A lens into cognition:
Topology and geometry of neural systems

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Emergent phenomena across scales

Brain networks and control
- Tang et al., *Nature Comm* 2017
- Tang & Bassett, *Rev Mod Phys* 2018

Information in fluid flows
- Tang & Golestanian, *arXiv* 2019

Effective learning
- Tang et al., *Nature Neuro* 2019

Topological phases of matter
- Tang et al., *Phys Rev Lett* 2012
Learning: an out-of-equilibrium process

Janus sphere with controllable orientation
Golestanian & Ebbens groups, *Nat Comm* 2015

As we gain understanding and control of active systems, can we “teach” them what to do?
Machine learning is successful but opaque and expensive

Cost to train a new model

Strubell, Ganesh & McCallum, *Proc. 57th Comp. Ling.* 2019

Huge number of parameters
Biological learning is quick and efficient

A baby who sees their parent use cell phones

What are underlying principles of learning?
Probing learning in humans is difficult

- No ground up theory for cognition: we can model neuron dynamics, but coarse-graining methods lacking
- No controlled experiments
- Sample sizes are small
- Data is noisy; has side effects from other physiological processes

What features in neural data can distinguish between cognitive states?
Coarse-grained feature of a multi-dimensional dataset

Learning engages complex dynamics: coordination over different modalities including sensory, attentional, memory

Given noisy data, won’t study specific dynamics

Hypothesis: there exists a suitable dimension for computational complexity
Neural data can be separated along a dimension

Trained to categorize cats and dogs

Activity in lateral prefrontal cortex of monkeys could be classified according to animal type

Freedman et al., Science 2001
Effective dimension can be lower than that of measurement space.

Probe the appropriate dimension for successful learning.

Rigotti et al., Nature 2013
Combinatorial approach to estimate dimension for noisy data

Given $n$ types of data (shapes):

Assign binary labels (blue or red):

1D:

2D:

$n$ categories

$2^n$ ways

Their linear separability (over different assignments) estimates the dimension

Unlike spectral analysis: does not depend on a metric

Rigotti et al., Nature 2013
Experiment with complex cognitive stimuli

Computer-generated shapes with similar statistical properties

Shapes have value drawn from a Gaussian with fixed mean

Participants had to associate dollar values to each new shape
Adult participants learned the values of these shapes through feedback. 

3 training sessions a day, over 4 days.
Their neural patterns scanned using fMRI throughout the experiment

Each session: 140 pairs shown

Blood-oxygen-level dependent activation measured

Coarse-grained approach:
83 regions parcellation of whole-brain
Neural responses across all regions form a geometric representation

140 shapes each contribute a point in data cloud

For $n$ categories: $2^n$ ways to assign binary labels

Average separability over hyperplanes is a proxy for dimension – large combinatorics allows method to be robust to noise
Fast learners have a higher dimensional representation of neural data. Higher dimensional neural representations are associated with effective learning on this task.

Monotonic relation to physical dimension

$r = 0.56$
Test result reliability using a null model

Without theory, and without repeated experiments to fit – need to identify “no result”

Obtain a baseline comparison from similar data without task information

1. Shuffle task labels of data points
2. Repeat analysis
Null model shows result is significant

Separability dimension on Day 4 vs. response accuracy end of Day 1:

- Correlation: $r = 0.56$, $p < 0.001$

Null data (1000 bootstrapped samples):

Correlation between dimension and accuracy.
Null data also shows smaller dimension for fast learners

Negative correlation between null data dimension and learning accuracy

Larger $m$ more reliable from averaging over $2^m$
Fast learners have higher task-based dimension and lower embedding dimension.

Fast learners have an efficient representation: high ratio of information-coding to resources used.

Analogous results seen in data manifolds of neural networks

Analysis of the shape of manifold representations in neural networks

Two kinds of dimensionality that can behave in different ways with training

Chung, Lee & Sompolinsky, *PRX* 2018
Virtual lesioning: data-driven approach to identify which brain regions contribute most

Brain regions are removed one at a time: result recalculated
Largest change (in correlation of accuracy with dimension) due to:

Left hippocampus

Associated with rapid learning of stimulus associations

Squire, *Psych Rev* 1992

Right temporal pole

Represents information about abstract conceptual properties (such as value)

Peelen & Caramazza, *J Neuroscience* 2012
Recapitulation of effect on smaller voxel-level in some regions

Study 5 regions in each hemisphere with 300 (of fewer) voxels

Left anterior cingulate cortex has strongest result and known role in reward-based learning

Followed by left V1 and right posterior fusiform

Bush et al., PNAS 2002
The geometry of neural activity reflects cognitive performance

1. Fast learners have higher dimensional representations of neural activity.

2. This allows objects of different value to be more easily distinguished.

3. Fast learners also have lower embedding dimension: hence they have more efficient representations with a high ratio of information-coding to resources used.
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How does brain structure subserve dynamics and function?

Control theory and dynamical models to probe the role of connectivity; and in changes across development.

Towards understanding function: e.g. children are more spontaneous while adults are better at cognitive control.
Network control theory models dynamics in heterogeneous real-world systems

Liu et al., *Nature* 2011
Models the driving of dynamical changes across neurons or neural systems

External Input
Stimulation
Neurofeedback

Internal Control
Cognitive Control
Homeostasis

Bassett et al., *Ann Rev Biomed Eng* 2017
Topology of brain connectivity mapped with non-invasive neuroimaging

Identify brain regions on the mesoscale

Glasser et al., *Nature* 2016

White matter pathways inferred from movement of water molecules diffusing along tracts

Tuch et al., *Neuroimage* 1997
Build brain network which estimates strength of connections between regions.

