Physical Interpretation on the Spontaneous Radiation of Inertia-gravity Waves Using the Renormalization Group Method

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Introduction

Recent studies indicate that inertia-gravity waves (GWs) are radiated from an approximately balanced flow. The present study derives a new theory describing the spontaneous radiation by using the renormalization group (RG) method. The new theory is verified by numerical simulations and gives physical interpretations on the radiation mechanism.

1. A New Mechanism

Vortical flow field
- Slaved components : slow time scale
- Doppler-shifted GWs : ~~fast~~ slow time scale

Time scale matching → quasi-resonance

GWs are radiated through a quasi-resonance with components slaved to the vortical flow

2. RG Method (Chen et al. 1994, 1996)

- RG method is a singular perturbation method (SPM).
- Most SPMs are regarded as the RG method.

Derived time-evolution equations (RG equations)

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Envelope equations

(Kunihiro 1995, 1997)

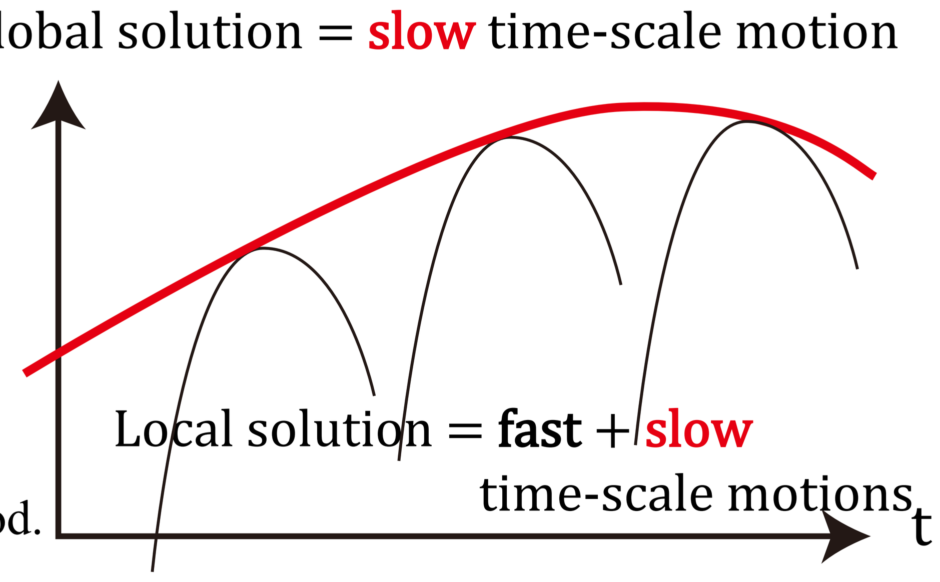


Fig. 1. Schematic illustration for the RG method.

3. Application to the Hydrostatic Boussinesq Eqs.

[1] Dependent variables are transformed.

u, v, w, b, p → q, δ, γ

a slow variable — $q \equiv \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + \frac{\partial b}{\partial z},$

fast variables —
$$\delta \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y},$$
$$\gamma \equiv \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) p.$$

[2] Diagnostic and GW components are introduced.

$(\delta^{diag}, \gamma^{diag})$: slowly vary by nonlinear effects, which are diagnostically obtained.
= slaved components + GW radiation reactions

$(\delta^{GW}, \gamma^{GW})$: slowly vary by Doppler shift, which are spontaneously-radiated GWs.
= eigenmodes against a given vortical flow

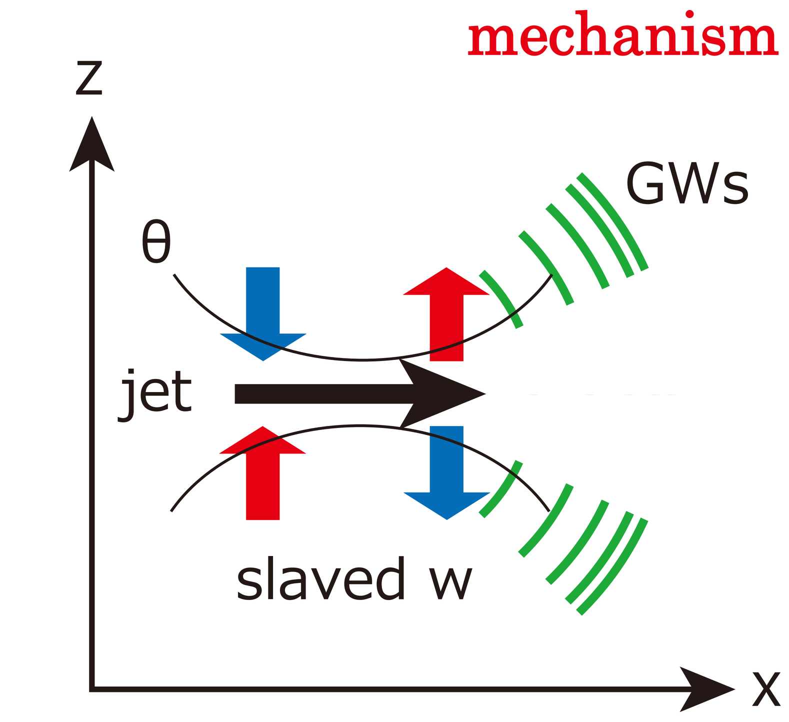
[3] The RG method is applied to the above system.

