Do I have to draw you a diagram? A tale of quantum gravity

KITP Public Lecture

April 20, 2016

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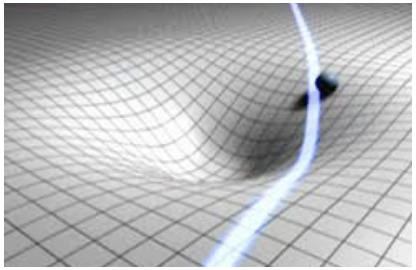
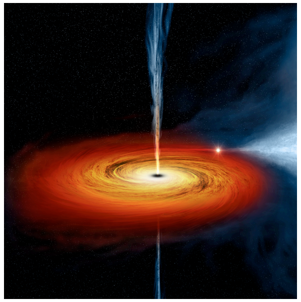
UCLA & KITP

A Tale of Two Theories

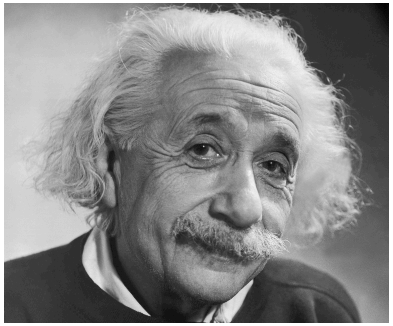
General Relativity is Einstein's theory of gravity



curved space-time

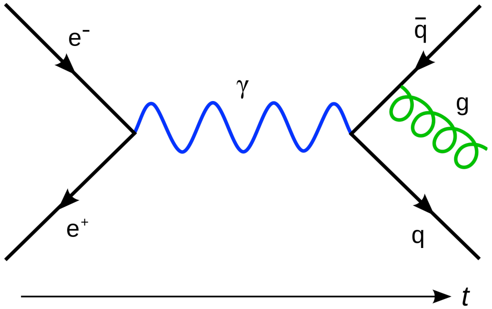


black holes



Albert Einstein

Quantum field theory:



Feynman diagram

Richard Feynman

Julian Schwinger

Sin-Itiro Tomonaga

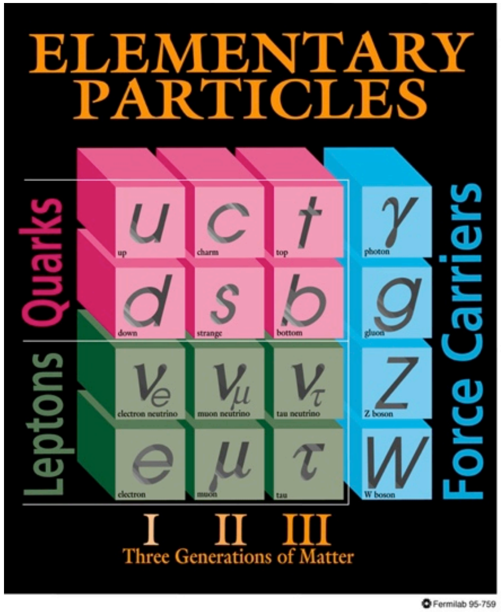
This lecture will be a tale about bringing these two theories together.

Yang-Mills Gauge Theory

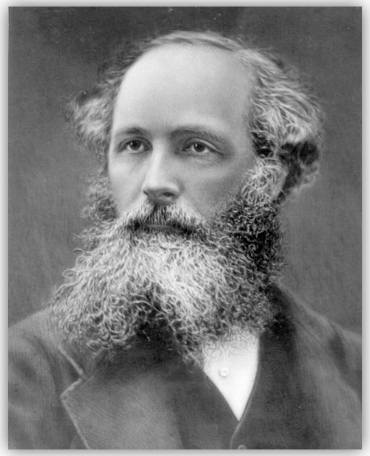
3 of 4 known forces in Nature are described by gauge theory:

1. Electromagnetism.
2. Strong nuclear force (QCD).
3. Weak nuclear force.

Yang-Mills gauge theory



- Yang-Mills theory generalizes Maxwell's electromagnetism.
- Instead of electric charge, "color" charge. (Just a cute name!)



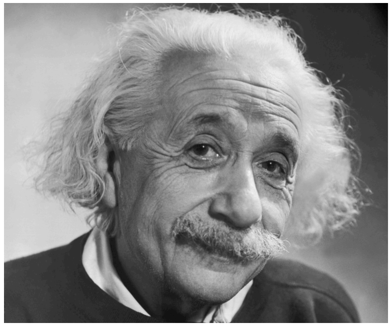
James Clerk Maxwell

C. N. Yang and Robert Mills

- Photons
- Gluons
- *W*, *Z* bosons

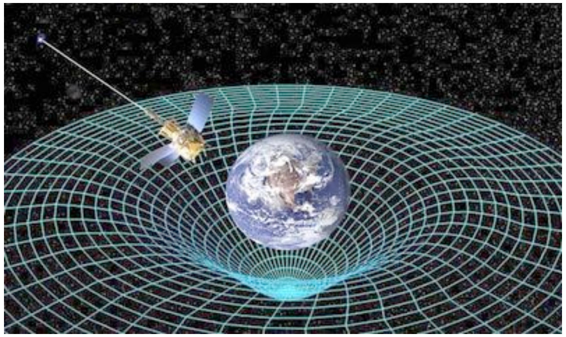
Spin 1 particles

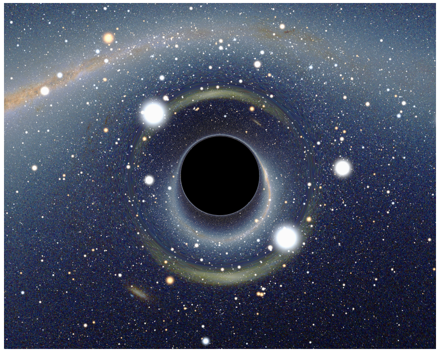
Gravity



Gravity seems different:

- Not based on a Yang-Mills theory.
- Based on curved space-time.
- Marvelous concepts such a black holes.





