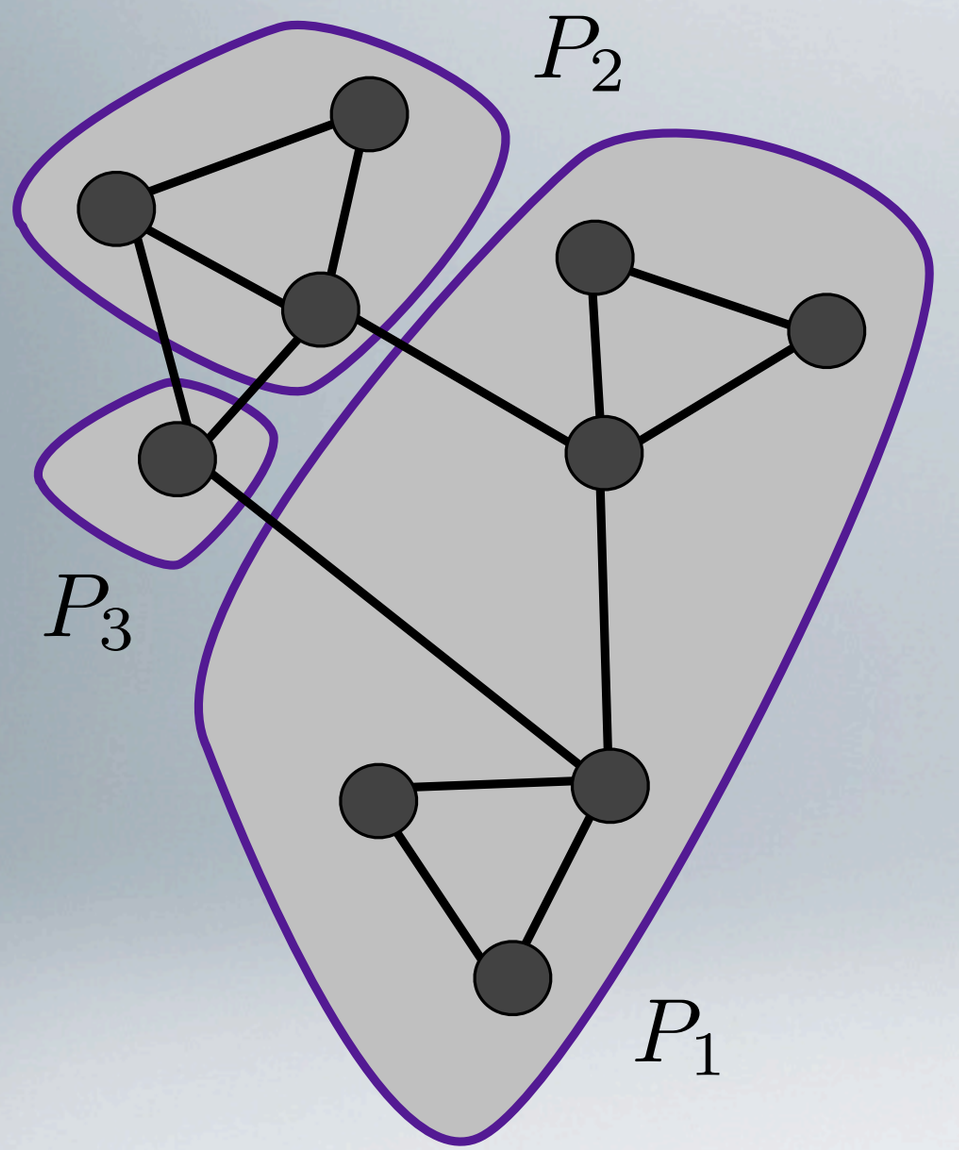


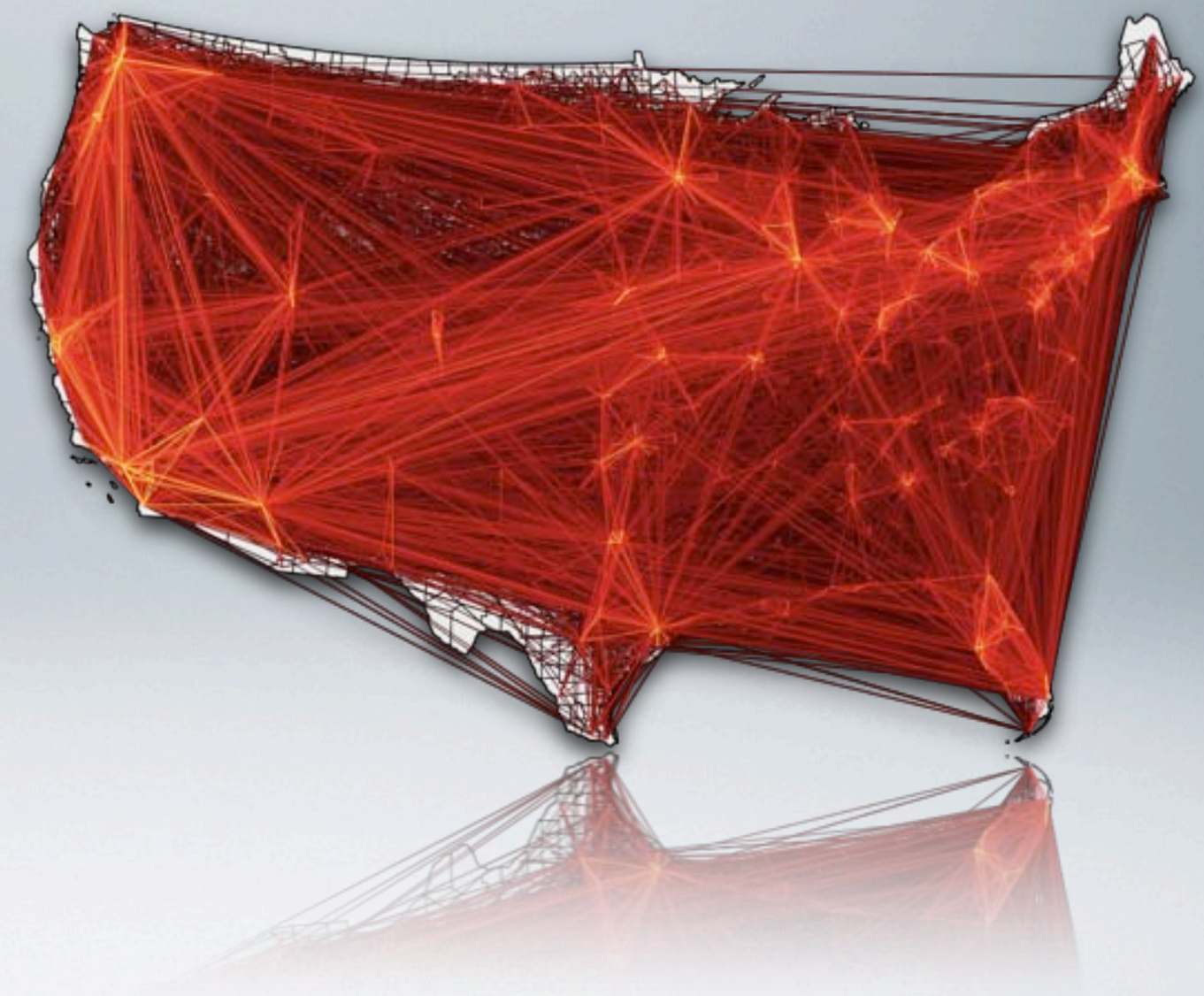
FINDING COMMUNITY STRUCTURE IN NETWORKS



