Holographic Mesons: Adding Flavor to the Gauge/Gravity Duality

Kruczenski, Mateos, Winters + RCM
hep-th/0304032;
(Hovdebo, Thomson, ....)

Anti-de Sitter/Conformal Field Theory Correspondence:

d=10 Type IIb superstrings in AdS$_5 \times$ S$^5$
with N units of RR flux
equivalent to
d=4 $\mathcal{N}$ =4 U(N) super-Yang-Mills

\[ \left( \frac{R^2}{\alpha'} \right)^2 = g_{YM}^2 N \equiv \lambda \]
\[ 4\pi g_\sigma = g_{YM}^2 \]
Decoupling limit of D3-branes provides “derivation” of AdS/CFT correspondence

N coincident D3-branes:

• decouples asymptotic strings
• throat geometry: AdS$_5$ X S$^5$
• reduces brane theory to field theory: $\mathcal{N}=4$ SYM

\( \alpha' E^2 \rightarrow 0 \)

\( ds^2 = \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \)

Symmetries: SO(4,2) X SO(6)
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\[ \mathcal{L} = \frac{1}{g^2} \text{Tr} \left[ -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - i \bar{\lambda} \sigma^{\mu} D_\mu \lambda - i \bar{\psi}_i \sigma^{\mu} D_\mu \psi_i 
\right. \\
+ (D_\mu \phi_i)^\dagger (D^\mu \phi_i) - \frac{1}{2} [\phi_i^\dagger, \phi_i]^2 + [\phi_j^\dagger, \phi_k^\dagger][\phi_j, \phi_k] \\
- i \sqrt{2} [\lambda, \psi_i] \phi_i^\dagger - i \sqrt{2} [\bar{\lambda}, \bar{\psi}_i] \phi_i \\
- \sqrt{2} i \epsilon_{ijk} (\phi_i \psi_j \psi_k - \bar{\psi}_k \bar{\psi}_j \phi_i^\dagger) \]

\begin{align*}
\text{vector:} & \quad A_\mu \\
\text{4 Weyl fermions:} & \quad \lambda, \psi_{1,2,3} \\
\text{3 complex scalars:} & \quad \phi_{1,2,3}
\end{align*}

\begin{itemize}
\item conformally invariant: $\text{SO}(3,1) \rightarrow \text{SO}(4,2)$
\item R-symmetry: $\text{SO}(6)$
\end{itemize}

\textbf{Anti-de Sitter/Conformal Field Theory Correspondence:}

\[ \text{d=10 Type IIB superstrings in } \text{AdS}_5 \times S^5 \]
\[ \text{with } N \text{ units of RR flux} \]
\[ \text{equivalent to} \]
\[ \text{d=4 } \mathcal{N} = 4 \text{ U}(N) \text{ super-Yang-Mills} \]

\[ \left( \frac{R^2}{\alpha'} \right)^2 = g_Y^2 N \equiv \lambda \gg 1 \]
\[ 4 \pi g_s = g_Y^2 \]

supergravity limit $\sim$ large-$N$ with strong `t Hooft coupling
**Holographic Mesons**

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**Adding Static “Quarks”:**

external sources in fundamental representation

Motivation: remove a D3-brane to infinity breaking $U(N+1) \rightarrow U(N) \times U(1)$; massive gauge bosons transform as bifundamental of residual gauge group.

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**Adding Dynamical “Quarks”:**

need a place/brane where strings can end at finite $r$ → add $k$ D7-branes to AdS background

$m_q = \frac{L}{2\pi \alpha'}$
Decoupling limit of N D3-branes with k D7-branes

N coincident D3-branes:

![Diagram of D3 and D7 branes]

\[ r = L \]

\[ d = 4 \text{ U}(N) \text{ o.s., } d = 8 \text{ U}(k) \text{ o.s.} \]

Low-energy limit with

\[ g^2 \alpha' E^2, L^2/\alpha' \rightarrow 0 \]

Gauge coupling for D7 string:

\[ g_7^2 = \frac{2qL^2}{2\pi \alpha'} \]

- throat geometry: \( AdS_5 \times S^5 \) with k D7's
- brane theory \( \mathcal{N} = 4 \) U(N) SYM with fundamental matter

Field theory:

