Differential Rotation in the Sun

(Modeling with the ASH Code)

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Outline

• The Solar Rotation

• Modeling with ASH

• The Deep Convection Zone

• The Upper Shear Layer

• The Tachocline Rotation

• (MASH)

• (CASH)

• (SLASH)
The Solar Rotation

- Latitudinal shear in the envelope but little in the interior
- Vertical shear near the top and bottom of the convection zone
- Angular velocity increasing outward (with slow poles!)
- Smooth and Steady

Where does the Differential Rotation come from?

Assume Lorentz forces and viscous dissipation are negligible:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\rho (\mathbf{v} \cdot \nabla) \mathbf{v} - \nabla P + \rho \nabla \Phi + 2 \rho \mathbf{v} \times \boldsymbol{\Omega}$$

Average the zonal component over longitude and time

(Assume a statistically steady state)

$$\mathbf{L} = r \sin \theta (\Omega r \sin \theta + \langle v_\phi \rangle)$$

$$\nabla \cdot \mathbf{F} = 0$$

$$F_r = \langle \rho v_r \rangle \mathbf{L} + r \sin \theta \left( \langle \rho v_r \rangle - \langle \rho v_r \rangle \langle v_\phi \rangle \right) \left( v_\phi - \langle v_\phi \rangle \right)$$

$$F_\theta = \langle \rho v_\theta \rangle \mathbf{L} + r \sin \theta \left( \langle \rho v_\theta \rangle - \langle \rho v_\theta \rangle \langle v_\phi \rangle \right) \left( v_\phi - \langle v_\phi \rangle \right)$$
Where does the Differential Rotation come from?

- Reynolds stresses vs Meridional Circulation
- Meridional Circulation contribution can also be written as:

\[ \nabla \cdot (\langle \rho v_M \rangle \mathbf{L}) = \langle \rho v_M \rangle \cdot \nabla \mathbf{L} \]

Streamlines = angular momentum contours! Not like the Sun!

- Reynolds stresses (no mystery here!)

Rotation induces systematic velocity correlations in the convection!

What Else Influences the Rotation Profile?

\[ \rho \frac{\partial \mathbf{v}}{\partial t} = -\rho (\mathbf{v} \cdot \nabla) \mathbf{v} - \nabla P + \rho \nabla \Phi + 2\rho \mathbf{v} \times \Omega \]

Take the curl, average over longitude and time (assume steady state)

\[ \nabla \times \langle \mathbf{v} \times (2\Omega + \omega) \rangle = \frac{\nabla \rho \times \nabla P}{\rho^2} \]

Now make the following approximations:

\[ R_o = \frac{\omega_{rms}}{2\Omega} \ll 1 \quad S = C_P \ln \left( \frac{P^{1/\gamma}}{\rho} \right) \quad \nabla P \approx -\rho g \hat{\theta} \]

And you come up with: **Thermal Wind**

\[ \Omega \cdot \nabla \langle u_\phi \rangle = \frac{g}{2\tau C_P} \frac{\partial S}{\partial \theta} \]
Modeling Strategy = Brute Force!

- 3D, Nonlinear, Anelastic fluid equations
  + biggest computers we can find
  = high resolution, low dissipation = turbulence!

- Shave off granulation layer and deep interior for practical reasons

- Investigate turbulent transport
  - Reynolds Stresses
  - Heat Flux

The Anelastic Spherical Harmonic Code

- Anelastic Approximation:
  \[ \nabla \cdot (\hat{\rho} \mathbf{v}) = 0 \]
  \[
  \hat{\rho} \frac{D\mathbf{v}}{Dt} = -\nabla P + \rho g + 2\hat{\rho}(\mathbf{v} \times \Omega) - \nabla \cdot \mathbf{D} - \left[ \nabla \hat{P} - \hat{\rho} g \right]
  \]
  \[
  \hat{\rho} \frac{D}{Dt} \left( \hat{S} + S \right) = \nabla \cdot \left[ \kappa \beta \hat{\mathbf{r}} \nabla (\hat{S} + S) + \kappa_r \beta C_P \nabla (\hat{T} + T) \right] + \Psi
  \]