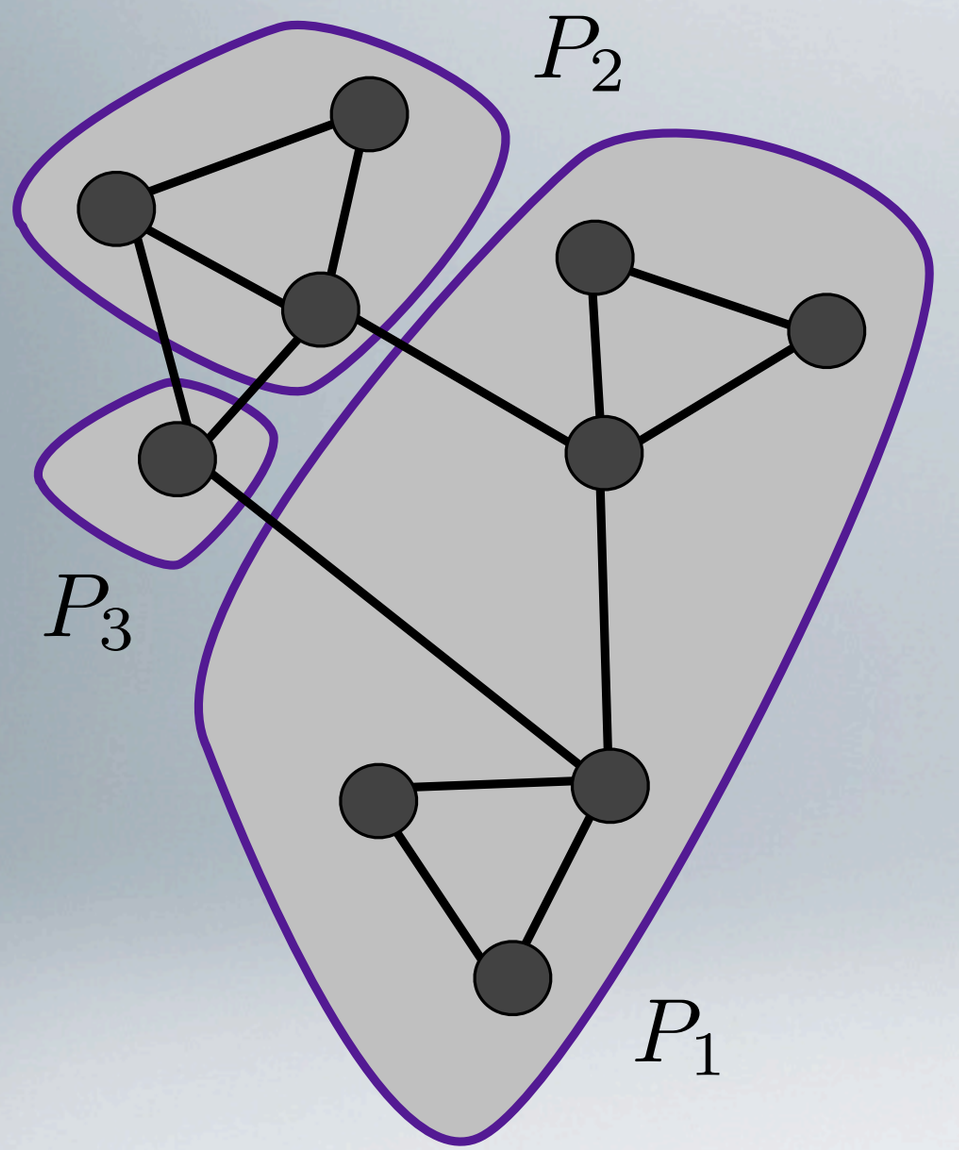


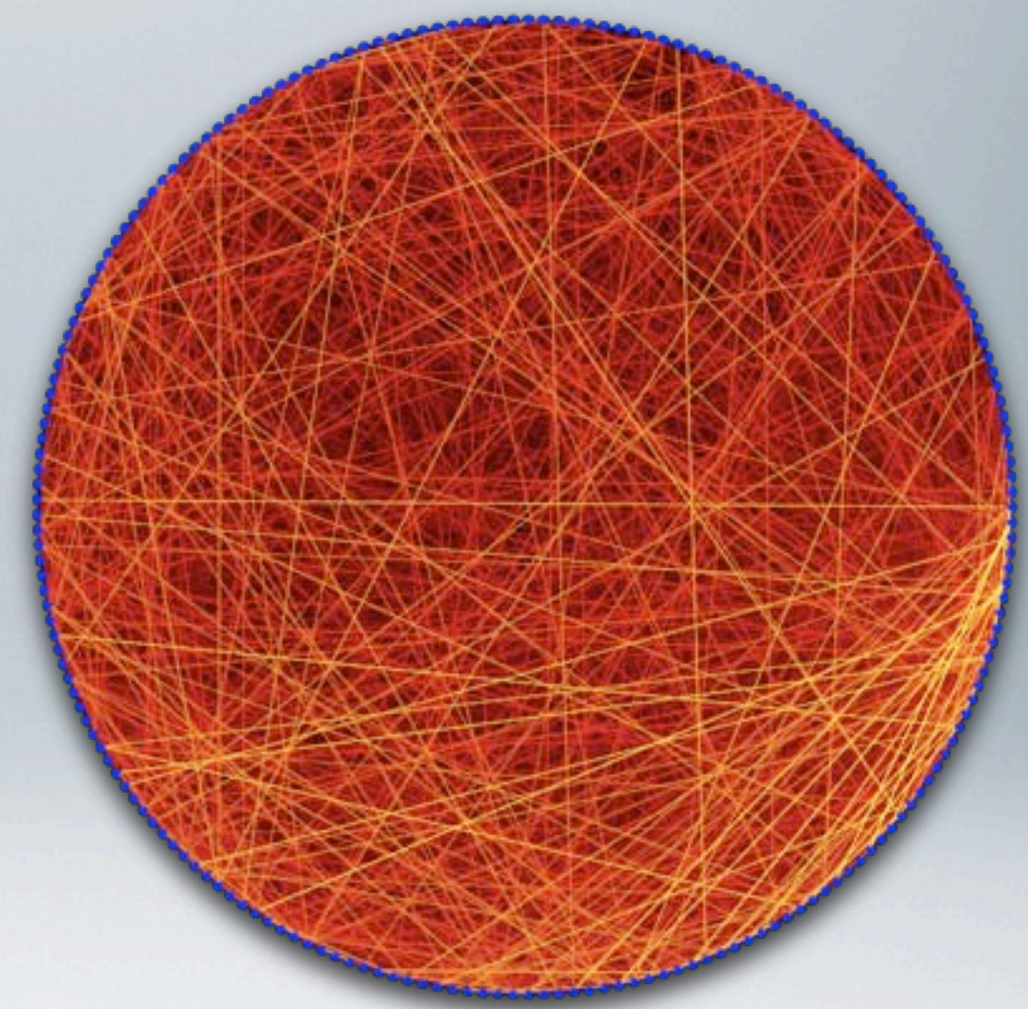
FINDING COMMUNITY STRUCTURE IN NETWORKS



