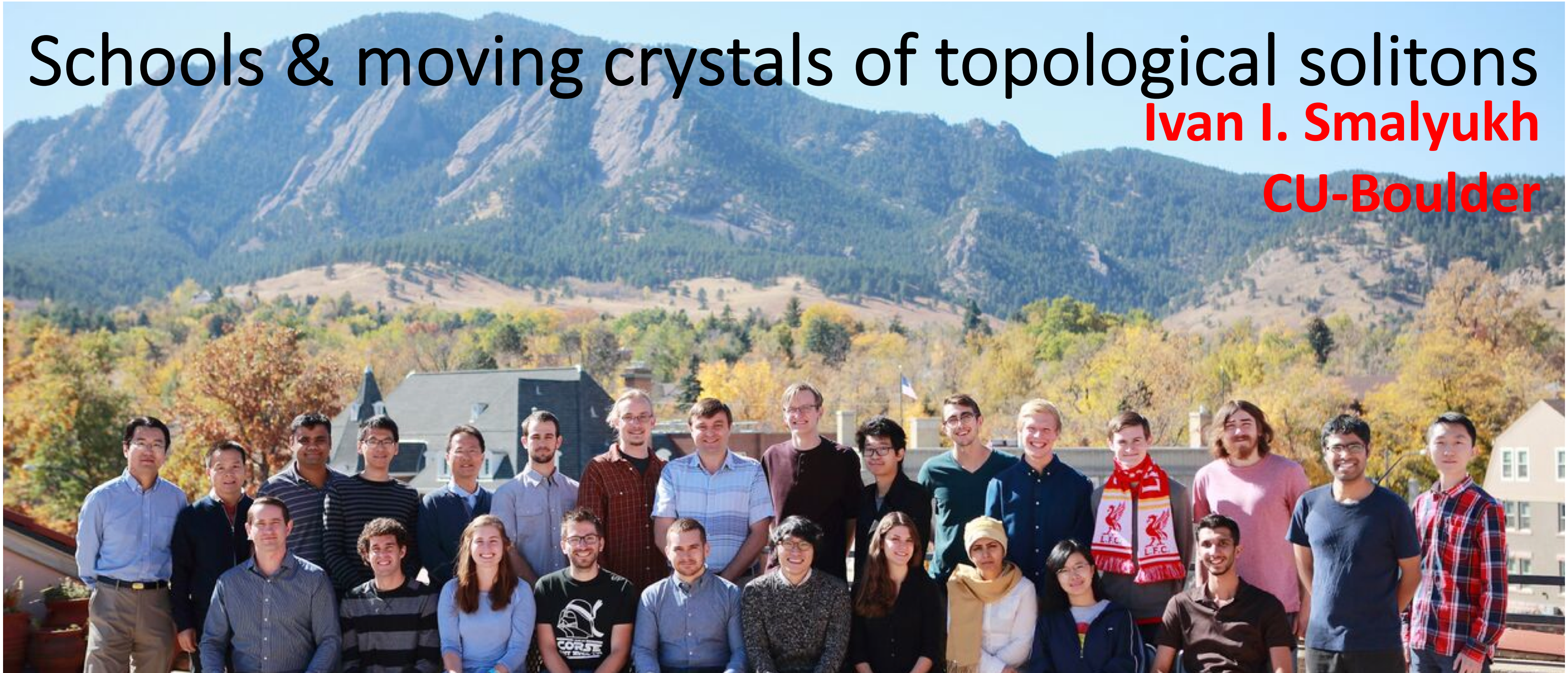
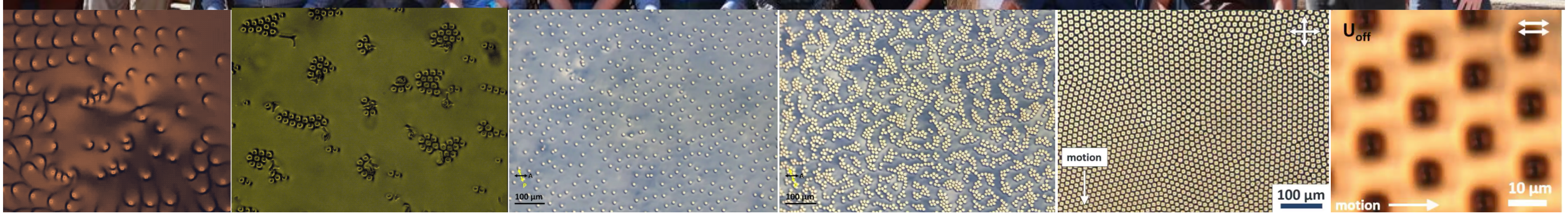
Schools & moving crystals of topological solitons

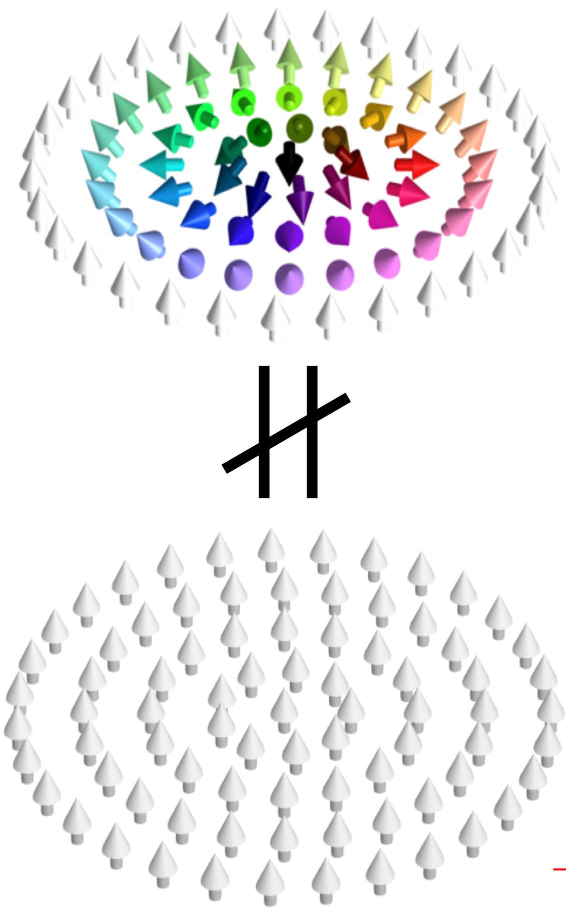
Ivan I. Smalyukh

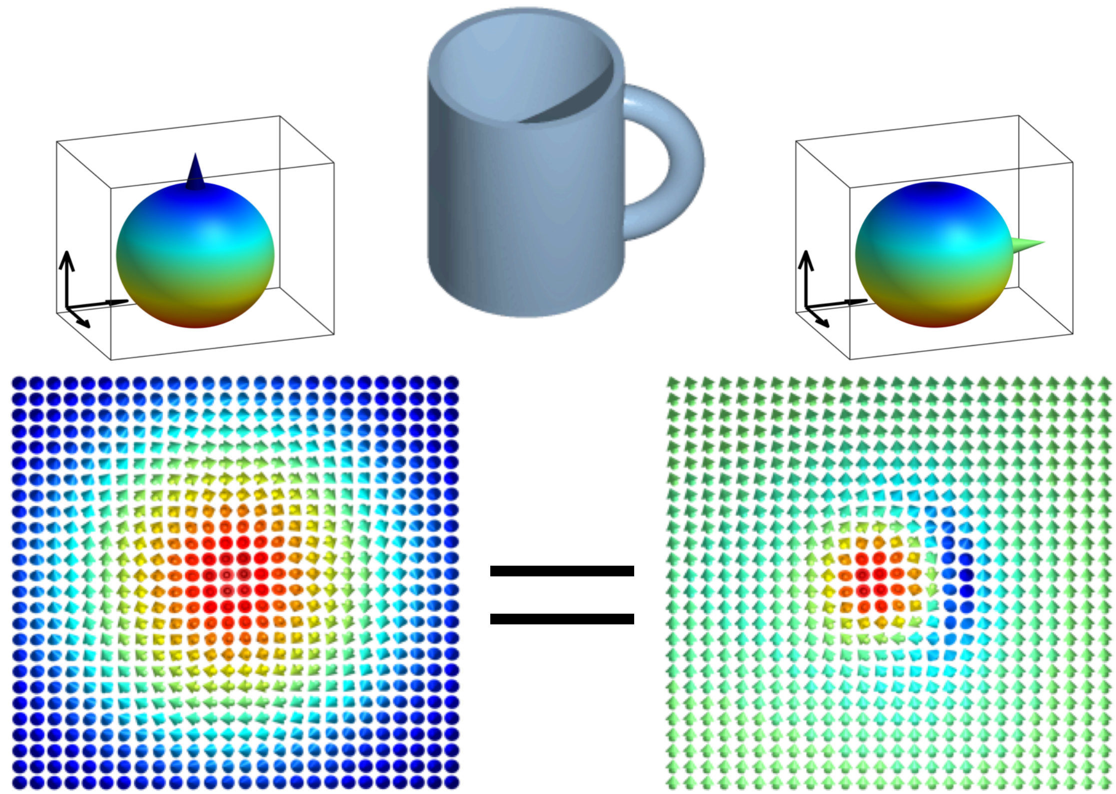
CU-Boulder





Continuous but topologically protected field structures

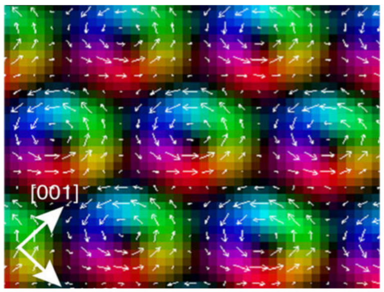




→Like particles, but just topology-protected structures in fields
→Topology-preserving morphing during motions

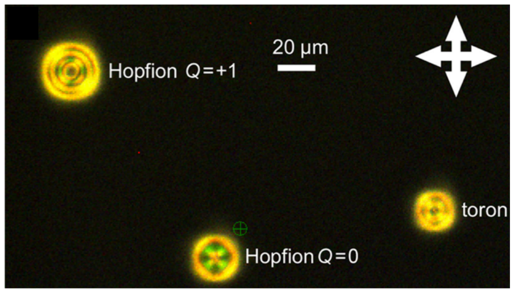
Topological solitons: “baby” skyrmions in condensed matter

- Magnets



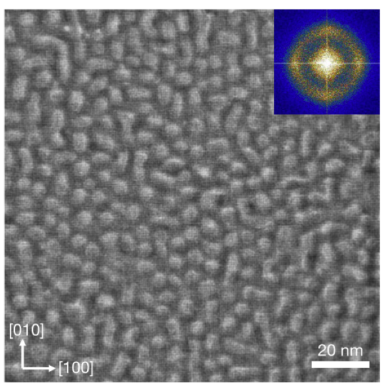
Yu et al., *Nature* **465**, 901-904 (2010)

- Liquid crystals



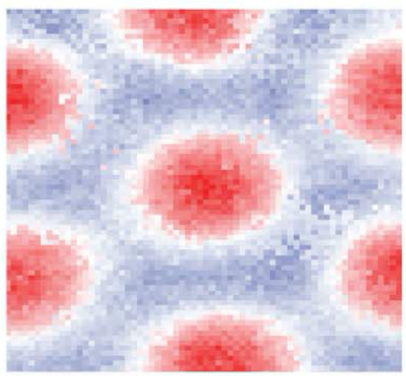
Ackerman & Smalyukh. *PRX* **7**, 011006 (2017)

- Ferroelectrics



Das et al., *Nature* **568**, 368 (2019)

- Electromagnetic fields



Tsesses et al. *Science* **361**, 993 (2018)

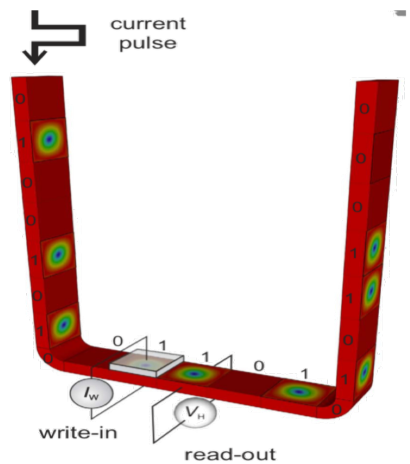
- Fundamental & applied interest

⇨ conservation of the topology

⇨ (meta)stable configurations

⇨ particles/information carriers, spintronics applications

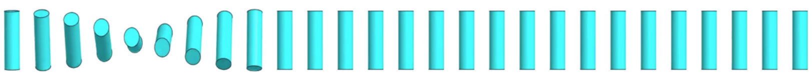
⇨ Topological phases of matter



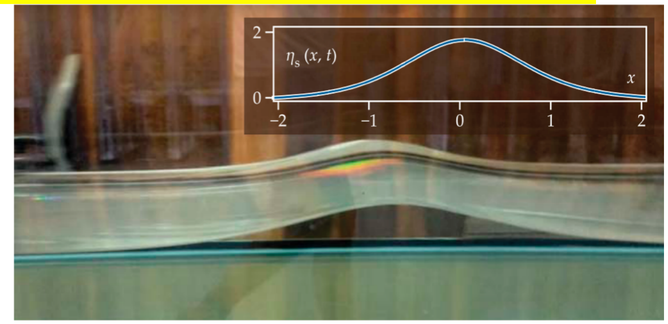
Nagaosa & Tokura. *Nat. Nano*. **8**, 899 (2013)

Waves, solitons & topological twist

Topology-protected 1D nematic twist wall in $\mathbb{R}^1$

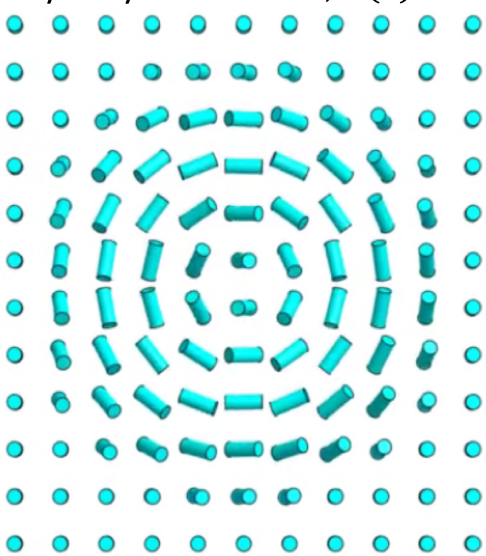


U. Of Maine wave basin



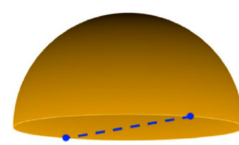
N. Pizzo, "Shallow water wave"; *Phys. Today* 73, 44 (2020)

2D "baby" Skyrmion in $\mathbb{R}^2$; $\mathbf{n}(\mathbf{r})$continuously vectorized

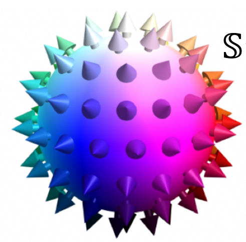


→Can be continuously decorated by a unit vector field
→Cannot be eliminated through continuous deformations

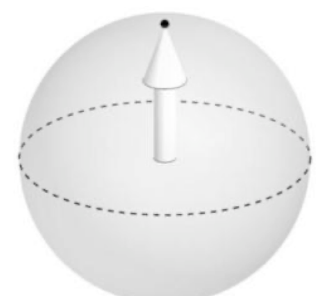
$\mathbb{S}^2/\mathbb{Z}_2$



Configuration space



$\mathbb{S}^2$



→Why "**baby**" Skyrmions in magnets & LCs?

P.J. Ackerman, T. Boyle, I.I. Smalyukh. Squirming motion of baby skyrmions. *Nature Comm* **8**, 673 (2017)

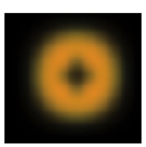
Sphere-sphere maps, homotopy theory & solitons

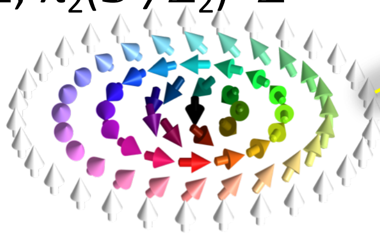
→$\pi_3(\mathbb{S}^2) = \mathbb{Z}$ & $\pi_3(\mathbb{S}^2/\mathbb{Z}_2) = \mathbb{Z}$ (Hopfions)

$\mathbb{R}^1 \cong \mathbb{S}^1$, $\mathbb{R}^2 \cong \mathbb{S}^2$ & $\mathbb{R}^3 \cong \mathbb{S}^3$ when the far-field is uniform

1D wall solitons
$\pi_1(\mathbb{S}^1/\mathbb{Z}_2) = \mathbb{Z}$; $\pi_1(\mathbb{S}^1) = \mathbb{Z}$

2D "baby" skyrmions
$\pi_2(\mathbb{S}^2) = \mathbb{Z}$; $\pi_2(\mathbb{S}^2/\mathbb{Z}_2) = \mathbb{Z}$





$$N_{\text{sk}} = \frac{1}{4\pi}\int dxdy\,\boldsymbol{n}(\boldsymbol{r}) \cdot \left(\partial_x \boldsymbol{n}(\boldsymbol{r}) \times \partial_y \boldsymbol{n}(\boldsymbol{r})\right)$$

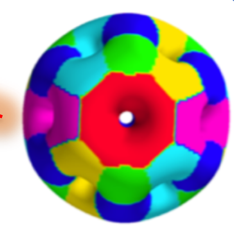
	π_1	π_2	π_3	π_4	π_5
$\mathbb{S}^0$	0	0	0	0	0
$\mathbb{S}^1$	$\mathbb{Z}$	0	0	0	0
$\mathbb{S}^2$	0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\mathbb{S}^3$	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\mathbb{S}^4$	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$
$\mathbb{S}^5$	0	0	0	0	0

$\pi_i(\mathbb{S}^n)$ – "ways" $\mathbb{S}^i$ maps to $\mathbb{S}^n$
$\mathbb{Z}$: classified by integer $Q \in \mathbb{Z}$

T Skyrme

Ed Witten

Skyrme solitons
in high energy
physics $\pi_3(\mathbb{S}^3) = \mathbb{Z}$



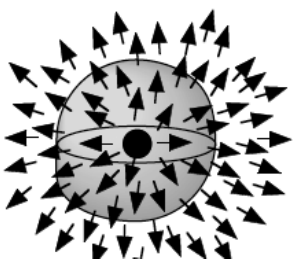
SU(2)

Singular defects, 2D skyrmions & torons

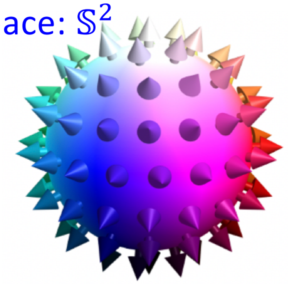
$\boldsymbol{n}(\boldsymbol{r})$: $\boldsymbol{r} \in$ configuration space $\longrightarrow \boldsymbol{n} \in$ OP space (target space)

Point defects in 3D

$\mathbb{S}^2$

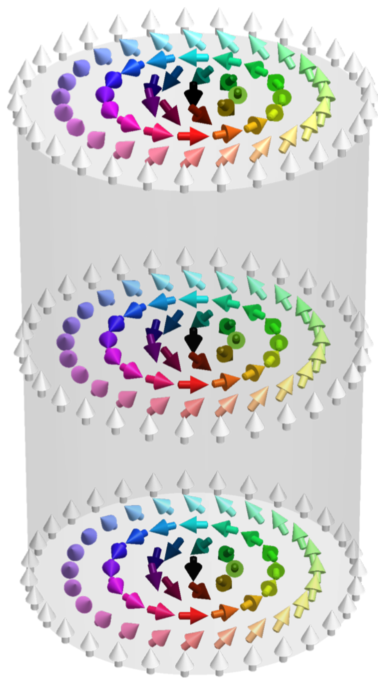


OP space: $\mathbb{S}^2$

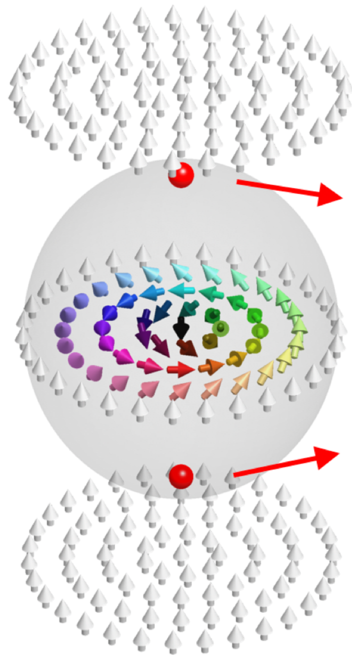


$\pi_2(\mathbb{S}^2) = \mathbb{Z}$

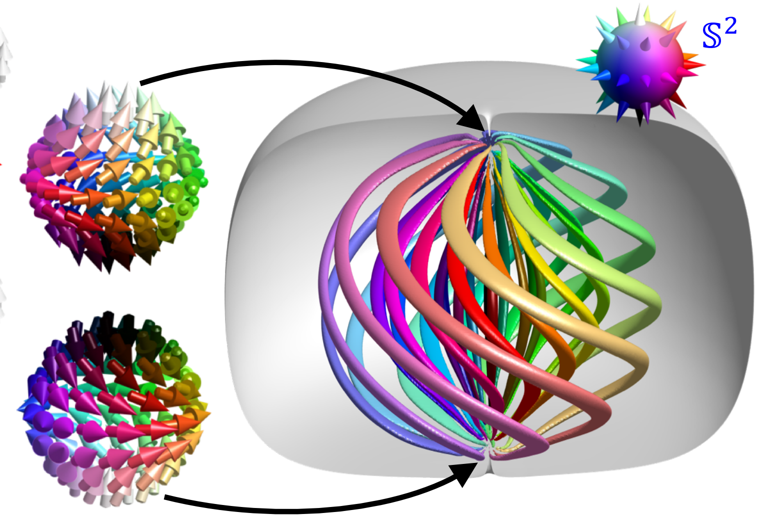
Translationally invariant $\pi_2(\mathbb{S}^2) = \mathbb{Z}$ skyrmion in 3D



