Tensor Networks $\rightarrow$ Holography
via Integral Geometry

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Why should tensor networks relate to holography?
What is holographic duality?

- an auxiliary representation of a conformal field theory as a gravity theory
- which makes scale transformations (RG) manifest
- by introducing an additional dimension

Sounds just like tensor networks!
Two descriptions of a state

- an additional dimension
- which encodes RG transformations
- entropies represented by minimal cuts ↔ geodesics
  (Ryu-Takayanagi, 2006)
Ingredients:
- the causal structure of MERA
- the minimal cut prescription
- entanglement entropy

Strategy:
- construct a metric that captures this structure of MERA
- relate that metric to the holographic geometry
Choosing a metric: causal structure

- identify null coordinates
- point \((u,v)\) sits on top of a unique **minimal cut**, which identifies a field theory interval \((u,v)\)
- the isometry at \((u,v)\) completes the compression of the state on interval \((u,v)\)

**Kinematic Metric:**

\[ ds^2 = (...) dudv \]
Why is the metric Lorentzian?

- **timelike** separated \((u,v)\): \((u,v)\) contains \((u,v)\)
- **spacelike** separated \((u,v)\): neither contains the other
- **null** separated: common endpoint left \((u = u)\) or right \((v = v)\)

- **Past**: all intervals contained in \((u,v)\)
- **Future**: all intervals containing \((u,v)\)

**Kinematic Metric:**

\[ ds^2 = (...) \, du \, dv \]
Conditional mutual information

- strong subadditivity of entropy: $I(A,C | B) \geq 0$
- because of cancellations, this quantity localizes in the network
- it counts the number of isometries in a causal diamond

$$I(A,C | B) = S(AB) + S(BC) - S(ABC) - S(B)$$
Choosing a metric: volume element

Let volume count isometries!

\[ I(A, C | B) = S(AB) + S(BC) - S(ABC) - S(B) \]

Kinematic Metric:

\[ ds^2 = \frac{\partial^2 S_{\text{ent}}}{\partial u \partial v} \, \, du \, dv \]
- **Causal diamonds** are conditional mutual informations.
- **Diamonds that extend all the way to the bottom** are mutual informations.
- **Past causal diamonds of kinematic points** characterize the isometric embedding of a compressed state in the Hilbert space.
Because volume is additive, consistency requires:

\[ I(A,D|BC) + I(B,D|C) = I(AB,D|C) \]

This is an identity: the chain rule for conditional mutual information.
Kinematic Space → Holographic Geometry

Point \((u,v)\) uniquely selects:
- a minimal cut (dropped along the causal cone)
- a boundary interval \((u,v)\)

If the state has a holographic description obeying the Ryu-Takayanagi proposal...

Kinematic Space is the space of oriented geodesics ↔ boundary intervals!

\[ ds^2 = \frac{\partial^2 S_{\text{ent}}}{\partial u \partial v} \, du \, dv \]
Convex curves are kinematic regions

\[
\frac{\text{circumference}}{4G} = \int_{\text{intersect}} \frac{\partial^2 S(u, v)}{\partial u \partial v} \, du \, dv = \text{volume in kinematic space}
\]
\[
\frac{\text{circumference}}{4G} = \int_{\text{intersect}} \frac{\partial^2 S(u,v)}{\partial u \partial v} \, du \, dv = - \int d\theta \frac{\partial S(u,v)}{\partial u} \bigg|_{v=v(u)}
\]

works in every geometry that obeys the Ryu-Takayanagi proposal.
Why only spacelike cuts are allowed

- Timelike cuts do not effect coarse-graining
- They introduce spurious degrees of freedom
- Timelike cuts fail to define bulk curves

What goes wrong?
Curves with common endpoints

- length = volume of intersecting geodesics
- every geodesic intersecting orange also intersects yellow
- \textit{kinematic volume} \leq \textit{kinematic volume} \iff orange curve is a geodesic

length - length = \textit{volume} of geodesics that intersect yellow but not orange

length - length = \# isometries in this isometric embedding of states
We show that MERA corresponds to a discretization of de Sitter space. Beny, 2011
A quantitative connection between tensor networks (MERA) and a holographic geometry, mediated by integral geometry

**Assumptions:**
- Geometry: the Ryu-Takayanagi proposal
- Tensor networks: 
  \[
  \text{#cuts in MERA} \rightarrow \text{EE}
  \]

Not just the vacuum or thermal state

Does holography (large $N$) imply that we can construct a MERA-like tensor network with this property?
Understand the kinematic space directly in the language of the path integral (cf. Tensor Network Renormalization)

Understand the action of the conformal group on kinematic space

Kinematic space versus cMERA
(Haegeman, Osborne, Verschelde, Verstraete, 2011)
Kinematic Space in higher dimensions?

- **1+1-dimensional CFT:**
- **d+1-dimensional CFT:**

Boundary regions have shapes.
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arXiv:1505.05515
arXiv:1506.0xxxx

THANK YOU!