STOCHASTIC AVERAGING, JET FORMATION AND BISTABILITY IN TURBULENT PLANETARY ATMOSPHERES

F. Bouchet, C. Nardini and T. Tangarife

École Normale Supérieure de Lyon and CNRS, France

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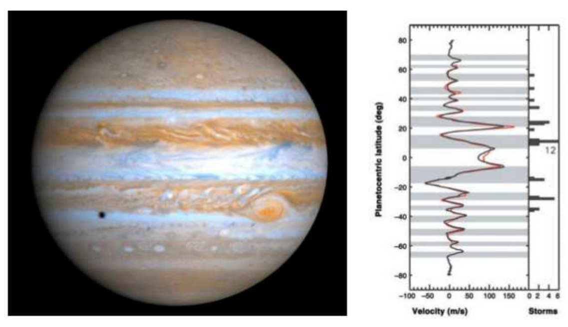
Motivation: zonal jets slow evolution and rare transitions

ZONAL JETS VELOCITY PROFILES

Zonal jets are observed in upper planetary atmospheres (Earth, Jupiter, Saturn, ...).



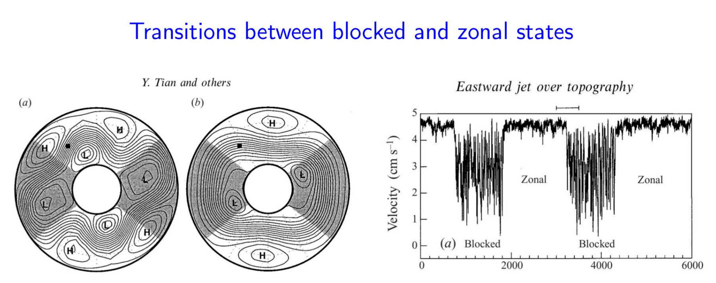
Left: snapshot of Jovian upper atmosphere. Right: zonally averaged velocity profile obtained from the data analysis of the fly-by of Cassini (2000) and Voyager (1979). Figures are taken from [1], see also [2].

They evolve very slowly with respect to turbulent fluctuations and most of the energy is in zonal degrees of freedoms (about 93% on Jupiter, [2])

How to predict these velocity profiles?

MULTIPLE ATTRACTORS AND BISTABILITY

Real data, experiments and DNS reveal the possibility of **sporadic transitions** among different attractors after a long period of apparent stability. This behaviour is observed in many geophysical flows and turbulence problems, for example the Earth magnetic field reversal, paths of oceanic currents, atmospheric flows.



Experimental idealisation [3] of the Earth polar jet stream in a rotating tank with topography. Left: blocked and zonal states. Right: Rare transitions showing bistability with different average residence times and fluctuations properties around the attractors.

How to predict the statistical properties of rare transitions?
average residence time, fluctuations properties

Stochastic barotropic equation on a beta-plane

Simplest model for mid-latitude large-scale dynamics

$$\partial_t q + \mathbf{v}\cdot\nabla q = \nu\Delta\omega - \lambda\omega + \sqrt{2}\eta(\mathbf{r},t) \qquad \mathcal{D} = [0, 2\pi l_x L]\times[0, 2\pi L]$$
$$q = \omega + \beta y \qquad \mathbf{v} = e_z \times \nabla\psi \qquad \omega = \Delta\psi$$

The stochastic forcing η represents the effect of unresolved features of the model (radiation, baroclinic instability, effect of different layers, ...). η is Gaussian with correlation

$$\mathbb{E}[\eta(\mathbf{r},t)\,\eta(\mathbf{r}',t')] = C(\mathbf{r}-\mathbf{r}')\,\delta(t-t')$$

NON-DIMENSIONAL UNITS

Denoting by ϵ the energy injection per unit of mass, we have the following typical values: jet velocity: $U = \sqrt{\epsilon/2\lambda}$; time-scale of the zonal jet: $\tau_z = 1/\lambda$; time-scale of fluctuations $\tau_m = U/L$

We rescale barotropic equations setting to unity τ_m and L

$$\partial_t \mathbf{q} + \mathbf{v}\cdot\nabla\mathbf{q} = \nu\boldsymbol{\Delta}\omega - \alpha\omega + \sqrt{2\alpha}\eta(\mathbf{r},\mathbf{t}) \qquad \alpha = \frac{\tau_\mathbf{m}}{\tau_\mathbf{z}}$$

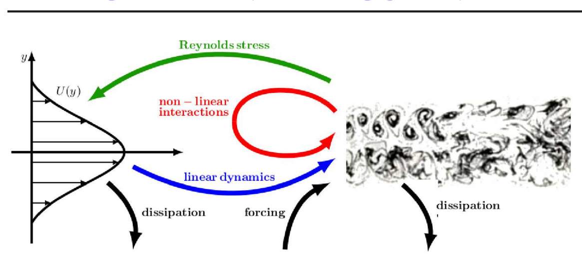
INERTIAL LIMIT: $\nu \ll \alpha \ll 1$. Slow evolution of the zonal jet with respect to the evolution time-scale of turbulent fluctuations