Terminating a skyrmion by singular pointy defects



Elementary toron: skyrmion terminated on point defects



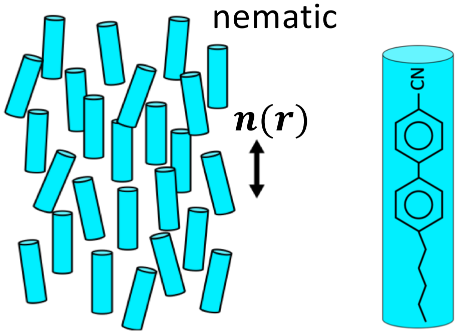
$\pi_2(\mathbb{S}^2) = \mathbb{Z}$

Preimage of $\boldsymbol{n}(\boldsymbol{r})$: region of a constant $\boldsymbol{n}$

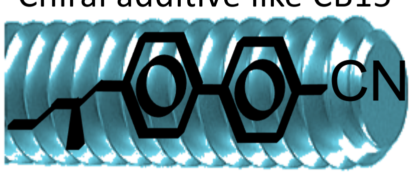
J.-S. B. Tai and I. I. Smalyukh. *Phys Rev E* 191, 042702 (2020)

→Point defects: source & sink, opposite charge

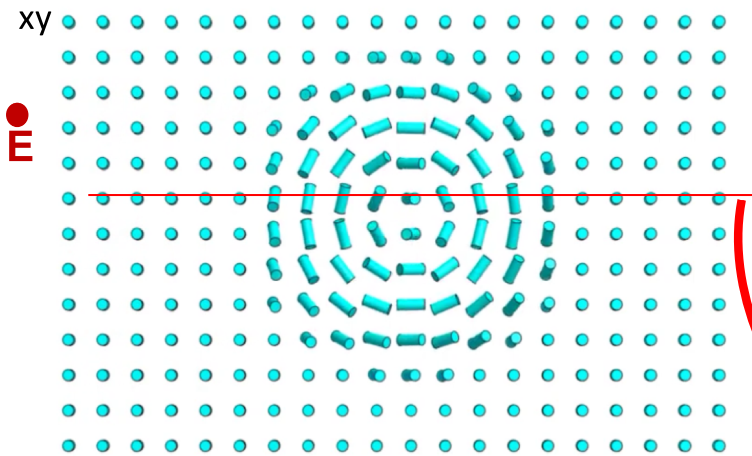
Chiral nematic liquid crystals (LCs) & experiment geometry



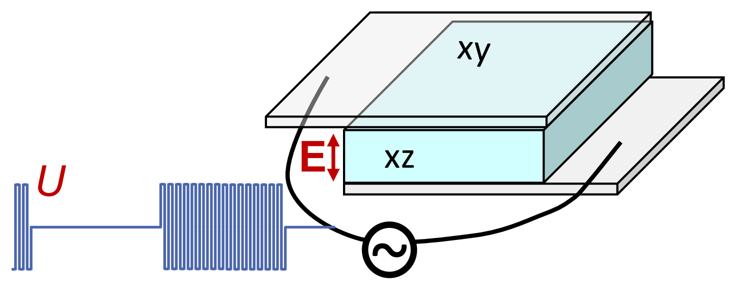
Chiral additive like CB15



Like in displays, TVs, smartphones

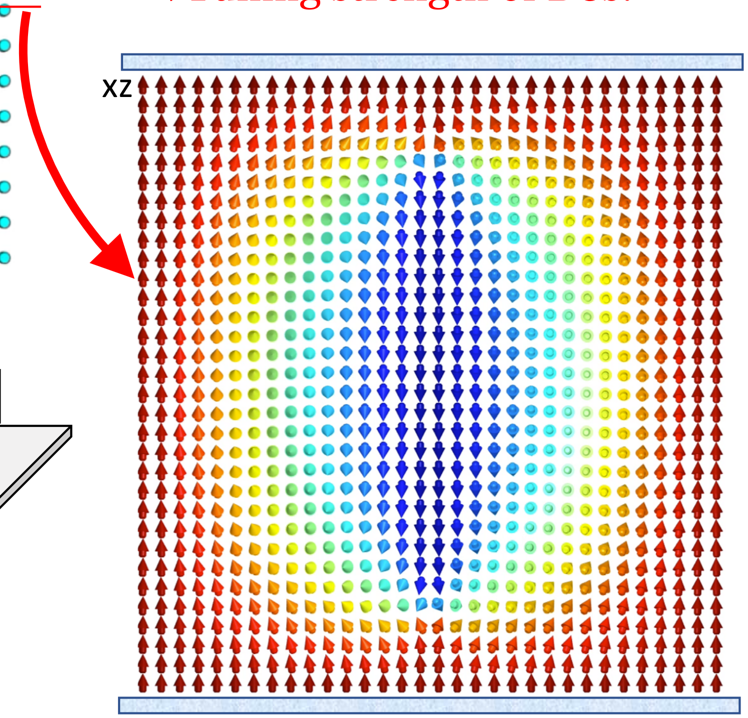


Geometry like in VA displays



→Negative dielectric anisotropy
→Changing surface anchoring W: 10^{-4} to 10^{-6} J/m^2

→ Tendency to twist
→Localized skyrmion & toron structures with twist
→Tuning strength of BCs:



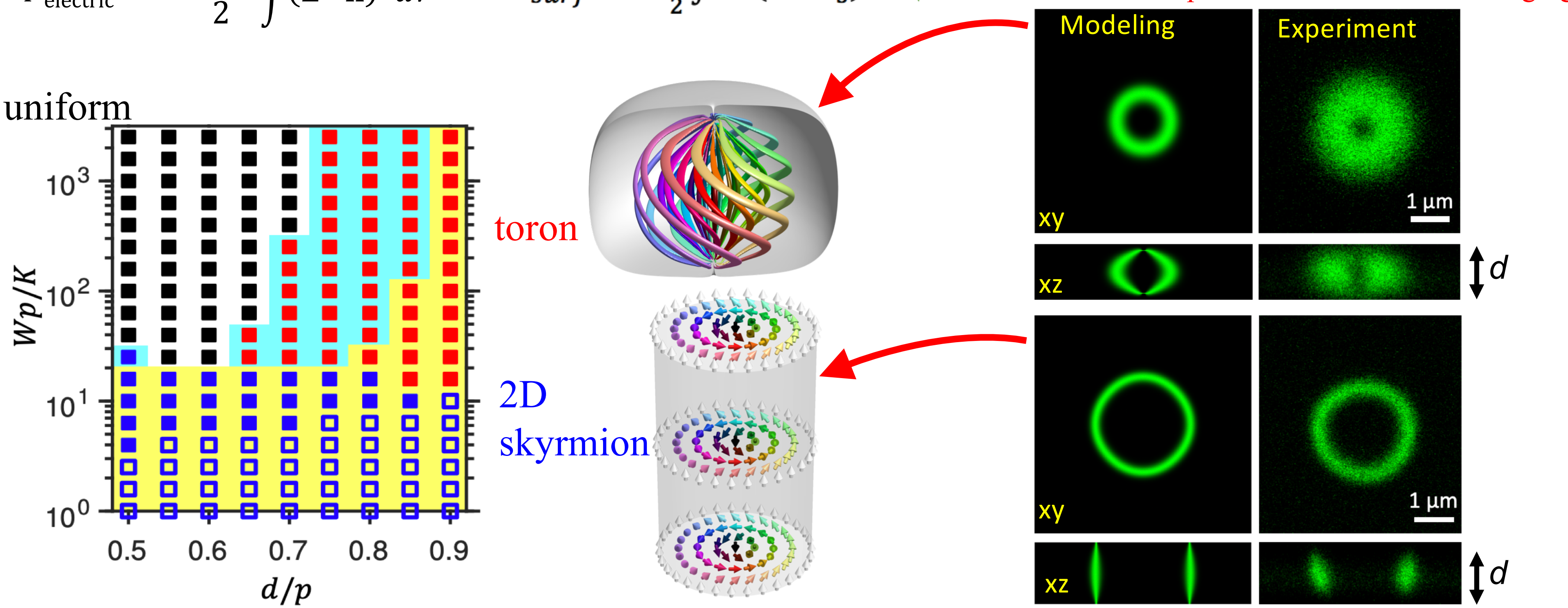
Numerical modeling: skyrmion vs toron stability

→FD based total free energy F_{total} minimization for experimental parameters:

$$F_{elast} = \int \left\{ \frac{K_{11}}{2} (\nabla \cdot \mathbf{n})^2 + \frac{K_{22}}{2} [\mathbf{n} \cdot (\nabla \times \mathbf{n}) + q_0]^2 + \frac{K_{33}}{2} [\mathbf{n} \times (\nabla \times \mathbf{n})]^2 - K_{24} [\nabla \cdot [\mathbf{n}(\nabla \cdot \mathbf{n}) + \mathbf{n} \times (\nabla \times \mathbf{n})]] \right\} dV$$

$$F_{\text{electric}} = -\frac{\varepsilon_0 \Delta\varepsilon}{2} \int (\mathbf{E} \cdot \mathbf{n})^2 dV \qquad F_{surf} = -\frac{1}{2} \int W (\mathbf{n} \cdot \mathbf{n}_s)^2 ds$$

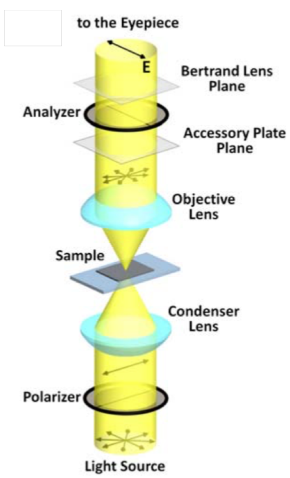
→3D nonlinear polarized luminescence imaging

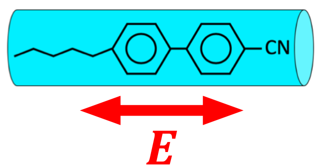


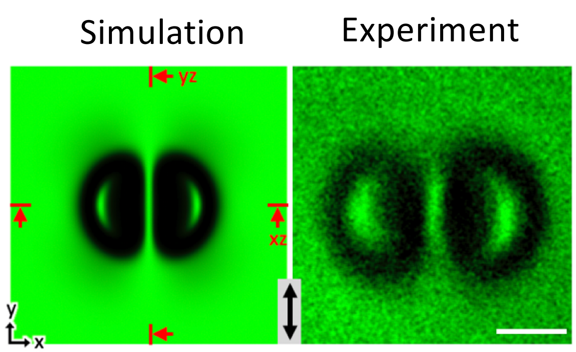
Experimental methods

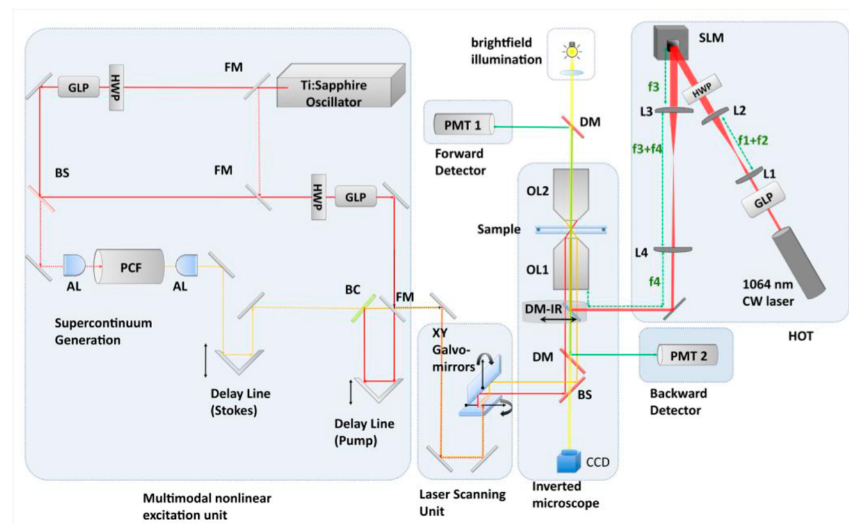
- 3D imaging of director orientation, 3PEF-PM
- Polarizing optical microscopy & videomicroscopy
- Laser tweezers
- Reconstruct & control director fields

POM



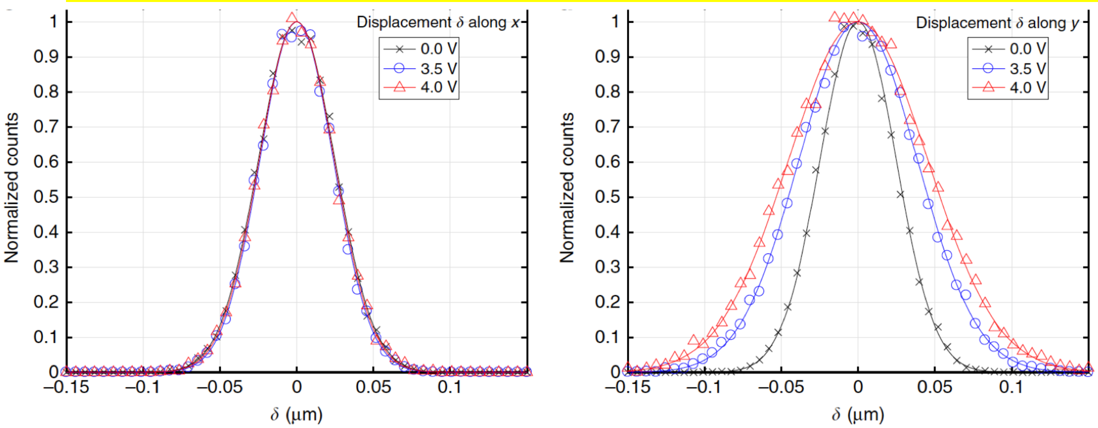






Trivedi & Smalyukh. *Optics Express* **18**, 27658 (2010)

Voltage dependent particlelike Brownian motion & drift

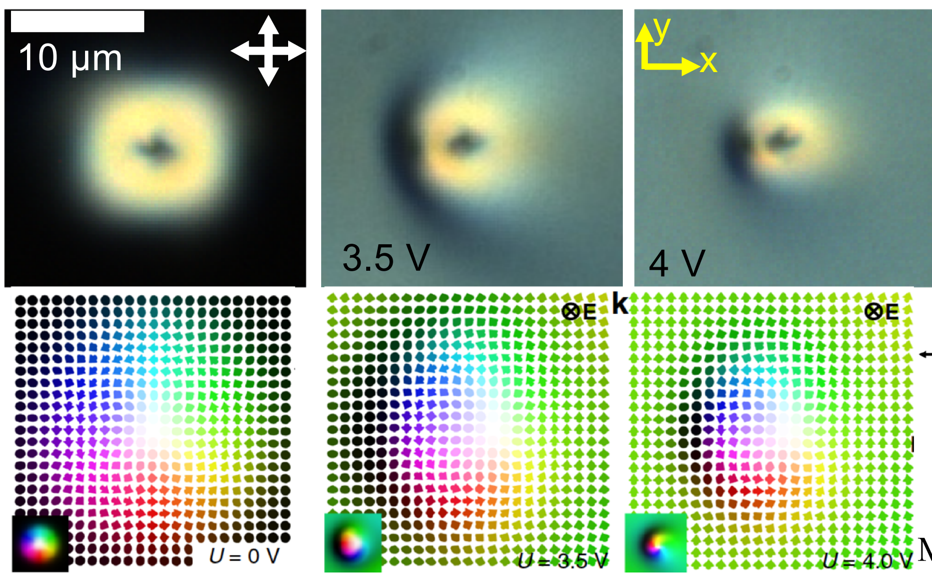


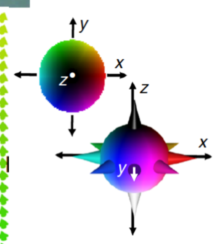
→ Skyrmion diffusion driven by unbalanced orientational thermal fluctuations of **n**(**r**)

→Direction-independent diffusion at U=0

→Lateral diffusivity D & viscous drag versus U:

	Along *x*		**Along *y***	
***U* (V)**	***D* (μm² s⁻¹)**	**ζ (kg s⁻¹)**	***D* (μm² s⁻¹)**	**ζ (kg s⁻¹)**
0.0	0.0187	2.174×10^{-7}	0.0185	2.195×10^{-7}
3.5	0.0172	2.356×10^{-7}	0.0447	9.081×10^{-8}
4.0	0.0180	2.259×10^{-7}	0.0616	6.590×10^{-8}

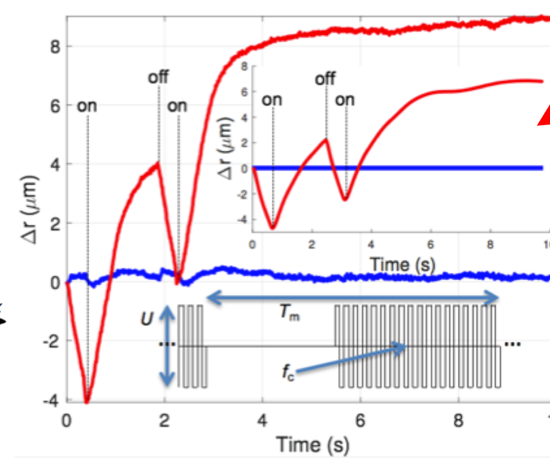




Experiment

Modeling

→ Drift with turning voltage on & off:

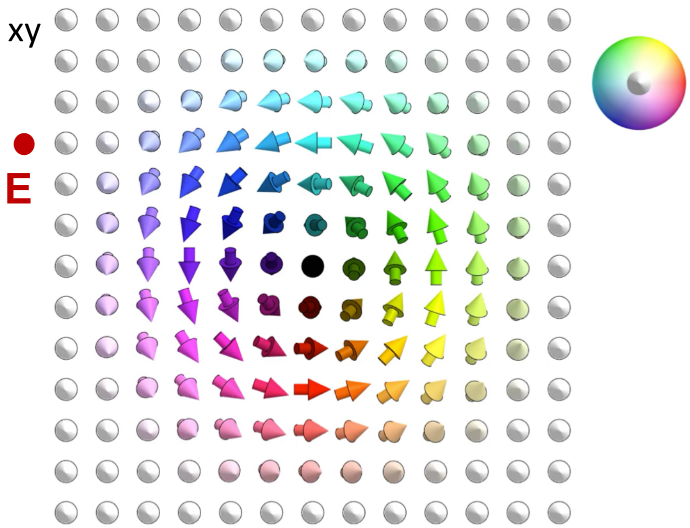


Experiment

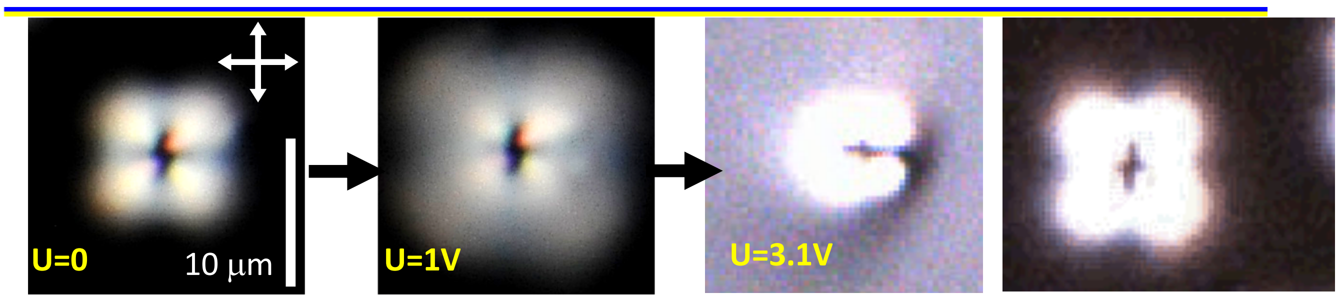
$$\gamma \frac{\partial n_i}{\partial t} = -\frac{\delta F_{total}}{\delta n_i}$$

