The role of criticality on the horizontal and vertical scales of extratropical eddies in a dry GCM

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1. Introduction

The theory of the extratropical eddy scale L_e can be regarded as falling between two end-members.
(a) Linear regime: eddy scale is similar to that of the most unstable wave. For Eady problem, most unstable wavelength $L_e \sim \lambda \approx 3.9 L_R$.
(b) Turbulent regime: eddy scale is determined by geostrophic turbulence. In many cases, $L_e \sim L_\beta = \sqrt{U/\beta}$.
The regimes in which above two strands of theory are relevant can be separated by a non-dimensional number, the criticality ξ.
(a) Low criticality: inverse cascade is not important
$\xi \leqslant 1 \rightarrow L_e \sim L_\beta \sim L_R$ (Schneider and Walker 2006)
(b) Highly supercritical: there can be substantial inverse cascade
$\xi \gg 1 \rightarrow L_e \sim L_\beta \sim \xi L_R$ (Held and Larichev 1996)

Criticality is a measure of isentropic slope and it can be defined in layered and continuously stratified quasi-geostrophic (QG) and primitive equation (PE) models. In two-layer QG model,

$$\xi = U/(\beta L_R^2).$$

In PE models,

$$\xi = \left|\frac{f\partial_y \theta}{\beta \Delta_v \theta}\right|,$$

where $\Delta_v\theta$ is the bulk stability. In spherical geometry,

$$f\partial_y\theta/\beta = \Delta_h \approx a\partial_y\theta,$$

where Δ_h approximate the potential temperature difference between the subtropics and the pole.

In PE model, criticality is difficult to change and stays close to 1.
- Stone's (1978) baroclinic adjustment hypothesis.
- Schneider (2004) and Schneider and Walker (2006) derived constraints $\xi \approx 1$.

Recent studies showed that this constraint can be violated.
- Zurita-Gotor (2008) first increased criticality above one by changing diabatic heating rate in a PE dry GCM.
- Jansen and Ferrari (2012, 2013a,b) suggested Schneider's (2004) formulation of averaging along isentropes in the surface layer is not consistent with the turbulence diffusive closure, and their scaling clearly suggested that criticality can vary with changes in radiative forcing, and to a greater extent, with changes in the size and rotation rate of the planet, as

$$\xi \sim (f\tau)^{-1/5}(a/L_R)^{3/5}.$$

Motivation: filling the gap between linear and turbulent regimes
- **What gives the eddy scale? (a mean field estimate)**
- **What is the role of inverse cascade?**

Interestingly, Zurita-Gotor (2008) showed that when criticality is increased, the eddy scale *increases* while the Rossby radius *decreases*. However, inverse cascade seems not to be significant.