Zonal degrees of freedom, stochastic averaging

To extract zonal degrees of freedom: $f_z(y,t) = \langle f\rangle = \int \frac{dx}{2\pi l_x} f(\mathbf{r},t)$

$$q = q_z(y,t) + \sqrt{\alpha}\omega_m \qquad \mathbf{v} = U(y,t)\mathbf{e}_x + \sqrt{\alpha}\mathbf{v}_m$$

EXACT DYNAMICS IN TERMS OF U AND ω_m



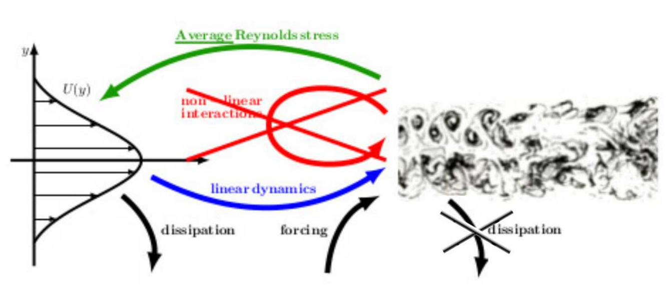
$$\begin{cases} \partial_t U = \alpha\left\langle v_m^{(y)}\omega_m\right\rangle - \alpha U + \nu\partial_y^2 U \\ \partial_t\omega_m + L_U[\omega_m] = -\sqrt{\alpha}\mathbf{v}_m.\nabla\omega_m + \sqrt{\alpha}\left\langle \mathbf{v}_m.\nabla\omega_m\right\rangle \\ \qquad + \sqrt{2}\eta - \alpha\omega_m + \nu\Delta\omega_m \end{cases}$$

$L_U[\omega_m] = U(y)\partial_x\omega_m + \partial_y q_z\,\partial_x\psi_m$: advection operator linearized around U

STOCHASTIC AVERAGING

We aim at a description of the evolution of U tracing out the turbulent fluctuations contained in $\mathbf{v}_m$. We employ a perturbative expansion in the INERTIAL LIMIT called in literature stochastic averaging. This is a well known technique in mathematics [4] and very similar to kinetic theories for plasma and self-gravitating systems [5,6]. The technique is applied to the functional Fokker-Planck associated to the equations for U and ω_m above [7].

Effective dynamics at order α



$$\begin{cases} \partial_t U = \alpha\mathbb{E}_U[\langle\mathbf{v}_m^{(y)}\,\omega_m\rangle] - \alpha U + \nu\partial_y^2 U \\ \partial_t\omega_m + L_U[\omega_m] = \sqrt{2}\eta \end{cases}$$

- absence of eddy-eddy interactions (quasi-linear theory)
- Reynolds' stresses suitably averaged trough $\mathbb{E}_U$:

$$\mathbb{E}_\mathbf{U}[\langle\mathbf{v}_\mathbf{m}^{(\mathbf{y})}\,\omega_\mathbf{m}\rangle] = \lim_{\mathbf{t}\to\infty}\mathbb{E}[\langle\mathbf{v}_\mathbf{m}^{(\mathbf{y})}\,\omega_\mathbf{m}\rangle]$$

Our effective equations are similar to others already introduced phenomenologically [8,9,10] going under the names of S3T (Farrell, Ioannou, et al.), CE2 (Marston, Schneider, Young, et al.), quasi-linear theory. However:

- our equations are obtained from first principles
- the difference mainly stands in how the average of Reynolds' stresses is defined

The theory is self-consistent

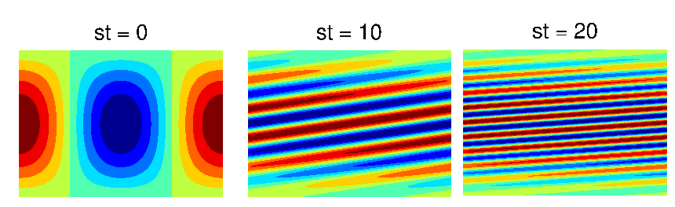
A perturbative expansion in an infinite dimensional system may suffer of ultraviolet divergences which would invalidate the development.