From temporal evolution computer-simulated

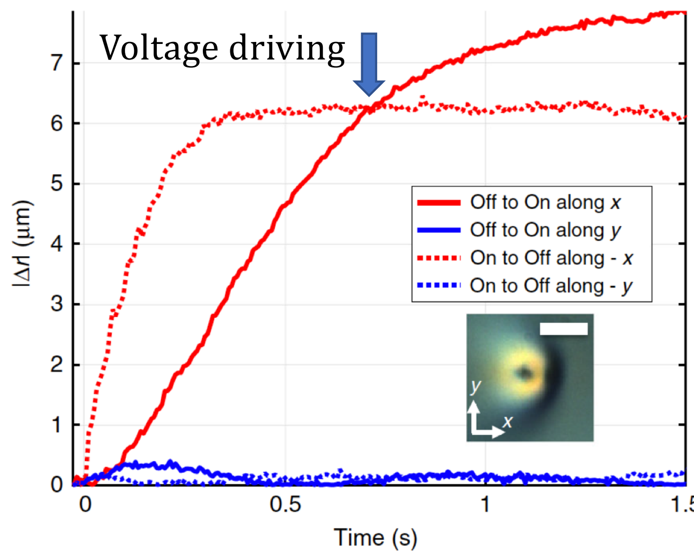
Random axial symmetry breaking & shift during switching



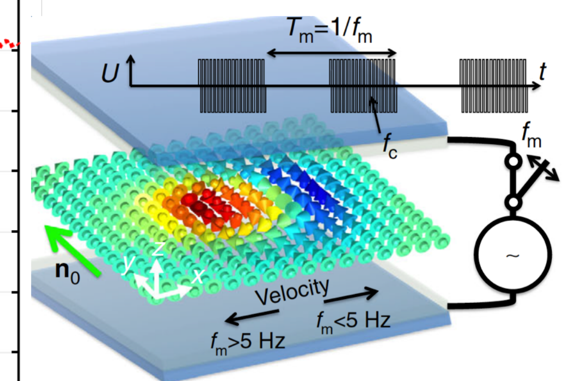
Voltage on & off: topology-preserving



→**n**(**r**,t) rotation not invariant upon reversal of time
→Skyrmion drifts with turning voltage on & off



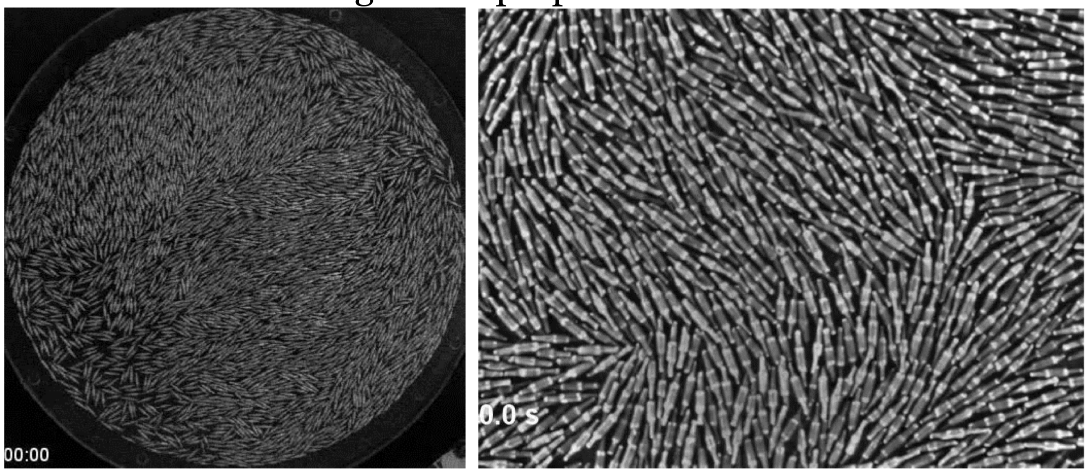
Reversal modulation frequency



→Negative dielectric anisotropy LC
→Director tends to align orthogonally to **E**
→Electric morphing breaks axial symmetry!

Supplying energy to solitons by an oscillating electric field?

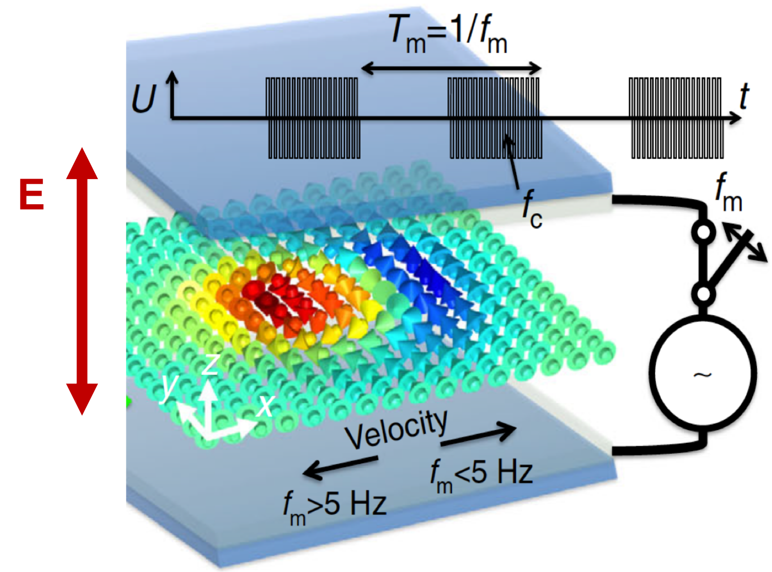
→ Mechanical agitation perpendicular to motions



Narayan, Ramaswamy & Menon, *Science* **317**, 105-108 (2007).

→Skyrmion move in a plane orthogonal to **E**
→Selection of direction of motion is spontaneous
→**A threshold effect from torque competition**
→Only **n**(**r**) within Skyrmion morphs at U=1V<U_c
→Background **n**(**r**) switching at U>U_c>2.5V
→Energy converted to motion by individual skyrmions

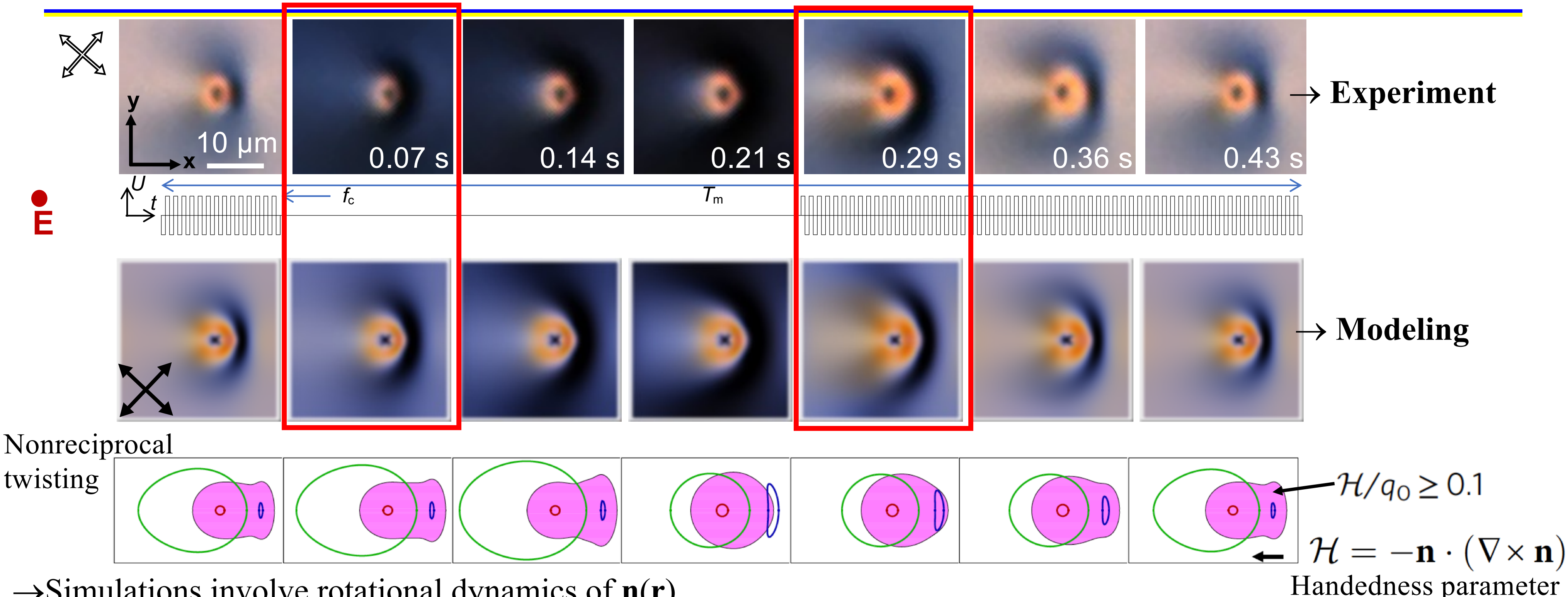
Cell with perpendicular BCs & a skyrmion
VA far-field



Negative dielectric anisotropy LC

$$F_{\text{electric}} = -\frac{\varepsilon_0 \Delta\varepsilon}{2} \int (\mathbf{E} \cdot \mathbf{n})^2 dV$$

Videomicroscopy of squirming motion of “baby” skyrmions



→Simulations involve rotational dynamics of $\mathbf{n}(\mathbf{r})$

→Electric, viscous & elastic torques

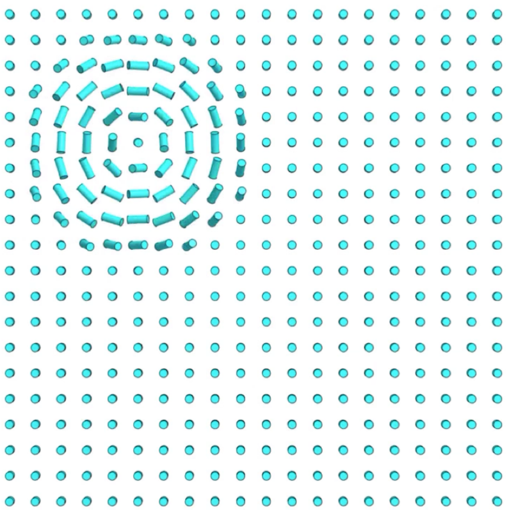
→Asymmetric non-reciprocal topology-preserving soliton transformation

→Locally transform electric energy to motion!

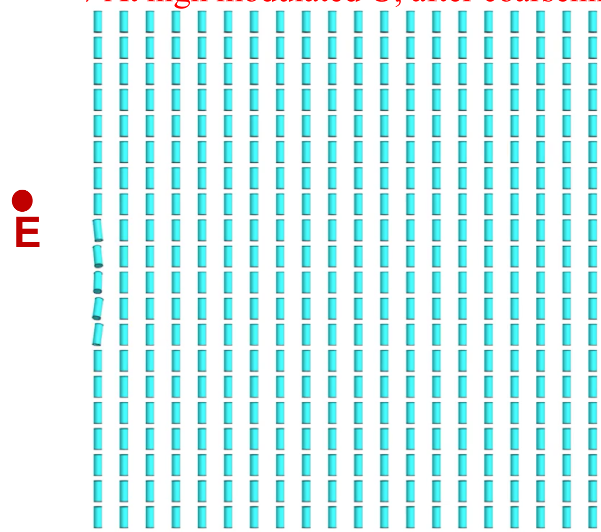
P.J. Ackerman, T. Boyle, I.I. Smalyukh. *Nature Comm* **8**, 673 (2017)

Passive liquid crystal with “activated” solitons

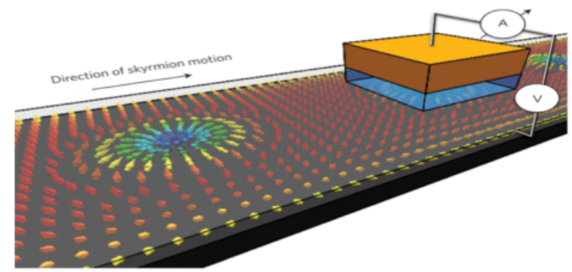
→ At low modulated U

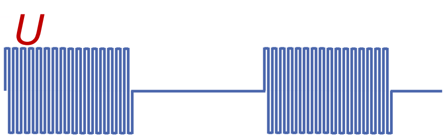


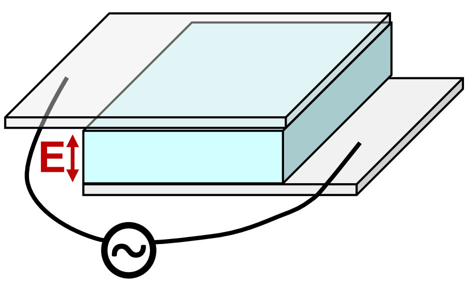
→ At high modulated U, after coarsening



→ Like skyrmions in solid-state racetrack memory







→Skyrmions have no mass, overdamped motion

→Law Ericksen number here

→Could be high?

$$Er = \frac{\mu v L}{K}$$

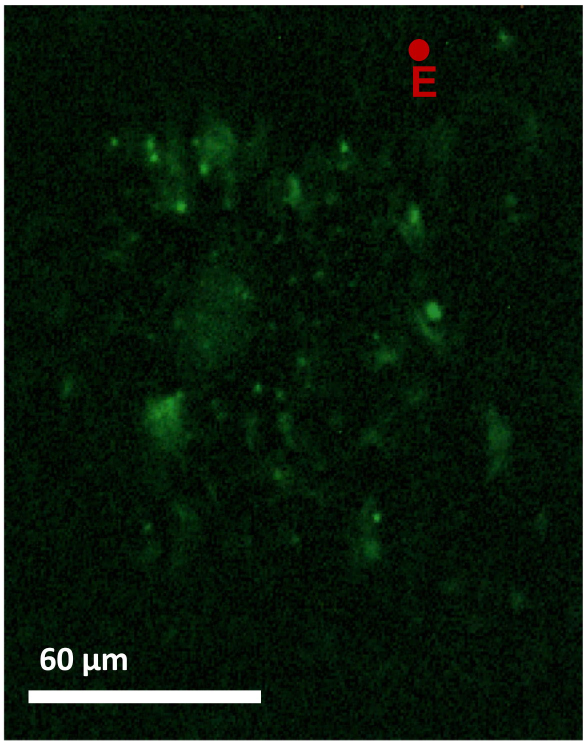
→Local conversion of energy

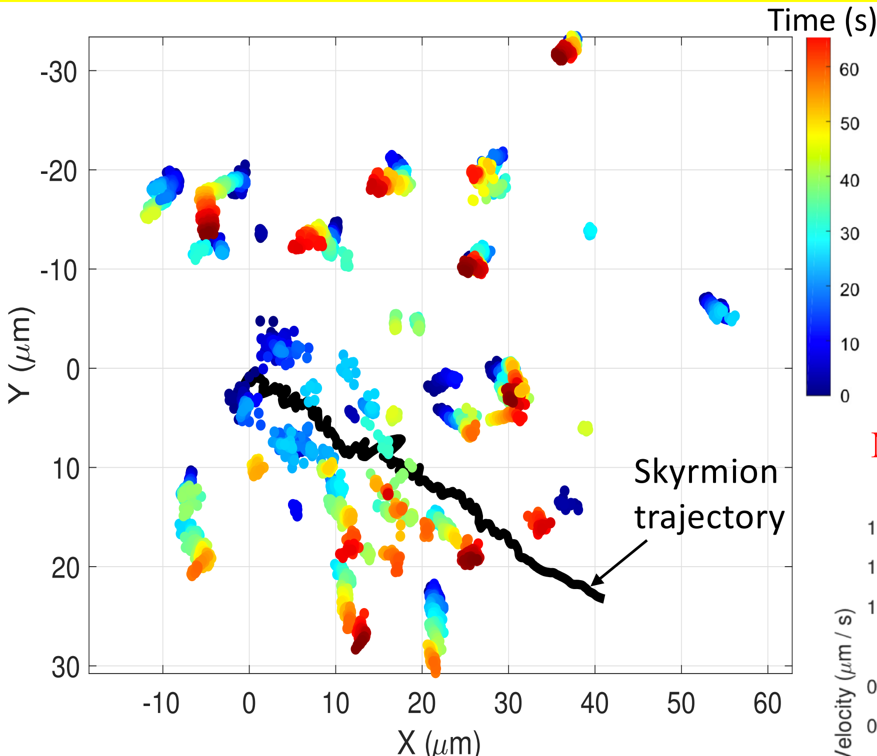
→Rotational dynamics alone can lead to skyrmion translation;

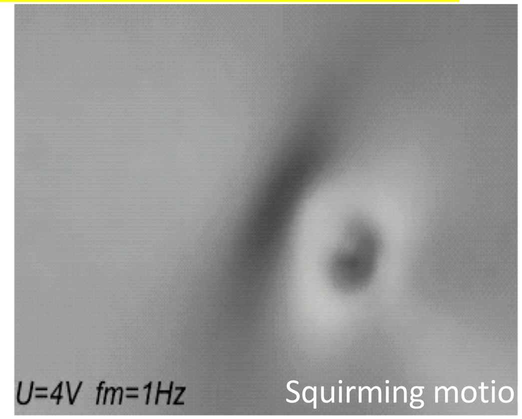
→Skyrmion motion does not rely on flows

→… but are there flows generated when skyrmions move?

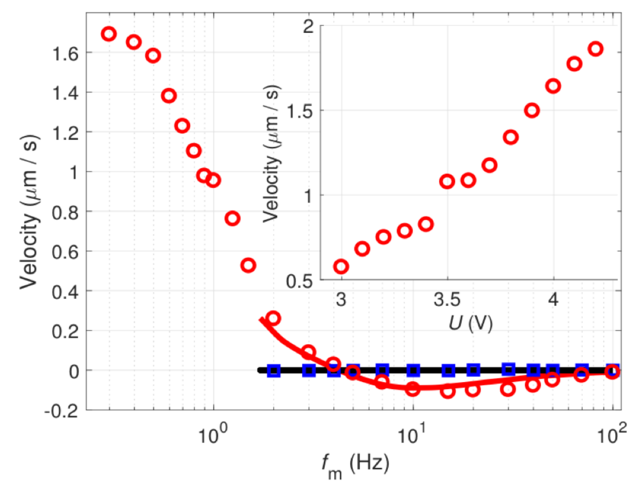
Tracer particles reveal no significant mass transport or flows







Modulation frequency dependence

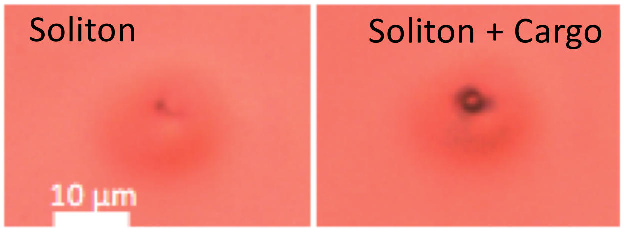


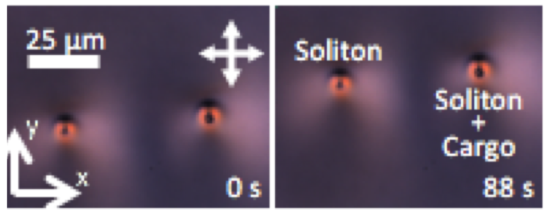
→ Tracer particles: upconversion luminescence nanocubes (R~10nm)
→ Tracers undergo Brownian motion as the solitons move past
→ Backflows induced by director rotations are present but weak

Sohn, Ackerman, Smalyukh. *Phys Rev E* **97**, 052701 (2018).

