

# Surface Wave Effects on Oceanic Fronts, Filaments, and Turbulence

**Baylor Fox-Kemper (Brown Geo.)**

with Jim McWilliams (UCLA), Qing Li (Brown Geo), Nobu Suzuki (Brown Geo), and Sean Haney (CU-ATOC),

Expanding on past work with:

Peter Hamlington (CU-Boulder), Luke Van Roekel (Northland College),  
Adrean Webb (TUMST), Keith Julien (CU-APPM), Greg Chini (UNH),  
Peter Sullivan (NCAR), Mark Hemer (CSIRO)

Kavli Institute for Theoretical Physics

Sponsors: NSF 1258907, 1245944, 0934737, NASA NNX09AF38G

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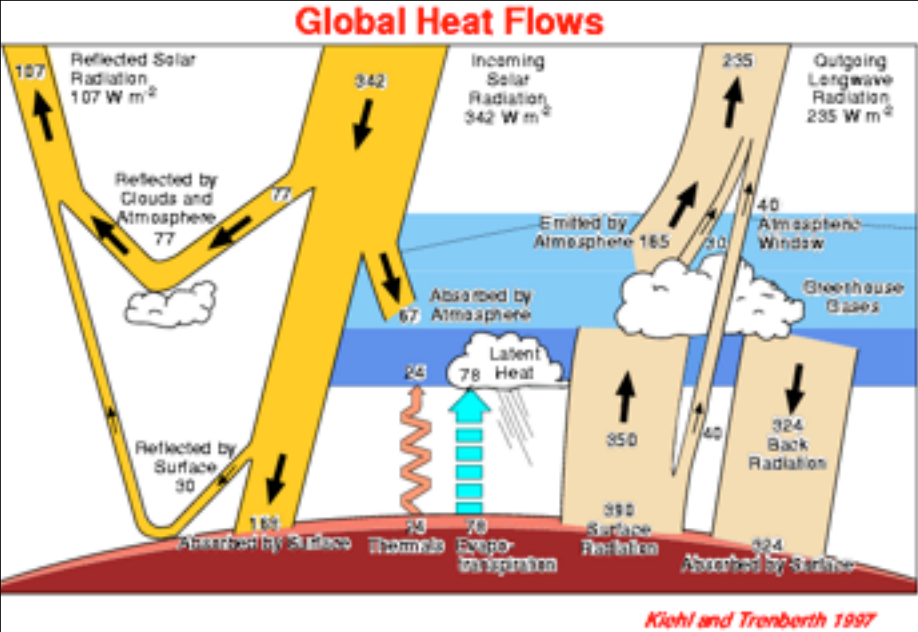
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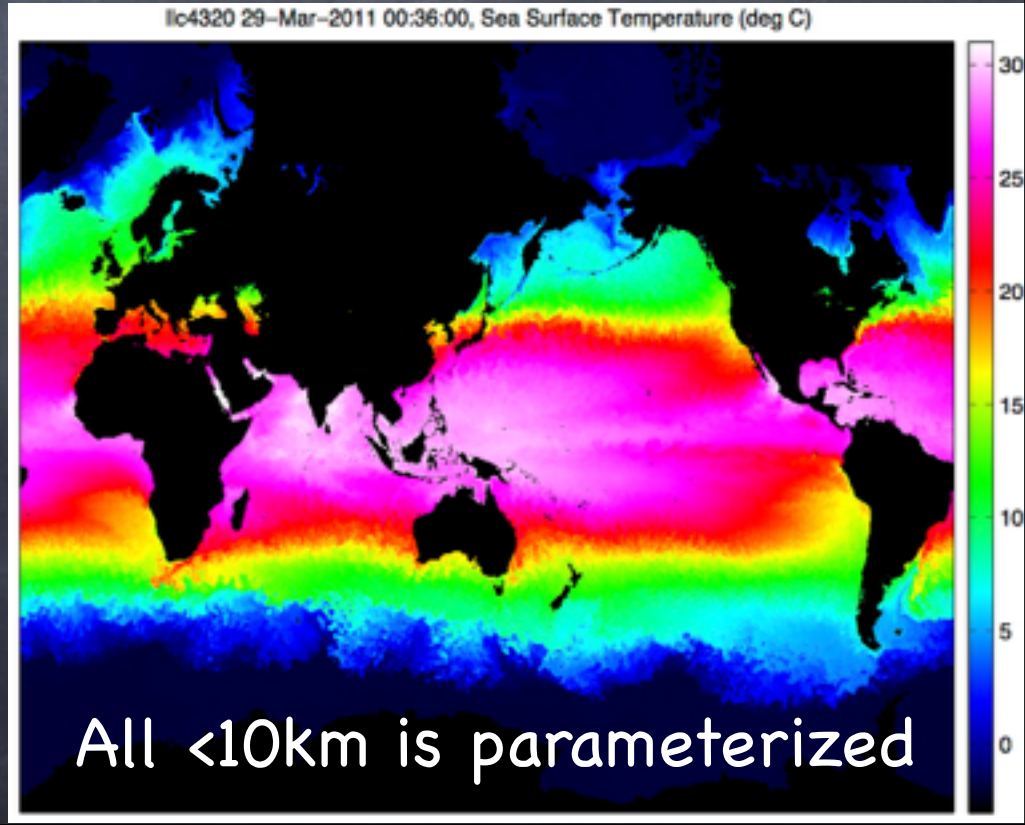
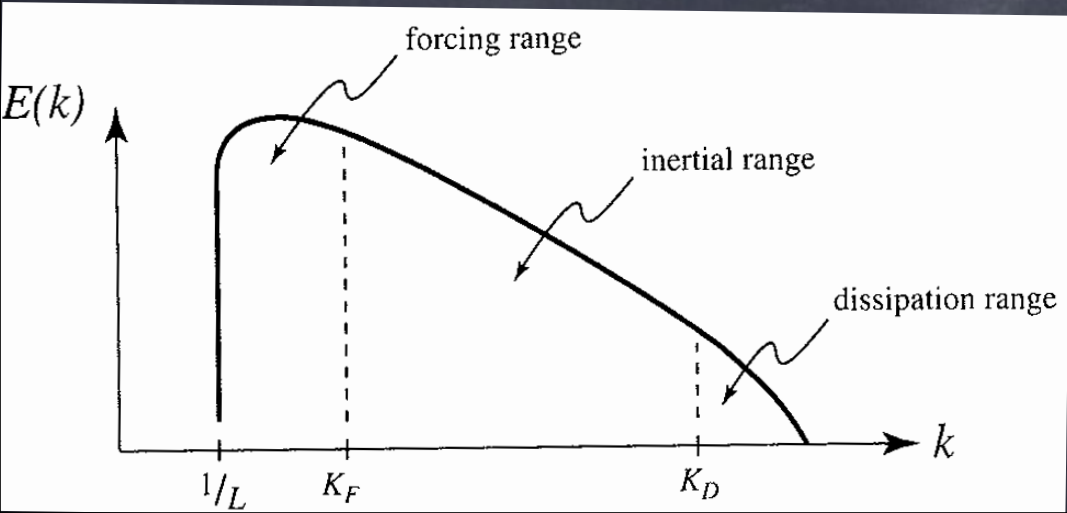


The Earth's Climate System is forced by the Sun on a global scale (24,000km)



Next-gen. ocean climate models simulate globe to 10km: Mesoscale Ocean Large Eddy Simulations (MOLES)

Turbulence cascades to scales about 10 billion times smaller  $O(1\text{mm})$

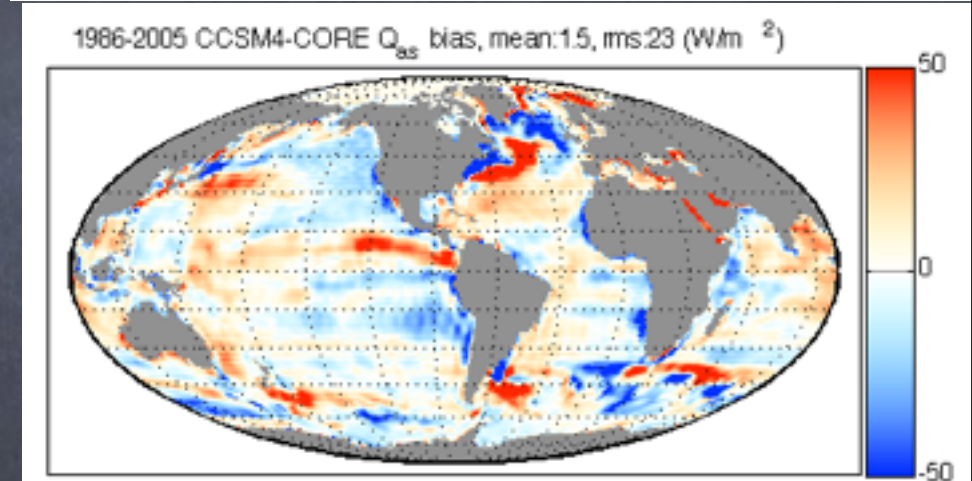
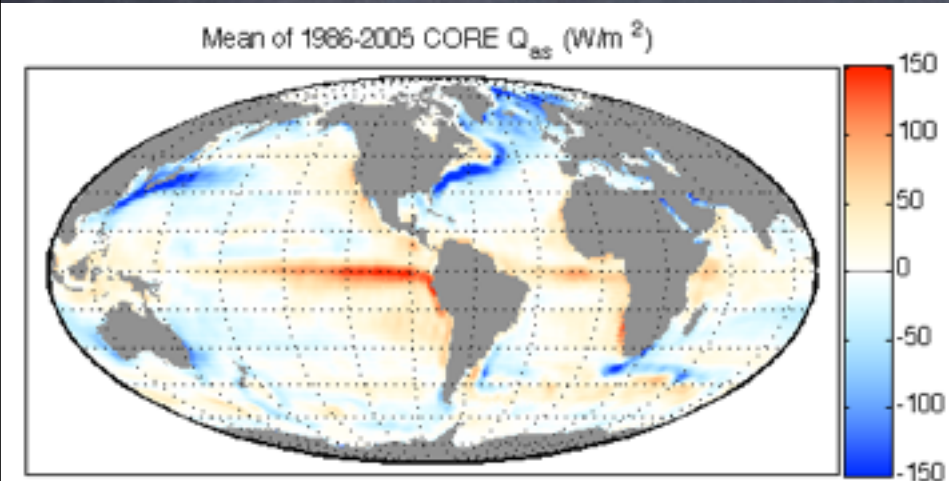
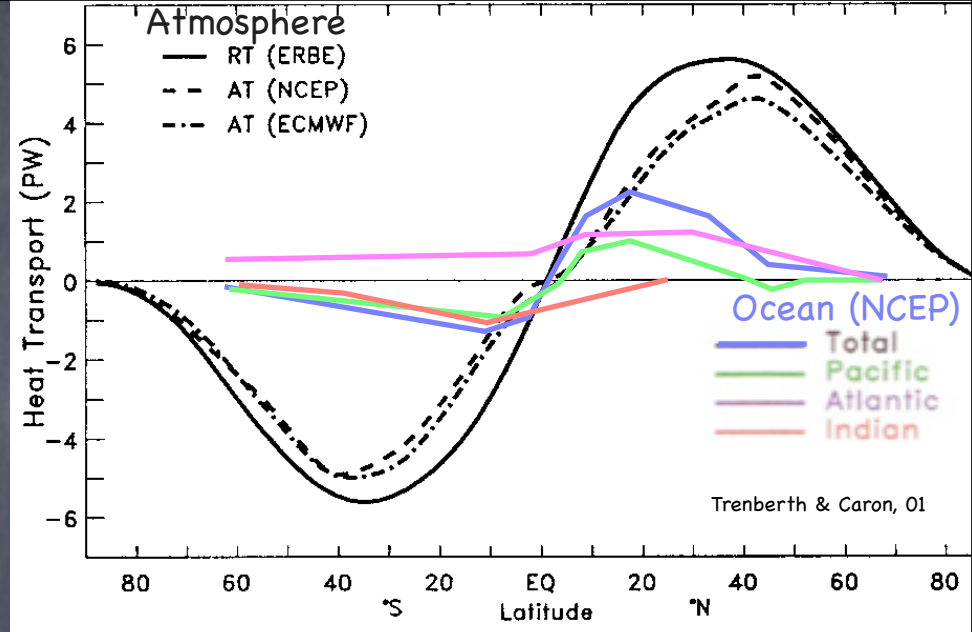


# Air-Sea Flux Errors vs. Data

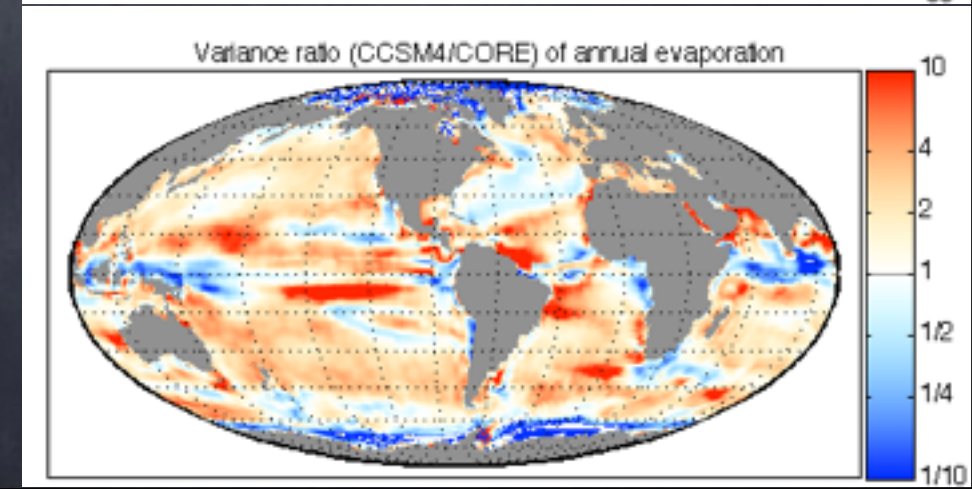
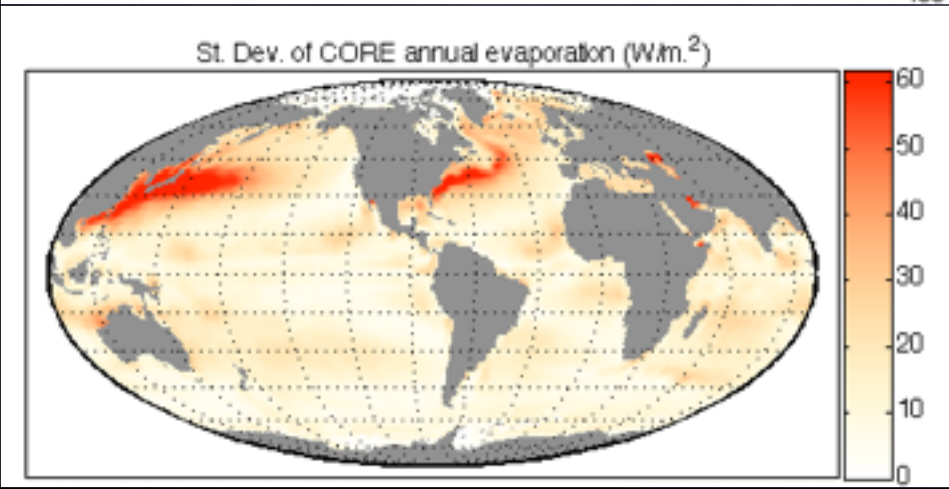
Heat capacity & mode of transport is different in A vs. O

>90% of GW is oceanic, 10m O=whole A

S. C. Bates, B. Fox-Kemper, S. R. Jayne, W. G. Large, S. Stevenson, and S. G. Yeager. Mean biases, variability, and trends in air-sea fluxes and SST in the CCSM4. *Journal of Climate*, 25(22):7781-7801, 2012.



Mean



Annual  
9-15mo

With nearly incompressible (small density variations)  
 approximation & approximated rotating Earth:  
 A set of 5 vars

## Summary of Boussinesq Equations

$$\frac{D?}{Dt} \equiv \frac{\partial ?}{\partial t} + \mathbf{v} \cdot \nabla ?$$

The simple Boussinesq equations are, for an inviscid fluid:

momentum equations: 
$$\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla\phi + b\mathbf{k}, \quad (\text{B.1})$$

mass conservation: 
$$\nabla \cdot \mathbf{v} = 0, \quad (\text{B.2})$$

buoyancy equation: 
$$\frac{Db}{Dt} = \dot{b}. \quad (\text{B.3})$$

Vallis, 06

If you want, it's easy to distinguish buoyancy into contributions from Temperature and from Salinity

Don't blame me that they are inviscid—they are the vallis equations...

# Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models  
inhabits a special distinguished limit:

Inviscid ( $Re \gg 1$ ), rapidly rotating ( $Ro \ll 1$ ), and thin\* ( $L \gg H$ )

## Full Momentum

$$\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla\phi + b\mathbf{k} + \nu\nabla^2\mathbf{v}$$

$$Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri = \frac{\frac{\partial b}{\partial z}}{\left(\frac{\partial U}{\partial z}\right)^2} \quad \alpha = H/L$$

\*closely related to strong stratification & ocean dimensions

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(Horizontal) Geostrophic Balance

$$\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla\phi + b\mathbf{k} + \nu\nabla^2\mathbf{v}$$

$$Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri = \frac{\frac{\partial b}{\partial z}}{\left(\frac{\partial U}{\partial z}\right)^2} \quad \alpha = H/L$$

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(Vertical) Hydrostatic Balance

$$\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla\phi + b\mathbf{k} + \nu\nabla^2\mathbf{v}$$

$$Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri = \frac{\frac{\partial b}{\partial z}}{\left(\frac{\partial U}{\partial z}\right)^2} \quad \alpha = H/L$$

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# Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models  
inhabits a special distinguished limit:

Inviscid ( $Re \gg 1$ ), rapidly rotating ( $Ro \ll 1$ ), and thin\* ( $L \gg H$ )

(Combined) Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b$$

Taken together with the forcing (air-sea) of buoyancy  
and the advection of buoyancy by this flow--you have  
the tools to study large-scale ocean physics!

# Dimensionless Boussinesq

## Spanning Mesoscale to Stratified Turbulence

McWilliams (85)

$$Ro [v_{i,t} + v_j v_{i,j}] + \frac{M_{Ro}}{Ri} w v_{i,z} + \epsilon_{izj} v_j = -M_{Ro} \pi_{,i} + \frac{Ro}{Re} v_{i,jj}$$

$$\frac{\alpha^2}{Ri} \left[ w_{,t} + v_j w_{,j} + \frac{M_{Ro}}{Ro Ri} w w_{,z} \right] = -\pi_{,z} + b + \frac{\alpha^2}{Re Ri} w_{,jj}$$

$$b_t + v_j b_{,j} + \frac{M_{Ro}}{Ro Ri} w b_z + w = 0$$

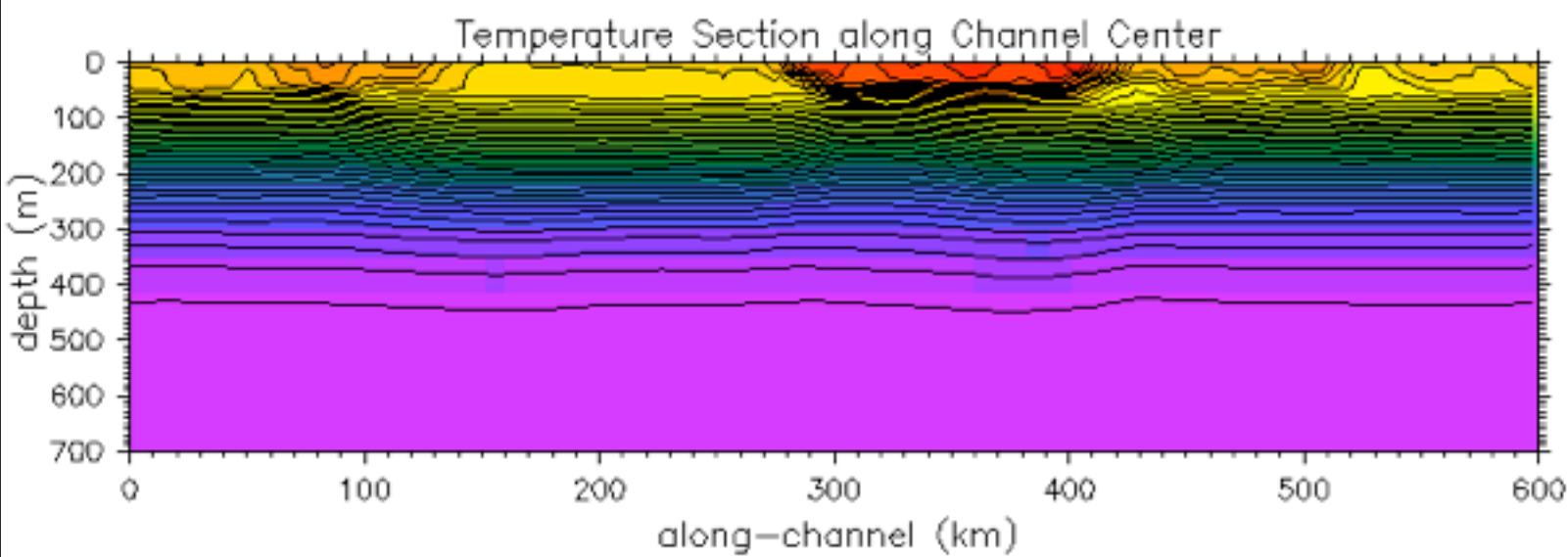
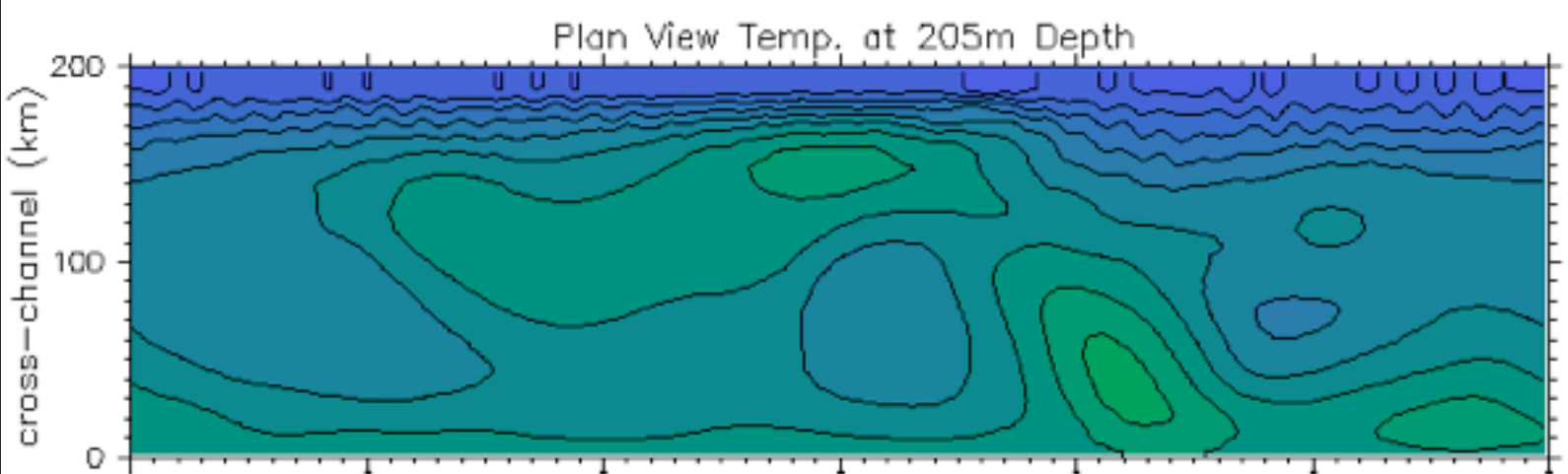
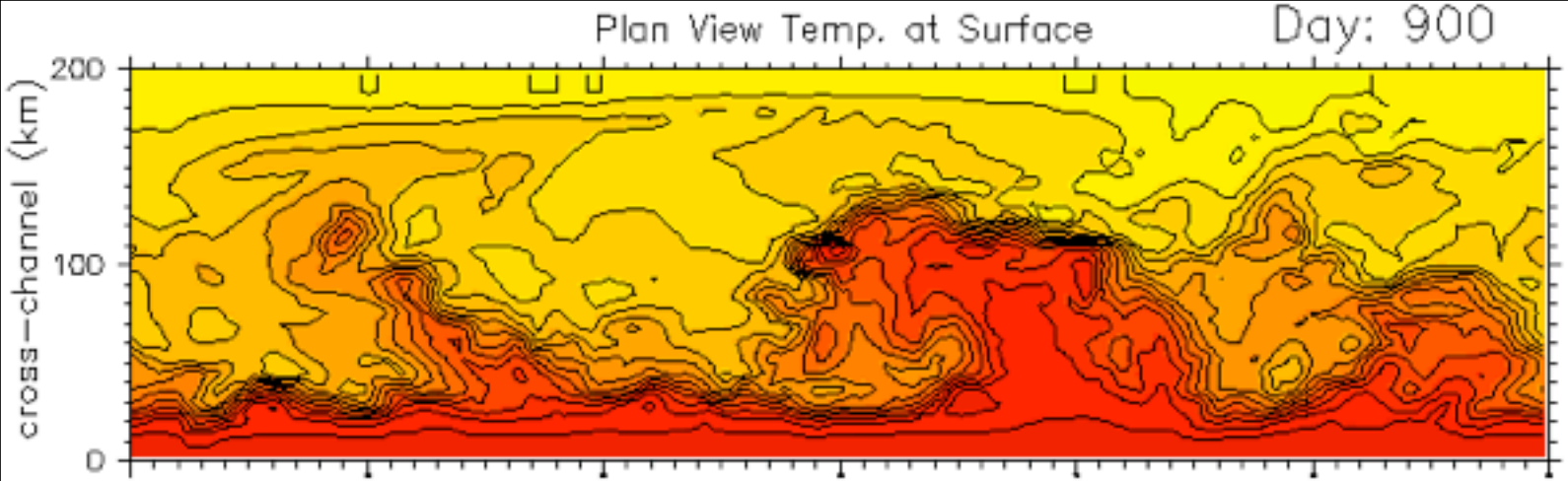
$$v_{j,j} + \frac{M_{Ro}}{Ro Ri} w_z = 0$$

Plus boundary conditions

$$Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri = \frac{N^2}{(U_{,z})^2} \quad \alpha = H/L$$

$$M_{Ro} \equiv \max(1, Ro) \quad v = \text{horiz. vel.} \quad w = \text{vert. vel.}$$

