Chern-Simons Theories and AdS/CFT

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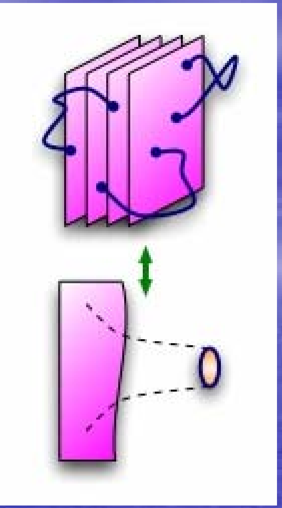
Introduction

- Recent progress has led to realization that coincident membranes in M-theory are described by Chern-Simons gauge theories coupled to massless matter.
- This appears to solve a long-standing problem which was harder than the description of D-branes in string theory that is known explicitly at small string coupling.
- But M-theory is inherently strongly coupled: one can think of it as the strong coupling limit of a 10-dimensional superstring theory. What to do?

D-Branes vs. Geometry



- Dirichlet branes (Polchinski) realize maximally supersymmetric gauge theories.
- A stack of N D3-branes realizes $\mathcal{N}$=4 supersymmetric SU(N) gauge theory. It also creates a curved background of 10-d theory of closed superstrings (artwork by E.Imeroni)

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(-(dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right)$$

which for small r approaches $AdS_5 \times \mathbf{S}^5$

whose radius is related to the coupling by $L^4 = g_{\rm YM}^2 N \alpha'^2$

- For example, two calculations of absorption of massless states agree exactly. I.K.

Super-Conformal Invariance

- In the $\mathcal{N}=4$ SYM theory there are 6 scalar fields (it is useful to combine them into 3 complex scalars: Z, W, V) and 4 gluinos interacting with the gluons. All the fields are in the adjoint representation of the SU(N) gauge group.
- Comparing with QCD, the Asymptotic Freedom is canceled by the extra fields; the gauge coupling g_{YM} does not depend on the Energy. The theory is invariant under scale transformations $x^{\mu} \to a\, x^{\mu}$. It is also invariant under space-time inversions. Such a theory is called (super) conformal.

The AdS_5/CFT_4 Duality

Maldacena; Gubser, IK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the $\mathcal{N}$=4 SYM theory this compact space is a 5-d sphere.
- The geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- The AdS_d (hyperbolic) space is

$$(X^0)^2 + (X^d)^2 - \sum_{i=1}^{d-1} (X^i)^2 = L^2 \ .$$

with metric
$$ds^2 = \frac{L^2}{z^2}\left(dz^2 - (dx^0)^2 + \sum_{i=1}^{d-2} (dx^i)^2\right)$$

- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large: $\frac{L^2}{\alpha'} \sim \sqrt{g_{\mathrm{YM}}^2 N}$

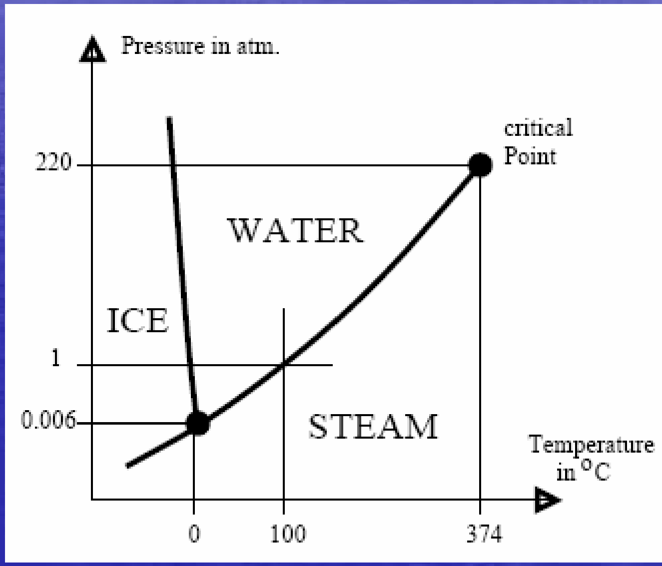
- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of $\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$

- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak coupling the dual string theory becomes difficult.

