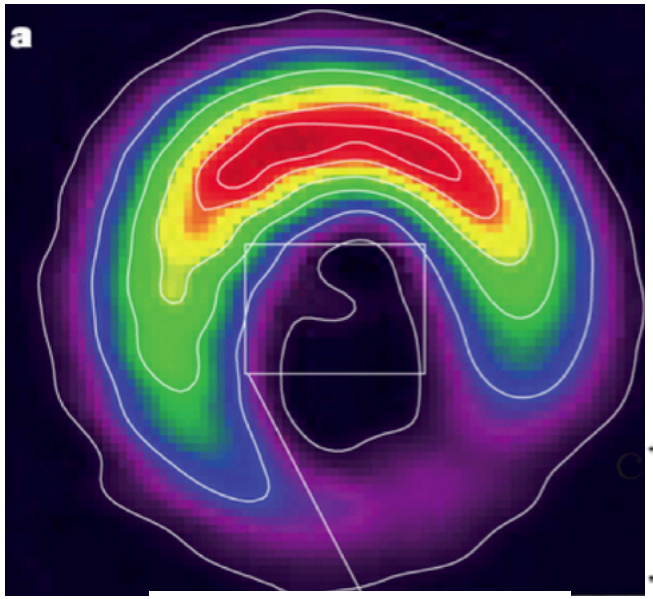
Vortices in Protoplanetary Disks

Hui Li (LANL)

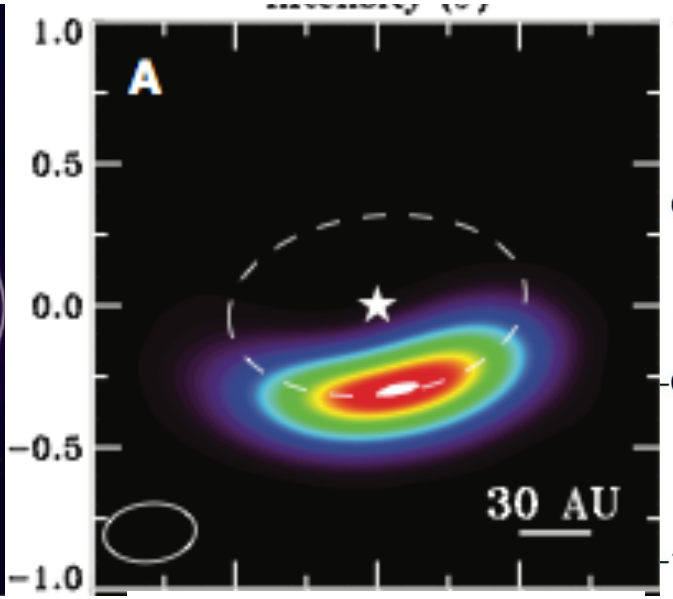
Collaborators:

Rossby wave instability: Lovelace, Finn, Colgate, …

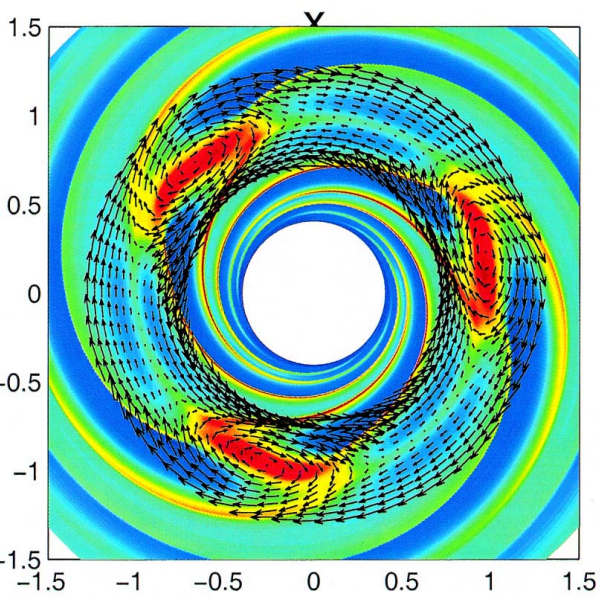
Protoplanetary disk: S. Li, W. Fu, S. Lubow, D. Lin, …



Casassus et al., Nature, 2013



van der Marel et al., Science, 2013



Li et al, ApJ, 2001

A Bit of History …

$$v_\varphi^2 = \frac{GM}{r} + \frac{r}{\Sigma}\frac{\partial p}{\partial r}$$

Keplerian velocity

Correction for pressure gradient

Stirling: Can we build paddle wheels in disks to transport angular momentum?

"Paddle Wheels" in Disks

Paper 1

ROSSBY WAVE INSTABILITY OF KEPLERIAN ACCRETION DISKS

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Received 1998 March 19; accepted 1998 October 15

Paper 2

ROSSBY WAVE INSTABILITY OF THIN ACCRETION DISKS. II. DETAILED LINEAR THEORY

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Received 1999 July 20; accepted 1999 December 3

Paper 3

ROSSBY WAVE INSTABILITY OF THIN ACCRETION DISKS. III. NONLINEAR SIMULATIONS

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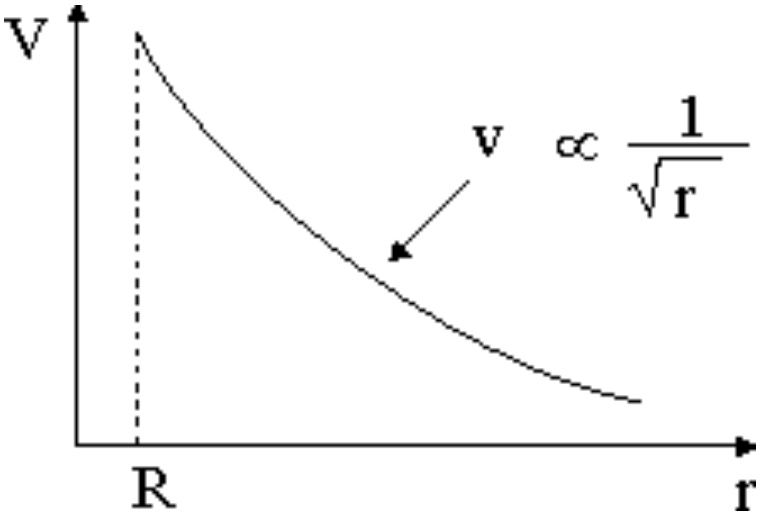
Received 2000 October 18; accepted 2000 December 19

Disk Basics

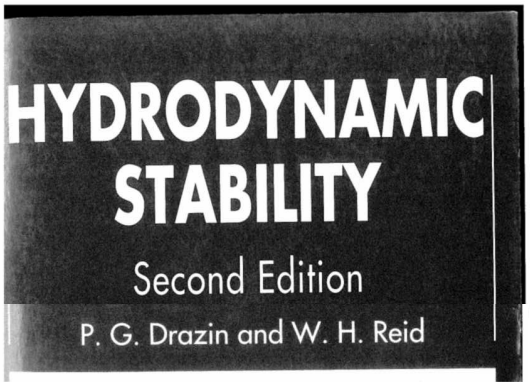


*** Modified Keplerian Rotation**

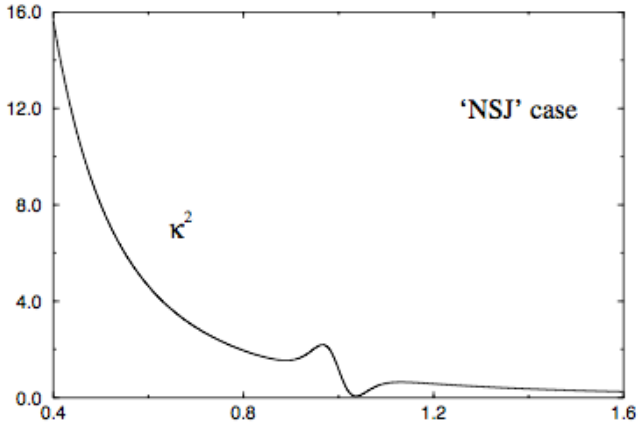
$$v_\varphi^2 = \frac{GM}{r} + \frac{r}{\Sigma}\frac{\partial p}{\partial r}$$

Rayleigh Criterion: $\kappa^2 \equiv \frac{1}{r^3}\frac{d(r^4\Omega^2)}{dr}$ > 0 i.e., stable

Solberg-Hoiland: $\kappa^2(r) + N^2(r) \geq 0,$ where $N^2 \equiv \frac{1}{\Sigma}\frac{dP}{dr}\left(\frac{1}{\Sigma}\frac{d\Sigma}{dr} - \frac{1}{\Gamma P}\frac{dP}{dr}\right)$



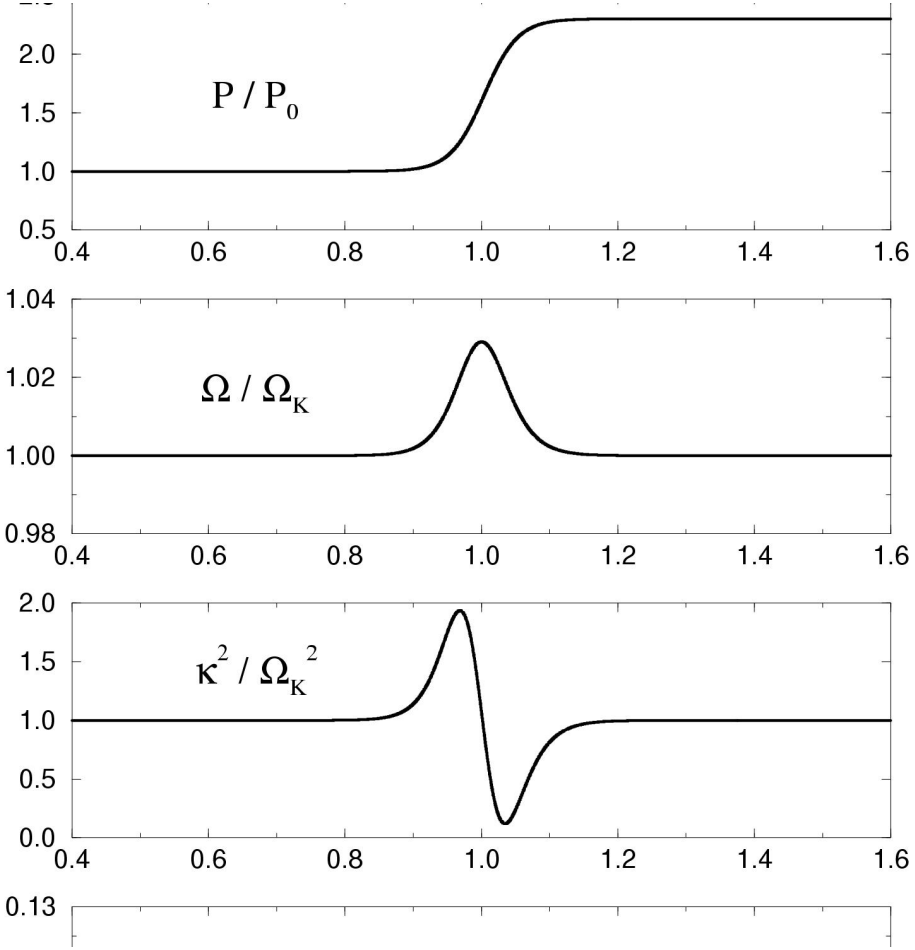
p81:
Rayleigh's inflexion -point theorem:
extreme in potential vorticity profile



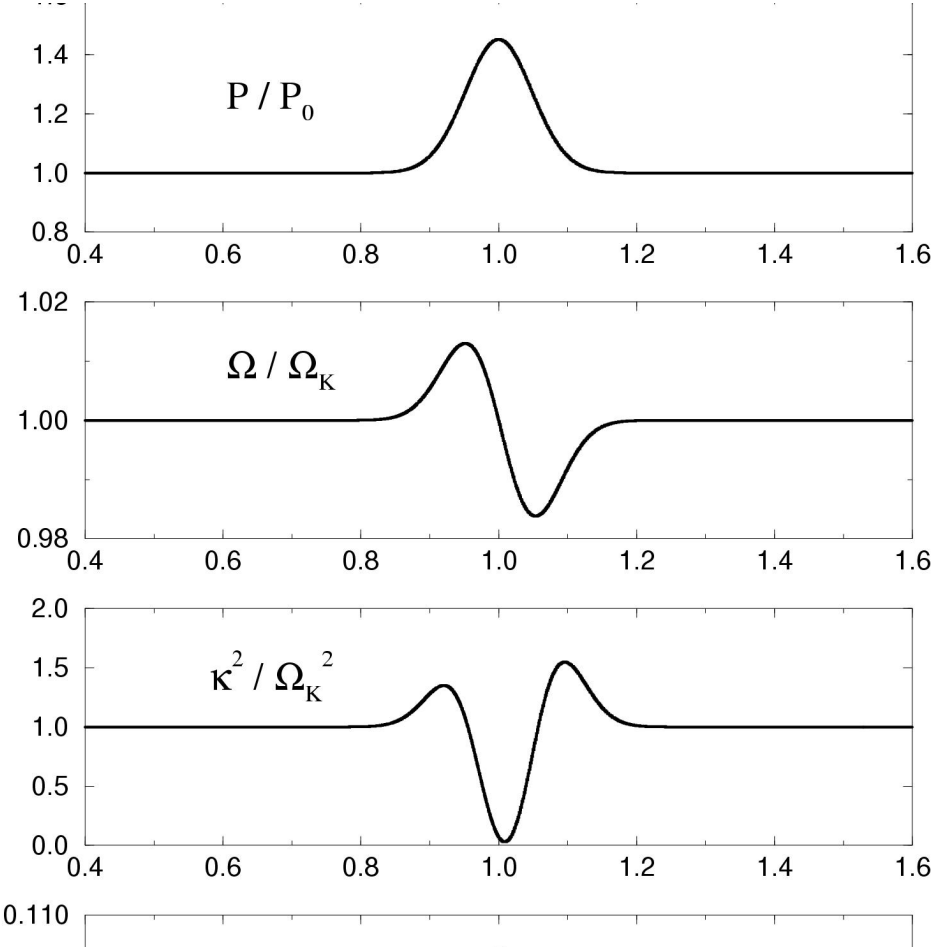
Q: What happens if there is strong density gradients in disks?

$$v_\varphi^2 = \frac{GM}{r} + \frac{r}{\Sigma}\frac{\partial p}{\partial r}$$

Density/Pressure step



Density/Pressure bump



Papaloizou & Pringle Instability ('84, 85)

The dynamical stability of differentially rotating discs with constant specific angular momentum

The dynamical stability of differentially rotating discs – II

J. C. B. Papaloizou *Theoretical Astronomy Unit, School of Mathematical Sciences, Queen Mary College, London E1 4NS*

J. E. Pringle *Institute of Astronomy, Madingley Road, Cambridge CB3 0HA*

See also
Blases'85; Goldreich+'86; Kato'87; Narayan+'87;
Hawley'91, etc.

Linear Theory Derivations *(Lovelace et al. 1999; Li et al. 2000)*

Linearized Eqs:

$$\frac{D\tilde{\Sigma}}{Dt} + \tilde{\Sigma}\nabla\cdot\tilde{\boldsymbol{v}} = 0\,,$$

$$\frac{D\tilde{\boldsymbol{v}}}{Dt} = -\frac{1}{\tilde{\Sigma}}\nabla\tilde{P} - \nabla\Phi\,,$$

$$\frac{D}{Dt}\left(\frac{\tilde{P}}{\tilde{\Sigma}^{\Gamma}}\right) = 0\,,$$

1) Using variable: $\Psi \equiv \delta P/\Sigma$

$$\frac{1}{r}\left(\frac{r\mathscr{F}}{\Omega}\Psi'\right)' - \frac{k_\phi^2\mathscr{F}}{\Omega}\Psi = \frac{\Sigma\Psi}{c_s^2} + \frac{2k_\phi\mathscr{F}'}{\Delta\omega}\Psi + \left[\frac{\mathscr{F}}{\Omega L_s^2} + \frac{1}{r}\left(\frac{r\mathscr{F}}{\Omega L_s}\right)' + \frac{4k_\phi\mathscr{F}}{\Delta\omega L_s} + \frac{k_\phi^2 c_s^2\mathscr{F}}{\Delta\omega^2\Omega L_s L_p}\right]\Psi$$

2) Key eq.

$$\Psi'' + B(r)\Psi' + C(r)\Psi = 0$$

$$B(r) = \frac{1}{r} + \frac{\mathscr{F}'}{\mathscr{F}} - \frac{\Omega'}{\Omega}\,,$$

$$C(r) = -c_1 - c_2\,,$$

with

$$c_1 = k_\phi^2 + \frac{\kappa^2 - \Delta\omega^2}{c_s^2} + 2k_\phi\frac{\Omega}{\Delta\omega}\frac{\mathscr{F}'}{\mathscr{F}}\,,$$

$$c_2 = \frac{1 - L_s'}{L_s^2} + \frac{B(r) + 4k_\phi\Omega/\Delta\omega}{L_s} + \frac{k_\phi^2 c_s^2/\Delta\omega^2 - 1}{L_s L_p}$$

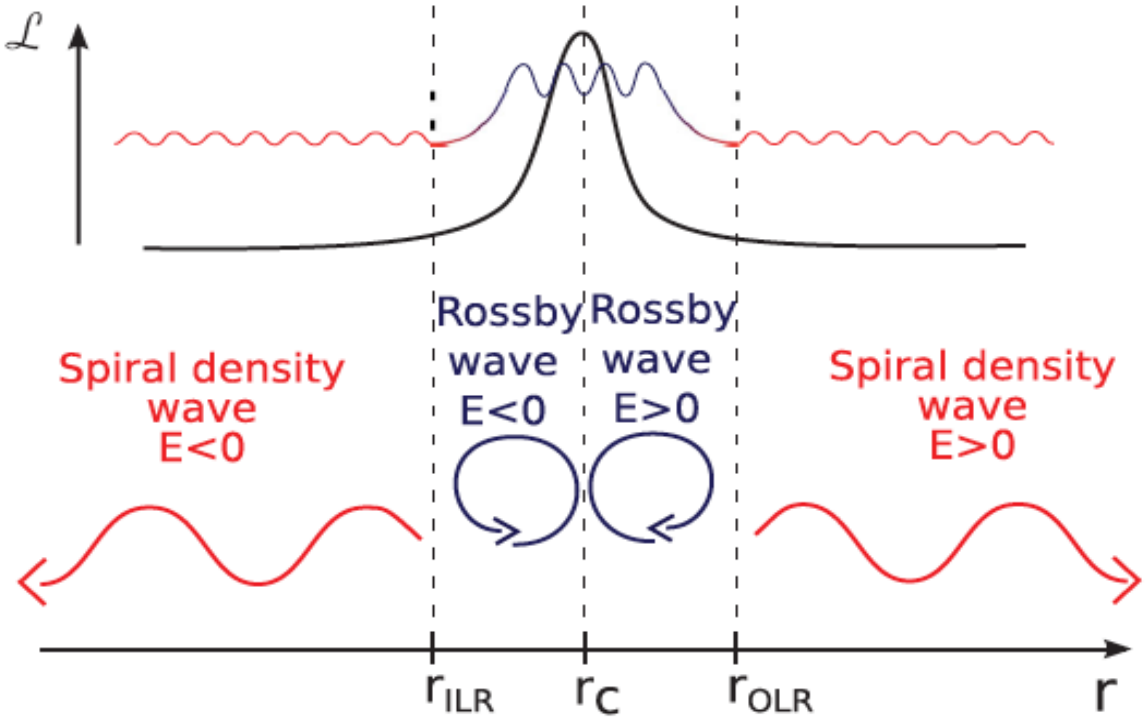
Linear Theory Derivations *(Lovelace et al. 1999; Li et al. 2000)*

3) Potential Vorticity :

$$\mathscr{F}(r) \equiv \frac{\Sigma\Omega}{\kappa^2 - \Delta\omega^2 - c_s^2/(L_s L_p)} \qquad \mathcal{L} = \frac{\Sigma\Omega}{\kappa^2}(p\Sigma^{-\gamma})^{2/\gamma}$$

4) Rossby wave dispersion:

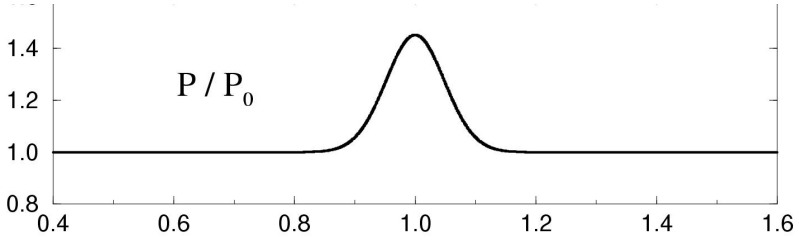
$$\Delta\omega = -\frac{k_\phi c_s^2/\Omega}{1 + k^2 h^2}\left[(\ln \mathscr{L})' \pm \sqrt{[(\ln \mathscr{L})']^2 - \frac{1 + k^2 h^2}{L_s L_p}}\right],$$

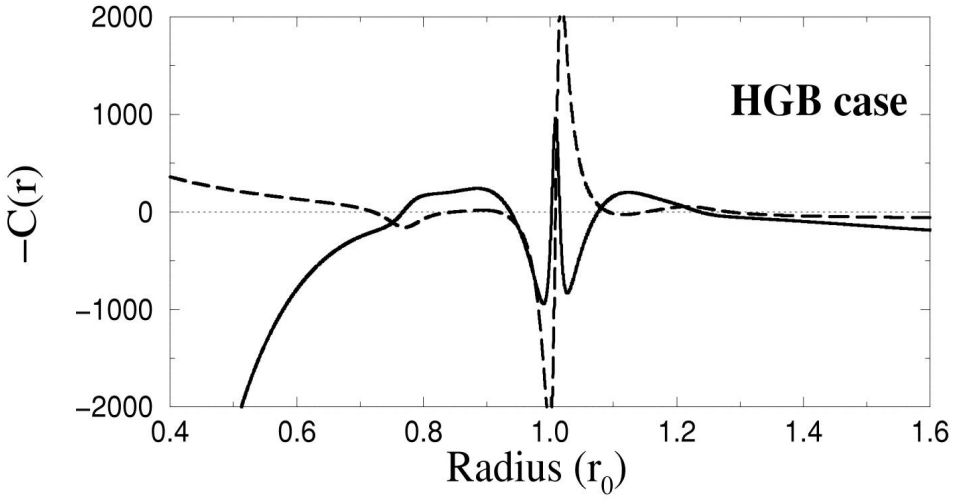


(See also Tagger'01, Tsang & Lai'08, Meheut et al.'13, etc.)