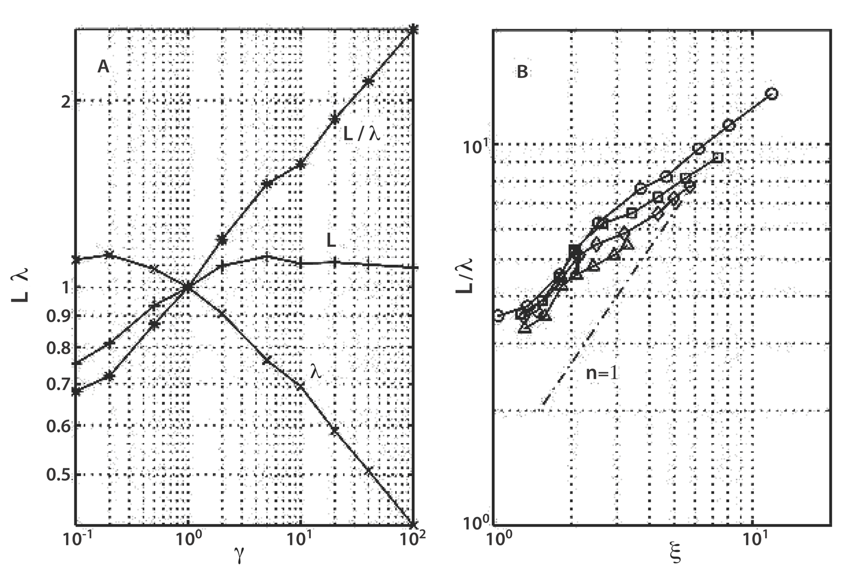
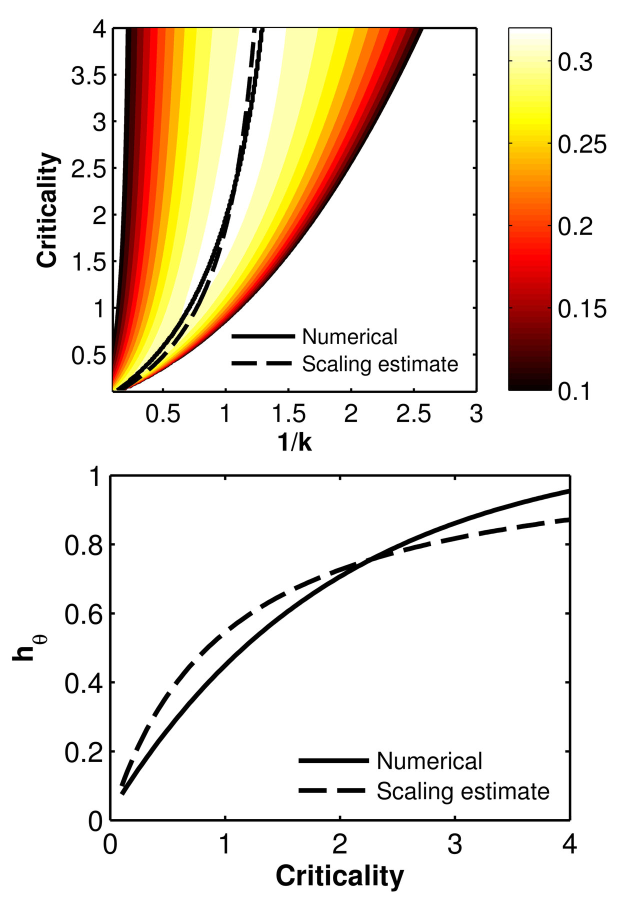


Figure 1 (a) When varying thermal forcing rate γ, eddy scale (pluses), Rossby radius (crosses), and their ratio (stars), all normalized with their control values. (b) Ratio between eddy scale and Rossby radius, as a function of the criticality for all runs. (From Zurita-Gotor 2008.)

2. Linear theory

Revisit Charney problem: most unstable wave's dependency on criticality



Charney problem can be formulated using a single parameter, criticality:

$$\xi = \frac{f^2 \partial_z U}{\beta N^2 H_\rho}.$$

By simple rescaling, we derived a simple approximate estimate for the most unstable wave's horizontal and vertical scales' dependency on criticality.

$$L_m \approx C\frac{\xi}{\xi+1}L_R$$

$$h_m \approx D\frac{\xi}{\xi+1}H_\rho$$

The vertical scale is defined from e-folding scale of heat flux.

Figure 2 Comparison of scaling estimates and numerical results. (Upper) Contour plot showing the growth rate against the inverse wavenumber $1/k$ and criticality ξ for the Charney model. Our scaling estimate is $1.54\xi/(1+\xi)$. (Lower) Vertical scale of the most unstable wave vs. criticality: our scaling estimate is $1.09\xi/(1+\xi)$. The horizontal and vertical scales are nondimensionalized by Rossby radius and density scale height respectively.

