Talk Outline

Introduction
- Review of accreting neutron stars, X-ray bursts, and burst oscillations
- Comparisons between magnetic and non-magnetic neutron stars

Theory of Surface Modes on Neutron Stars
- Review of shallow gravity waves
- Bursting neutron star surfaces and the resulting modes
- Predictions, comparisons with observations, and constraints on the properties of neutron star crusts

Conclusion
- Lingering mysteries, future work, and other comparisons with observations
Low Mass X-ray Binaries (LMXBs)

A “typical” neutron star is 1.4 solar masses and 10 km. Orbital periods of these binaries are a few hours to half a day.

The neutron star accretes from its donor star at

$$\dot{M} \approx 10^{-10} - 10^{-8} M_\odot \text{ yr}^{-1}$$

$$L \approx \frac{GM\dot{M}}{R} \approx 10^{36} - 10^{38} \text{ erg s}^{-1}$$

$$\frac{GMM_p}{R} \approx 200 \text{ MeV nucleon}^{-1}$$

Nuclear burning on its surface releases

$$\approx 5 \text{ MeV nucleon}^{-1}$$

This can never be seen…right?

Yes…when the nuclear energy is released all at once!
Type I X-ray Bursts

Nuclear fuel is stored up and then burns rapidly!

- Unstable Helium ignition (triple-alpha) as predicted by Hansen & van Horn ‘75
- Observed by Belian et al ‘76; Grindlay et al ‘76 and identified by Woosley & Taam ‘76; Maraschi & Cavaliere ‘77; Joss ‘77, ‘78; Lamb and Lamb ‘78
- Bursts repeat every few hours to days (timescale to accrete an unstable column)
- Energy release:
  \[ E_{\text{burst}} \approx 5 \times 10^{39} \text{ ergs} \]

X-ray Bursts and Surface Composition

- Burst length and \( \alpha \)-value indicate composition of bursting fuel

\[
\alpha \equiv \frac{\langle L_{\text{acc}} \rangle}{\langle L_{\text{burst}} \rangle} \approx \frac{200 \text{ MeV}}{5 \text{ MeV}} \approx 40
\]

Mixed H/He burns to A~100 heavy ashes

Pure He burns to \( \alpha \)-elements like \( ^{28}\text{Si}, ^{40}\text{Ca}, ^{64}\text{Zn}, \text{etc.} \)

\[
\alpha \approx \frac{200 \text{ MeV}}{1.6 \text{ MeV}} \approx 100
\]

\~100 sec cooling characteristic of rp-process burning of mixed H/He

GS 1826-24; Galloway et al. ‘04
Burst Oscillations from LMXBs

4U 1702-429; Strohmayer & Markwardt '99

- Frequency and amplitude during rise are consistent with a hot spot spreading on a rotating star (Strohmayer et al. '97)
- Angular momentum conservation of surface layers (Strohmayer et al. '97) underpredicts late time drift (Cumming et al. '02)

Oscillation during rise

~1-5 Hz Drift

~10 sec cooling tail characteristic of Helium bursts

The asymptotic frequency is characteristic to each object

- Frequency stable over many observations (within 1 part in 1000; Muno et al. '02)

It must be the spin…right?

<table>
<thead>
<tr>
<th>Source</th>
<th>Asymptotic Freq. (Hz)</th>
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<tr>
<td>4U 1608-522</td>
<td>620</td>
</tr>
<tr>
<td>SAX J1750-2900</td>
<td>600</td>
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<td>MXB 1743-29</td>
<td>589</td>
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<td>SAX J1808.4-3658</td>
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<td>4U 1926-053</td>
<td>270</td>
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<td>EXO 0748-676</td>
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</table>
Neutron Stars Speed Limit?

- Enough time and accretion to spin the neutron star up to the breakup $\sim 1200$ Hz

- No observational bias against fast or slow rotators so small range is meaningful

- Chakrabarty et al. ‘03 found highest frequency $< 730$ Hz at 95% confidence!

Probably interesting physics behind explanation. Magnetic spin equilibrium (Ghosh & Lamb ‘79)? Accretion torques balanced by gravitational waves (Bildsten ‘98, Wagoner ‘84, Andersson ‘98)?

Accreting Millisecond Pulsars

Accreting magnetic neutron stars with spin rates of $\sim 180$-600 Hz (first discovered was SAX J1808, Wijnands & van der Klis 1998)

$B > 1.6 \times 10^8 G \left( \frac{M}{1.4M_\odot} \right)^{1/4} \left( \frac{10 \text{ km}}{R} \right)^{3/2} \times \left( \frac{\dot{M}}{10^{-10} M_\odot \text{yr}^{-1}} \right)$
Burst Oscillations from Pulsars

- Burst oscillation frequency = spin!
- No frequency drift, likely due to large B-field (Cumming et al. 2001)
- ~ 100 sec decay like H/He burst!

What Creates Burst Oscillations in Non-pulsar Neutron Stars?