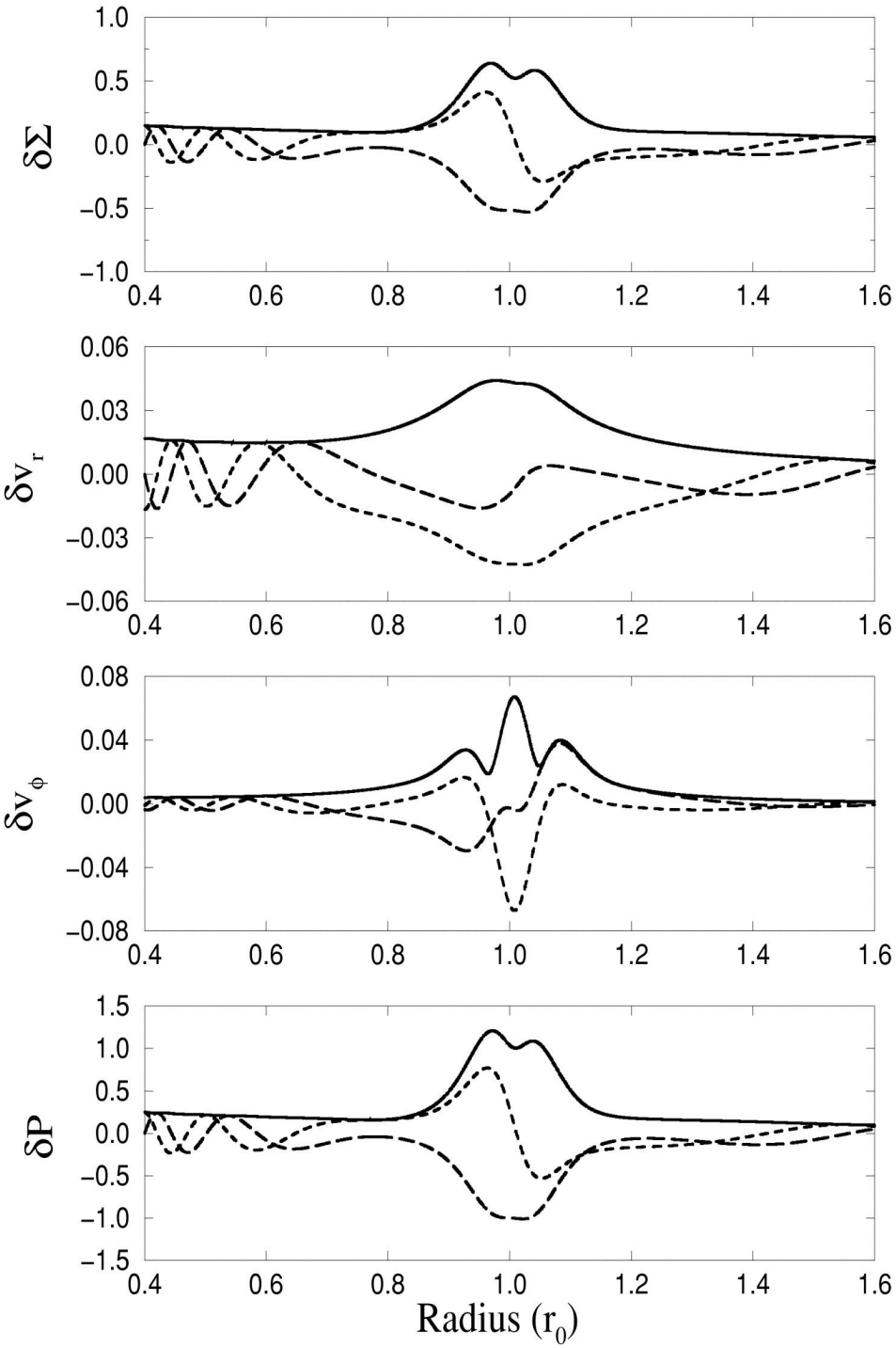
Rossby Wave Instability

$$\Psi'' + B(r)\Psi' + C(r)\Psi = 0$$

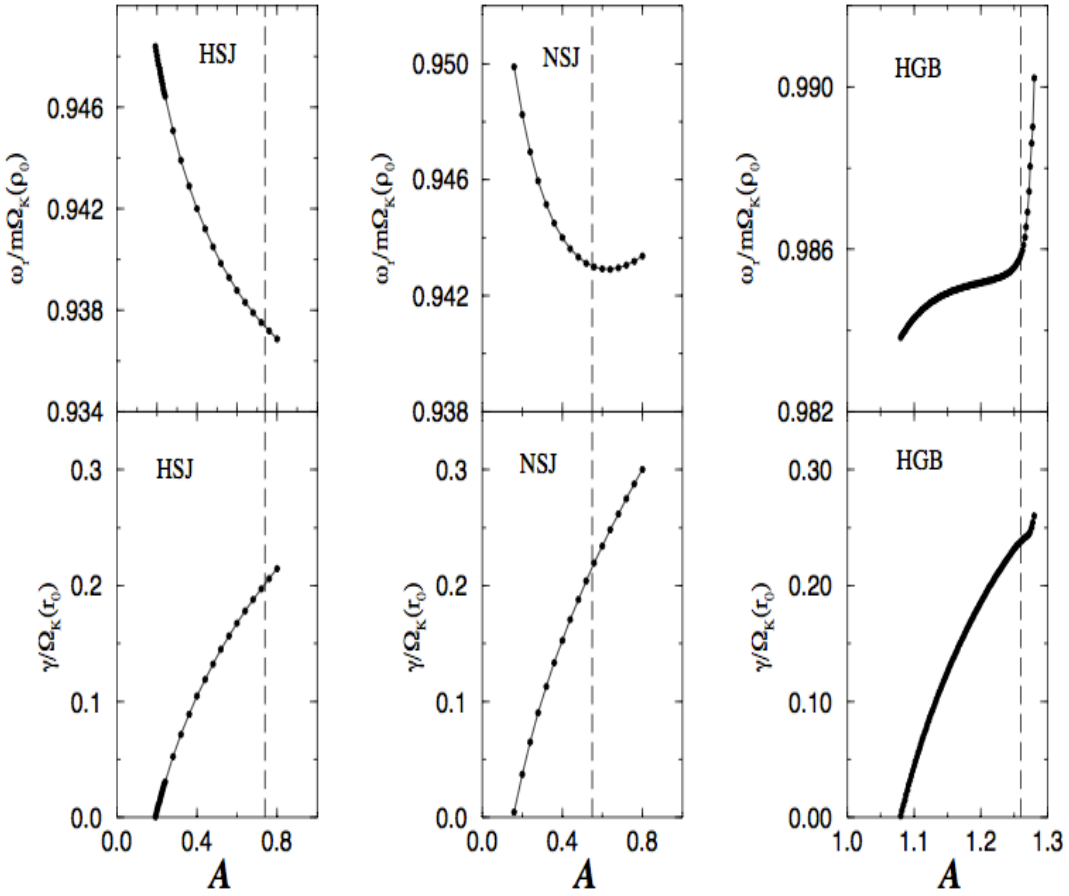


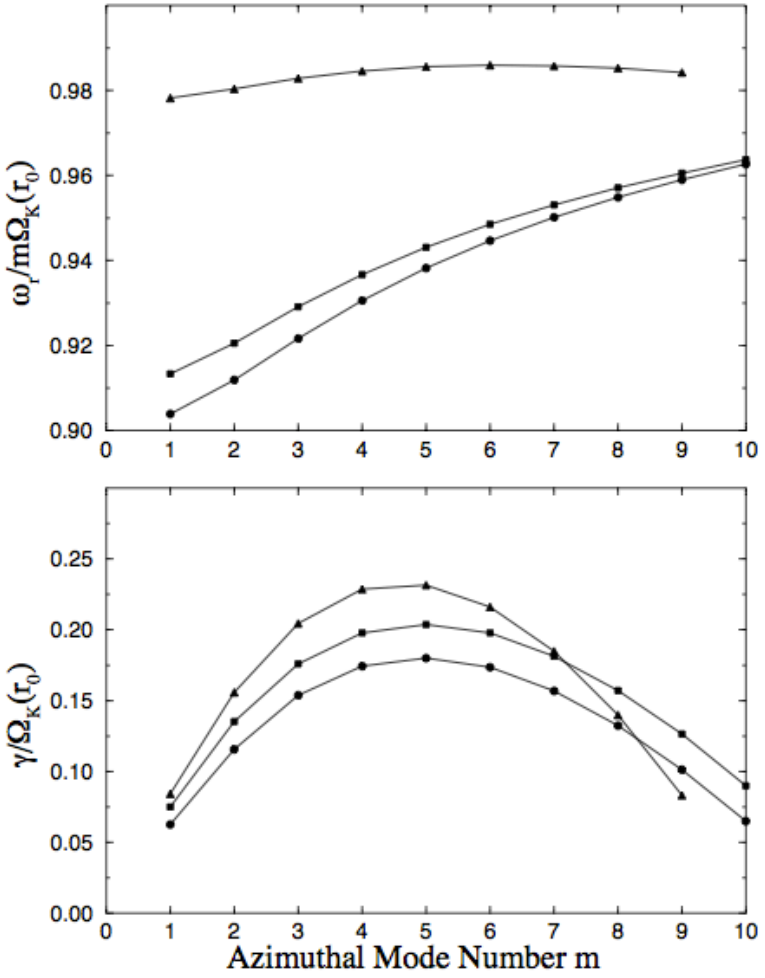


Li et al. 2000



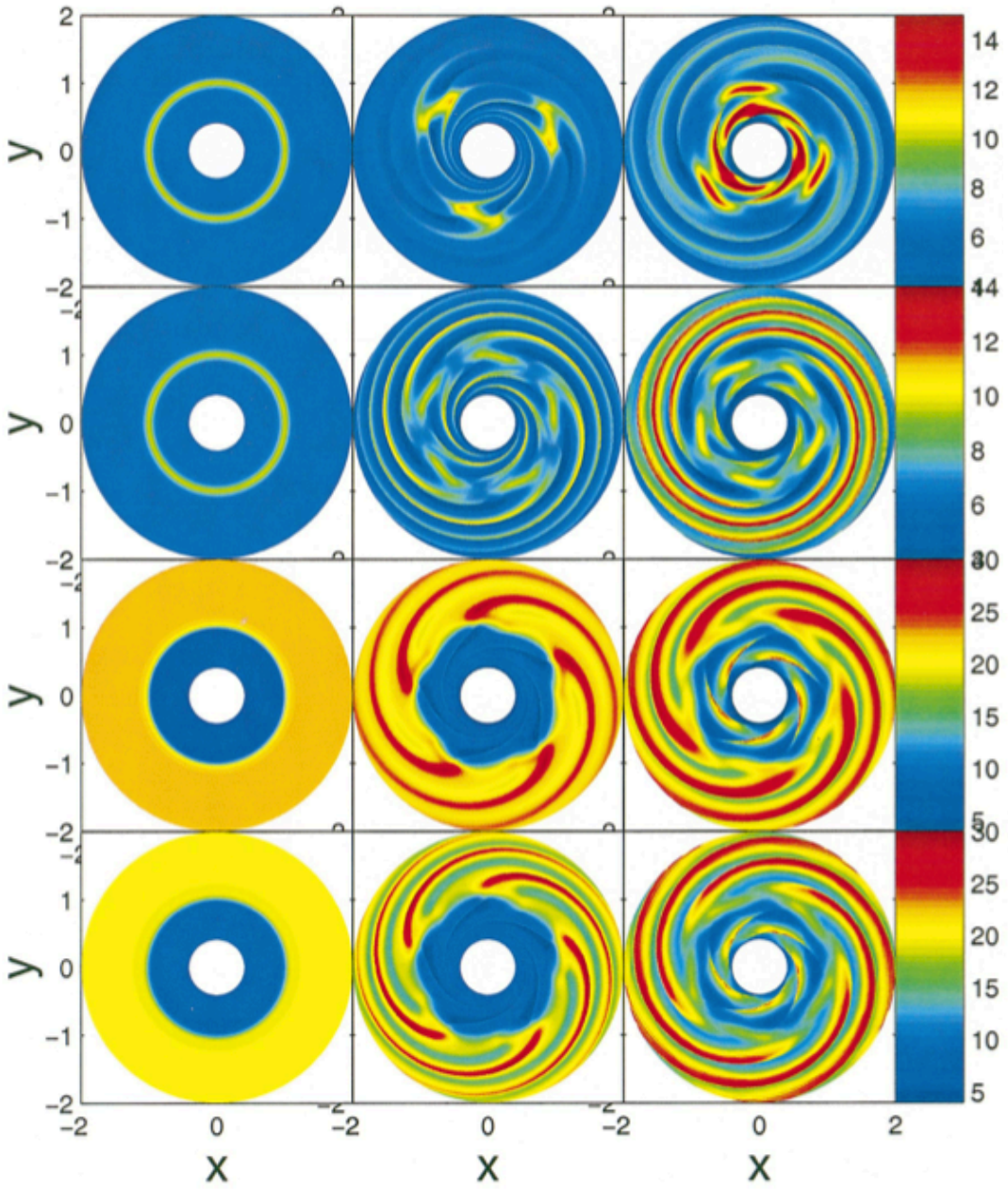
Rossby Wave Instability

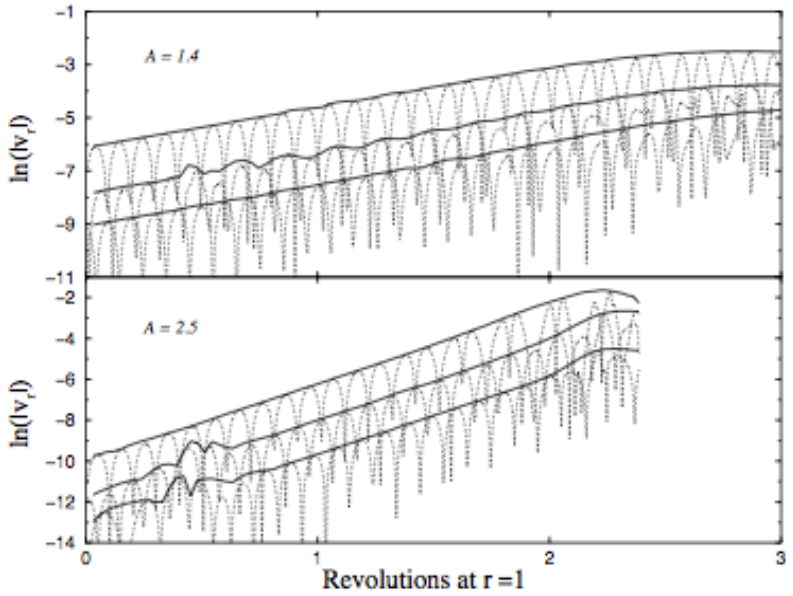




Li et al. 2000

Rossby Wave Instability -- Nonlinear Evolution

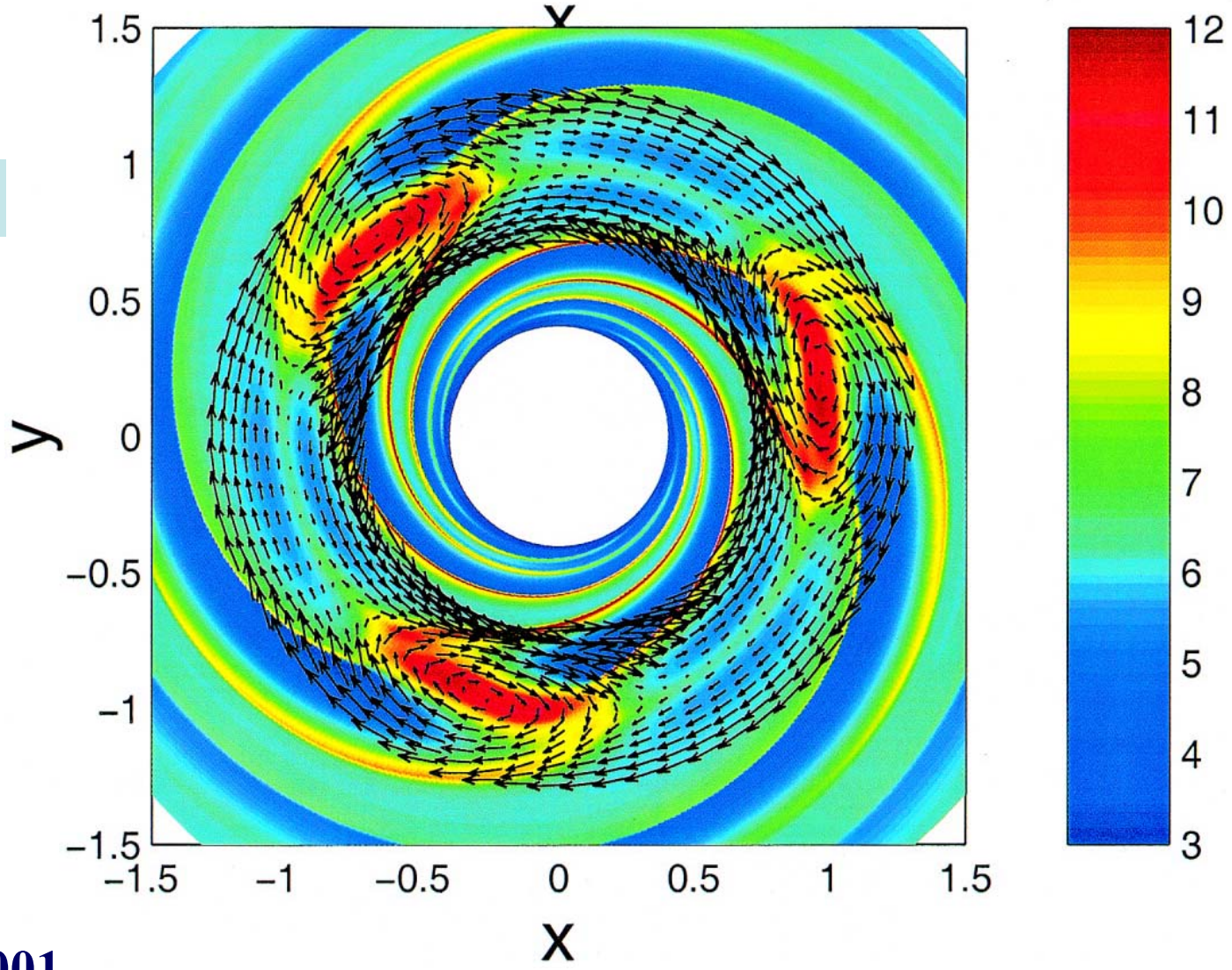




Li et al. 2001