- The research on AdS_5/CFT_4 has rekindled interest in the maximally super-symmetric 4-d gauge theory and provided a host of information about its strongly coupled limit. See the January 2009 Physics Today article by I.K., J.Maldacena.
- This conformal gauge theory is becoming `The Harmonic Oscillator of 4-d Gauge Theory' in that it may be exactly solvable.
- It has served as a `hyperbolic cow' approximation, for example, to some phenomena observed in Heavy Ion Colliders.

AdS_4/CFT_3

- Besides describing all of known particle physics, Quantum Field Theory is important for understanding the vicinity of certain phase transitions, such as the all-important water/vapor transition.
- Here we are interested in a 3-d (Euclidean) QFT.



- This transition is in the 3-d Ising Model Universality Class.
- Other common transitions are described by 3-d QFT with O(N) symmetry.
- 3-d CFT’s are also important in describing 2-d quantum critical systems, such as those in the high-Tc superconductors, Quantum Hall Effect, etc.
- Can we find a `Harmonic Oscillator’ of 3-d Conformal Field Theory ?

O(N) Sigma Model

- Describes 2nd order phase transitions in statistical systems with O(N) symmetry.

$$S=\int d^3x\left[\frac{1}{2}(\partial_\mu\phi^a)^2+\frac{\lambda}{2N}(\phi^a\phi^a)^2\right]$$

- IR fixed point can be studied using the Wilson-Fisher expansion in $\epsilon=4-d$.
- The model simplifies in the large N limit since it possesses conserved currents with all even spin

$$J_{(\mu_1\cdots\mu_s)}=\phi^a\partial_{(\mu_1}\cdots\partial_{\mu_s)}\phi^a+\ldots$$

Higher Spin Gauge Theory

- An AdS_4 dual of the large N sigma model was proposed. I.K., Polyakov (2002)
- It is the Fradkin-Vasiliev gauge theory of an infinite number of interacting massless gauge fields with all even spins.
- Large N makes the dual theory semi-classical, but there is no small AdS curvature limit. This makes the theory difficult to study in the dual AdS formulation.

M2 Brane Theory

- The theory on N coincident M2-branes has N=8, the maximum possible supersymmetry in 3 dimensions.
- When N is large, its dual description is provided by the weakly curved $AdS_4 \times S^7$ background in 11-dimensional M-theory which is essentially described by Einstein gravity coupled to other fields.
- This dual description is tractable and makes many non-trivial predictions.

- A general prediction of the AdS/CFT duality is that the number of degrees of freedom on a large number N of coincident M2-branes scales as $N^{3/2}$

  I.K., A. Tseytlin (1996)

- This is much smaller than the N^2 scaling found in the 4-d SYM theory on N coincident D3-branes (as described by the dual gravity). The normalization of entropy is ¾ of that in the free theory.
  Gubser, I.K., Peet (1996)

What is the M2 Brane Theory?

- It is the Infrared limit of the D2-brane theory, the N=8 supersymmetric Yang-Mills theory in 2+1 dimensions, i.e. it describes the degrees of freedom at energy much lower than $(g_{YM})^2$
- The number of such degrees of freedom $\sim N^{3/2}$ is much lower than the number of UV degrees of freedom $\sim N^2$.
- Is there a more direct way to characterize the Infrared Scale-Invariant Theory?

The BLG Theory

- In a remarkable recent development, Bagger and Lambert, and Gustavsson formulated an SO(4) Chern-Simons Gauge Theory with manifest N=8 superconformal gauge theory. In Van Raamsdonk's SU(2)xSU(2) formulation,

$$X^* = -\varepsilon X \varepsilon$$

$$\mathcal{S} = \int d^3x \, \mathrm{tr}\Big[-(\mathcal{D}^\mu X^I)^\dagger \mathcal{D}_\mu X^I + i\bar{\Psi}^\dagger \Gamma^\mu \mathcal{D}_\mu \Psi$$
$$- \frac{2if}{3}\bar{\Psi}^\dagger \Gamma^{IJ}(X^I X^{J\dagger}\Psi + X^J \Psi^\dagger X^I + \Psi X^{I\dagger} X^J) - \frac{8f^2}{3}\mathrm{tr}\, X^{[I}X^{\dagger J}X^{K]}X^{\dagger[K}X^J X^{\dagger I]}$$
$$+ \frac{1}{2f}\epsilon^{\mu\nu\lambda}(A_\mu \partial_\nu A_\lambda + \frac{2i}{3}A_\mu A_\nu A_\lambda) - \frac{1}{2f}\epsilon^{\mu\nu\lambda}(\hat{A}_\mu \partial_\nu \hat{A}_\lambda + \frac{2i}{3}\hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda)\Big]$$

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- X^I are the 8 fields transforming in (2,2), which is the 4 of SO(4)

$$X^I = \frac{1}{2}(x_4^I \mathbb{1} + i x_i^I \sigma^i)$$

- It was suggested that this theory describes two coincident M2-branes, but some of the details were hard to pin down.