\( \mathcal{N} = 4 \) U(N) super-Yang-Mills

- vector: \( A_\mu \)
- 4 Weyl fermions: \( \lambda, \psi_{1,2,3} \)
- 3 complex scalars: \( \phi_{1,2,3} \)

adjoint in U(N)

![Diagram of U(N) super-Yang-Mills]

coupled to: k massive \( \mathcal{N} = 2 \) hypermultiplets

- 2 complex scalars: \( \xi^\pm \)
- 2 Weyl fermions: \( x^\pm \)

fund. in U(N) & global U(k)

- SUSY: \( \mathcal{N} = 4 \rightarrow \mathcal{N} = 2 \)
- \( SO(6) \rightarrow SO(4) = SU(2)_L \times SU(2)_R \{ X U(1)_R \}_{m=0} \)

\[ Q_L (0, \frac{1}{2}, 1); \quad q^\pm (0, \frac{1}{2}, 0); \quad x^\pm (0, 0, \mp 1) \]
**Geometry:**

- $r = \infty$: AdS boundary
- $r = L$: equator
- $r = 0$: horizon

SO(4) = rotational symmetry of $S^3$

\{ U(1)$_R$ = additional rot. sym. for L=0 \}

**Probe approximation:**

This construction does not take into account the “gravitational” back-reaction of the D7-branes!

→ considering large-N limit with $k$ fixed
  (in fact, $k=1$ in following)

(see, however: Burrington et al)

Note: even with $m_3=0$, hypermultiplets introduce non-vanishing $\beta$-function - however, running of `t Hooft coupling vanishes in large-N

$$\beta(g) \sim k \, g^3, \quad \beta(\lambda) \sim \frac{k}{N} \lambda^2 \rightarrow 0$$
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“Mesons”:
bound states of fundamental fields dual to
open string states supported by D7-brane

Consider two cases:
- mesons with low J (and arbitrary R-charge)
- mesons with large J (and no R-charge)

**Mesons with J=0,1** (and arbitrary R-charge)

lowest lying open string states are excitations of the
massless modes on D7-brane: vector, scalars (& spinors)

their dynamics is governed by usual worldvolume action:

\[ S_{D7} = -\frac{1}{(2\pi)^7 g_s \alpha'^4} \int d^8 x \sqrt{-\text{det} \left( P[C]_{ab} + 2\pi \alpha' F_{ab} \right)} \]
\[ + \frac{1}{2(2\pi)^5 g_s \alpha'^2} \int P[C^{(4)}] \wedge F \wedge F \]

free spectrum:
- expand action to second order in fluctuations
- solve linearized eq’s of motion by separation of variables

discrete spectrum

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**Meson spectrum:**

$$M^2(n, \ell) = \frac{4L^2}{R^4}(n + \ell + 1)(n + \ell + 2)$$

- $n = \text{radial AdS quantum}$
- $\ell = \text{angular quantum}$ on $S^3 = \text{R-charge}$

$A_\mu$:
- one vector in the $(\ell, \ell)$ with $n \geq 0$, $\ell \geq 1$
- one scalar in the $(\ell, \ell)$ with $n \geq 0$, $\ell \geq 1$
- one scalar in the $(\ell, \ell + 2)$ with $n \geq 0$, $\ell \geq 0$
- one scalar in the $(\ell, \ell - 2)$ with $n \geq 0$, $\ell \geq 2$

$\Phi^a$:
- two scalars in the $(\ell, \ell)$ with $n \geq 0$, $\ell \geq 0$

$\psi^a$:
- Dirac fermion in the $(\ell, \ell + 1)$ with $n \geq 0$, $\ell \geq 0$
- Dirac fermion in the $(\ell, \ell - 1)$ with $n \geq 0$, $\ell \geq 1$

massive supermultiplets with $B(\ell + 1)$ bosons and fermions

$$n = \ell = 0 : \quad m_{\text{gap}} = 2\sqrt{2} \frac{R}{L} = 2m_\text{q}\sqrt{\frac{2\pi}{g_\text{N}}}$$

