Quantum criticality in diluted 2D antiferromagnets

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Simulations of dimer-diluted S=1/2 Heisenberg models

- Quantum criticality in the presence of disorder
- Geometric/quantum criticality at percolation point
Background: Random antiferromagnets

1D random $J>0$ Heisenberg chain: 

$$H = \sum_i J_i \vec{S}_i \cdot \vec{S}_{i+1}$$

Infinite-randomness fixed point; random-singlet phase:

Spins form singlets on all length-scales at $T=0$

Ma, Dasgupta, Hu, 1979; D. Fisher, 1994

Renormalization procedure:

successive decimation of strongest-coupled pairs

$$\chi \sim \frac{1}{T \ln^2(T)}$$

Dynamic exponent: $Z = \infty$

$$\omega = k^Z, \Delta_L = L^{-Z}$$
**Dimerized chain:** \( H = \sum_i [J_i + \delta(-1)^i] \vec{S}_i \cdot \vec{S}_{i+1} \)

**Griffiths fixed point:** \( Z \propto 1/\delta \)

Igloi, Juhasz, Rieger (2000)

**2D random transverse Ising model:** \( Z = \infty \)


**2D random J Heisenberg models:**

**RG procedure \( \Rightarrow \) conventional scaling**

Lin, Melin, Rieger, and Igloi (2003)
Site-diluted 2D Heisenberg model

Cuprates: Cu → Zn substitution
Fraction $p$ of non-magnetic sites

Sublattice magnetization vs $p$

Is $p_c$ less than percolation point $p^* \approx 0.41$?

- Kato et al. (2000); $p_c = p^*$
- Sandvik (2001); percolating cluster ordered

Classical percolation transition
QMC studies of percolating cluster (d=91/46)

Comparison with neutron scattering; La$_2$(Zn,Mg)$_x$Cu$_{1-x}$O$_4$

Vajk, Greven, Mang, Lynn, Gehring (2001)

Order slightly suppressed; likely other interactions present
Quantum-criticality in dimerized 2D systems

Singlet-formation at strong bonds ("dimers") $\Rightarrow$ antiferromagnetic to spin-gapped transition

$g_c \approx 2.5$ in both bilayer and single-layer model
Quantum-critical diluted systems

Unpaired spins in site-diluted gapped system ⇒ localized moments form; order antiferromagnetically ⇒ no quantum-critical point

Dimer-diluted models:
Bilayer model

Sandvik (2002); Vajk, Greven (2002)

Geometric percolation transition expected for \( g < g^* \)

Disorder relevant at transition for \( g > g^*, p < p^* \)
(Harris criterion; \( \nu < 2/d \))

Confirmed in simulations of 3D classical model
with columnar defects (Sknepek, Vojta, Vojta); \( Z = 1.310(6) \)
Uniform magnetic susceptibility

$T > 0$ quantum-critical form: $\chi = a + b T^{d/z-1}$

$T=0; \chi = 0$ for finite system. Use $\chi(q \to 0^+)$

$\xi \sim T^{-z}; \chi = a + b \xi^{z-d} \quad \xi \to L$

$\chi(T = 0, q \to 0^+) = a + b L^{z-d}$

$Z=1.36(2)$
No sign of quantum-critical to renormalized classical cross-over

Pure 2D antiferromagnet (Chakravarty, Halperin, Nelson)

Low-T cross-overs controlled by spin-stiffness/gap

Conjecture: Stiffness = 0 on percolating cluster; no RC cross-over
Check of scaling (Z) using antiferromagnetic quantities

\[ q = (\pi, \pi) \]

\[ S(q) = \langle S_{-q}^z S_q^z \rangle = \int d\omega S(q, \omega) \sim L^{1-\eta} \]

\[ \chi(q) = \int_0^{1/T} d\tau \langle S_{-q}^z (\tau) S_q^z (0) \rangle = \int d\omega \frac{1}{\omega} S(q, \omega) \sim L^{1+z-\eta} \]

\( 1 - \eta = \gamma_S \)

\( 1 + z - \eta = \gamma\chi \)

Data at \( g \) close to \( g^* \) consistent with

\( z = \gamma\chi - \gamma_S \approx 1.36 \)
Correlation length measured by neutron scattering in $\text{La}_2(\text{Zn},\text{Mg})_x\text{Cu}_{1-x}\text{O}_4$

Vajk, Greven, Mang, Lynn, Gehring (2001)

Close to percolation: $\xi \sim 1/T^z$, $z \approx 1.4$

Could this be due to multi-critical point ($g^*$, $p^*$) in extended parameter space?

How universal is the behavior found for the diluted bilayer antiferromagnet?

Can similar behavior be observed in a single layer?
Single layer with staggered dimers

Expected $T=0$ phase diagram

$g = J_2/J_1$

Dimers form triangular lattice; $p^*=1/2$

Brick-wall lattice when $g=0$; different percolation point

$P^*(g = 0) \approx 0.29$
Uniform susceptibility at $p^*(g>0)$

- Indicates critical point; $g_c \approx 1.247$
- Finite susceptibility at critical point
- Divergent susceptibility for $g < g_c$
- Divergence follows closely $T^{-1/2}$ for $g > 1$
- Faster divergence for smaller $g$
- Curie form for $g=0$, due to “broken clusters”
Extracting $z$ using the form

$$\chi = aT^{-\alpha} = aT^{d/z-1}$$

$T=0$ scaling consistent with $T>0$ scaling

$$\chi(q \rightarrow 0^+) \sim L^{z-d}$$
No long-range order

Finite-size scaling at $g^*$

$Z \approx 1.7$

Line of critical points with continuously varying exponents

$S(\pi, \pi), \chi(\pi, \pi)$
\( S(\pi, \pi), \chi(\pi, \pi) \) Finite-size scaling at \( g=1.0 \)

\( Z \approx 3.0 \)

Disagrees with \( Z\approx 4.0 \) from uniform susceptibility

None-asymptotic behavior?

Or problems related to disorder averaging?

\( z \) from \( \frac{\langle S \rangle}{\langle \chi \rangle} \) or \( \langle \frac{S}{\chi} \rangle \)?
Conclusions

Novel quantum-critical behavior discovered numerically in dimer-diluted systems

Conventional scaling for the $p<p^*$ transition; $z \approx 1.31$

Why does the single layer have a line of critical points?

Conditions under which the percolating cluster orders?