$\lim_{t\to\infty}\mathbb{E}[\langle\mathbf{v}_m^{(y)}\,\omega_m\rangle] < \infty$? YES if U has no unstable nor neutral modes

The average Reynolds' stresses can be written through the deterministic evolution of $\tilde{\omega}_m$ with appropriate initial conditions (that depends on $C(\mathbf{r}-\mathbf{r}')$):

$$\mathbf{E}_U\left[\left\langle v_m^{(y)}\omega_m\right\rangle(y)\right] = \int_0^\infty \tilde{\omega}(y,u)\,\tilde{v}^{(y)*}(y,u)\,\mathrm{d}u$$

ORR MECHANISM



$$\partial_t\tilde{\omega}_m + L_U[\tilde{\omega}_m] = 0$$

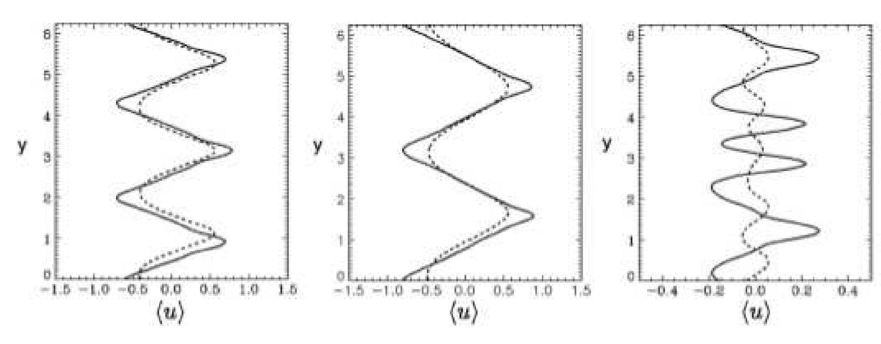
For large times, vorticity creates finer and finer structures, but velocity decays to zero!

Vorticity evolution: $U(y) = sy$ and $\beta = 0$ $\qquad \tilde{\omega}_m \underset{t\to\infty}{\sim} e^{ikU'(y)t} \qquad \tilde{v}_m^{(y)} \underset{t\to\infty}{\sim} \frac{1}{t^2}$

Proven in [11] for any base flow $U(y)$ and $\beta = 0$

Comparison with DNS (figures taken from [9])

The theory should exactly predict the profile $U(y,t)$ in the inertial limit. In literature [8,9] there are good hints. Further tests are necessary



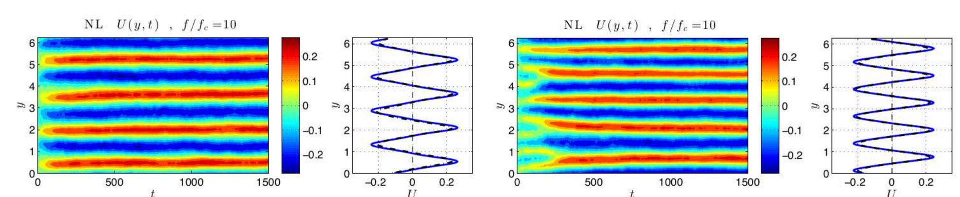
$\alpha = 0.068 \qquad \alpha = 0.068 \qquad \alpha = 1.45$

Stationary velocity profiles U from DNS and from the effective dynamics as obtained by Tobias and Marston in [9]. The accuracy of the prediction increases when α get smaller

At order α, the effective dynamics describes the deterministic relaxation to an attractor

Bistability: effective dynamics at order α^2

Multiple attractors are sometimes found between different jets configurations



Same physical parameters, different initial condition. Figure taken from [12]

A single long run will put in evidence bistability between these two attractors

EFFECTIVE DYNAMICS AT ORDER α^2

The stochastic averaging can be carried on at order α^2

$$\partial_t U = \alpha\mathbb{E}_U[\langle\mathbf{v}_m^{(y)}\,\omega_m\rangle] - \alpha U + \zeta[U] + \mathcal{O}(\alpha^2)$$

Deterministic $\sim\alpha$
Stochastic $\sim\alpha^2$

$\zeta[U]$ is a zero average Gaussian noise whose correlation can be explicitly computed

$$\mathbb{E}[\zeta[U](y_1,t_1)\,\zeta[U](y_2,t_2)] = \alpha^2\,\delta(t_1-t_2)\int_0^\infty \mathrm{d}t\,\mathbb{E}_U\left[\left[\left\langle v_m^{(y)}\omega_m\right\rangle(y_1,t)\left\langle v_m^{(y)}\omega_m\right\rangle(y_2,0)\right]\right]$$

Work in progress: apply large deviations techniques to the effective evolution to explain statistical properties of bistability

CONCLUSIONS

- effective dynamics of zonal jet velocity profile through a perturbative approach
- very good (exact?) predictions in the inertial limit $\nu \ll \alpha \ll 1$

PERSPECTIVES

- bistability: large deviations applied to the effective evolution
- effective evolution in presence of neutral modes
- precise numerical validation of the theory

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The results described in this poster are mainly contained in [7]