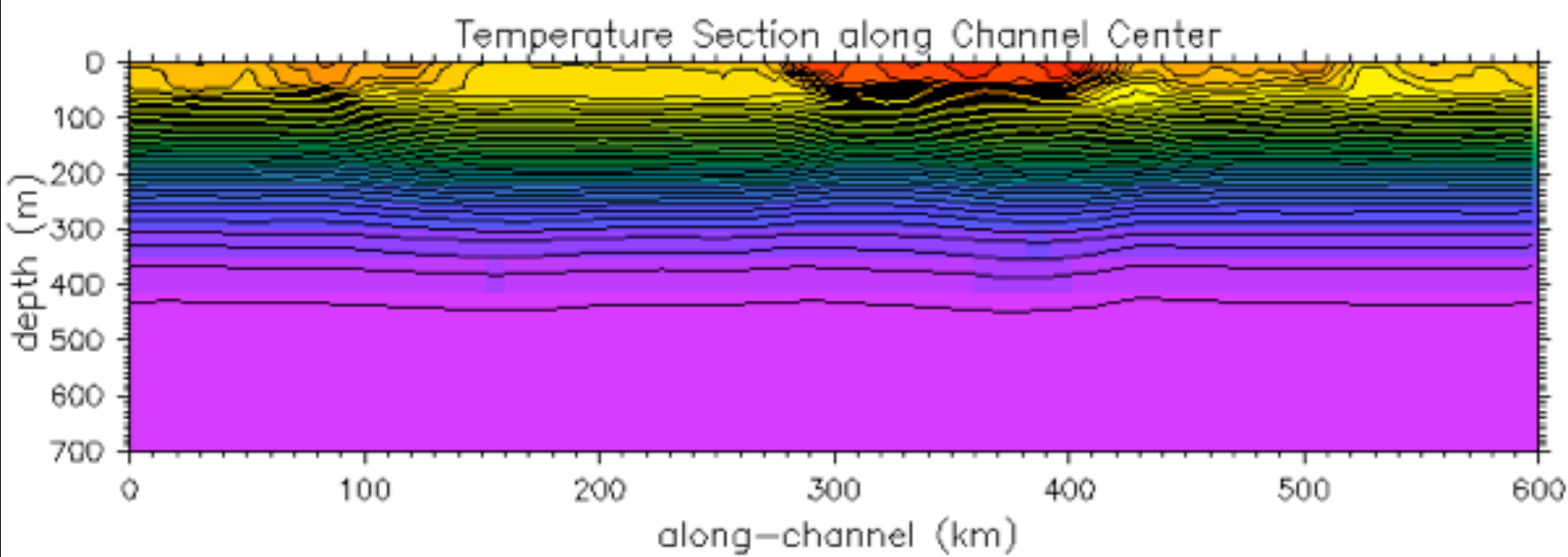
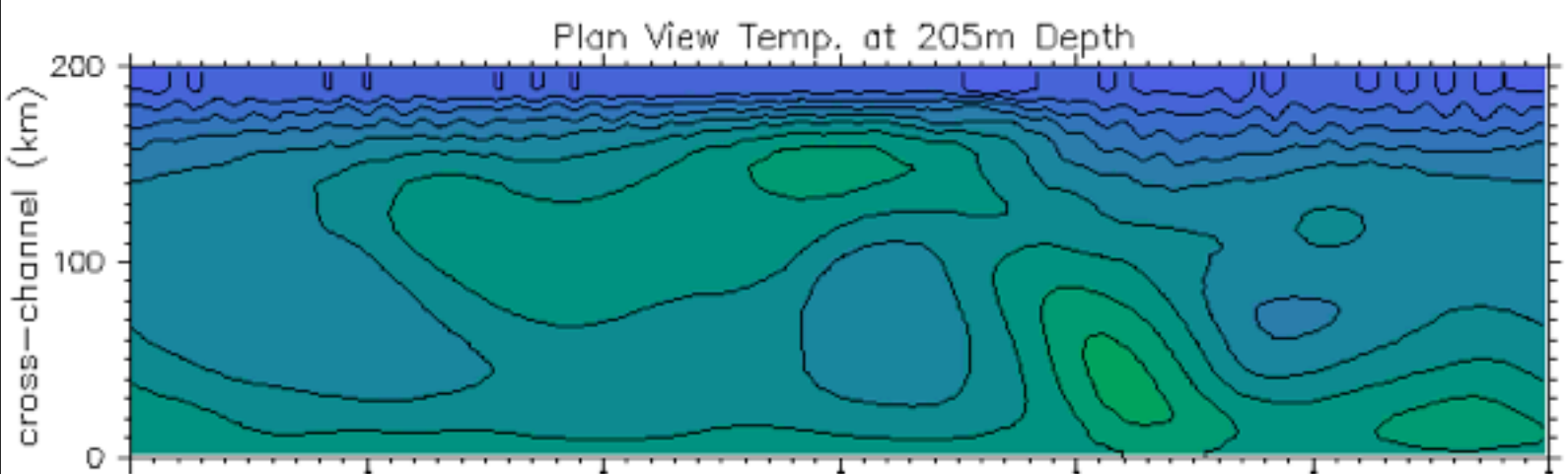
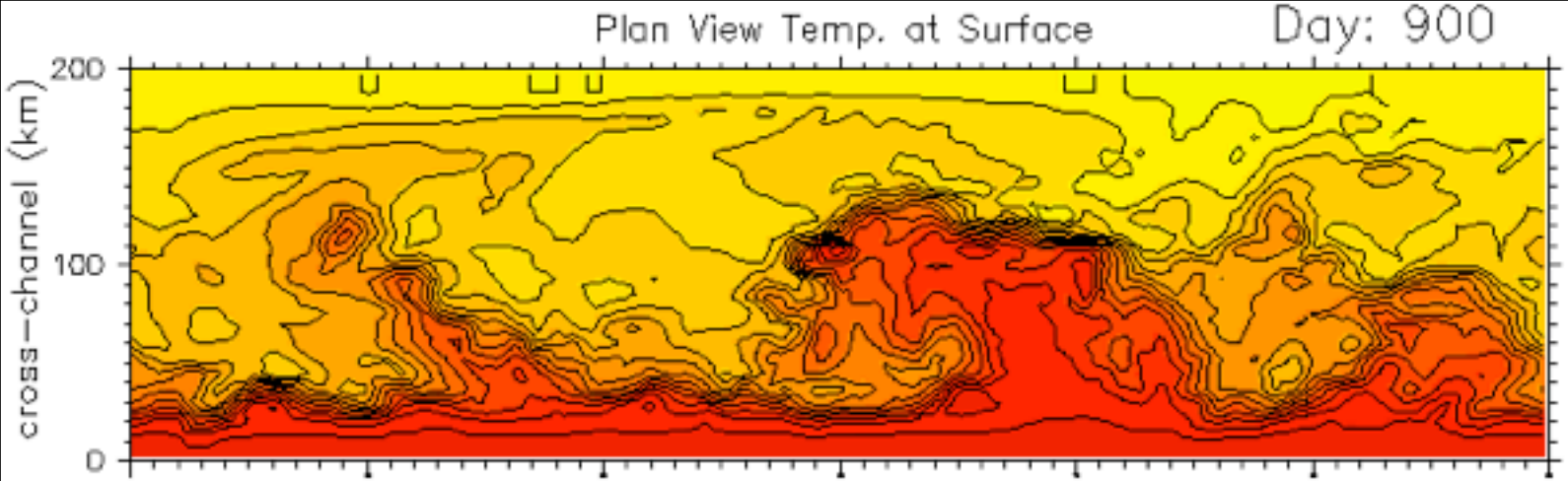
Let's see some examples of  
Bousinesq, Hydrostatic Models  
at work in the  
mesoscale (10–100km) &  
submesoscale (100m–10km)



Big, Deep  
(mesoscale)  
w/  
Little,  
Shallow  
(submeso)

Note mixed  
layer heat  
capacity!

B. Fox-Kemper, R. Ferrari,  
and R. W. Hallberg.  
Parameterization of mixed  
layer eddies. Part I: Theory  
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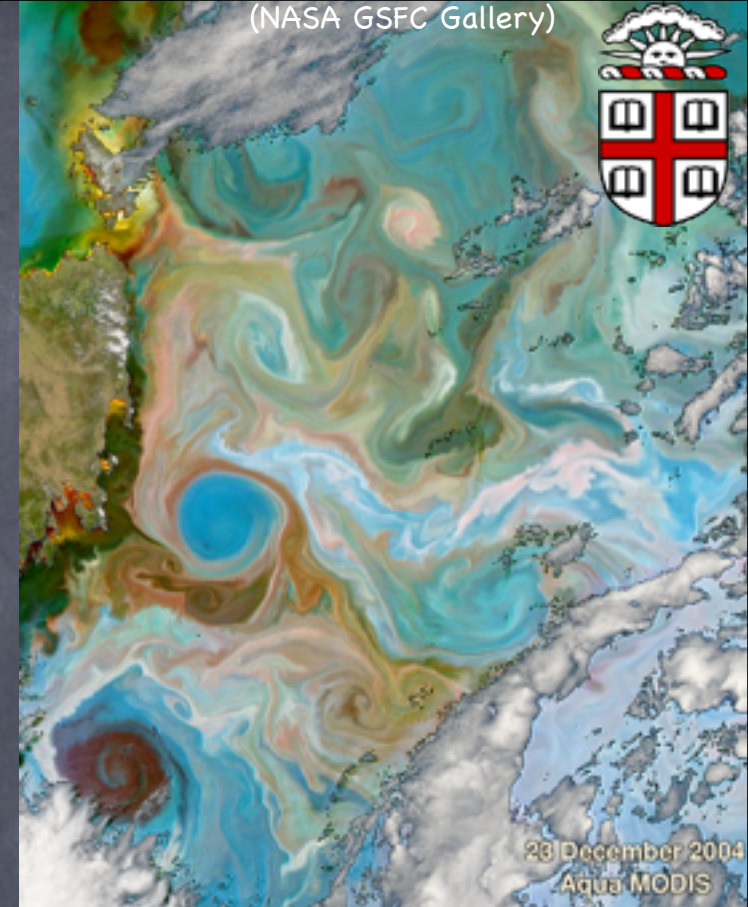
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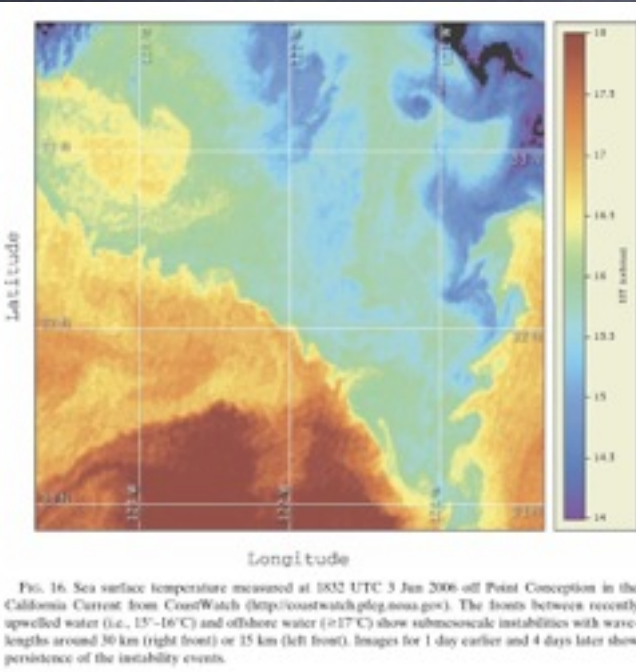
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# The Character of the Mesoscale

← 100 km



(Capet et al., 2008)



- Boundary Currents
- Eddies
- $Ro=O(0.1)$
- $Ri=O(1000)$
- Full Depth (4km)
- Eddies strain to produce Fronts
- 100km, months

Eddy processes mainly **baroclinic & barotropic instability**. Parameterizations of baroclinic instability (GM, Visbeck...), will be routinely resolved in climate models in 2040

# The Character of the Submesoscale

(Capet et al., 2008)

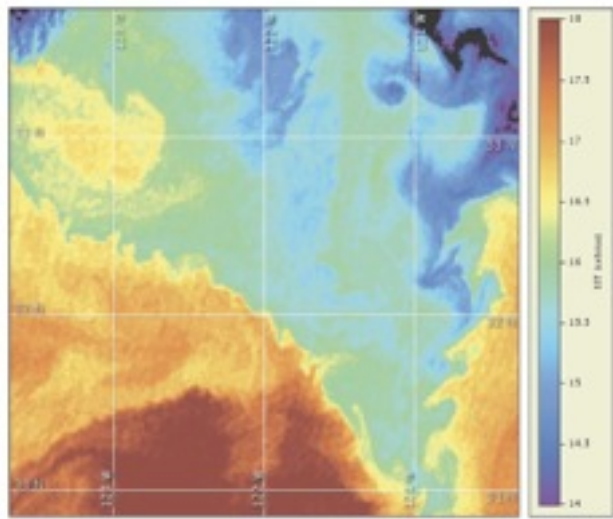
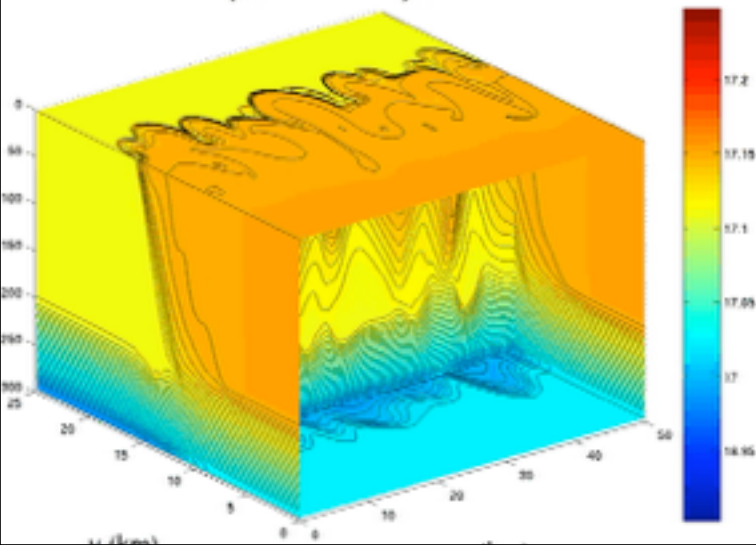


FIG. 16. Sea surface temperature measured at 1832 UTC 3 Jun 2006 off Point Conception in the California Current from CoastWatch (<http://coastwatch.pfeg.noaa.gov>). The fronts between recently

Temperature on day:17.375



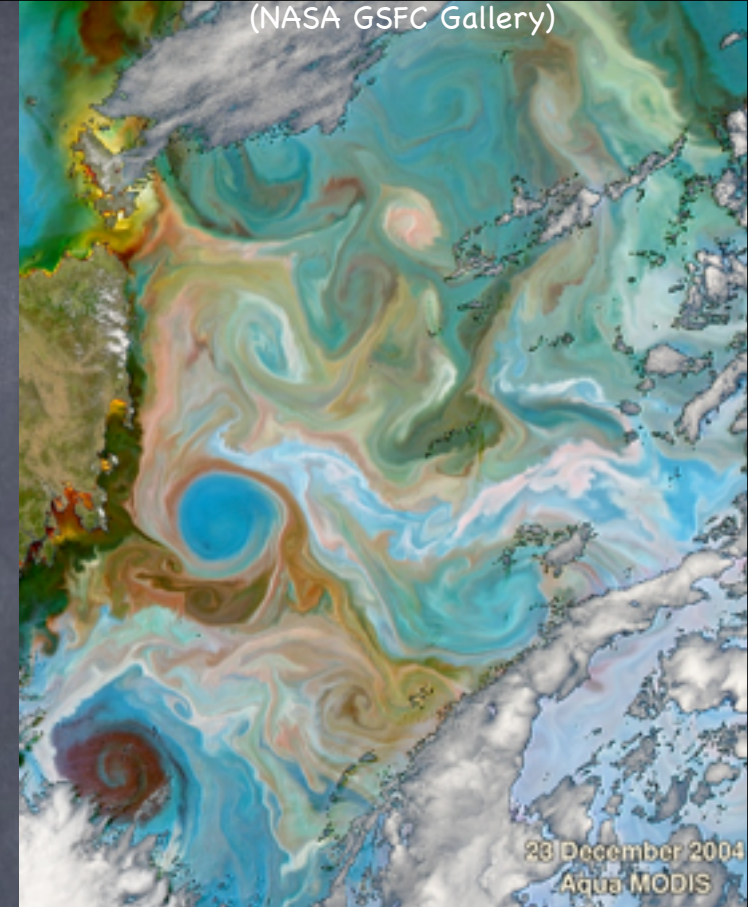
- Fronts
- Eddies
- $Ro=O(1)$
- $Ri=O(1)$
- near-surface ( $H=100m$ )
- 1-10km, days

Eddy processes often **baroclinic instability**

Parameterizations = F-K et al (08-11).

Routinely resolved in 2100

←  
10  
km



B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. *Journal of Physical Oceanography*, 38(6):1145-1165, 2008

S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. *Ocean Modelling*, 64:12-28, 2013

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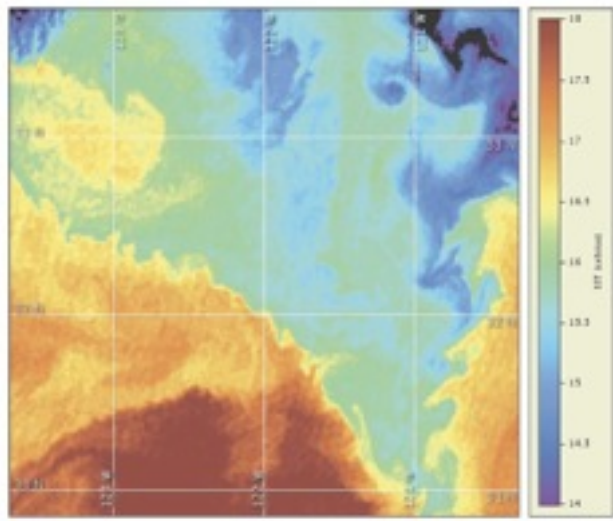
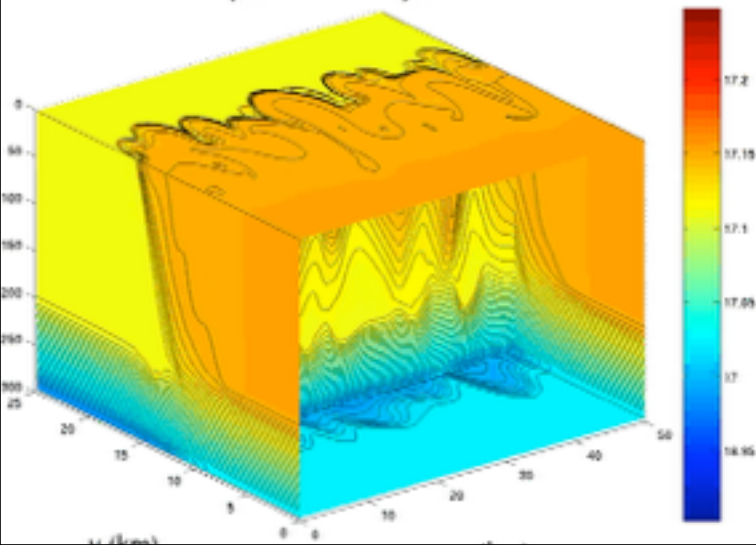


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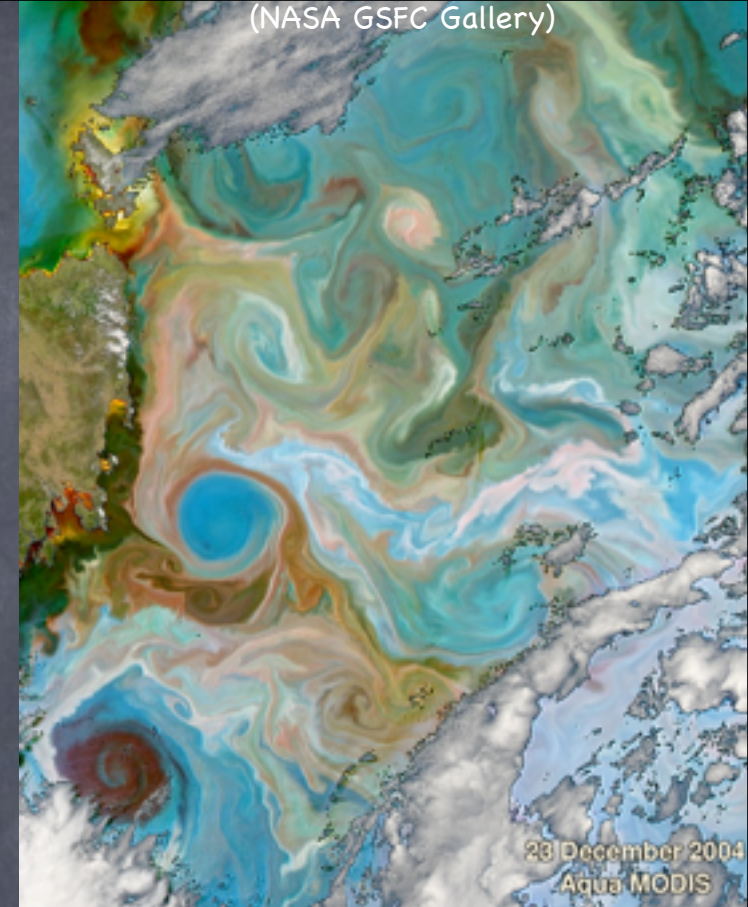
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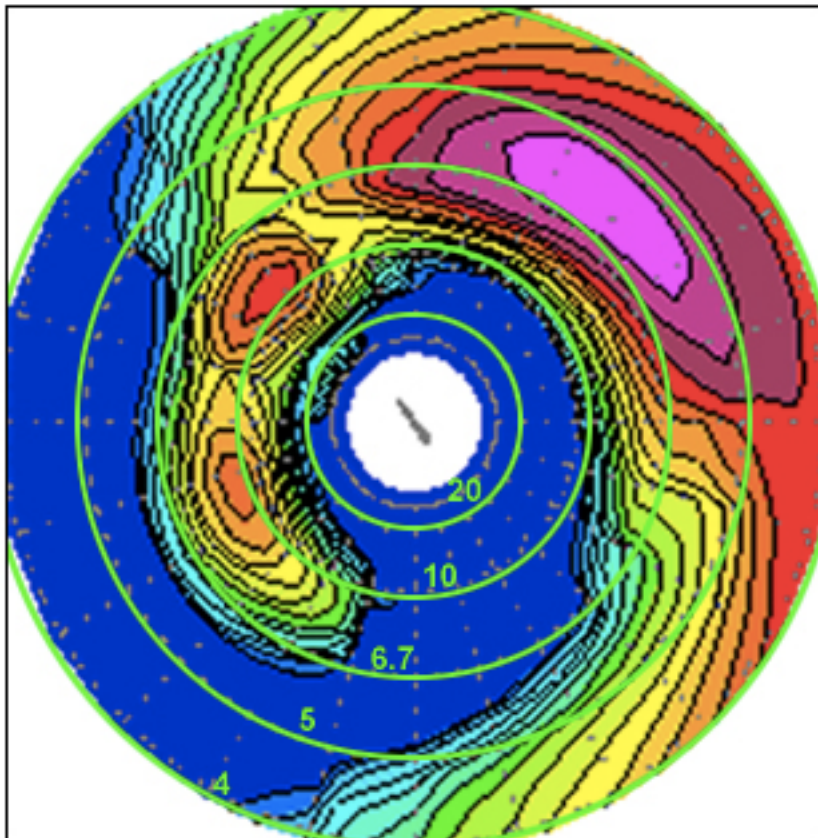


- We can study a small-scale system, derive parameterizations, and then use them to improve climate models & assess impact globally
  - This process often relies heavily on thermal wind scaling relationships
- But, what about the effects of things that aren't geostrophic & hydrostatic?
  - For example, waves and near-surface 3d turbulence

# Surface Waves are...

fast, small, irrotational solutions of the Boussinesq Equations

NWW3 Polar Plot of Wave Energy Spectrum at ILM01



24 hr fcst Valid 0000 UTC 26 Apr 2002

NOAA / NWS / NCEP / MMAB

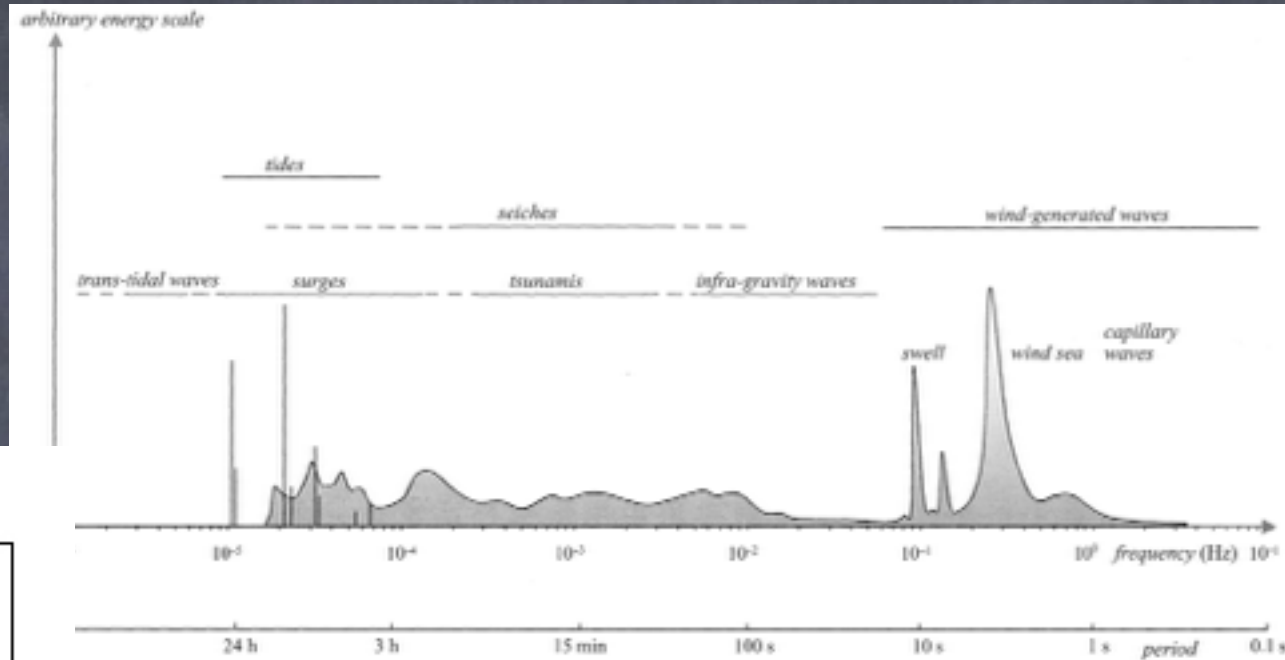


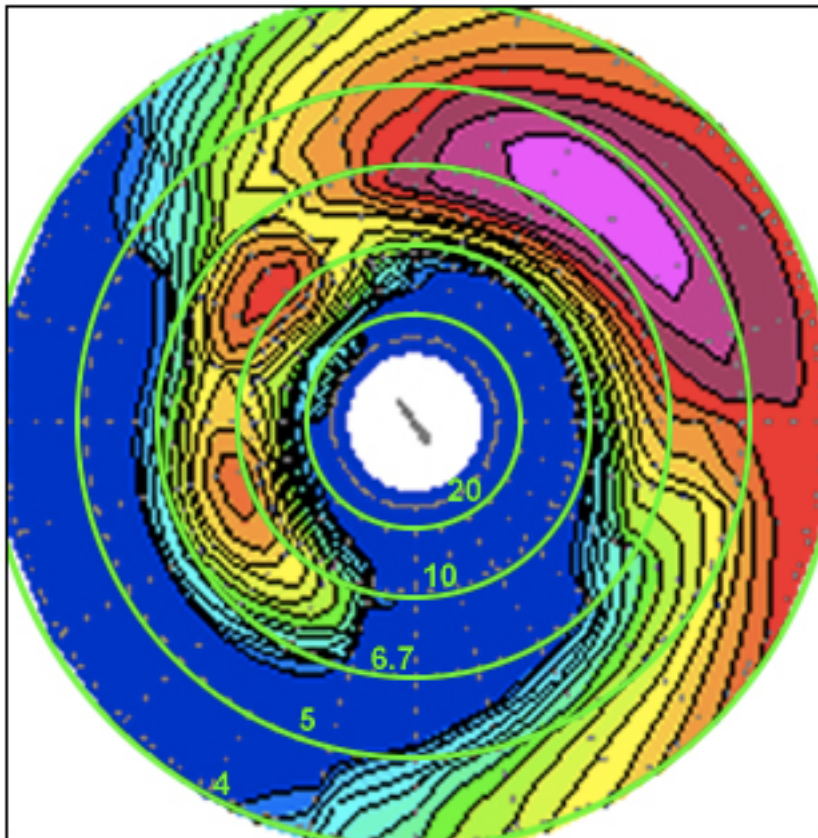
Illustration of wave spectra from different types of ocean surface waves (Holthuijsen, 2007)



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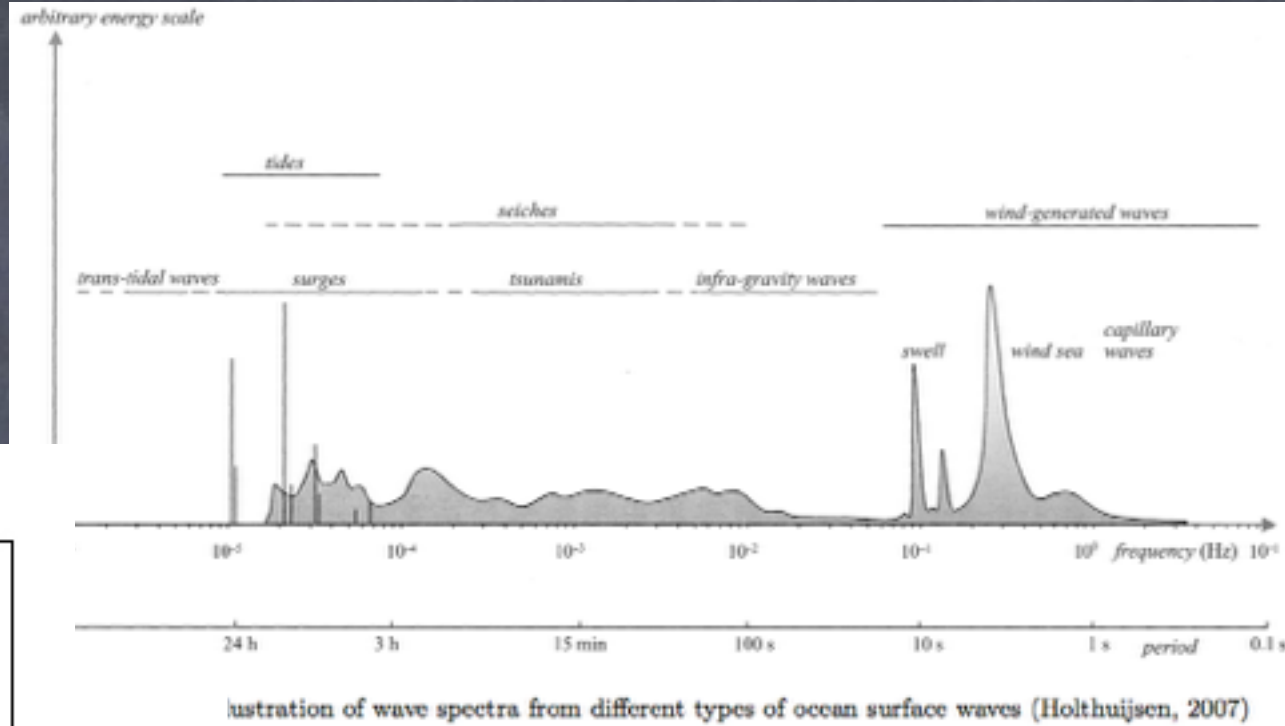


Illustration of wave spectra from different types of ocean surface waves (Holthuijsen, 2007)



# Craik-Leibovich Boussinesq Or Wave-Averaged Eqtns

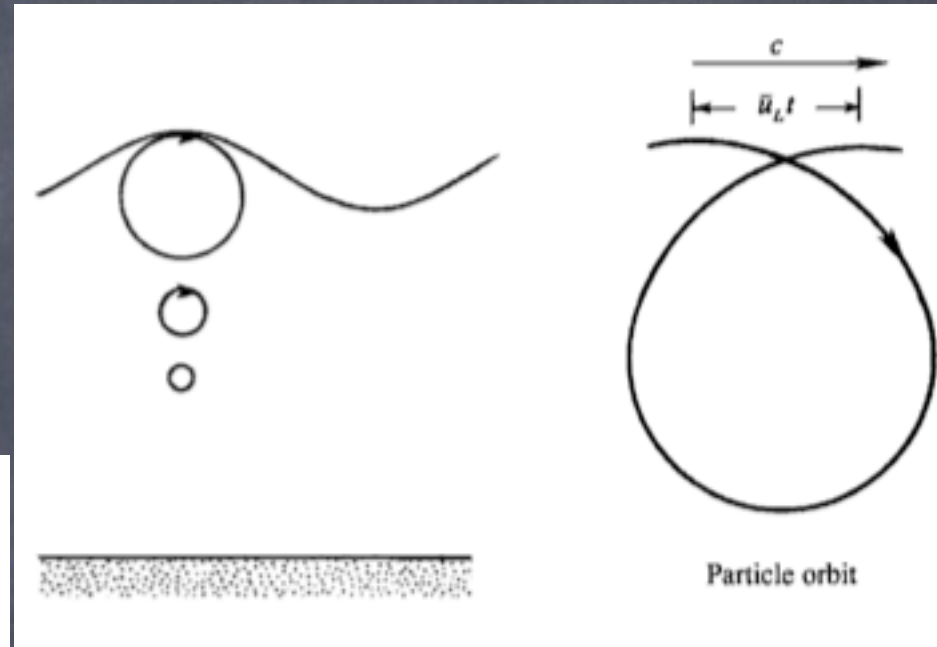
- Formally a multiscale asymptotic equation set:
  - 3 classes: Small, Fast; Large, Fast; Large, Slow
  - Solve first 2 types of motion in the case of limited slope ( $ka$ ), irrotational  $\rightarrow$  Deep Water Waves!
  - Average over deep water waves in space & time,
  - Arrive at Large, Slow equation set.

