Tonks Girardeau Gas in an Optical Lattice

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Outline

• Introduction
• SF-Mott transition – a reminder
• Tonks-Girardeau gas in an optical lattice (Nature in press)
• Conclusion and outlook
Status of the Experiments in Mainz

After disentangling classical objects...

And rather not wanting to speak, hear or see anything about the move...
The Experiment Finally Moves to Mainz

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Current Status of Experiments

• BEC machine operational

• 3D Lattices almost completed and gearing up for new round of experiments
3D Lattice Potential

- Resulting potential consists of a simple cubic lattice
- BEC coherently populates more than 100,000 lattice sites

\( V_0 \text{ up to } 40 \ E_{\text{recoil}} \)

\( \omega_r \text{ up to } 2\pi \times 45 \ kHz \)

\( n \approx 1-5 \text{ atoms on average per site} \)
The SF-Mott Insulator Transition

\[ H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1) \]

Characteristic parameter for the behaviour of the system

\[ \frac{U}{J} \]


Superfluid Limit

\[ H = -J \sum_{i,j} \hat{a}^+_i \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1) \]

Atoms are delocalized over the entire lattice!
Macroscopic wave function describes this state very well.

\[ \left| \Psi_{SF} \right\rangle \propto \left( \sum_{i=1}^{M} \hat{a}^+_i \right)^N \left| 0 \right\rangle \]

Atom number distribution after a measurement
"Atomic Limit" of a Mott-Insulator

\[ H = -J \sum_{i,j} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1) \]

Strong repulsion between atoms leads to a kind of "fermionization"

Repulsion mimics Pauli principle, but connection still vague

Atom number distribution after a measurement
Short Resume of Work with the Mott State

Quantum Information

- Spin dependent potentials for individual atoms

- Collaps and revival of the matter wave field of a BEC

- Controlled collisions – massively parallel quantum gate array – controllable Ising interaction

Atomic/Molecular Physics

- Entanglement Interferometry – Precision measurement of atomic scattering properties
  A. Widera et al., PRL 92, 160406 (2004)

- State Selective Production of Molecules in Optical Lattices
  T. Rom et al. (submitted)
The Tonks-Girardeau Gas
– A Fermionized 1D Quantum Gas –

Requirements (I): 1D bosonic quantum gas, tightly confined in two dimensions and only weakly confined along the axial direction

\[ \mu \ll \hbar \omega_\perp \quad \text{and} \quad T < T_a \approx N \hbar \omega_{ax} \]

In Experiments here: aspect ratio typically 100-200

Experiments with 1D condensates:
more recently:
2D Lattice Potential

- Resulting potential consists of an array of tightly confining potential tubes
- BEC is split into up to 10,000 1D quantum gases (radial motion confined to zero point oscillations)

\[ V_0 \text{ up to } 30 \, E_{\text{recoil}} \]
\[ \omega_r \text{ up to } 2\pi \times 35 \, \text{kHz} \]
\[ n \approx \text{up to 20 atoms per tube} \]
**Requirements (2):** Strong repulsive interactions between atoms

\[ \gamma \approx \frac{I}{K} \gg 1 \]

**Homogeneous case**

- **Interaction energy** \( I \sim ng \)
- **Kinetic energy** \( K \sim \frac{\hbar^2 n^2}{m} \)

\[ \gamma = \frac{mg}{\hbar^2 n} \]


*General Theory of „Luttinger Liquids“* (see work of Haldane) can be applied to these quantum gases for arbitrary \( \gamma \)
In 1D for strongly interacting bosons, the many-body wave function can be mapped on to the one of non-interacting fermions.

(M.D. Girardeau, J. Math. Phys. 1960) This lies at the heart of a TG gas!

\[
\Psi_B(x_1, \ldots, x_N) = |\Psi_F(x_1, \ldots, x_N)|
\]

For example:

\[
\Psi_B(x_1, \ldots, x_N) = \left| \det \left[ \phi_i(x_j) \right] \right| \quad i, j = 1 \ldots N
\]

- Slater determinant ensures that two particles cannot be placed at the same position in space!
- Absolute value ensures symmetrization.
Bosons behave like Fermions – Not Quite

**Density distribution:**
\[ |\Psi_B(x)|^2 = |\Psi_F(x)|^2 \]
identical to the one of free fermions!
(absolute value of det does not matter)

**Correlation function:**
\[ g^{(1)}(x) = \langle \Psi_B^\dagger(0) \Psi_B(x) \rangle \neq \langle \Psi_F^\dagger(0) \Psi_F(x) \rangle \]
different to the one of free fermions!
(absolute value of det matters)

**Momentum Distribution:**
\[ n(p) \propto \int e^{-ipx} g^{(1)}(x) \, dx \]
different to the one of free fermions!
(FT of correlation function)

*Momentum distribution is characteristic for a Tonks-Girardeau gas!*
Status of Experiments

So far, experiments in 2D optical lattices have achieved $\gamma \approx 0.5-1$,

Still 1D mean-field regime (see H. Moritz et al. PRL (2003)), although correlations begin to be modified (see B. Laburthe Tolra et al. cond-mat/0312003)

\[
\gamma = \frac{mg}{\hbar^2 n}
\]

Ways to increase $\gamma$:

1. Increase Interaction strength
   \[ g = 2 a \hbar \omega_\perp \]

2. Decrease density
   \[ n \]

3. Increase of mass
   \[ m \]
Increasing the Mass

Addition of lattice along the axial direction leads to an increase in the effective mass $m^*$!