Important differences:

- Non-pulsars only show oscillations in short (~ 2-10 s) bursts, while pulsars have shown oscillations in longer bursts (~ 100 s)
- Non-pulsars show frequency drifts often late into cooling tail, while pulsars show no frequency evolution after burst peak
- Non-pulsars have highly sinusoidal oscillations (Muno et al. ‘02), while pulsars show harmonic content (Strohmayer et al. ‘03)
- The pulsed amplitude as a function of energy different between the two types of objects (Muno et al. ‘03; Watts & Strohmayer ‘04)

These differences support the hypothesis that a different mechanism may be acting in the case of the non-pulsars.
Perhaps Nonradial Oscillations?
Initially calculated by McDemott & Taam (1987), BEFORE burst oscillations were discovered. Hypothesized by Heyl (2004).

- It’s the most obvious way to create a late time surface asymmetry in a liquid.
- It is supported by the HIGHLY sinusoidal nature of oscillations
- The angular and radial eigenfunctions are severely restricted by the main characteristics of burst oscillations.
- Heyl (2004) identified that the angular structure must be an $m = 1$ buoyant $r$-mode (we’ll come back to this later)

Let’s first consider radial part…

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Surface Gravity Waves
Consider a liquid layer with depth $H$ above a rigid floor

$$g = \frac{GM}{R^2}$$

$$\text{wavelength} = \frac{2\pi}{k}$$

The frequency of the oscillation is given by the dispersion relation:

$$\omega^2 = gk \tanh kH$$
Deep and Shallow Limits

The general dispersion relation is $\omega^2 = gk \tanh kH$ but it has important deep and shallow limits.

Deep layer, $kH \gg 1$, $\tanh kH \rightarrow 1$

\[
\begin{align*}
\omega^2 &= gk \\
v_p &= \frac{\omega}{k} = \sqrt{\frac{g}{k}} \\
v_g &= \frac{d\omega}{dk} = \frac{1}{2} \sqrt{\frac{g}{k}}
\end{align*}
\]

Dispersive!

Shallow layer, $kH \ll 1$, $\tanh kH \rightarrow kH$

\[\omega^2 = gHk^2 \quad v_p = v_g = \sqrt{gH}\]

Important formula!

No dispersion

Just like a Tsunami!

Cooling Neutron Star Surface

- We construct a simple cooling model of the surface layers
- The composition is set from the He-rich bursts of Woosley et al. ’04
- Profile is evolved forward in time using finite differencing (Cumming & Macbeth ’04)

Time steps of 0.1, 0.3, 1, 3, & 10 seconds
Modes On Neutron Star Surface

<table>
<thead>
<tr>
<th>Depth</th>
<th>Density</th>
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<tbody>
<tr>
<td>&lt; 1 m</td>
<td>10^4 g cm⁻³</td>
</tr>
<tr>
<td>H_b ≈ 2 m</td>
<td>10^6 g cm⁻³</td>
</tr>
<tr>
<td>H_c ≈ 20 m</td>
<td>10^9 g cm⁻³</td>
</tr>
</tbody>
</table>

- Shallow surface wave
  \[ \omega_s^2 = g H_b k^2 \frac{\Delta \rho}{\rho} \]
  \[ k^2 = \frac{\lambda}{R^2} \]

- Crustal interface wave
  \[ \omega_c^2 = g H_c k^2 \frac{\mu}{P} \]
  \[ \frac{\mu}{P} \approx 10^{-2} \]

(Piro & Bildsten '05)

Shallow Surface Wave

\[ \omega_s^2 = g H_b k^2 \frac{\Delta \rho}{\rho} \]

\[ H_b = \frac{k_B T_b}{\mu_b m_p g} \]
\[ k^2 = \frac{\lambda}{R^2} \]
\[ \frac{\Delta \rho}{\rho} = 1 - \frac{T_c}{T_b} \frac{\mu_b}{\mu_c} \]

If the neutron star is not rotating then \( \lambda = l(l + 1) \). We instead use \( \lambda = 1/9 \approx 0.11 \) which we later explain from the effects of rotation.

\[ \omega_s \propto T_b^{1/2} \]

\[ \frac{\omega_s}{2\pi} = 10.8 \text{ Hz} \left( \frac{2Z_b}{A_b} \frac{T_b}{10^9 \text{ K}} \frac{\lambda}{0.11} \right)^{1/2} \left( \frac{10 \text{ km}}{R} \right) \left( 1 - \frac{T_c}{T_b} \frac{\mu_b}{\mu_c} \right)^{1/2} \]

(This mode was studied by McDermott & Taam 1987.)
Crustal Interface Wave
McDermott, Van Horn & Hansen '88; Piro & Bildsten '05

Easy case!…
a solid bottom boundary
results in a frequency:

$$\omega^2 = gH_c k^2$$

Crustal Interface Wave
McDermott, Van Horn & Hansen '88; Piro & Bildsten '05

A non-zero bottom displacement
decreases frequency dramatically!