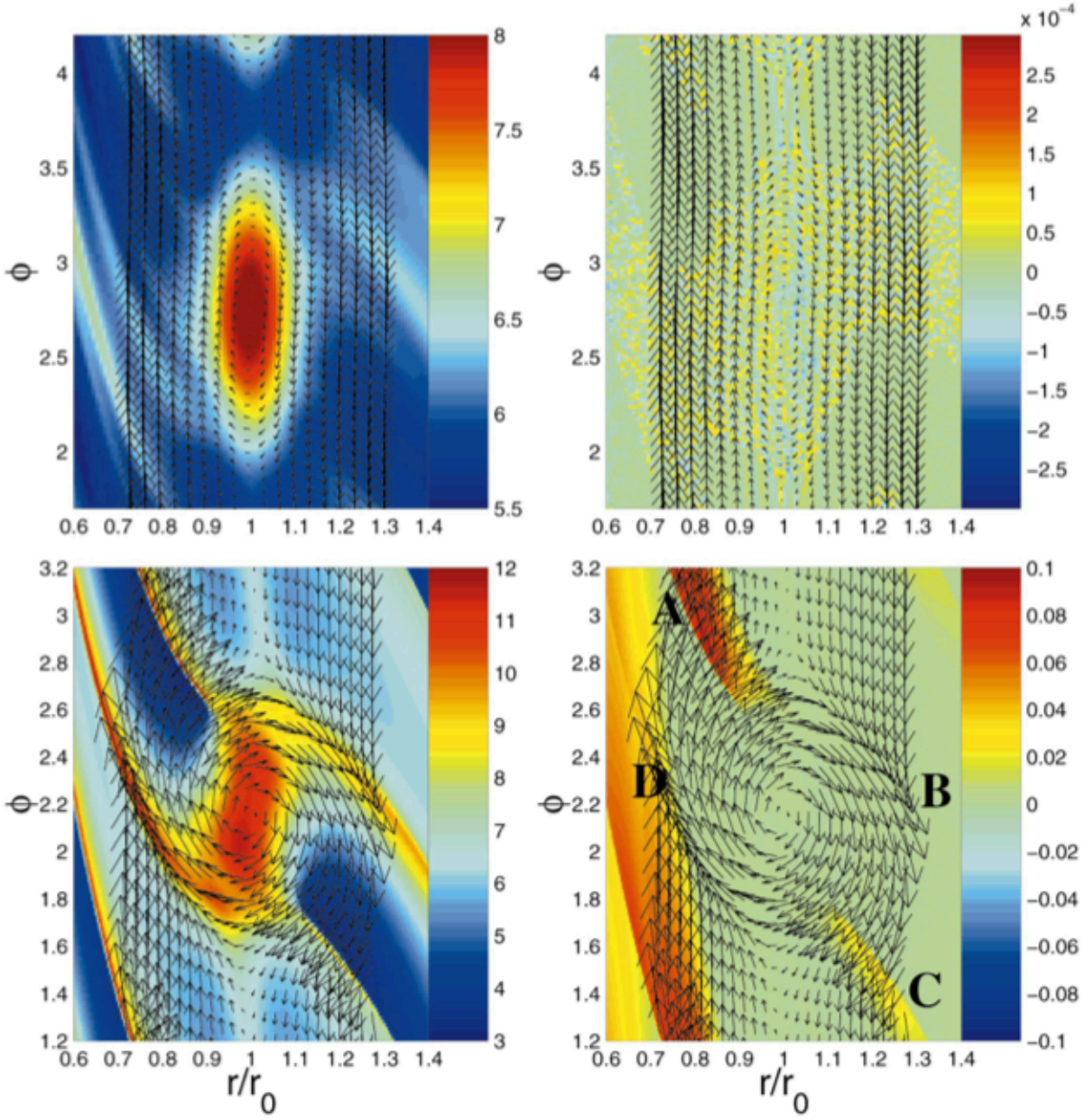
Rossby Vortices

Pressure



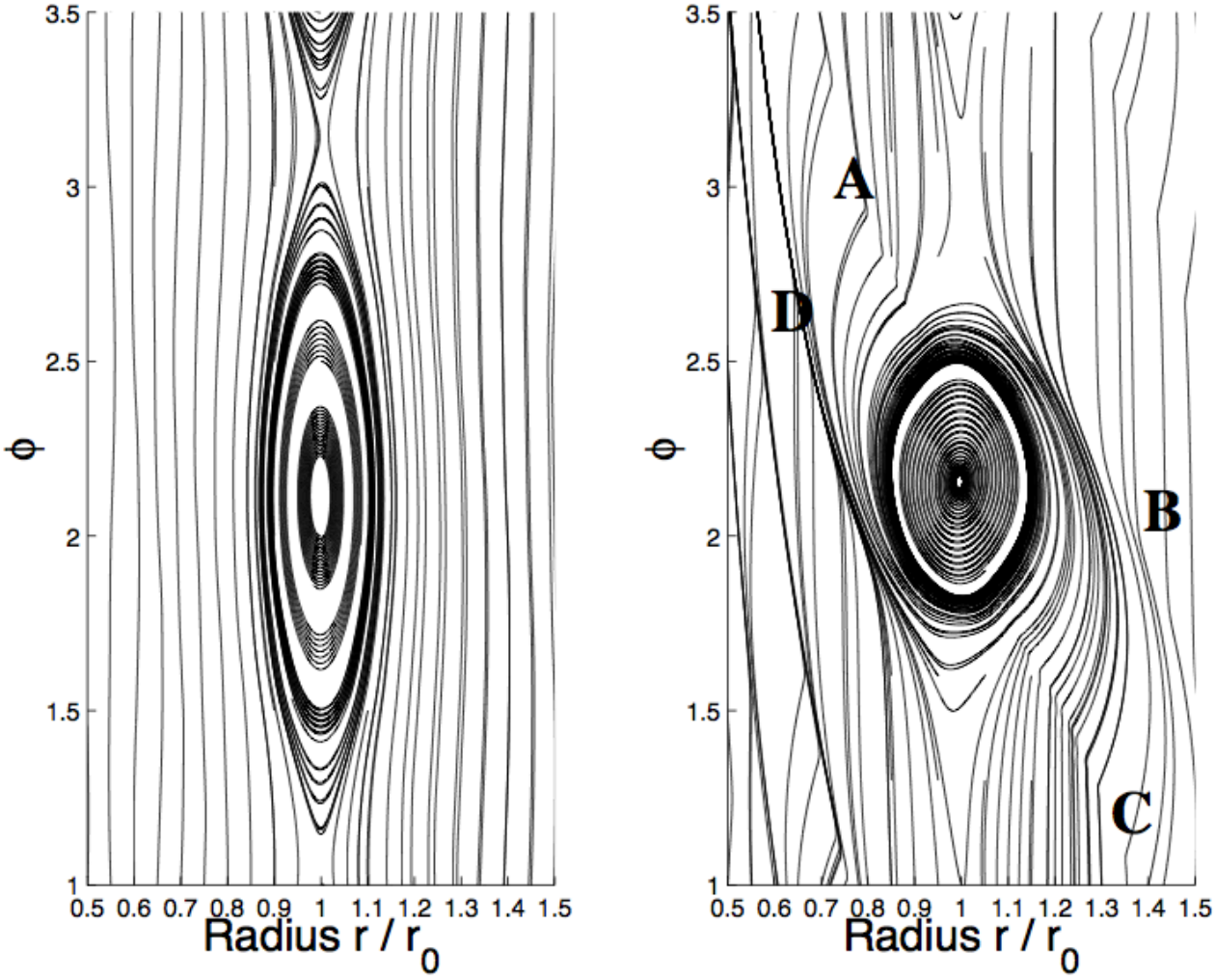
Li et al. 2001

Saturation

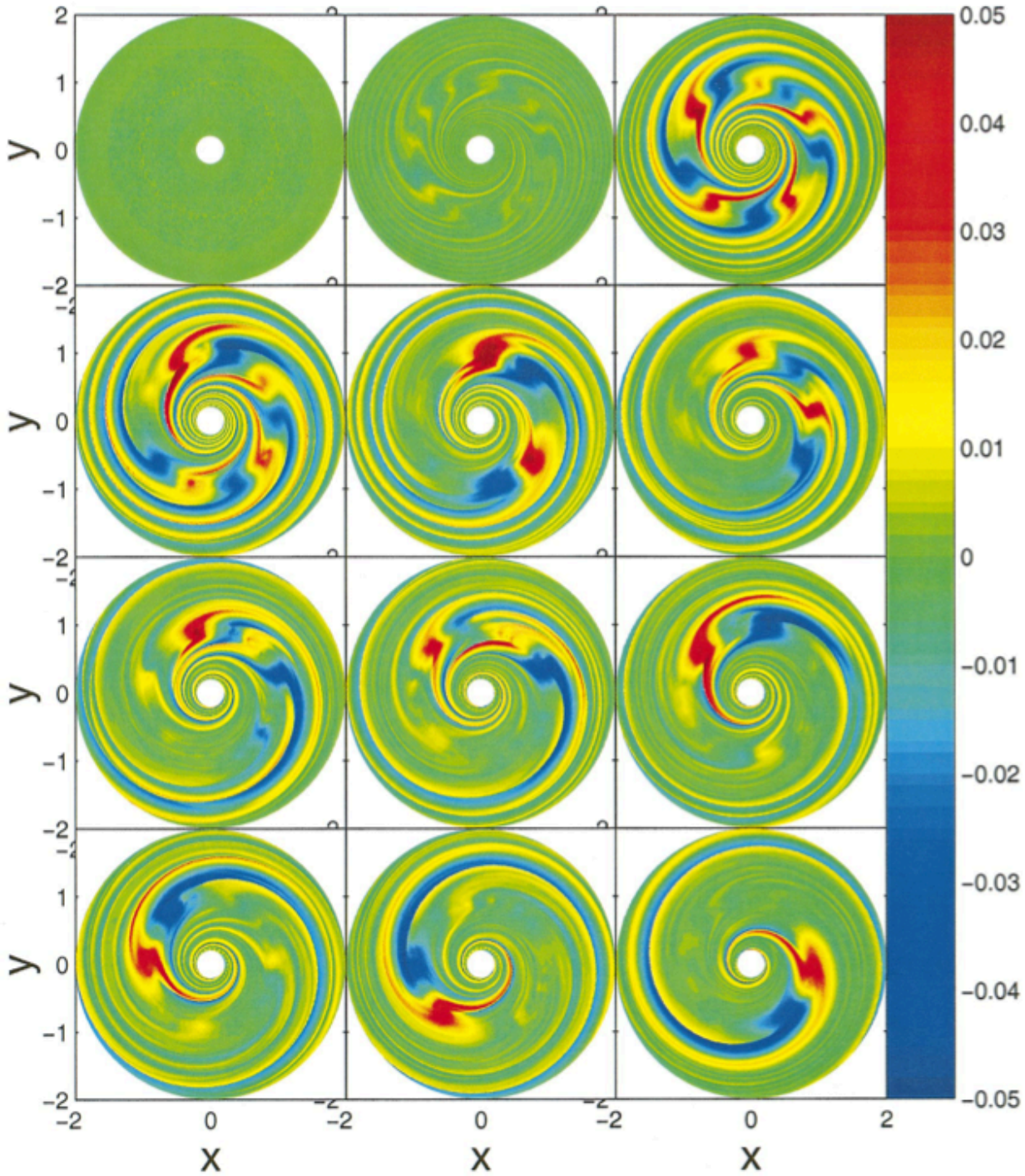


Li et al.'01

Saturation



Li et al.'01



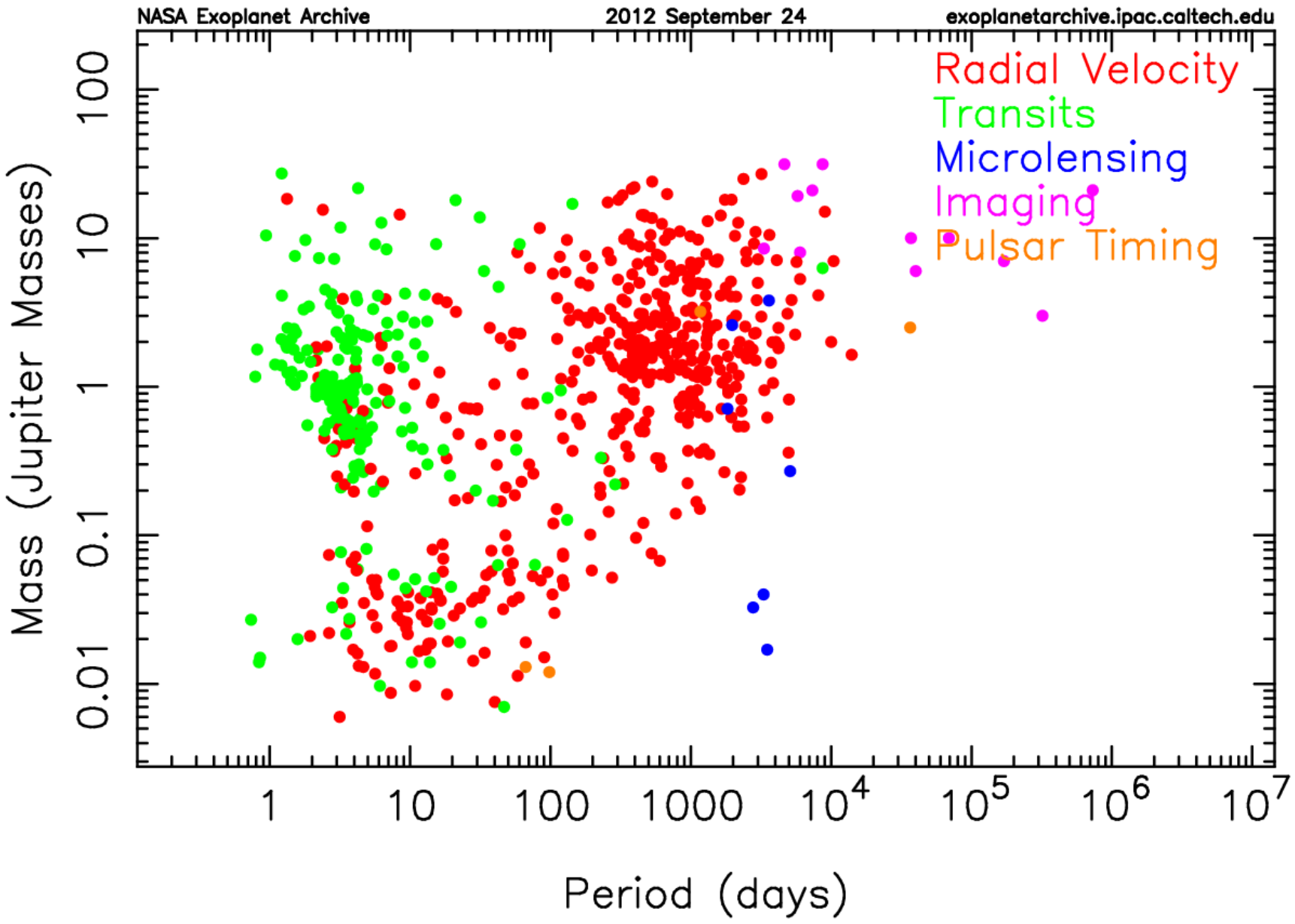
Merging and Migration of vortices

Work on RWI mostly went un-noticed (except a few groups in Europe) …

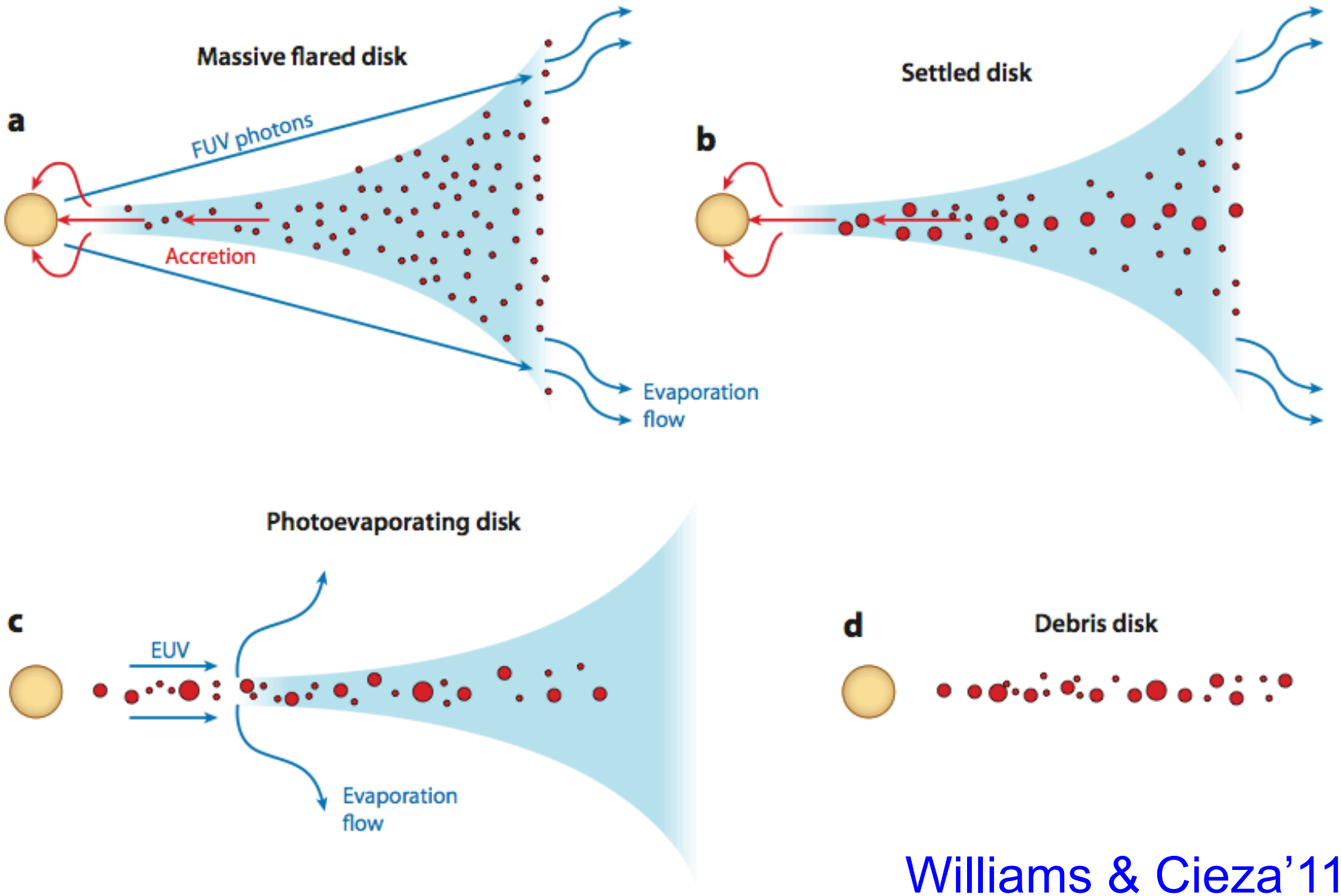
(Tagger+, Varniere+, Barge+, Meheut+, Lesur+, Lai+, Lin+, …)