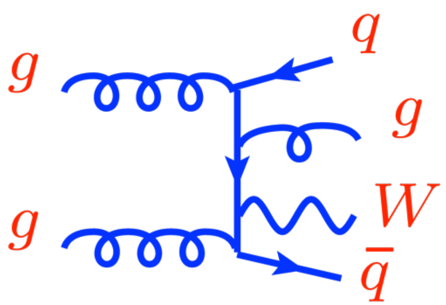
Quantum field theory:

- Gravitons analogous to photons.
- Untamed infinities. Not encountered with other forces.
- String theory is most popular solution of untamed infinities.

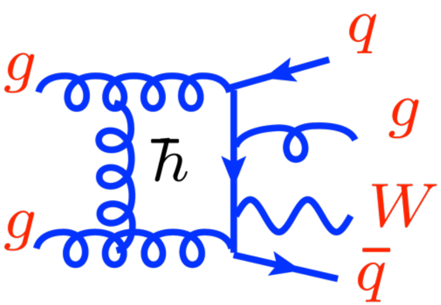
Feynman Diagrams of Quantum Field

Every graduate student in particle physics learns to use Feynman diagrams.

Diagrams represent precise formulas.



Tree Diagrams



Loop Diagrams– Quantum correction

Feynman diagrams give us a perturbative expansion in Planck's constant $\hbar$

Gravity similar but much much more complicated.

Precision of Quantum Field Theory

Quantum field theory is the most precise theory known.

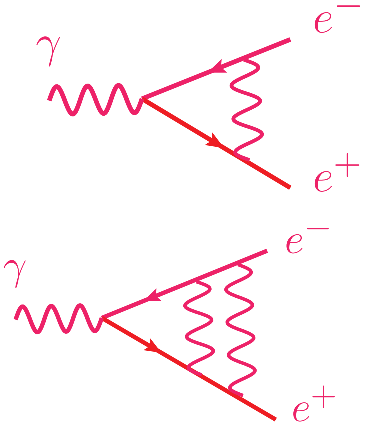
Anomalous magnetic moment of the electron.
Magnetic moment compared to spin.
Triumph of quantum field theory!

$$a_e(\mathrm{exp}) = 1.159\,652\,180\,73(28) \times 10^{-3} \quad [0.24 \text{ ppb}]$$

$$a_e(\mathrm{th}) = 1.159\,652\,181\,78(80) \times 10^{-3} \quad [0.66 \text{ ppb}]$$

Julian Schwinger

Toichiro Kinoshita



- **This uses Quantum Electrodynamics for electrons and photons.**
- **Here will be using Quantum Chromodynamics (QCD) or Yang-Mills theory which describes quarks and gluon.**

Quantum Field Theory of Gravity

Often repeated statement:

"Einstein's theory of General Relativity is incompatible with quantum field theory."

A bit of a silly statement, but to a large extent this is based on another often repeated statement:

"All quantum field theories of gravity have untamed infinities." Known as the non-renormalizability problem.

- **Where do these statements come from?**
- **More importantly: are they true?**

Infinities

Consider the sum:

$$1 + 2 + 3 + 4 + \ldots = \infty$$

Don't google this sum!
You'll find it is –1/12.

Consider the integral: $$\int_1^\infty \frac{dx}{x} = \infty$$

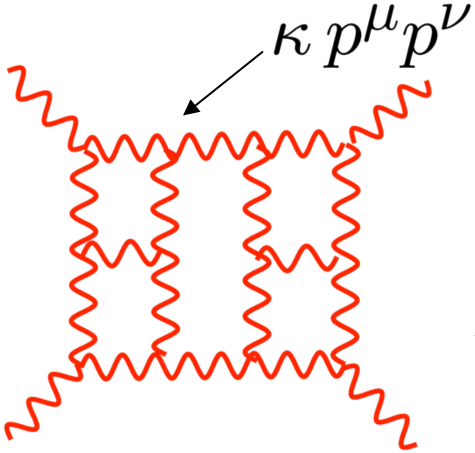
In quantum field theory we encounter infinities all the time:

$$\int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2(p-k)^2} = \infty$$

Some infinities are our friends: "Renormalizable"
Some infinites are our enemies: "Nonrenormalizable"

"Ultraviolet" or High Energy Behavior

$\kappa = \sqrt{32\pi G_N}$ ← **Dimensionful coupling**



Einstein gravity: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots (\kappa p_j^\mu p_j^\nu) \cdots}{\text{propagators}}$$

Yang-Mills theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots (g p_j^\nu) \cdots}{\text{propagators}}$$

Easy to argue that individual Feynman diagrams of gravity have badly behaved infinities.

Origin of statement that all quantum field theories of gravity must have bad infinities.

Hidden cancellations between diagrams if you add them up? Are we sure there must be bad infinities after adding up diagrams?

Supersymmetry

The most promising theories to look at are supersymmetric.

Supersymmetry relates forces (bosons) and matter (fermions).

Supergravity combines supersymmetry with gravity

Ferrara, Freedman, van Nieuwenhuizen

graviton $\longleftrightarrow$ gravitino $\qquad$ $N = 1$ supergravity

Can have more supersymmetry up to the maximum "$N = 8$" supersymmetry.

graviton $\longleftrightarrow$ 8 gravitinos $\qquad$ $N = 8$ supergravity

Supersymmetry helps tame infinities.
Bosons plus sign. Fermions minus sign.

In the late 70's and early 80's supergravity was seen as the most promising means for unifying gravity with other forces, until string theory came to prominence.

Test case: $N = 8$ Supergravity

We will consider maximal $N = 8$ supergravity.

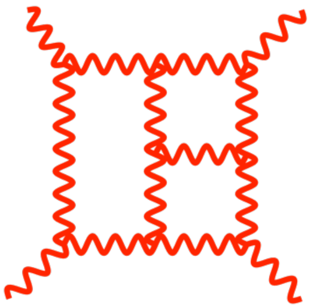
Einstein gravity + 254 other particles

Reasons to focus on $N = 8$ supergravity:

- **With more supersymmetry expect better high-energy properties.**
- **High symmetry implies technical simplicity.**

Opinions from the 80's

If certain patterns that emerge should persist in the higher orders of perturbation theory, then … $N = 8$ supergravity in four dimensions would have ultraviolet divergences starting at three loops. Green, Schwarz, Brink, (1982)



Aside: Ultraviolet means high energy

It is therefore very likely that all supergravity theories will diverge at three loops in four dimensions… The final word on these issues may have to await further explicit calculations.