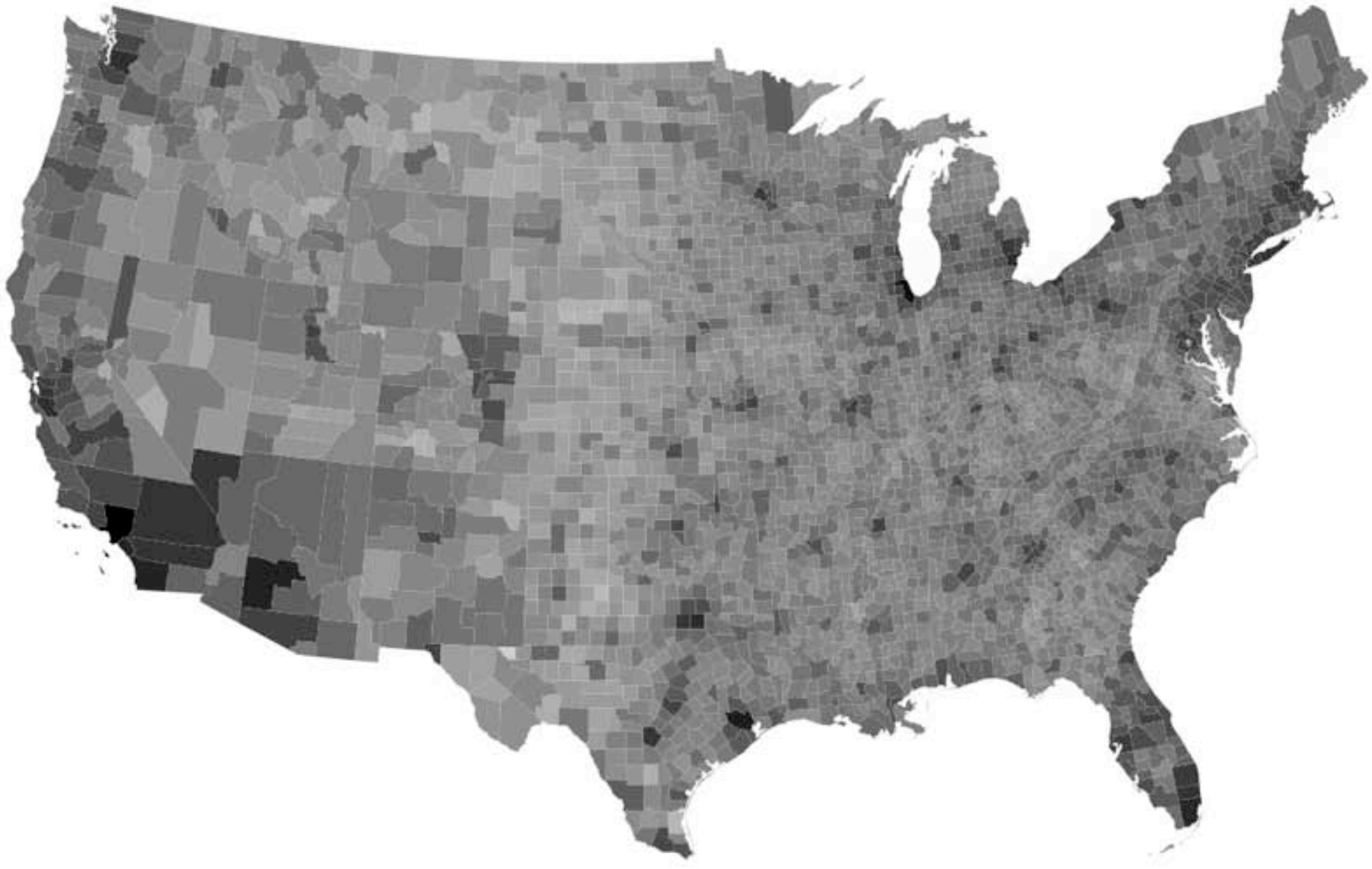


FINDING COMMUNITY STRUCTURE IN NETWORKS

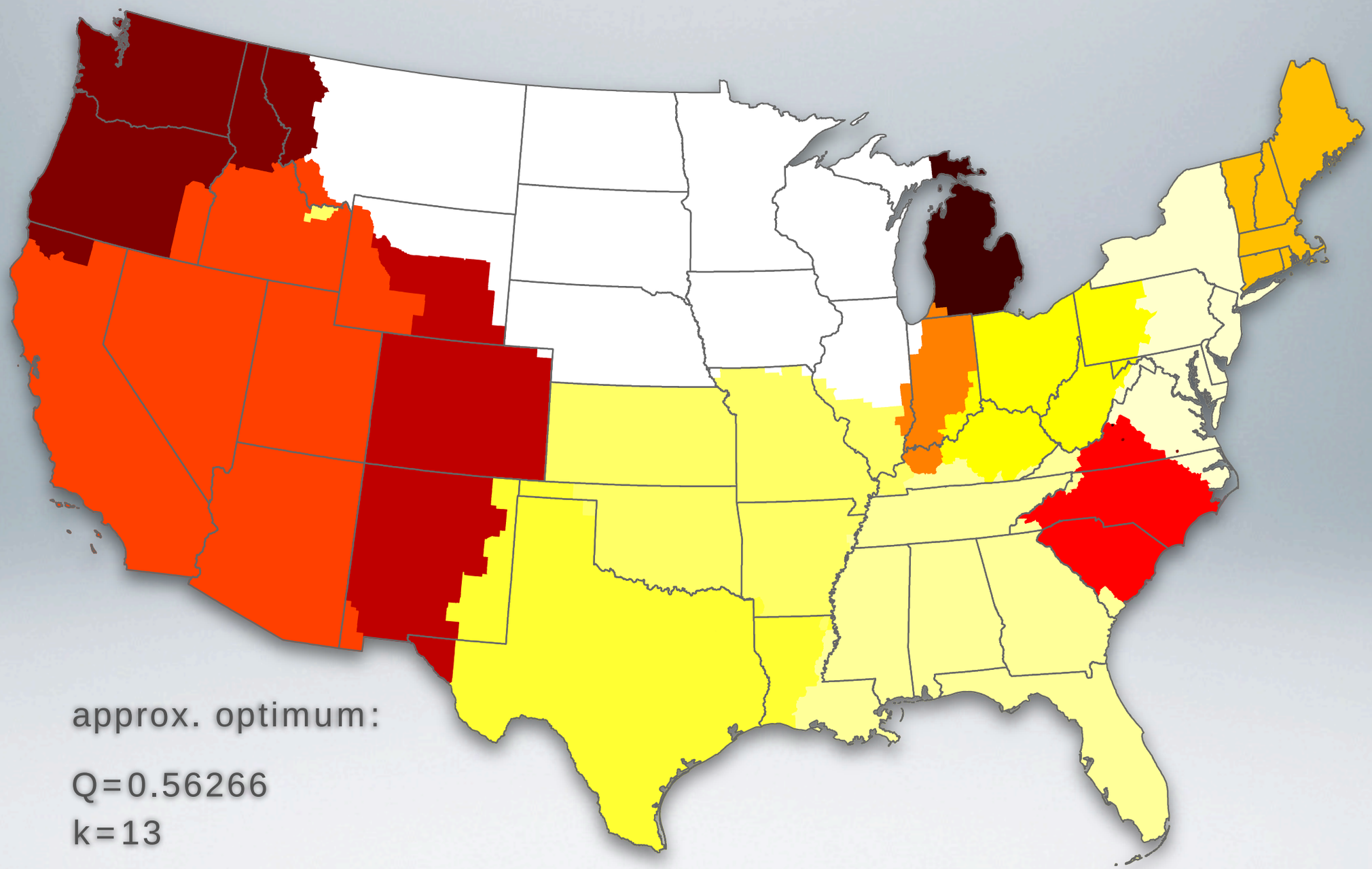