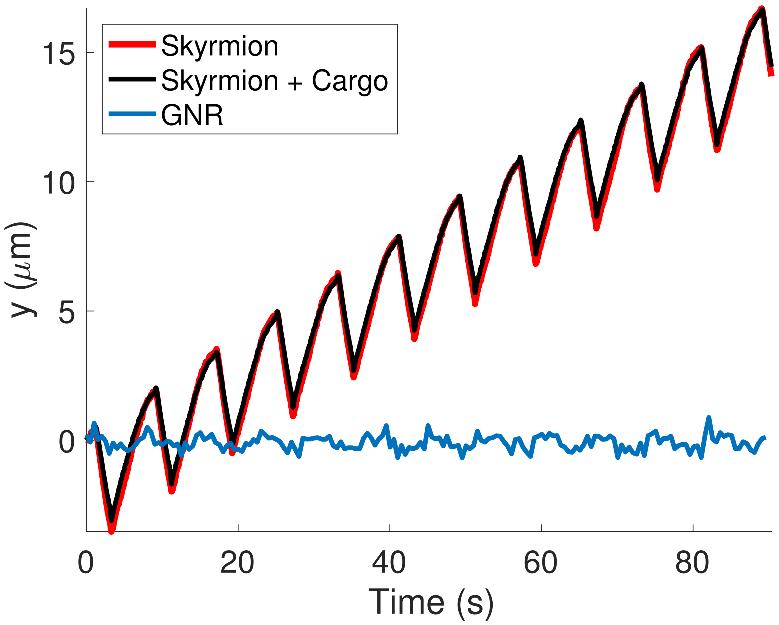
Solitons capable of transporting colloidal microcargo

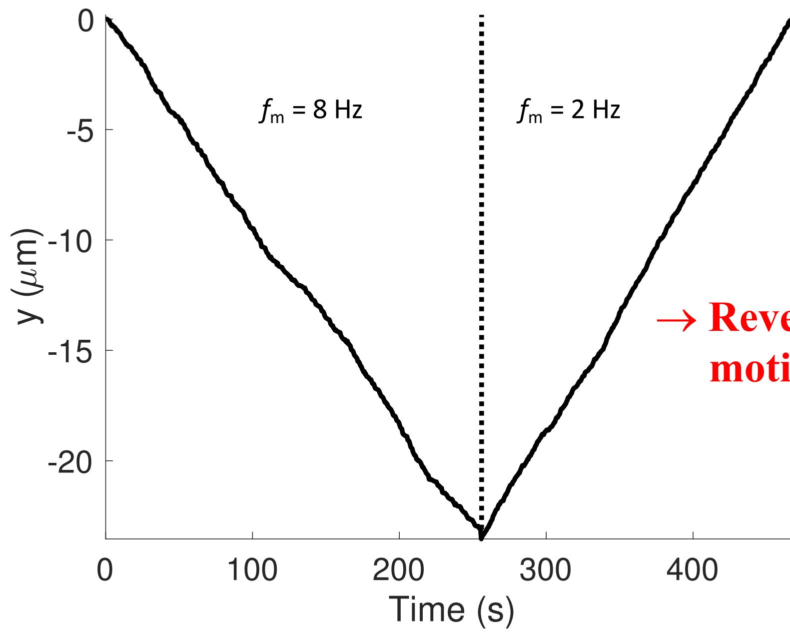
→ **The soliton can entrap & transport a cargo microparticle**





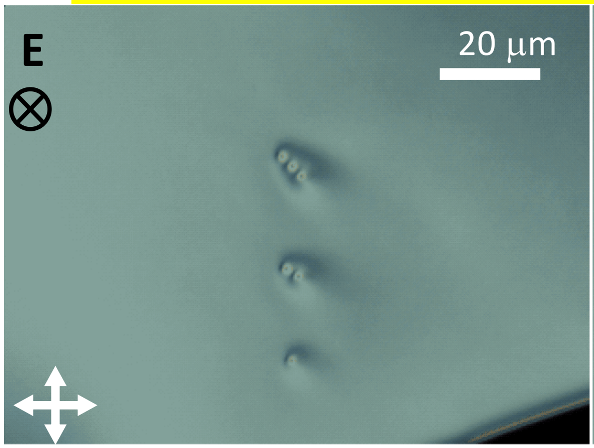
→ **The soliton with cargo moves nearly as fast as without**

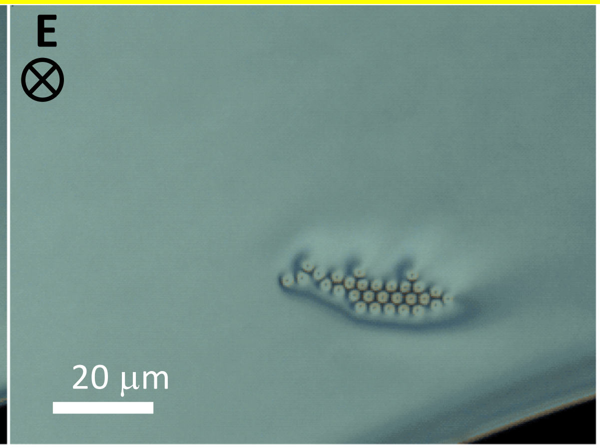


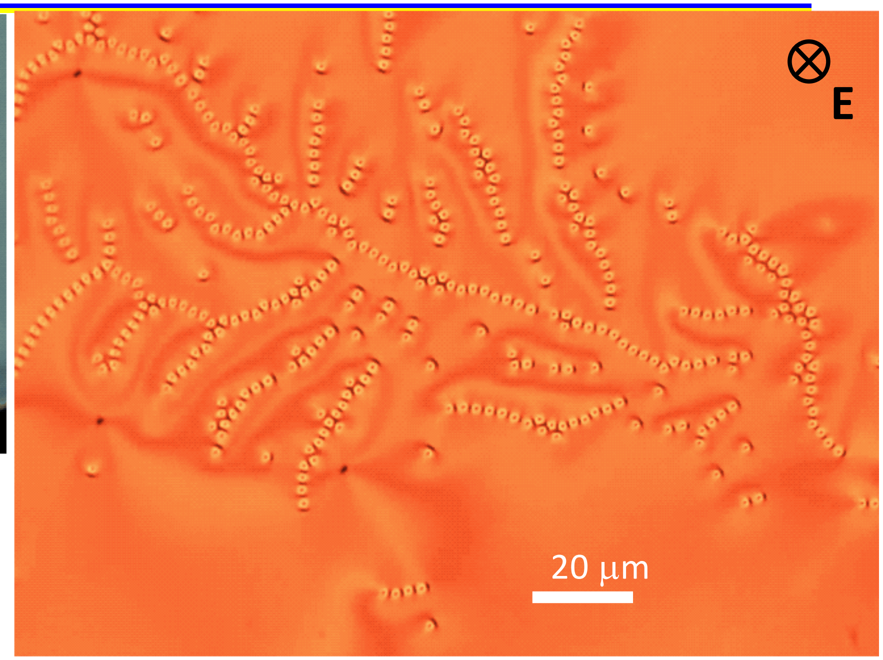


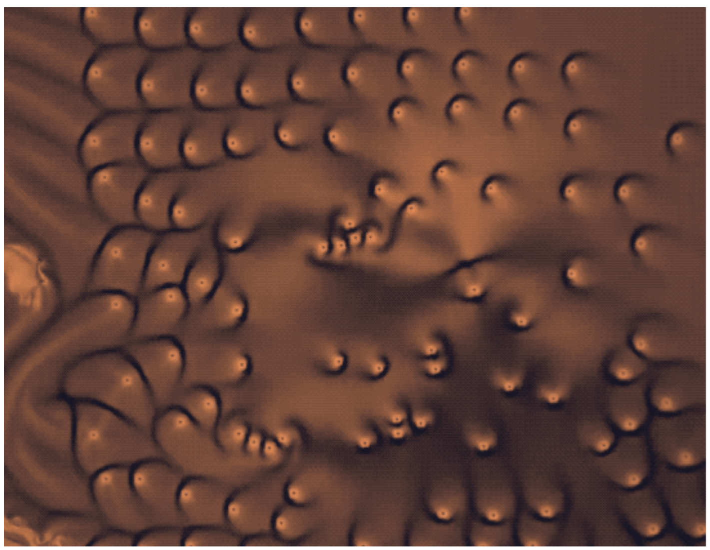
→ **Reverse-directional motion with cargo**

From one to many: energy conversion & collective motions









→Motion directions selected spontaneously, not along **E**
→Electric energy converted to motion on single skyrmion basis
→Out-of-equilibrium elastic interactions in action!

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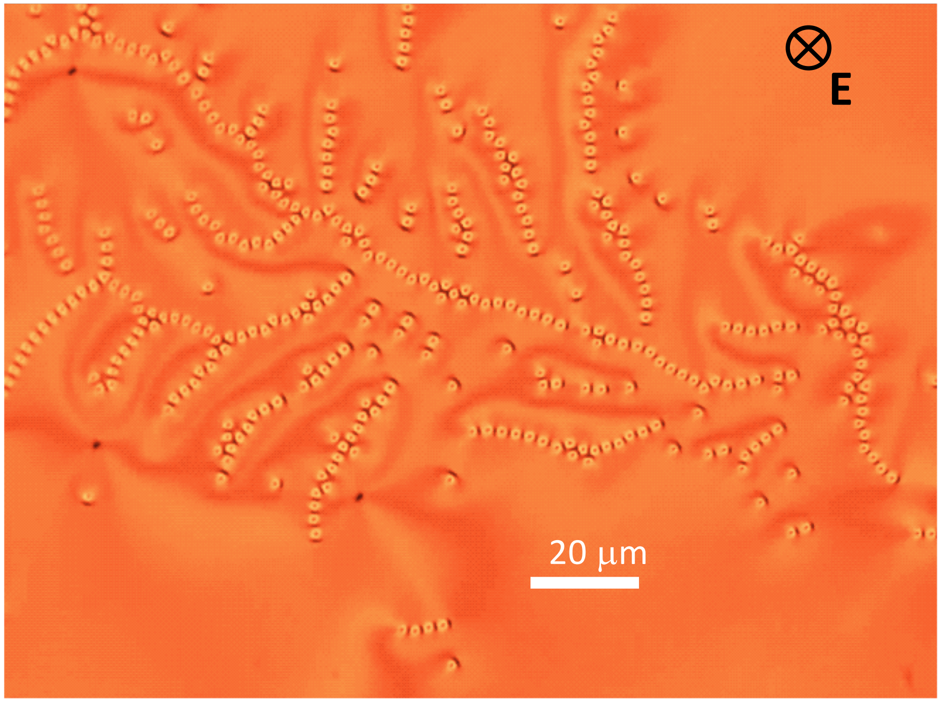
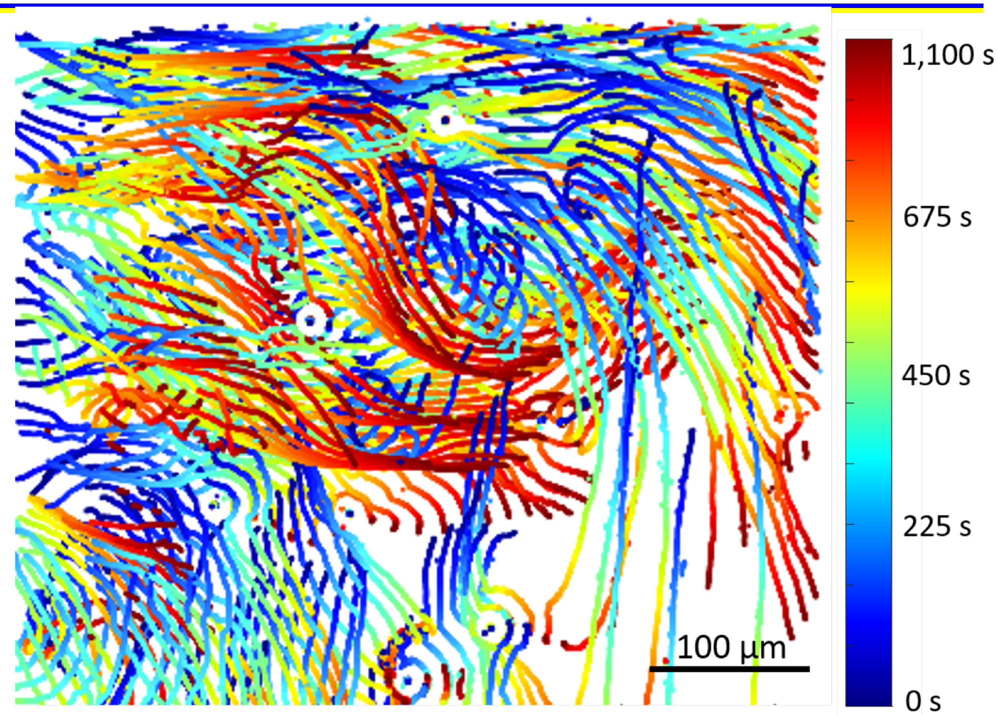
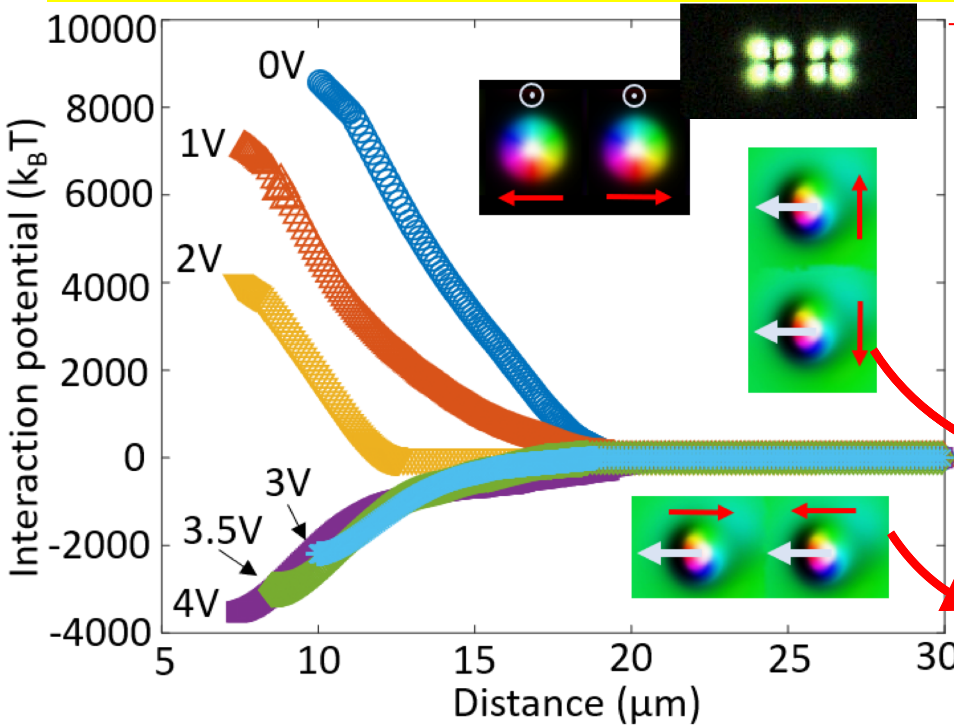
No repetition, emergent dynamics



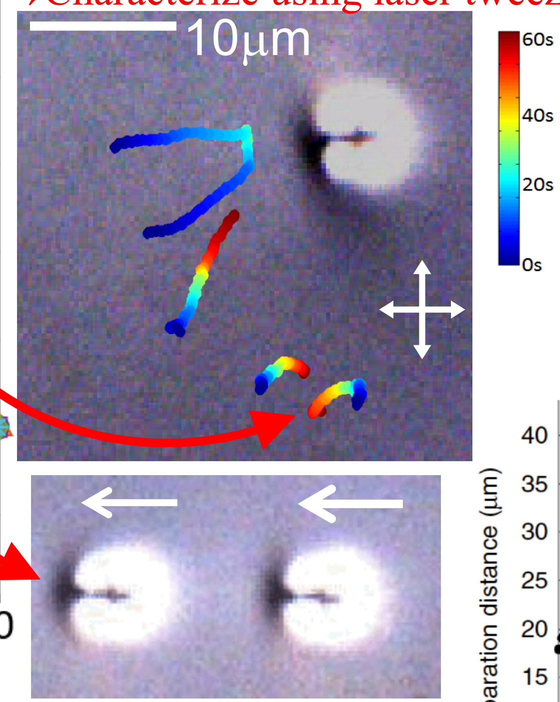


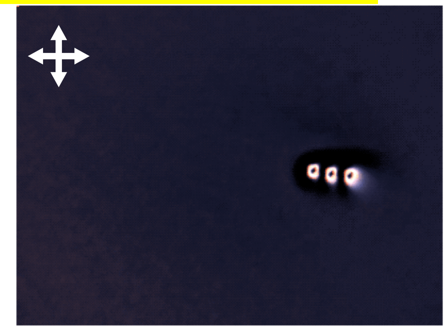
→Highly reconfigurable out-of-equilibrium dynamics
→Vary voltage & frequency to tune behavior
→Emergent phenomenon that relies on inter-skyrmion interactions

Long-range out-of-equilibrium pair interactions

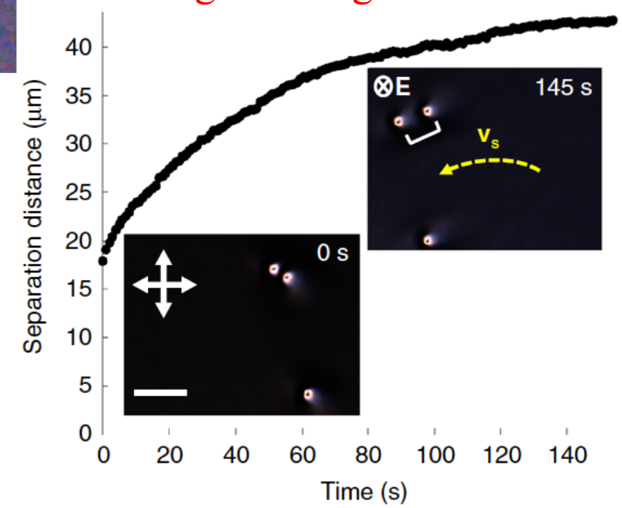


→Characterize using laser tweezers





→Inter-skyrmion distance changes during motion



→Interaction between 2 moving skyrmions
→Elastic dipoles with variable tilt to a 2D plane
→From repulsions to anisotropic attractions

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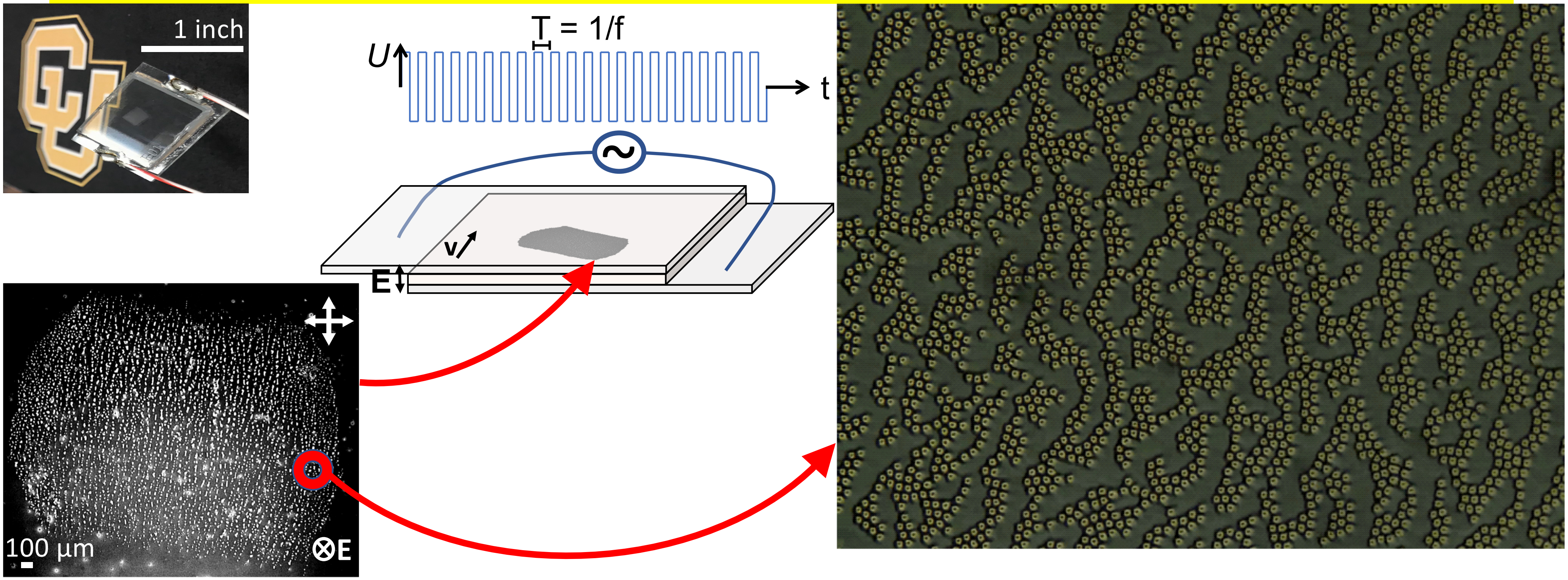
Emergent order: schools with 10^2-10^7 coherently moving skyrmions



→Field applied to entire sample
→At a scale of skyrmions energy transforms to motion
→At large numbers, orderly polar motion emerges from interactions