Marcus, Sagnotti (1985)

The idea that *all* supergravity theories diverge at 3 loops had been widely accepted wisdom for over 25 years, with a only handful of contrarian voices.

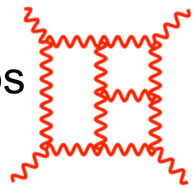
Status of Divergences

… it is not clear that general relativity, when combined with various other fields in supergravity theory, can not give a sensible quantum theory. Reports of the death of supergravity are an exaggeration. One year everyone believed that supergravity was finite. The next year the fashion changed and everyone said that supergravity was bound to have divergences even though none had actually been found.

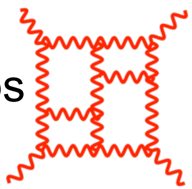
— Stephen Hawking, 1994

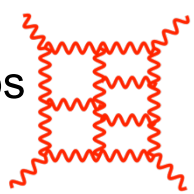
Feynman Diagrams for Gravity

SUPPOSE WE WANT TO CHECK IF CONSENSUS OPINION IS TRUE

3 loops  $\sim 10^{20}$ TERMS

No surprise it has never been calculated via Feynman diagrams.

4 loops  $\sim 10^{26}$ TERMS

5 loops  $\sim 10^{31}$ TERMS

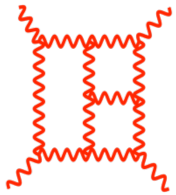
More terms than atoms in your brain!

- Calculations to settle this seemed utterly hopeless!
- Seemed destined for dustbin of undecidable questions.

With Modern Ideas

Z B, Carrasco, Dixon, Johansson, Roiban

For $N = 8$ supergravity.

3 loops  ~10 TERMS

4 loops  $\sim 10^2$ TERMS

Much more manageable!

5 loops  $\sim 10^4$ TERMS (Not yet done—1000 diagrams)

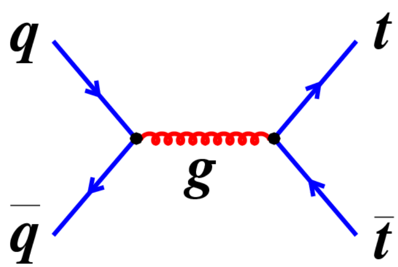
We now have the ability to settle the 35 year debate and determine the true high energy behavior gravity theories.

Scattering Amplitudes

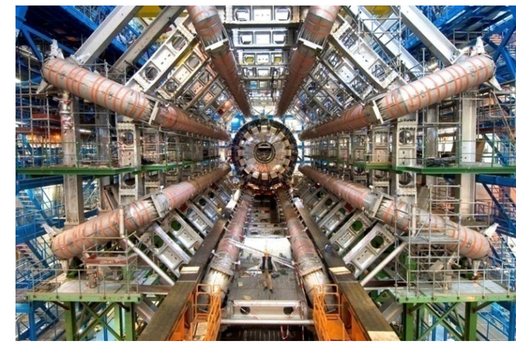
To understand how we attack the question of quantum gravity first need a detour through scattering amplitudes.

Scattering amplitudes

Scattering of elementary particles is fundamental to our ability to unravel microscopic laws of nature.

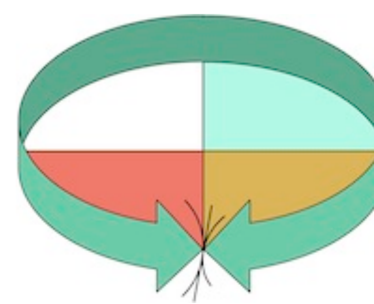


LHC: 27 km ring



ATLAS particle collider

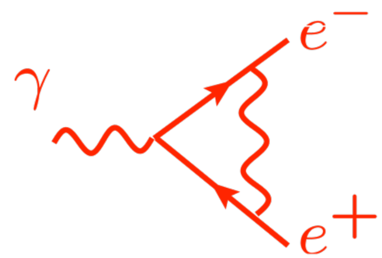
- **Feynman diagrams give scattering amplitudes.**
- **Needed for making predictions at the Large Hadron Collider at CERN.**



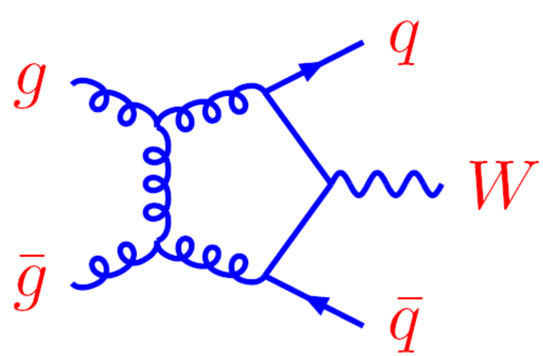
KITP programs on LHC physics

State-of-the-Art Feynman Diagram Calculations

In 1948 Schwinger computed anomalous magnetic moment of the electron.



60 years later at 1 loop only 2 or 3 legs more than Schwinger, if we use traditional Feynman diagrams.

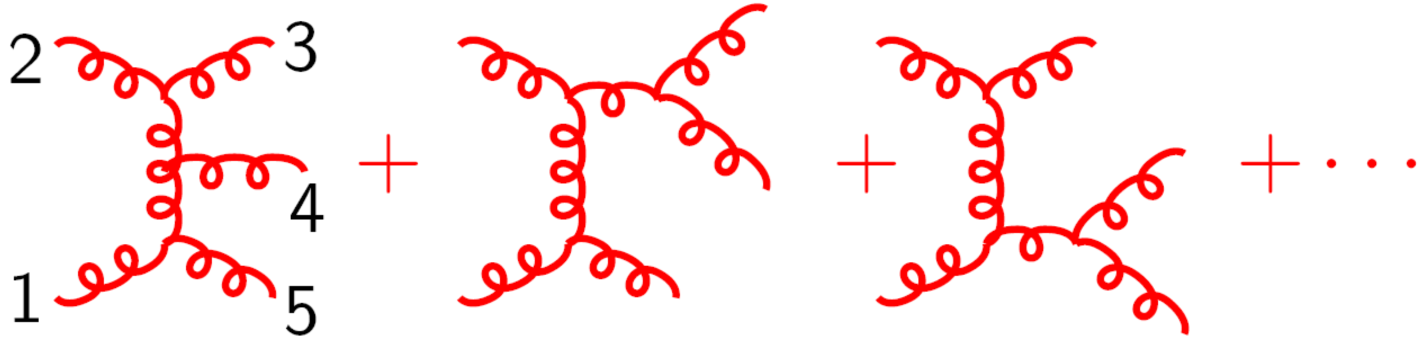


g **: gluon -- carrier of strong force**
q**: quark -- matter**
W, Z **: Weak boson carrier of weak interaction**

Tree-level example: Five gluons

Force carriers in QCD are gluons. Similar to photons of QED except they self interact.