- dominate low energy physics

**Meson interactions:**

Continue expansion of D7-brane action beyond 2\text{nd order}

- substitute $\Phi = \phi(x)\psi_{n,\ell}(r)Y_\ell(\Omega)$ and integrate out $r$ and $S^3$

$$\mathcal{L}_{\text{eff}} \sim -\frac{1}{2} \left[ (\partial \phi)^2 + M^2 \phi^2 \right] + \frac{2\pi \alpha'}{L} \frac{1}{\sqrt{N}} \phi (\partial \phi)^2 + \ldots$$

$$g_{\phi(\partial \phi)^2} \sim \frac{2\pi \alpha'}{L} \frac{1}{\sqrt{N}} \frac{1}{m_\text{q}\sqrt{N}}$$

4\text{th order}:

$g_{\phi^4} \sim \frac{1}{\lambda N}$, $g_{\phi^2(\partial \phi)^2} \sim \frac{1}{N m_\text{q}^2}$, and $g_{(\partial \phi)^4} \sim \frac{\lambda}{N m_\text{q}^4}$

- agrees with standard large $N$

of course, for fixed $\lambda \sim g_8 N$

- glueballs $g_8 \sim \frac{1}{N}$, $g_8 \sim \sqrt{g_8} \sim \frac{1}{\sqrt{N}}$ for mesons
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**Very large R-charge:**

radial profile \( \sim \frac{(r^2 - L^2)^{\ell/2}}{r^{2\ell}} \)

with narrow peak at \( r = \sqrt{2}L \)

null geodesics or time-like geodesics with large J/M orbiting S\(^3\) in *induced* D7-brane geometry sit at precisely this radius

\( \rightarrow \) PP-wave analog for open strings??

**Mega-mesons:** for very large \( \ell \), also expect to see brane expansion effects as for giant gravitons

??

**Mesons with large J** (and no R-charge)

**Classical rotating open strings attached to D7-brane**

Nambu-Goto action: \( S = \frac{1}{2\pi \alpha'} \int dr d\sigma \sqrt{-\det G_{\mu \nu} \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\nu}} \)

Ansatz: \( t = \tau, \rho = \sigma, \theta = \omega \tau, r = R^2/z(\sigma) \)

\[
S = -\frac{R^2}{2\pi \alpha'} \int d^2 \sigma \frac{1}{z^2} \sqrt{\left(1 - \omega^2 \sigma^2\right)\left(1 + z'\right)}
\]

eom: \( \frac{z''}{1 + z'^2} + \frac{2}{z} - \frac{\omega^2 \sigma z'}{1 - \omega^2 \sigma^2} = 0 \)

bc: \( z' \rightarrow \infty \) (string ends \( \perp \) to D7)

\[
E = \omega \frac{\partial L}{\partial \dot{\omega}} - L = \frac{R^2}{2\pi \alpha'} \int d\sigma \frac{1}{z^2} \sqrt{\frac{1 + z'^2}{1 - \omega^2 \sigma^2}}
\]

\[
J = \frac{\partial L}{\partial \dot{\omega}} = \frac{R^2}{2\pi \alpha'} \int d\sigma \sigma^2 \frac{1 + z'^2}{z^2} \frac{1 - \omega^2 \sigma^2}{1 - \omega^2 \sigma^2}
\]
Meson spectrum:
solve $E(J)$ numerically:

\[
E \propto \sqrt{J}
\]
\[
\delta E \propto 1/J^2
\]

Case I: proper size $\ll R$ ($\omega \to \infty$)

\[
lr \sim \frac{L^2}{R^2\omega}
\]

\[
\rho_{\text{max}} \sim \frac{R^4}{L^4\omega}
\]

\[
r = L
\]

\[
r = 0
\]

Regge behavior:

\[
J^2 \approx \frac{R^2 \alpha'}{L^2} B^2 = \frac{g_s N}{2\pi^3/2m_q^2} B^2
\]

\[
T_{\text{eff}} = \frac{1}{2\pi\alpha'} \frac{L^2}{R^2} = \sqrt{\pi} \frac{m_q^2}{\sqrt{g_s N}} \left( \approx \sqrt{g_s N} m_{\text{gap}}^2 \right)
\]

Analysis requires: $\frac{L}{R} \rho_{\text{max}} \ll R \rightarrow 1 \ll J \ll \sqrt{g_s N}$
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**Case II**: proper size $\gg R$ ($\omega \to 0$, $J \gg \sqrt{g_s N}$)

\[
r_0 \simeq R^2 \frac{\omega^{2/3}}{L^{1/3}}
\]

with

\[
E' \sim 2m_q - \kappa^4 \frac{m_q}{4J^2}
\]

and

\[
\kappa^2 = \frac{8\pi^2 \sqrt{2\pi}}{\Gamma(1/4)^4} \sqrt{g_s N}
\]

Binding energy for Coulomb-like potential, $V = -\frac{\kappa^2}{r}$

matches precisely static quark potential!