3. Experiments and results

Vary criticality

Criticality is varied by varying thermal forcing. To vary thermal forcing by several orders without damping out the eddies, Zurita-Gotor (2008) suggests only varying the forcing time scale on the zonal mean flow, as

$$\frac{\partial T}{\partial t} = \ldots - k_T T' - \gamma k_T(\bar{T} - T_{eq}).$$

Reduced GCM: without eddy-eddy interactions

In order to isolate the effects of inverse cascade, we compare results from *full* GCM runs with the GCM without nonlinear eddy-eddy interactions introduced by O'Gorman and Schneider (2007). Equations are modified, for example,

$$\frac{\partial T}{\partial t} = \ldots - \bar{v}\frac{\partial \bar{T}}{\partial y} - \bar{v}\frac{\partial T'}{\partial y} - v'\frac{\partial \bar{T}}{\partial y} - \overline{v'\frac{\partial T'}{\partial y}}$$

Series of simulations

For each meridional temperature difference δ_y = 30, 60, 90, and 120 K, consider 9 values of γ = (0.1, 0.2, 0.5, **1** [control run], 2, 5, 10, 20, 40).

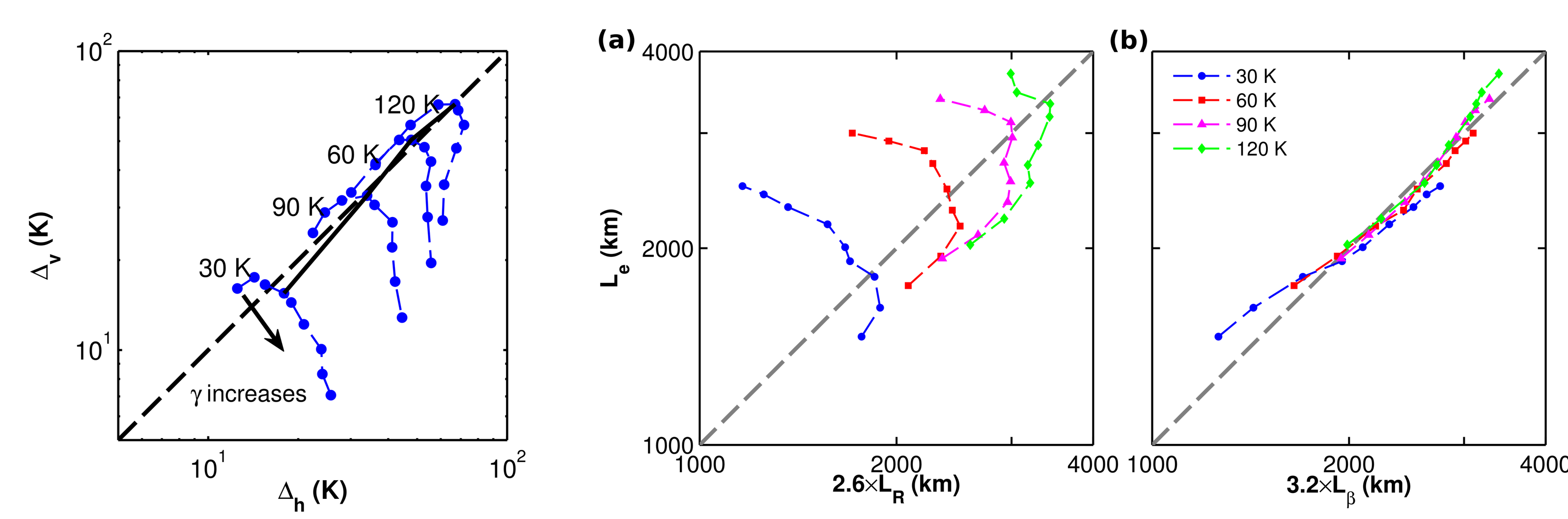


Figure 3 (Left) Extratropical bulk stability $\Delta_v\theta$ vs. scaled meridional temperature gradient $f\partial_y\theta/\beta = \Delta_h$. (Right) (a) Best Rossby radius estimate $C_R L_R$ and (b) Rhines scale estimate $C_\beta L_\beta$ for eddy scale.

Does inverse cascade cause eddy scale to increase?