Consider the five-gluon process:



Used in calculation of scattering processes at the LHC

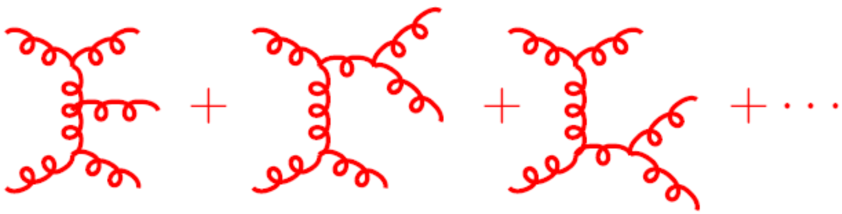
If you evaluate this following textbook Feynman rules you find…

Result of evaluation (actually only a small part of it):

$$k_1 \cdot k_4\, \varepsilon_2 \cdot k_1\, \varepsilon_1 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5$$

Messy combination of momenta and gluon polarization vectors.

Reconsider Five-Gluon Process



With a little Chinese magic:

Xu, Zhang and Chang
and many others

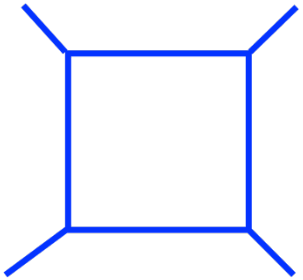
$$A_5^{\text{tree}}(1^\pm, 2^+, 3^+, 4^+, 5^+) = 0$$

$$A_5^{\text{tree}}(1^-, 2^-, 3^+, 4^+, 5^+) = i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

$$A_5^{\text{tree}}(1^-, 2^+, 3^-, 4^+, 5^+) = i\frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

Same physical content as previous slide.

Example of loop difficulty



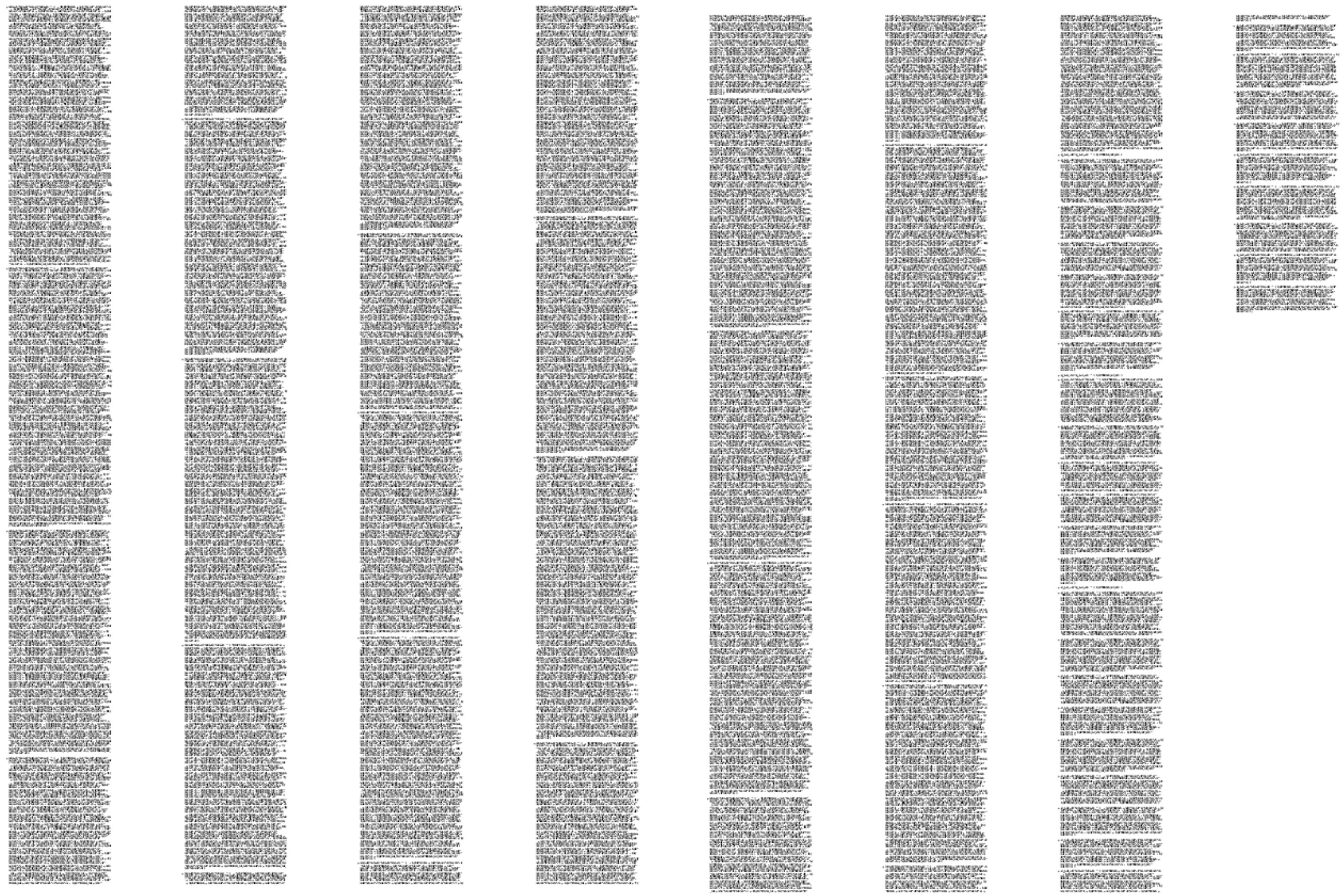
Consider the integral:

$$\int \frac{d^{4-2\epsilon}\ell}{(2\pi)^{4-\epsilon}} \frac{\ell^\mu \, \ell^\nu \, \ell^\rho \, \ell^\lambda}{\ell^2 \, (\ell - k_1)^2 \, (\ell - k_1 - k_2)^2 \, (\ell + k_4)^2}$$

Note: this is trivial on modern computer. Non-trivial for larger numbers of external particles.

