Photon Emission from QGP

What is the emission rate $\frac{dI_y}{d^3k}$?

(Over) simplifications:

Equilibrium plasma

Very hot - $\alpha_s(T) \ll 1$

On-shell hard photon - $k \sim T$

Leading order in $\alpha_s$ only
(neglect $1/a_0/T$, $m_q/T$)

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Kapusta, Lichard, Seibert
& Boier, Nakagawa, Niagawa, Redlich:

\[ \frac{dG}{dk} = \frac{2}{3\pi^2} \alpha_s^2 \frac{1}{k} \left[ \frac{\ln T}{m_\omega} + \frac{1}{2} \ln \frac{2k}{T} + C_{2\omega_2}(k/T) \right] \]

\( m_\omega = \text{asymptotic thermal quark mass} = g_s^2 T / \sqrt{3} \)

\( \lim_{k/T \to 0} C_{2\omega_2}(k/T) = -0.361 \ldots \)

\[ g \sim g T \] soft + collinear enhancements
\( \Theta \sim \Theta \] compensate extra explicit \( \alpha_s \)
\( \Theta \sim \Theta \] compensation. Extra explicit \( \alpha_s \)

virtuality \( SE = O(g_s^2 T) \Rightarrow \) photon formation time \( \Rightarrow O(g_s g T) \)

But: mean free time \( \left[ \text{for } O(g_T) \text{ collisions} \right] \) is also \( O(g_s g T) \).

\[ \vdots \] Multiple scattering during photon emission is important.
Leading order calculation requires complete treatment of LPM effect = interference among multiple collisions

Ex: \[ \text{Re} \left( \begin{array}{c} \text{\includegraphics[width=0.2\textwidth]{diagram1}} \end{array} \right) \]

Complications:
- Frequency dependent soft scattering
- Non static scattering centers
- Non-Abelian gluon interactions
- Sensitivity to non-perturbative \[ O(g^2T) \] interactions \[ [AGZ] \]

Diagrammatic analysis

\[
\frac{d\Gamma_k}{\beta_k} = \frac{1}{(2\pi)^3 2k} \epsilon^\mu_k (k) \epsilon^\nu_k (k) \left< J_\mu (k) J_\nu (k) \right>
\]

\[ = \sum \begin{array}{c} \text{\includegraphics[width=0.3\textwidth]{diagram2}} \end{array} \]

Detailed power counting of real time thermal diagrams

- All ladder diagrams with HTL resummed propagators contribute
- Crossed ladders, vertex corrections, ... do not contribute
- \[ g \ll gT \] exchanges cancel \[ g \ll gT \] self-energies

\[ \text{Leading order emission rate insensitive to non-perturbative physics} \]
Sum ladders ⇒ linear integral equation

\[ \frac{d\Gamma_{\mu}}{d^2k} = \frac{2}{\pi^2} \int \frac{d^2p_+}{2\pi^2} \frac{d^2p_-}{2\pi^2} n_s(p_+ + k) \left[ 1 - n_s(p_-) \right] \]

\[ \cdot \frac{1}{(2\pi)^2} \left[ \frac{1}{k^2} - \frac{1}{k^2 - m^2} \right] \]

\[ 2 \frac{\vec{p}_+}{\vec{p}_-} \cdot \text{Re} \left( \mathcal{F}(p_+, p_-, k) \right) \]

\[ \mathcal{F}(p_+, p_-, k) = \frac{k}{2} \text{SE} \left( \mathcal{F}(p_+, p_-, k) + \frac{\vec{p}_+}{(2\pi)^2} \right) \]

\[ 2 \frac{\vec{p}_+}{\vec{p}_-} k \left( \frac{p_+^2 + m^2}{2p_+ (k + p_-)} \right) \approx E_{\vec{p}_+} + \frac{1}{k} k - E_{\vec{p}_-} k \]

collision kernel

\[ C(p_{\perp}) = \frac{3}{2\pi} \int d^2q_{\perp} \delta(k - p_{\perp}) \left( A^+(q_{\perp}) A^-(q_{\perp}) \right) \]

soft gauge field variance

\[ \left\langle A^+(q_{\perp}) A^-(q_{\perp}) \right\rangle \approx \frac{m^2 T}{8} \left\{ \frac{2}{g^2 - \frac{q^2 T^2}{T_L^2(q_{\perp})} + \frac{q^2 T^2}{T_T^2(q_{\perp})}} \right\} \]

Amazing sum rule \( C(0_{\perp}) \) ⇒

\[ C(0_{\perp}) \propto \frac{1}{8_{\perp}^2} - \frac{1}{8_{\perp}^2 + m^2} \]

Solve integral egn. using variational formulation
(or convert to local Schrodinger egn in impact parameter)

Results:

\[ \frac{d\Gamma_{\mu}}{d^2k} = \frac{2}{\pi^2} \alpha_s \alpha_s \frac{n_s(k)}{k} \left[ \ln \frac{T}{\Lambda_{\text{QCD}}} + C_{\text{tot}} \left( \frac{k}{T} \right) \right] \]

\[ C_{\text{tot}} \left( \frac{k}{T} \right) = \frac{1}{2} \ln \frac{2k}{T} + C_{\text{brem}} \left( \frac{k}{T} \right) + C_{\text{ann}} \left( \frac{k}{T} \right) \]

domain of validity:

\[ k \gg m_Y = eT/\sqrt{3} \]

near collinear processes:

\( > 50\% \) of emission rate for all \( k \),
bremsstrahlung dominant for \( k \leq 2T \),
collinear annihilation dominant for \( k \geq 10T \).

LPM suppression:

\( \leq 30\% \) effect for \( 2T < k < 10T \),
large effect \( (\sim T^2/k) \) for \( k \leq T \),
large effect \( (\sim \sqrt{k}/T) \) for \( k \geq 20T \).
Generalizations:

- Off shell photons (= dilepton rates)
- straightforward
- need to include longitudinal polarization
- numerical evaluation in progress [Geli, Narce]

Gluon emission

- straightforward generalization of analysis
- "3-way" ladders
- similar linear integral equation
**Fate of a quasi-particle?**

![Quasi-particle](image)

A. **Small angle (soft) scattering**
   \[ g \rightarrow g \]  
   mean free time \( \approx O(1/2g^2T) \)  
   negligible change in momentum  
   big change in color  
   relevant for color conductivity, non-pert.  
   \( \& \)  
   irrelevant for transport of energy, flavor

B. **Large angle (hard) scattering**
   \[ g \rightarrow \gamma \]  
   mean free time \( \approx O(1/2g^2T) \)  
   relevant for transport coefficients

C. **Near collinear fission/fusion**
   \[ g \rightarrow g \]  
   mean free time \( \approx O(1/2g^2T) \)  
   big change in \( 1/p \), negligible change in \( \hat{p} \)  
   relevant for transport coefficients

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**Calculation of transport coefficients**

- viscosity, conductivity, diffusivity

**Leading order evaluation requires**

- complete treatment of both  
- hard + LPM suppressed near-collinear processes

\[ \Rightarrow \] effective kinetic theory with

- \( 1 \rightarrow 2 \) and \( 2 \rightarrow 1 \) processes  
- in addition to usual \( 2 \leftrightarrow 2 \) scatterings.

valid for time scales \( \gamma \gg 1/2g^2T \)  
(not just \( \gamma \gg 1/T \))

explicit evaluation - in progress
Typical diagram for QCD Shear at Leading Order

- Blue: Hard, On-Shell (within $g^2 T$)
- Green: Hard, Off-Shell
- Red: Soft, Spacelike, HTL re-summed
- X: New insertion

[Scalar theory analog]