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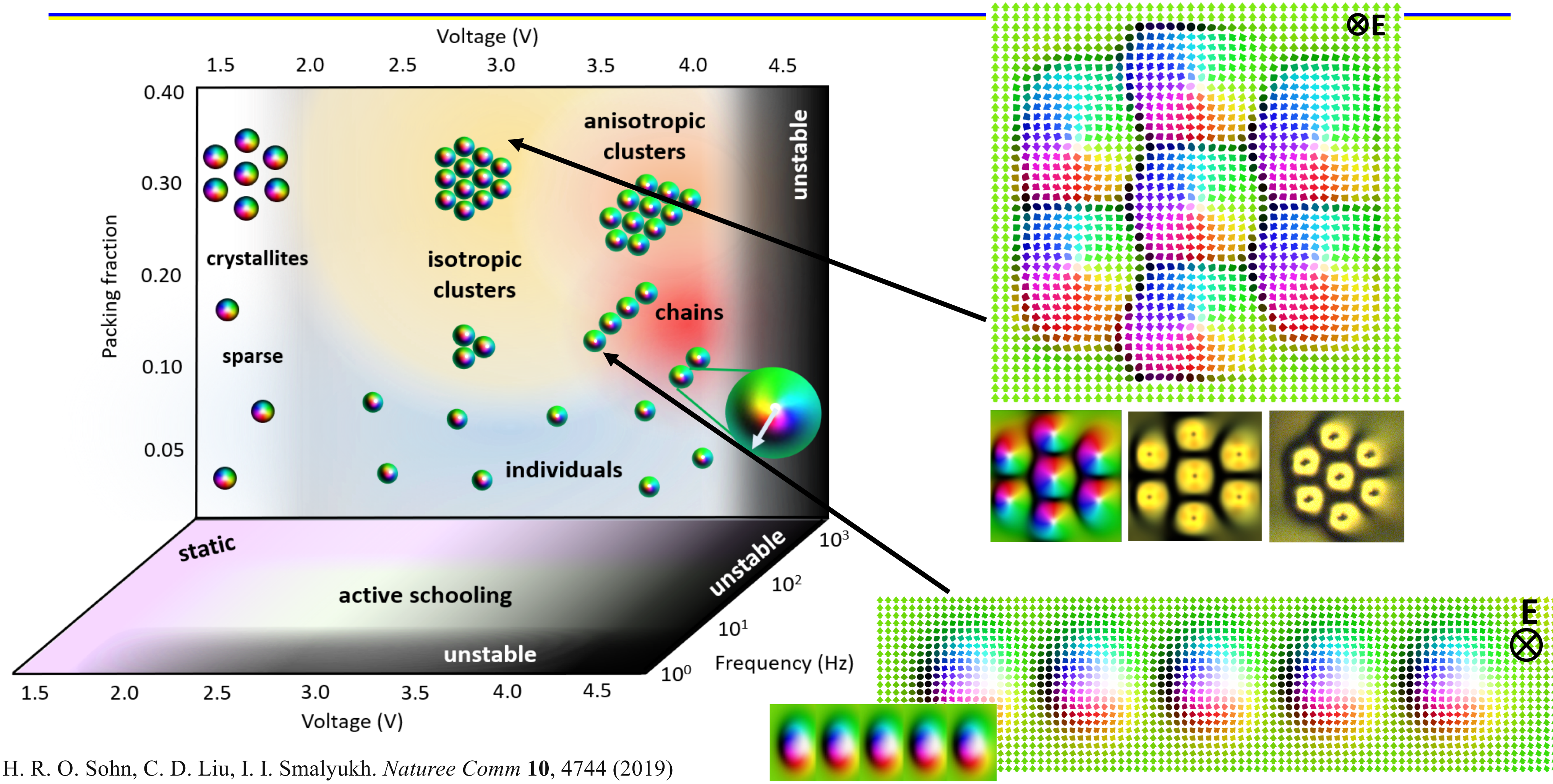
Diagram of dynamic & static states



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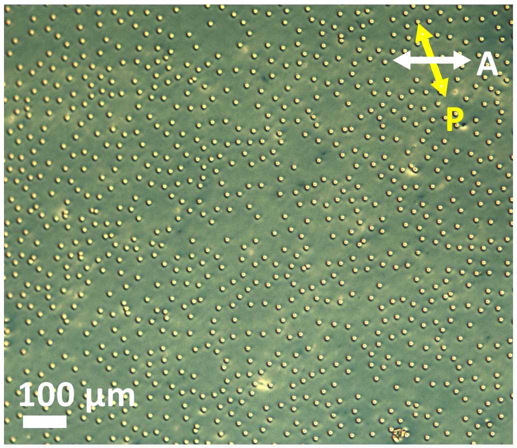
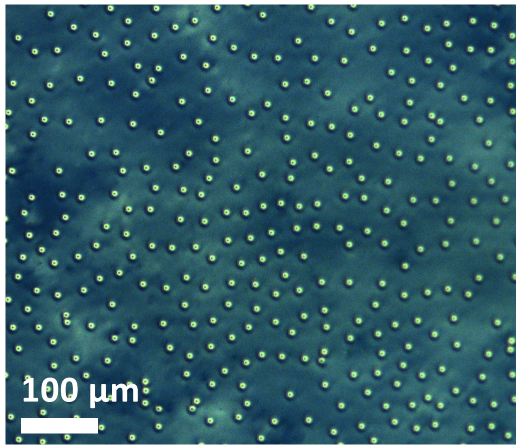
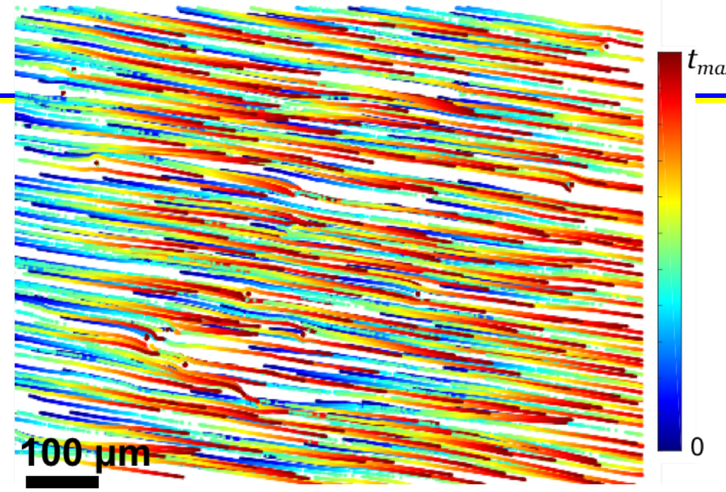
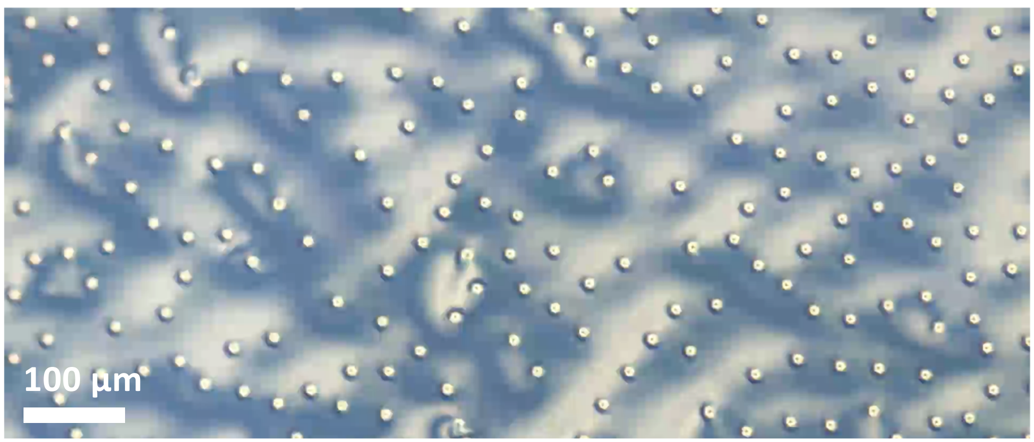
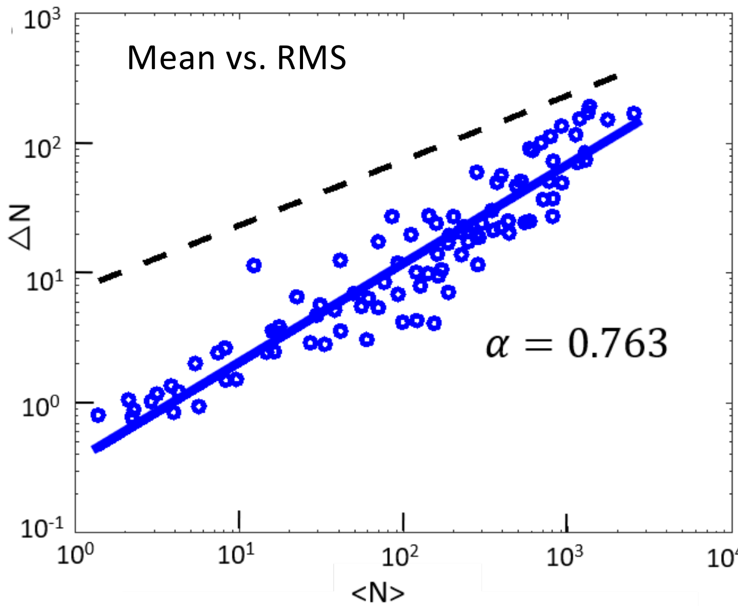
Schools of individual Skyrmions











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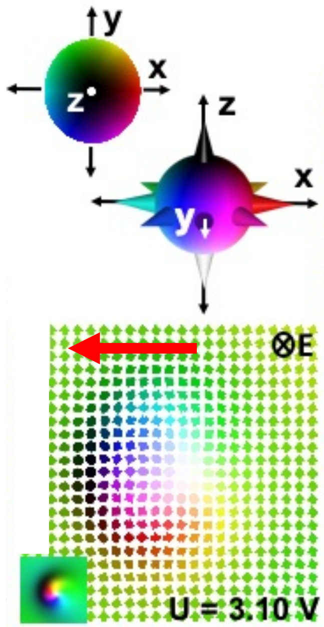
Emergent velocity and polar order

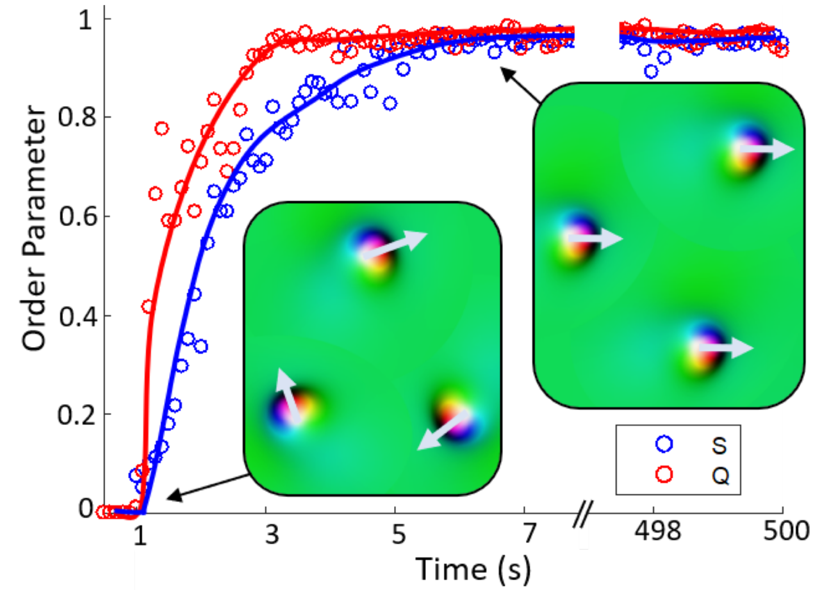


Polar order:
$\mathbf{p}_i$ represents south-to-north-pole unit vector

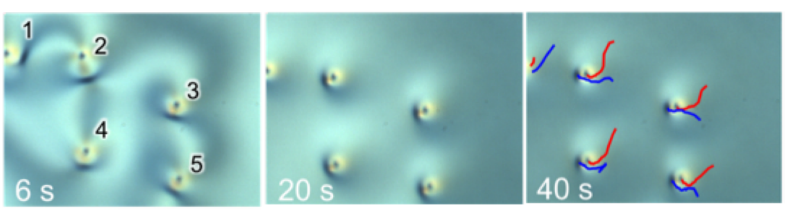
$$Q = |\sum_i^N \boldsymbol{p_i}| / N$$

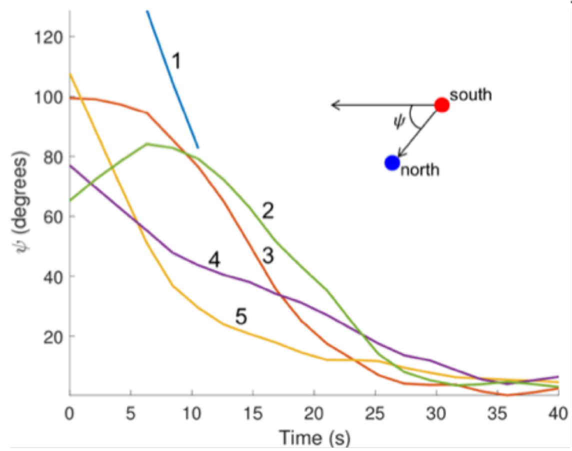
Velocity order:
$\mathbf{v}_i$ represents directional velocity unit vector
v_s represents velocity of the school with N skyrmions

$$S = |\sum_i^N \boldsymbol{v_i}| / (N \cdot \mathbf{v}_s)$$



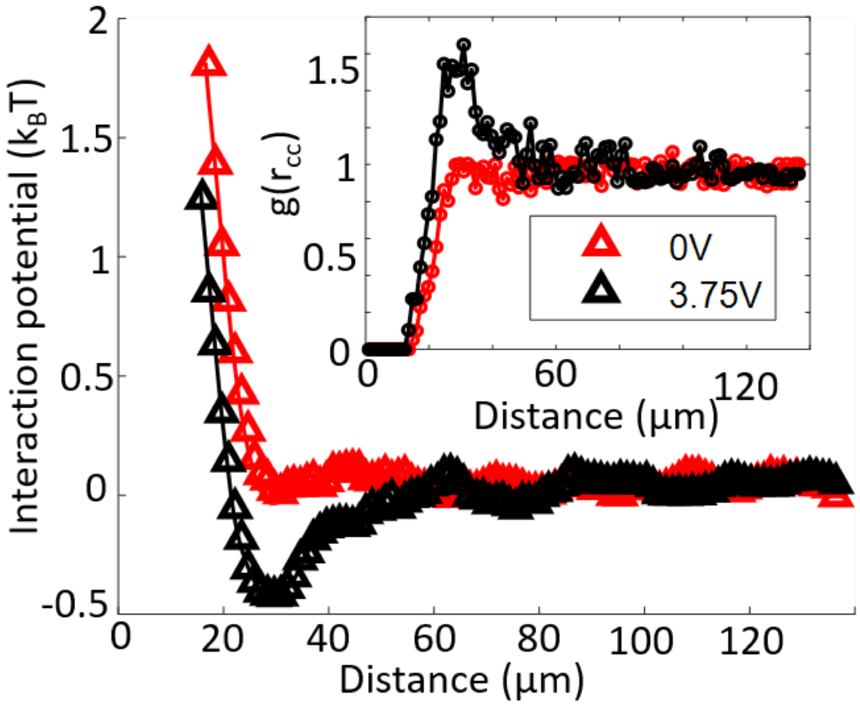
Emergence of polar order of preimage vectors





Effective cohesion with schools of sckyrmions

→Different from pair interactions!
→Many-body elastic interactions within schools



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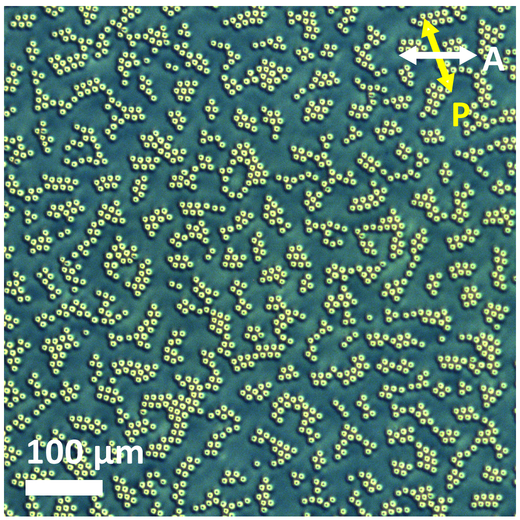
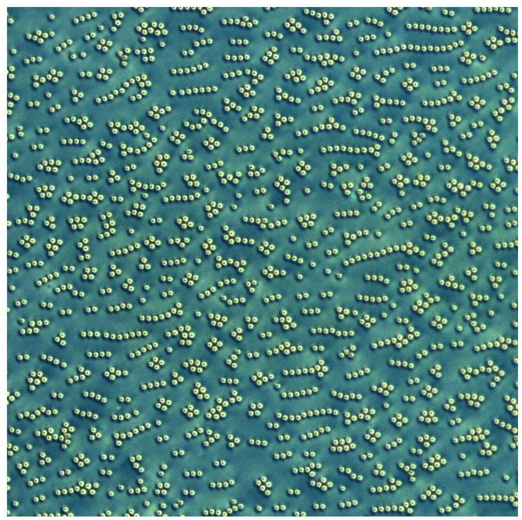
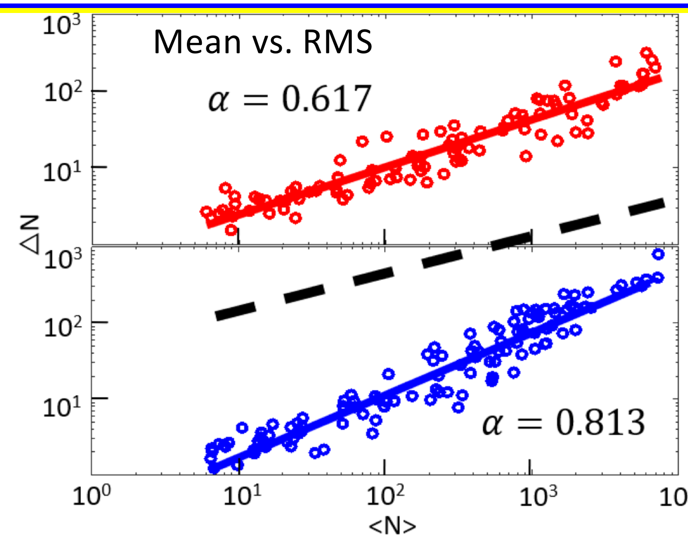
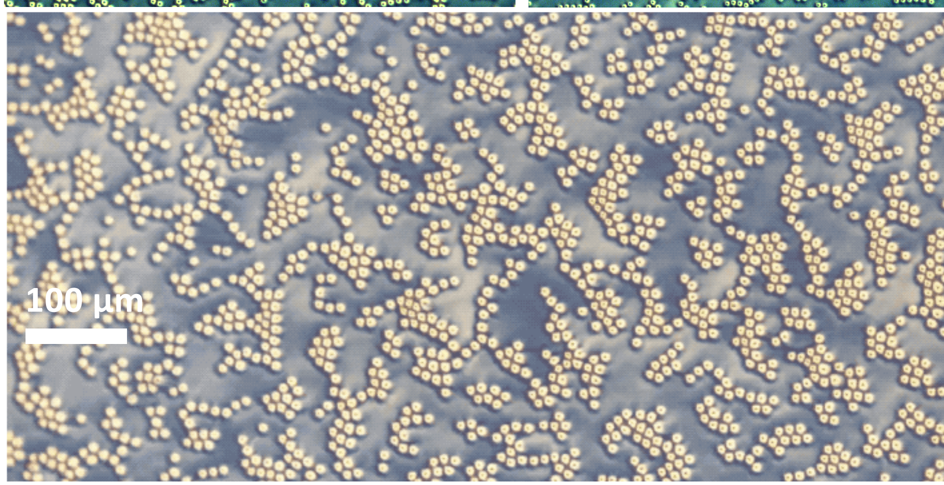
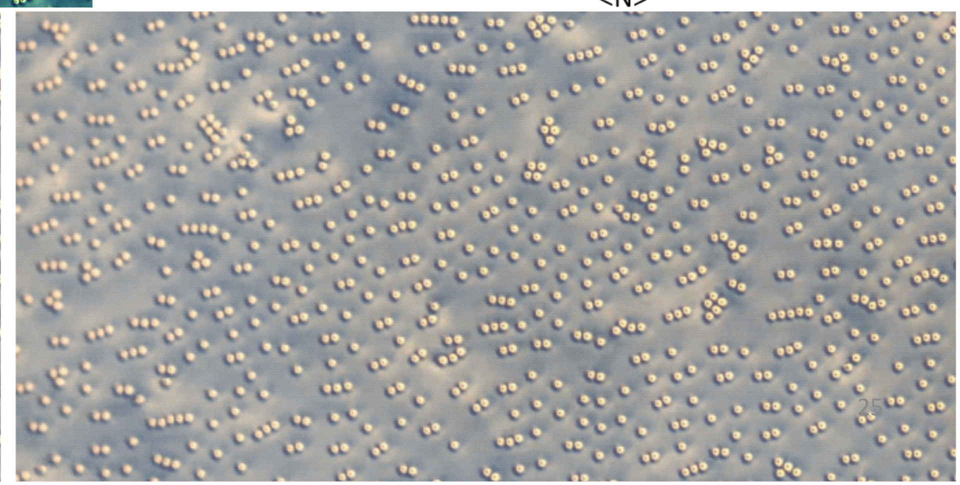
Clusters & chains within schools of skyrmions