Evaluate this integral via standard methods.
Result is …

Result of performing the integration



Calculations explode for larger numbers of particles or loops. Clearly, there should be a better way!

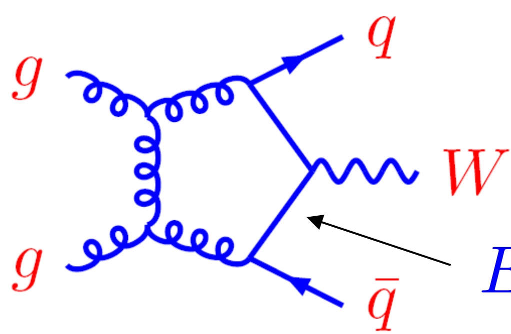
Why are Feynman diagrams difficult for high-loop or high-multiplicity processes?

Physical particles satisfy Einstein's mass energy relation.

$$E = mc^2,\ \vec{p} = 0 \qquad\qquad E^2 = \vec{p}^{\,2}c^2 + m^2c^4$$

"On-shell"

$$\int \frac{d^3\vec{p}\,dE}{(2\pi)^4}$$



Individual Feynman diagrams unphysical.
"Off-shell"

$$E^2 \neq \vec{p}^{\,2}c^2 + m^2c^4$$

Vertices and propagators involve unphysical "off-shell" states.

An important origin of the complexity.

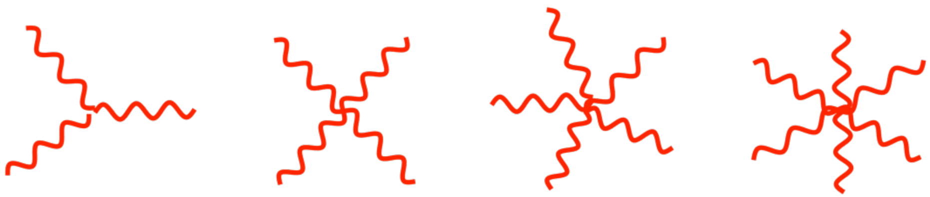
Don't violate Einstein's relation. Stay on-shell:

Modern Unitarity Method. ZB, Dixon, Dunbar, Kosower (1998)

On-shell recursion. Britto, Cachazo, Feng and Witten (2005)

Gravity vs Gauge Theory

Consider Einstein gravity interaction of gravitons

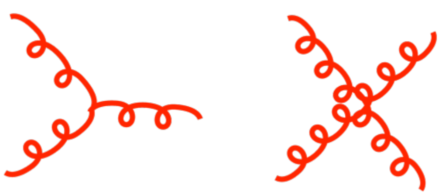


$+ \cdots$

Infinite number of complicated interactions

terrible mess

Compare to Yang-Mills gluon interactions

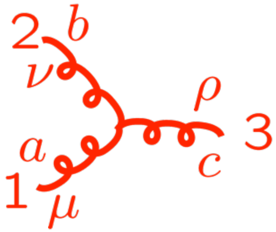


Only three and four point interactions

Gravity seems so much more complicated than gauge theory.

Three Vertices

Standard Feynman diagram approach.



Three-gluon vertex:

$$V_{3\,\mu\nu\sigma}^{abc} = -gf^{abc}(\eta_{\mu\nu}(k_1-k_2)_\rho+\eta_{\nu\rho}(k_1-k_2)_\mu+\eta_{\rho\mu}(k_1-k_2)_\nu)$$

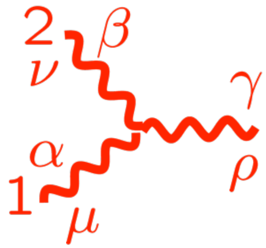
Three-graviton vertex:

off shell

$$E^2 \neq \vec{p}^{\,2}c^2 + m^2c^4$$

$$G_{3\mu\alpha,\nu\beta,\sigma\gamma}(k_1,k_2,k_3) =$$

$$\begin{aligned}&\mathrm{sym}[-\tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \tfrac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + \tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma})\\ &\quad + P_6(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma})\\ &\quad + P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + 2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma})\\ &\quad + 2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1\cdot k_2\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})]\end{aligned}$$



About 100 terms in three vertex

Naïve conclusion: Gravity is a nasty mess.

Simplicity of Gravity Amplitudes

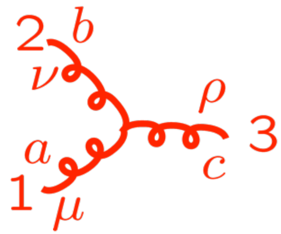
People were looking at gravity the wrong way. On-shell viewpoint much more powerful.

use Einstein's relation:

$$E^2 = \vec{p}^{\,2} c^2 + m^2 c^4$$

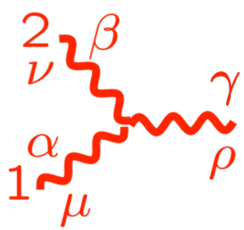
***On-shell* three vertices contains all information:**

Yang-Mills gauge theory:



$$-g f^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$

Einstein gravity:



$$i\kappa(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$
$$\times (\eta_{\alpha\beta}(k_1 - k_2)_\gamma + \text{cyclic})$$

"square" of Yang-Mills vertex.

Using modern methods, any gravity scattering amplitude constructible solely from *on-shell* 3 vertex. Higher-point vertices irrelevant!

Gravity just as simple as gauge theory!