- Since the ♏ matrix makes sense only for the SU(2) gauge group, it was also not clear how to generalize this construction to more than two M2-branes.

The ABJM Theory

- Aharony, Bergman, Jafferis and Maldacena argued that the correct description of a pair of M2-branes is slightly different. It involves U(2) x U(2) gauge theory.
- Let us form unconstrained complex matrices

$$Z^1 = X^1 + iX^5 \,, \qquad W_1 = X^{3\dagger} + iX^{7\dagger}$$
$$Z^2 = X^2 + iX^6 \,, \qquad W_2 = X^{4\dagger} + iX^{8\dagger}$$

- This breaks the manifest global symmetry to SU(2)xSU(2), but in fact the symmetry is higher.

- For N M2-branes ABJM theory easily generalizes to U(N) x U(N) gauge group. The theory with Chern-Simons coefficient k is then conjectured to be dual to AdS_4 x S^7/Z_k supported by N units of flux. This corresponds to N M2-branes placed at the orbifold C^4/Z_k which multiplies each of the 4 complex coordinates by $e^{2□i/k}$ $y^A \to e^{2\pi i/k} y^A$
- For k>2 this theory has N=6 supersymmetry, in agreement with this conjecture. In particular, the theory has manifest SO(6)~SU(4) R-symmetry.

$SU(4)_R$ Symmetry

- The classical action of this theory indeed has this symmetry. Benna, IK, Klose, Smedback

$$V^{\text{bos}} = -\frac{L^2}{48}\,\mathrm{tr}\Big[Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + 4Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger - 6Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger\Big]$$

$$V^{\text{ferm}} = \frac{iL}{4}\,\mathrm{tr}\Big[Y_A^\dagger Y^A \psi^{B\dagger}\psi_B - Y^A Y_A^\dagger \psi_B \psi^{B\dagger} + 2Y^A Y_B^\dagger \psi_A \psi^{B\dagger} - 2Y_A^\dagger Y^B \psi^{A\dagger}\psi_B - \epsilon^{ABCD} Y_A^\dagger \psi_B Y_C^\dagger \psi_D + \epsilon_{ABCD} Y^A \psi^{B\dagger} Y^C \psi^{D\dagger}\Big]\,.$$

- Y^A, A=1,...4, are complex N x N matrices:

$$Y^A = \{Z^1, Z^2, W^{1\dagger}, W^{2\dagger}\}$$

- Since L=8□/k, the gauge theory is perturbative when N/k is small. The dual gravity is reliable when it is large.

- The SU(4) symmetry currents have the standard structure

$$j^A_{\mu B} = \mathrm{Tr}\left[Y^A \mathcal{D}_\mu Y^\dagger_B - (\mathcal{D}_\mu Y^A) Y^\dagger_B + i\psi^{\dagger A}\gamma^\mu \psi_B\right]$$

- In fact, there is also a U(1) current (the trace part) which enhances the symmetry to U(4).
- For k=1 or 2 the global symmetry should enhance to SO(8) according to the ABJM conjecture. In order to write the 12 additional currents we have to employ the `monopole operators' such as $(\mathcal{M}^{-2})^{\hat{a}\hat{b}}_{ab}$

$$J^{AB}_\mu = \mathcal{M}^{-2}\left[Y^A \mathcal{D}_\mu Y^B - \mathcal{D}_\mu Y^A Y^B + i\psi^{\dagger A}\gamma^\mu \psi^{\dagger B}\right]$$

Monopole Operators

- They modify the behavior of fields near an insertion point to create a U(1) magnetic flux through a 2-sphere surrounding the point

Borokhov, Kapustin, Wu

Kapustin, Witten

$$A = \frac{H}{2} \frac{\pm 1 - \cos\theta}{r} \mathrm{d}\varphi$$

- For a U(N) gauge theory, the generator H describes the U(1) embedding. Due to Dirac quantization, it is labeled by a set of integers:

$$H = \mathrm{diag}(q_1, \ldots, q_N) \qquad q_1 \geq q_2 \ldots \geq q_N$$

- With Chern-Simons level k, these operators transform under U(N) as a representation with a Young tableaux with $kq_1, kq_2, \ldots, kq_N$ rows.