OPTIMIZING MODULARITY



KEY FEATURES

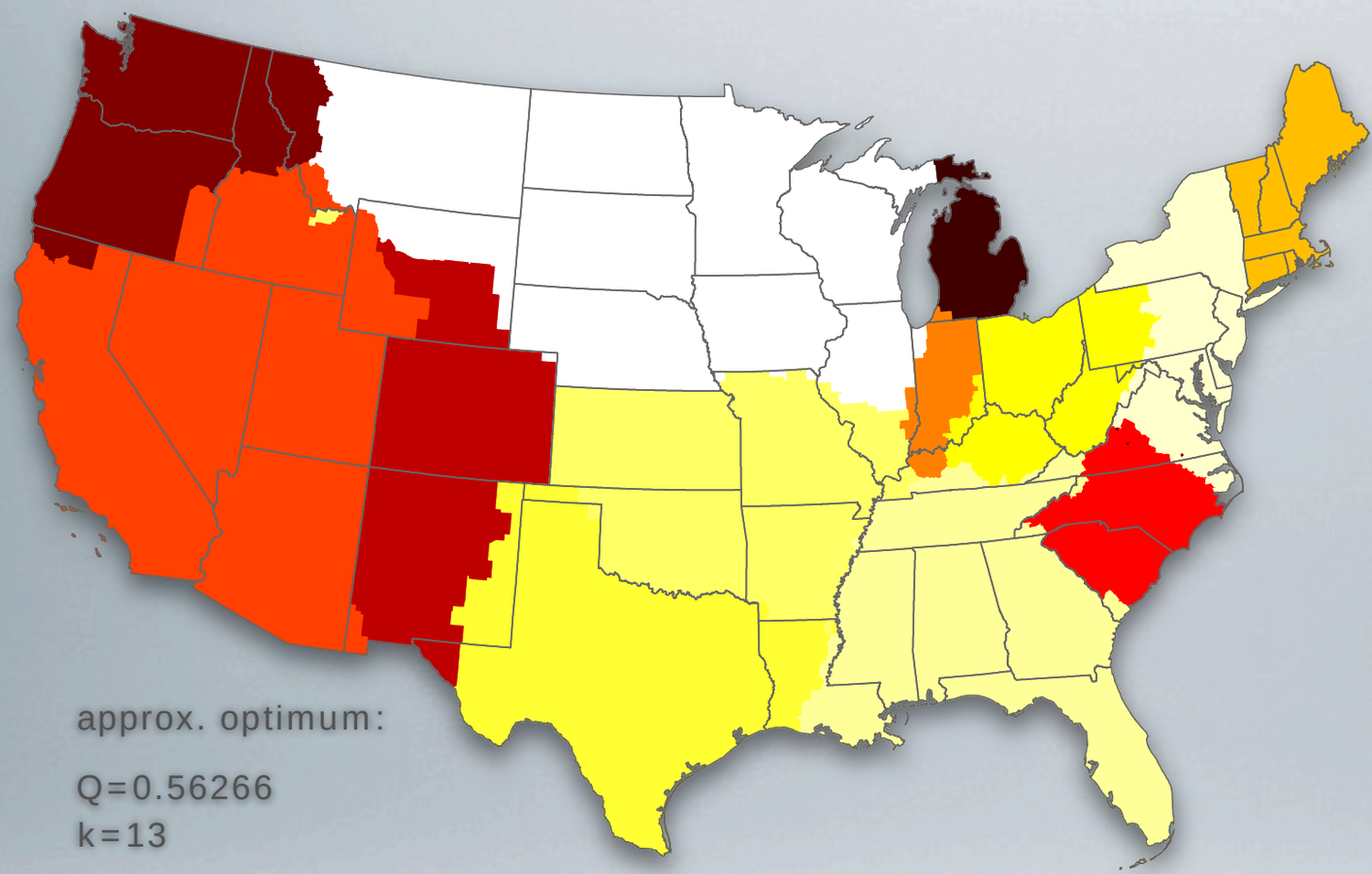


approx. optimum:

Q=0.56266
k=13

F. J. Theis, P. Gruber and M. Vingron, *Soft Clustering finds hard partitioning*, (2008, submitted)

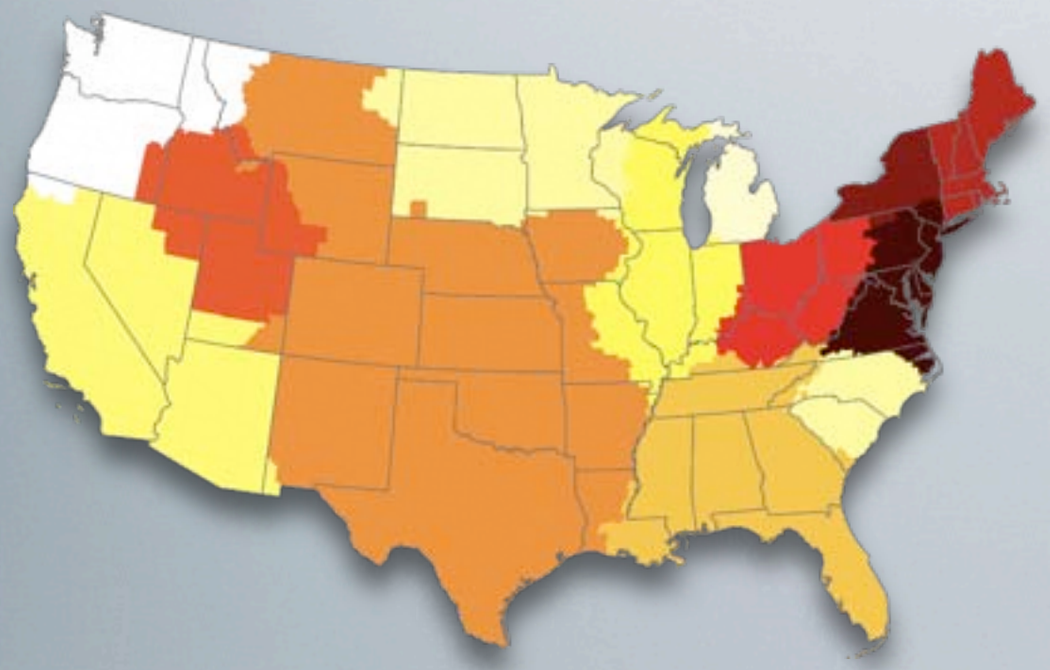
KEY FEATURES

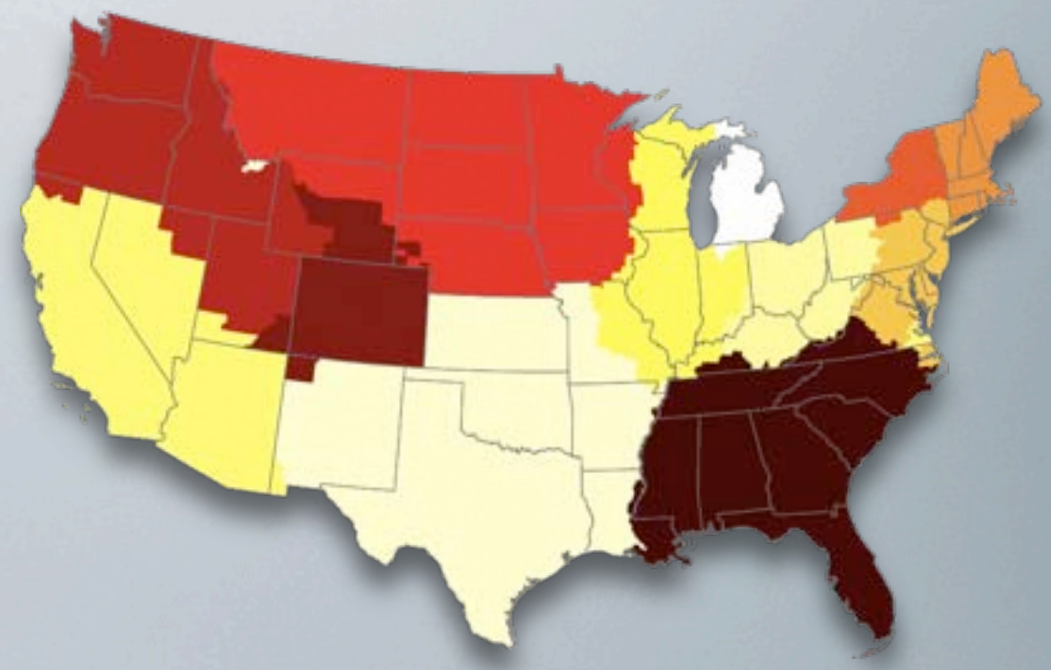


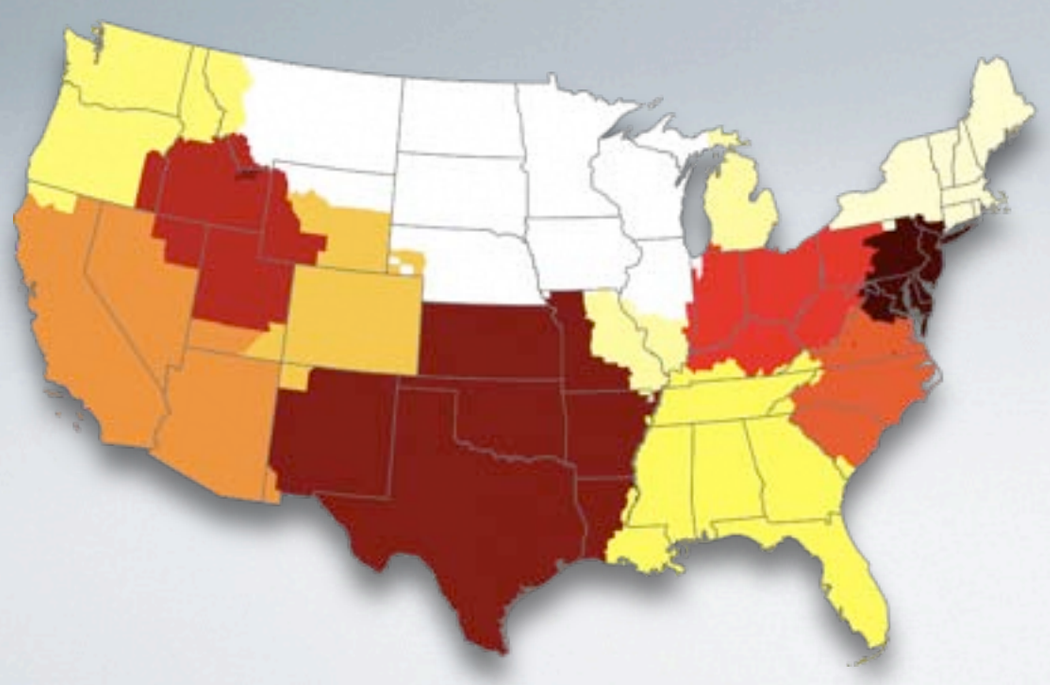
- global units are compact
- regional correlation with geographical features (rivers, mountains)
- regional correlation with state boundaries