All Wave-Mean coupling terms  
involve the Stokes Drift

# What is Stokes Drift?

Take wave solns, compare the velocity of trajectories vs. Eulerian velocity, Taylor Expand, calculate:

$$\begin{aligned} \mathbf{u}^L(\mathbf{x}_p(t), t) - \mathbf{u}^E(\mathbf{x}_p(t_0), t) &\approx [\mathbf{x}_p(t) - \mathbf{x}_p(t_0)] \cdot \nabla \mathbf{u}^E(\mathbf{x}_p(t_0), t) \\ &\approx \left[ \int_{t_0}^t \mathbf{u}^E(\mathbf{x}_p(t_0), s') ds' \right] \cdot \nabla \mathbf{u}^E(\mathbf{x}_p(t_0), t). \end{aligned}$$



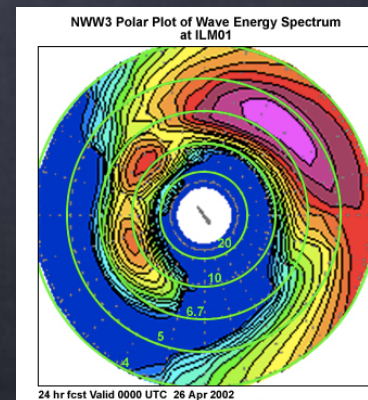
Examples:

Monochromatic:

$$\mathbf{u}^S = \hat{\mathbf{e}}^w \frac{8\pi^3 a^2 f_p^3}{g} e^{\frac{8\pi^2 f_p^2}{g} z} = \hat{\mathbf{e}}^w a^2 \sqrt{gk^3} e^{2kz}.$$

Spectrum:

$$\mathbf{u}^S = \frac{16\pi^3}{g} \int_0^\infty \int_{-\pi}^\pi (\cos \theta, \sin \theta, 0) f^3 S_{f\theta}(f, \theta) e^{\frac{8\pi^2 f^2}{g} z} d\theta df.$$



# Wave-Averaged Equations

following McWilliams & F-K (13)

and Suzuki & F-K (14)

(for horizontally uniform Stokes drift)

$$\varepsilon = \frac{V^s H}{f L H_s}$$

$$Ro [v_{i,t} + v_j^L v_{i,j}] + \frac{M_{Ro}}{Ri} w v_{i,z} + \epsilon_{izj} v_j^L = -M_{Ro} \pi_{,i} + \frac{Ro}{Re} v_{i,jj}$$

$$\frac{\alpha^2}{Ri} \left[ w_{,y} + v_j^L w_{,j} + \frac{M_{Ro}}{Ro Ri} \right] = -\pi_{,z} + b + \boxed{\varepsilon v_j^L v_{j,z}^s} + \frac{\alpha^2}{Re Ri} w_{,jj}$$

$$b_t + v_j^L b_{,j} + \frac{M_{Ro}}{Ro Ri} w b_z + w = 0$$

$$v_{j,j} + \frac{M_{Ro}}{Ro Ri} w_z = 0$$

Plus boundary  
conditions

LAGRANGIAN (Eulerian+Stokes) advection of Eulerian momentum

Coriolis Effect is on LAGRANGIAN velocity

Stokes Shear Force is NEW in vertical momentum equation.

# The Character of the Langmuir Scale

image:  
Thorpe, 04

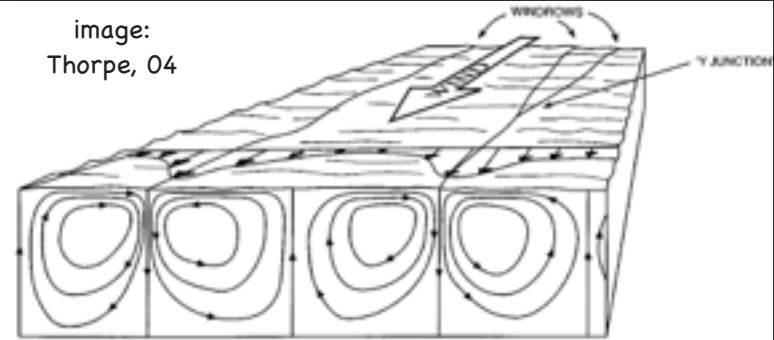


Figure 1 Sketch showing the pattern of mean flow in idealized Langmuir circulation. The windrows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). In practice the flow is turbulent, especially near the water surface, and the windrows (Figure 2) amalgamate and meander in space and time. Bands of bubbles or buoyant algae may form within the downward-going (or downwelling) flow (see Figure 3).

- Near-surface

- Langmuir Cells & Langmuir Turb.

- $Ro \gg 1$

- $Ri < 1$ : Nonhydro

- 1-100m ( $H=L$ )

- 10s to 1hr

- $w, u = O(10\text{cm/s})$

- Stokes drift

- Eqtns: Craik-Leibovich

- Params: McWilliams & Sullivan, 2000, Van Roekel et al. 2011

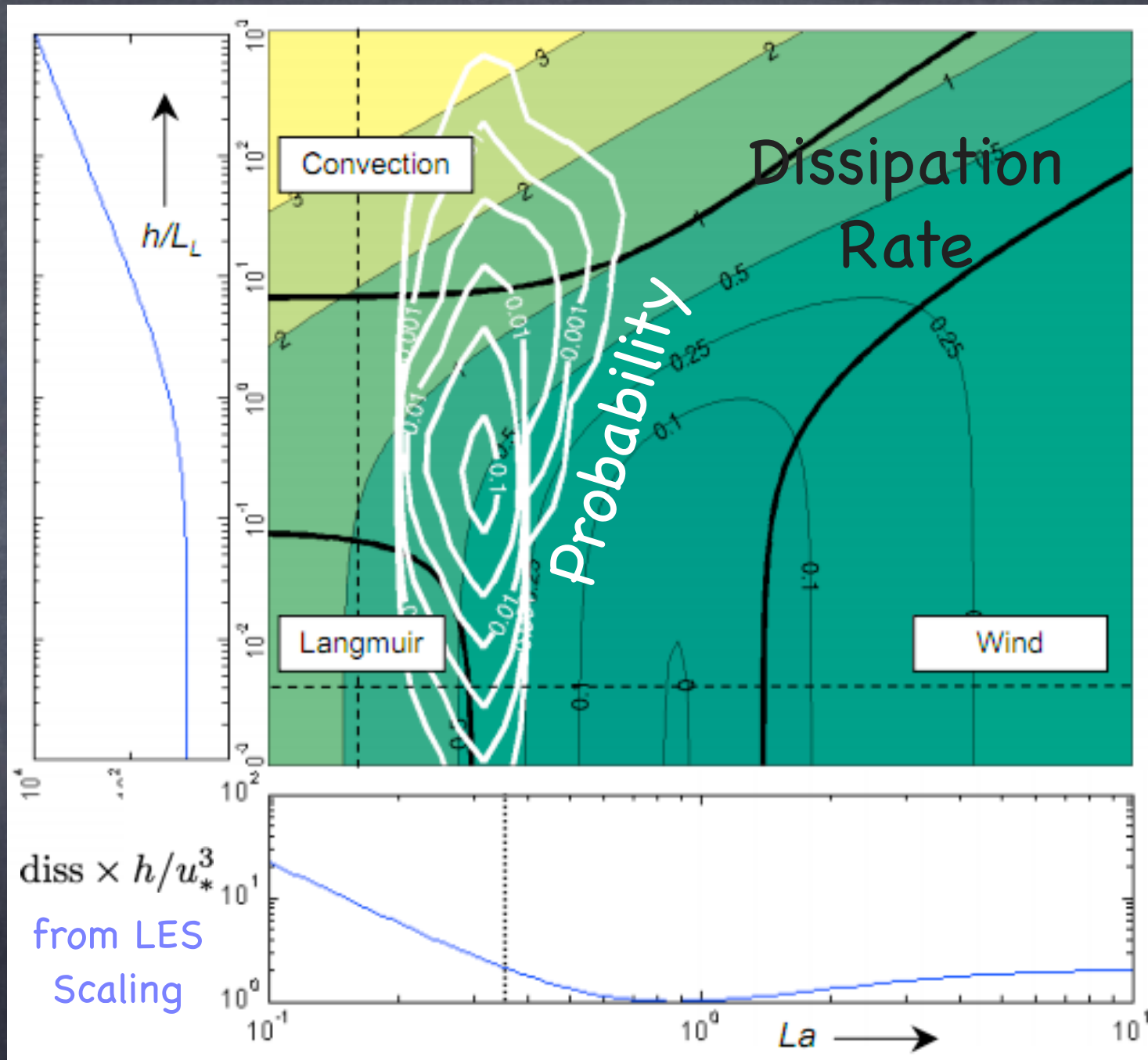
- Resolved routinely in 2170

Image: NPR.org,  
Deep Water  
Horizon Spill

Data + LES scaling,  
Southern Ocean  
mixing energy:

One way to  
estimate

So, waves  
can drive  
mixing via  
Stokes drift  
(combines  
with cooling  
& winds)



S. E. Belcher, A. A. L. M. Grant, K. E. Hanley, B. Fox-Kemper, L. Van Roekel, P. P. Sullivan, W. G. Large, A. Brown, A. Hines, D. Calvert, A. Rutgersson, H. Petterson, J. Bidlot, P. A. E. M. Janssen, and J. A. Polton. A global perspective on Langmuir turbulence in the ocean surface boundary layer. *Geophysical Research Letters*, 39(18):L18605, 9pp, 2012.

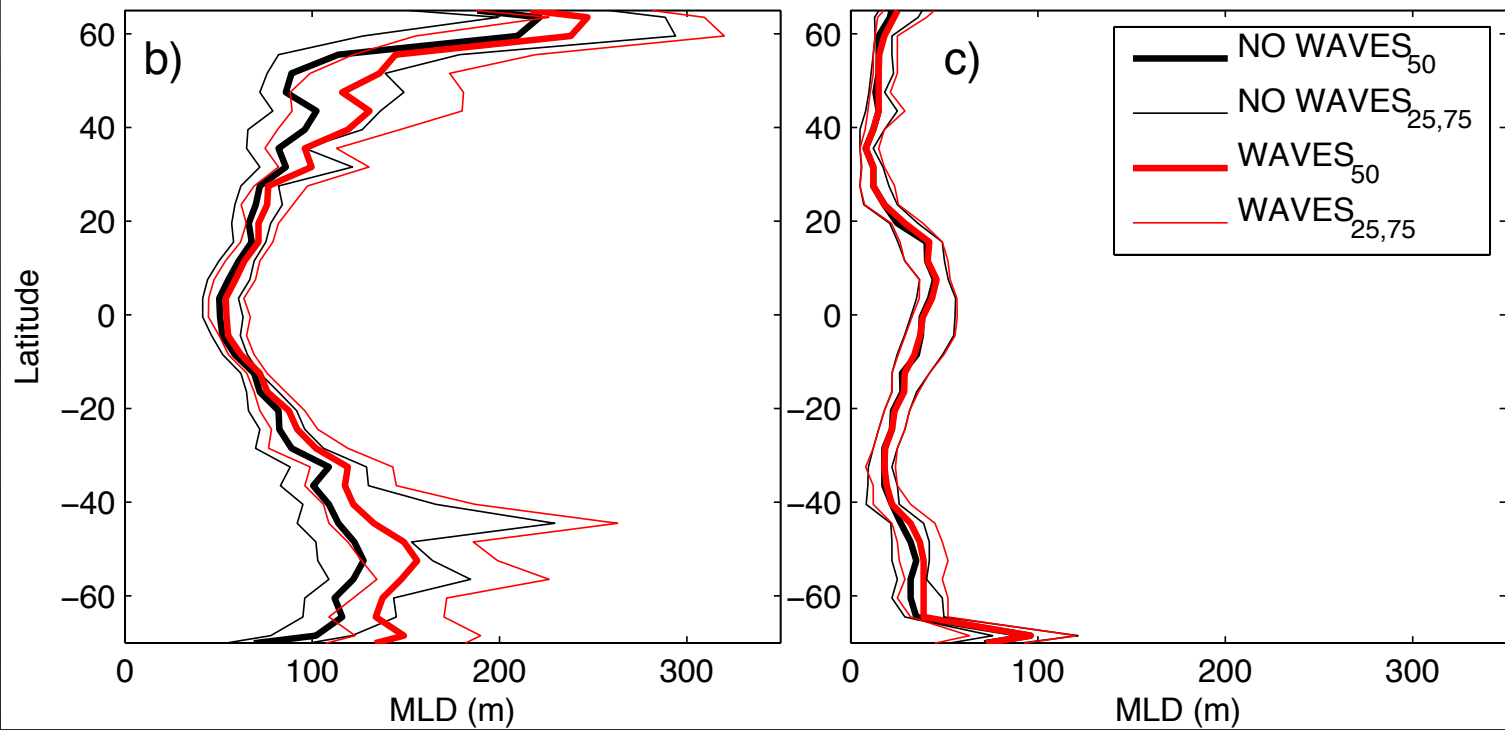
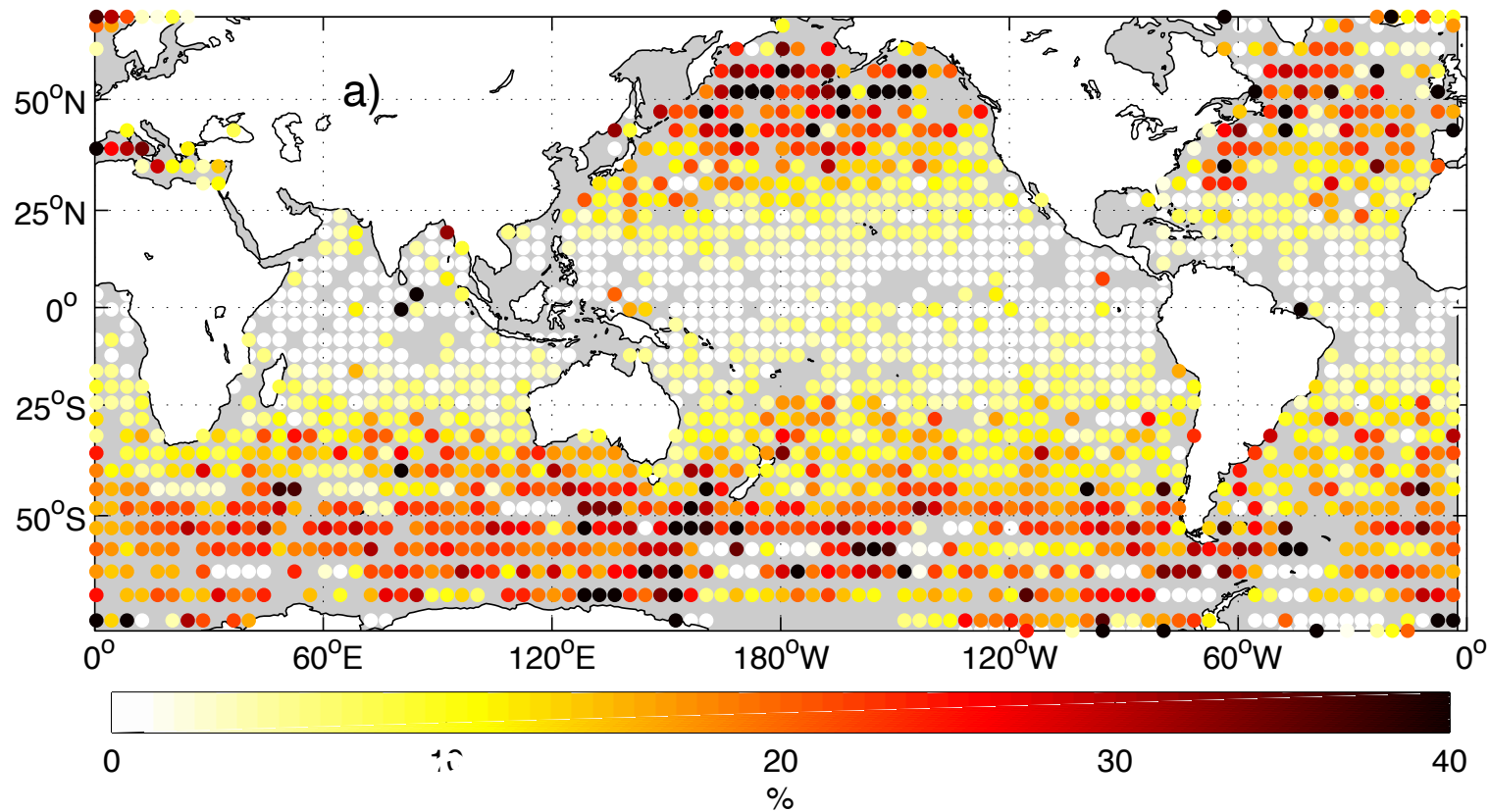


Data-driven offline  
parameterization:

Another way to  
estimate

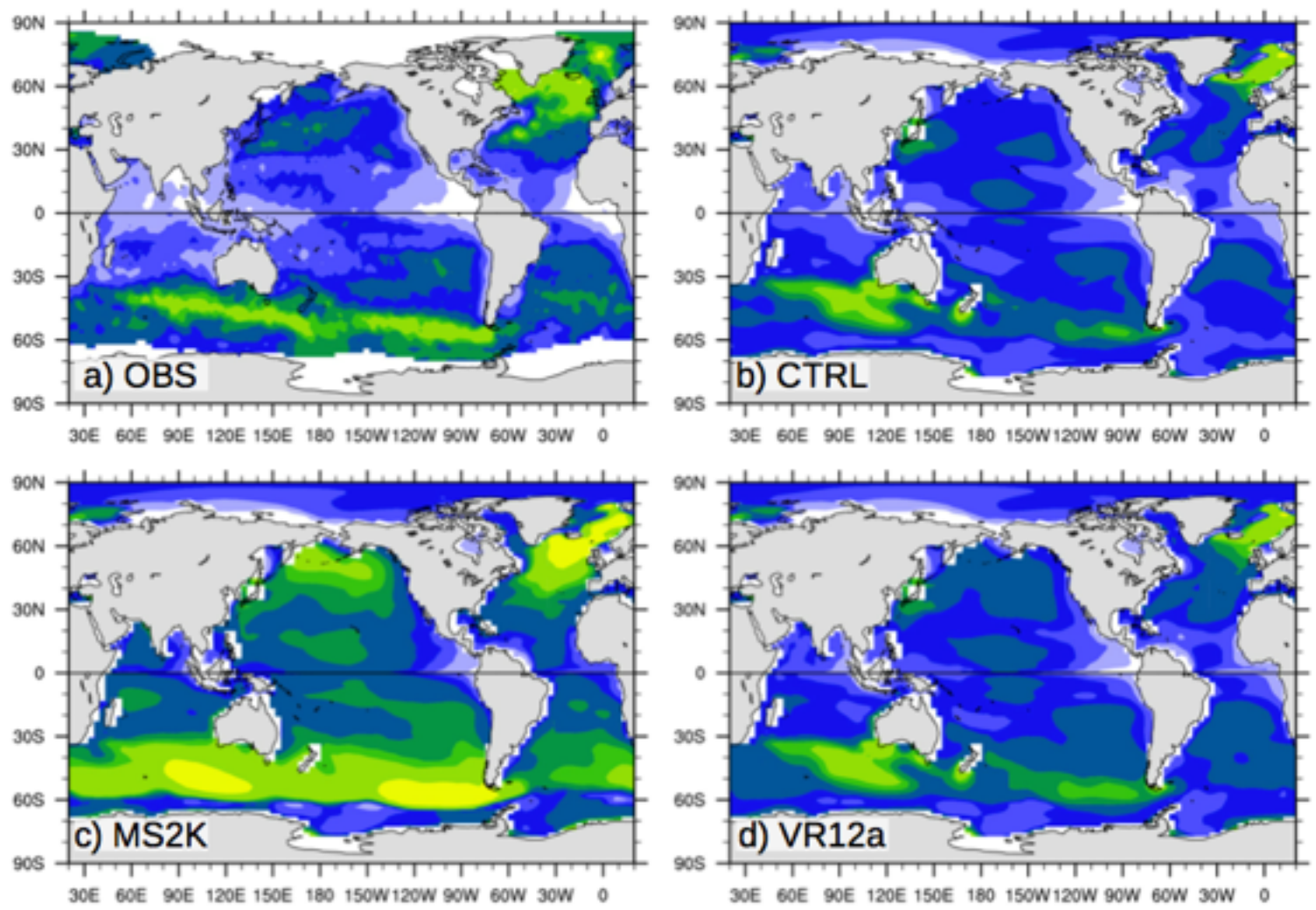
Including  
Wave-driven  
Mixing  
(Harcourt 2013)  
Deepens the  
Mixed Layer!

E. A. D'Asaro, J. Thomson, A.  
Y. Shcherbina, R. R. Harcourt,  
M. F. Cronin, M. A. Hemer,  
and B. Fox-Kemper.  
Quantifying upper ocean  
turbulence driven by surface  
waves. *Geophysical  
Research Letters*, 41(1):  
102-107, January 2014.



Parameterization in a  
climate model:  
Better estimate

# Winter Waves in NCAR Community Earth System Model

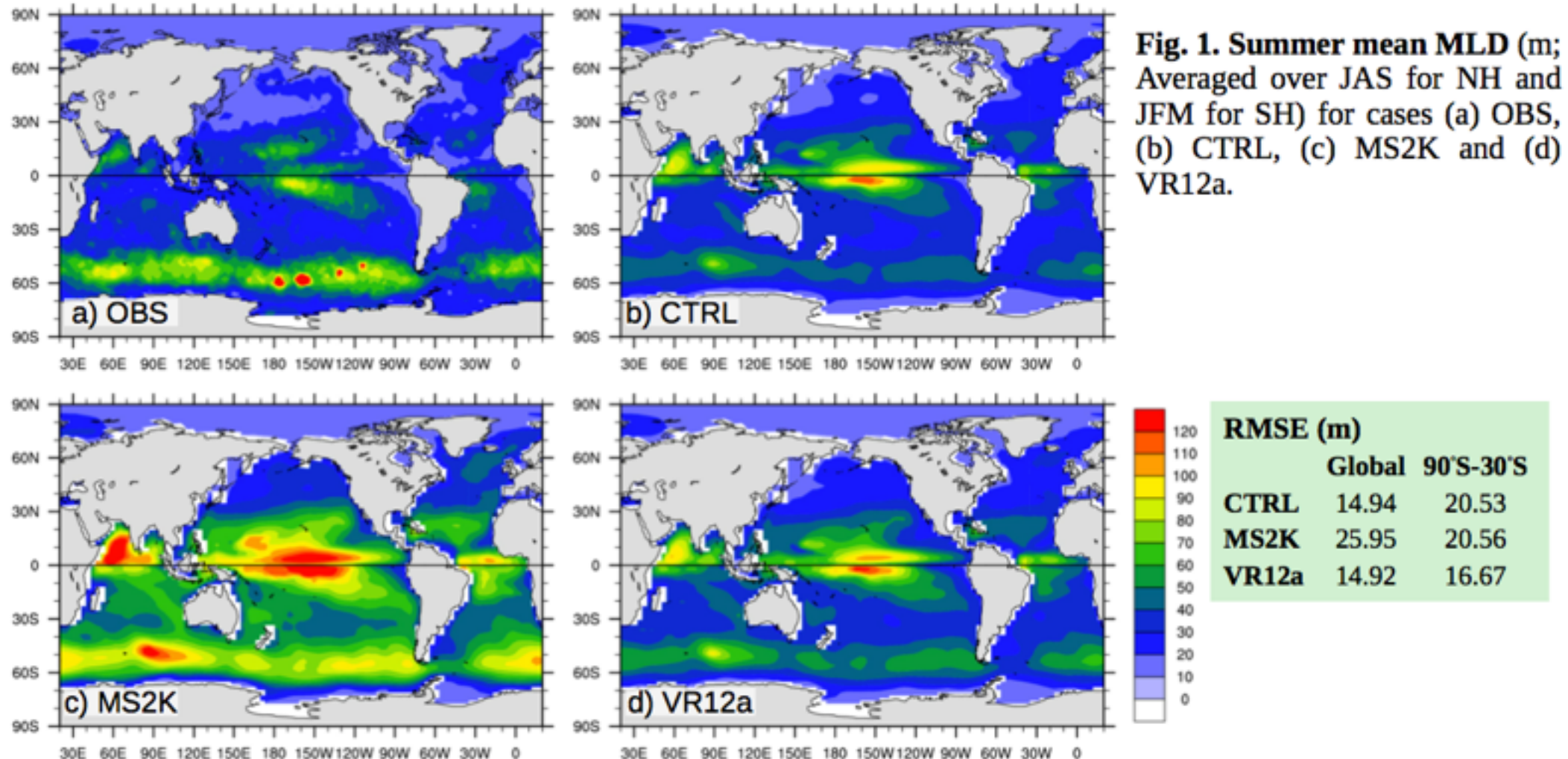


**Fig. 2. Winter mean MLD (m; Averaged over JFM for NH and JAS for SH) for cases (a) OBS, (b) CTRL, (c) MS2K and (d) VR12a.**

	RMSE (m)	
	Global	90°S-30°S
CTRL	59.57	63.46
MS2K	135.55	184.87
VR12a	54.86	53.93

Li et al., in prep.

# Summer Waves in NCAR Community Earth System Model



Li et al., in prep.

# Multi-Model Ensemble: Best Estimate

Results similar in pattern and magnitude to ours (in NCAR CESM) were found by Fan & Griffies (2014) using the NOAA Geophysical Fluid Dynamics Laboratory CM2M model.

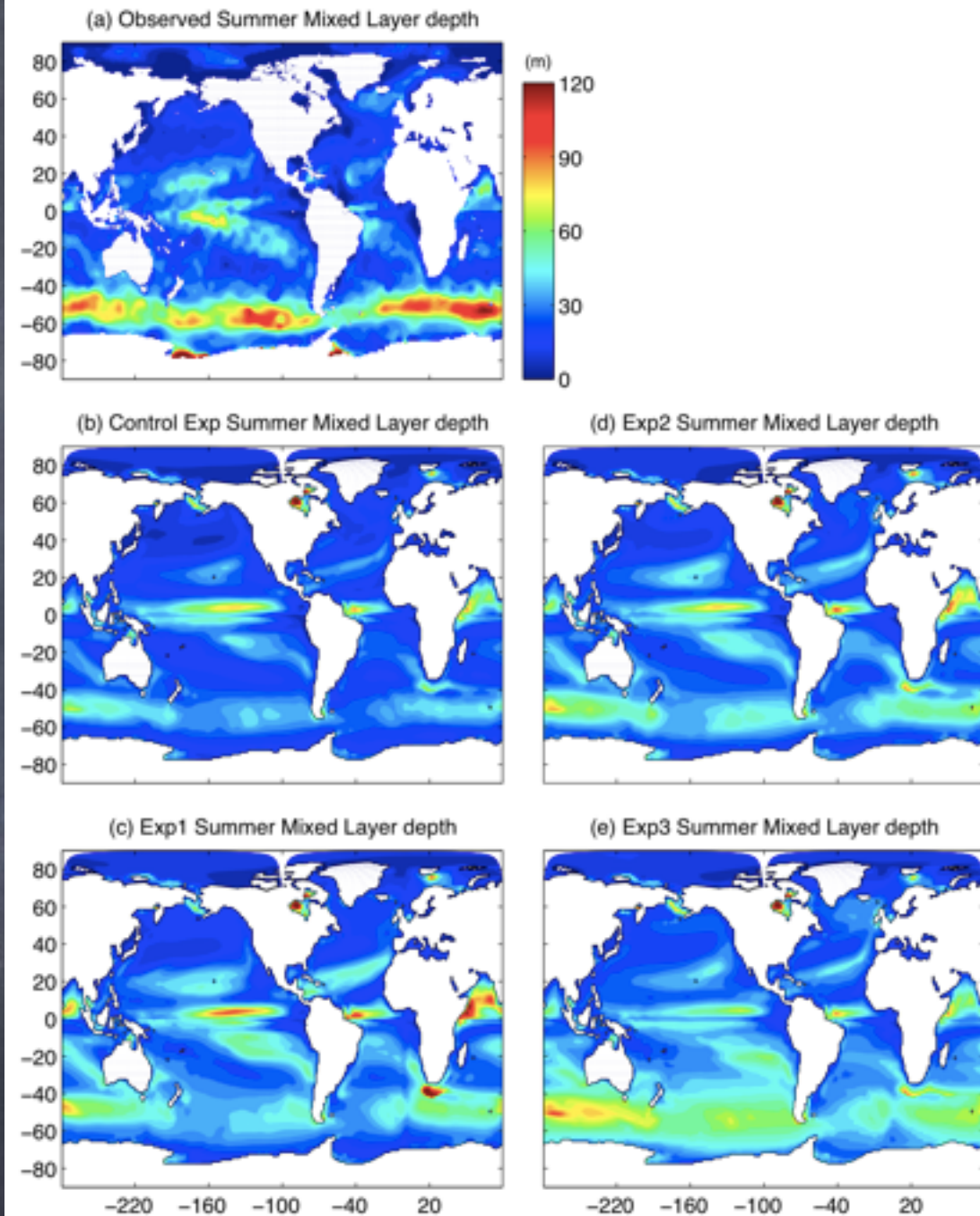


Figure 3. Summer mean (July – September and January – March averages in the Northern and Southern Hemispheres, respectively) mixed layer depth from (a) observational estimate based on world ocean atlas data obtained from the national oceanographic data center (<http://data.nodc.noaa.gov/woa/WOA09/>), and the (b) Control, (c) Exp1, (d) Exp2, and (e) Exp3 experiments.

