Planar Negative Refractive Index Metamaterials Based on L-C Loaded Transmission Lines

George V. Eleftheriades
A. Iyer, A. Grbic, O. Siddiqui

Electromagnetics Group
Dept. of Electrical and Computer Engineering
The University of Toronto

METAMATERIALS

META="BEYOND" IN GREEK
Materials with unusual properties, not encountered in nature

TYPE#1:
ARTIFICIAL DIELECTRICS SYNTHESIZED BY PERIODICALLY LOADING A HOST TRANSMISSION-LINE MEDIUM WITH R,L,C ELEMENTS (lumped or distributed):
PERIODICITY << 1.

TYPE#2:
DIELECTRIC OR METALLIC PHOTONIC-BANDGAP-MATERIALS (PBGs) FOR WHICH PERIODICITY ~ 1.
Planar Negative Refractive Index Metamaterials Based on L-C Loaded Transmission Lines

**BACKGROUND:** \( \varepsilon < 0 \) AND \( \mu < 0 \)

**METAMATERIALS**

Veselago, 1967

\( \varepsilon > 0, \mu > 0 \)  

\( \varepsilon < 0, \mu < 0 \)  

**Backward Waves**

Regular Materials (right-handed)  

Left-Handed Materials  

\[ n = -\sqrt{\varepsilon \mu} \]

**Negative Refraction**

**Negative Refractive Index Media**
Focusing from Planar n<0 Slabs

- Flat but homogeneous lens
- Point-to-point focusing
- Unlike any other lens it offer sub-wavelength resolution!

Pendry 2000

HOW CAN ONE MAKE $\varepsilon<0$ AND $\mu<0$ METAMATERIALS?

3-D Arrangement of Split-Ring Resonators (SRR) and Straight Wires

- Bulky: 3-D structure
- Distributed cells: Large for usage at RF frequencies
- Operates around resonances: Narrowband
- Unit cells not connected; Difficult System Integration

R. A. Shelby, D.R. Smith, S. Schultz
Science, 2001 Demonstrated Negative Refraction at Microwave Frequencies
RECONCEPTUALIZING $\varepsilon < 0$ AND $\mu < 0$

**METAMATERIALS**

Start from the transmission line representation of normal dielectrics:

$$j\omega L = j\omega X \Rightarrow \mu = \frac{L}{\Delta S}$$

$$j\omega C = j\omega B \Rightarrow \varepsilon = \frac{C}{\Delta S}$$

**How to synthesize $\varepsilon < 0$, $\mu < 0$?**

Simply: Make the series reactance $X$ and shunt susceptance $B$ both negative!

$$j\omega L = j\omega X \Rightarrow \mu = \frac{L}{\Delta S}$$

$$j\omega C = j\omega B \Rightarrow \varepsilon = \frac{C}{\Delta S}$$

Planar Negative Refractive Index Metamaterials Based on L-C Loaded Transmission Lines

CONTINUOUS LIMIT

\[ \beta = \frac{1}{\omega \sqrt{LC}} \]

This practically yields a very large bandwidth over which \( n < 0 \)

PERIODICALLY L-C LOADED TRANSMISSION LINES

Backward Wave \( v_p v_g < 0 \)

For short interconnecting lines \( kd << 1 \) and small phase-shifts per-unit-cell \( \beta d << 1 \)

\[ \varepsilon_{\text{eff}} \equiv \varepsilon - \frac{1}{\varepsilon} \]
\[ \mu_{\text{eff}} \equiv \mu - \frac{1}{\mu} \]

Finite LHM 2-D Unit Cell Perioded

\[ f_{\text{series}} = \frac{\pi}{L} \sqrt{\mu \varepsilon} \]
\[ f_{\text{shunt}} = \frac{1}{L} \sqrt{\mu \varepsilon} \]
\[ f_{\text{Bragg}} = \frac{\pi}{L} \sqrt{\mu \varepsilon} \]
Negative Refraction

Such “Dual” L-C Networks Support Backward Waves

Explaining Negative Refraction (Key is phase matching)