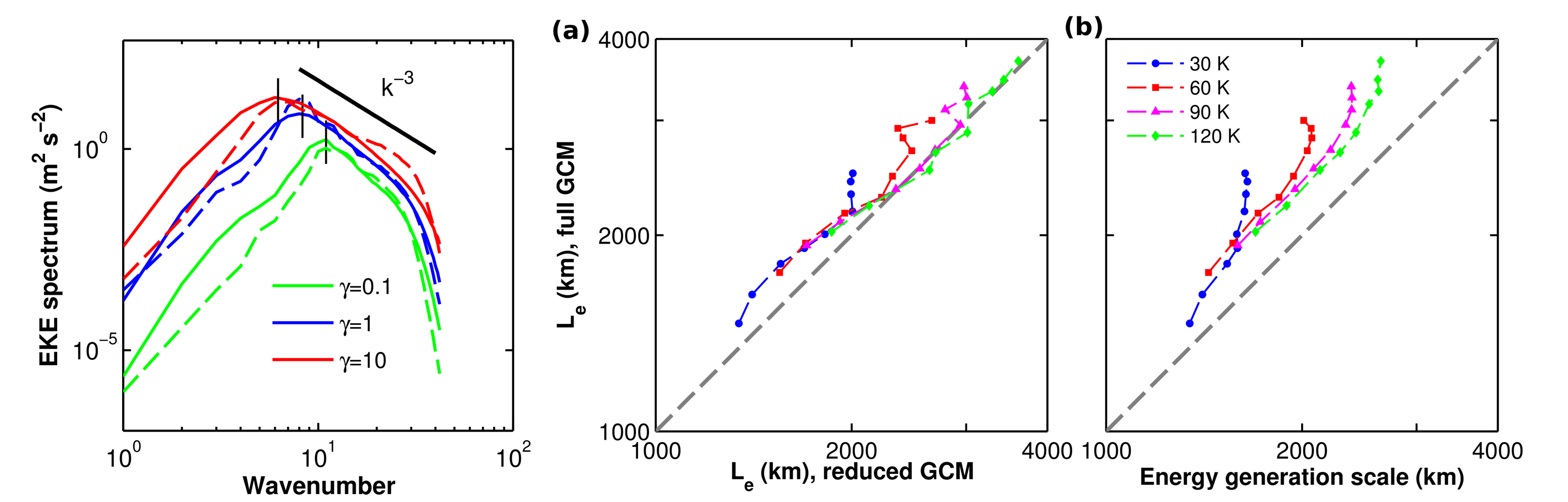


Figure 4 (Left) For the Earth-like settings with δ_y = 60 K, and varying thermal forcing γ = 0.1, 1, 10, comparison of EKE spectrum between the full (solid lines) and reduced (dashed lines) GCMs. (Right) (a) Comparison of eddy scale in full and reduced GCMs, and (b) comparison of eddy scale and eddy kinetic energy generation scale in full GCMs.

Change of eddies' vertical scale with criticality

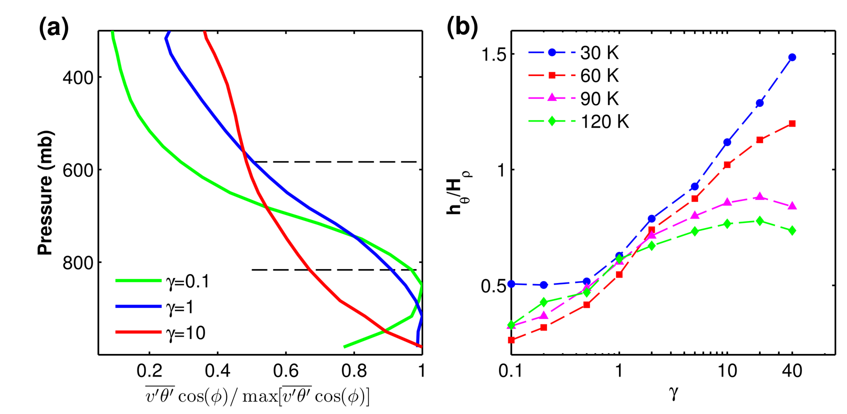


Figure 5 (a) The vertical structure of eddy potential temperature flux averaged over the baroclinic zone and normalized by its maximum value from simulations with δ_y=60 K and γ = 0.1, 1, 10. (b) The vertical scale of eddies.