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$$\frac{\alpha^2}{Ri} \left[ w_{,y} + v_j^L w_{,j} + \frac{M_{Ro}}{Ro Ri} \right] = -\pi_{,z} + b + \boxed{\varepsilon v_j^L v_{j,z}^s} + \frac{\alpha^2}{Re Ri} w_{,jj}$$

$$b_t + v_j^L b_{,j} + \frac{M_{Ro}}{Ro Ri} w b_z + w = 0$$

$$v_{j,j} + \frac{M_{Ro}}{Ro Ri} w_z = 0$$

Plus boundary  
conditions

LAGRANGIAN advection of Eulerian momentum

Coriolis Effect is on LAGRANGIAN velocity

Stokes Shear force is NEW in vertical momentum equation.

So, Waves can Drive turbulence that affect larger scales indirectly:

What about direct effects of waves on larger scales?

(Combined) Lagrangian Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial}{\partial z} (\mathbf{v} + \mathbf{v}_s) = \mathbf{f} \times \frac{\partial \mathbf{v}_L}{\partial z} = -\nabla b$$

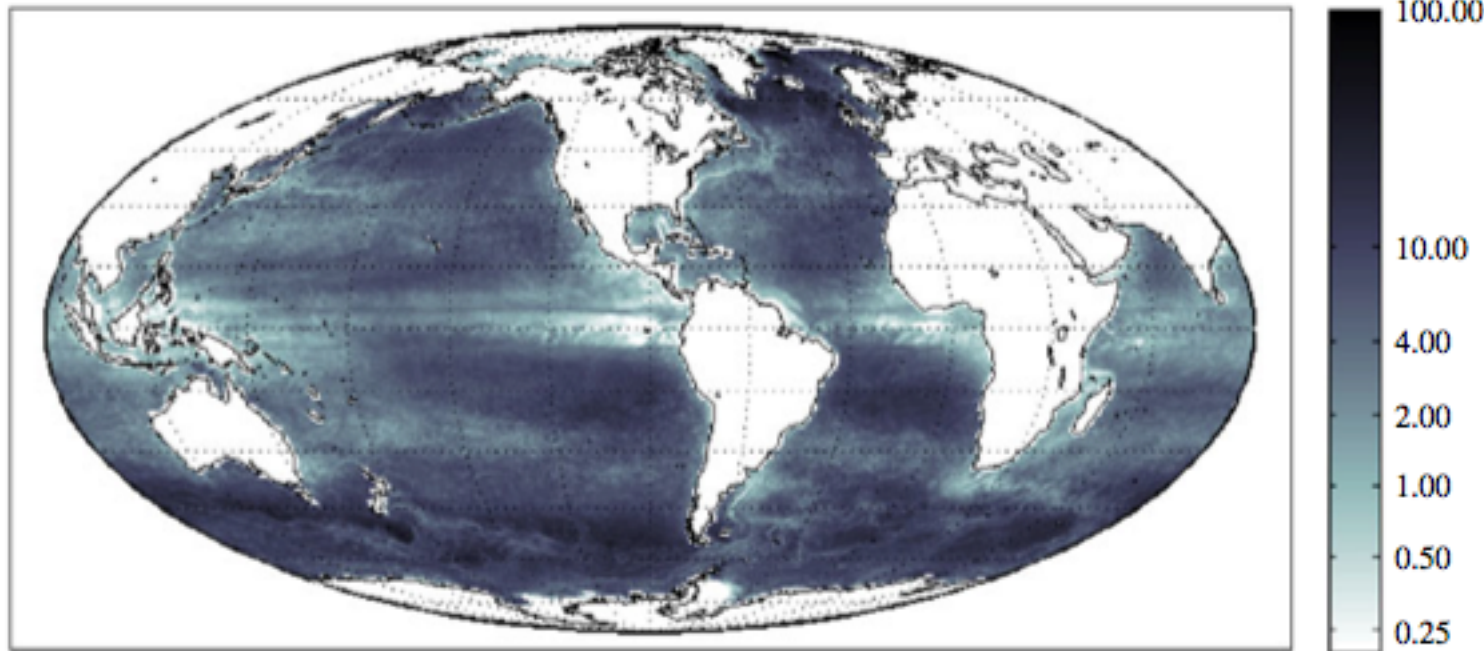
Now the temperature gradients govern the Lagrangian flow, not the not the Eulerian!

# Estimated importance of Stokes vortex on (sub)mesoscale: McWilliams & F-K (13)

466

*J. C. McWilliams and B. Fox-Kemper*

$\epsilon/\mathcal{R}$



$$\epsilon = \frac{V^s H}{f L H_s}$$

$$Ro = \frac{U}{f L}$$

FIGURE 1. (Colour online) Estimated ratio  $\epsilon/\mathcal{R} \approx (|\mathbf{u}_s \cdot \mathbf{u}|h) / (|\mathbf{u}|^2 h_s)$  governing the relative importance of Stokes effects versus nonlinearity. Eulerian velocity ( $\mathbf{u}$ ) is taken as the AVISO weekly satellite geostrophic velocity or  $-\mathbf{u}_s$  (for anti-Stokes flow) if  $|\mathbf{u}_s| > |\mathbf{u}|$ . The front/filament depth ( $h$ ) is estimated as the mixed layer depth from the de Boyer Montégut *et al.* (2004) climatology. An exponential fit to the Stokes drift of the upper 9 m projected onto the AVISO geostrophic velocity provides  $\mathbf{u}_s \cdot \mathbf{u}$  and  $h_s$ . Stokes drift is taken from the Wave Watch 3 simulation described in Webb & Fox-Kemper (2011).  $\mathbf{u}$ ,  $\mathbf{u}_s$ , and  $h_s$  are all for the year 2000, while  $h$  is from a climatology of observations over 1961–2008. The year 2000 average of  $\epsilon/\mathcal{R}$  is shown.

# Wave-Averaged Equations

following McWilliams & F-K (13)

$$\varepsilon = \frac{V^s H}{f L H_s}$$

and Suzuki & F-K (14)

(for horizontally uniform Stokes drift)

$$Ro [v_{i,t} + v_j^L v_{i,j}] + \frac{M_{Ro}}{Ri} w v_{i,z} + \epsilon_{izj} v_j^L = -M_{Ro} \pi_{,i} + \frac{Ro}{Re} v_{i,jj}$$

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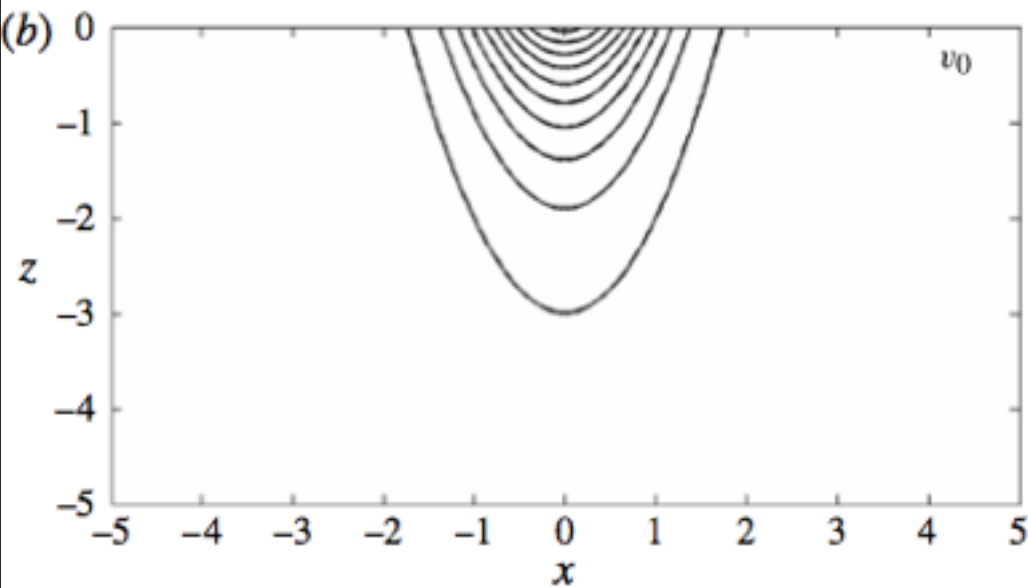
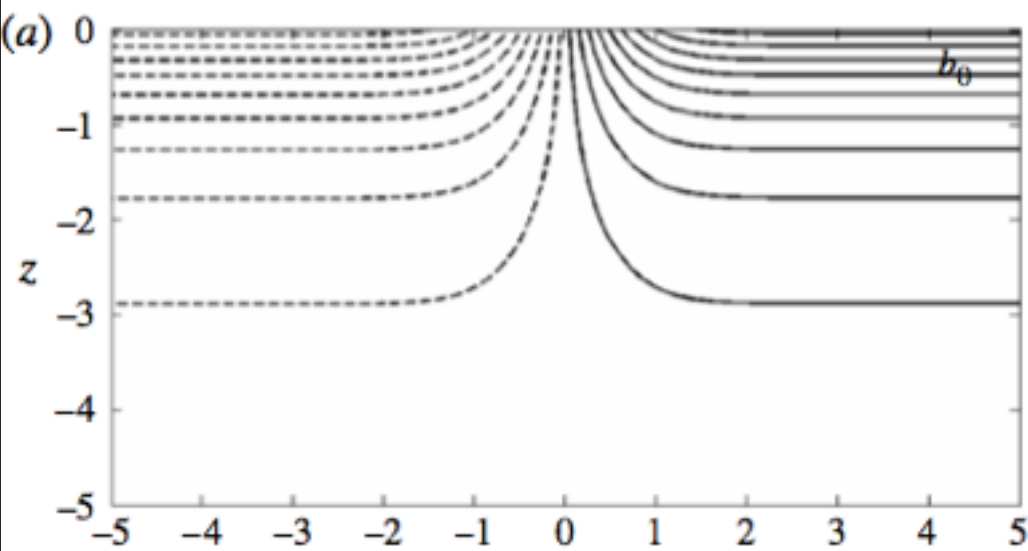
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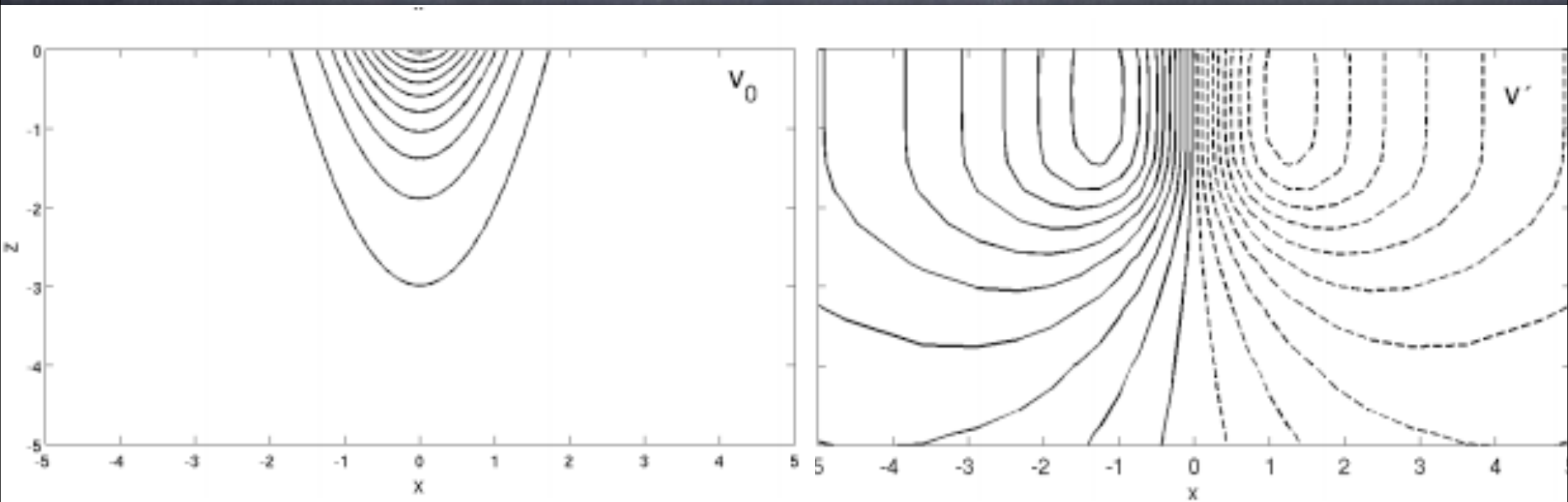
- 1) Consider a plane parallel balanced flow with  $b(y,z)$ ,  $q(y,z)$
- 2) Perturb this flow by introducing Stokes forces (turn on waves)
- 3) Adiabatically rearrange the  $b(y,z)$  and  $q(y,z)$  until new forces are balanced
- 4) This solution amounts to solving for  $b'$  and  $v'$  in:

$$\partial_x^2 b' + \partial_z^2 \left( \frac{b'}{N^2} \right) = \mathcal{F}' + \partial_z \left( \frac{\epsilon \mathcal{Q}'}{N^2} \right) - \partial_x \epsilon \mathcal{P}'$$

$$v' = - \int_{-\infty}^x \left[ \partial_z \left( \frac{b'}{N^2} \right) + \frac{S_s}{N^2} \partial_z v_0 + \epsilon S_s \frac{\partial_x b'}{N^2} \right] dx'$$

# Waves (Stokes Drift Vortex Force) ->

Submeso, Meso, an example for  $\varepsilon \ll 1$   
near the "sweet spot"



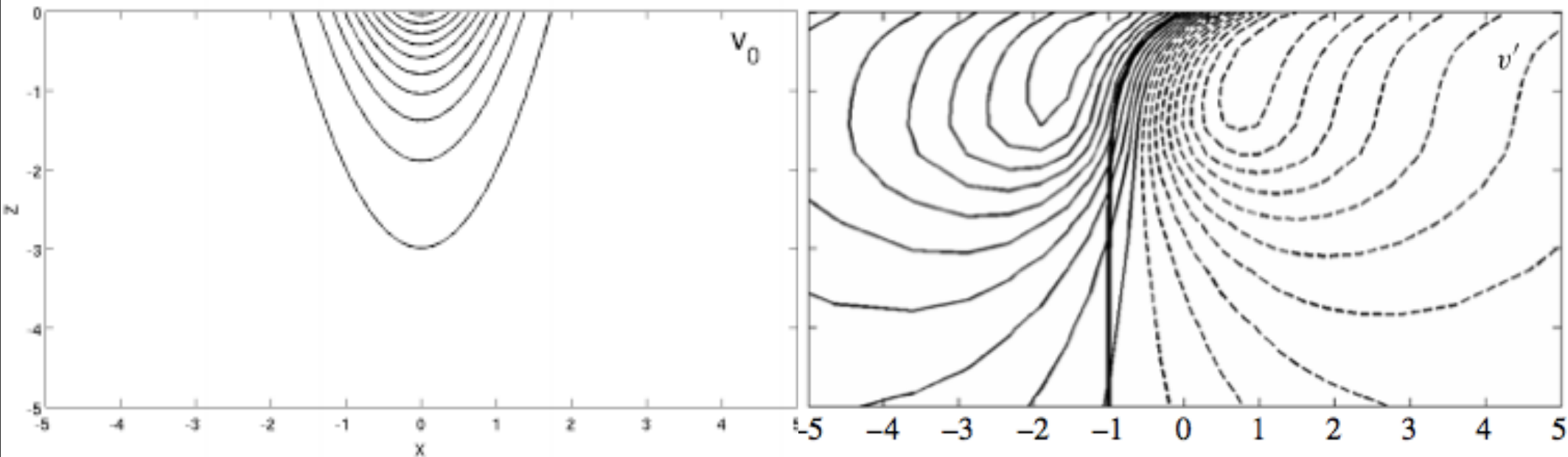
Initial Submeso Front

Max: 1

Perturbation on that scale  
due to waves

Max:  $7.1\varepsilon$

# Waves (Stokes Drift Vortex Force) → Submeso, Meso, an example for finite waves



Initial Submeso Front

Perturbation on that scale  
due to waves

Max: 1

Max:  $6.8\epsilon = 13.6$

# LES of Langmuir-

## Submeso Interactions?

Perform large eddy simulations (LES) of Langmuir turbulence with a submesoscale temperature front

Use NCAR LES model to solve Craik-Leibovich equations (Moeng, 1984, McWilliams et al, 1997)

$$\frac{\partial \rho}{\partial t} + \mathbf{u}_L \cdot \nabla \rho = \text{SGS}$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\boldsymbol{\omega} + f\hat{\mathbf{z}}) \times \mathbf{u}_L = -\nabla \pi - \frac{g\rho\hat{\mathbf{z}}}{\rho_0} + \text{SGS}$$

Computational parameters:

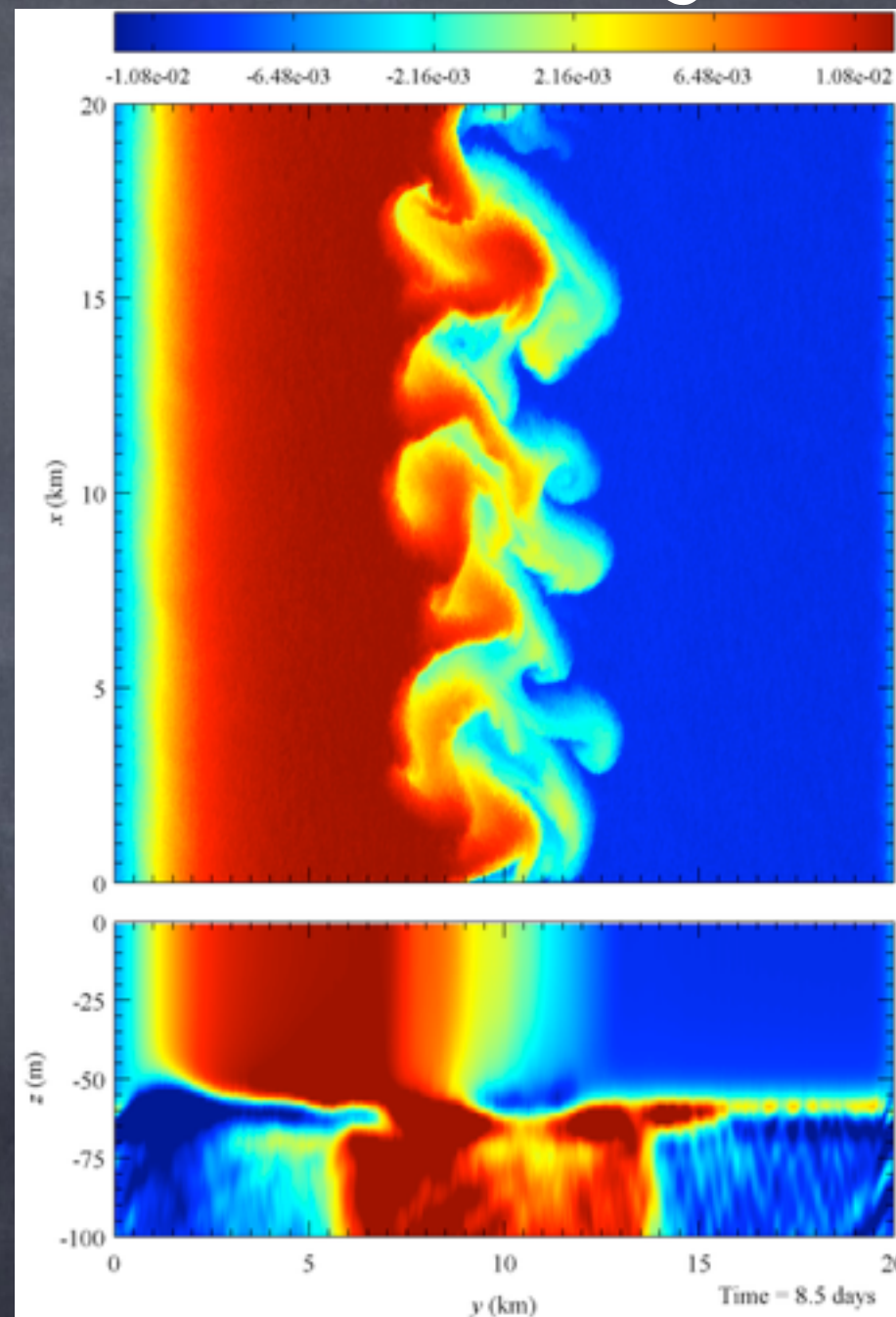
Domain size: 20km x 20km x -160m

Grid points: 4096 x 4096 x 128

Resolution: 5m x 5m x -1.25m

1000x more gridpoints than CESM

Movie: P. Hamlington



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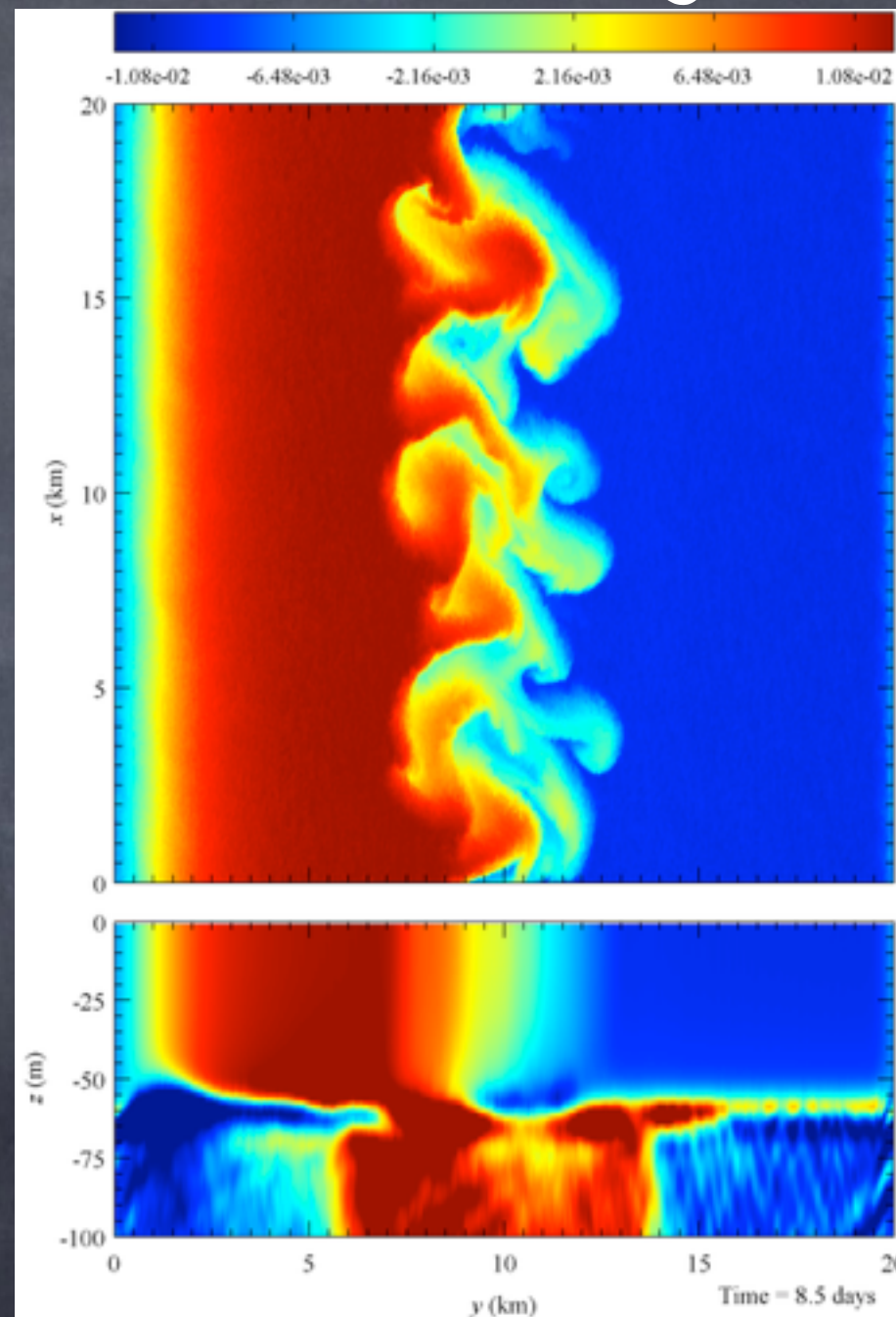
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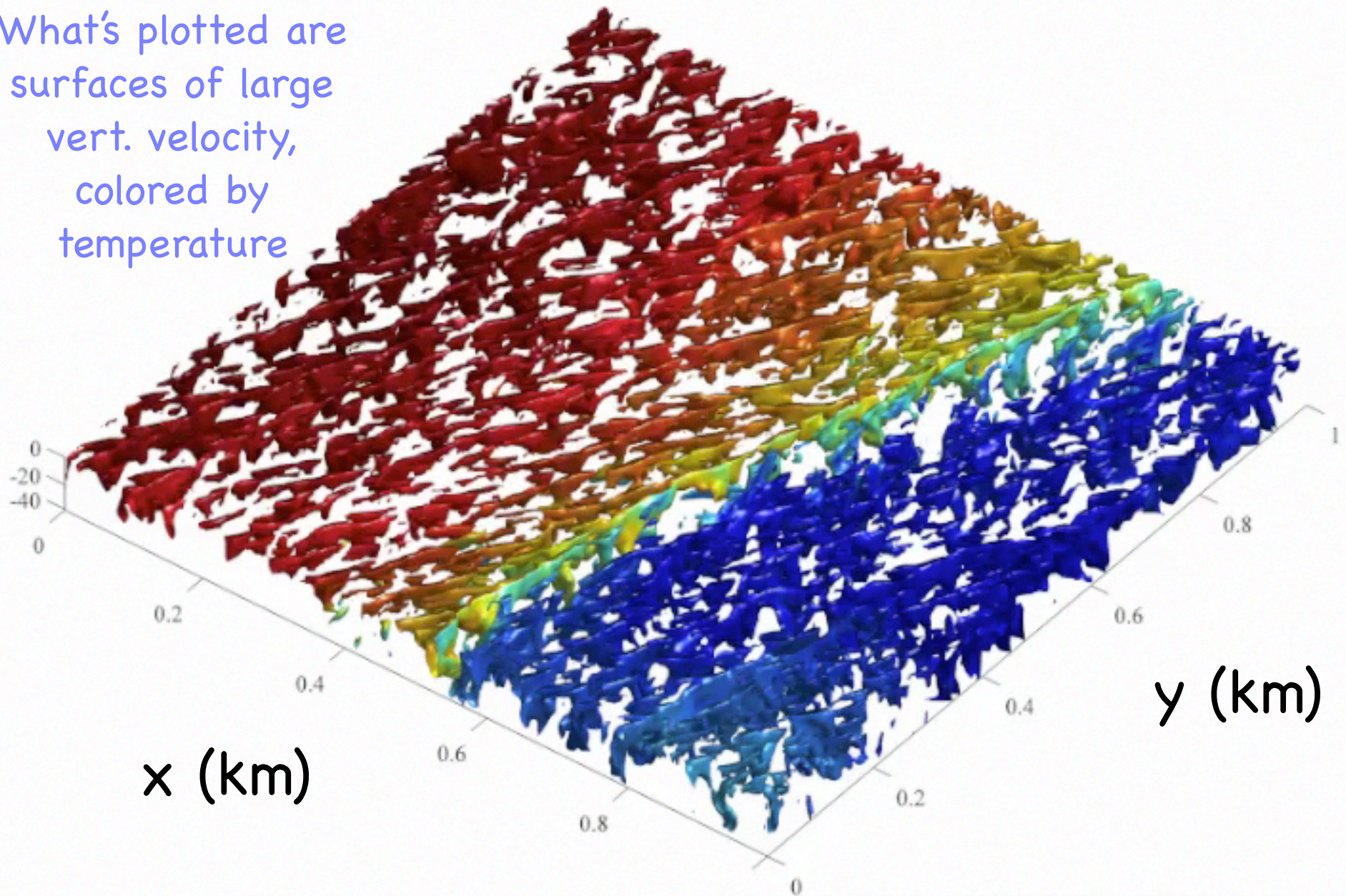
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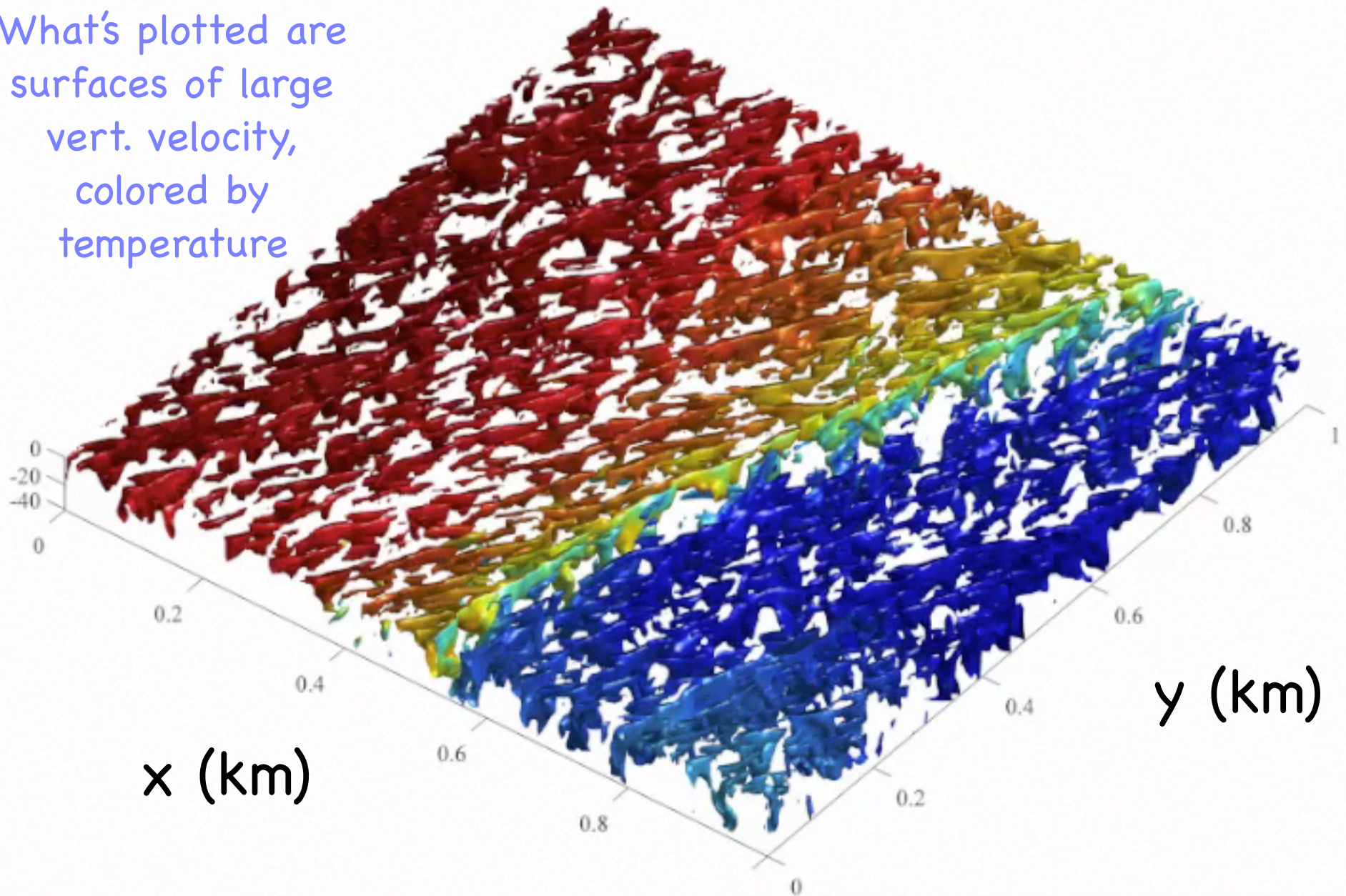
# Zoom: Submeso-Langmuir Interaction!