But in the mean time …

Observations of Exoplanets

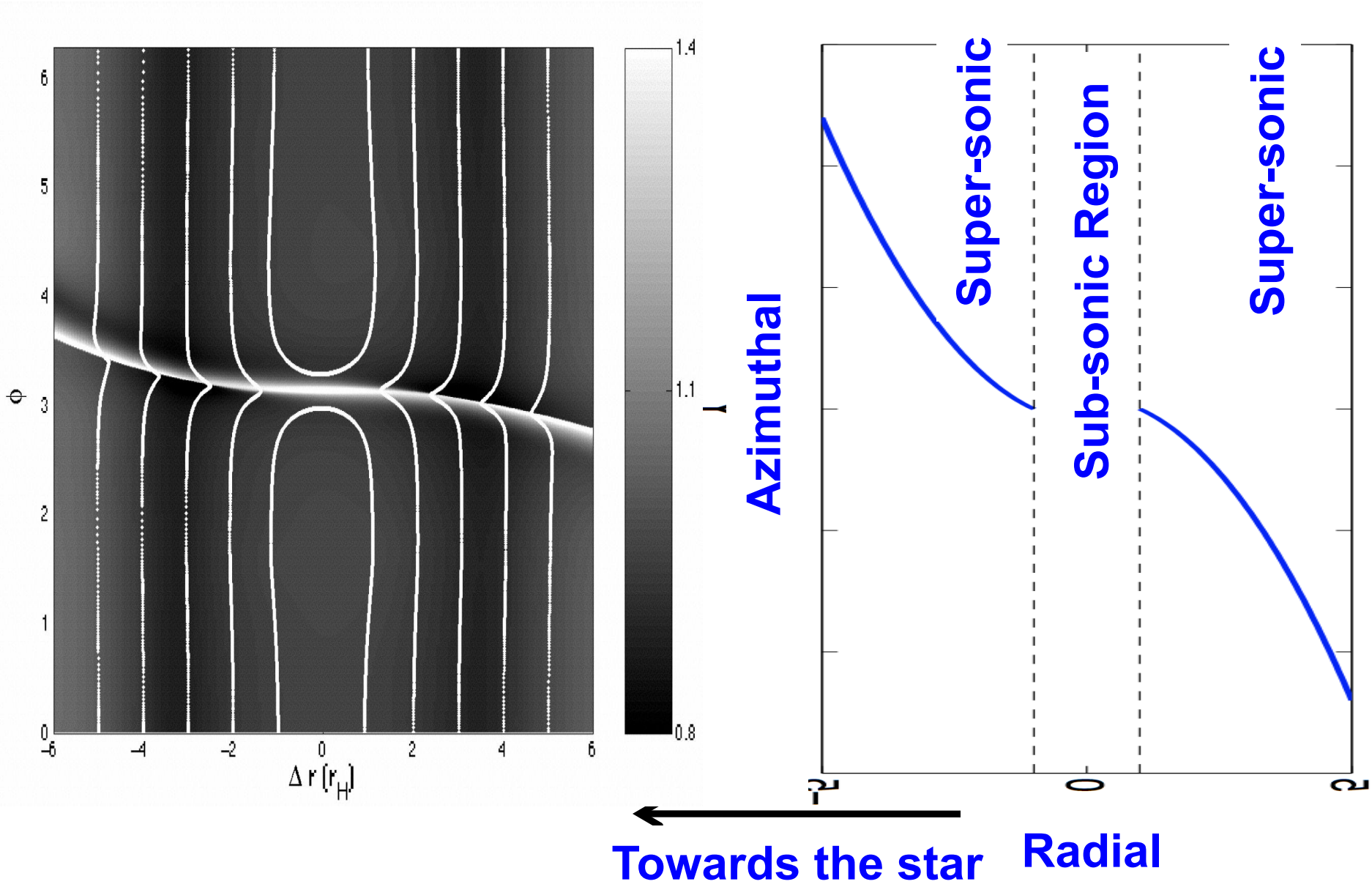


Evolution of a Typical Disk

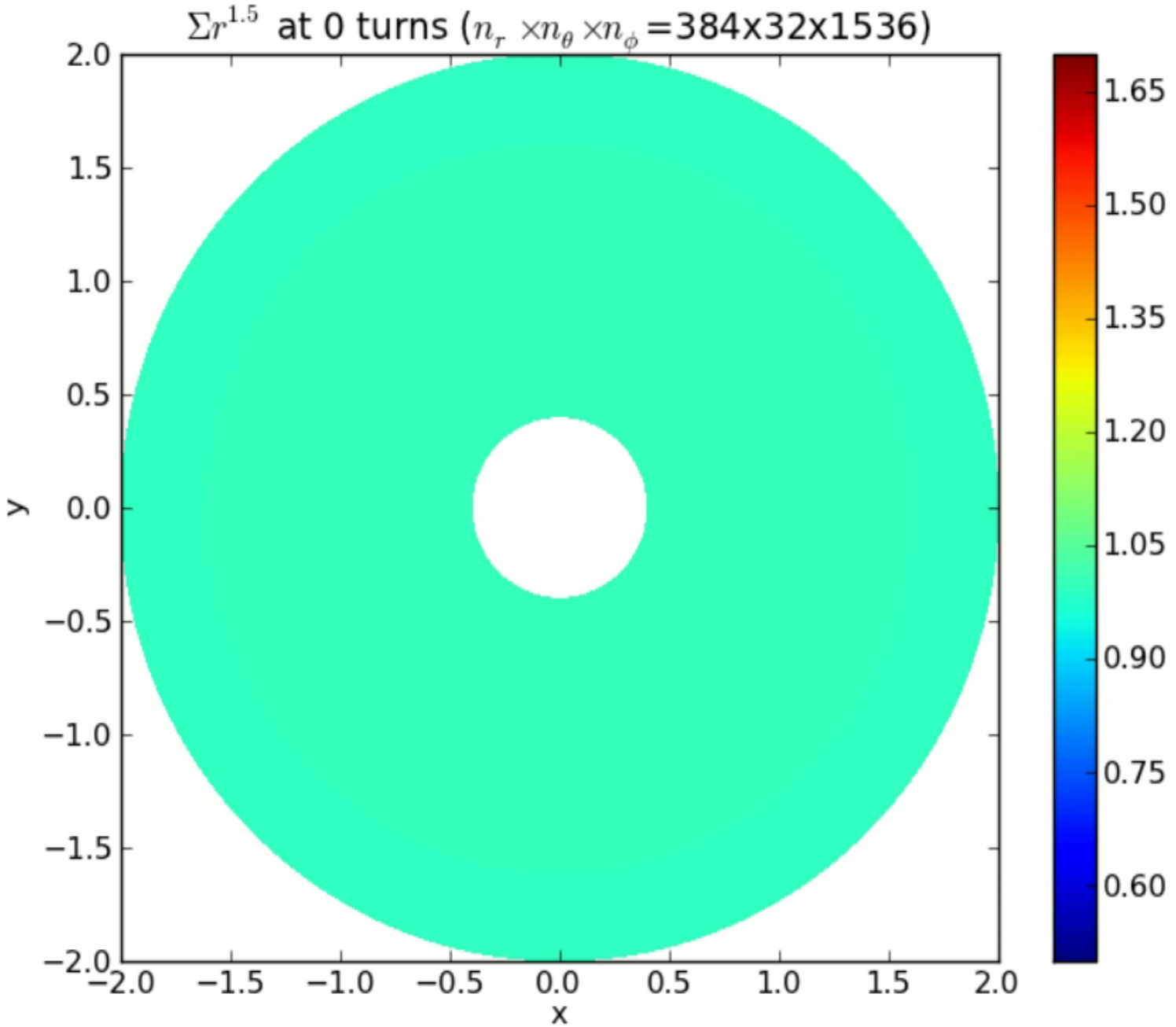


Williams & Cieza'11

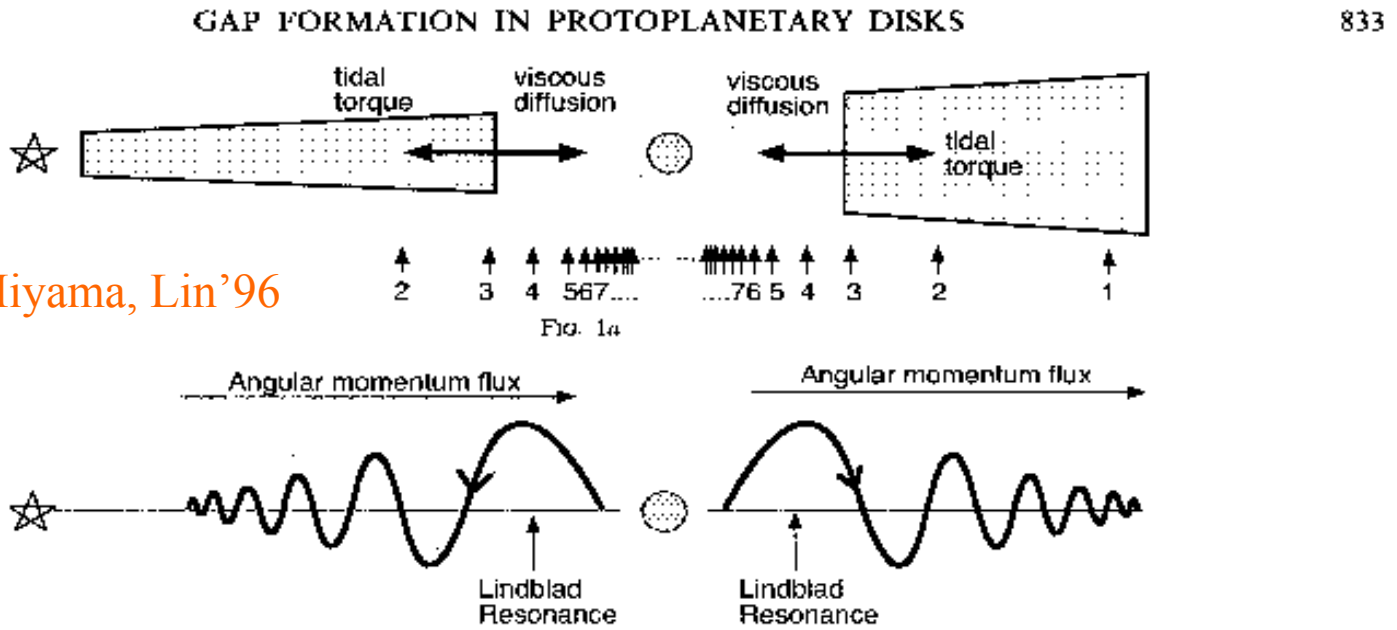
Disk+Planet Basics



Disk+planet Interaction



Disk-Planet Tidal Interaction



Takeuchi, Miyama, Lin'96

- Planet excites density waves in the disk at the Lindblad resonances:
  $w = W \pm k/m$, k: epicyclic freq., $r_L = (1 \pm 1/m)^{2/3} r_p$.
- Ingoing (outgoing) waves carry a negative (positive) angular momentum flux as they move away from planet into the disk interior (exterior).
- Waves dissipate, net angular momentum exchange between disk and planet.
- Inner (outer) disk loses (gain) angular momentum, matter move away from the planet, forming a gap.

Potential Vorticity

- Potential Vorticity (PV or vortensity):

$$\zeta\,\hat{\mathbf{z}} = \frac{(\nabla \times \mathbf{v})_z}{\Sigma}$$

- PV along streamlines for inviscid flow:

$$\frac{D(\zeta\,\hat{\mathbf{z}})}{Dt} = (\zeta\,\hat{\mathbf{z}} \cdot \nabla)\mathbf{v} + \frac{1}{\Sigma^3}\nabla\Sigma \times \nabla p$$

  - If barotropic p=p(Σ), baroclinic term

$$\nabla\Sigma \times \nabla p = 0$$

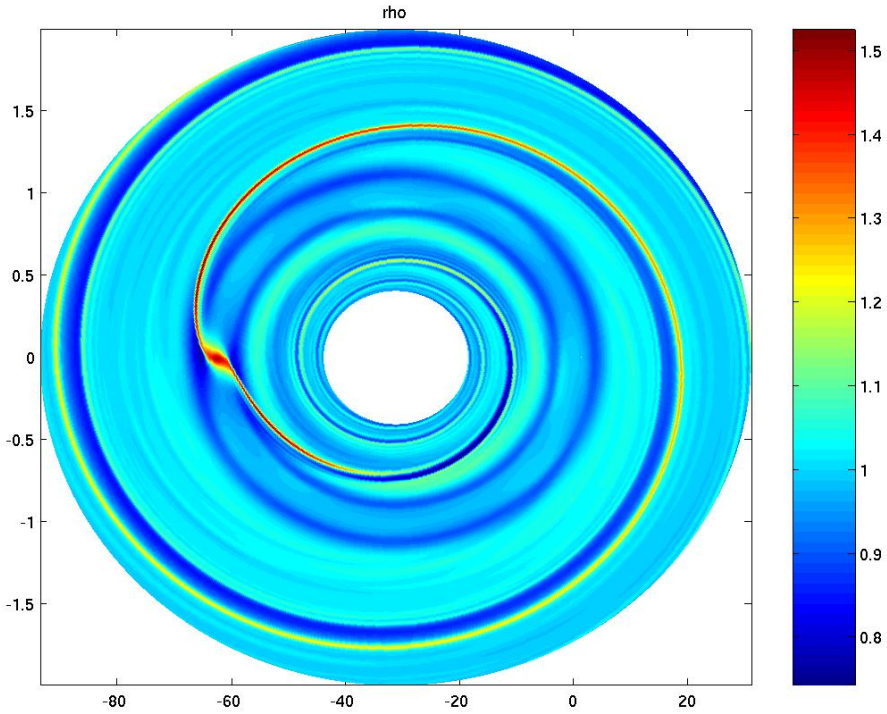
  - and in 2D ($\hat{\mathbf{z}} \cdot \nabla = 0$):

$$\boxed{\frac{D\zeta}{Dt} \equiv \mathbf{0}}$$

Break the conservation of Potential Vorticity

$$\frac{D\zeta}{Dt} = \textbf{viscosity} + \textbf{shocks} + \textbf{non-adiabatic forcing}$$

- Viscosity: either imposed or numerical
- Spiral shocks: cutting through the whole disk
- “Switch-on” of the planet



PV Changes due to Shocks

- Vorticity jump across shock: Truesdell (‘52), Lighthill (’ 57), Hayes(’ 57),…, Kevlahan (’ 97)

$$\delta\omega = \frac{q^2}{1+q}\frac{\partial Cr}{\partial S} + baroclinic\ term + q\omega_a$$

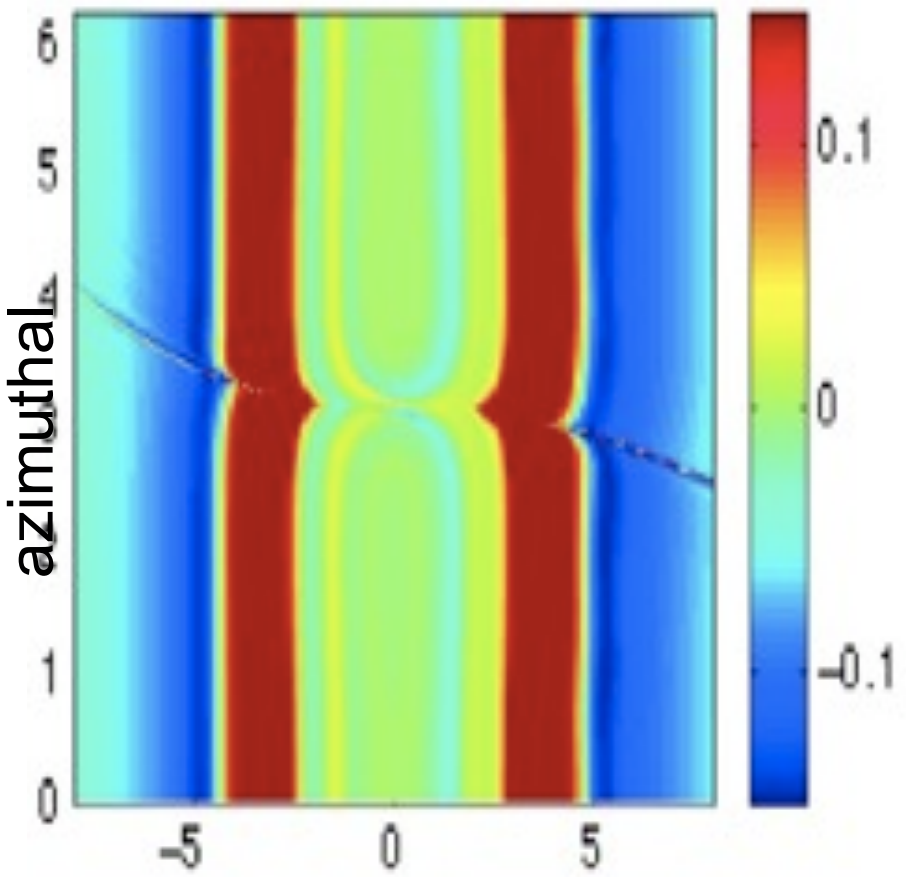
$$where \quad q = \frac{\Sigma_b}{\Sigma_a} - 1,\ Cr = -V_\perp (in\ shock\ frame)$$

- For isothermal shock, potential vorticity jump:

$$\delta\varsigma = \frac{1}{\Sigma_b}\frac{q^2}{1+q}\quad\frac{\partial Cr}{\partial S}$$