$$\omega^2 \approx gH_c k^2 \left| \frac{\xi_{z,t}}{\xi_{z,c}} \right| \approx gH_c k^2 \frac{\mu}{P}$$

$$\left| \frac{\xi_{z,t}}{\xi_{z,c}} \right| \approx \frac{\mu}{P} \sim 10^{-2}$$

$$\frac{\omega_c}{2\pi} = 4.3 \text{ Hz} \left( \frac{64 \ T_{c,8}}{A_c} \frac{\lambda}{3 \ 0.11} \right)^{1/2} \times \left( \frac{10 \text{ km}}{R} \right)$$
The First 3 Radial Modes
(using $\lambda = 0.11$)

- Mode energy is set to $5 \times 10^{36}$ ergs

10$^{-3}$ of the energy in a burst (Bildsten '98)

- Estimate radiative damping time using “work integral” (Unno et al. '89)

- Surface wave (single node) has best chance of being seen (long damping time + large surface amplitude)

Avoided Mode Crossings

The two modes meet at an avoided crossing
Avoided Mode Crossings

What Angular Eigenfunction?

Heyl ('04) identified crucial properties:

- Highly sinusoidal nature (Muno et al. '02) implies $m = 1$ or $m = -1$

- The OBSERVED frequency is

$$\omega_{\text{obs}} = |m\Omega - \omega|$$

If the mode travels RETROGRADE ($m = -1$) a DECREASING frequency is observed

$$\omega_{\text{obs}} = \Omega + \omega$$

If the mode travels PROGRADE ($m = 1$) an INCREASING frequency is observed

$$\omega_{\text{obs}} = \Omega - \omega$$
Rotational Modifications

Since layer is thin and buoyancy is very strong, Coriolis effects ONLY alter ANGULAR mode patterns and latitudinal wavelength (through $\lambda$) and NOT radial eigenfunctions! (Bildsten et al. ’96)

**Inertial R-modes**

$$\omega = \frac{2m\Omega}{l(l+1)}$$

Only at slow spin.
Not applicable.

**Rotational Modifications**

Since layer is thin and buoyancy is very strong, Coriolis effects ONLY alter ANGULAR mode patterns and latitudinal wavelength (through $\lambda$) and NOT radial eigenfunctions! (Bildsten et al. ’96)

**Inertial R-modes** $l = m$, **Buoyant R-modes**

$$\omega = \frac{2m\Omega}{l(l+1)}$$

Only at slow spin.
Not applicable.

$$\lambda \sim \left(\frac{2\Omega}{\omega}\right)^2 \sim 10 - 10^3$$

Too large of drifts and hard to see.
Rotational Modifications

Since layer is thin and buoyancy is very strong, Coriolis effects ONLY alter ANGULAR mode patterns and latitudinal wavelength (through $\lambda$) and NOT radial eigenfunctions! (Bildsten et al. '96)

$l = 2, m = 1$

Inertial R-modes $l = m$, Buoyant R-modes Buoyant R-mode

$\omega = \frac{2m\Omega}{l(l+1)}$

Only at slow spin. Not applicable.

$\lambda \sim \left(\frac{2\Omega}{\omega}\right)^2 \sim 10 - 10^3$

Too large of drifts and hard to see.

$\lambda = 0.11$

Just right. Gives drifts as observed and nice wide eigenfunction

Observed Frequencies

400 Hz neutron star spin

$\omega_{\text{obs}} = |m\Omega - \omega|$

- Lowest order mode that matches burst oscillations is the $l = 2, m = 1$, r-mode

$\lambda \approx 1/9 \approx 0.11$

- Neutron star still spinning close to burst oscillation frequency (~ 4 Hz above)

All sounds nice...but can we make any predictions?
Comparison with Drift Observations

- The observed drift is just the difference of
  \[ \frac{\omega_s}{2\pi} \approx 9.5 \text{ Hz} \]
  \[ \frac{\omega_c}{2\pi} \approx 4.3 \text{ Hz} \left( \frac{64}{A_c} \frac{T_{c,8}}{3} \right)^{1/2} \]
  
  Hot crust = small drift!

- Use \( T_c \) from crust models courtesy of E. Brown.

- We compared these with the observed drifts and persistent luminosity ranges.

- Comparison favors a Fe-like crust, consistent with He-rich bursts.

Could other modes be present during X-ray bursts?

- Nothing precludes the other low-angular order modes from also being present.

- Such modes would show 15-100 Hz frequency drifts, so they may be hidden in current observations.
Conclusions and Discussions

• We propose a surface wave transitioning into a crustal interface wave as the burst oscillations. Only ONE combination of radial and angular eigenfunctions gives the correct properties!

• This is the first explanation for burst oscillations that fits both the frequencies and the drifts, and provides testable predictions.

• Why short (~ 2-10 sec) bursts only?

• Why the $m = 1$ buoyant r-mode? Need to understand excitation mechanism!

What can we now learn?

• Provides constraints on ocean/crust compositions (using drifts)

• Shows that neutron stars are indeed spinning at ~ 270-620 Hz (4-5 Hz ABOVE burst oscillation frequency)