What's plotted are  
surfaces of large  
vert. velocity,  
colored by  
temperature

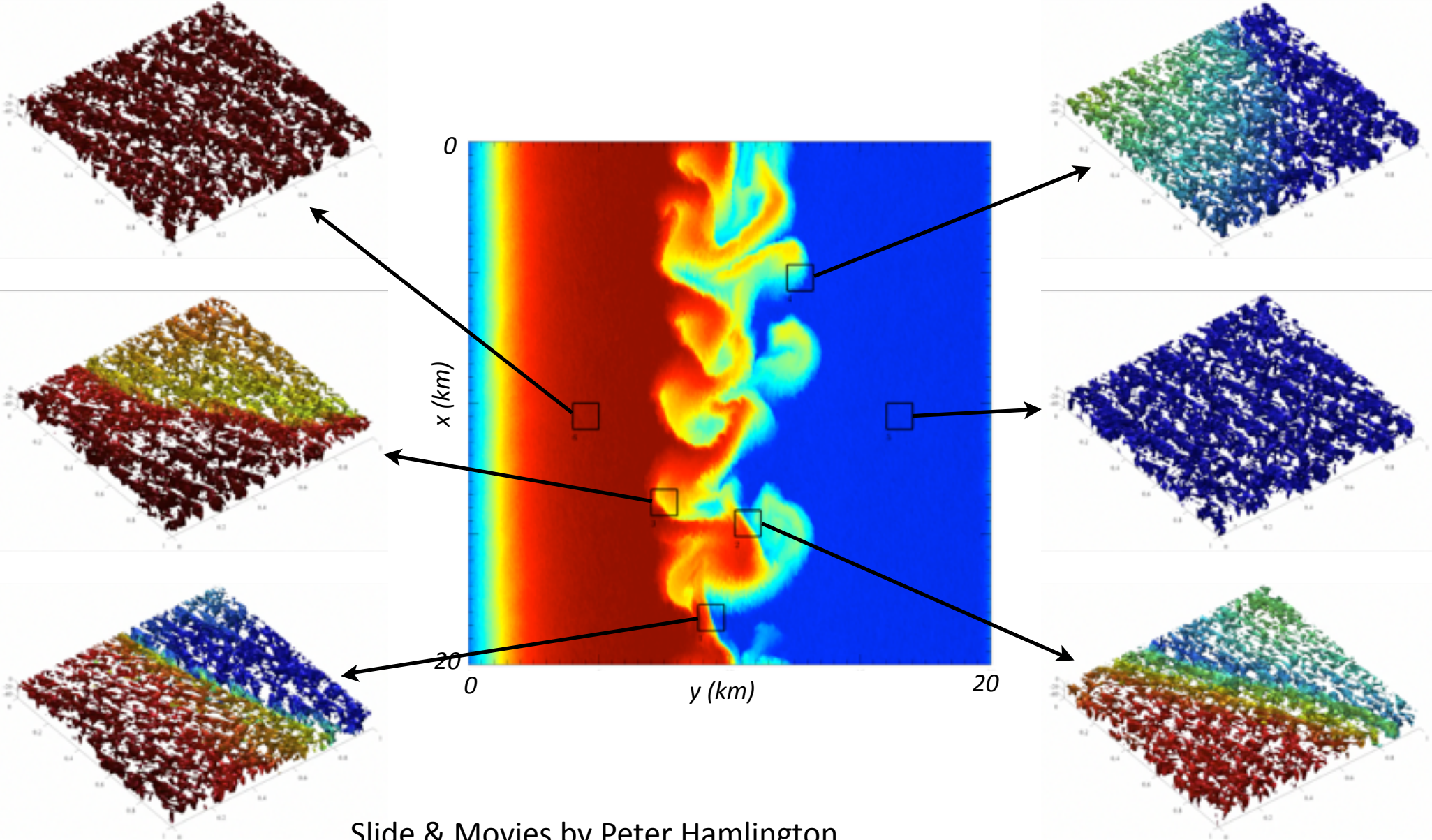


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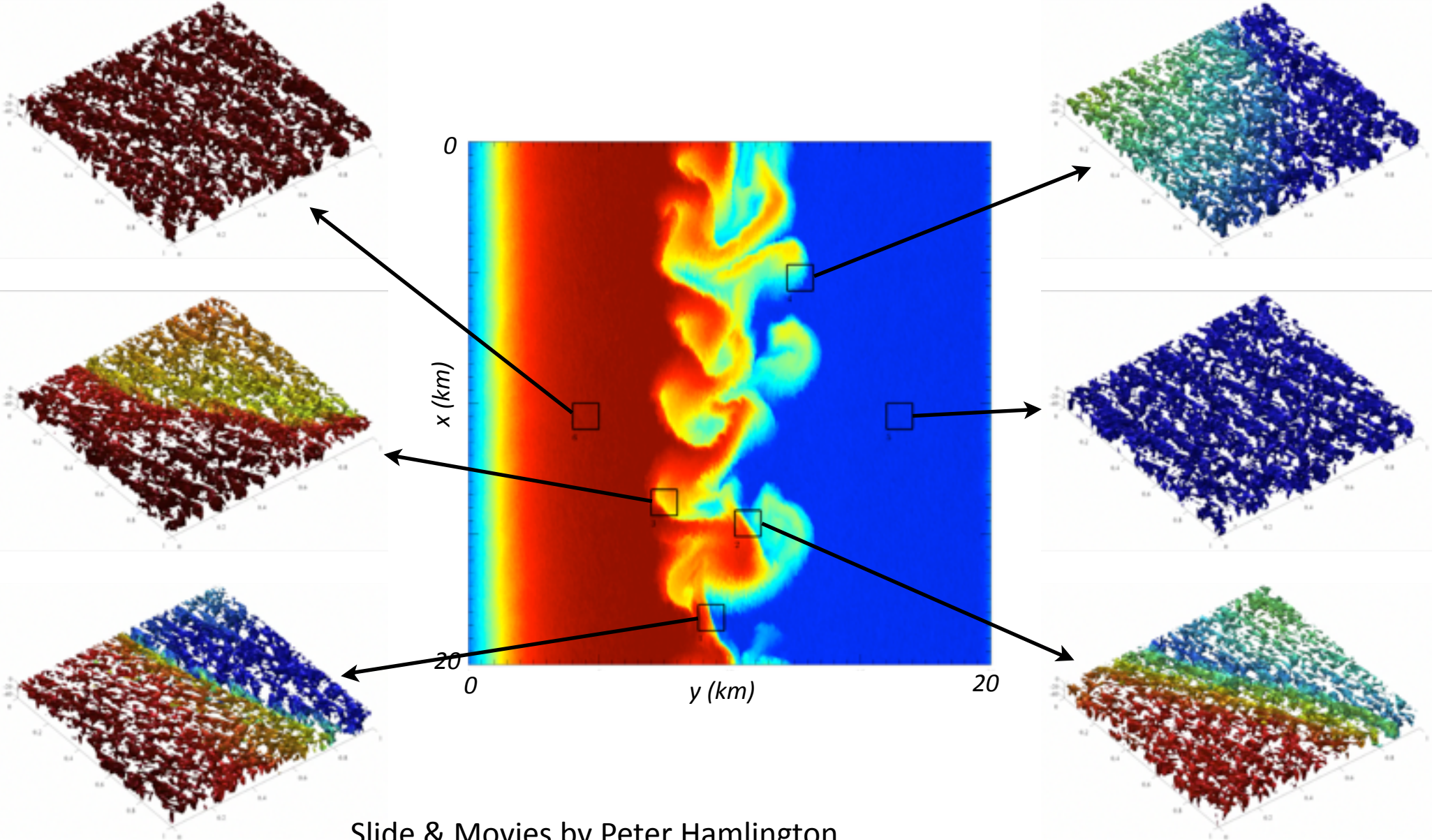
# Diverse types of interaction



Slide & Movies by Peter Hamlington



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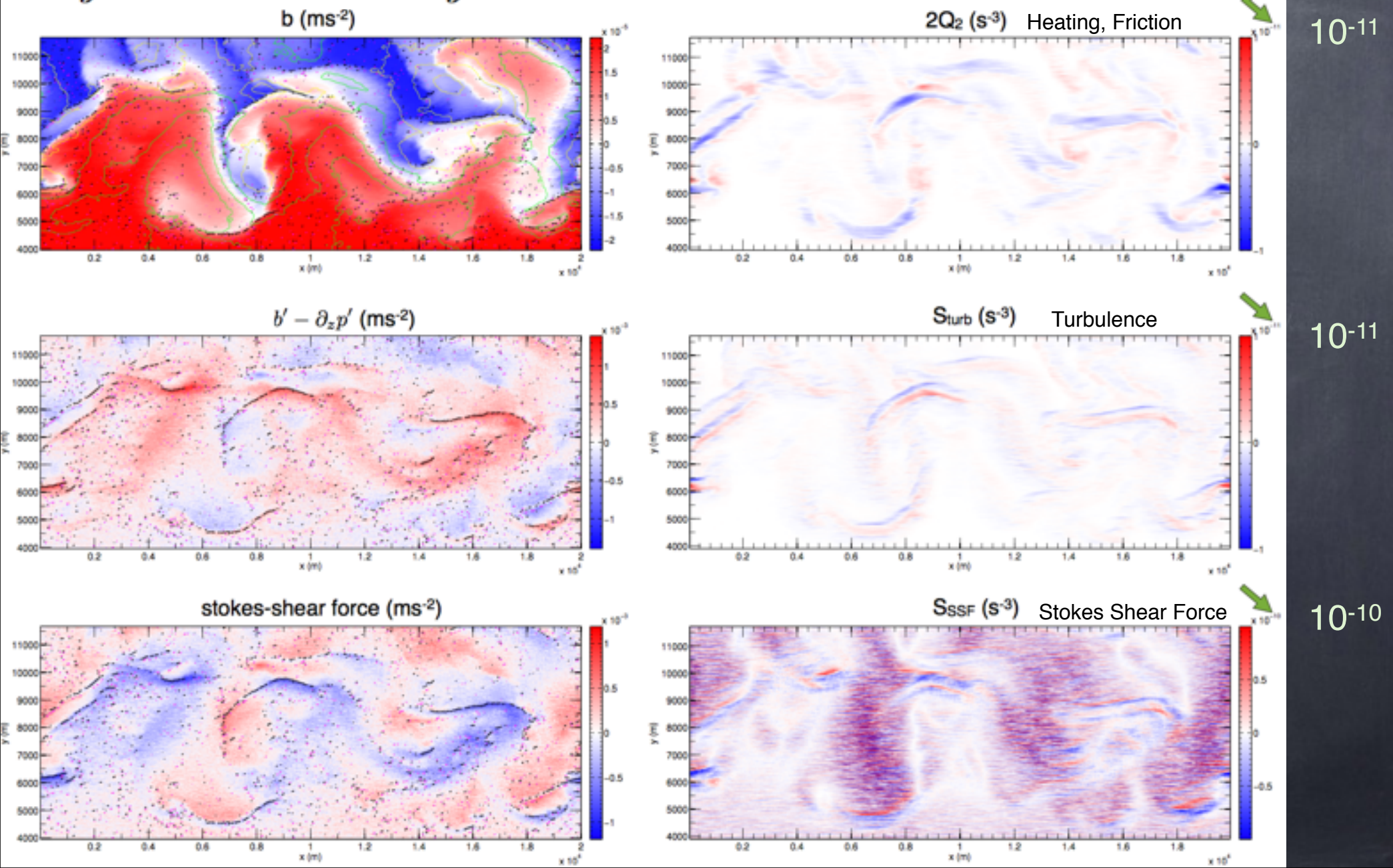


Slide & Movies by Peter Hamlington

# Wave-influenced Sawyer-Eliassen eq.

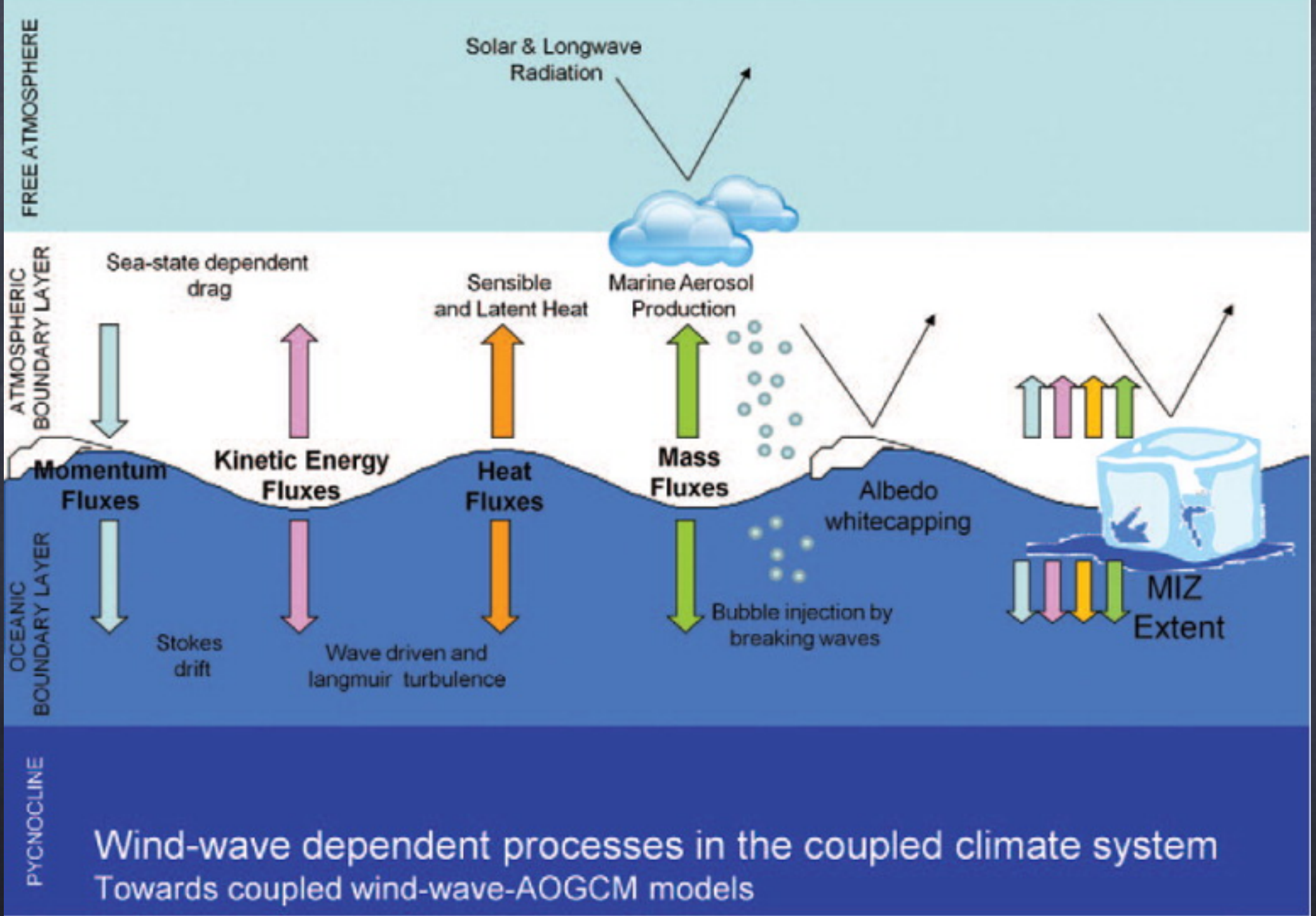
Suzuki &  
F-K in prep

$$N_*^2 \frac{\partial^2 \psi}{\partial y^2} + F_*^2 \frac{\partial^2 \psi}{\partial z^2} + 2M_*^2 \frac{\partial^2 \psi}{\partial y \partial z} = 2Q_2 + S_{turb} + S_{SSF}$$



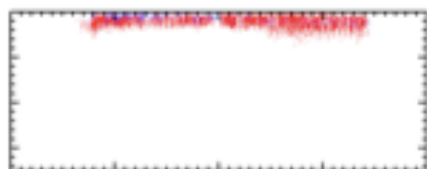
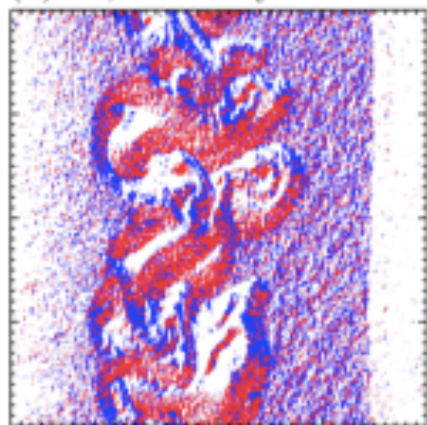
# Conclusions

- Climate modeling is challenging partly due to the vast and diverse scales of fluid motions
- In the upper ocean, horizontal scales as big as basins, and as small as meters contribute non-negligibly to the air-sea exchange and climate
- Interesting transition occurs on the Submeso to Langmuir scale boundary, as nonhydro. & ageostrophic effects begin to dominate
- The effects of the Stokes forces on mesoscale and submesoscale dynamics are under-appreciated.
- All papers at: [fox-kemper.com/pubs](http://fox-kemper.com/pubs)

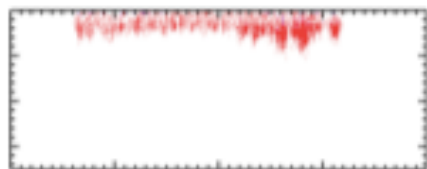
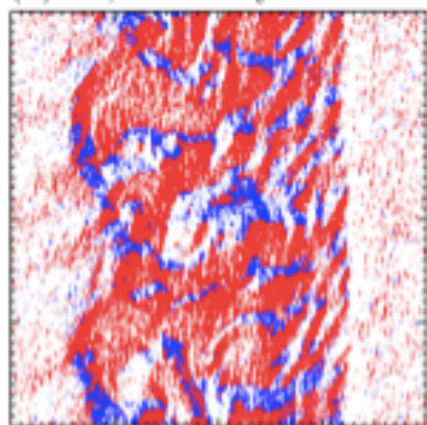


L. Cavaleri, B. Fox-Kemper, and M. Hemer. Wind waves in the coupled climate system. Bulletin of the American Meteorological Society, 93(11):1651-1661, 2012.

(d) LT, Instability



(h) ST, Instability



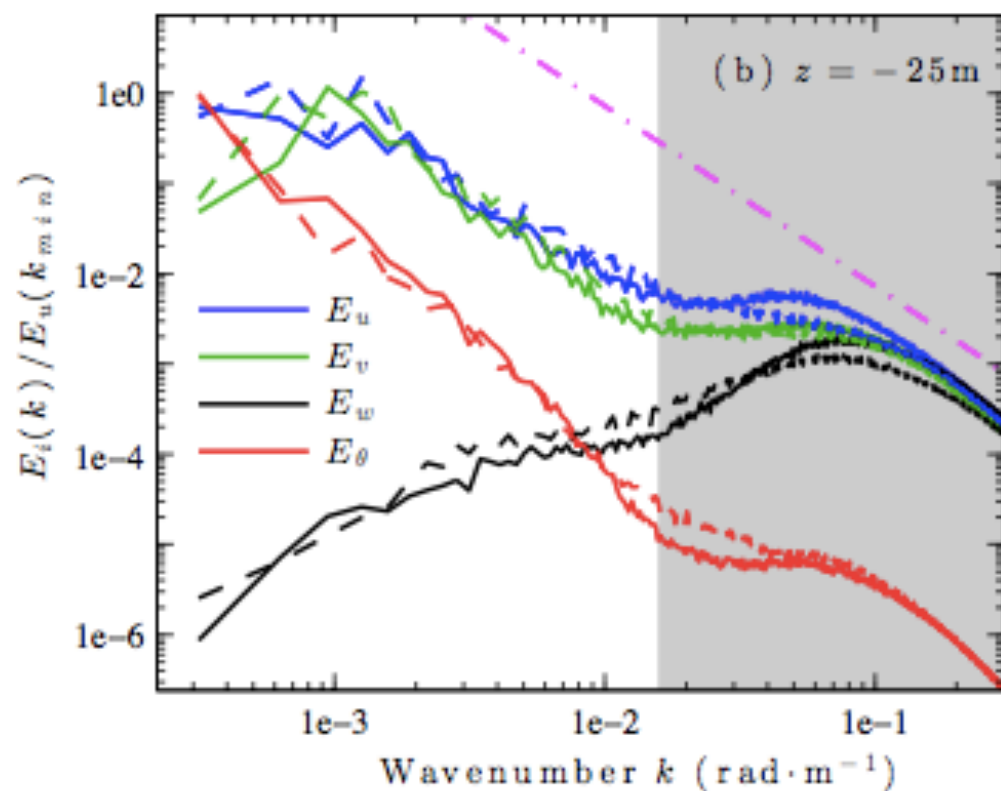
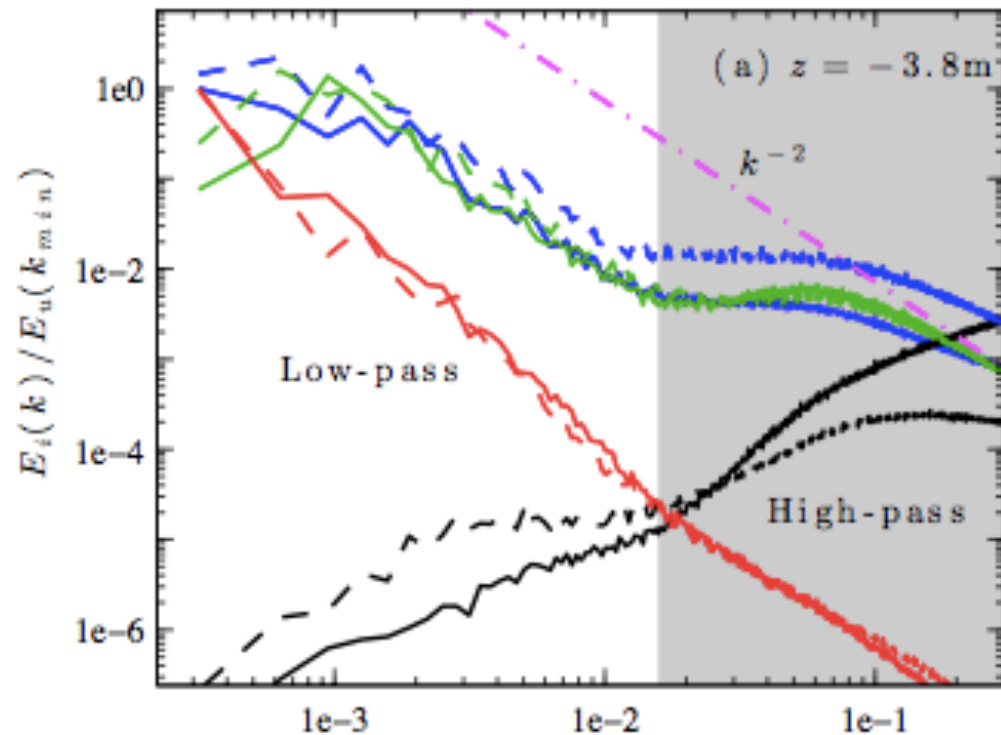
0 5 10 15 20

y (km)



S I/SI SI SI/G G

P. E. Hamlington, L. P. Van Roekel, B. Fox-Kemper, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. *Journal of Physical Oceanography*, 2013. Submitted.



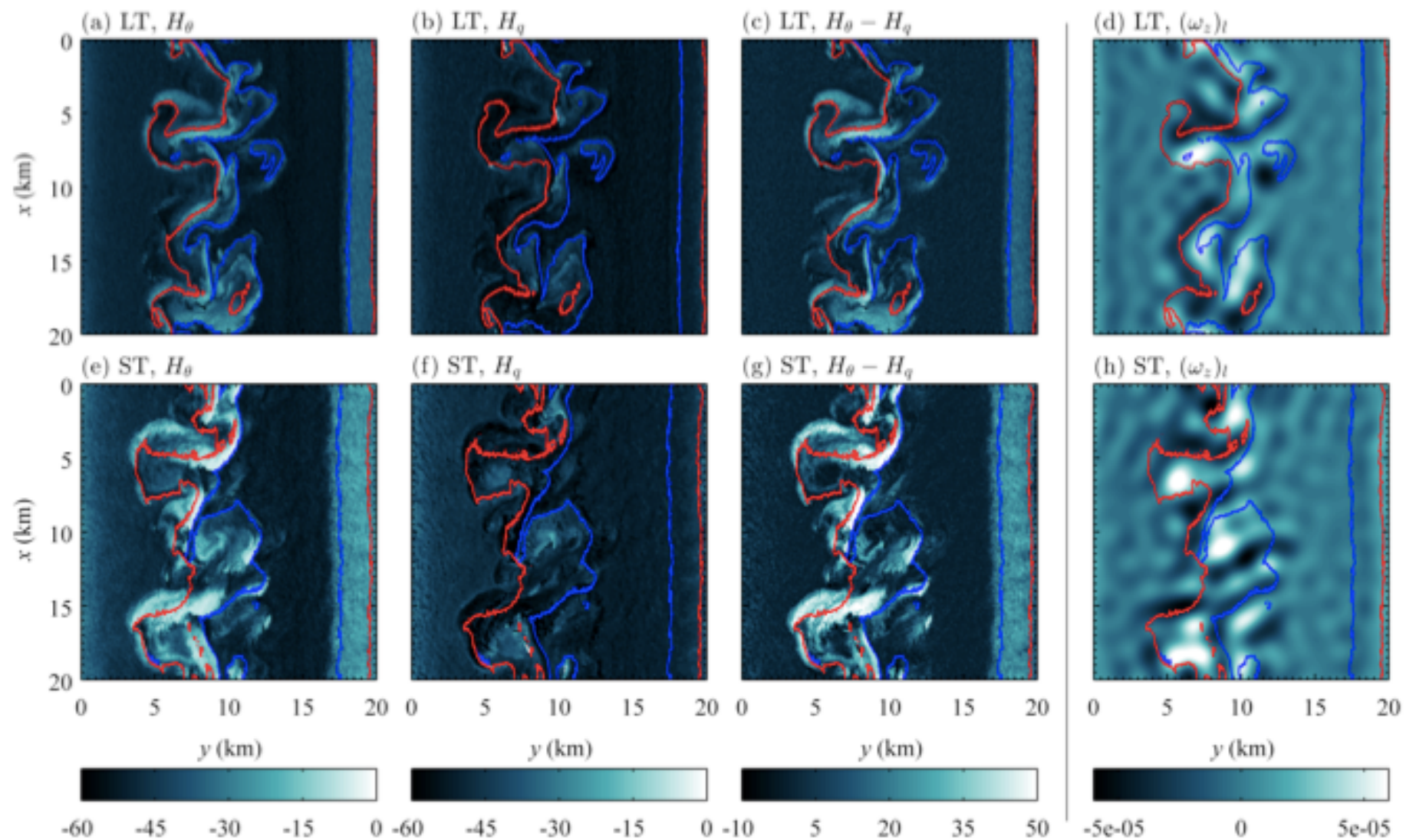


FIG. 13. Fields of the mixed layer depth (in m) based on temperature, denoted  $H_\theta$ , (a,e) and on potential vorticity, denoted  $H_q$ , (b,f) for the LT (a,b) and ST (e,f) cases. The difference  $H_\theta - H_q$  is shown in (c,g) and low-pass (submesoscale) vertical vorticity fields are shown in (d,h), where the filter cutoff for the vorticity fields is at 2km. Contour lines correspond to temperature contours taken from Figure 2.