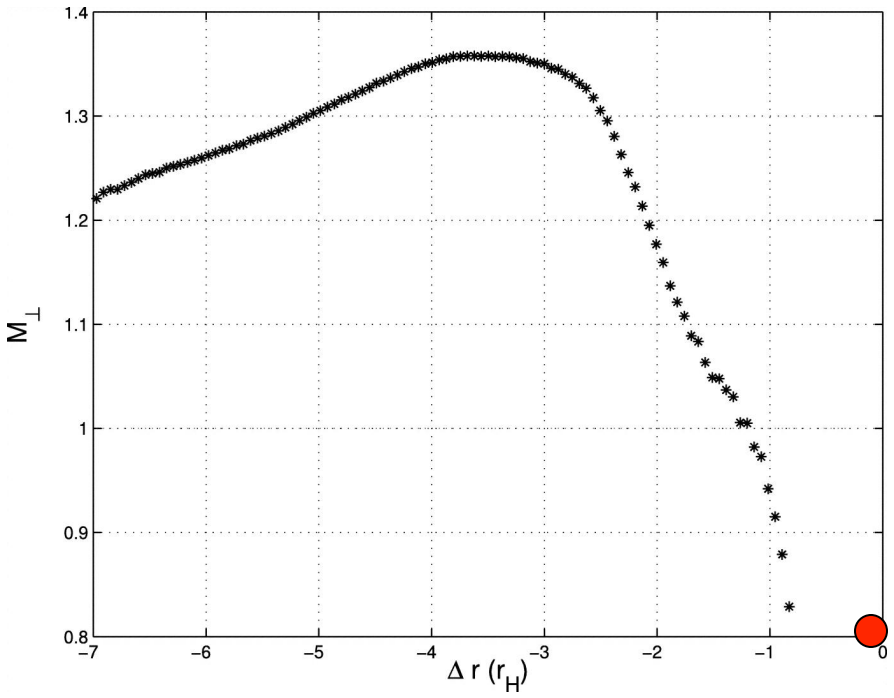
density_factor curvature_factor

Flow Lines, Horse-shoe Region and Shocks



radius

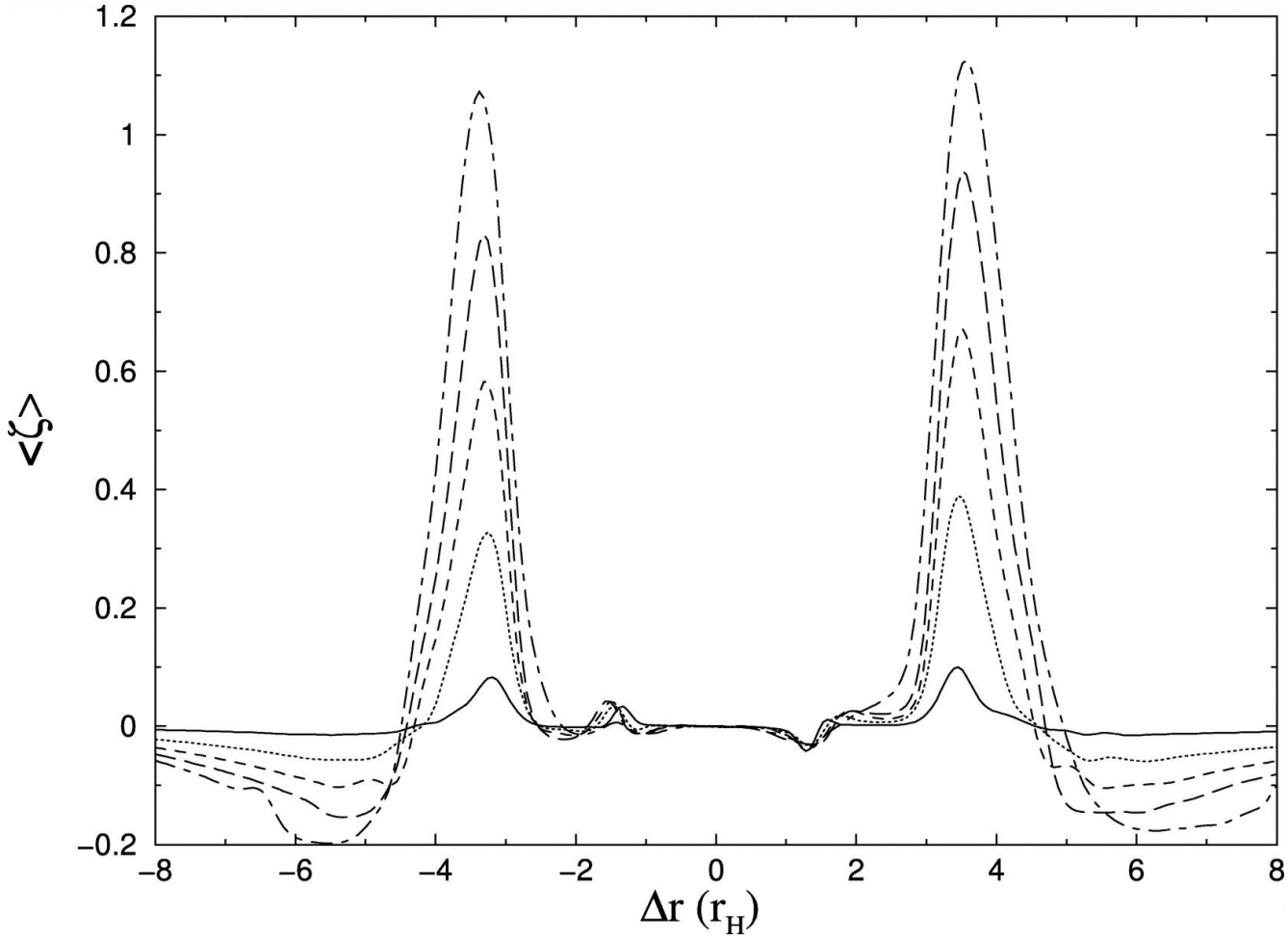
Perpendicular Mach number



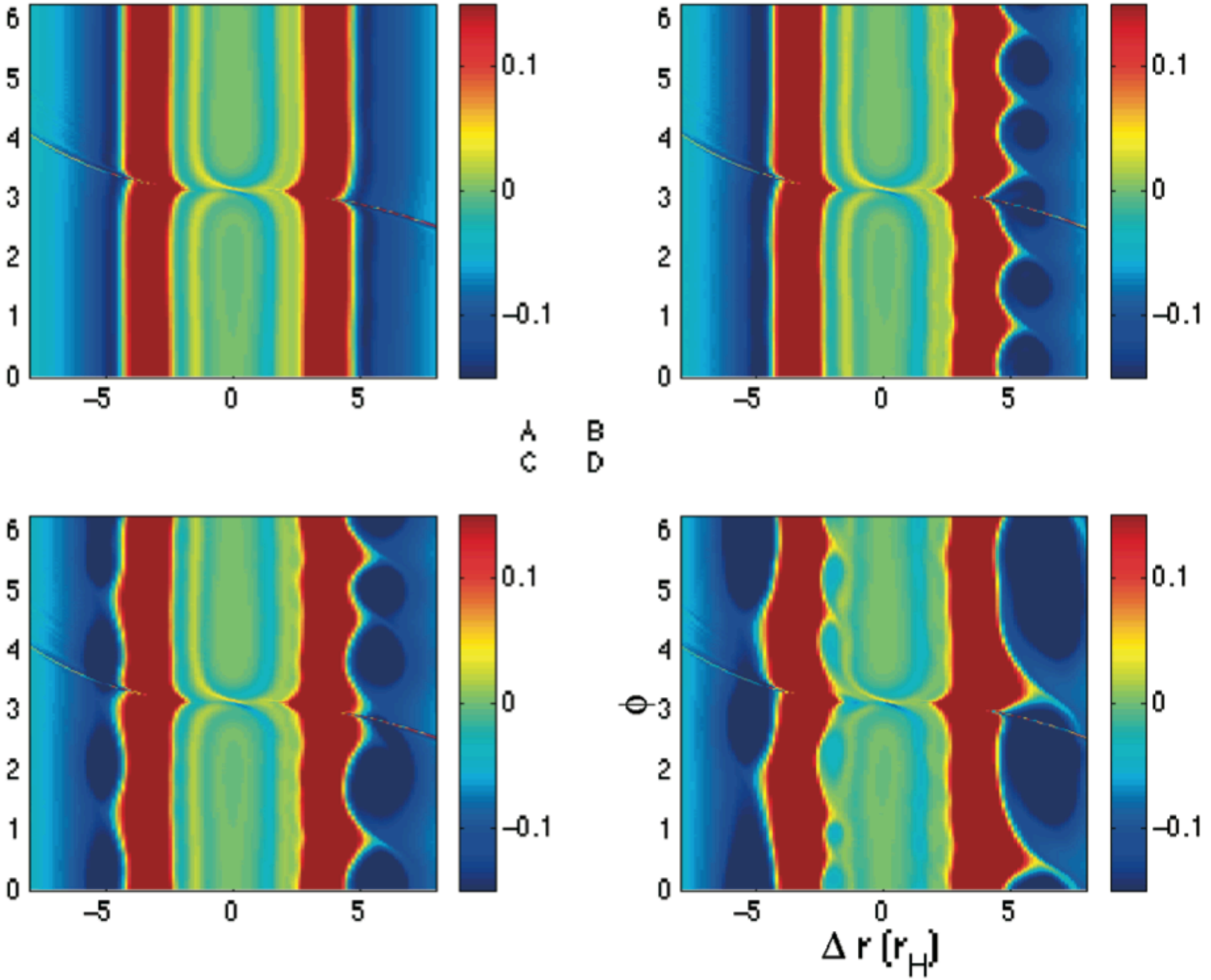
Distance away from planet

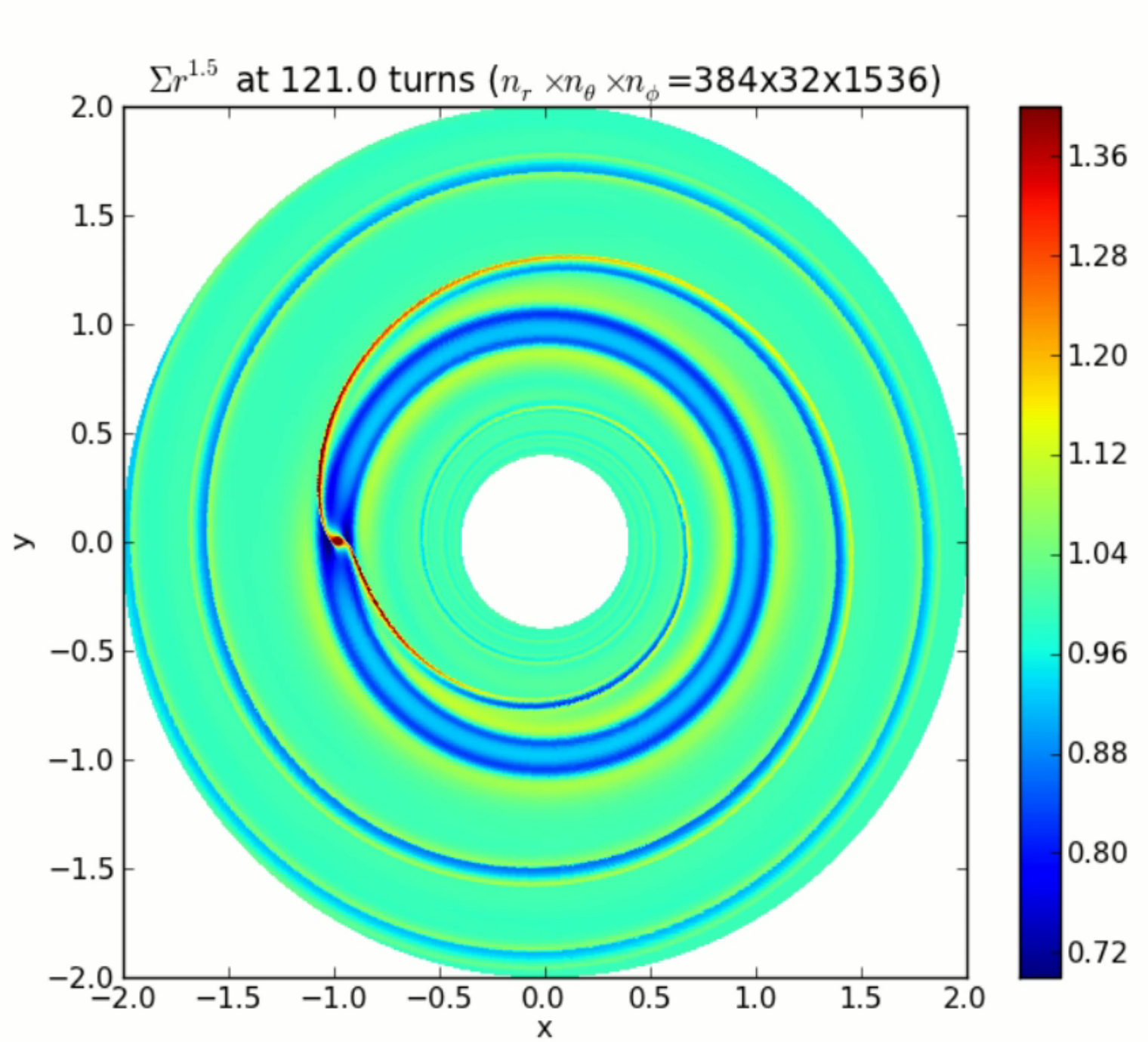
Li et al. 2005

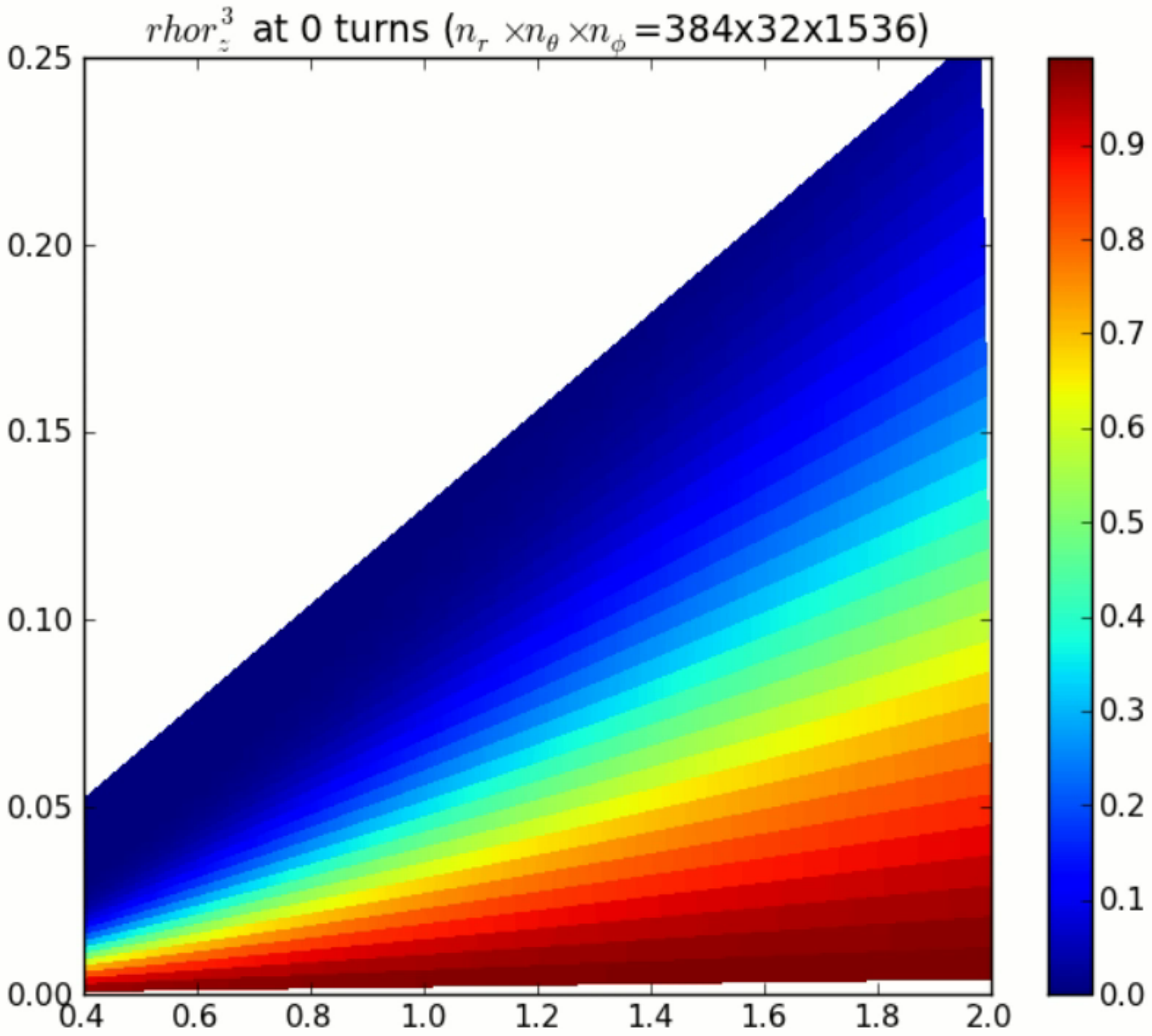
Potential Vorticity Evolution



Li et al. 2005


A
B
C
D
0
1
2
3
4
5
6
−5
0
5
0.1
0
−0.1
ϕ
Δ r (r_H)


$\Sigma r^{1.5}$ at 121.0 turns ($n_r \times n_\theta \times n_\phi$ =384x32x1536)
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x
1.36
1.28
1.20
1.12
1.04
0.96
0.88
0.80
0.72

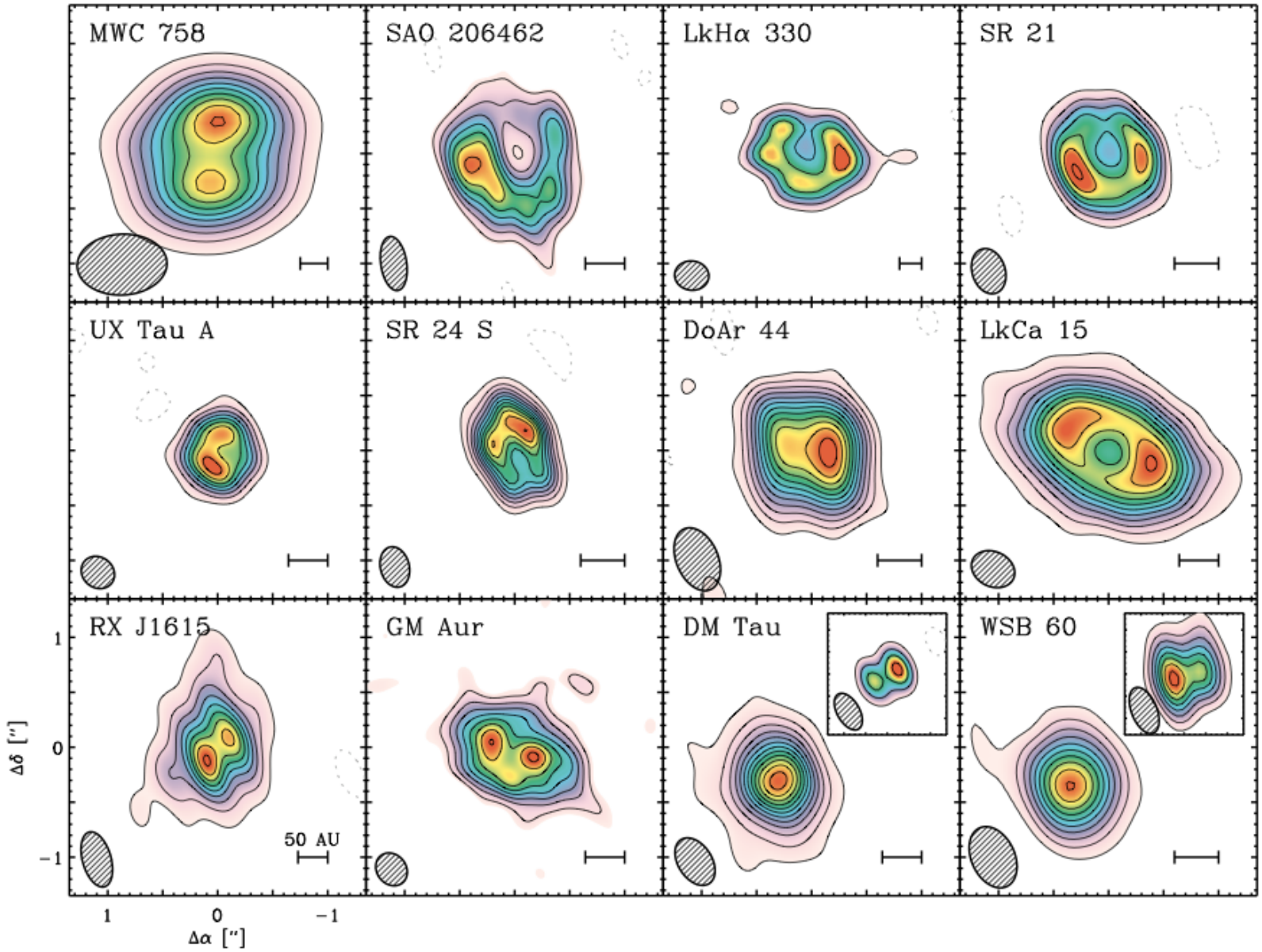


3D Disk

10 M_{Earth}

Q:
Observational signatures of the joint evolution of Disks + Planets?