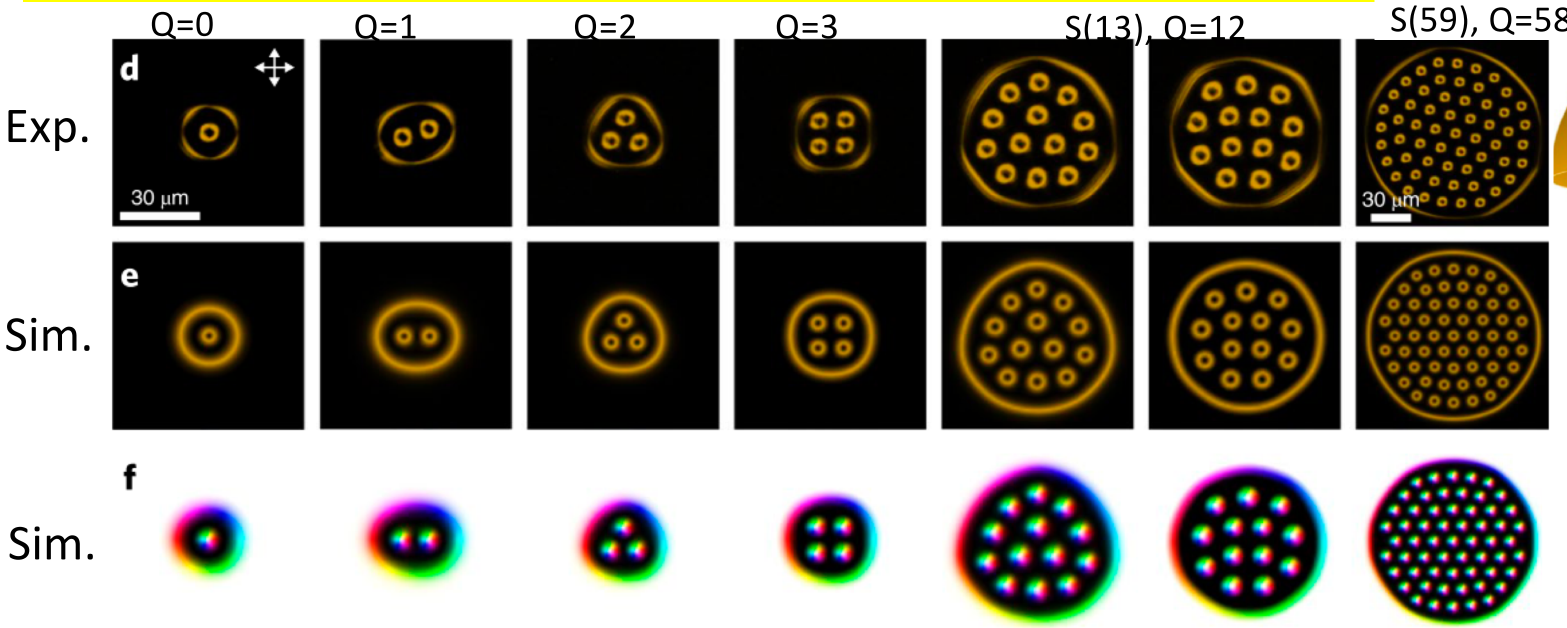


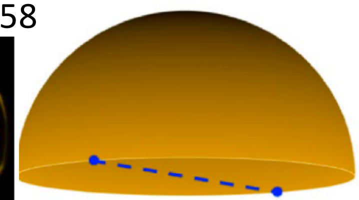




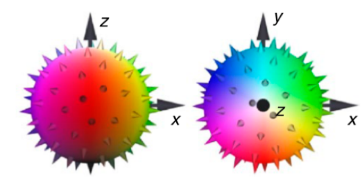


High-charge self-assembled clusters, “skyrmion bags”

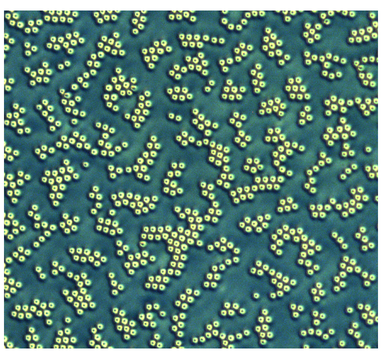




→Skyrmion bags in magnets & LCs

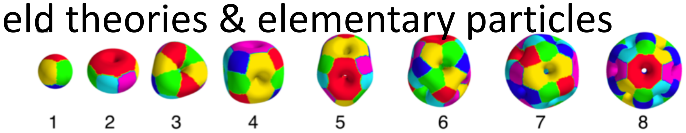


D Foster, C Kind, PJ Ackerman, JSB Tai, MR Dennis, II Smalyukh, *Nature Physics* **15**, 655 (2019).



→Large Skyrmion number clusters
→From out of equilibrium self-assembly!
→Not accessible to equilibrium condensed matter
→Like in nuclear physics skyrmions

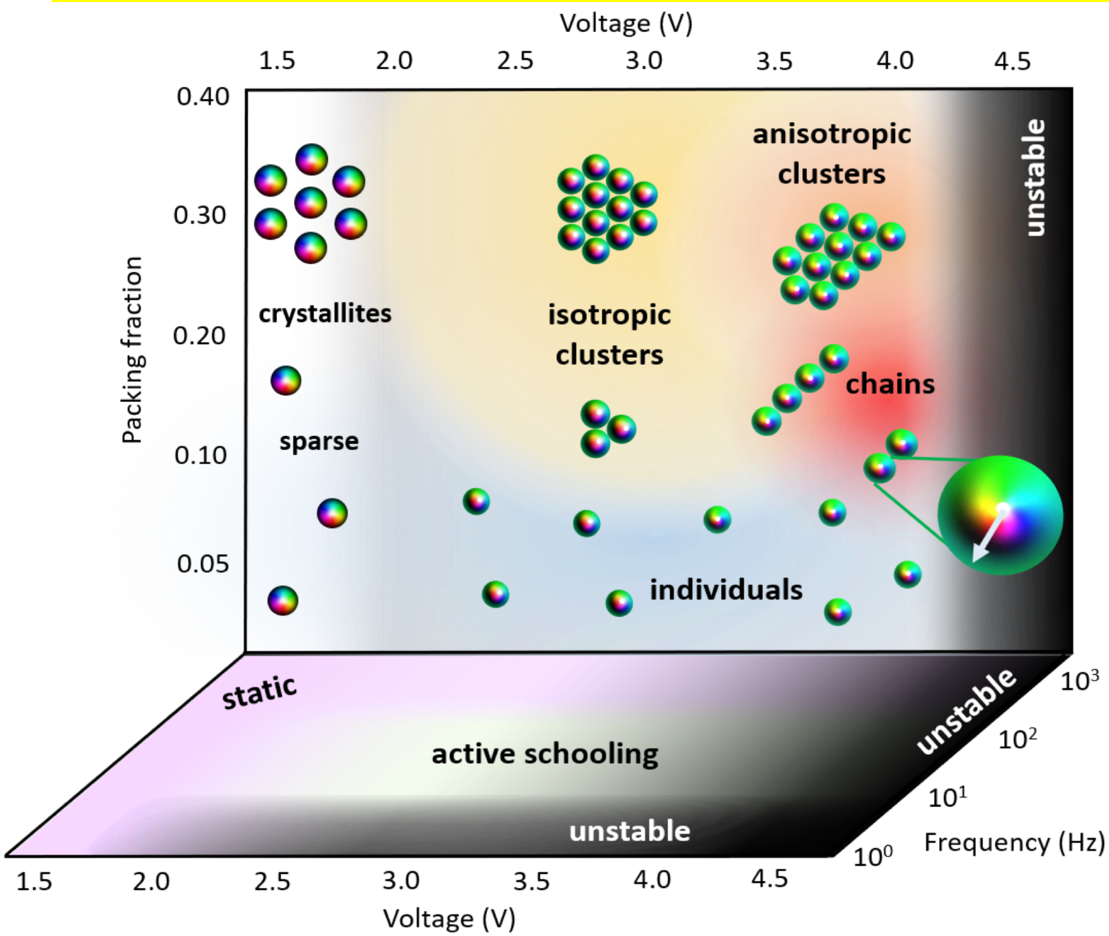
Field theories & elementary particles

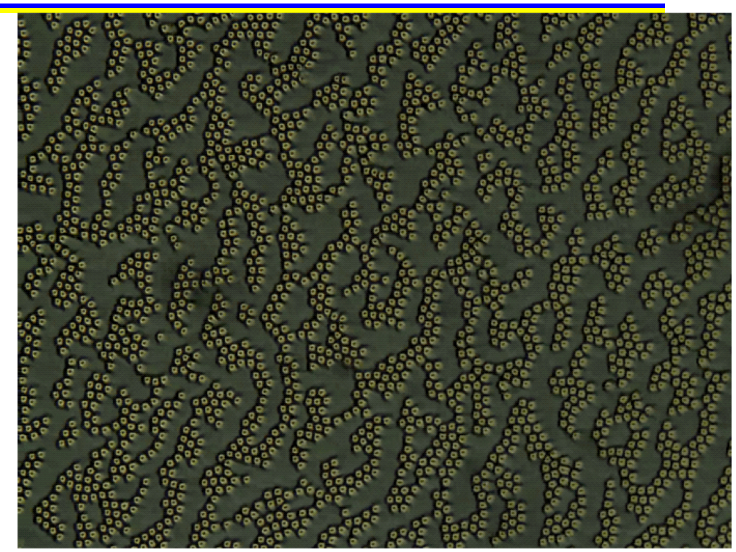


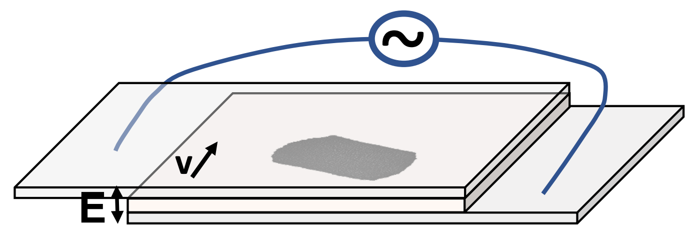
PRL 121, 232002 (2018)

→Very different from singular defect counterparts!

Diagram of dynamic & static states







→Behavoir at edges, effects of obstacles?

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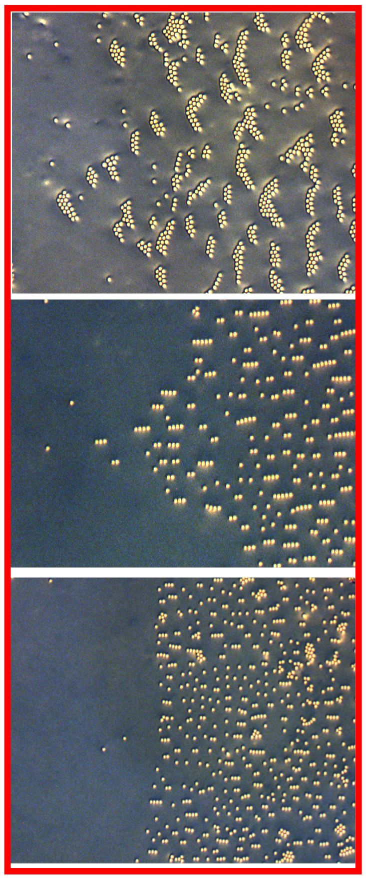
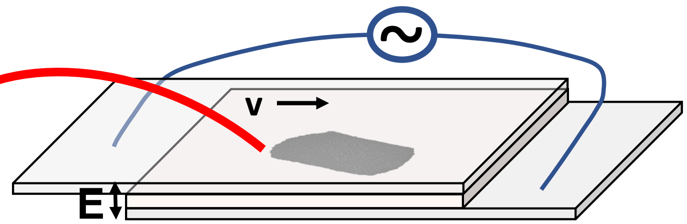
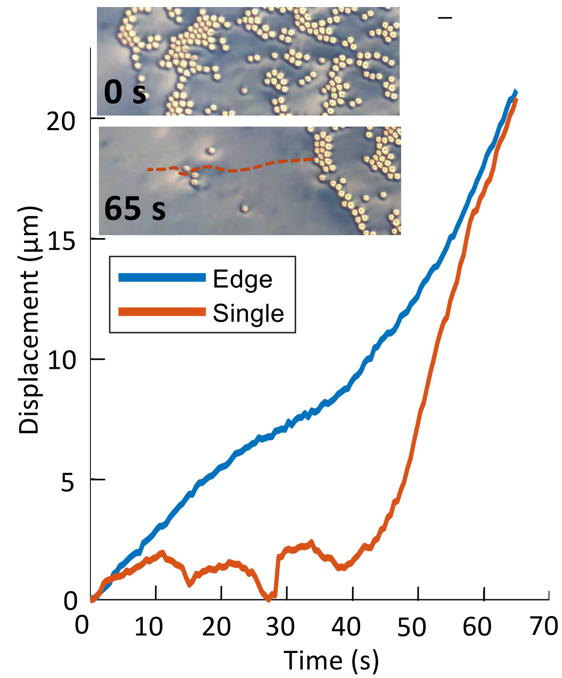
Edges of moving schools: individuals catch up

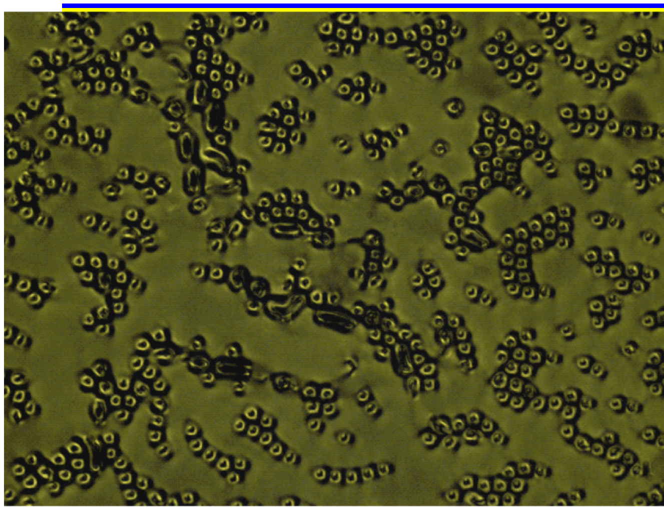


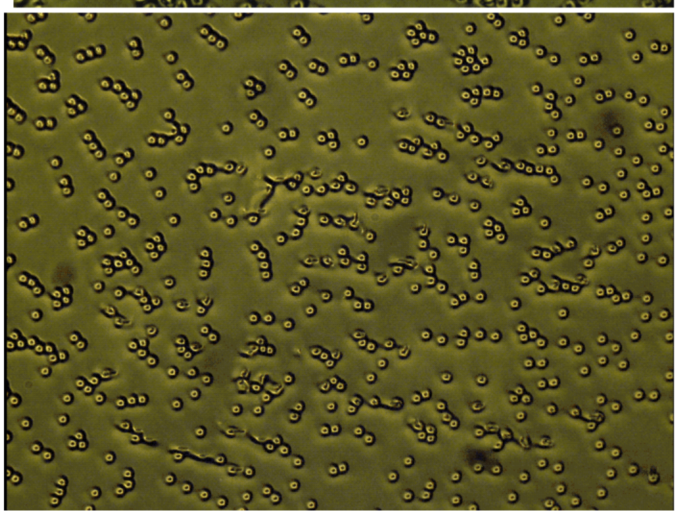




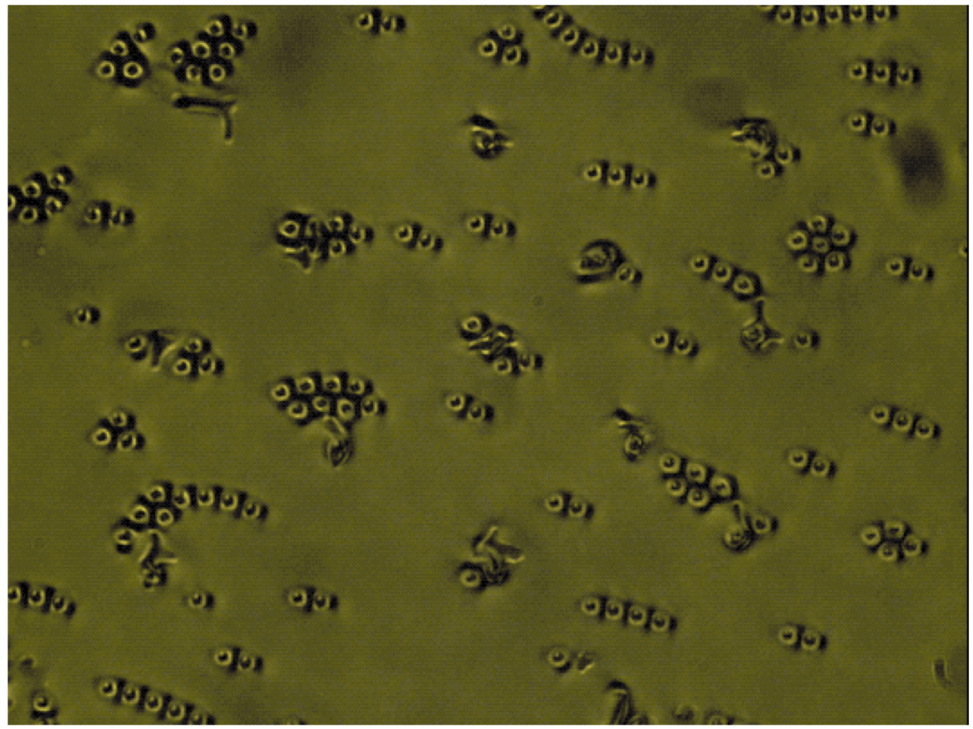
→ Skyrmions left behind are the fastest & catch up with the school

Obstacles, crowding and jamming behavior



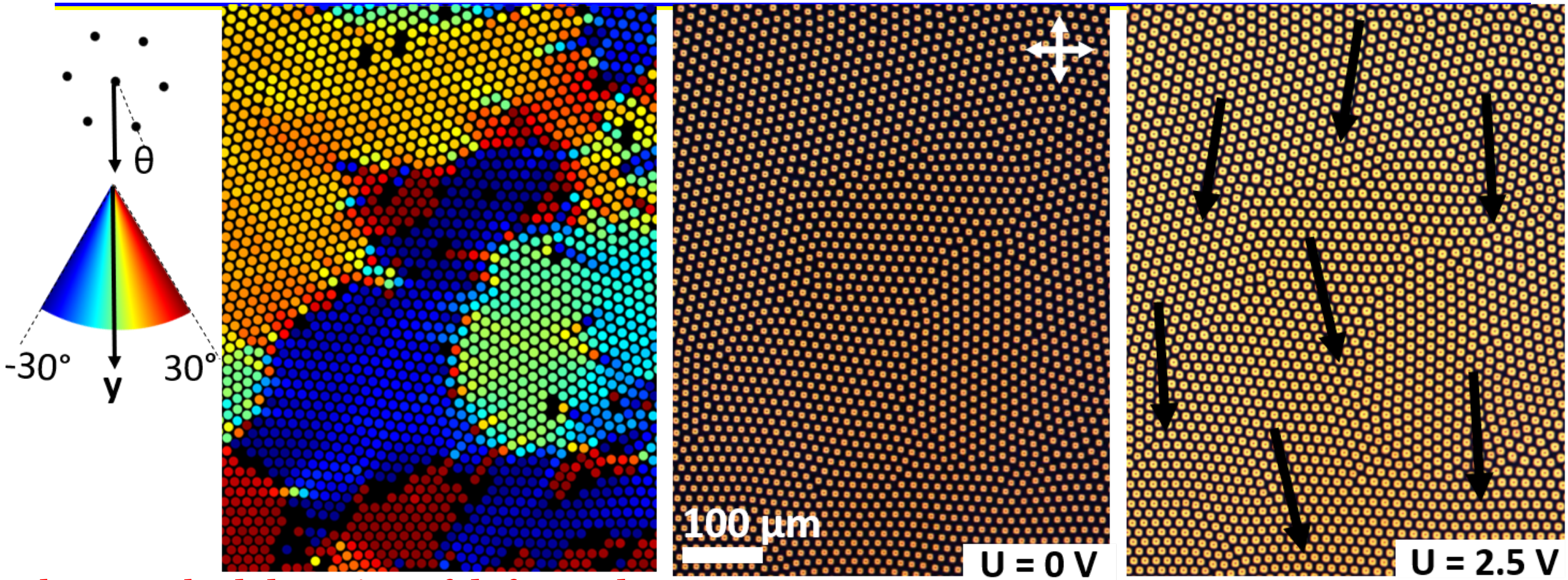


Electrostriction overcomes jamming
as voltage is increased

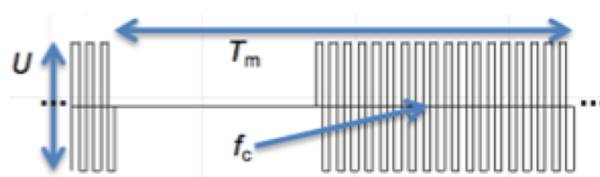


Optically laser-pinned skyrmions act as
obstacles to promote jamming behavior

Dense packing: moving crystallites?