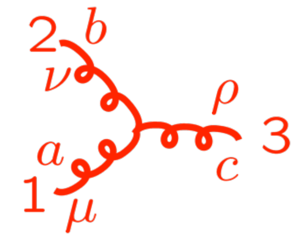
Duality Between Color and Kinematics

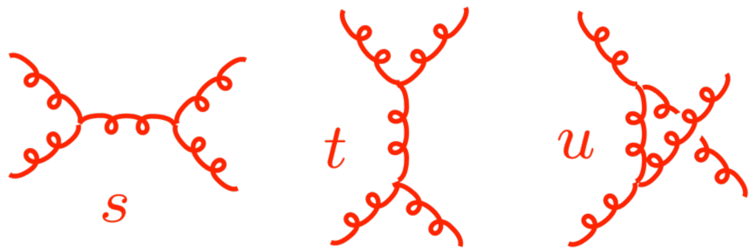
coupling constant

color charge factor

momentum dependent kinematic numerator factor

$$-gf^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$





$$c_u = c_s - c_t$$ **Color Jacobi Identity. Well known.**

$$n_u = n_s - n_t$$ **Kinematic numerator Jacobi Identity. Who ordered this?**

A really strange idea that even experts don't fully understand.

Gravity and Gauge Theory

ZB, Carrasco, Johansson

color charge factor

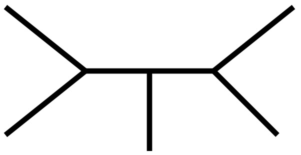
kinematic numerator factor

Yang-Mills gauge theory: $$\mathcal{A}_n^{\text{tree}} = i g^{n-2} \sum_i \frac{c_i\, n_i}{D_i}$$

Feynman denominators

If duality holds, to get gravity, replace: $c_i \rightarrow n_i$

Einstein Gravity: $$\mathcal{M}_n^{\text{tree}} = i \kappa^{n-2} \sum_i \frac{n_i^2}{D_i}$$



sum over diagrams with only 3 vertices

This is known as the "double copy construction".

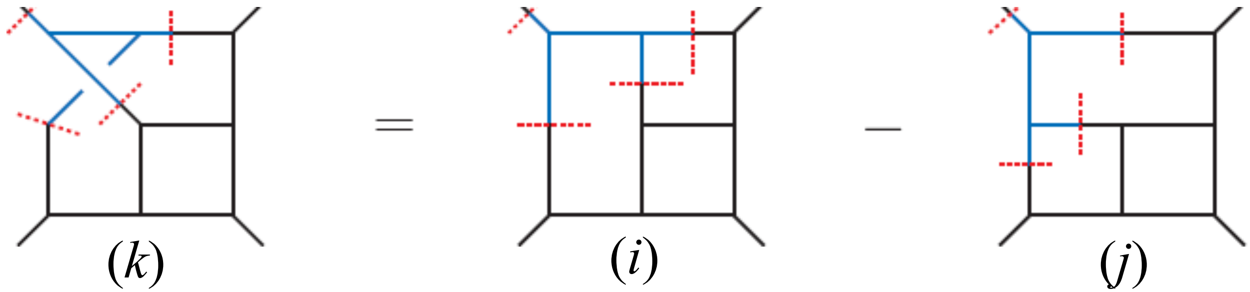
Cries out for a unified description of the sort given by string theory.

Chiodaroli, Gunaydin, Johansson and Roiban,; Johannsson, Ochirov; O'Connell, Montiero, White; ZB, Davies, Nohle; Boels, Isermann, Monteiro, and O'Connel; Mogull and O'Connell, He, Monteiro, and Schlotterer, etc.

BCJ

Gravity loop integrands are free!

Ideas conjectured to generalize to loops:



color factor

$$c_k = c_i - c_j$$
$$n_k = n_i - n_j$$

kinematic numerator

If you have a set of duality satisfying numerators.

To get:

gauge theory ⟶ gravity theory

simply take

color factor ⟶ kinematic numerator

$$c_k \longrightarrow n_k$$

Double copy construction

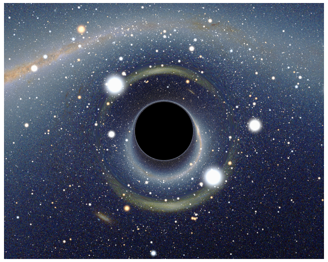
Gravity loop integrands follow from gauge theory!

Applications to Black-Hole Physics

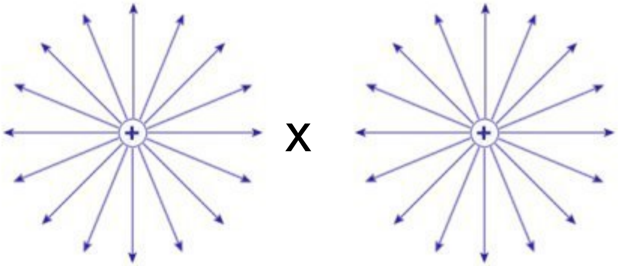
Wouldn't it be really cool if every classical solution in gravity is equivalent to a double gauge theory solution?

Where to start? **Obviously the coolest place possible: black holes.**

Monteiro, O'Connell and White



black hole



point electric charges

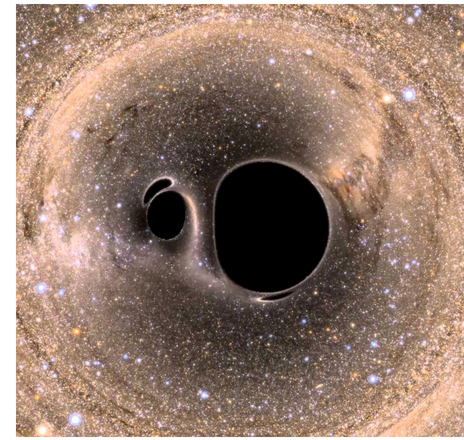
Schwarzschild ~ (Coulomb)^2

Applications to Black Hole Physics

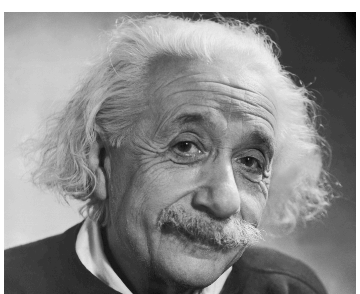
Luna, Monteiro, O'Connell and White;
Luna, Monteiro, Nicholsen, O'Connell and White;
Ridgway and Wise.

A variety of other cases:

- **Kerr (rotating) black hole.**
- **Solutions with cosmological constant.**
- **Radiation from accelerating black hole.**



It may be possible to extend this to more general cases. Maybe even gravity waves from colliding black holes!