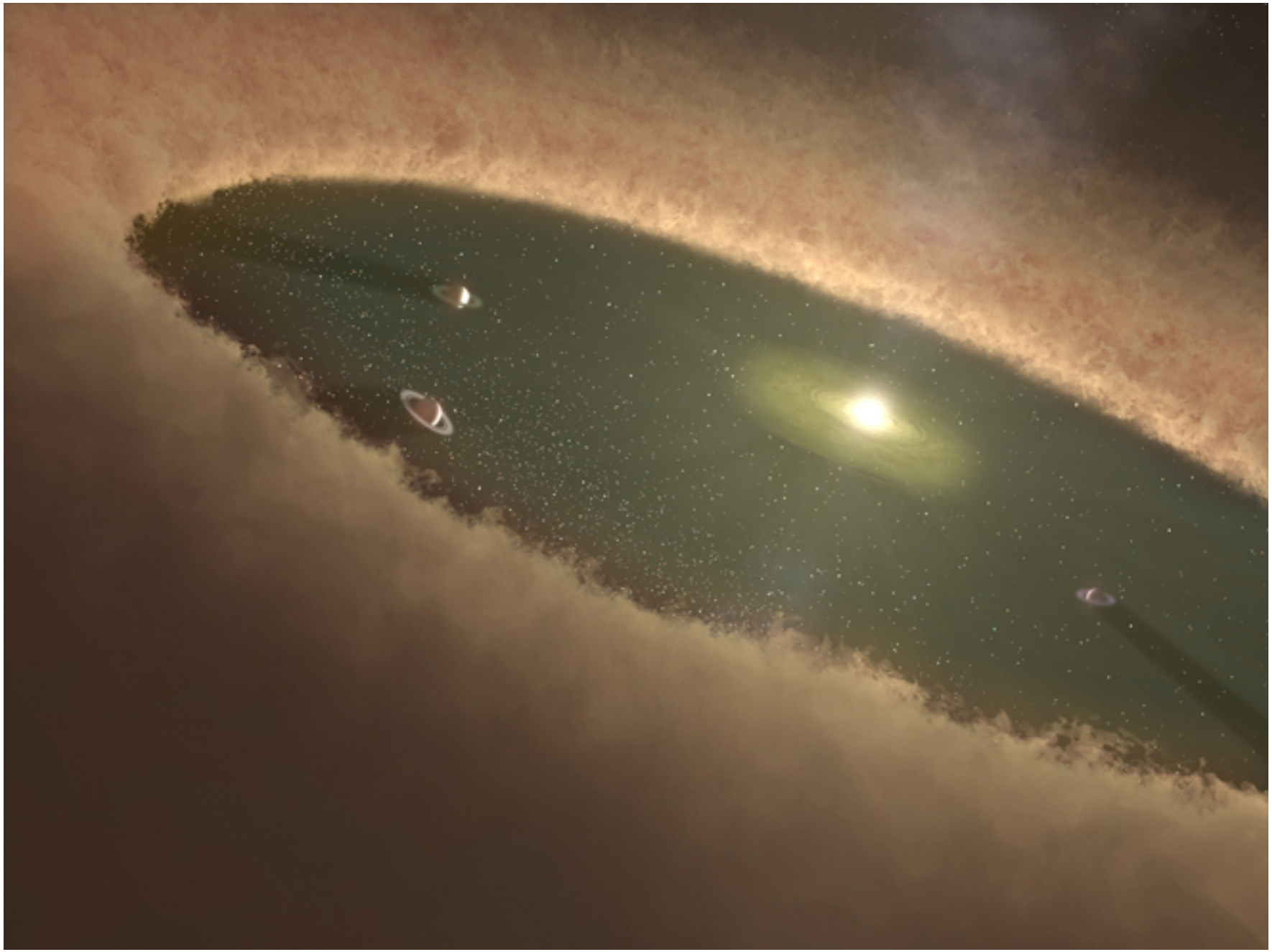
Transition Disks



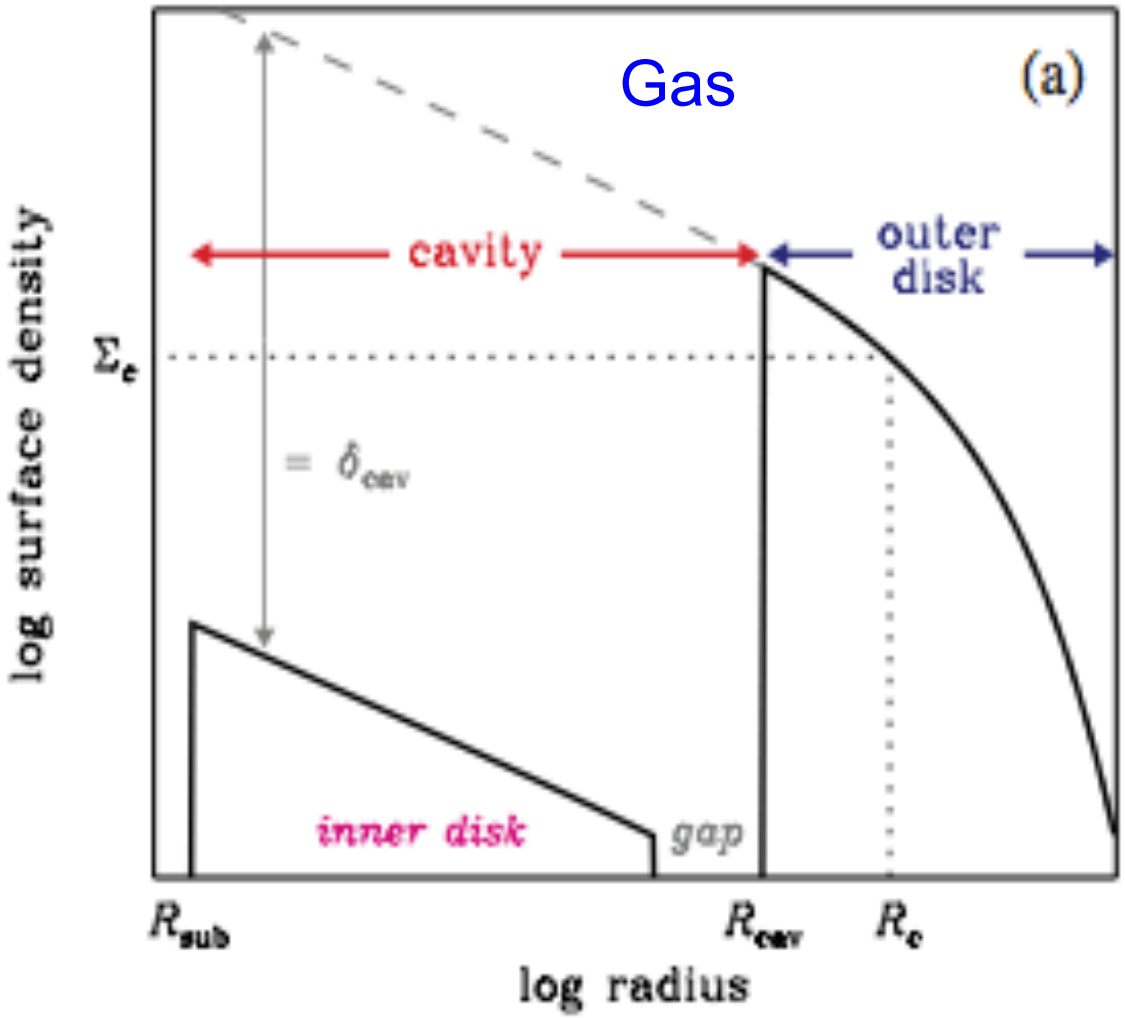
SMA Obs.

Andrews et al' 11

Figure 1. SMA aperture synthesis maps of the 880 μm continuum emission from this sample of transition disks. Each panel is 2″.7 on



Transition Disks: Idealized Structures



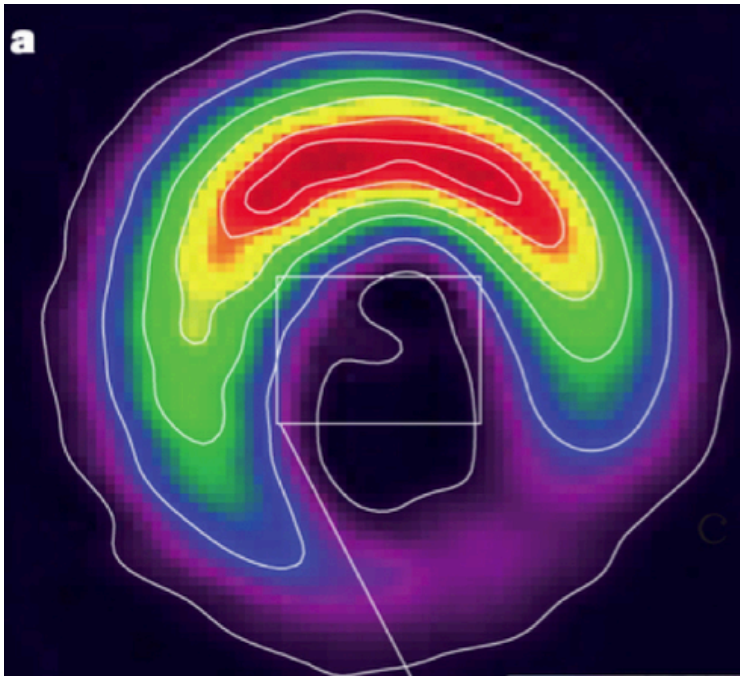
Q: Should be quite unstable???

Andrews et al. 2011

ALMA

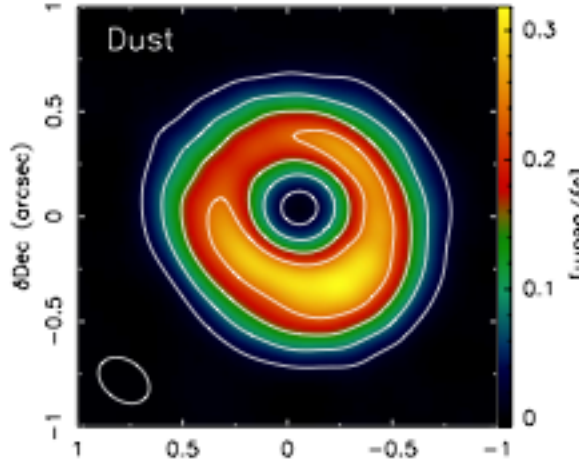
ALMA Observations of Transition Disks (so far…)

HD 142527

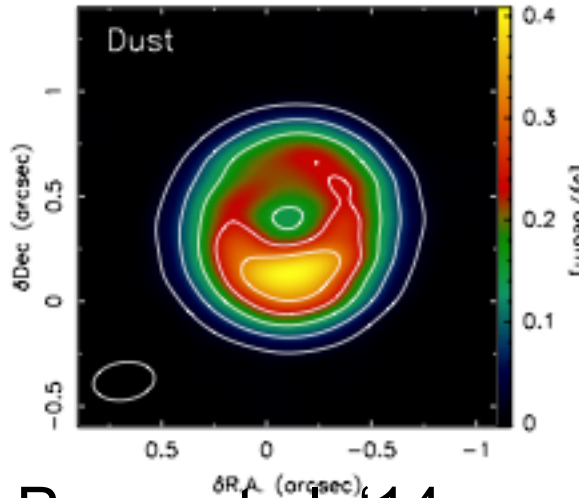


Casassus et al. ’13

SAO 206462

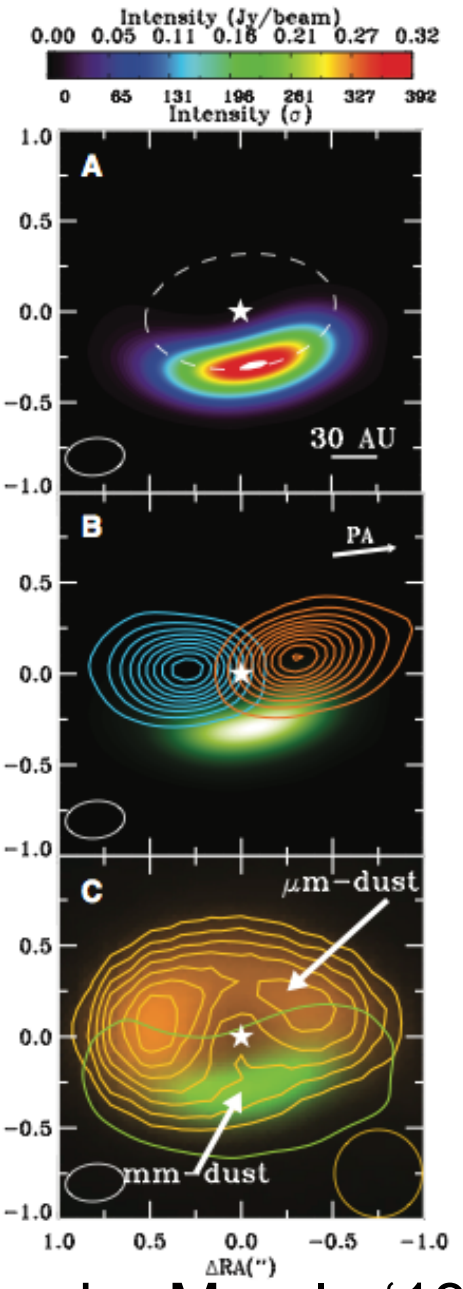


SR 21



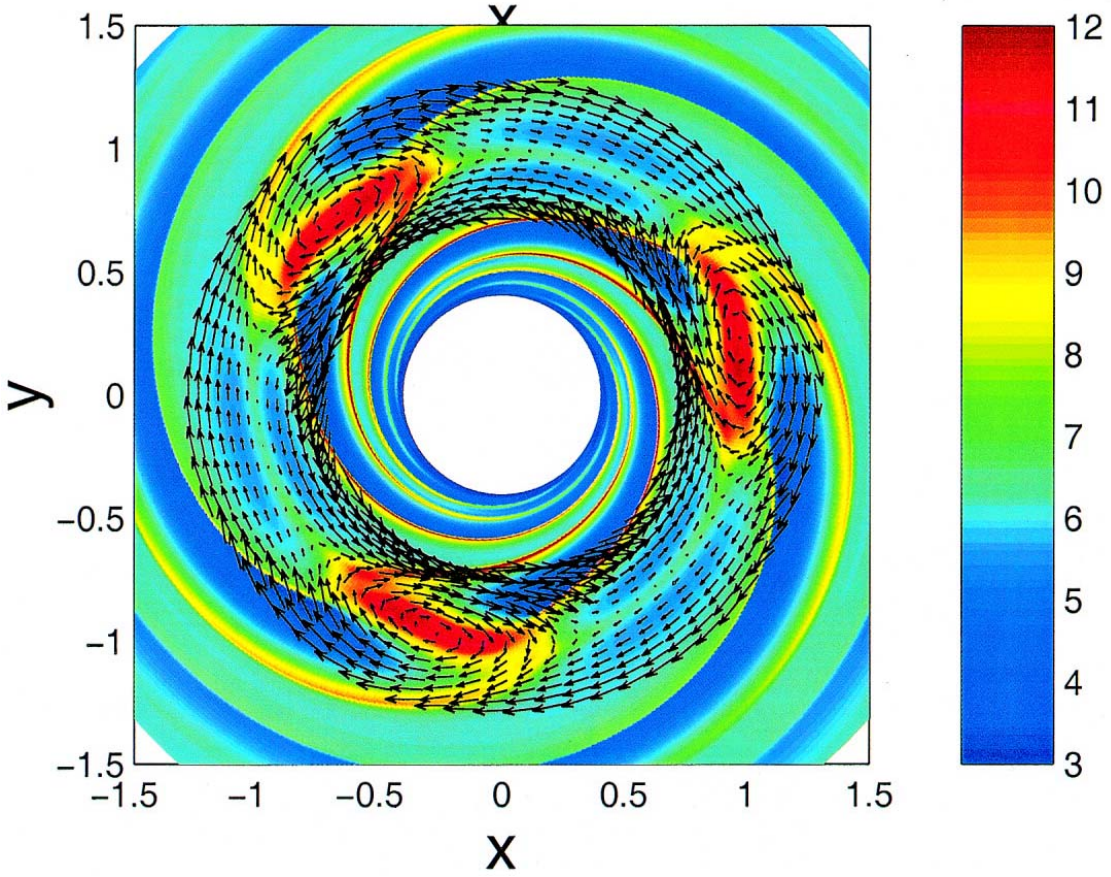
Perez et al. ‘14

IRS 48

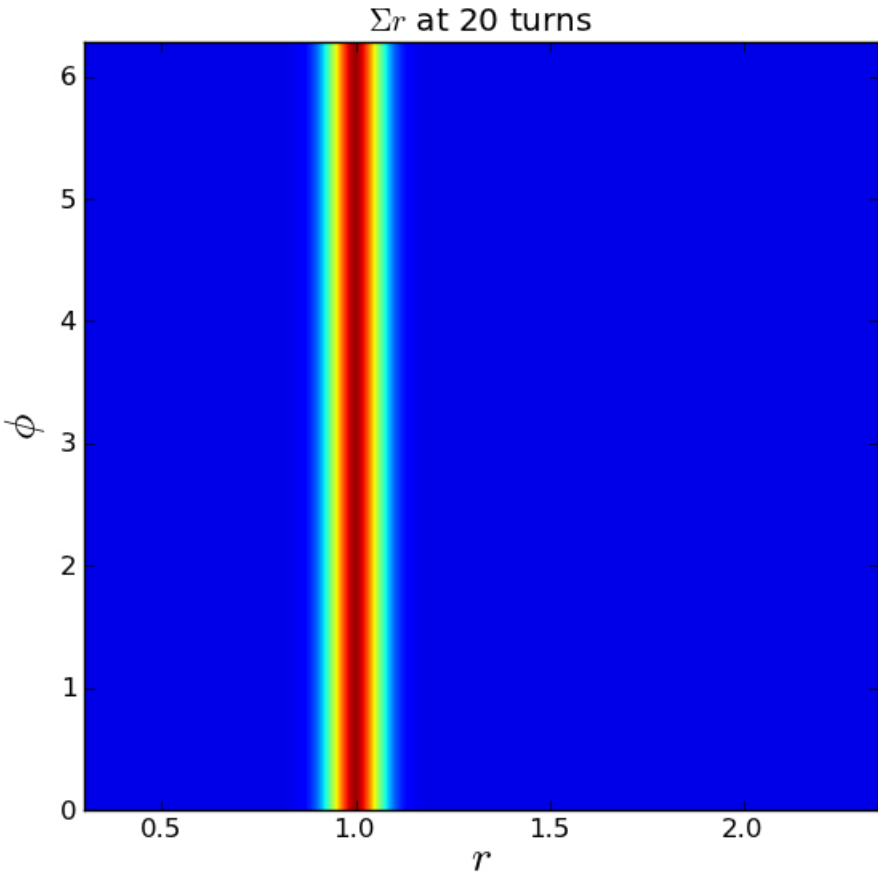


van der Marel+ ‘13

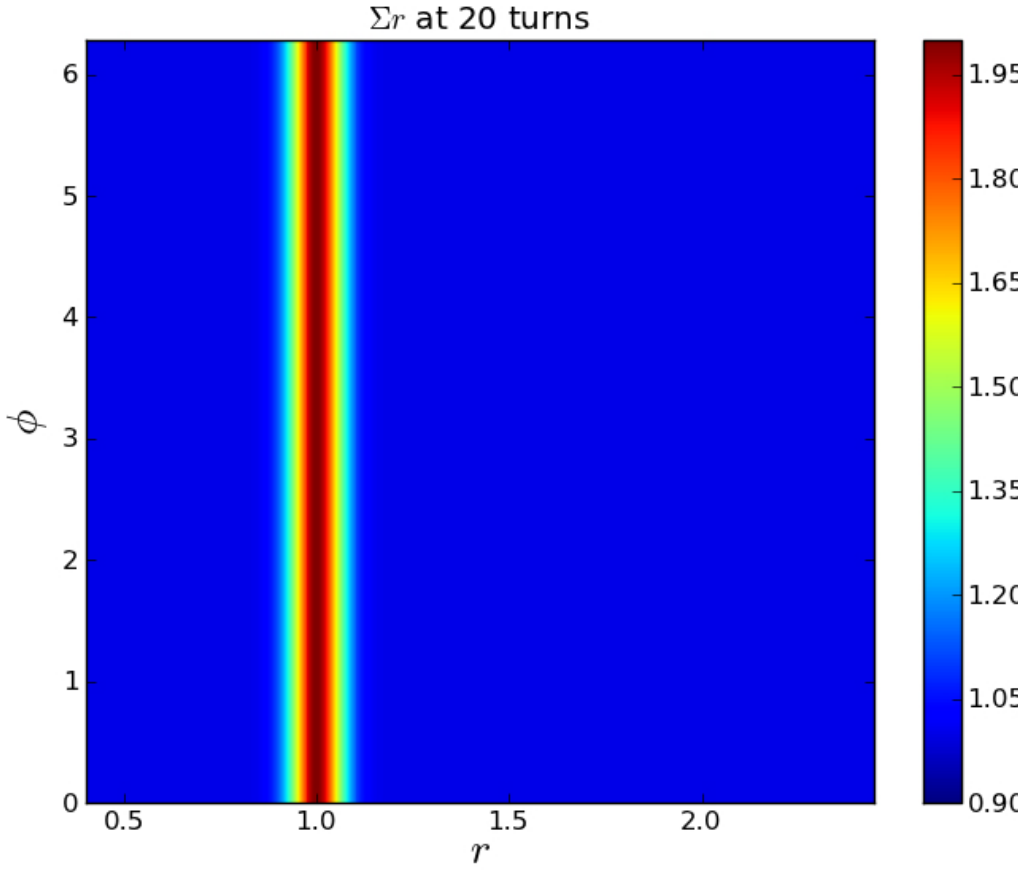
Q: How long can vortices survive?