Case 1: \( n > 0 \)

Case 2: \( n < 0 \)

2-D Microstrip Implementation of \( \varepsilon < 0 \) AND \( \mu < 0 \) Metamaterials

The blue wires represent inductors, and the gaps capacitors

2-D (coplanar) propagation

The gaps can be loaded with chip capacitors and the vias with chip inductors to lower the operating frequency.
**ADVANTAGES/UNIQUE FEATURES**

- Low profile 2-D operation
- Connected unit cells/Easy system integration
- Broadband $n<0$ bandwidth
  
  By inserting lumped L-C elements, the operating frequency can be lowered for RF applications/Scalability
  
  By using variable L-C elements and/or switches, controllable materials can be synthesized/Tunability

**SIMULATION RESULTS**

**Negative Refraction**

- Microwave circuit simulation
- Negative refraction observed at interface
- Snell’s Law verified for $n<0 \sin\theta_{\text{RH}}/\sin\theta_{\text{LHM}} = n_{\text{LHM}}/n_{\text{RH}} < 0$
2-D (coplanar) Focusing

\[ n_1 > 0 \quad n_2 > 0 \]

\[ n_1 > 0 \quad n_2 < 0 \]

Focusing (Details)

\[ f_1(\theta_{inc}, \theta_{refr}) = \sin \theta_{inc} \cos \theta_{refr} \]

\[ f_2(\theta_{inc}, \theta_{refr}) = \frac{d}{f_1(\theta_{inc}, \theta_{refr})} \]

Planar Focusing

Inherent Aberration

FULL-WAVE MoM SIMULATIONS

Microstrip Implementation: f=1.5 GHz, periodicity=5mm
Substrate: Rogers 6002 (εr=2.94, t=1.55mm)

EXPERIMENTS
A LEAKY BACKFIRE ANTENNA

\[ \cos(\theta) = \frac{c}{v_0} \]

Analogous to Reversed Cherenkov Radiation

Implementation

F=15GHz

\( \lambda = 20\text{mm} \)
Period<\( \lambda/6 \)
Planar Negative Refractive Index Metamaterials Based on L-C Loaded Transmission Lines

Patterns

MoM Simulation

F=15GHz Measured


Focusing

$n_1 > 0$ $n_2 < 0$

Electromagnetics Group

Slide 21

Slide 22
RF-Lens Device

- NRI metamaterial prototype fabricated/Interfaced with a parallel-plate waveguide
- Vertical E-field probed over metamaterial surface
- Scattering parameter data (transmission) collected from 0-3GHz

Full-Wave Simulation/Experiment

- Full-wave (thin-wire MOM) simulation
- Designed for -2.5<n_{REL}<-1.5 @ 1.5 GHz
- Focusing demonstrated
- TM_{z} mode predominant
FURTHER EXPERIMENTAL RESULTS

Measured Phase

- n < 0 observed from 1-2 GHz (broadband)
- Confined focal region near 1.5 GHz (-2.5 < n_{rel} < -1.5)
- 15dB distinction between peak and edges over an area of 4cm x 3cm, \( \lambda = 20\) cm
- Focal region recedes to interface as frequency increases (n reduces)
1-D BACKWARD-WAVE LINE MEASUREMENTS

Transmission Loss: -1.7 dB

1-D LHM Line (11x5mm cells)

Gaussian Pulse Propagation in a Negative Refractive Index Medium supporting Anomalous Dispersion

\[ v_p > 0, v_g > 0, \quad n > 0 \]
\[ v_p < 0, v_g < 0, \quad n < 0 \]
CONCLUSIONS

By loading planar transmission-line networks periodically with L-C elements, a new generation of NRI metamaterials can be implemented

Low profile 2-D operation
No SRR resonators/Broadband $n<0$ bandwidth
Connected unit cells/Easy system integration
By inserting lumped L-C elements, the operating frequency can be lowered for RF applications/Scalability

Note: Provisional patents filed, May 2002