Scaling of Stick-Slip Instabilities in Granular Materials

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Overview

Goal: investigate physics of stick-slip in sheared granular materials

• Stick-slip is the closest laboratory analog to the seismic cycle -- periods of slow loading followed by rapid failure

• Examine how aspects of stick-slip scale with experimental conditions, varying:
  - Normal stress
  - Layer thickness
  - Driving velocity

• Find that dilation plays an important role in failure processes, look at scaling to understand the physics of dilation and failure

• Make quantitative connections to constitutive laws to try and understand implications for larger scale deformation and failure
Stick-Slip Instabilities

Goal: investigate physics of stick-slip in sheared granular materials

- Similar to seismic cycle, with long, slow loading cycles and short periods of rapid failure
- Challenge is to connect microscopic grain scale physics to macroscopic dynamics of friction
- Although I focus on granular materials here, many common features with deformation of other amorphous materials

Experiment

Discrete Element

Consitutive Law
Examples:

glass

biological tissue

silicon panels

bulk metallic glass

dense colloids/emulsions

foam

grain

fault gouge

sand
Microscopic Deformation in Amorphous Materials

Experiments:
Sheared 2D granular material, image particle displacements and force network using photoelastic beads

Simulations:
2D molecular dynamics simulation of a glass, shows where plastic deformation occurs
But… limited length and time scales
=> Develop a constitutive model
Constitutive Laws

In seismology, usually use laboratory derived Dieterich-Ruina friction laws.

Phenomenological fits to data, incorporates rate dependence of friction. Not derived from microscopic physics, so provides little physical insight.

Problem: conditions in lab experiments are vastly different than those in the earth

- Normal stress is much larger (~5 MPa vs. ~100 MPa)
- Layers are thicker (~4 mm vs. ~1 m)
- Imposed driving rate is much slower (~microns/second vs. ~cm/year)

We don’t know what is different when at conditions that can’t be replicated in the laboratory

State variable in DR friction (history dependence of friction) interpreted as lifetime of surface asperity contacts, observed experimentally Dieterich and Kilgore (1994)
Constitutive Laws

In physics, we have STZ Theory, which incorporates observations (mostly from simulations)

Basic premise:

\[ \dot{\gamma} = f(\tau) \exp(-1/\chi) \]

- Stress determines rate at which STZs rearrange
- Effective temperature $\chi$ determines number density of STZs (flow defects), more configurational disorder = more STZs

Incorporates same observations as DR, plus captures dynamics of strain localization
Constitutive Laws

Assume effective temperature follows a heat equation (i.e. include terms for dissipation and diffusion):

\[
\frac{\partial \chi}{\partial t} = \frac{\gamma \tau}{c_0 \tau_y} \left( 1 - \frac{\chi}{\dot{\chi}(\dot{\gamma})} \right) + \frac{\partial}{\partial z} \left( \gamma D \frac{\partial \chi}{\partial z} \right)
\]

Dissipation drives \( \chi \) towards steady state

Most of STZ Theory based on simulations, with few connections with experiments.

Goal here is to improve this link between physical constitutive laws and experimental stick-slip

Shi et al., PRL 2007
Differences Between DR and STZ Theory

Primary difference is in the frictional length scale. Constant in DR law, varies with slip rate in STZ Theory (in agreement with experiments -- Mair and Marone, 1999)

But frictional length scale also might scale with thickness, normal stress. Need to look at data to assess this.
Length Scale in Earthquakes

Why should we care about this length scale?
If this varies with slip rate, can impact rupture propagation, peak slip rate, and earthquake nucleation (Daub and Carlson, JGR, 2008)
Stick-Slip Instabilities

Goal: look at laboratory stick-slip to address questions about deformation and failure

Can we provide constraints on the physics that should go into constitutive models? Can we say anything about effective temperature/free volume from experiments?

Are there any implications for earthquakes in this?
Dataset

Data from Penn State Rock Mechanics Lab

• Shear layers of 125 µm spherical glass beads
• Constant Load Point rate $V_0$ (5 µm/s)
• Variable normal stress $\sigma$ (2-8 MPa)
• Variable thickness (layers thin as they shear)
• Also an experiment with varying rate (not fully analyzed -- noisy, other difficulties)

Experiment with Variable Normal Stress

![Graph showing shear stress vs load point displacement](image)
Stick-Slip Instabilities

Measure shear stress and layer thickness throughout experiment.

Stick-slip cycles include stick, creep, and slip phases.

Also see dilation and compaction of layer during cycle -- grains must dilate in order to slip rapidly.
Assume stress evolution is (ignores inertia of center block, which is negligible):

\[
\frac{d\tau}{dt} = k (V_0 - V)
\]

- Know \( V_0 \)
- Determine stiffness \( (k) \) from data after each failure event
- Stress \( \tau \) measured as a function of time, differentiate
- Can calculate \( V \) this way
- Integrate \( V \) to get slip
Failure Mechanism?