Case 1: Evolution of RVI



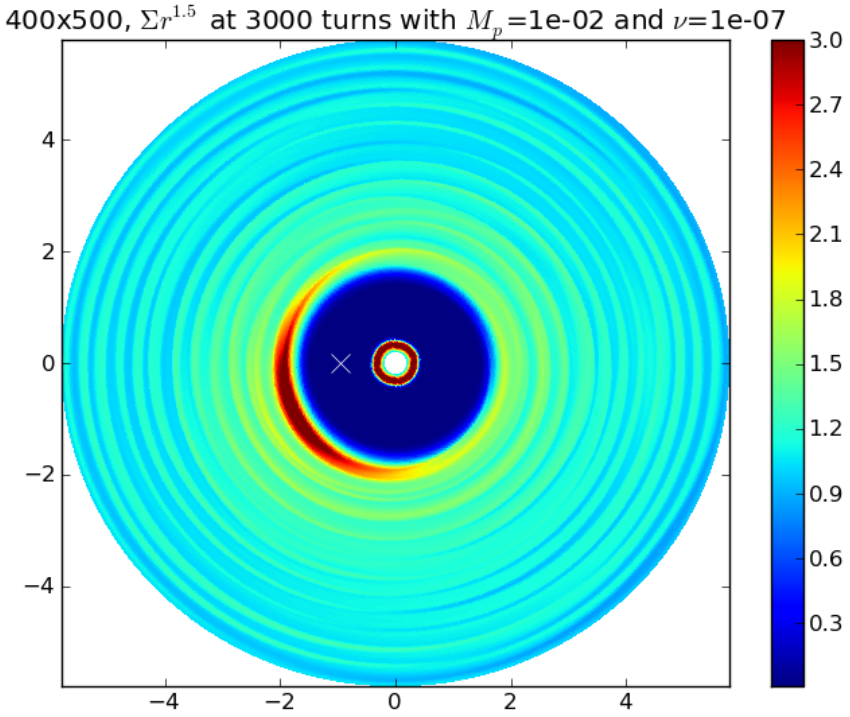
High viscosity
alpha = 1e-4

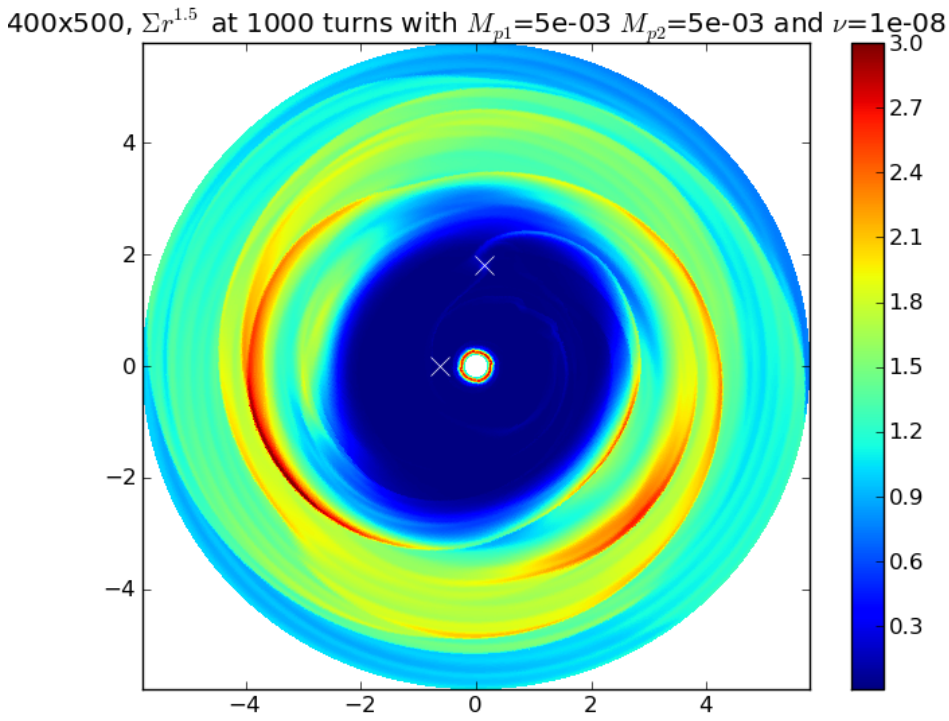


Low viscosity
alpha = 2.7e-6

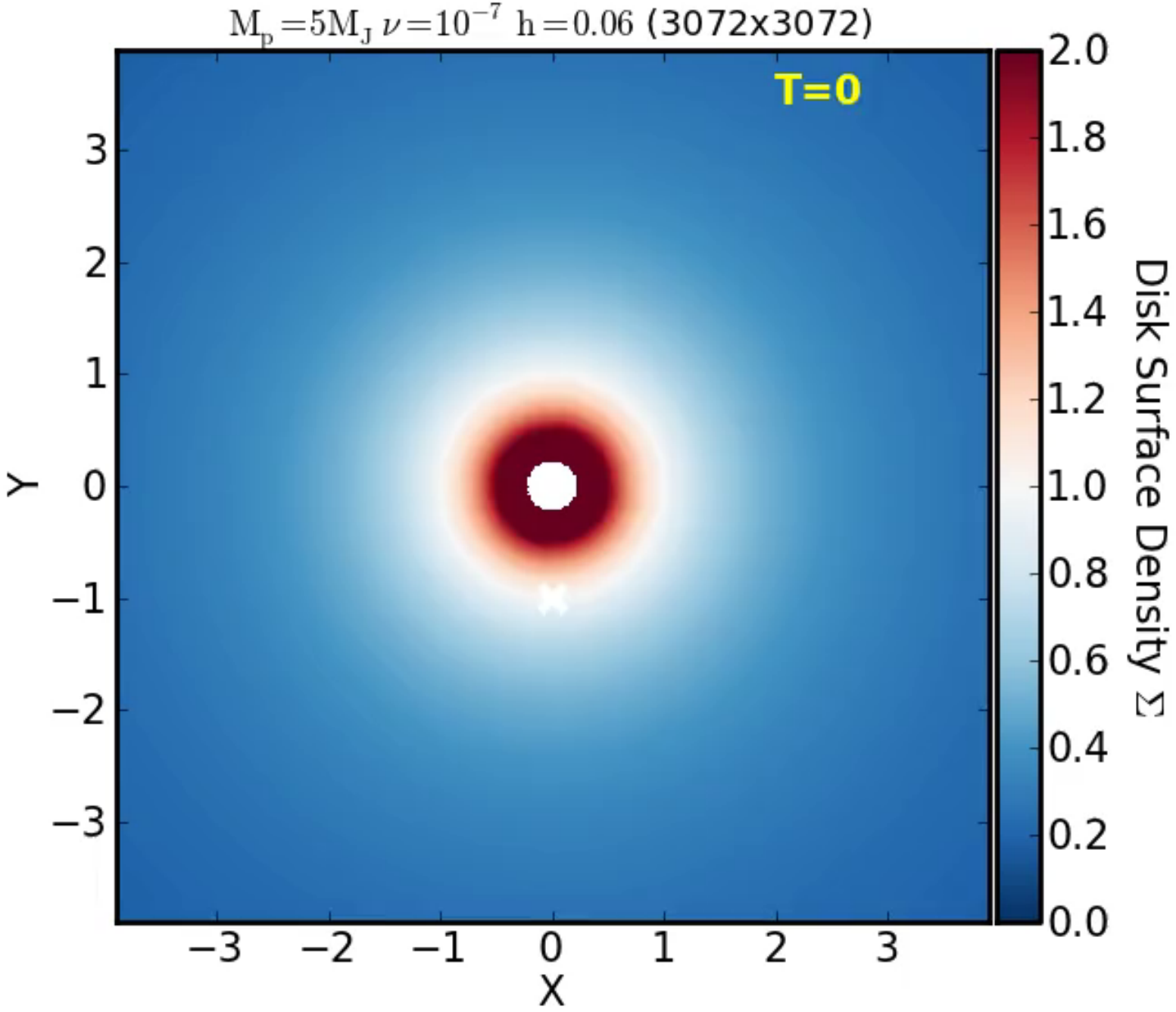
Li et al. 2001; Li et al. 2014

Case 2: Gap and Vortex from massive planet(s)

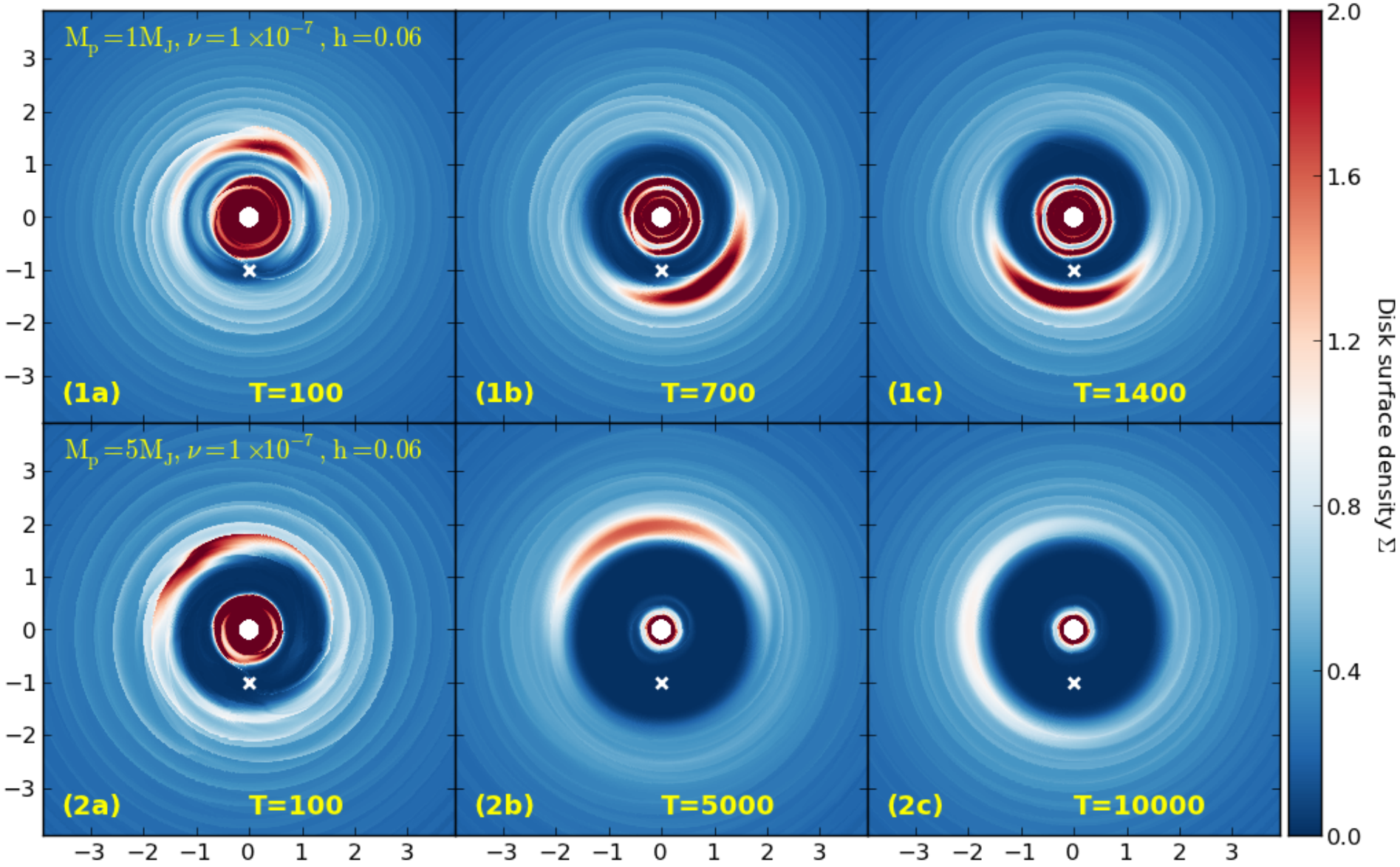




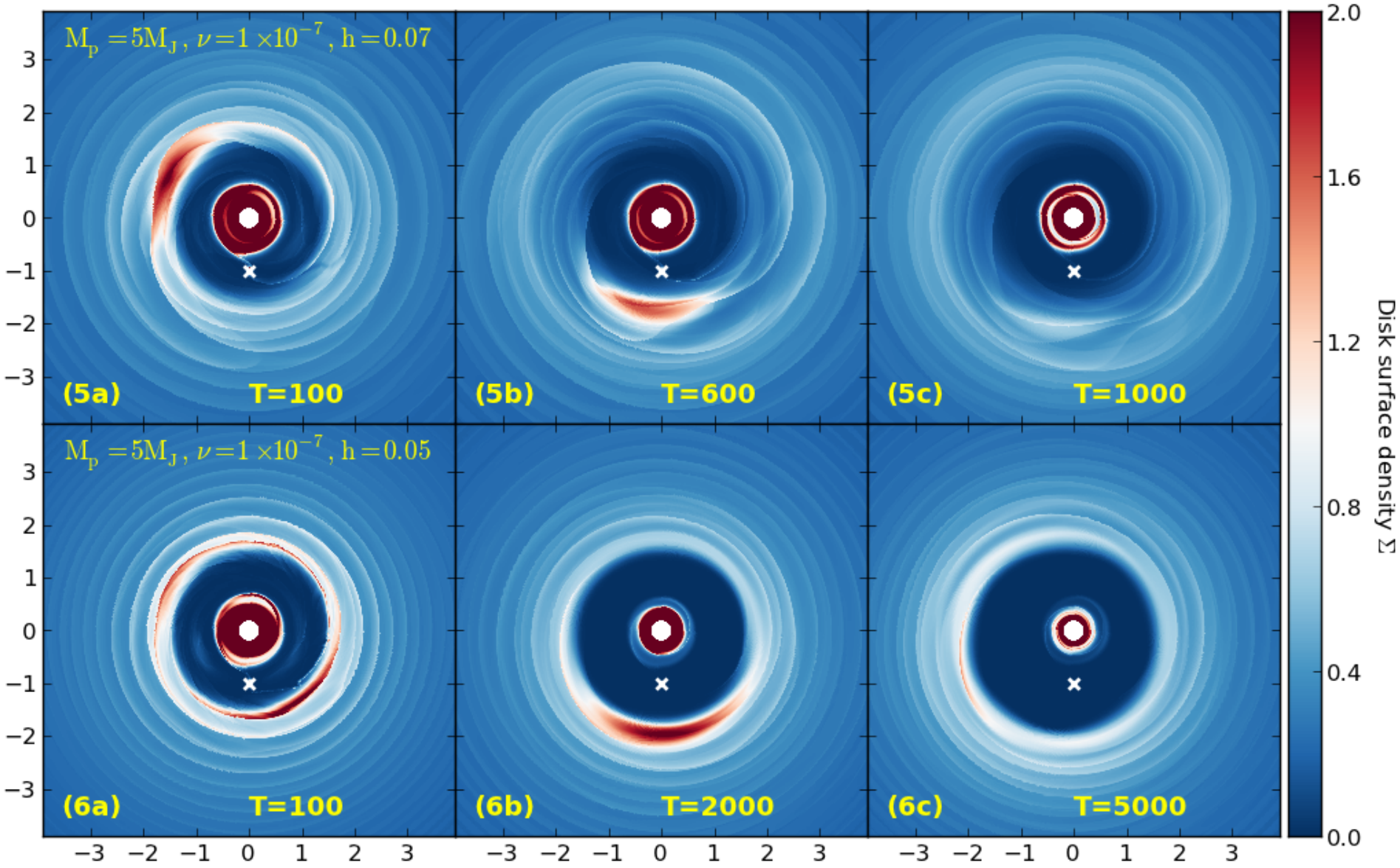
Fu, HL, et al. unpublished


$M_p = 5 M_J$ $\nu = 10^{-7}$ h = 0.06 (3072x3072)
T=0
Y
X
Disk Surface Density Σ
3
2
1
0
−1
−2
−3
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0