Einstein

=

Yang and Mills

x

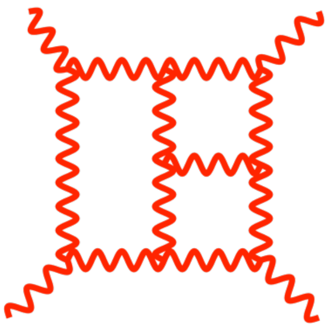
Yang and Mills

Applications to Quantum Gravity

N = 8 Supergravity Three-Loop Result

ZB, Carrasco, Dixon, Johansson, Kosower, Roiban (2007);
ZB, Carrasco, Dixon, Johansson, Roiban (2008)

Using modern methods we decided to check whether *N* = 8 supergravity is divergent at 3 loops.



This is finite contrary to consensus opinion of 70's and 80's

More Recent Opinion

In 2009 Bossard, Howe and Stelle had a careful look at the question of how much supersymmetry can tame UV divergences.

In particular … suggest that maximal supergravity is likely to diverge at four loops in $D = 5$ and at five loops in $D = 4$ …

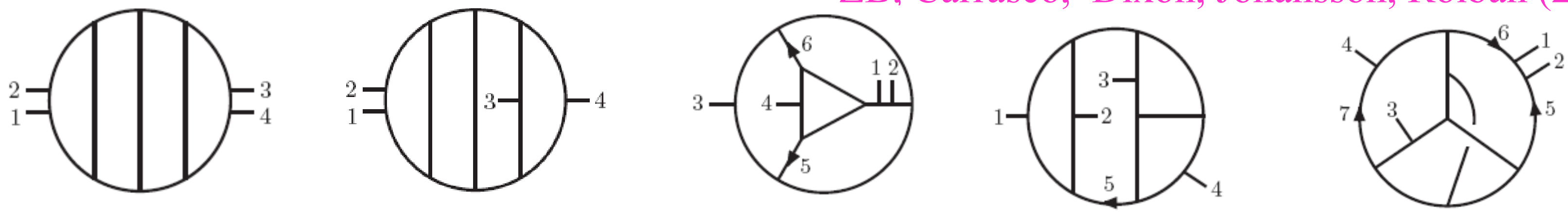
Bossard, Howe, Stelle (2009)

Bottles of wine were at stake!

We had tools to collect the wine.

$N = 8$ Supergravity Four-Loop Calculation

ZB, Carrasco, Dixon, Johansson, Roiban (2010)



50 distinct planar and non-planar diagrammatic topologies

"I'm not shaving until we finish the calculation"
— John Joseph Carrasco

John Joseph shaved!

UV finite for $D = 4$ and 5

Very very finite.

A New Consensus from Experts

More recent papers argue that trouble starts at 5 loops and by 7 loops we have valid potential UV divergence in $D = 4$.

Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Bjornsson ; Bossard , Hillmann and Nicolai; Kallosh; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger

***D* is space-time dimension**

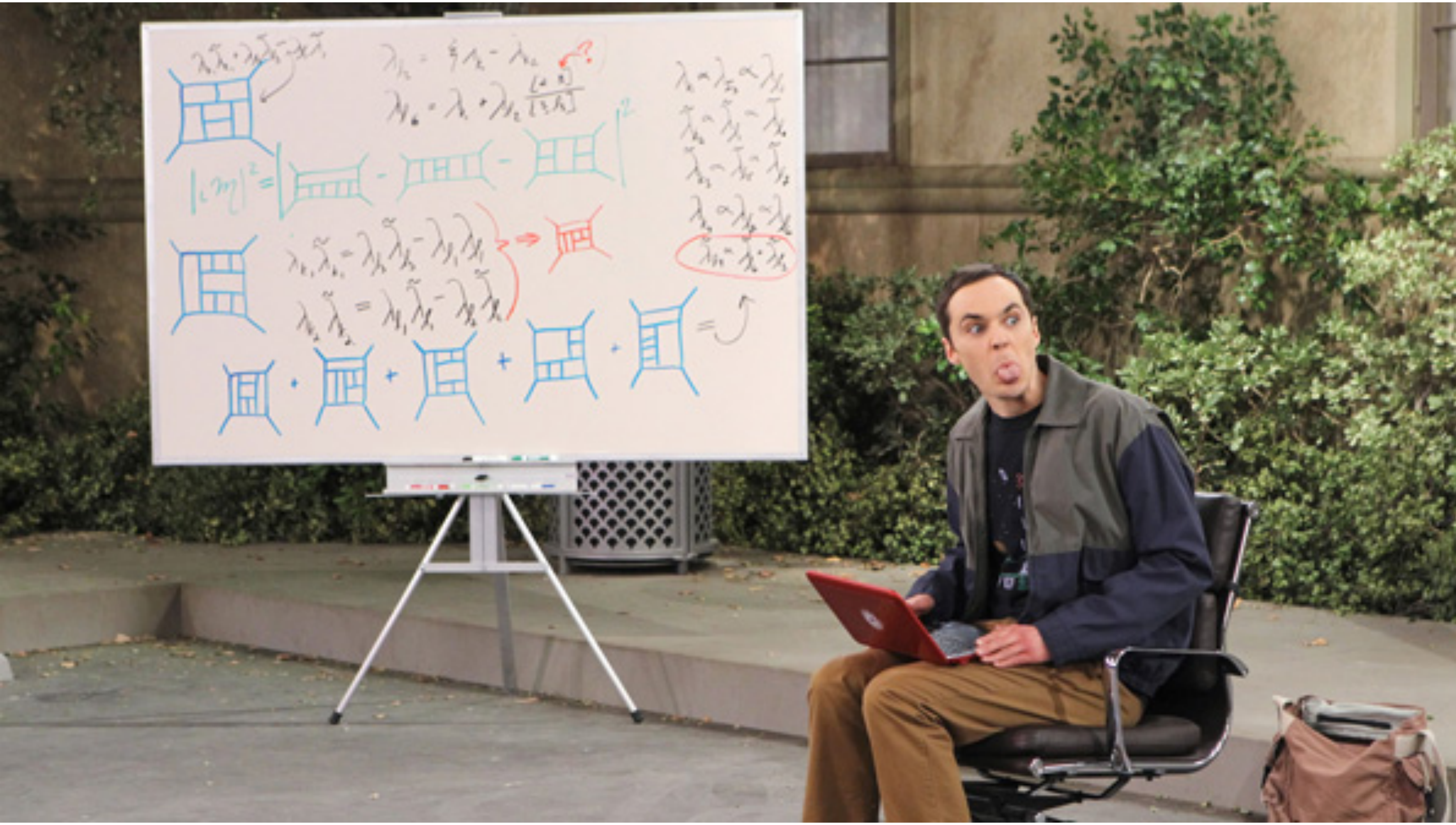
All previous calculations explained and divergences predicted.