Eddy scale estimate using Rossby radius and criticality

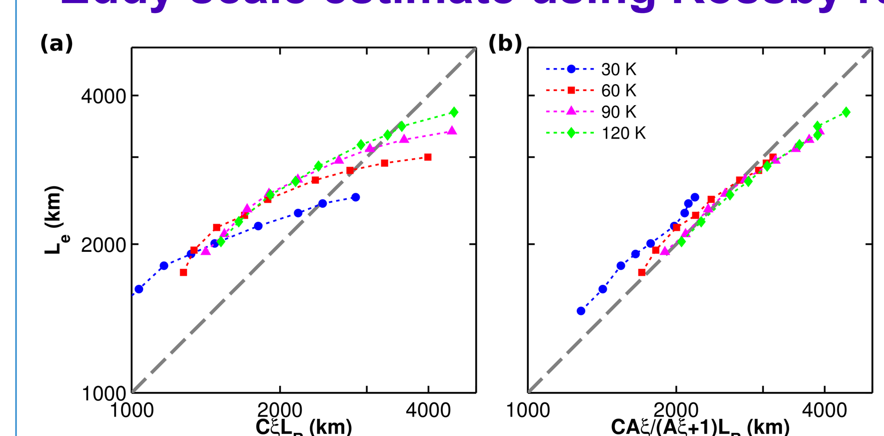


Figure 6 (a) Best estimate using $C\xi L_R$ vs. observed eddy scale in all runs, with C = 1.8. (b) Best estimate using $CL_R(A\xi)/(A\xi+1)$ vs observed eddy scale in all runs, with C = 8.7 and A = 0.36.

Obtaining the high criticality limit: increasing planetary radius

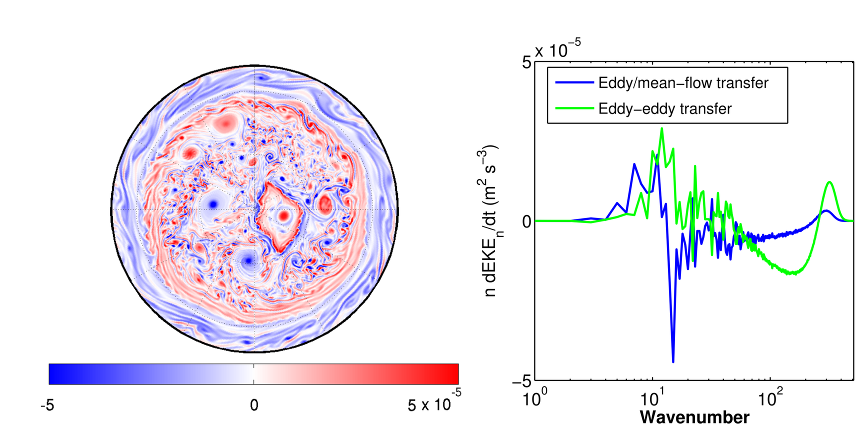


Figure 7 Results from a run with similar Earth's parameters but 10 times of Earth's radius. (a) A snapshot of relative vorticity field. (b) Spectral EKE budget analysis showing eddy-eddy nonlinear energy transfer and eddy/mean-flow energy transfer.

4. Conclusions

- When criticality varies near one, it is shown that there exists a weakly nonlinear regime in which the eddy scale increases with criticality without involving an inverse cascade, while at the same time the Rossby radius may in fact decreases.
- We reconcile the opposite trends of eddy scale and Rossby radius using Charney model, and obtain an estimate for the eddy scale in terms of Rossby radius and criticality.

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