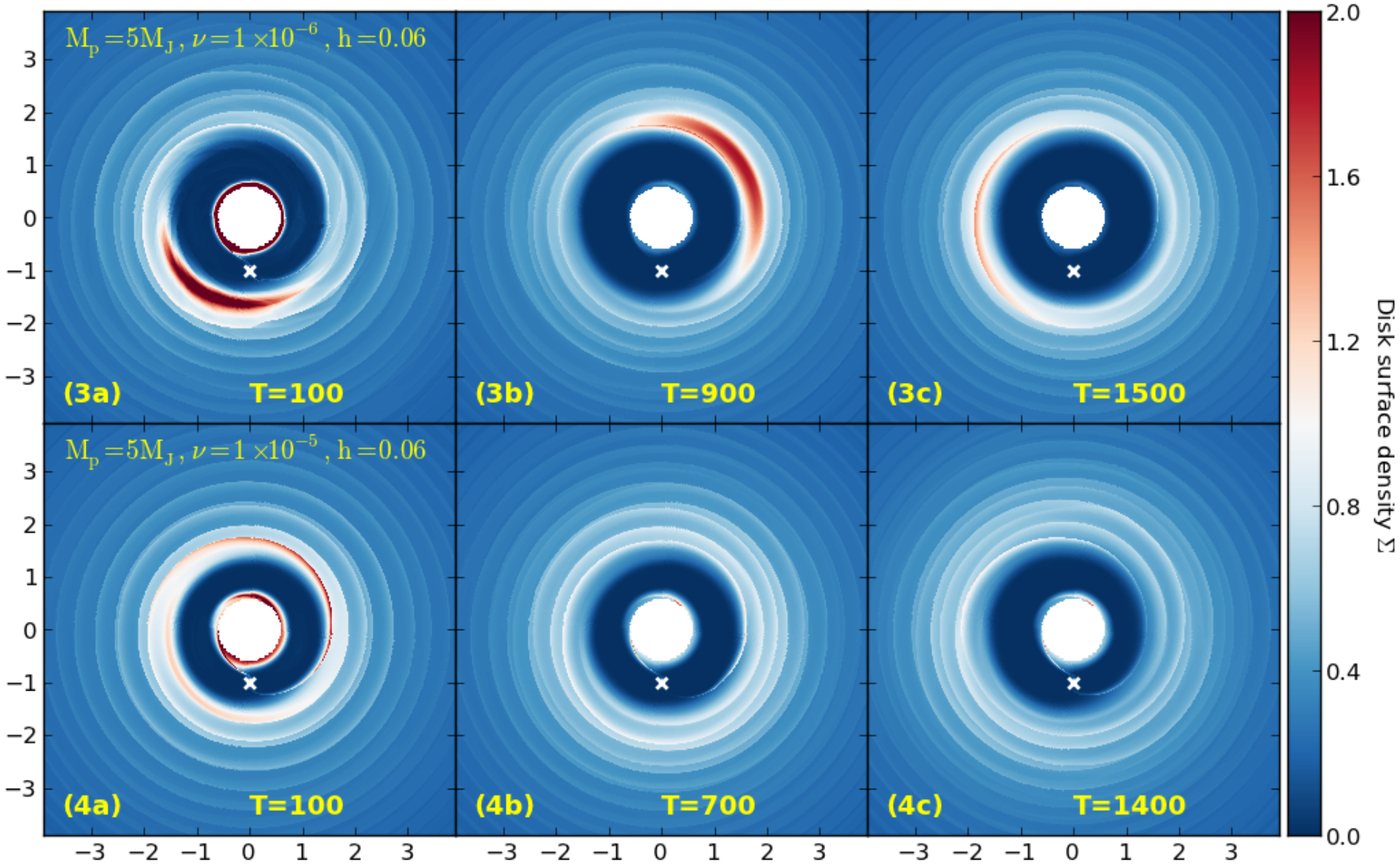
Effects of Planet Mass

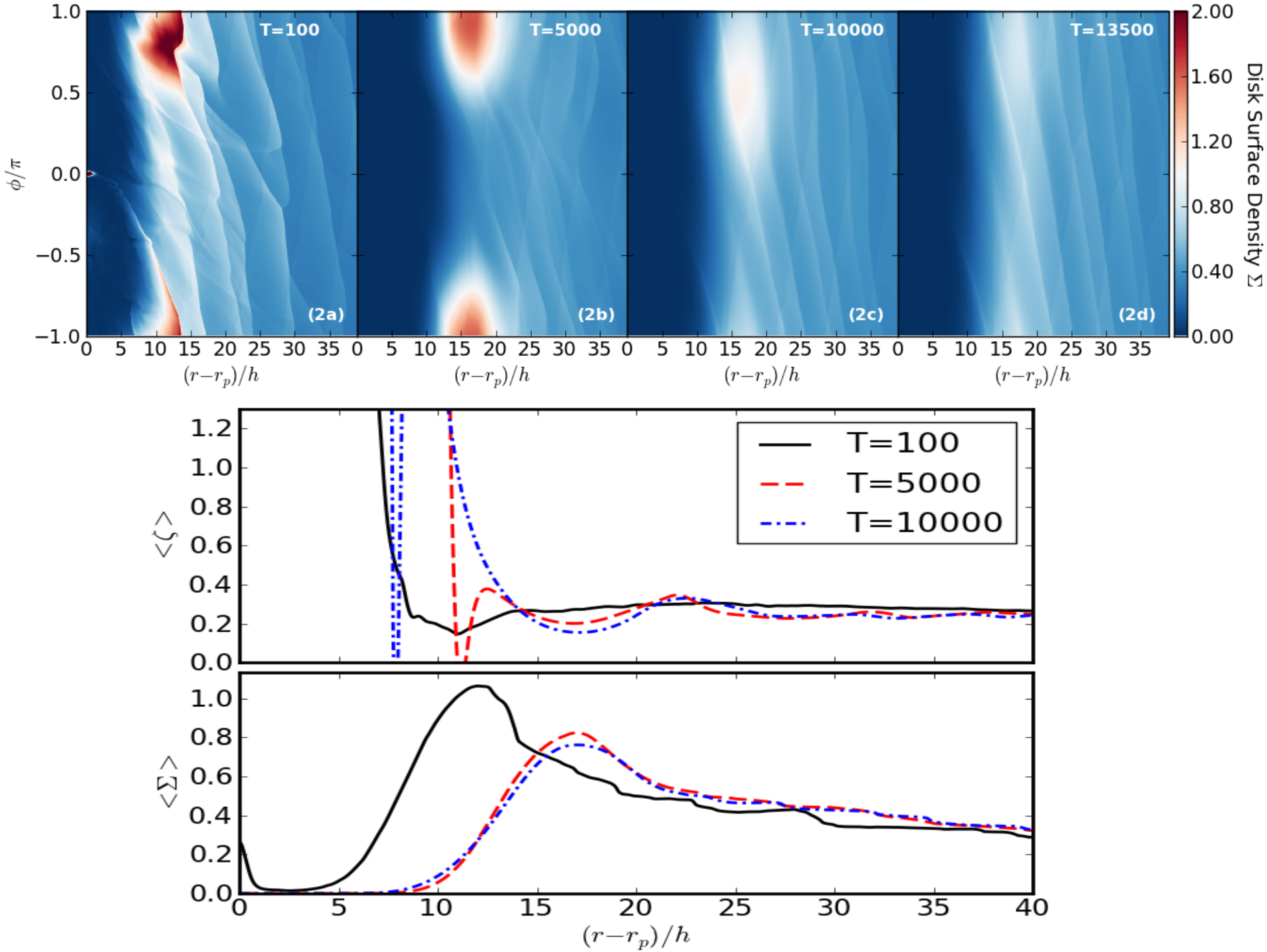


Effects of Disk Sound Speed

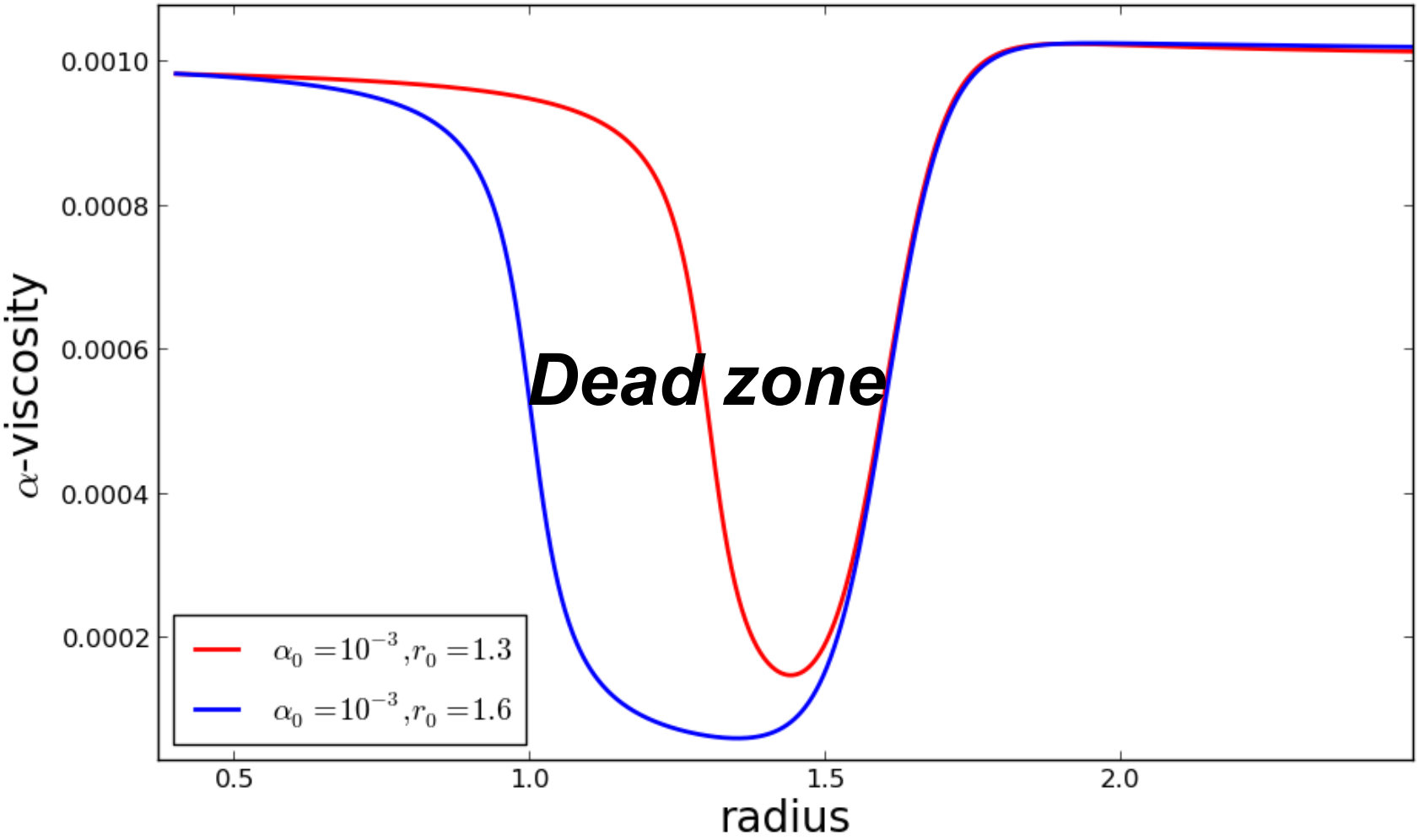


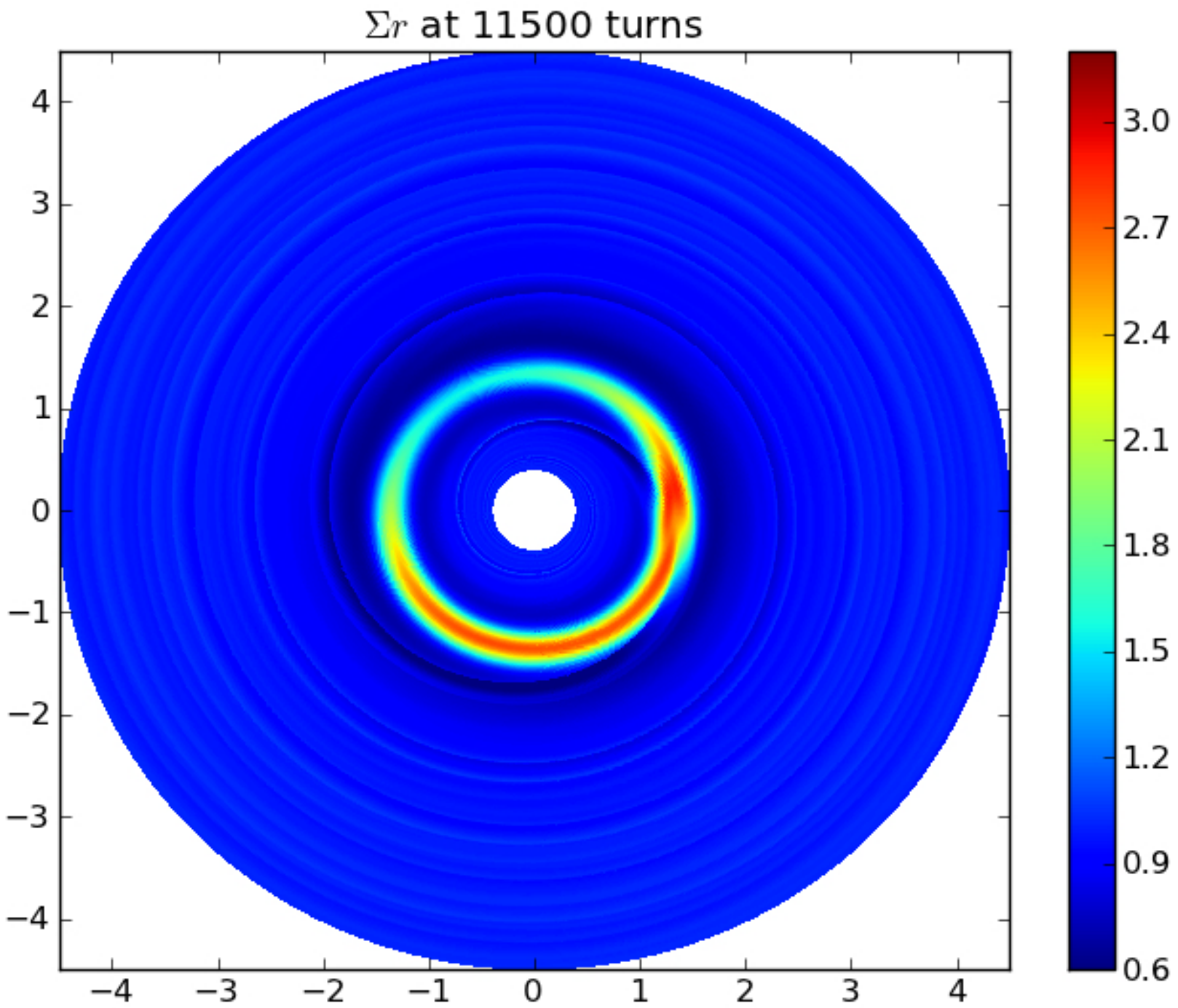
Effects of Disk (effective) Viscosity




T=100
T=5000
T=10000
T=13500
(2a)
(2b)
(2c)
(2d)
ϕ/π
$(r-r_p)/h$
Disk Surface Density Σ
$\langle\zeta\rangle$
$\langle\Sigma\rangle$

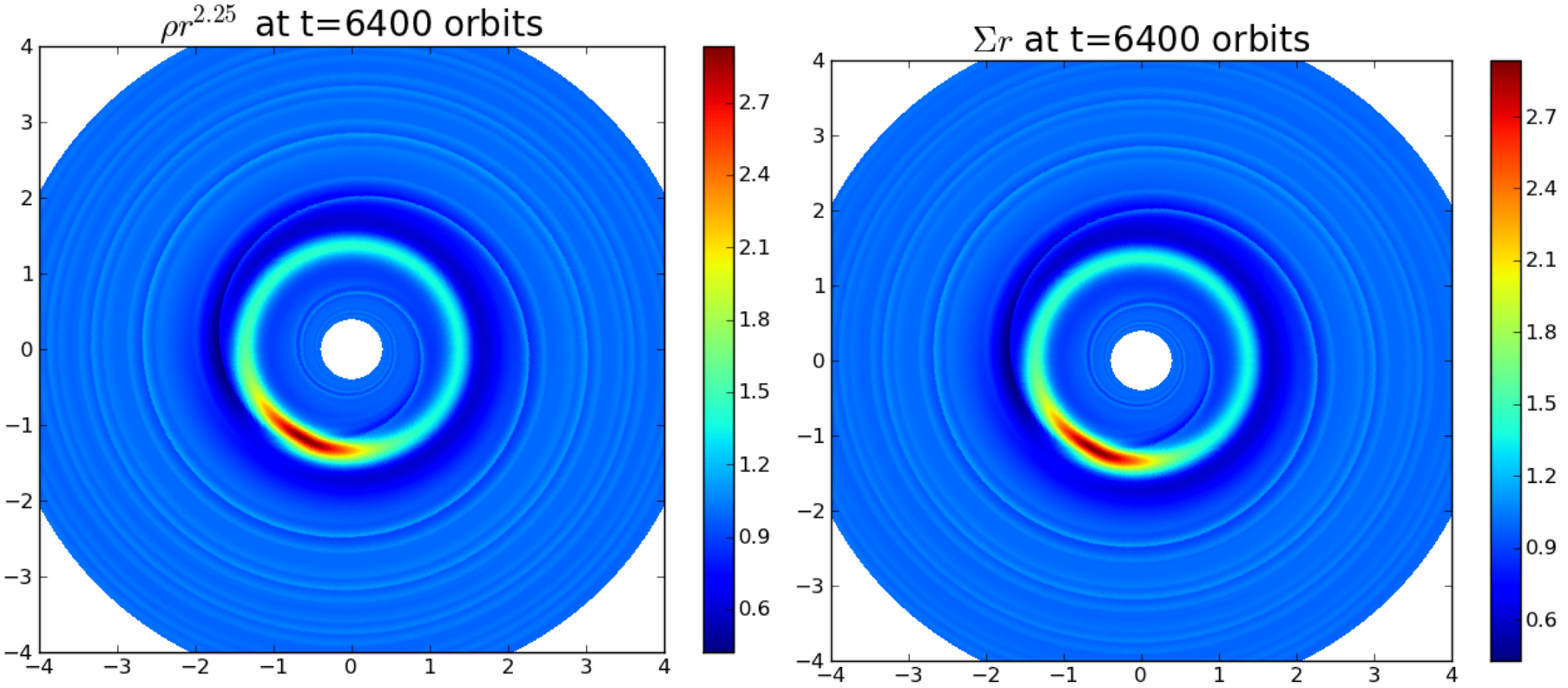
Case 3: Vortex from variable disk viscosity profiles




Σr at 11500 turns
4
3
2
1
0
−1
−2
−3
−4
−4
−3
−2
−1
0
1
2
3
4
3.0
2.7
2.4
2.1
1.8
1.5
1.2
0.9
0.6

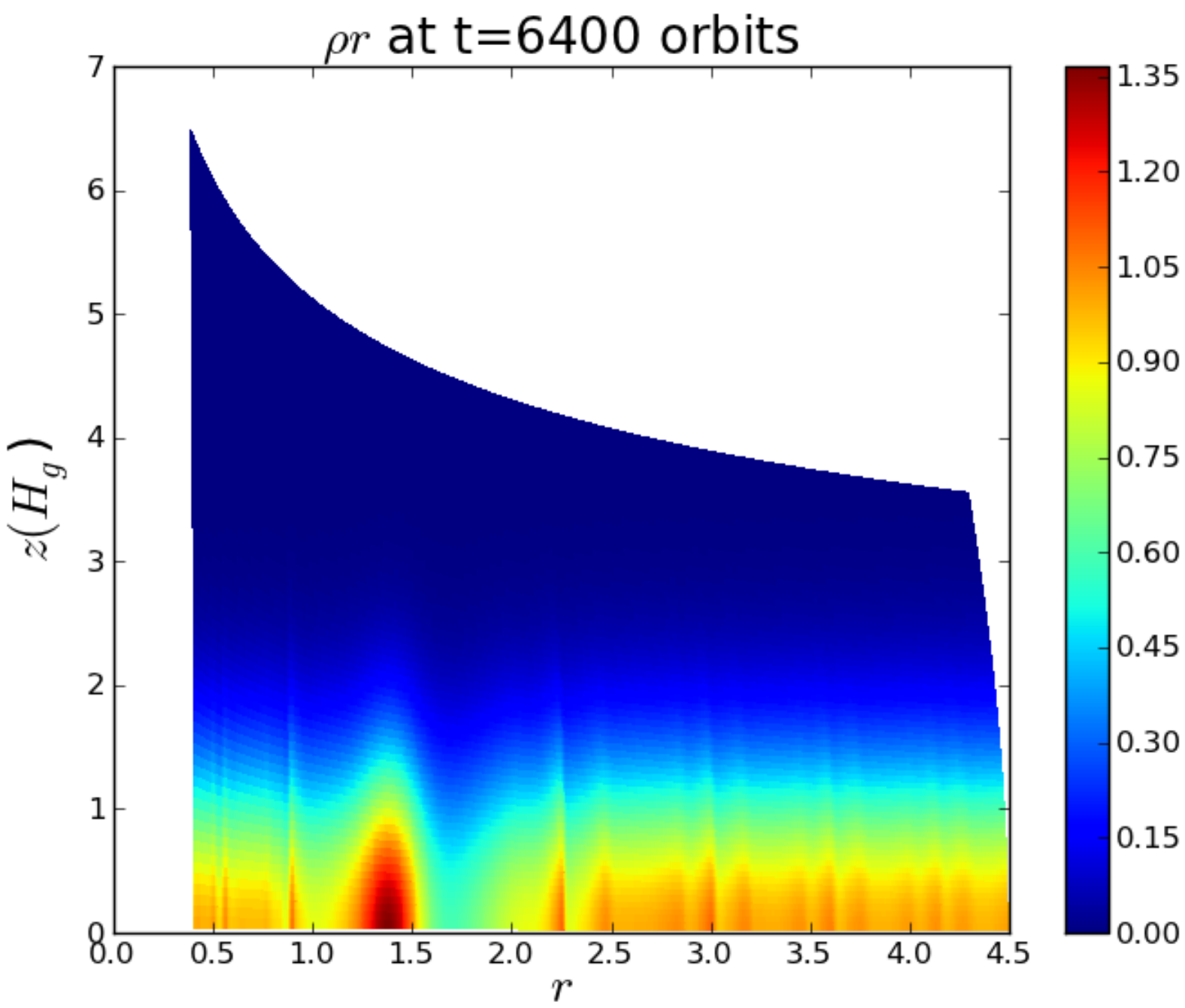
Case 4:
What happens in 3D?

Similar to 2D

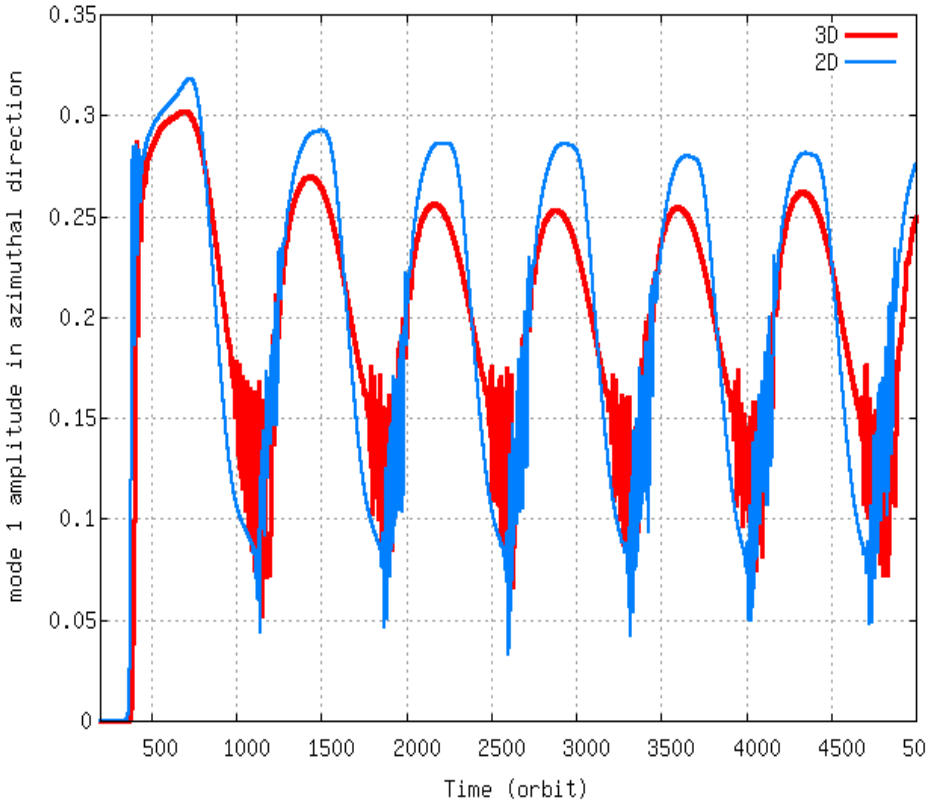


mid-plane density

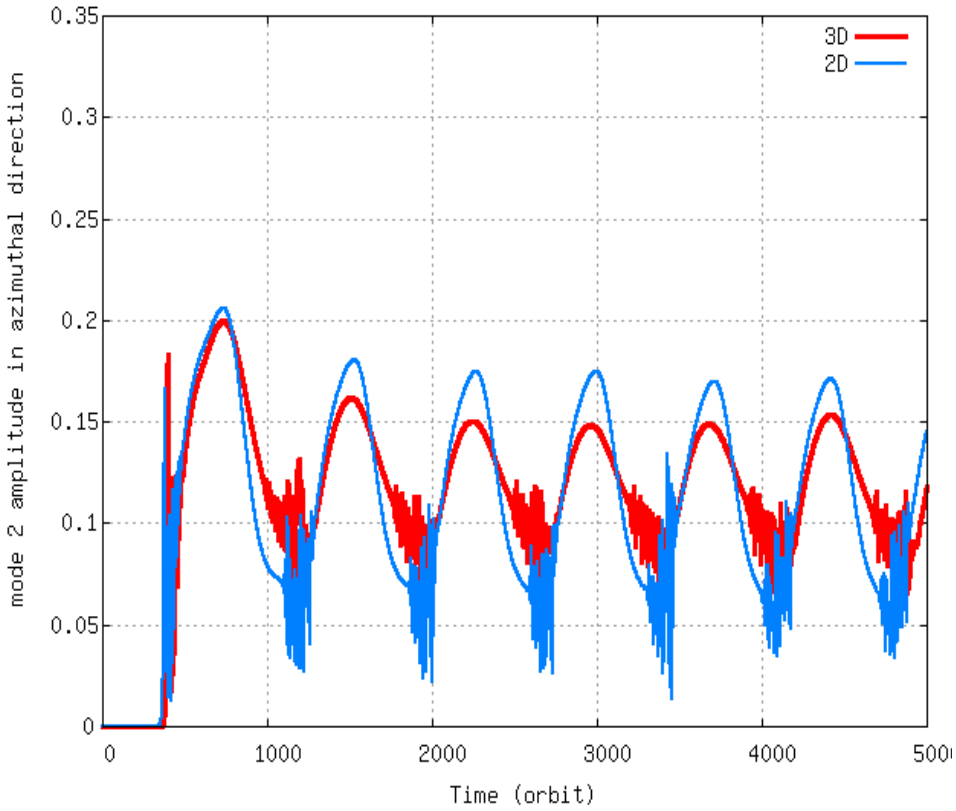
Surface density


ρr at t=6400 orbits
$z(H_g)$
r
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
0
1
2
3
4
5
6
7
1.35
1.20
1.05
0.90
0.75
0.60
0.45
0.30
0.15
0.00

Evolution of m=1 and m=2 modes in 2D vs. 3D



m = 1



m = 2

Summary

- Many transition disks show non-axisymmetric features
- These features facilitate dust accumulation, helpful for planet formation?
- Several mechanisms can produce non-axisymmetric features in disks: giant planets, variable disk viscosity
- Lifetime of vortices sensitive to disk conditions and planet masses, typically ~ 1000 orbits, rarely up to ~10,000 orbits
- Future ALMA observations will help to constrain the disk models