Maldecena; Rey & Yee

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Cross-over scale: $|x| \sim \frac{\sqrt{g_s N}}{m_q} \sim \frac{1}{m_{gap}}$

large strings; Coulomb bound state

small strings; Regge behavior

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Regge behavior:

![Graph showing excited strings]

Exotic states: Numerical integration also finds solutions with radial oscillations

![Graph showing radial oscillations]

decay suppressed in large-N: $g_s \sim 1/N$

hybrid states or glue-excited mesons
Regge behavior: compare slopes

\[(m_1/m_0)^2 = 2.99 \sim 3\]
\[(m_2/m_0)^2 = 4.97 \sim 5\]
\[(m_3/m_0)^2 = 6.90 \sim 7\]

1 node gives essentially 3 strings stretching between quarks
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1 node, $\omega$ small

$\rho = 1/\omega$

$r = L$

$r = 0$

“light cylinder”

Trajectory terminates when loop hits “light cylinder”

Interesting structure (loops) sink towards horizon (infrared) as $\omega \to 0$

→ limits applicability to QCD

Regge behavior:

Excited strings
Holographic Mesons

Regge behavior:

standard Regge trajectories “polluted” by “hybrid states” (amongst other things?)

Holography: “shadows on the wall”

AdS/CFT dictionary allows us to see “glue clouds” associated with quark/meson states

eg, SUGRA dilaton is dual to \( \langle F^2 \rangle \); so evaluate asymptotic dilaton sourced by particular (macroscopic) string configuration

for “static quark” (infinite straight string):

\[
\langle F^2 \rangle = \frac{1}{8\pi} \sqrt{g_s N} x^4
\]

in agreement with the Wilson loop calculation for the potential between such external quarks

Maldecena; Rey & Yee
**Holographic Mesons**

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→ add $k$ D7-branes to AdS background

\[ r = \infty \quad \rightarrow \quad D7 \]

\[ r = L \]

\[ r = 0 \quad \text{horizon} \]

\[ m_q = \frac{L}{2\pi \alpha'} \]

**Holography:**

for finite mass “quark” (straight string ending on D7):

\[ \langle F^2 \rangle_{p.2} = \frac{15}{64 \cdot 6 \pi N} \left( 1 + \frac{5 m^2}{3 \cdot 4 \cdot 2^2} \right) \ldots \]

where \[ M = \frac{L}{R^2} = \frac{\sqrt{\pi m_q}}{\sqrt{g_s N}} \sim m_{gap} \quad \text{for large } \|x\| \]

\[ T_{eff} \sim m_q / \sqrt{g_s N} \sim \sqrt{g_N m_{gap}^2} \]
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Profile A:

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Holographic Mesons

**Holographic Mesons:**

![Graph A](image1)

**Profile B:**

![Profile B](image2)
Holographic Mesons

Holographic Mesons:

Profile C:
Holographic Mesons
Conclusions:

Flavor physics is fun!

New directions:

- Lower dimensions:
  - new test of gauge/gravity duality
    (work in progress)
- Include back-reaction of D7-branes
  (Burrington et al; Erdmenger and Kirsch)
- Adding flavors to more interesting theories
  (Babington et al; Kruczenski et al; Evans & Schock; Nunez, Paredes & Ramallo; Sakai & Sonnenschein; . . .)
Holographic Mesons

Large-N SUSY gauge theories with DLCQ
in lower dimensions!

\( d=2 \quad N=(4,4) \quad U(N) \) SYM coupled to fundamental matter
dual to \( \rightarrow \) D5-brane probes in D1-background

\( r = \infty \): strong curvature

\( r = L \): low-J meson physics

\( r = 0 \): strong coupling

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• Adding flavors to more interesting theories
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Nunez, Paredes & Ramallo; Sakai & Sonnenschein; . . . )

\( \rightarrow \) See you at the discussion session!