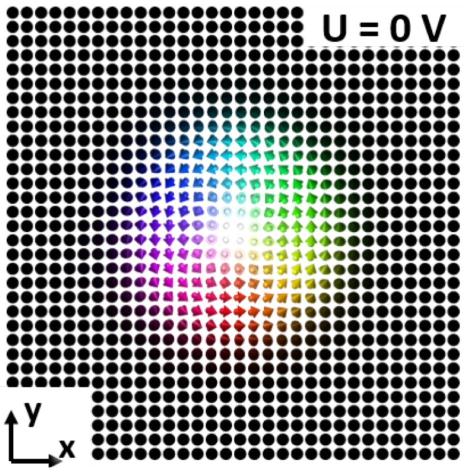
Close-packed domains of deformed quasi-hexagonal toron crystallites



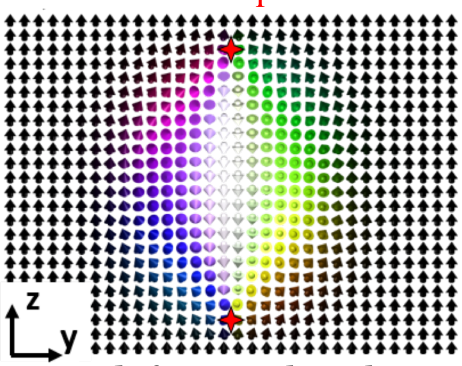
- Average direction of motion arises

Sohn & Smalyukh, *PNAS* **117**, 6437-6445 (2020)

2D moving crystallites of “soft” **toron** solitonic particles

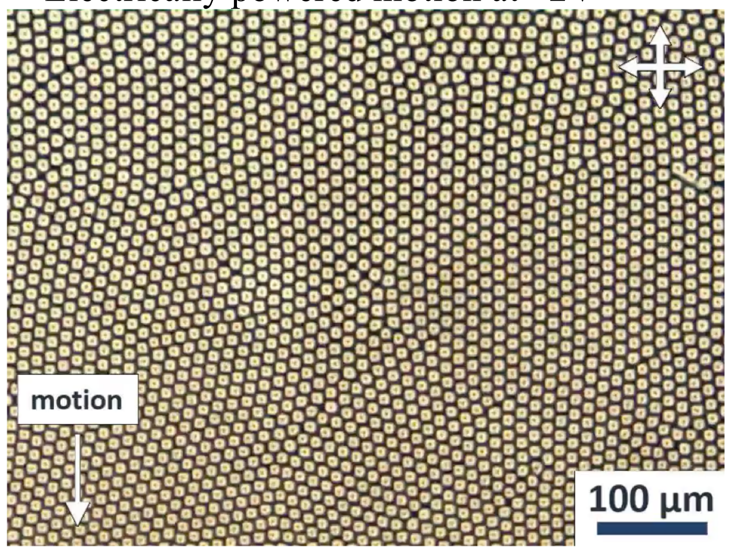


→Toron with 2 point defects

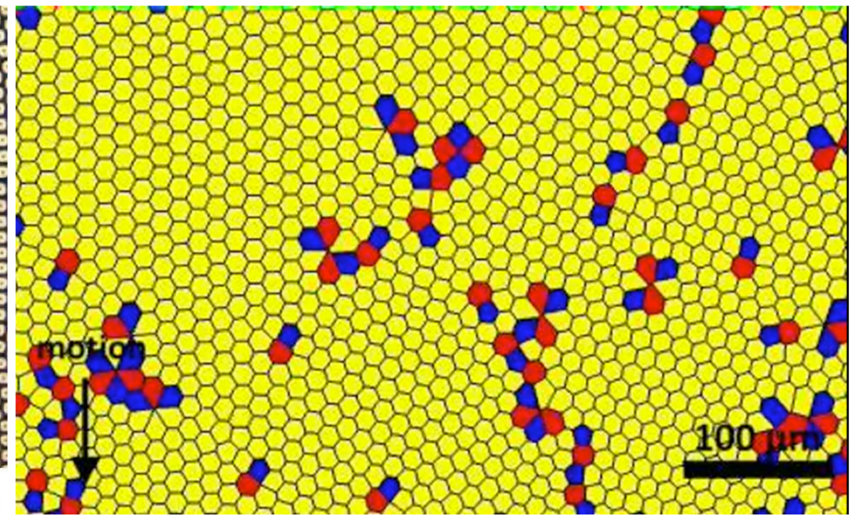


Point defects within the toron play an important role

Electrically powered motion at ~2V



Voronoi diagrams to analyze lattice defects

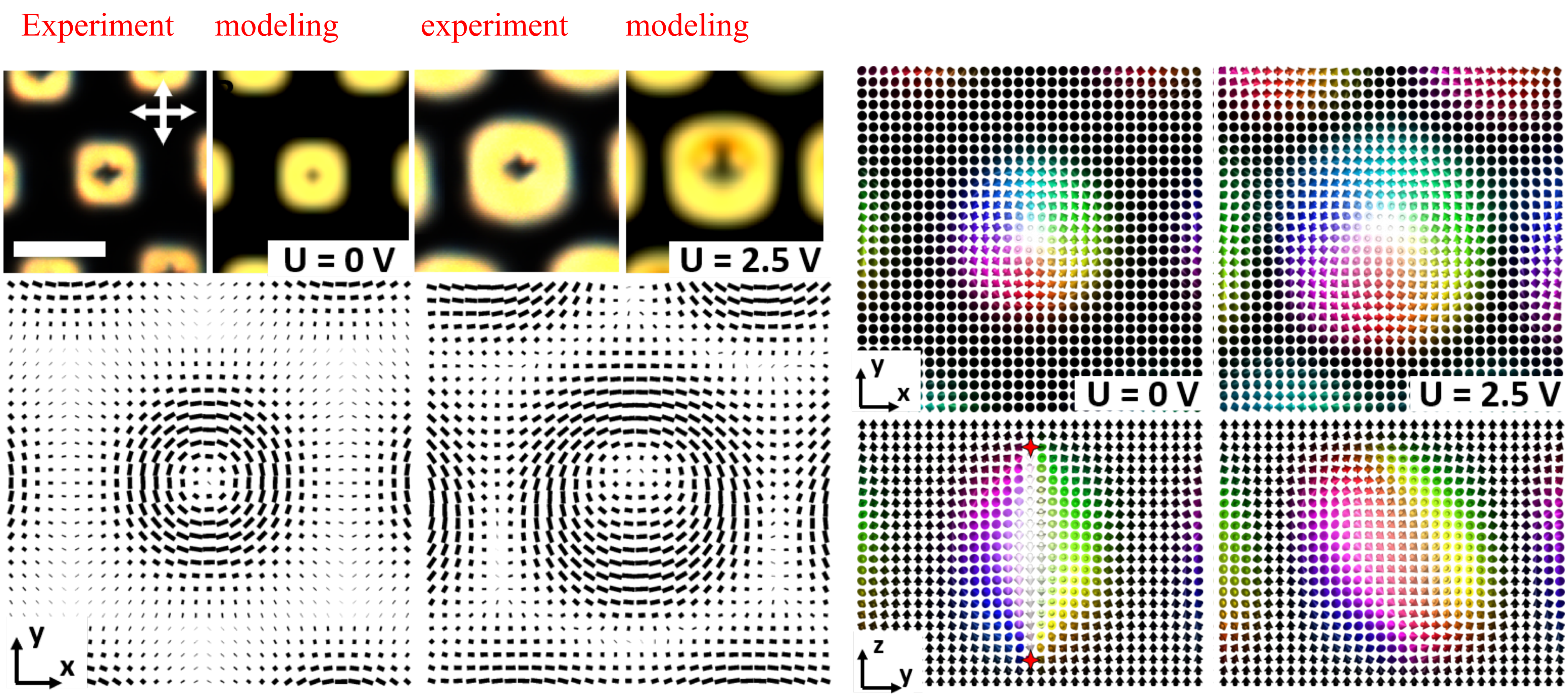


5 neighbors | 6 neighbors | 7 neighbors

→Bond orientational order parameter increases with motion
→Grain boundaries & 5-7 lattice defects exhibit complex dynamics
→Velocity order parameter reaches ~0.75 (lower than in skyrmion schools)

$$S = |\sum_i^N v_i| / (N \cdot v_s)$$

Imaging & modeling close-packed toron lattice

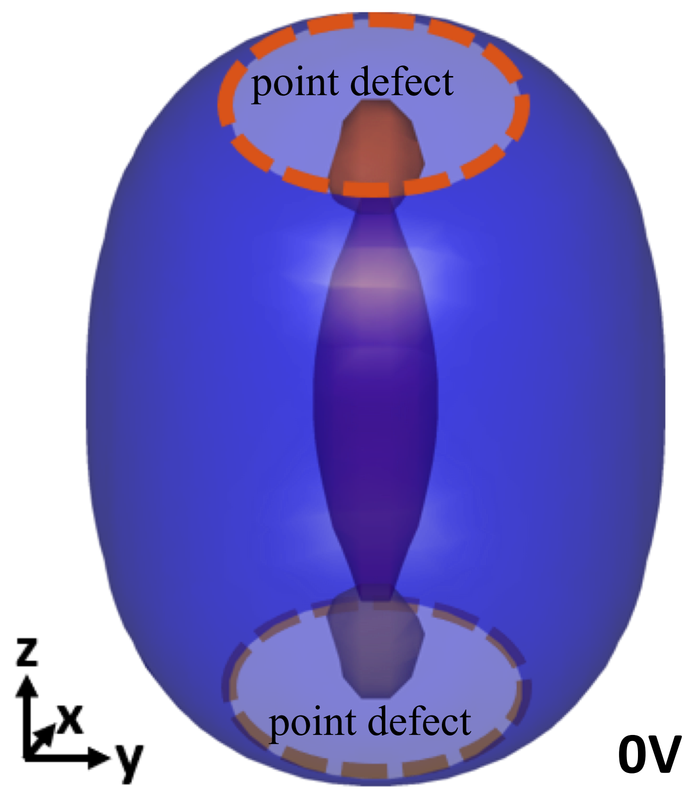


Sohn & Smalyukh, *PNAS* **117**, 6437-6445 (2020)

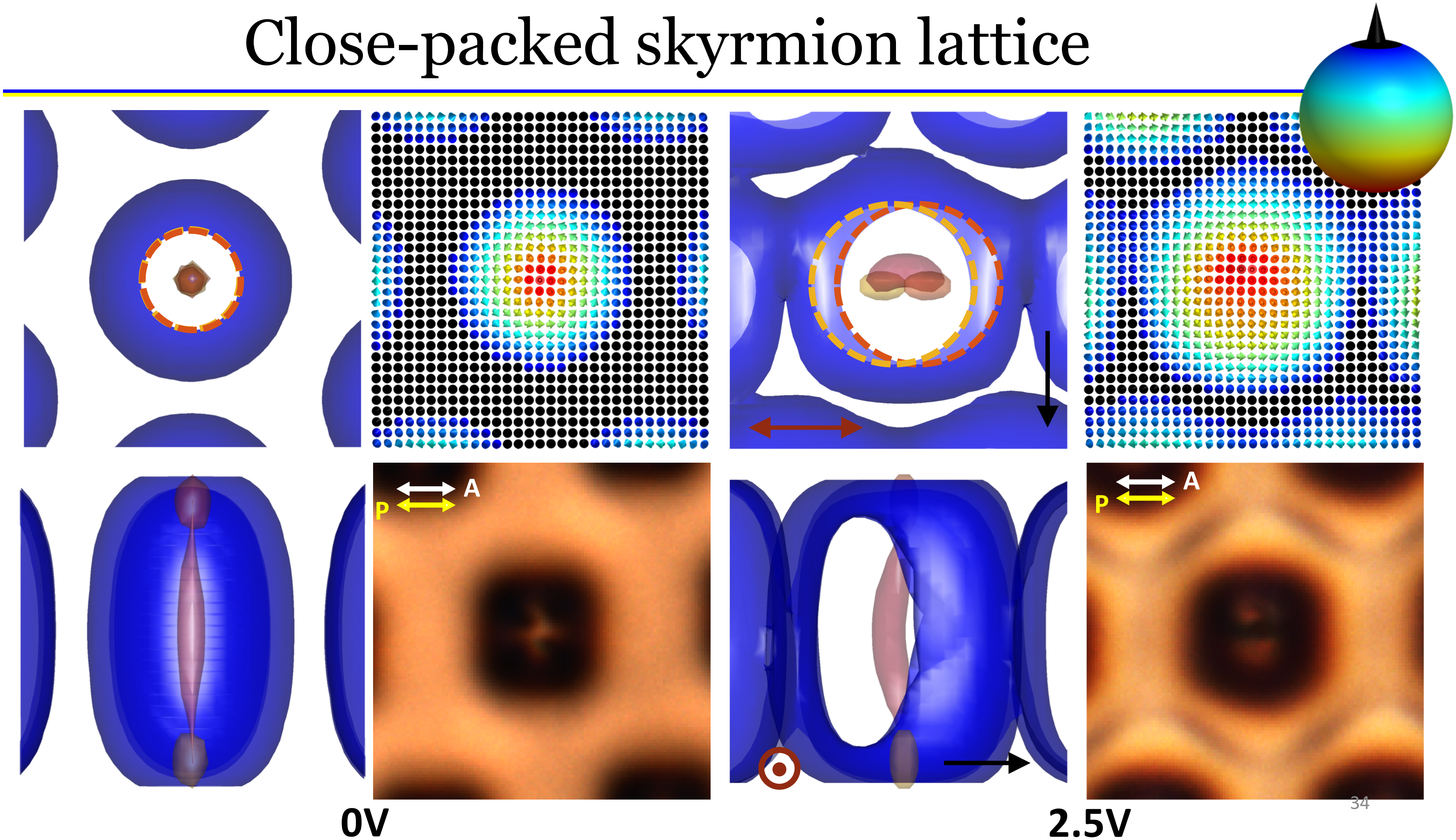
→Structures morph and self-shear upon application of voltage

Preimages & point defect visualization

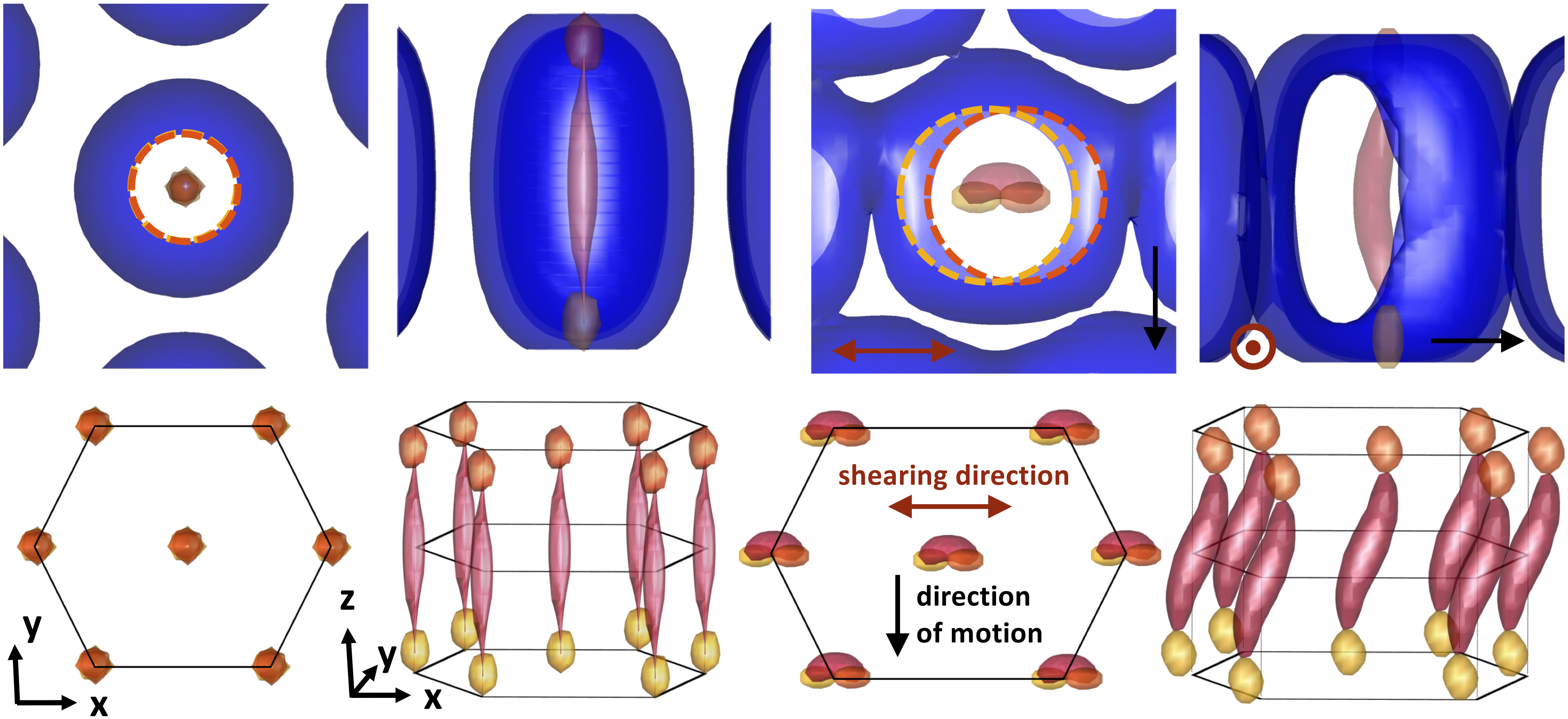
Visualize particle-like torons with skyrmions
& point defects within them



Close-packed skyrmion lattice

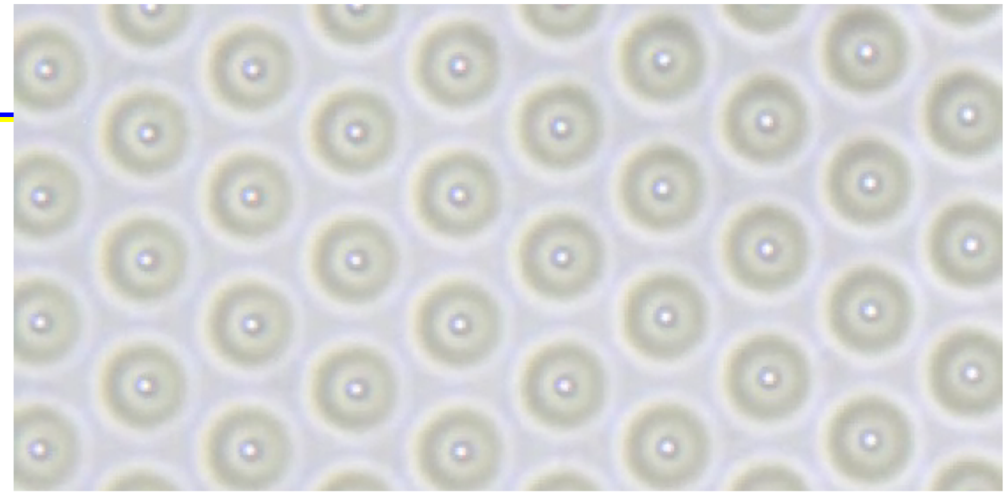


Preimage shearing

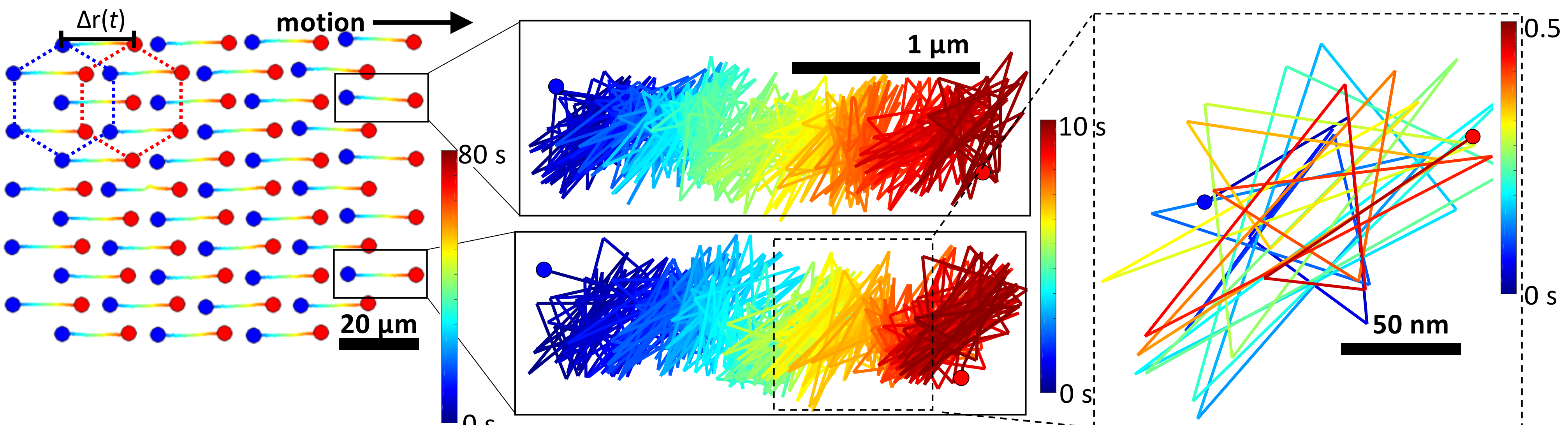


0V **2.5V**

Trajectories of motion



- Bright-field images of point defects
- Displacements on the order of thermal fluctuations result in net translational motion.