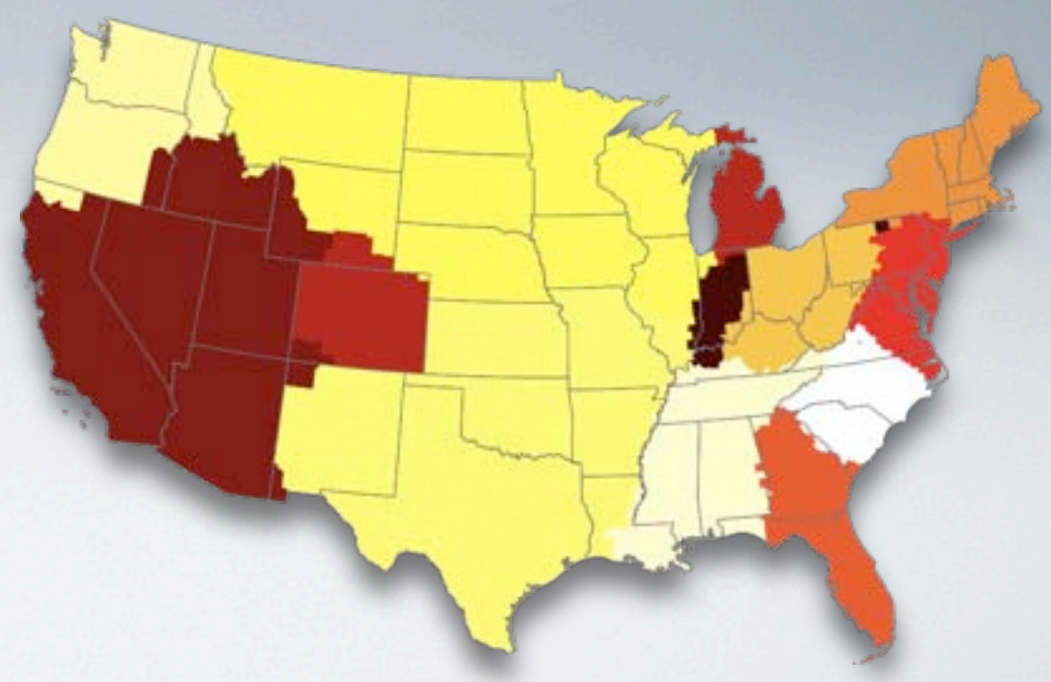
F. J. Theis, P. Gruber and M. Vingron, *Soft Clustering finds hard partitioning*, (2008, submitted)

SUPERPOSITION









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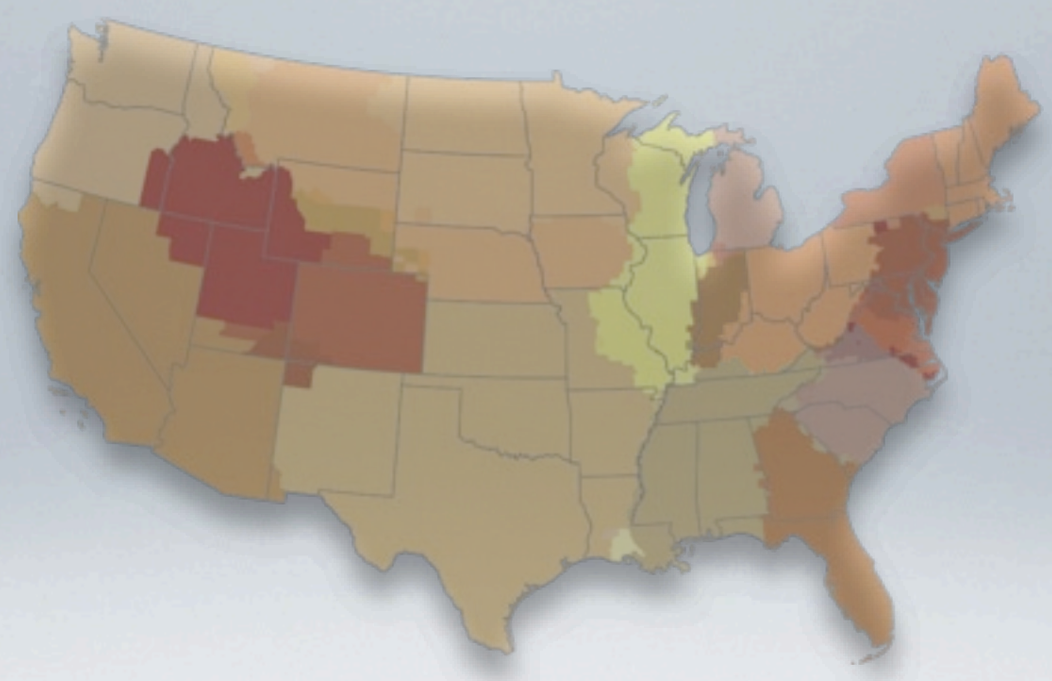
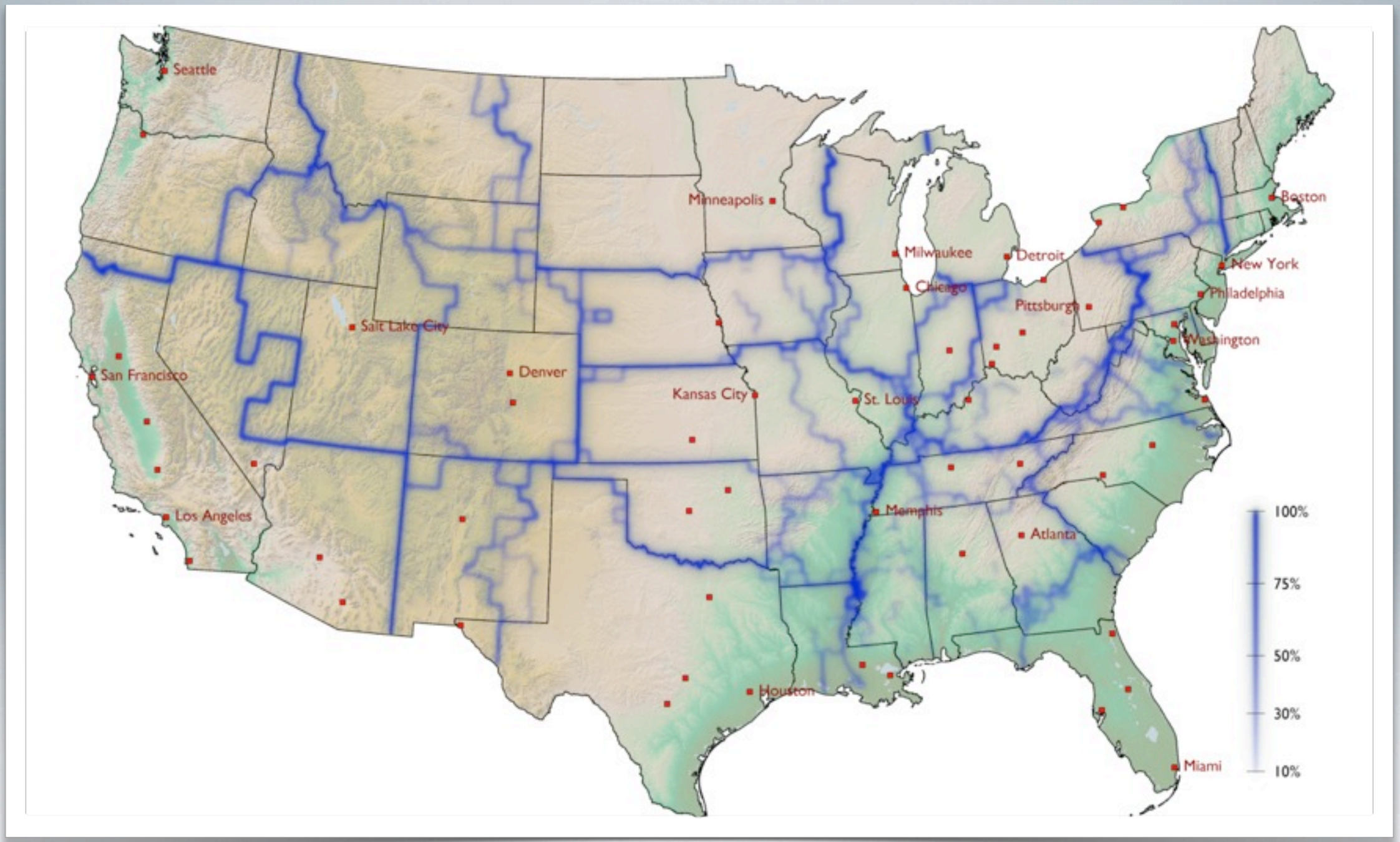