Based on this, a reasonable person would conclude that $N = 8$ supergravity almost certainly diverges at 7 loops in $D = 4$.

Same consensus methods also predict other divergences:

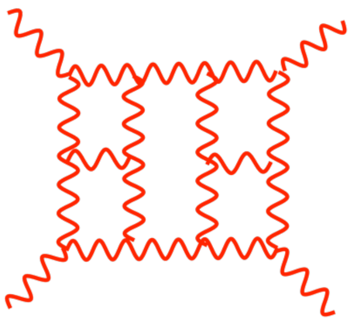
- **$N = 8$ sugra should diverge at 7 loops in $D = 4$**
- **$N = 8$ sugra should diverge at 5 loops in $D = 24/5$**
- **$N = 4$ sugra should diverge at 3 loops in $D = 4$**
- **$N = 5$ sugra should diverge at 4 loops in $D = 4$**

Scene from “The Big Bang Theory”



$N = 8$ Sugra 5 Loop Calculation

ZB, Carrasco, Johannson, Roiban



~1000 such diagrams with ~10,000s terms each

Being reasonable and being right are not the same.

Place your bets:

- **At 5 loops in $D = 24/5$ does $N = 8$ supergravity diverge?**
- **At 7 loops in $D = 4$ does $N = 8$ supergravity diverge?**

***D* is space-time dimension**

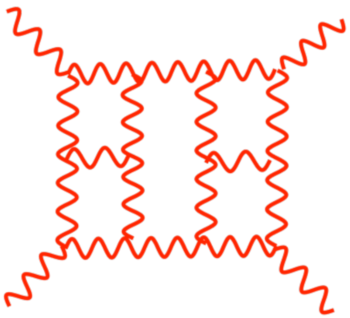
5 loops

Kelly Stelle:
English wine
"It will diverge"

Zvi Bern:
California wine
"It won't diverge"

$N = 8$ Sugra 5 Loop Calculation

ZB, Carrasco, Johannson, Roiban



~1000 such diagrams with ~10,000s terms each

Being reasonable and being right are not the same

Place your bets:

- **At 5 loops in $D = 24/5$ does $N = 8$ supergravity diverge?**
- **At 7 loops in $D = 4$ does $N = 8$ supergravity diverge?**

***D* is space-time dimension**

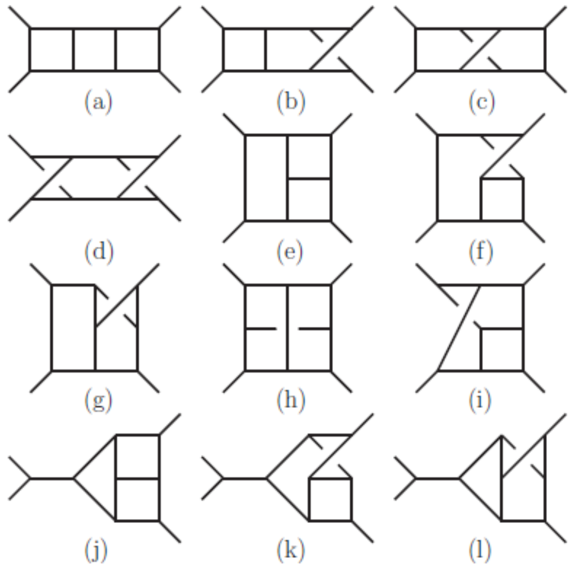
7 loops

David Gross:
California wine
"It will diverge"

Zvi Bern:
California wine
"It won't diverge"

$N = 4$ Supergravity UV Cancellation

ZB, Davies, Dennen, Huang



All **three-loop infinities cancel completely!**

Still no standard explanation, despite valiant attempt.

Bossard, Howe, Stelle; ZB, Davies, Dennen

This phenomenon is named "enhanced cancellations".

A pity we did not bet on this theory!

$\mathcal{N} = 5$ Supergravity at Four Loops

ZB, Davies and Dennen

We also calculated four-loop divergence in $\mathcal{N} = 5$ supergravity.

Industrial strength software needed: FIRE5 and C++

$\mathcal{N} = 5$ supergravity:

Crucial help from FIRE5 and $(\text{Smirnov})^2$

$\mathcal{N} = 5$ supergravity has no divergence at four loops.

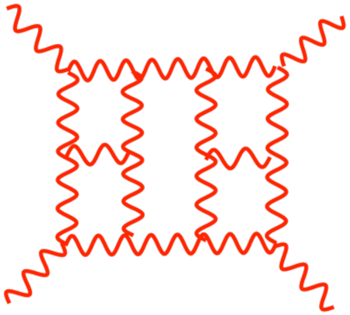
Another example of an "enhanced cancellation".

A pity we did not bet on this theory as well!

$\mathcal{N} = 4$ supergravity does have infinities, but they are weird and not generic. Quantum anomaly. Similar story for pure gravity.

ZB, Davies and Dennen, ZB, Cheung, Chi, Davies, Dixon and Nohle; ZB, Chi, Dixon, Edison (to appear)

Conclusions




- Good ideas can overcome "impossible" problems!

- "Reports of the death of supergravity are an exaggeration"

  Stephan Hawking (with help from Mark Twain)

- I don't know if this will lead to a completely satisfactory description of nature via supergravity. At least people are looking again at this possibility.

- I hope I convinced you we uncovered some interesting things along the way.

Further Reading

If you wish to read more see following non-technical descriptions.

Hermann Nicolai, *PRL Physics Viewpoint*, "Vanquishing Infinity"
http://physics.aps.org/articles/v2/70

Z. Bern, L. Dixon, D. Kosower,
May 2012 *Scientific American*,
"Loops, Trees and the Search for New Physics"

Anthony Zee, *Quantum Field Theory in a Nutshell*,
2nd Edition is first textbook to contain modern formulation of scattering and commentary on new developments. 4 new chapters.