- Pseudospectral: spherical harmonics and stacked Chebyshevs (or compact FD)
- Poloidal/Toroidal: \[ \hat{\rho} \mathbf{v} = \nabla \times \nabla \times (W \hat{\mathbf{r}}) + \nabla \times (Z \hat{\mathbf{r}}) \]
- Adams-Bashforth/Crank-Nicholson
- FORTRAN 90/MPI
Deep (Shell) Questions

• Can we reproduce the mean flows inferred from helioseismology?
• What should we expect the fluctuating flows to be like?
• What structures dominate the transport?
• How long do they live?
• Can we detect them?
• How important are the boundary layers?
• How are they influenced by rotation, stratification, magnetic field, ionization, etc
• How can all this mess produce a cyclic, large-scale magnetic field?

Differential Rotation
Three Challenges from helioseismology

• Nearly radial angular velocity contours at mid-latitudes (not cylindrical)
• Monotonic decrease in angular velocity from equator to pole (no polar spin-up)
• 30% contrast from equator-to-pole
Rogue’s Gallery
(Brun & Toomre 2002)

- Complexity increases as the viscosity and diffusivity are decreased
- Flows evolve on timescales of days and weeks
- Patterns propagate and are advected by the differential rotation
- The more turbulent simulations are dominated by intermittent downflow plumes and lanes
Keep the thermal diffusivity constant as you decrease the viscosity or you’ll lose your differential rotation!

(Brun & Toomre 2002)

Summary of Deep Shell Results

- Approaching consistency with helioseismic data: definite improvement over the pioneering (laminar) simulations of Gilman and Glatzmaier
- The most turbulent cases generally don’t give the best agreement with helioseismic inversions
- Thermal wind (dS/dtheta) important but not the whole story
- Flows are dominated by strong downflow lanes and plumes which exhibit substantial variation on timescales of weeks and even days
- Still not in the low-dissipation limit: results are sensitive to Reynolds and Prandtl numbers
The Upper Shear Layer

- Why does the radial angular velocity gradient become negative?
- What happens with the meridional circulation?
- What role do supergranules play?
- What other scales of motion are present?
- How do the convective patterns evolve over time and how might they be detected?
- How does this layer couple to the deep convection zone?
- Is this where poloidal field regeneration occurs? (the “alpha-effect”)

Derosa, Toomre & Gilman (2002)
Solar-like differential rotation imposed on the inner boundary with a stress-free top

DeRosa, Toomre, & Gilman (2002)
Radial Angular Velocity Profiles

Red: uniformly rotating
Inner boundary
Blue: Differentially Rotating
Inner boundary

- Negative radial gradient but smaller than what you’d expect from angular momentum conservation

Summary of results from the upper shear layer

- First global simulations to resolve super-granular scale motions
- Larger-scale (100-200 Mm) cells also present which advect and distort “supergranules”
- Flow structure dominated at the top by a rapidly evolving network of downflow lanes and at greater depths by intermittent plumes
- Negative radial angular velocity gradients maintained through an inward angular momentum flux by Reynolds stresses
**Tachocline Questions**

- Why is it so thin?
- Is turbulence generated by either shear instabilities or penetrative convection?
- If so, how does this turbulence feed back on the mean rotation profile?
- What is the dynamical importance of the magnetic field?
- Can we account for the inferred temporal variations?
- How does the tachocline couple to the convection zone?
- What role does it play in the solar dynamo?

**The Solar Tachocline**

- Stably stratified, rapidly rotating
  - Rossby modes (vertical vorticity)
  - Gravity modes (horizontal divergence)
- Differential rotation is maintained primarily by stresses from the overlying convective envelope.
- Why doesn’t the differential rotation spread to the interior?
  - Does turbulence in the tachocline wipe out the latitudinal gradient?