- In a recent paper, Benna, Klose and I studied the monopole operators in the U(N) x U(N) ABJM theory with $H = \hat{H}$
- For example, the operators like $(\mathcal{M}^{-2})^{\hat{a}\hat{b}}_{ab}$ appearing in the SO(8) R-symmetry currents correspond to $kq_1 = k\hat{q}_1 = 2$
- We have shown that the monopole insertion does not alter the `naïve' global charges and dimensions of these currents.

Relevant Deformations

- The M2-brane theory may be perturbed by relevant operators that cause it to flow to new fixed points with reduced supersymmetry. Benna, IK, Klose, Smedback; IK, Klose, Murugan; Ahn
- For example, a quadratic superpotential deformation, allowed for k=1, 2, may preserve SU(3) flavor symmetry while making one of the 4 superfields massive.

Squashed, stretched and warped

- The dual AdS_4 background of M-theory should also preserve N=2 SUSY and SU(3) flavor symmetry. Such an extremum of gauged 4-d supergravity was found 25 years ago by Warner. Upon uplifting to 11-d we find a warped product of AdS_4 and of a `stretched and squashed’ 7-sphere.
- Spectrum of multiplets in gauged SUGRA may be compared with the gauge theory.

	Scenario I	Scenario II
Hyper	$[n+2,0]_{\frac{n+2}{3}},\ [0,n+2]_{-\frac{n+2}{3}}$	$[n+2,0]_{-\frac{2n+4}{3}},\ [0,n+2]_{\frac{2n+4}{3}}$
Vector	$[n+1,1]_{\frac{n}{3}},\ [1,n+1]_{-\frac{n}{3}}$	$[n+1,1]_{-\frac{2n}{3}},\ [1,n+1]_{\frac{2n}{3}}$
Gravitino	$[n+1,0]_{\frac{n+1}{3}},\ [0,n+1]_{-\frac{n+1}{3}}$	$[n+1,0]_{-\frac{2n-1}{3}},\ [0,n+1]_{\frac{2n-1}{3}}$
Graviton	$[0,0]_{n},\ [0,0]_{-n}$	$[0,0]_{0},\ [0,0]_{0}$

- We find that Scenario I gives $SU(3)xU(1)_R$ quantum numbers in agreement with the proposed gauge theory dual where they are schematically given by

	Z^A	ζ^A	$Z_A^\dagger$	$\zeta_A^\dagger$	Z^4	ζ^4	$Z_4^\dagger$	$\zeta_4^\dagger$	x	θ	$\bar\theta$
SU(3)	$\mathbf{3}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Dimension	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{5}{6}$	1	$\frac{3}{2}$	1	$\frac{3}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$
R-charge	$+\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$+1$	0	-1	0	0	$+1$	-1

- In this theory, we find, for example fermionic operators of R-charge -1/3 and dimension 7/6. They are dual to fermionic fields in the AdS_4 whose masses and charges seem to fall in the `interesting range' pointed out in the work by Faulkner, Liu, McGreevy and Vegh; Schalm and Zaanen.

Spin-2 Perturbations

- Consider graviton perturbations in AdS with $h^i{}_i = 0\,, \qquad \partial^i h_{ij} = 0$

$\phi = h^i{}_j$ satisfy the minimal scalar equation

$\Box\phi = 0$ $\qquad \phi = \Phi(x^i, r)Y(y^\alpha)$ $\qquad \Box_4\Phi(r, x^i) - m^2\Phi(r, x^i) = 0$

For the (p,q) irrep of SU(3), we find the angular dependence IK, Pufu, Rocha

$$Y(y^\alpha) = a^{j_1 j_2 \cdots j_q}_{i_1 i_2 \ldots i_p} \left(\prod_{k=1}^{p} z^{i_k}\right)\left(\prod_{l=1}^{q} \bar{z}_{j_l}\right) w^{n_r}$$
$$\times \begin{cases} {}_2F_1(-j, 3+p+q+j+n_r; 3+p+q; 1-w\bar{w}) & \text{if } n_r \geq 0 \\ {}_2F_1(-j+n_r, 3+p+q+j; 3+p+q; 1-w\bar{w}) & \text{if } n_r < 0\,. \end{cases}$$