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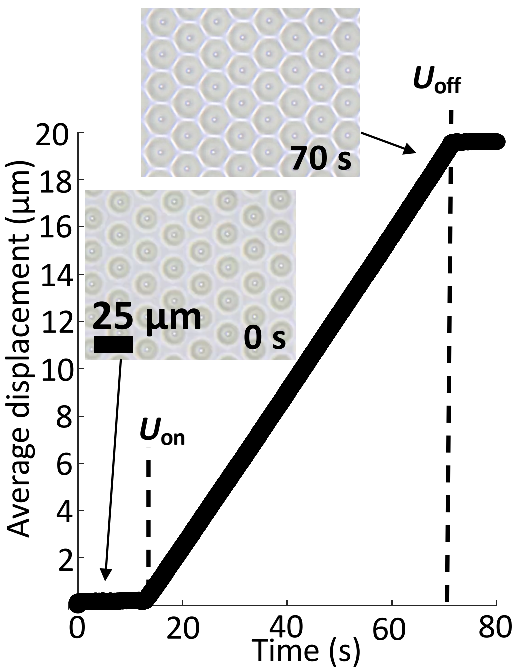
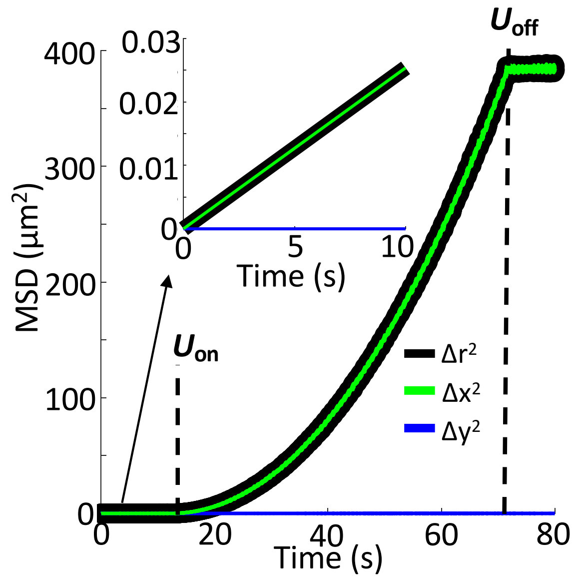
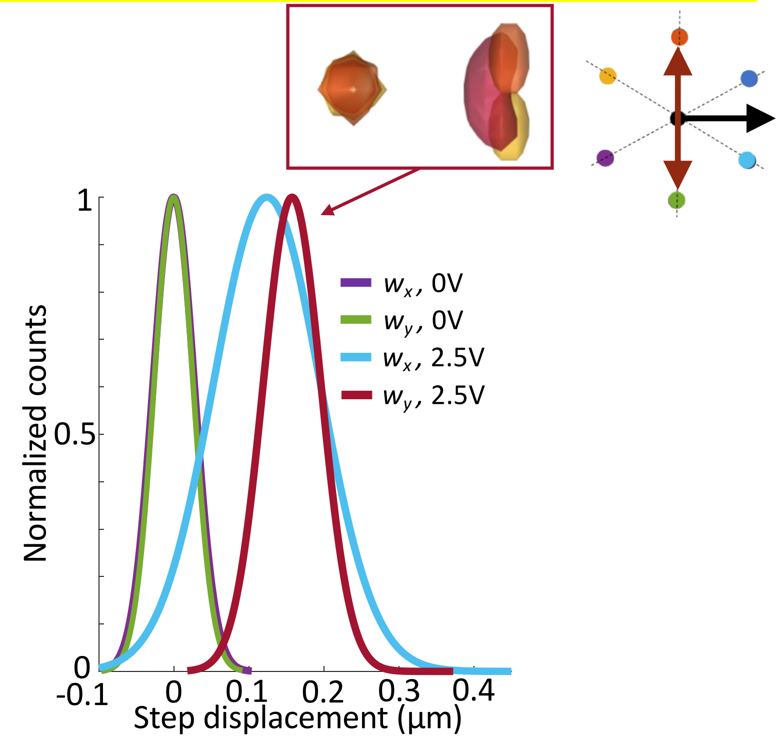
Displacement and motion of point defects







→ Directional motion when voltage is turned on
→ Point defects "walk" in the direction of motion

→ Crystallites shear perpendicular to motion

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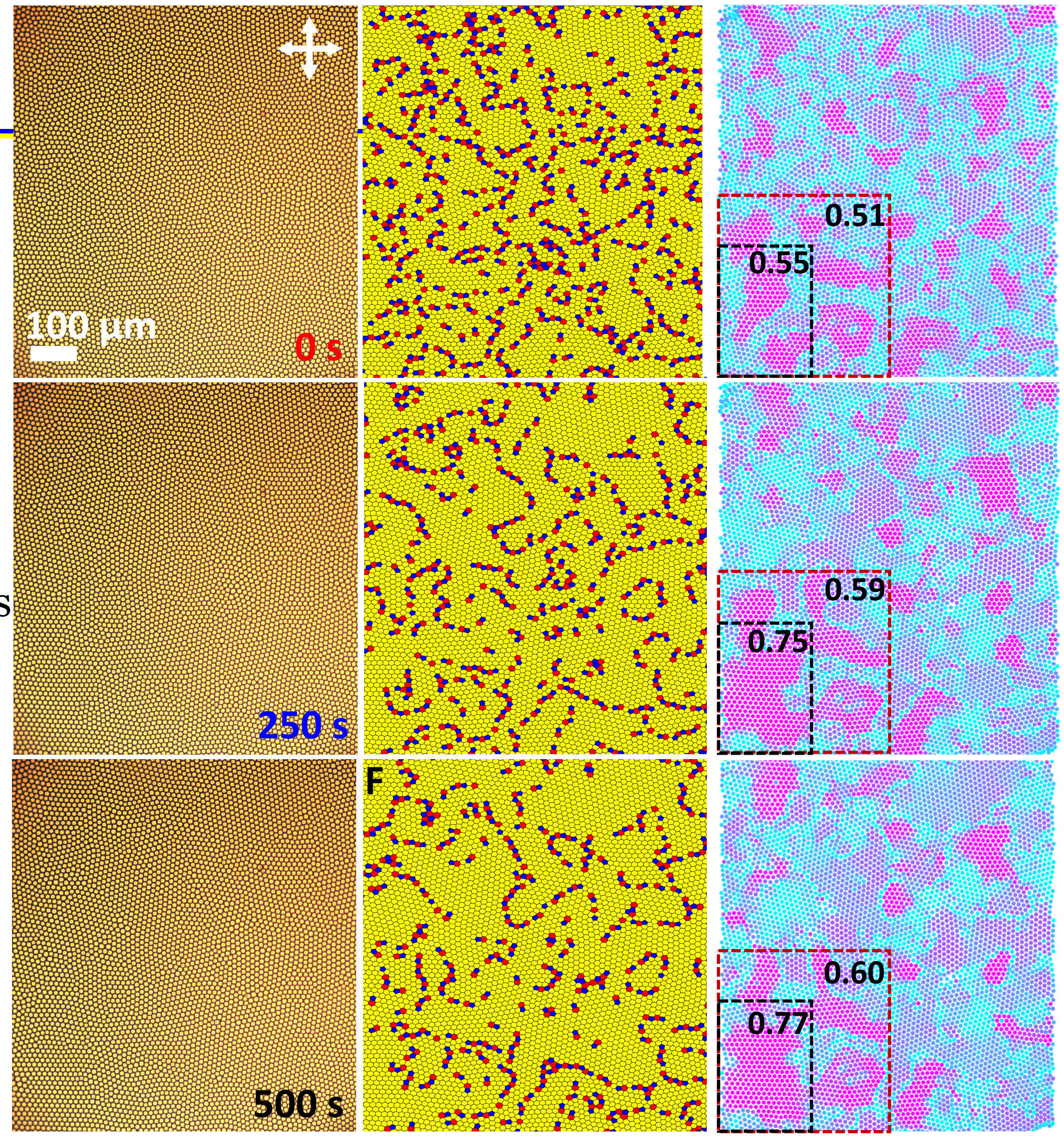
Evolution with motion



→ Annihilation of lattice defects to form larger crystallite domains

→ Hexatic order increases with motion

Local Bond Order

$$\psi_j = \frac{1}{n_j}\sum_{k=1}^{n_j} e^{im\theta_{jk}}$$

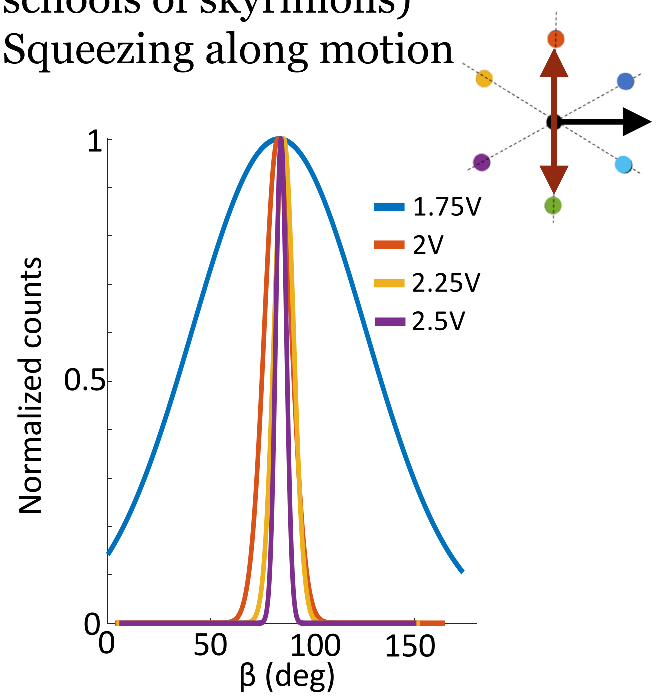
0 0.5 1

Hexatic order parameter

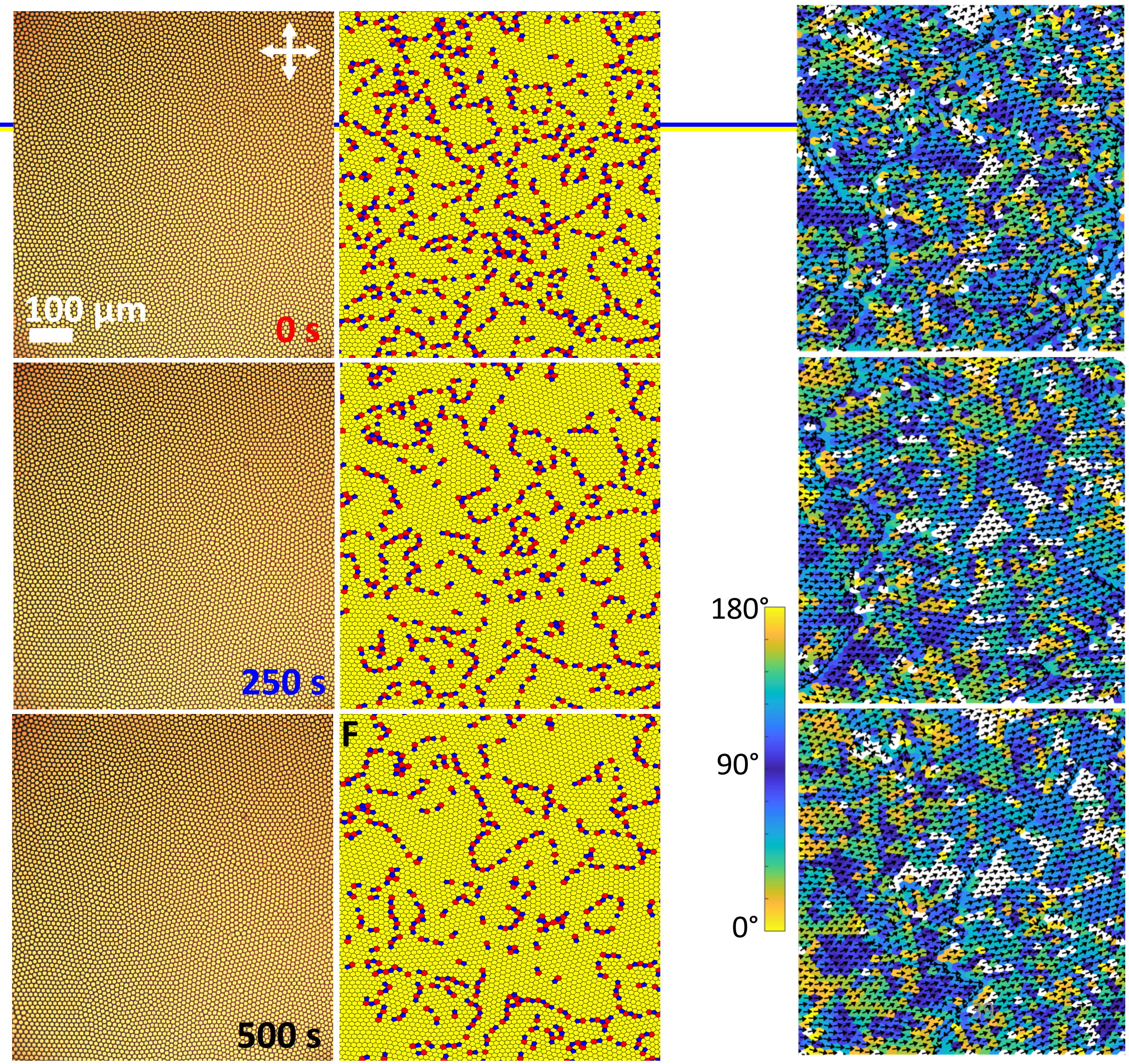
$$\Psi_6 = \left|\frac{1}{N}\sum_{j=1}^{N} \psi_j\right|$$

Evolution with motion

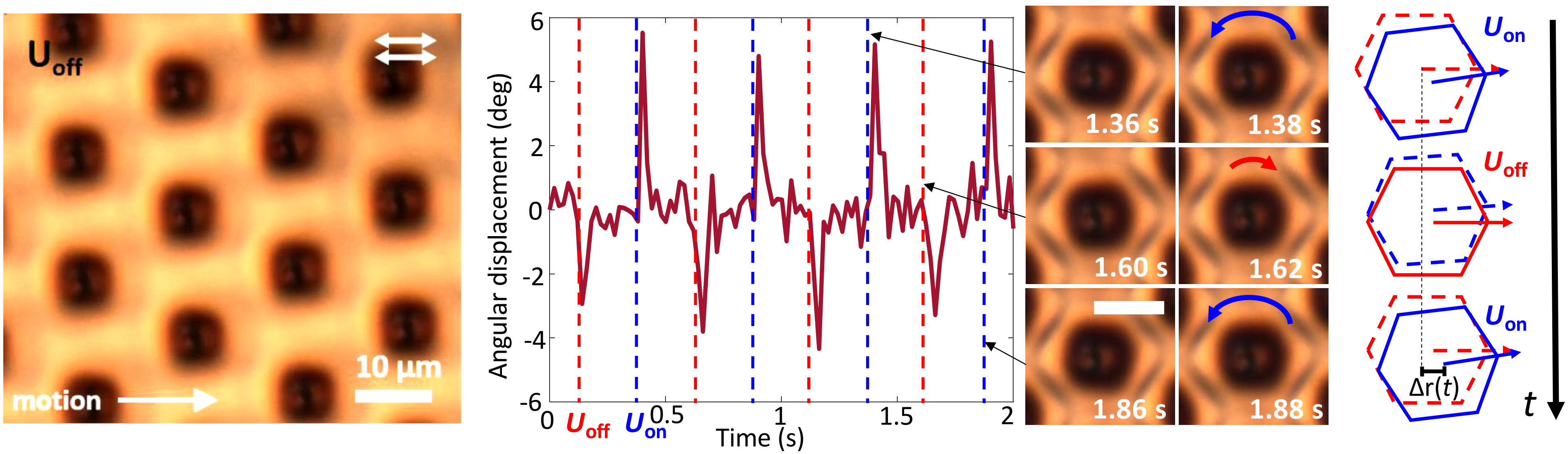
→ Synchronization of crystallite velocities & shearing directions
→ Velocity order parameter reaches ~0.75 (lower than in schools of skyrmions)
→ Squeezing along motion



→ Crystallites shear perpendicular to motion



Nonreciprocal behavior & motion



Evolution of **n(r)** not invariant upon time reversal.

Rotational dynamics of north-pole preimages within period of voltage modulation show nonreciprocity.

Sohn & Smalyukh, *PNAS* **117**, 6437-6445 (2020)

Topological carnival from rotational dynamics of **n**(**r**)

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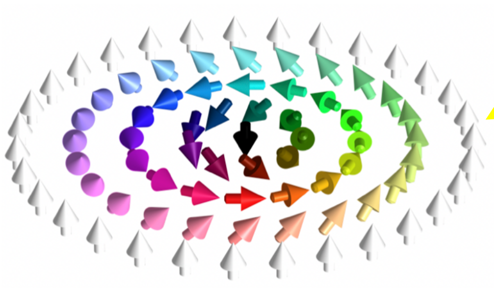
Evolution of **n(r)** not invariant upon time reversal upon instantaneous voltage turning on/off

Sohn & Smalyukh, *PNAS* **117**, 6437-6445 (2020)

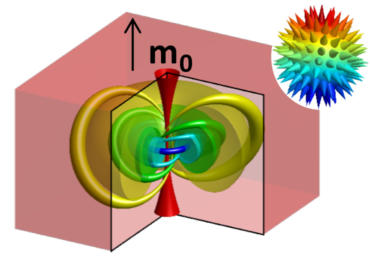
Other topological solitons in soft condensed matter?

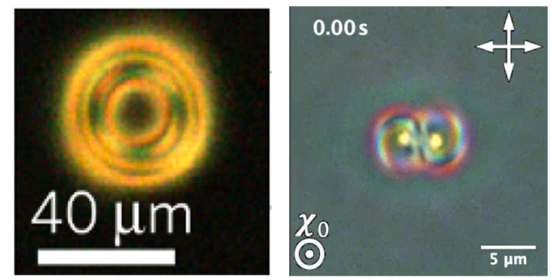
	π_1	π_2	π_3
$\mathbb{S}^0$	0	0	0
$\mathbb{S}^1$	$\mathbb{Z}$	0	0
$\mathbb{S}^2$	0	$\mathbb{Z}$	$\mathbb{Z}$
$\mathbb{S}^3$	0	0	$\mathbb{Z}$



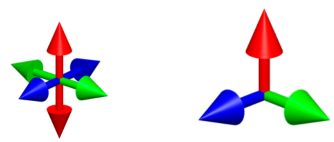
2D Skyrmions

Hopfions & heliknotons





PJ Ackerman and II Smalyukh. *Nat. Mat.* **16**, 426 (2017)
PJ Ackerman and II Smalyukh. *Phys. Rev. X* (2017)
JSB Tai, II Smalyukh, *Science* **365**, 1449 (2019).



$$\pi_3(\mathbb{S}^3/Q) = \pi_3(\mathbb{S}^3/\mathbb{Z}^2)$$
$$= \pi_3(\mathbb{S}^3) = \mathbb{Z}$$

Summary & conclusions

→Topological solitons in "passive" LCs can be "activated"

→Highly reconfigurable, mimic "soft" active particles

→Emergent dynamics of skyrmions & point defects

→Moving crystallites & lattice defects

→All in materials like in the LC displays

→Potential for LC electro-optics applications

→Extensions to other topological solitons

NSF *Thank you !!!* DEPARTMENT OF ENERGY • UNITED STATES OF AMERICA