# So, no problems?

## Just crunch away with CLB?

- Let's revisit our assumptions for scale separation:
  - CLB wave equations require limited \*wave steepness\* and irrotational flow
  - Real wind-waves are not monochromatic, but incorporate a spectrum of waves, and...



Power Spectrum  
of wave height

$$\langle \eta^2 \rangle = \int_0^{\infty} E(k) dk = C_0 + \int_{k_h}^{\infty} C_1 k^{-2} dk$$

Power Spectrum  
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steepness:  
**INFINITE!**

$$\langle k^2 \eta^2 \rangle = \int_0^{\infty} k^2 E(k) dk = D_0 + \int_{k_h}^{\infty} D_1 dk$$

Steep waves break  $\rightarrow$  vortex motion & small scale turbulence!

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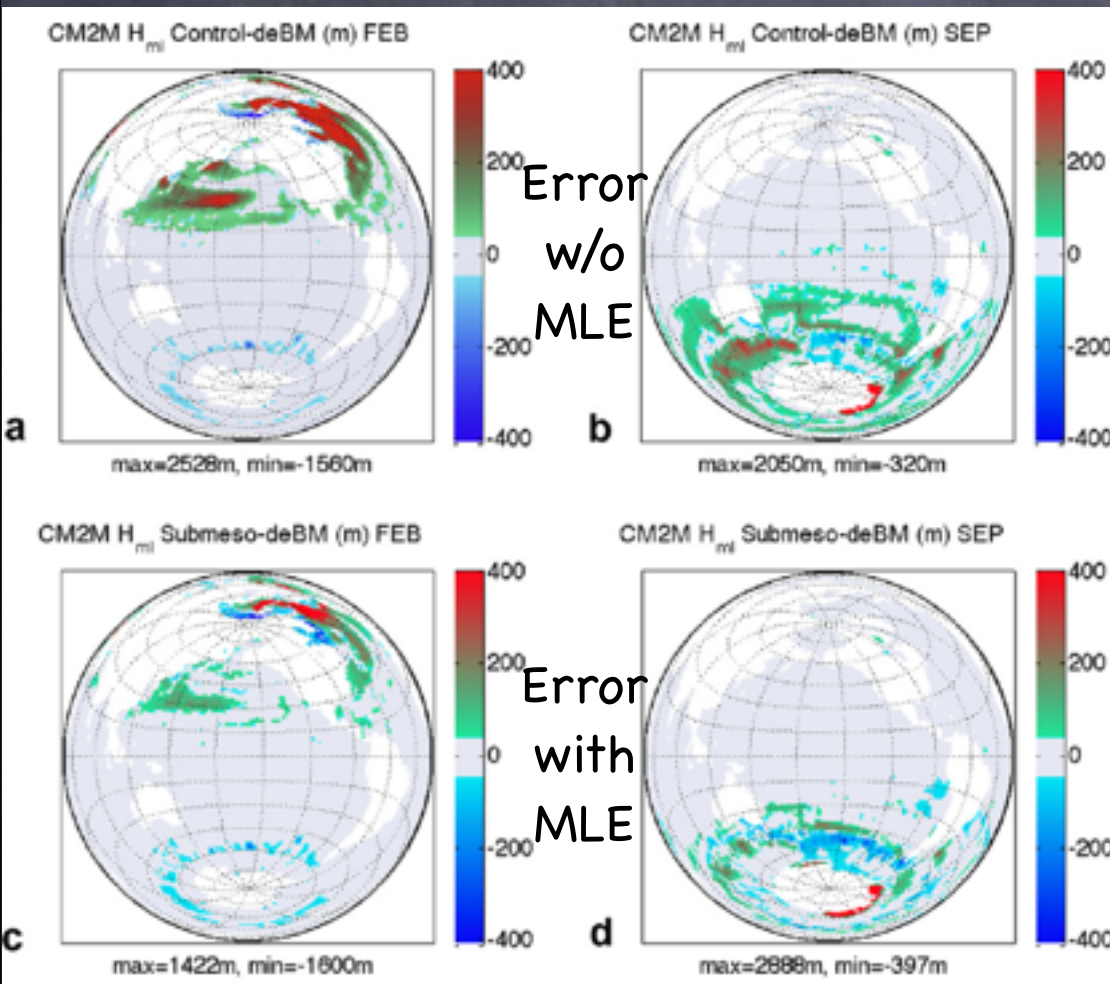
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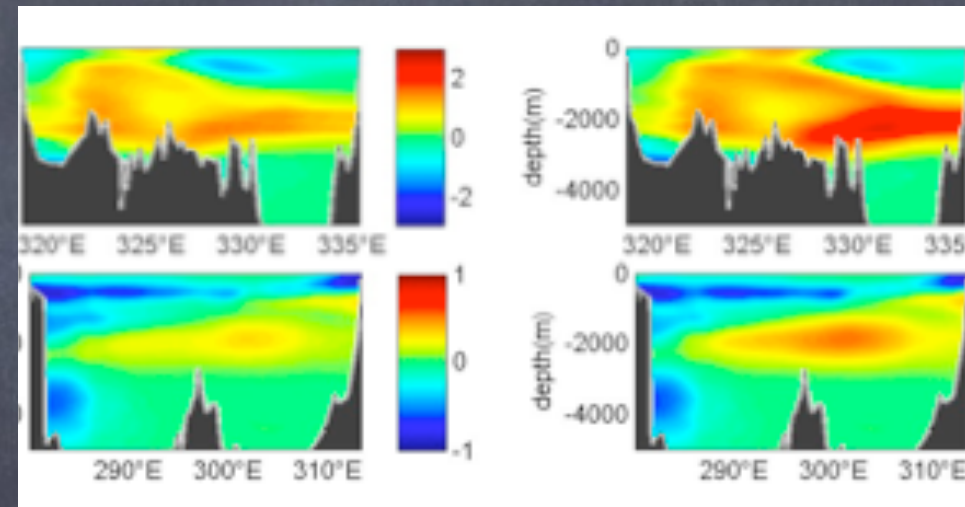


# Physical Sensitivity of Ocean Climate to MLE: (submeso) Mixed Layer Eddy Restratisation Improves CFCs (water masses)



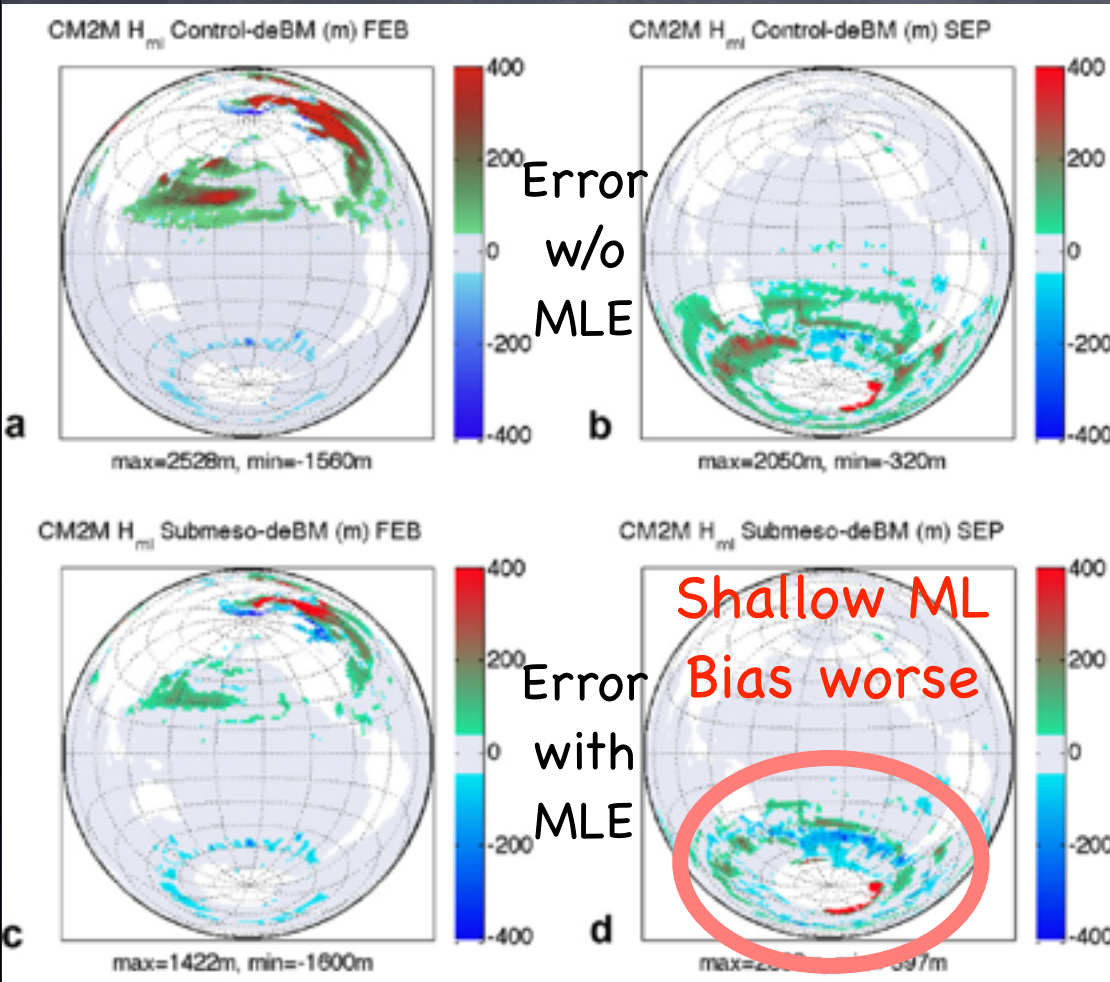
Bias with MLE

Bias w/o MLE



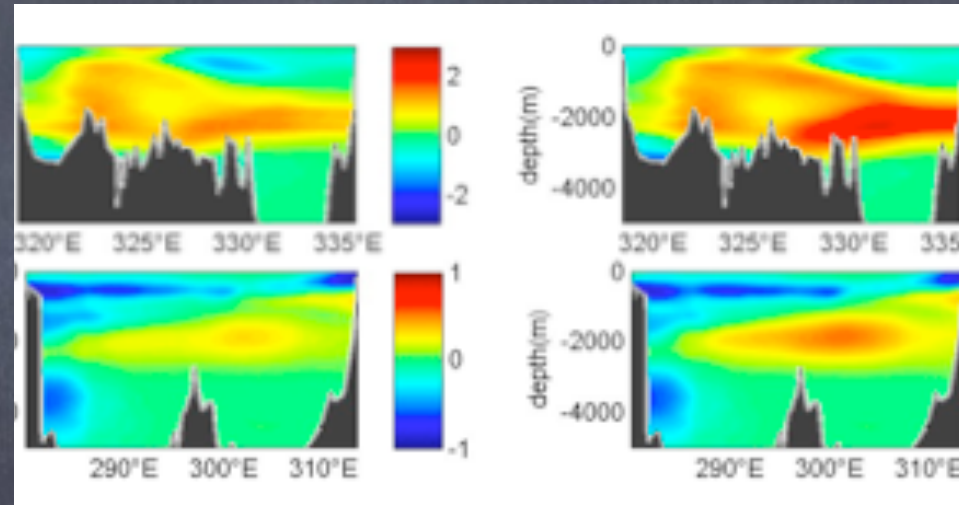
B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels.  
 Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

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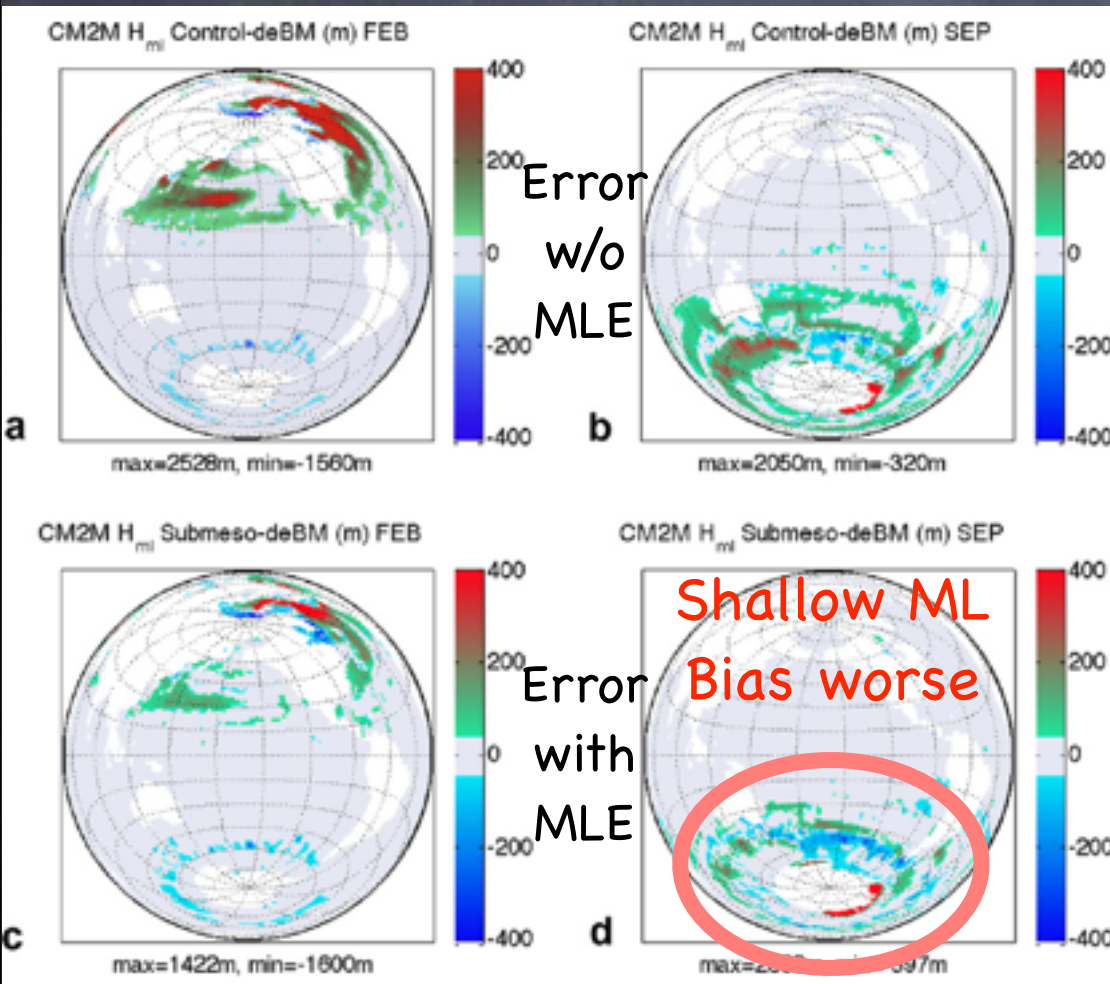
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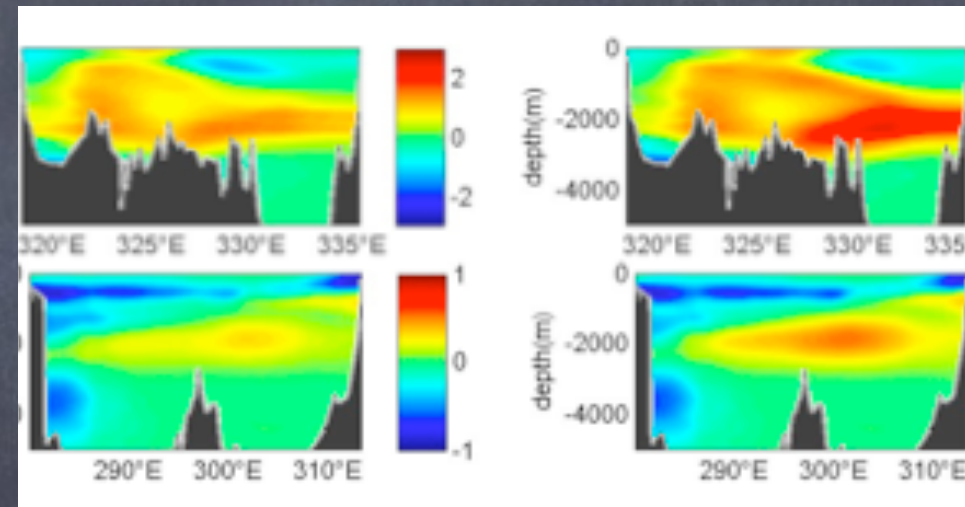
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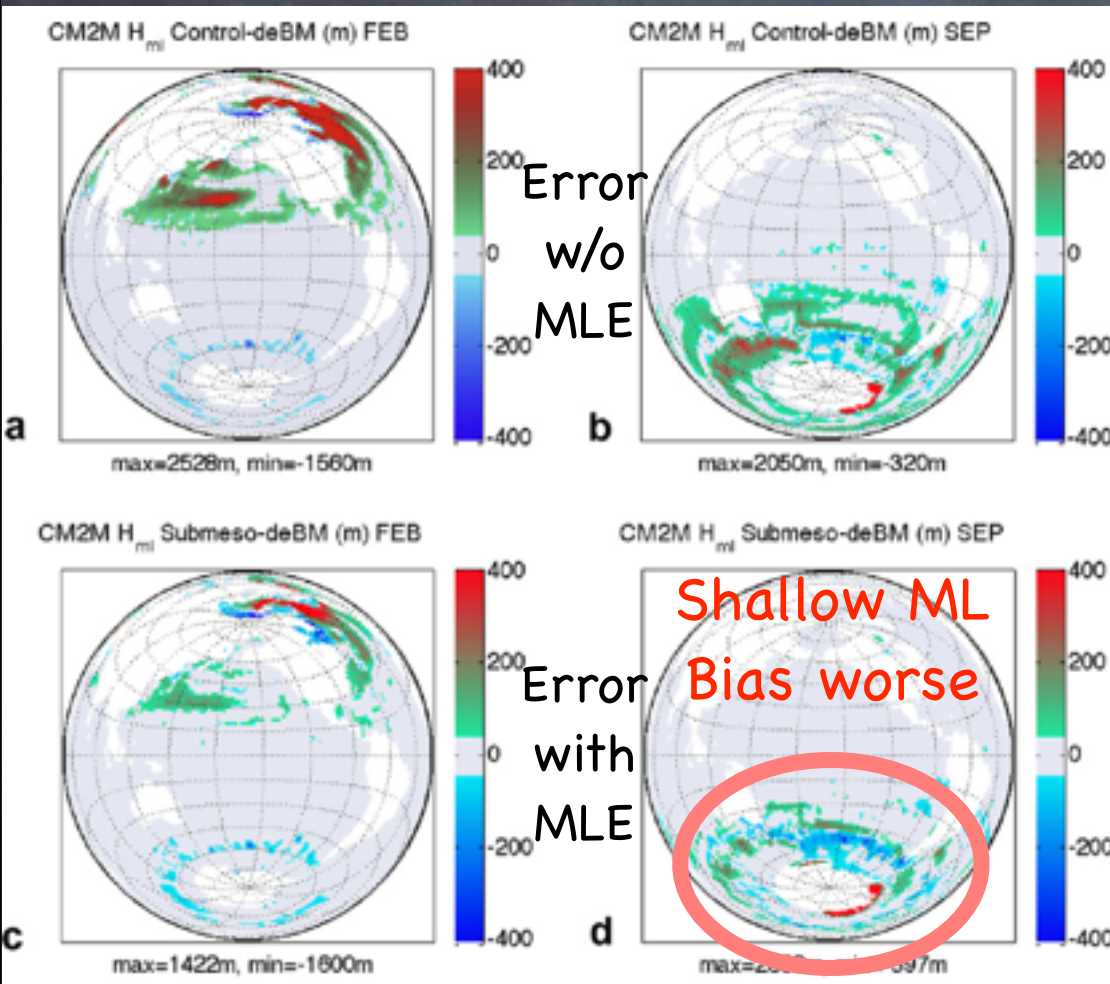


A consistently restratifying,

$$\overline{w'b'} \propto \frac{H^2}{|f|} |\nabla_H \bar{b}|^2$$

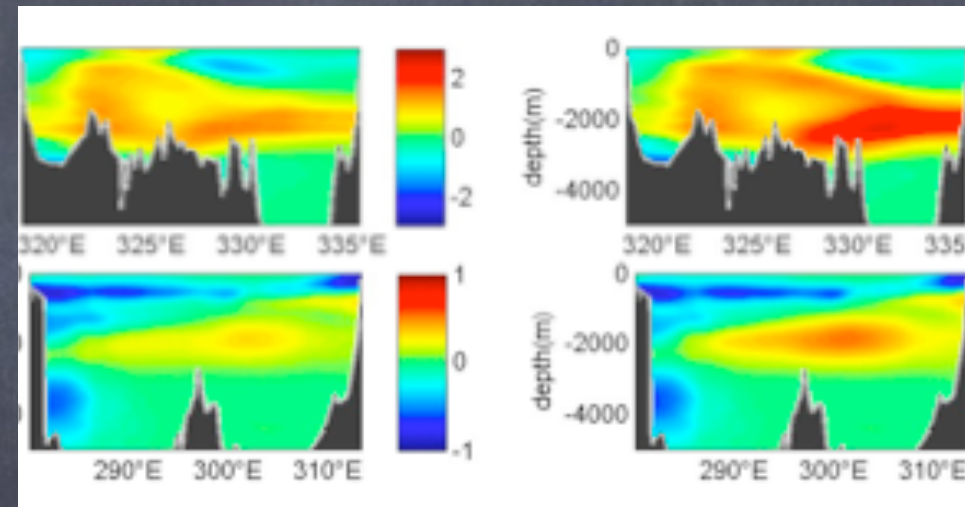
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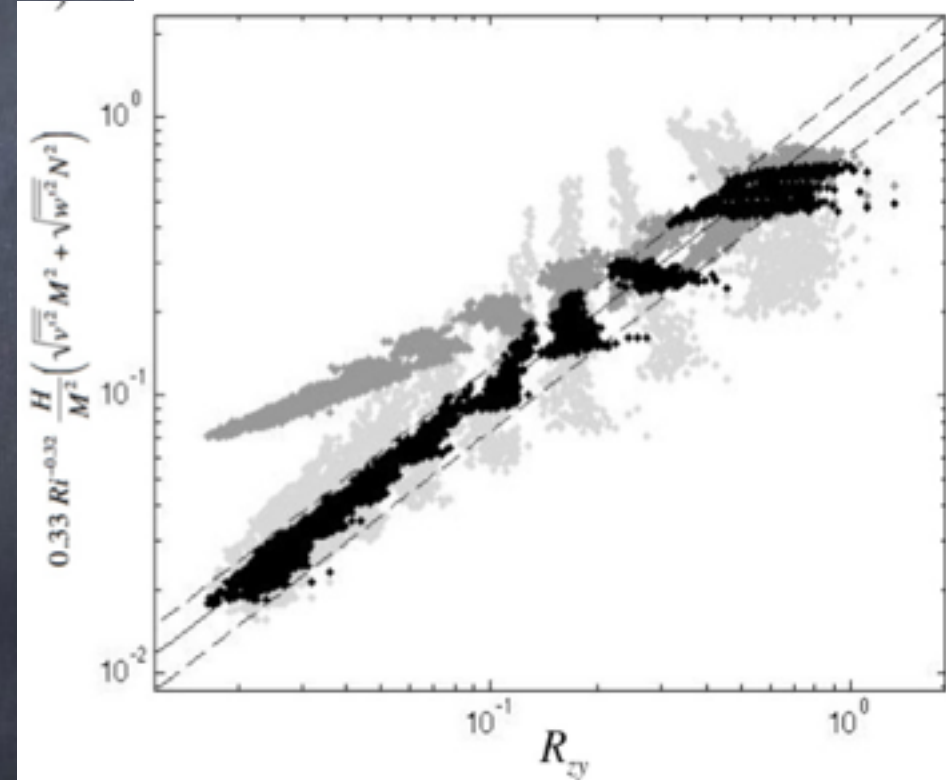
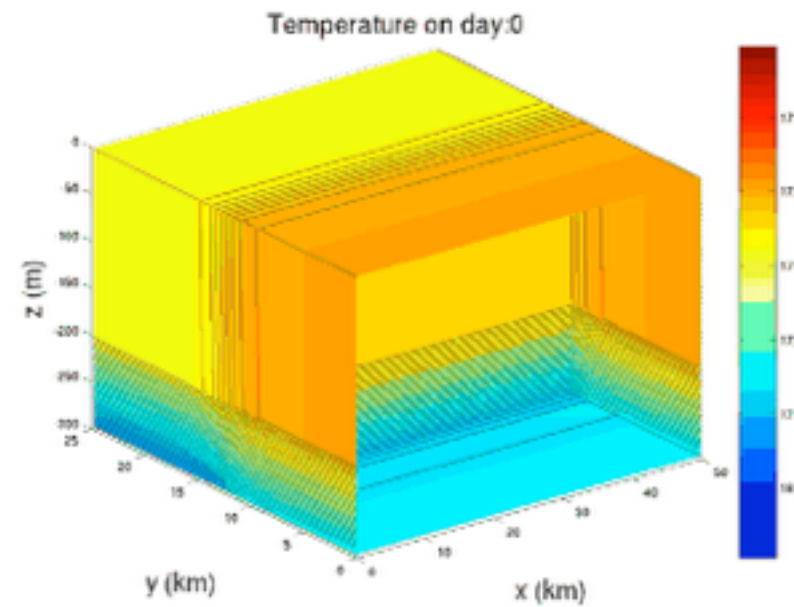
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Estimating eddy buoyancy/density fluxes:

$$\overline{\mathbf{u}'b'} \equiv \Psi \times \nabla \bar{b}$$

A submeso eddy-induced overturning:

$$\Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{\mathbf{z}}$$



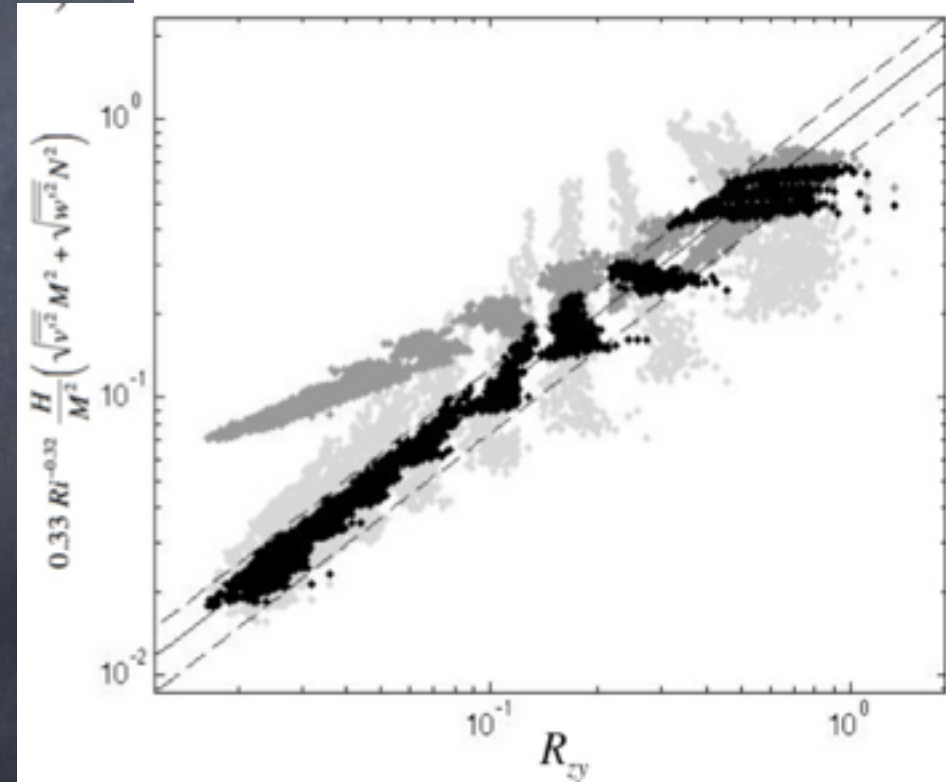
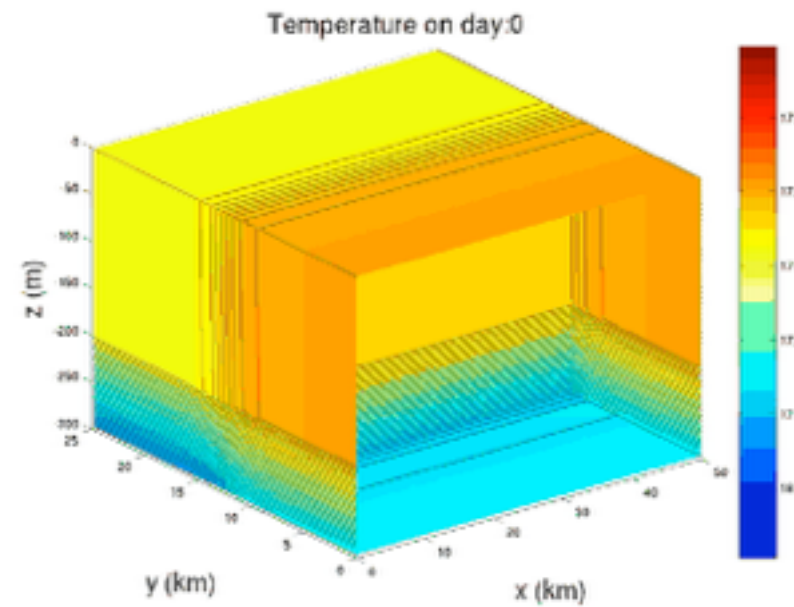
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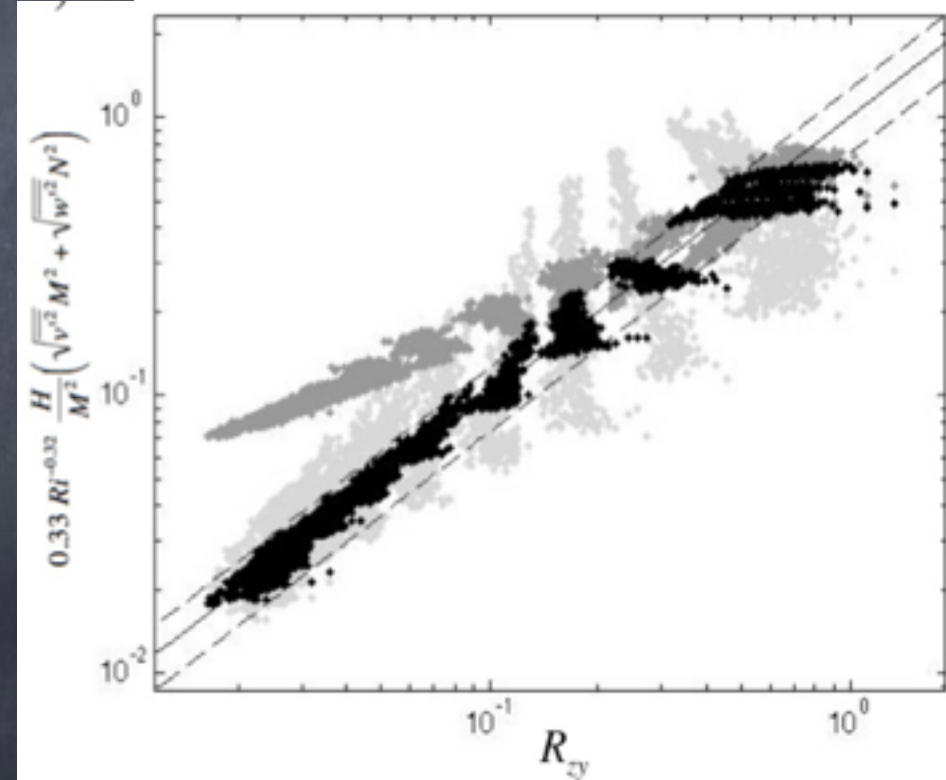
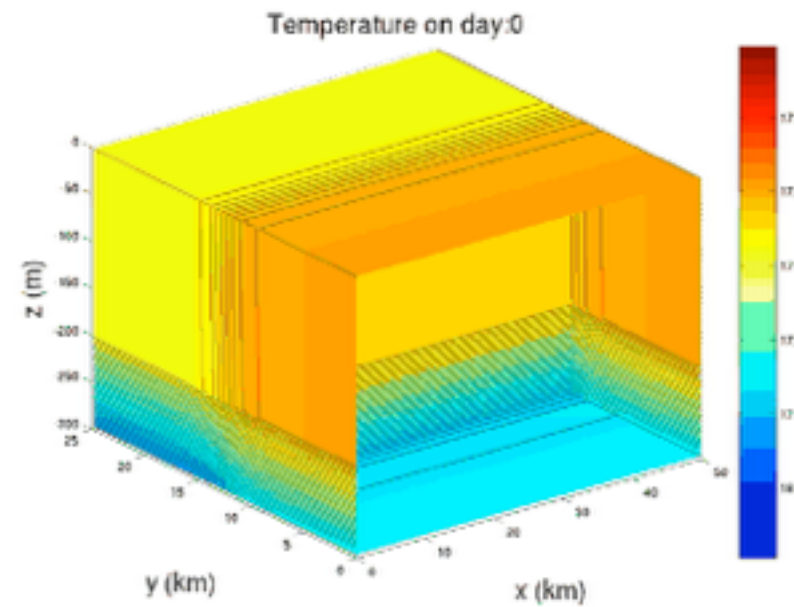
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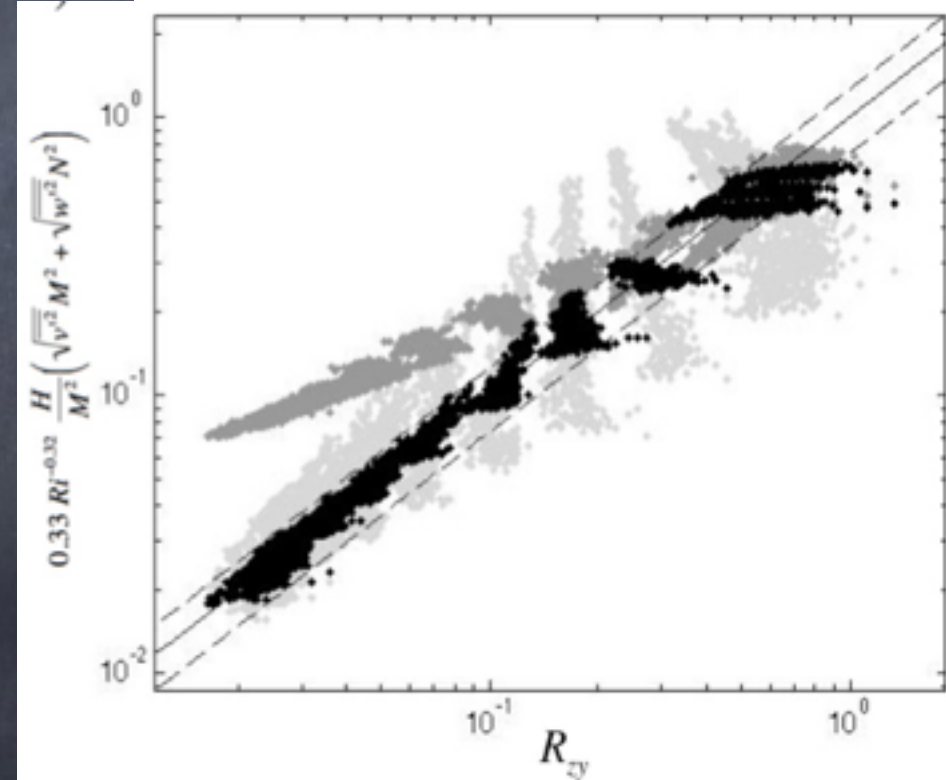
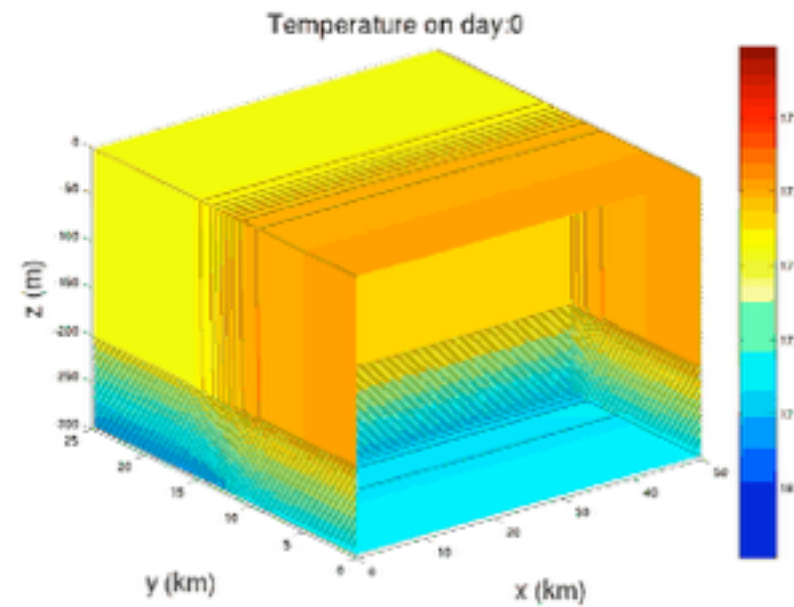
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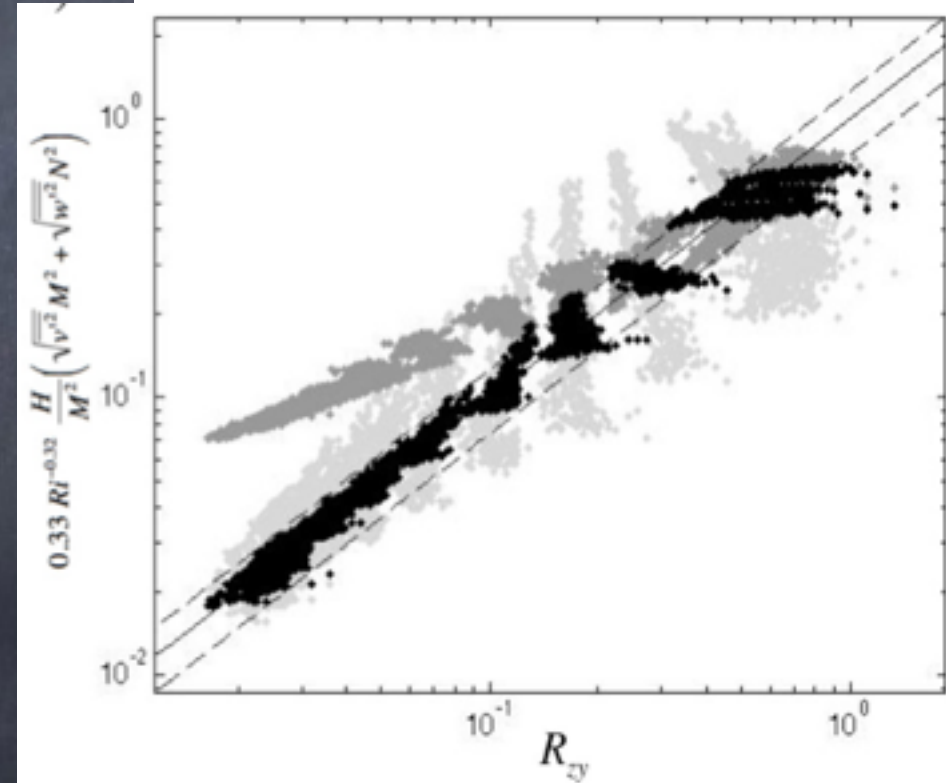
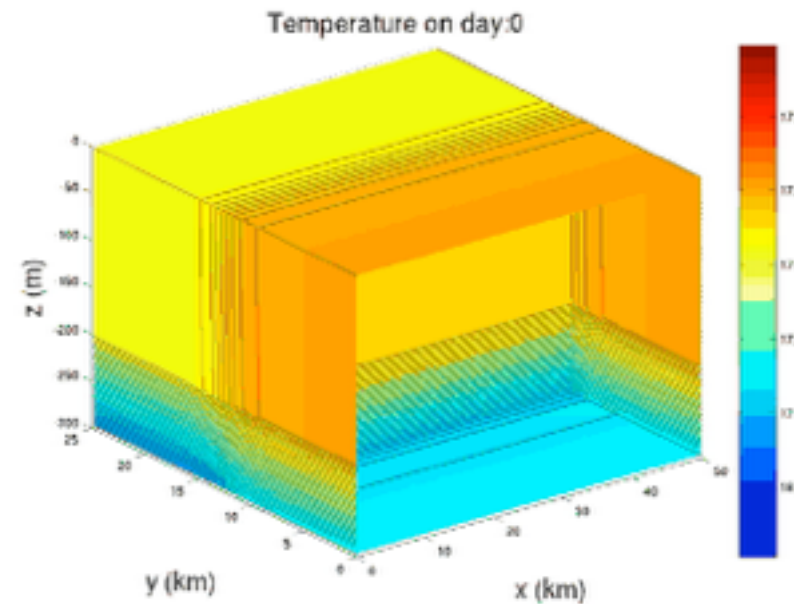
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S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eddy spindown. Ocean Modelling, 64:12-28, 2013

# Mixed Layer Eddy Res

Estimating eddy buoyancy/c

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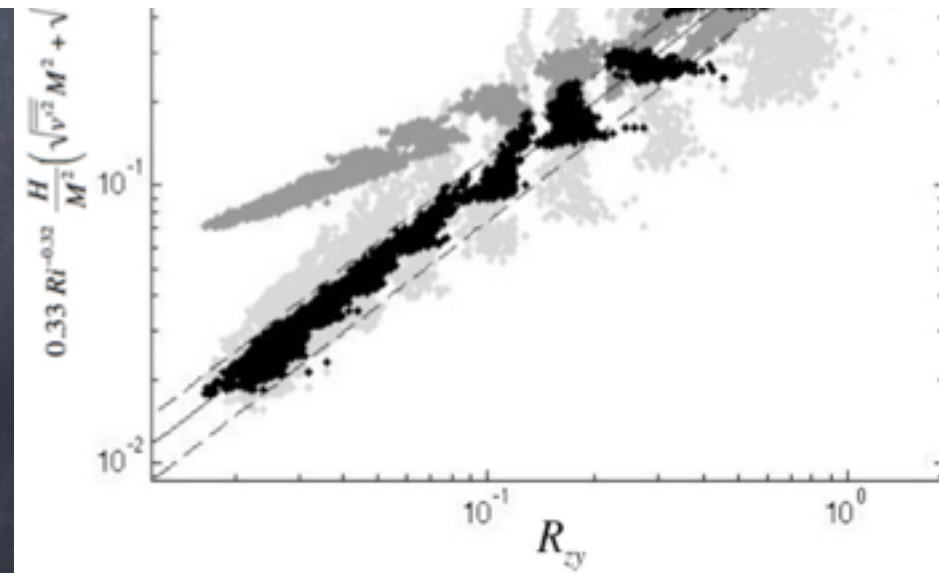
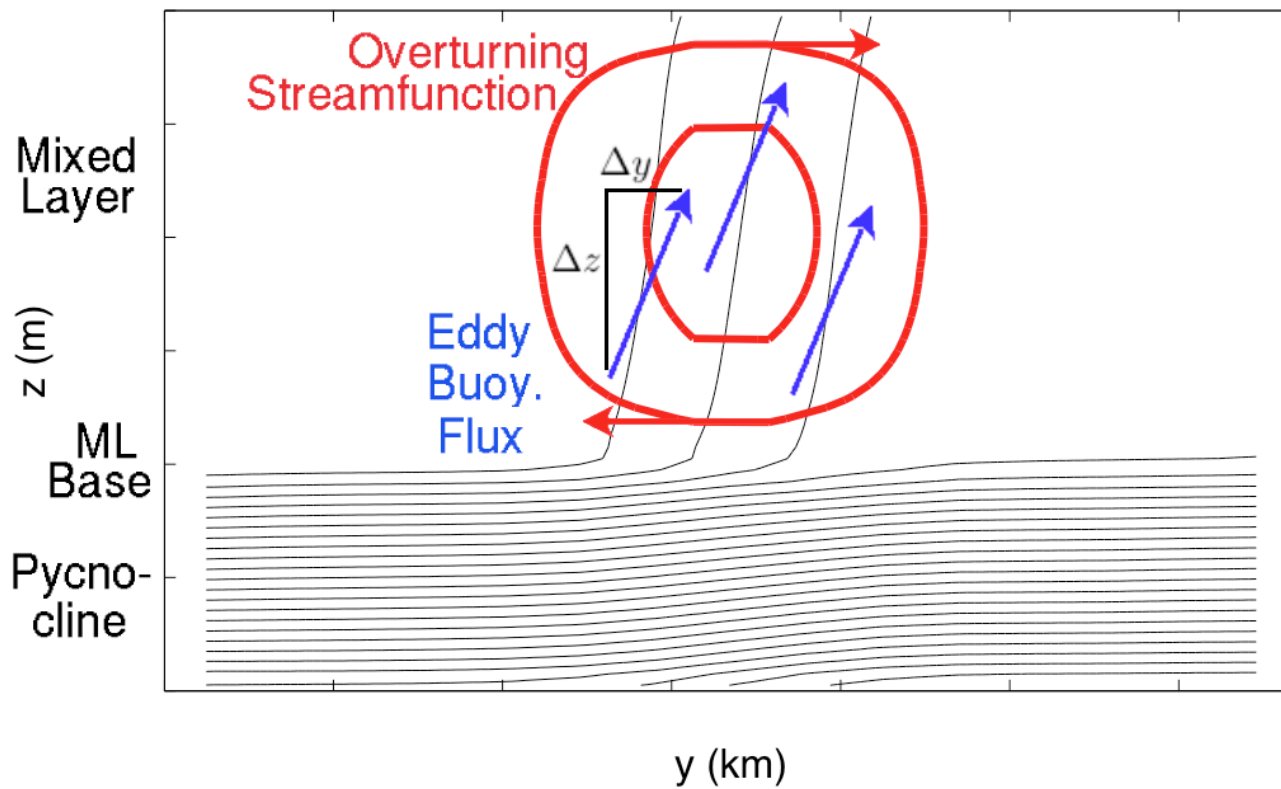
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and horizontally downgradient flux.

$$\overline{\mathbf{u}'_H b'} \propto \frac{-H^2 \frac{\partial \bar{b}}{\partial z}}{|f|} \nabla_H \bar{b}$$



S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eddy spindown. Ocean Modelling, 64:12-28, 2013

# Sensitivity of Climate to Submeso: AMOC & Cryosphere Impacts

May Stabilize AMOC

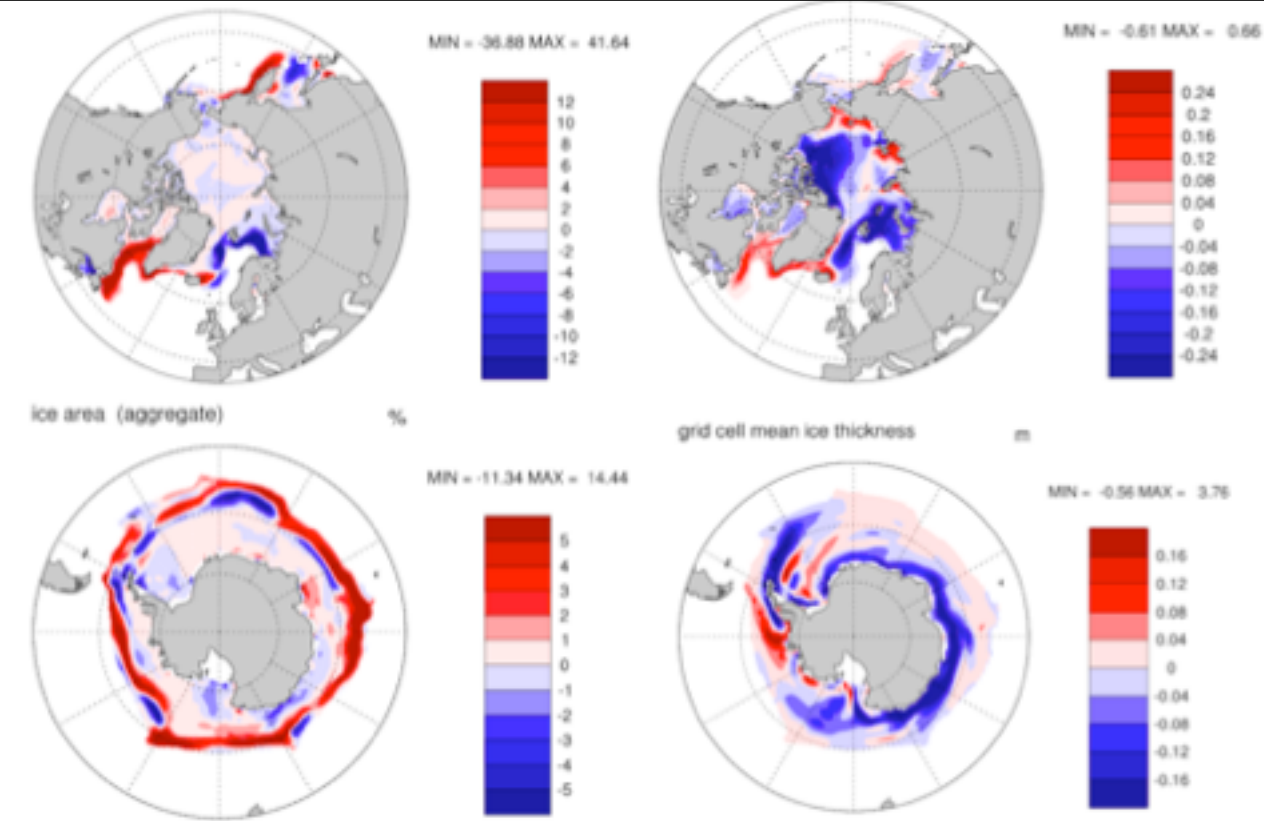


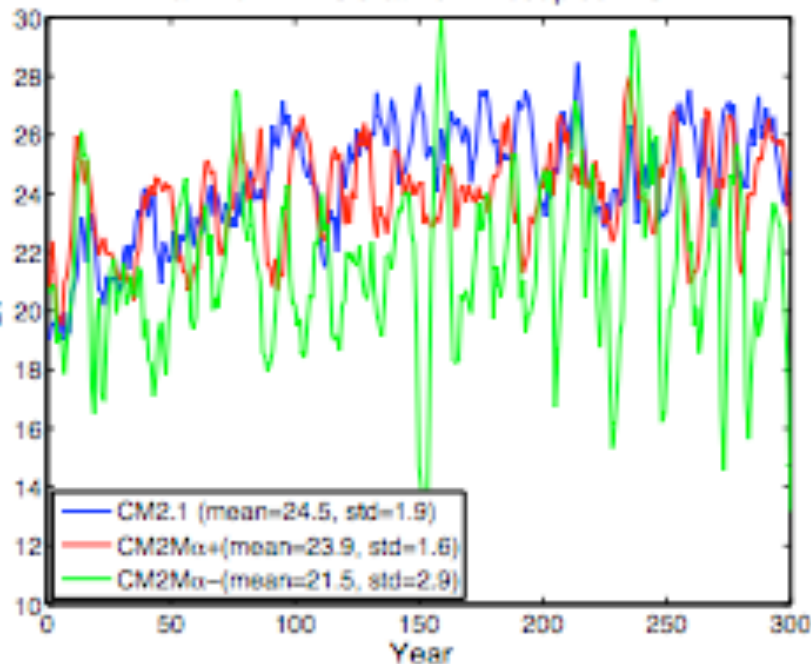
Figure 10: Wintertime sea ice sensitivity to introduction of MLE parameterization (CCSM<sup>+</sup> minus CCSM<sup>-</sup>): January to March Northern Hemisphere a) ice area and b) thickness and July to September Southern Hemisphere c) ice area and d) thickness.

Affects sea ice

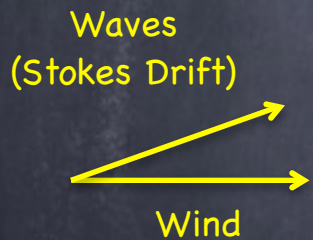
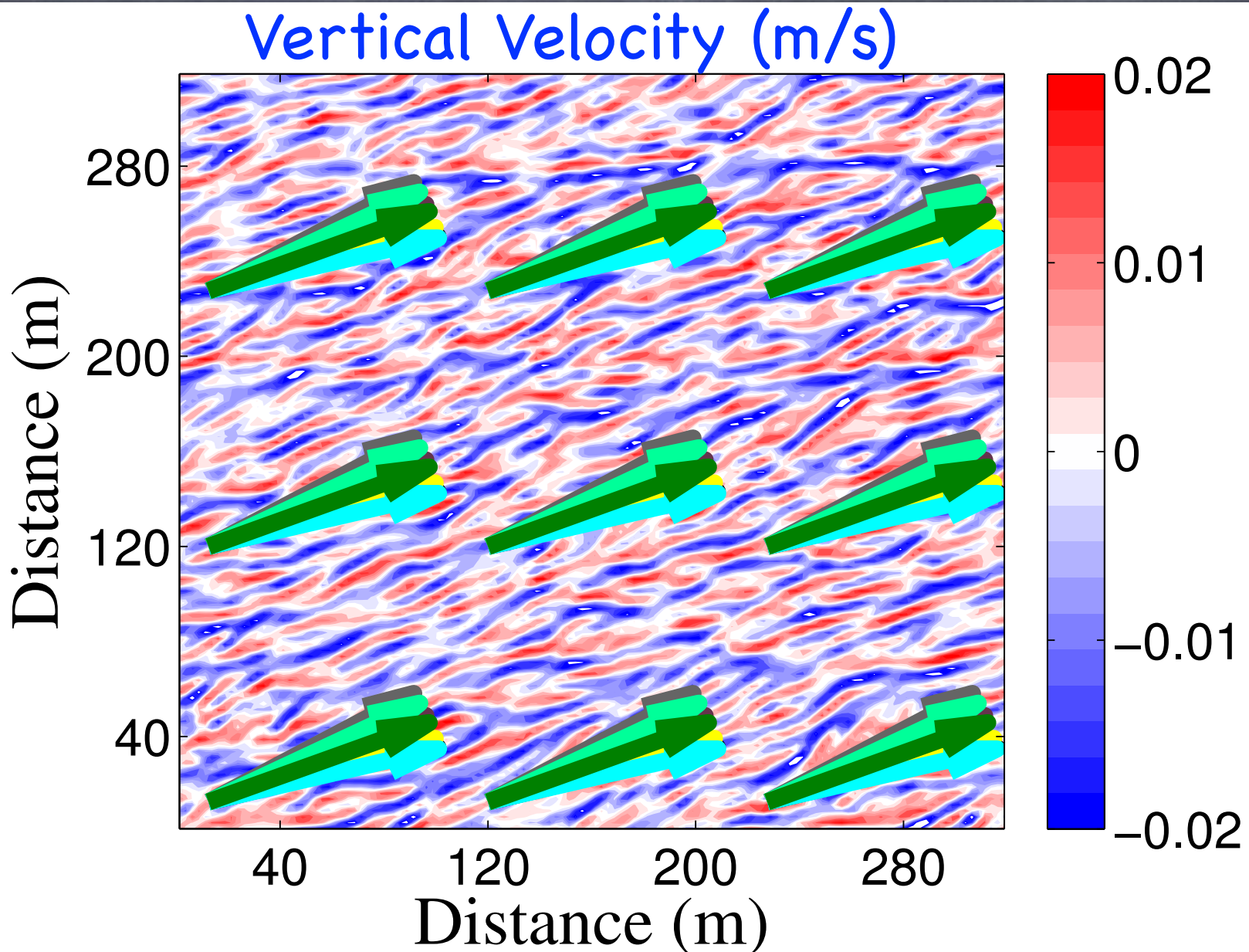
NO RETUNING  
NEEDED!!!