However, in order to apply Fermionization, we need to work in a regime, where:

$$\nu \leq 1$$


Tonks-Parameter in a lattice:

$$\gamma = \frac{U}{J}$$
Experimental Sequence to Prepare the 1D Quantum Gases

(1) Create array of 1D quantum gases

(2) Add lattice along axial direction

Experimental parameters:

$V_0$ (2D) approx 27 $E_r$

$V_{ax}$ = 0-19 $E_r$

Lattice Wavelengths 825 nm

Atom number < 3-4×10⁴

Harmonic confinements:

$\omega_{ax} = 2\pi \times 60$ Hz

$\omega_{\perp} = 2\pi \times 35$ kHz
**Typical Absorption Images After Time Of Flight**

*Observe fast expansion in radial direction*

Due to the low atom number we average horizontal profiles within the white dashed lines.

**Challenge:**

*Fully explain momentum distributions!*
Momentum Distribution of a (Lattice) 1D Gas

Important momentum scale
(1/average interparticle spacing):

\[ p_\nu = \hbar \times \frac{2\pi \nu}{\lambda} \]

\[ \nu : \text{filling factor} \]

(a) For \( p \ll p_\nu \) the slope tends to 1/2

\[ n(p) \propto \frac{1}{\sqrt{p}} \]

(b) For \( p \gg p_\nu \) the momentum distribution is affected by short range correlations, which tend to increase the slope

cp. M. Olshanii, PRL 91 (2003),
G.E. Astrakharchik & S. Giorgini
**Finite Temperature Effects in a (lattice) 1D gas**

**Important momentum scale**
(1/thermal phase coherence length):

\[
p_T = \hbar \times \frac{\pi}{L_\phi}
\]

\[
L_\phi \approx \frac{\lambda J}{k_B T \times \sin(\pi \nu)}
\]

\(\nu = 1/2, \ 1000 \text{ sites}\)

*Finite temperature affects low momenta (long range coherence) and broadens peaks*
Summary of Important Momentum Scales

\[ p_v / \hbar = \frac{2\pi v}{\lambda} \approx n \approx k_F \]

Short range – long range correlations
(change slope)

\[ p_T / \hbar = \frac{\pi}{L_\phi} \]

Thermal effects
(broaden momentum peaks)

\[ p_L / \hbar = \frac{\pi}{L} \]

Finite size effects

For our experimental parameters, we find:

\[ p_L < p_T \sim p_v \]

Finite size effects are dominated by
finite temperature effects!
Momentum & Density Distribution for a Fermionized 1D (lattice) gas

Single tube result: $N=15$

$V_{ax}=5\, E_r$

$\alpha=0.8,\, T=0$

Red curve $k_B\, T/J=0.75$

$V_{ax}=9.5\, E_r$

$\alpha=0.5,\, T=0$

$V_{ax}=12\, E_r$

$\alpha=0.2,\, T=0$
Increase of Lattice Depth Changes Filling Factor in the Inhomogeneous System

Even for \( U \to \infty \) the system spreads out due the kinetic energy \( J \)!

If \( J \) decreases (deeper lattice), the system shrinks until a Mott state with \( n=1 \) is formed in the center!

Fermionization describes all filling factor regimes up to \( n \leq 1 \), provided \( \gamma \gg 1 \)!
Simple Picture for Change in Filling Factor

Decrease of kinetic energy
(Increase of Lattice Depth)

\[
H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \frac{1}{2} m \omega^2 \left( \frac{\lambda}{2} \right)^2 \hat{i}^2 \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)
\]
Averaging over the Different 1D Gases

Atom number in potential tube $i,j$

$$N_{i,j} = N_{0,0} \left(1 - \frac{5}{2\pi} \frac{N}{N_{0,0}} (i^2 + j^2)\right)^{3/2}$$

Probability for finding tube with $M$ atoms

$$P(M) = \frac{2}{3} \frac{1}{N_{0,0}^{2/3} M^{1/3}}, \quad M \leq N_{0,0}$$

$N_{0,0}$ atom number in central tube!

*We use this probability distribution to average over the momentum distributions of different tubes.*
Comparison Experiment-Theory

Already for low axial lattice depths we observe a pronounced power-law decay with slopes <2
Comparison Experiment-Theory (2)

For whole series we use only two fit parameters $N_{0,0}=18$ and $T_0=0.5\text{ J}/k_B$!

Temperatures at higher lattice depths have been calculated from $T_0$ assuming an adiabatic evolution (conservation of entropy) of the system!
Example of Momentum Profile

\[ V_{ax}/E_r = 5.6 \]
\[ \gamma = 7.8 \]
Change in Densities

$V_0 = 4.6\ E_r$

$V_0 = 7.4\ E_r$

$V_0 = 12\ E_r$

$V_0 = 18.5\ E_r$
Comparison Theory-Experiment (All Series)

Theory has to predict:

- the shape of each momentum profile
- evolution of temperature (entropy) which is very different for an ideal (or weakly interacting) Bose and Fermi gas

Only two fit parameters for whole series!

\[ \frac{k_B T_0}{J} \approx 0.5 \]

\[ N_{0,0} = 18 \]
Conclusion & Outlook
- Fermionization -

1D Quantum gases

• We have been able to enter the Tonks-Girardeau regime in a 2D array of one-dimensional quantum gases
• Increase in effective mass good way to increase interactions
• For Fermionization to be applicable it is however important to work at low filling factors
• We observe excellent agreement with the theory based on a fermionization approach

First quantitative comparison of momentum distribution with theory

• Good agreement has allowed us to determine temperature of the quantum gases