RGE system
- Time evolution equations: RGE for q + RGE for $(\delta^{GW}, \gamma^{GW})$
- Diagnostic formulae for $(\delta^{diag}, \gamma^{diag})$

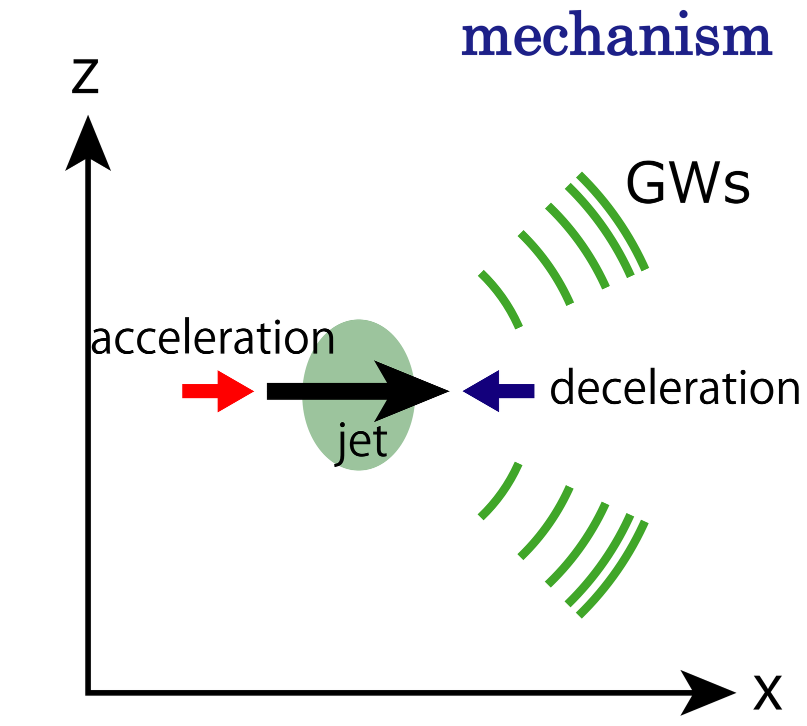
Conclusion

- The interaction between the vortical flow and Doppler-shifted GWs is formulated as RGEs.
- GWs are radiated through the quasi-resonance with slaved components regardless of their order.
- RGE is verified by the numerical simulation.
- Physical interpretations using RGEs can correspond to the mountain-wave-like mechanism (McIntyre 2009), or the velocity-variation mechanism (Viudez 2007).