These are impacts:  
bias change unknown

Maximum AMOC at 45n in coupled MOM

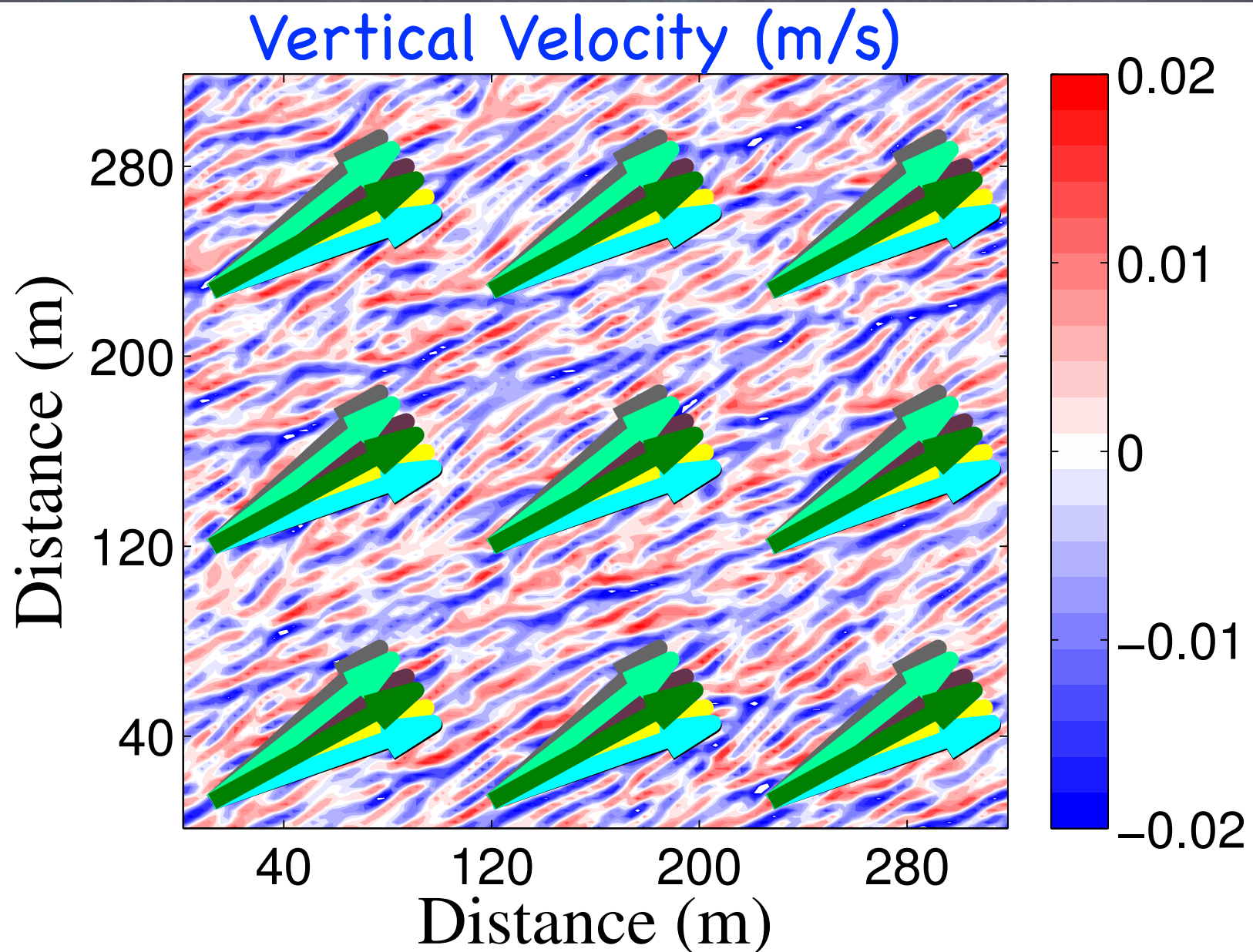


# CLB as equations for Large Eddy Simulations: Tricky: Misaligned Wind & Waves



L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney.  
The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

# Tricky: Misaligned Wind & Waves

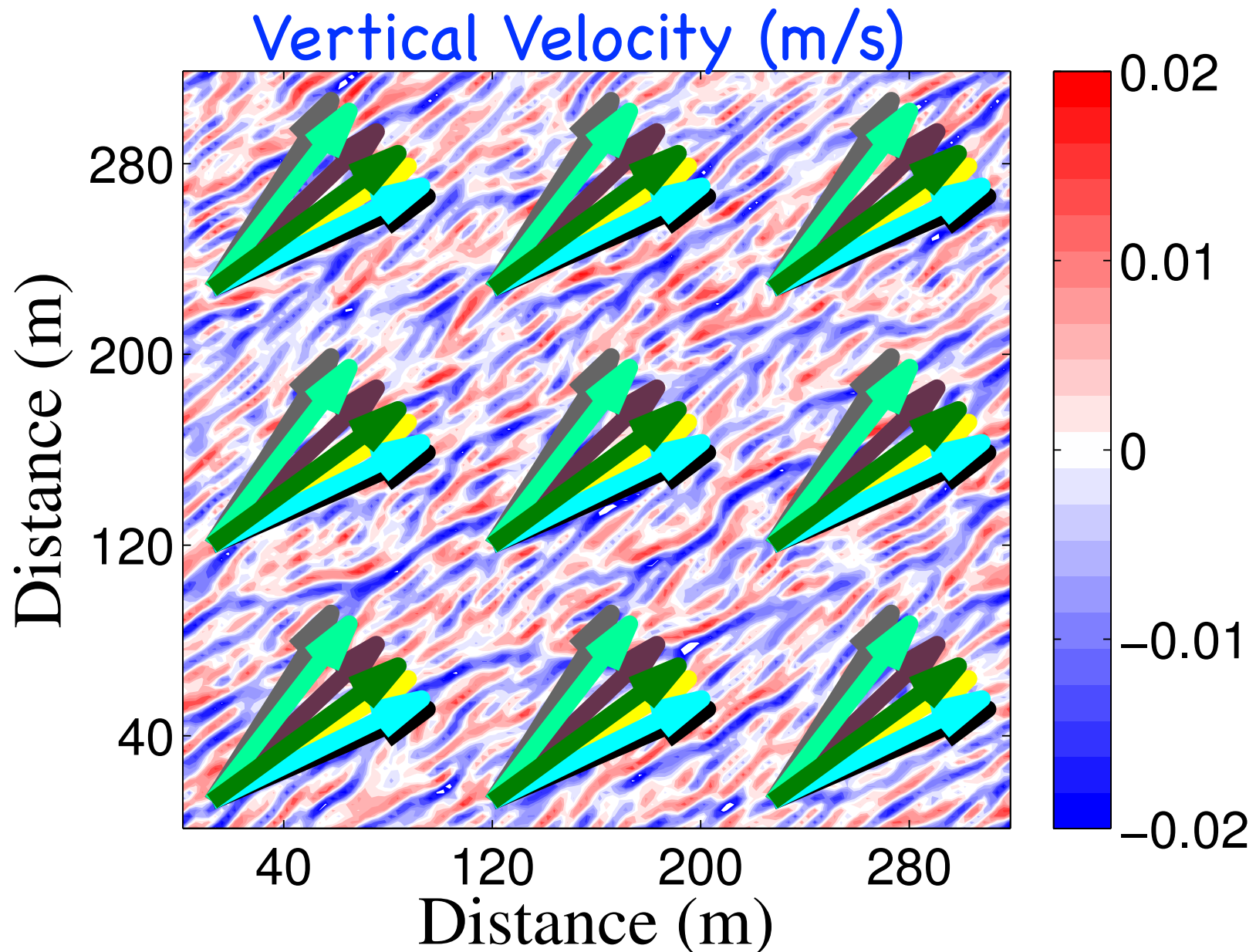


Waves  
(Stokes Drift)



L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney.  
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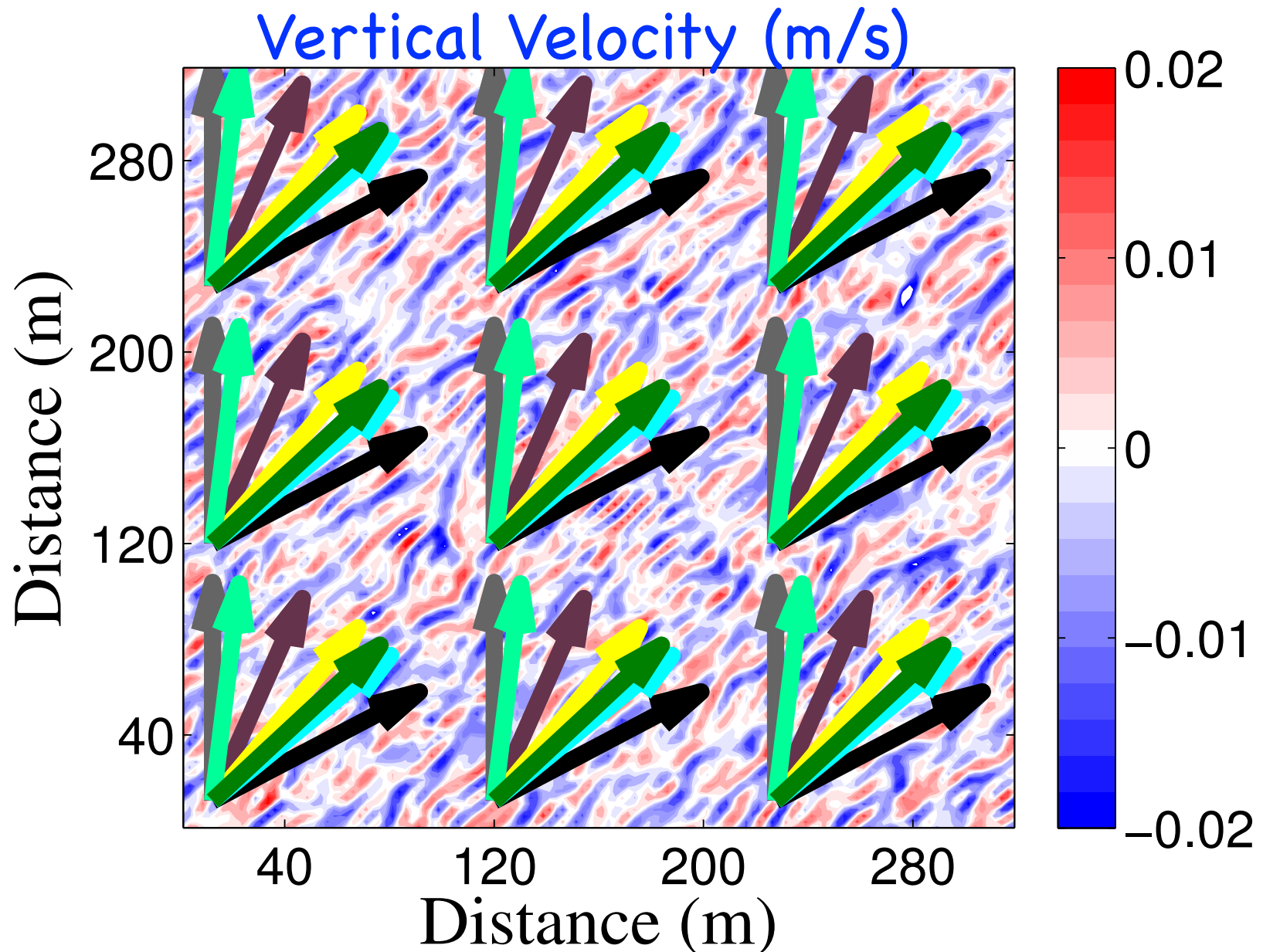


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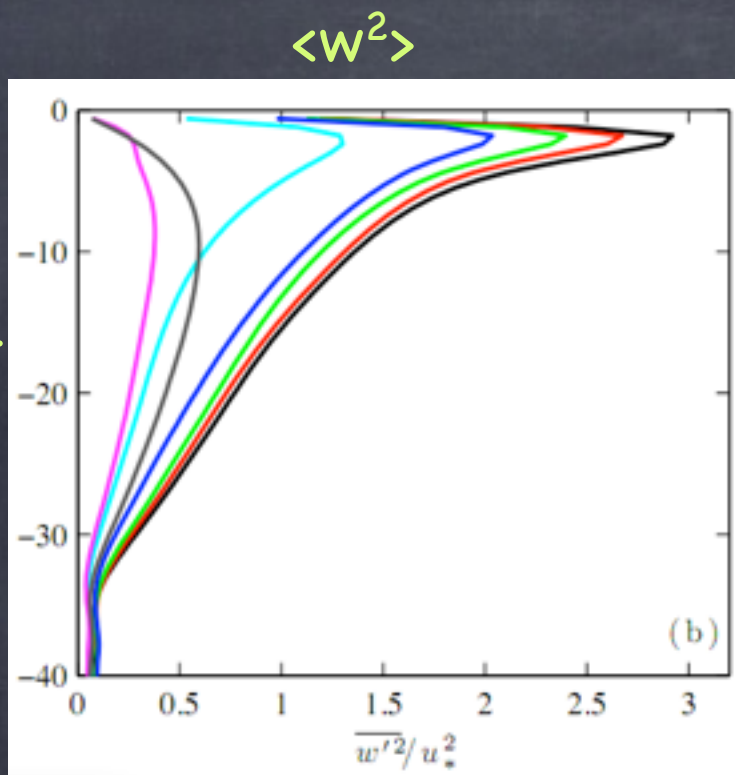


Waves  
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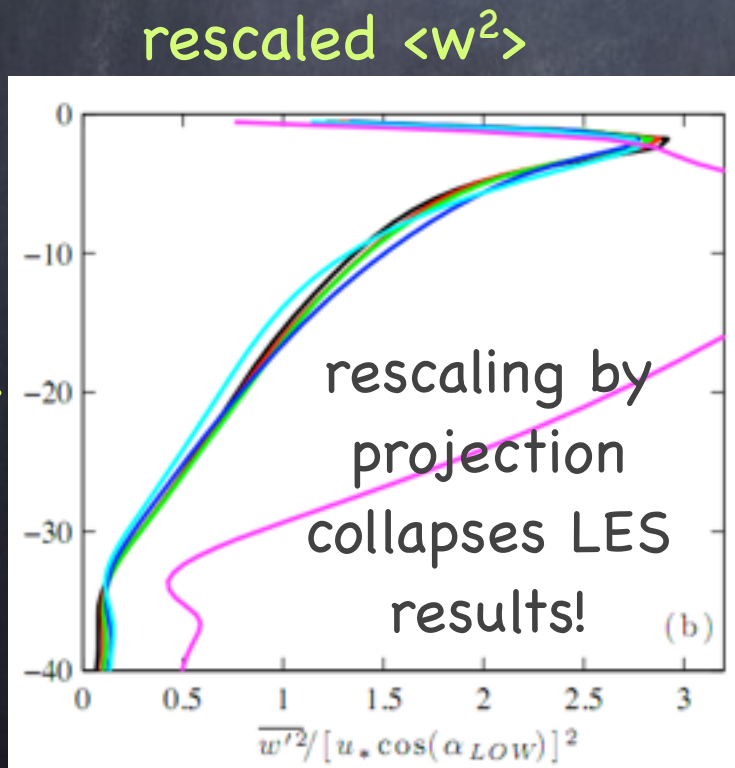


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The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

depth



depth



Generalized Turbulent Langmuir No.,  
Projection of  $u^*$ ,  $u_s$  into Langmuir Direction

$$\frac{\langle \overline{w'^2} \rangle_{ML}}{u_*^2} = 0.6 \cos^2(\alpha_{LOW}) [1.0 + (3.1 La_{proj})^{-2} + (5.4 La_{proj})^{-4}],$$

$$La_{proj}^2 = \frac{|u_*| \cos(\alpha_{LOW})}{|u_s| \cos(\theta_{ww} - \alpha_{LOW})},$$

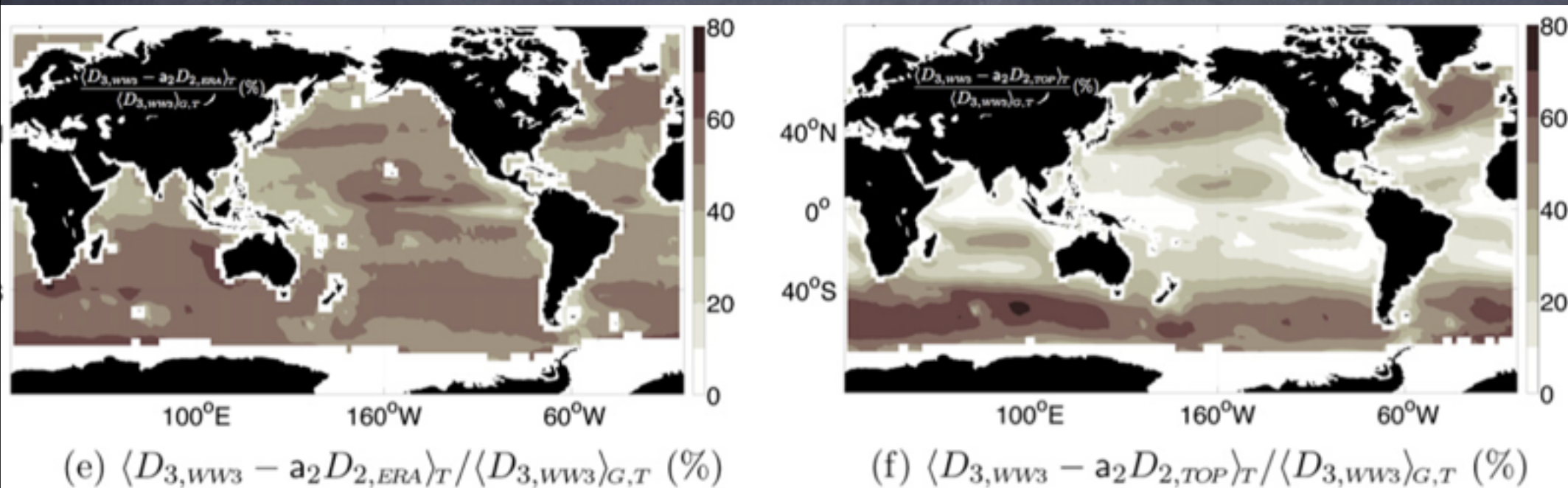
$$\alpha_{LOW} \approx \tan^{-1} \left( \frac{\sin(\theta_{ww})}{\frac{u_*}{u_s(0)\kappa} \ln \left( \left| \frac{H_{ML}}{z_1} \right| \right) + \cos(\theta_{ww})} \right)$$

A scaling for LC  
strength & direction!

L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, 2012.



# How well do we know Stokes Drift? <50% discrepancy



RMS error in measures of surface Stokes drift,  
2 wave models (left), model vs. altimeter (right)

Year 2000 data & models

# Why? Vortex Tilting Mechanism

In CLB: Tilting occurs in direction of  $\mathbf{u}_L = \mathbf{v} + \mathbf{v}_s$

Misalignment enhances degree of wave-driven LT

$$\frac{\partial \xi}{\partial t} + \underbrace{(\mathbf{u}_L \cdot \nabla)}_{AD} \xi = \underbrace{(\boldsymbol{\omega}_a \cdot \nabla)}_{TS} (\mathbf{u}_L \cdot \hat{\mathbf{x}}') + \underbrace{(\nabla b \times \hat{\mathbf{z}})}_{BV} \cdot \hat{\mathbf{x}}' + \text{SGS},$$

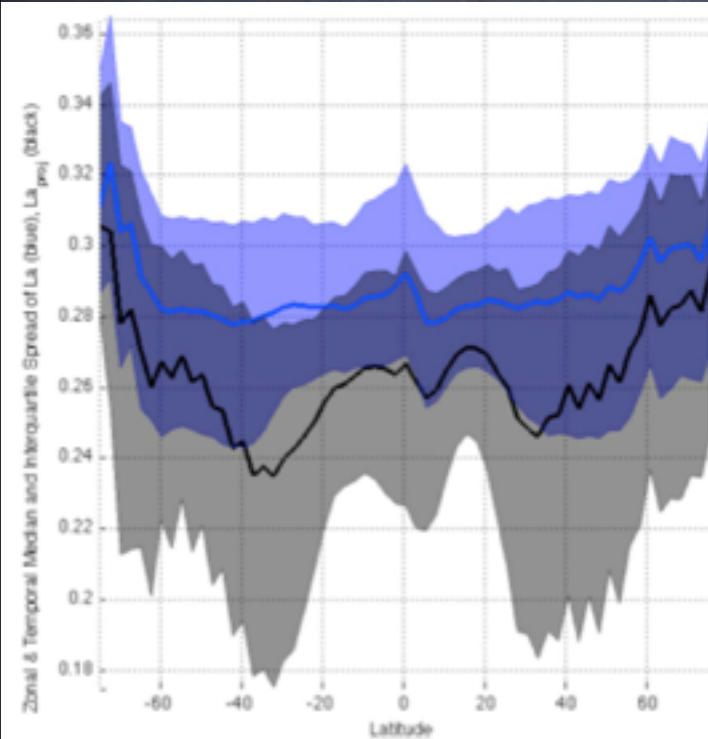


Figure 17. Temporal and zonal median and interquartile range of  $La_t$  and  $La_{proj}$  for a realistic simulation of 1994–2002 using Wave Watch III.

image:  
Thorpe, 04

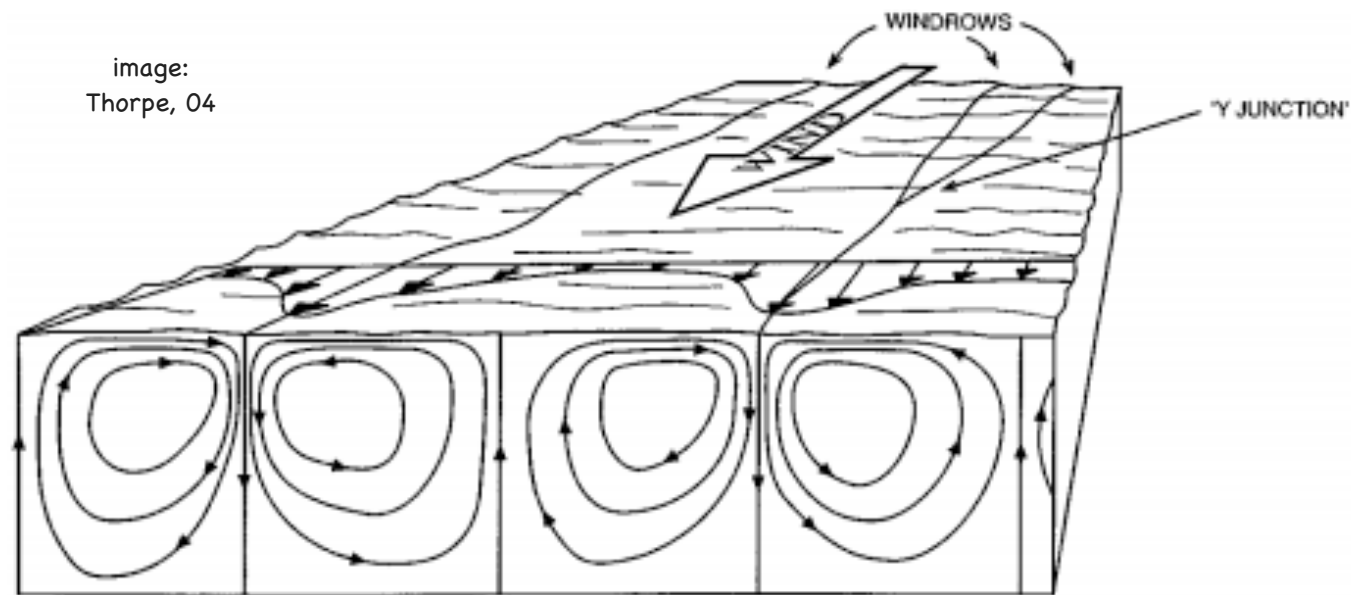


Figure 1 Sketch showing the pattern of mean flow in idealized Langmuir circulation. The windrows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). In practice the flow is turbulent, especially near the water surface, and the windrows (Figure 2) amalgamate and meander in space and time. Bands of bubbles or buoyant algae may form within the downward-going (or downwelling) flow (see Figure 3).

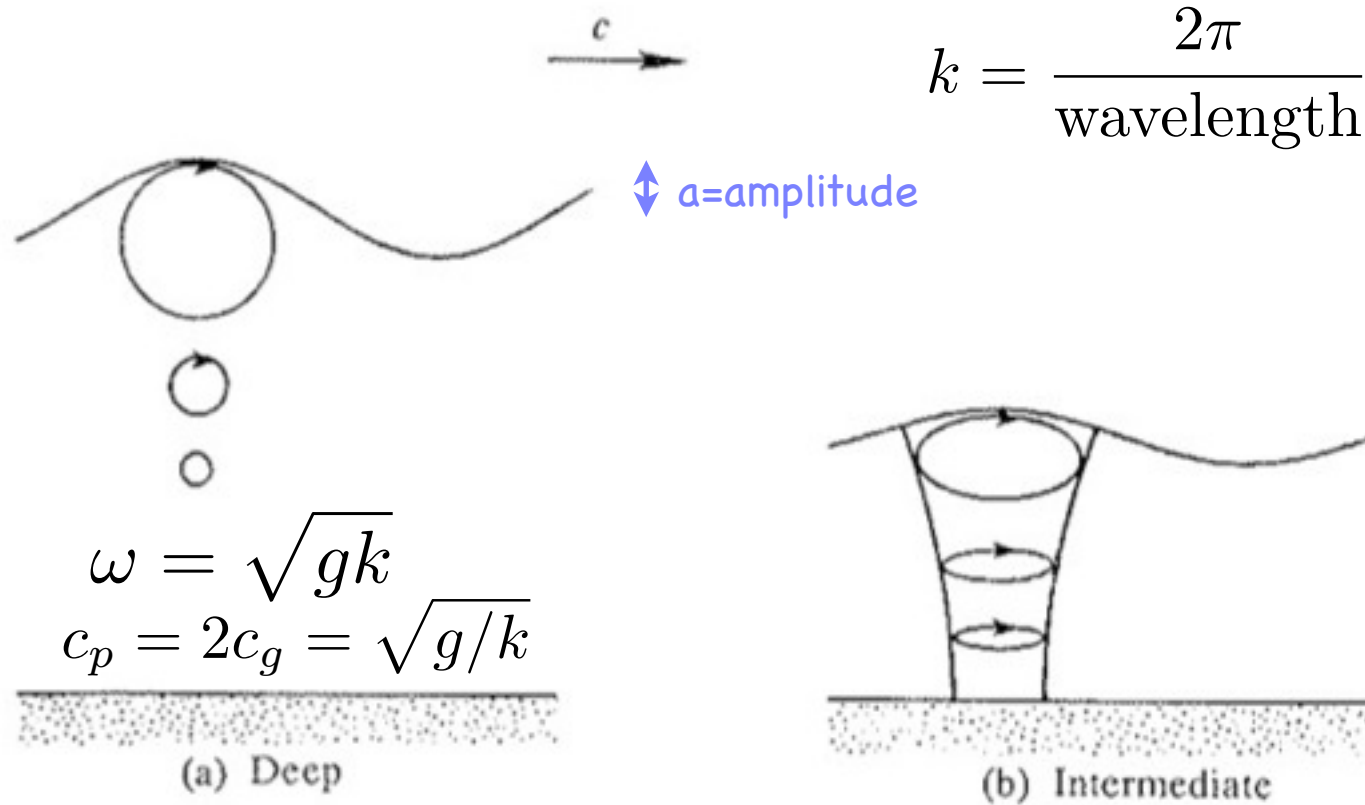
## Surface wave effects on oceanic fronts, filaments, and turbulence

Baylor Fox-Kemper

with contributions from Jim McWilliams (UCLA), Nobuhiro Suzuki (Brown), and Sean Haney (CU Boulder)

The upper ocean is home to many complex dynamical interactions—between the air and sea, between the waves, winds, and currents, between the gasses in the atmosphere and those dissolved in the ocean, and between spatiotemporal scales of variability. A standard approach to the upper ocean multiscale dynamical interaction was first proposed by Craik and Leibovich, and later improved by Holm, McWilliams, Lane, and Restrepo. The approach focuses on the Boussinesq equations averaged over the timescale of surface waves under the assumption that the fast scales are waves of limited steepness. Under this assumption, the leading order coupling between the fast and slow scales is through the Stokes drift of the surface waves, which affects the larger, slower scales through advection and the Coriolis and vortex forces. The approach is equivalent and complementary to the radiation stress theory of Longuet-Higgins. Analysis, theory, and Large Eddy Simulations of the wave-averaged equations will be presented that elucidate some effects of surface waves on upper ocean boundary layer turbulence and submesoscale fronts and filaments.

# Particle motions

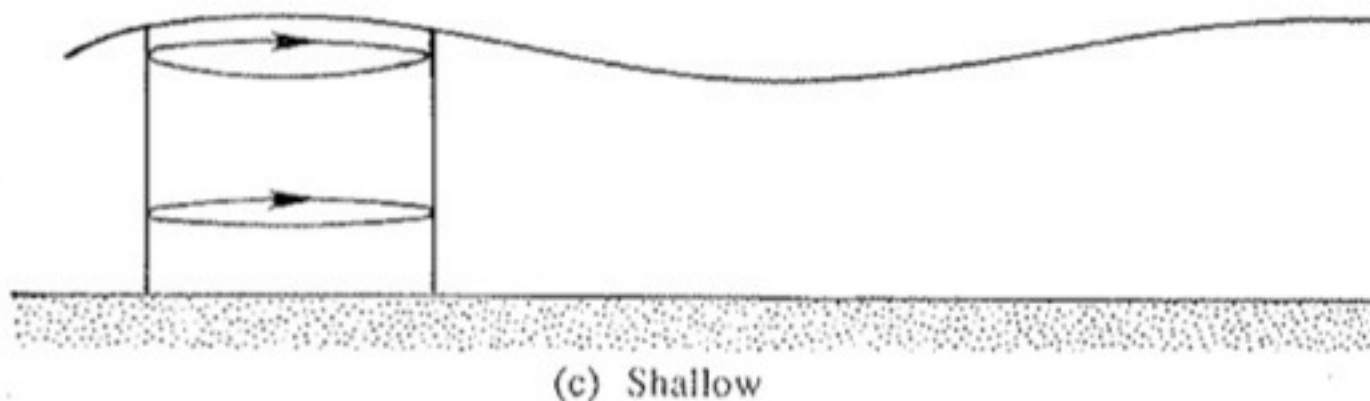


The  $u, v$ , decay exponentially toward the bottom with decay scale proportional to the wavelength.

Thus,  $kH$  is a measure of depth

$ka$  is a measure of steepness

Deep water waves don't "feel" the bottom. Implies nonhydrostatic ( ) & fast timescale ( $Ro \gg 1$ )



# So, no problems?

## Just crunch away with CLB?

- Let's revisit our assumptions for scale separation:
  - CLB wave equations require limited \*wave steepness\* and irrotational flow
  - Real wind-waves are not monochromatic, but incorporate a spectrum of waves, and...



Power Spectrum  
of wave height

$$\langle \eta^2 \rangle = \int_0^{\infty} E(k) dk = C_0 + \int_{k_h}^{\infty} C_1 k^{-2} dk$$

Power Spectrum  
of wave  
steepness:  
**INFINITE!**

$$\langle k^2 \eta^2 \rangle = \int_0^{\infty} k^2 E(k) dk = D_0 + \int_{k_h}^{\infty} D_1 dk$$

Steep waves break  $\rightarrow$  vortex motion & small scale turbulence!

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