What determines when failure occurs?

Slip occurs when friction coefficient is $\sim 0.375-0.4$, but 4 MPa data shows that system can slide at failure stress and not fail immediately. Note that this decreases frequency of failure at lower normal stress (by a factor of $\sim 2$).

If not shear stress, then what? Look at internal state of material (thickness).
Role of Dilation

Look at layer thickness -- does density of grains play a role in failure?

Dilation occurs during creep phase -- measure dilation prior to failure.

Dilation vs. Normal Stress

Internal configuration of granular layer important for determining failure.

Material requires a set amount of excess free volume in order to fail, which is independent of normal stress.
Role of Dilation

Look at layer thickness -- does density of grains play a role in failure?

Dilation occurs during creep phase -- measure dilation prior to failure.

Dilation vs. Layer Thickness

Layer dilation prior to failure is independent of thickness.

This is evidence for localization of inelastic deformation -- if the entire layer was deforming, expect to see more dilation for thicker layers.
Role of Dilation

Look at layer thickness -- does density of grains play a role in failure?

Dilation occurs during creep phase -- measure dilation prior to failure.

Dilation vs. Loading Rate

Dilation is rate dependent -- drive faster, less dilation required.

Note: this implies that internal configuration of the material evolves to a value that depends only on the strain rate

\[
\frac{\dot{\gamma} \tau}{c_0 \tau_y} \left( 1 - \frac{\chi}{\hat{\chi}(\dot{\gamma})} \right)
\]
Thickness Evolution

Idea: failure occurs once material dilates an amount dependent only on strain rate. Therefore, dilation rate determines the time/slip that occurs prior to failure.

What controls thickness evolution? Slip or time?

Dilation rate is controlled by slip -- time scale that controls dilation is the inverse plastic strain rate.

\[
\frac{\dot{\gamma}}{c_0 \tau_y} \left( 1 - \frac{\chi}{\hat{\chi} (\dot{\gamma})} \right)
\]
How does dilation rate with slip scale? Earthquake modelers often incorporate phenomenological fits to data in models (e.g. Segall and Rice, JGR, 1995), but don’t include any physical motivation.

Dilation rate scales ~linearly with normal stress.

Implies that rate of energy dissipation controls dilation. More inelastic work stirs up grains, forming less dense configurations.
Thickness Evolution -- Scaling with Layer Thickness

How does dilation rate with slip scale?

Dilation rate scales \( \sim \) inversely with layer thickness. Normal stress trend removed in plot.

Interesting thing to note: entire layer thickness important here. Dilation rate does depend on thickness, but total amount of dilation does not.

Short chains must dilate to both slip and support the shear stress

Long chains can deform to accommodate slip without dilating

Idea: length of force chains important for determining dilation rate. (i.e. Anthony and Marone, JGR, 2005)
Recurrence Time Scaling

Dilation important for failure in stick-slip events, dilation scales with normal stress and thickness. What does it mean for recurrence?

Recurrence time scales as \( \sim 1/\text{normal stress} \) and \( \sim \text{linearly with thickness} \). Scaling not perfect, as dilation is controlled by slip, not time.

When scaling from lab to earthquake faults, thickness scaling implies thicker (mature) faults should have longer recurrence intervals than immature faults, in agreement with observations.
Implications for Constitutive Models

- Find that layer dilation plays an important role in failure -- clearly an important state variable in a constitutive law

- Dilation prior to failure independent of normal stress, thickness, but does depend on driving rate

- Supports idea that free volume/effective temperature evolves towards a value that depends only on strain rate? Not entirely clear, as these measurements aren’t of steady-state behavior but of transient dynamics.

\[
\frac{\dot{\gamma} \tau}{c_0 \tau_y} \left( 1 - \frac{\chi}{\hat{\chi}(\dot{\gamma})} \right)
\]
Implications for Constitutive Models

- Dilation rate scales linearly with normal stress and inversely with layer thickness.
- This implies that the frictional length scale depends on both thickness and normal stress:

\[ L \propto \frac{h^2}{\sigma_n} \]

This implies that the frictional length scale for seismic faults should be larger than lab values.

Also confirms that \( L \) should be slip rate dependent, which is not captured by DR friction.
Implications for Constitutive Models

Interesting observation: see that dilation prior to failure is independent of thickness, but dilation rate with slip is not.

Evidence of localization, but also indications that the entire layer plays a role in dynamics.

Yu and Behringer

Inelastic deformation occurs here

But force network extends throughout the entire layer
Recap

- **Layer dilation** plays an important role in granular failure
- Dilation controlled by slip. Dilation rate scales **linearly with normal stress** and **inversely with layer thickness**
- Confirms that free volume/effective temperature evolution depends on dissipation rate
- Implies that recurrence time (time between events) scales approximately **inversely with normal stress** and **linearly with layer thickness**
- Thickness dynamics show evidence of localization, but we also find that the entire layer thickness plays a role