(a) Mountain-wave-like mechanism



(b) Velocity-variation mechanism



4. Verification by Numerical Simulation

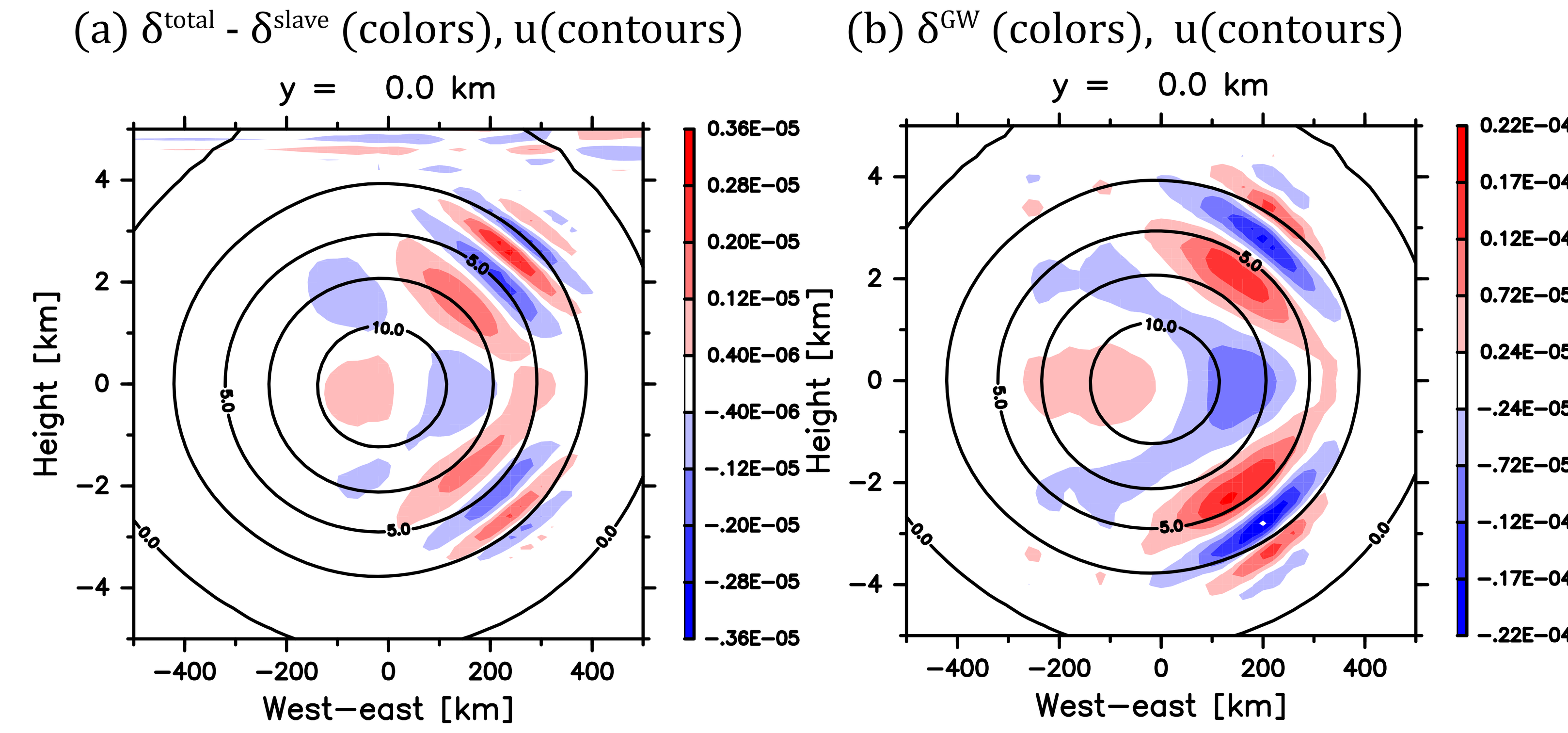
GW radiation is calculated by
- compressible nonhydrostatic equations.
- the linearized 2nd-order RGE.

compare

- Numerical Model : JMA-NHM (Saito and Coauthors 2006).
- Initial Condition : 3D modon solution (Berestov 1979).

Numerical model — (a) δ^{total} - δ^{slave} (colors), u(contours)

Linearized 2nd-order RGE — (b) δ^{GW} (colors), u(contours)



5. Physical Interpretation on Radiation Mechanism

(a) Mountain-wave-like mechanism $(\delta^{slave\ q\gamma(2)})$

$\delta^{slave\ q\gamma(2)}$ is generated by the vortical flow over deformed potential temperature surfaces due to Bernoulli effect.

$$\nabla^2 w^{\text{slave } q\gamma(2)} \approx -\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \tilde{\mathbf{u}}^q_{\rm H}\cdot\nabla_{\rm H}\, \tilde{b}^{\text{slave}(1)}$$

(b) Velocity-variation mechanism $(\gamma^{slave(1)}, \gamma^{slave\ q\gamma(2)})$

$\gamma^{slave(1)}$ and $\gamma^{slave\ q\gamma(2)}$ are generated by the horizontal divergence of the vortical flow acceleration.

$$\tilde{\gamma}^{\text{slave}(1)} \equiv \mathrm{Ro}\left[\frac{\partial(\tilde{\mathbf{u}}^q_{\rm H}\cdot\nabla_{\rm H}\tilde{u}^q)}{\partial x} + \frac{\partial(\tilde{\mathbf{u}}^q_{\rm H}\cdot\nabla_{\rm H}\tilde{v}^q)}{\partial y}\right] = \mathrm{Ro}\left(\frac{\partial}{\partial x}\frac{D^q}{Dt}\tilde{u}^q + \frac{\partial}{\partial y}\frac{D^q}{Dt}\tilde{v}^q\right)$$