MOBILITY BORDERS



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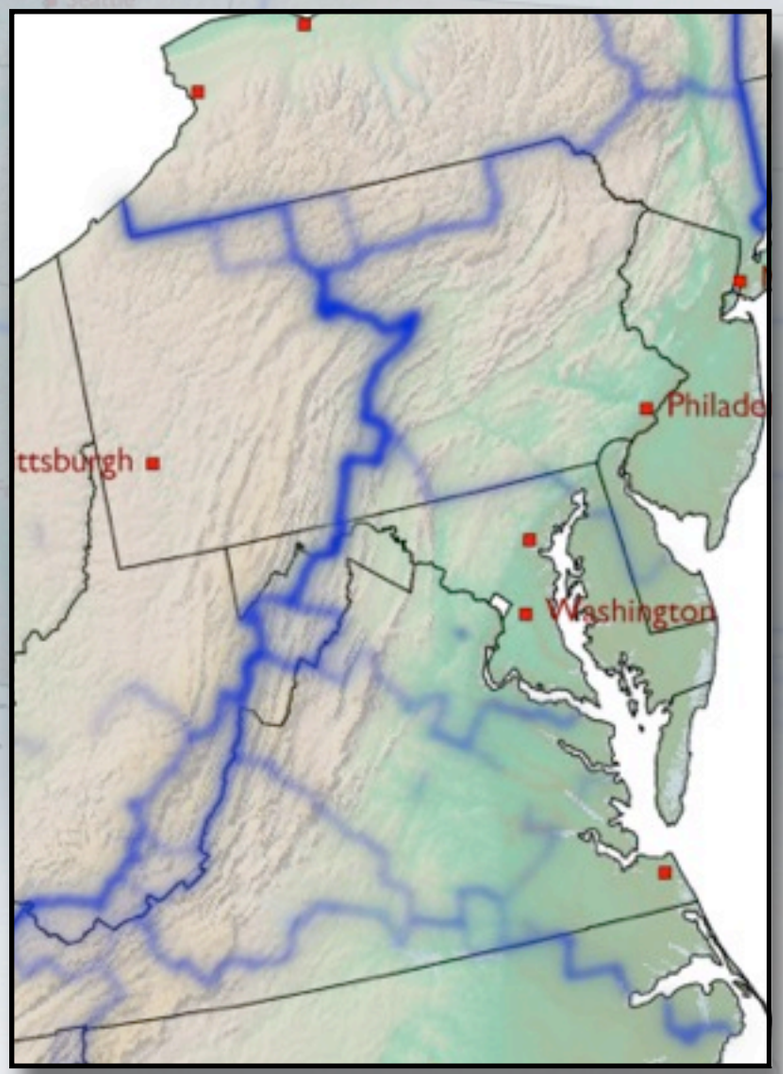

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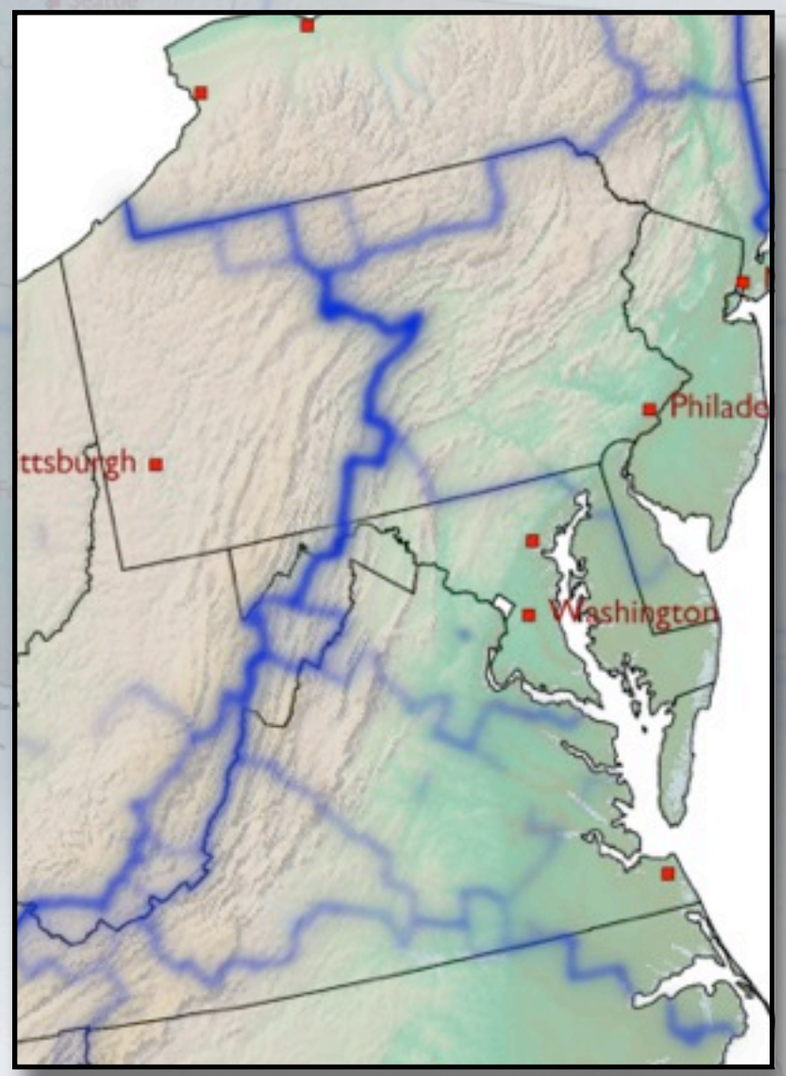