We represent the pattern of white matter tracts between brain regions as an undirected, weighted adjacency matrix.

Bullmore & Sporns, *Nat Rev Neurosci* 2009
Linear dynamical model + input into system

\[ x(t + 1) = Ax(t) + Bu(t) \]
Input into systems defines an energy landscape

\[ x(t + 1) = Ax(t) + Bu(t) \]

After \( T \) steps,

\[ x(T) = C_T \begin{bmatrix} u(T - 1) \\ \vdots \\ u(0) \end{bmatrix} ; \quad C_T := \begin{bmatrix} B & AB & \cdots & A^{T-1}B \end{bmatrix} . \]

Input energy

\[ E(u, T) := \sum_{t=0}^{T-1} \|u(t)\|^2 \]

Depends on brain network and input regions (structural)

Linearize around origin

Energy \( E \)
Structural network properties determine the minimum input energy

\[ \mathbf{x}(t + 1) = A \mathbf{x}(t) + B \mathbf{u}(t) \]

\[ \mathbf{x}(T) = C_T \begin{bmatrix} \mathbf{u}(T - 1) \\ \vdots \\ \mathbf{u}(0) \end{bmatrix} \]

Minimum input energy

\[ \begin{bmatrix} \mathbf{u}^*(T - 1) \\ \vdots \\ \mathbf{u}^*(0) \end{bmatrix} = C_T^T (C_T C_T^T)^{-1} \mathbf{x}_f \]

Kailath, *Linear Systems* 1980

\[ E^*(T) = \sum_{t=0}^{T-1} \| \mathbf{u}^*(t) \|^2 = \mathbf{x}_f^T (C_T C_T^T)^{-1} \mathbf{x}_f \]

Gramian

\[ W_T := C_T C_T^T = \sum_{t=0}^{T-1} A^t B B^T (A^T)^t \]
Network connectivity and strength determine possible dynamical transitions

Minimum input energy

\[ E^*(T) = x_f^T W_T^{-1} x_f; \quad W_T := \sum_{t=0}^{T-1} A^t B B^T (A^T)^t \]

When \( x_f \) is an eigenvector of \( W_T \) with eigenvalue \( \lambda \), \( E^*(T) = \lambda^{-1} \)

Average input energy (over all \( \{x_f\} \)): \( \text{Tr}(W^{-1}) \)

Ability to control with least average energy: \( \text{Tr}(W) \)

Pasqualetti et al., *IEEE TCNS* 2014

Gu et al., *Nat Comm* 2015

\( v_j \): \( j \)th eigenvector of \( A \) with eigenvalue \( \xi_j \).
If \( v_{ij} \) is small, then \( j \)th mode is poorly controllable from \( i \) (extension of PBH test).

“Worst-case” energy from \( E^*(T) \leq \lambda_{min}^{-1}(W_T) \)

Ability for modal control of most costly transition from \( i \):

\[ \sum_j (1 - \xi_j^2(A)) v_{ij}^2 \]
Use network control metrics on cohort of 882 youth from 8 to 22 years

Diffusion data from Philadelphia Neurodevelopmental Cohort

Roalf et al., Neuroimage 2016

Do brain networks show increasing control with age?
Topology of brain networks supports more dynamical transitions

Coarse metrics across whole-brain

Mean modal controllability per subject

Mean average controllability per subject
Brain networks increasingly support diverse dynamics with age

Older subjects have a larger range of possible dynamics from low to high energy transitions: increased specialization
What can a finer look tell us?

 Regions high in average controllability increase in controllability with age
Regional specialization of control with age or “super-controllers”

Seen in both average and modal control regions
Which brain regions have high control for effective cognition?

Subjects with high subcortical controllability exhibit poorer cognitive performance.

Consistent with evidence that segregation between neural systems is associated with improved cognitive ability

Wig, TICS 2017
Synchronous neural activity is often associated with pathology

Recording from a scalp electrode during a seizure

Taylor et al., *Front Neurosci*, 2015
Synchronizability measures network ability to sustain globally similar dynamics

Synchronous state described by Laplacian connectivity matrix and its eigenvalues \( \{ \lambda_j \} \)

Do brain networks have less susceptibility to synchronous (potentially pathological) dynamics with age?
Topology of brain networks is less synchronizable

Mean average controllability

Mean modal controllability

Structural connectivity

Null model preserving node strength

Null model preserving node degree

Mean average controllability

x_1

x_2
Brain networks less vulnerable to synchrony with development

Older subjects show less susceptibility to synchronous dynamics

Quantifies intuitions on emerging control and decreased susceptibility to global inputs with age
A phenomenological wiring rule that promotes healthy development?

If these findings suggest a mechanism for development

Rewire brain networks for higher controllability and lower synchronizability

Simulations recapitulate developmental arc

Tang et al., Nat Comm 2017
1. We develop dynamical models that link changes in white matter to predicted dynamics and function.

2. Our results formalize intuitions about increasing specialization of brain connectivity across development, at the expense of greater flexibility.

3. We identify regional changes and drivers of cognition.
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Time

Space

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