YES: Spiegel & Zahn 1992
NO: Gough & McIntyre 1998

Kitchatinov & Rudiger 1996
ASH Tachocline Model (Boussinesq, Thin-Shell)

\[
\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \nu) + \frac{1}{\sin \theta} \frac{\partial u}{\partial \phi} + \frac{\partial w}{\partial z} = 0
\]

\[
\frac{D \zeta}{Dt} = \left( \zeta + \frac{\cos \theta}{R_o} \right) \frac{\partial w}{\partial z} + \frac{\sin \theta}{R_o} \nu - \frac{\partial u \partial w}{\partial z \partial \theta} + \frac{\partial v}{\partial z} \left( \frac{1}{\sin \theta} \frac{\partial w}{\partial \phi} \right) + R \zeta + \frac{1}{R_e} \nabla^2 \zeta
\]

\[
\frac{\partial \Delta}{\partial t} = -\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left[ \sin \theta \nu \cdot \nabla u - u^2 \cos \theta \right] - \frac{\partial}{\partial \theta} \frac{\partial}{\partial \phi} \left[ \nu \cdot \nabla u + uv \cot \theta \right] - \nabla^2 \Delta
\]

\[
\delta \frac{D w}{Dt} = \frac{u^2 + v^2}{1 + \delta z} = -\frac{1}{\delta \frac{\partial}{\partial z}} + \frac{1}{R_o} \frac{T}{\delta R_e^2} + \frac{1}{R_o} u \sin \theta + \frac{\delta}{R_e} \nabla^2 w
\]

\[
\frac{DT}{Dt} + w = \frac{1}{\sigma R_e} \nabla^2 T
\]

Decaying Turbulence

What happens if we put in a spectrum of random velocity fluctuations and let it go?

Consider both vortex modes (Rossby waves) and horizontally divergent modes (gravity waves)
**Vertical Vorticity**

Unforced, random vortex initial conditions

**Non-Rotating**

QuickTime™ and a GIF decompressor are needed to see this picture.

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**Vertical Vorticity**

Unforced, random vortex initial conditions

**Rapidly Rotating (R_o = 0.1)**

QuickTime™ and a GIF decompressor are needed to see this picture.
Horizontal Divergence
Unforced, random wave initial conditions
**Rapidly Rotating** \((R_o = 0.1)\)

Randomly-Forced Simulations

What happens when you stir things up with random, high-wavenumber external forcing?
(intended to represent penetrative convection)

Consider forcing either the Rossby wave or the gravity wave component of the flow
Vertical Vorticity
Random vortex forcing l=10-12

QuickTime™ and a Video decompressor are needed to see this picture.

Vertical Vorticity
Random vortex forcing l=30-35

QuickTime™ and a Video decompressor are needed to see this picture.
Differential Rotation in the Sun (ITP Solar Magnetism Program 3/06/02)

Average Energy Spectra
Vortex Forcing

l=10-12

l=30-35

How would this turbulence interact with a background shear flow?

- Continue the randomly-forced simulations but now introduce a zonal shear flow
- Maintain this shear flow against viscous dissipation by also introducing a steady, axisymmetric forcing term to the vertical vorticity equation
- The imposed differential rotation is primarily latitudinal but the vertical shear is actually a bit larger due to the thin-shell geometry
- Shear flow kinetic energy comparable to turbulent kinetic energy
- Initially in hydrostatic and geostrophic balance (thermal wind)
Evolution of Differential Rotation Kinetic Energy

- Differential rotation is reduced by the turbulence
- Reduction is most efficient for the larger-scale forcing

Evolution of Angular Momentum Profiles

Dr. Marck Miesch, HAO/NCAR
Summary of Tachocline Results

- Strong coupling between Rossby and gravity wave components when the rotation is strong with equatorward-propagating wave modes
- Nonlinear interactions exhibit both upscale and downscale transfer and the upscale transfer is most efficient when the rotation and stratification are strong
- Randomly forced simulations with imposed shear produce angular momentum transport which is:

  Down-gradient (diffusive) in latitude and Counter-gradient (antidiffusive) in radius

Conclusion

- Where do we stand?
  - Simulations are beginning to look more realistic
  - Helioseismic comparisons are promising but questions remain
  - Tachocline simulations are still in preliminary stages
- Where do we go from here?
  - Still searching for more highly turbulent cases which produce mean flows like the Sun
  - Coupling between the bulk of the convection zone, the upper shear layer, and the tachocline requires much more investigation
  - What role does each play in the solar dynamo?
    - MHD shear instabilities in the tachocline