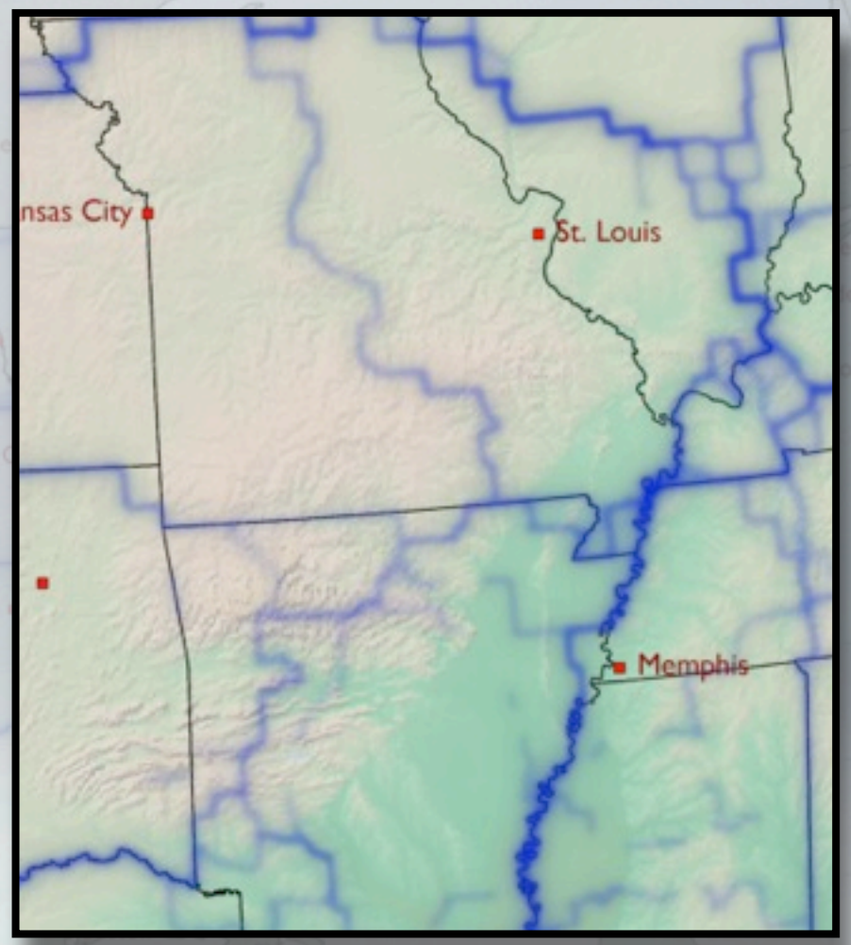

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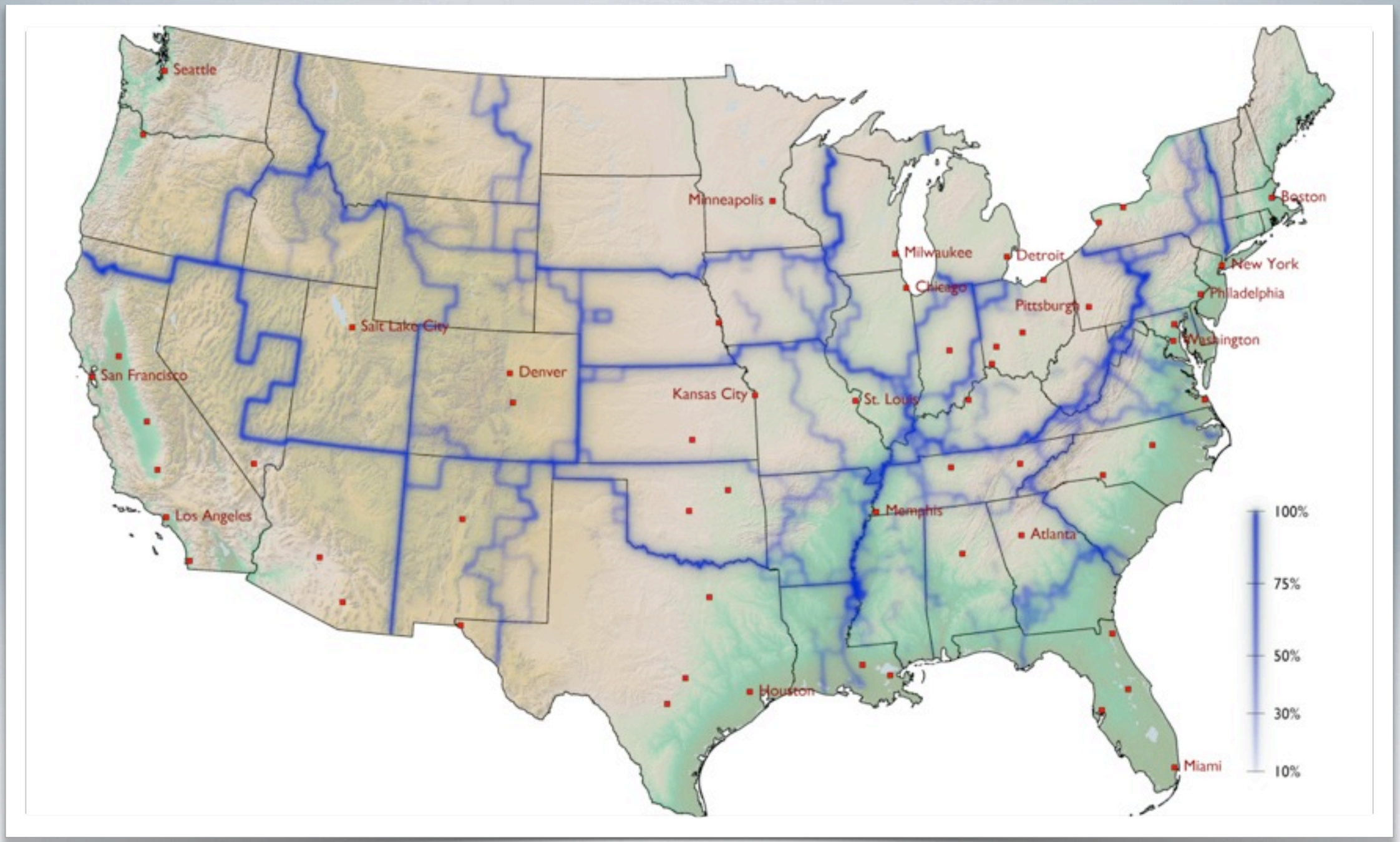

ACKNOWLEDGEMENTS

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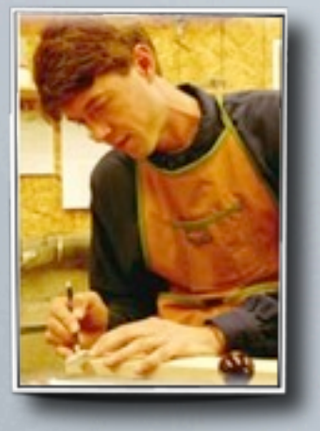

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Theo Geisel

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Olivia Woolley