- Here are the low lying operators

$$\mathcal{T}^{(0)}_{\alpha\beta} = \bar{D}_{(\alpha}\bar{\mathcal{Z}}_A D_{\beta)}\mathcal{Z}^A + i\bar{\mathcal{Z}}_A \overleftrightarrow{\partial}_{\alpha\beta}\mathcal{Z}^A$$

	$[p,q]_R$	j	n_r	Δ	m^2L^2	Operator
*	$[0,0]_0$	0	0	3	0	$\mathcal{T}^{(0)}_{\alpha\beta}$
*	$[0,0]_{\pm1}$	0	±1	4	4	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^4,\ \mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_4$
	$[0,1]_{-\frac{1}{3}},\ [1,0]_{\frac{1}{3}}$	0	0	$\frac{1}{6}\left(9+\sqrt{145}\right)$	$\frac{16}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_A,\ \mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^A$
*	$[0,0]_{\pm2}$	0	±2	5	10	$\mathcal{T}^{(0)}_{\alpha\beta}(\mathcal{Z}^4)^2,\ \mathcal{T}^{(0)}_{\alpha\beta}(\bar{\mathcal{Z}}_4)^2$
	$[0,0]_0$	1	0	$\frac{1}{2}\left(3+\sqrt{41}\right)$	8	$\mathcal{T}^{(0)}_{\alpha\beta}\left(1-4a^2\mathcal{Z}^4\bar{\mathcal{Z}}_4\right)$
	$[0,1]_{-\frac{4}{3}},\ [1,0]_{\frac{4}{3}}$	0	$-1,\ 1$	$\frac{1}{6}\left(9+\sqrt{337}\right)$	$\frac{64}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_A\bar{\mathcal{Z}}_4,\ \mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}_A\mathcal{Z}^4$
	$[0,1]_{\frac{2}{3}},\ [1,0]_{-\frac{2}{3}}$	0	$-1,\ 1$	$\frac{1}{6}\left(9+\sqrt{313}\right)$	$\frac{58}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_A\mathcal{Z}^4,\ \mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}_A\bar{\mathcal{Z}}_4$
	$[0,2]_{-\frac{2}{3}},\ [2,0]_{\frac{2}{3}}$	0	0	$\frac{1}{6}\left(9+\sqrt{217}\right)$	$\frac{34}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_{(A}\bar{\mathcal{Z}}_{B)},\ \mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^{(A}\mathcal{Z}^{B)}$
	$[1,1]_0$	0	0	4	4	$\mathcal{T}^{(0)}_{\alpha\beta}\left(\mathcal{Z}^A\bar{\mathcal{Z}}_B-\frac{1}{3}\delta^A_B\mathcal{Z}^C\bar{\mathcal{Z}}_C\right)$
	$[0,0]_{\pm1}$	1	±1	$\frac{1}{2}(3+\sqrt{65})$	14	$\mathcal{T}^{(0)}_{\alpha\beta}\left(2-5a^2\mathcal{Z}^4\bar{\mathcal{Z}}_4\right)\mathcal{Z}^4$, c.c.
*	$[0,0]_{\pm3}$	0	±3	6	18	$\mathcal{T}^{(0)}_{\alpha\beta}\left(\mathcal{Z}^4\right)^3,\mathcal{T}^{(0)}_{\alpha\beta}\left(\bar{\mathcal{Z}}_4\right)^3$
	$[1,0]_{-\frac{5}{3}},[0,1]_{\frac{5}{3}}$	0	$-2,+2$	$\frac{1}{6}(9+\sqrt{553})$	$\frac{118}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^A\left(\bar{\mathcal{Z}}_4\right)^2,\mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_A\left(\mathcal{Z}^4\right)^2$
	$[1,0]_{\frac{1}{3}},[0,1]_{-\frac{1}{3}}$	1	0	$\frac{1}{6}(9+\sqrt{505})$	$\frac{106}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^A\left(1-5a^2\bar{\mathcal{Z}}_4\mathcal{Z}^4\right)$, c.c.
	$[1,0]_{\frac{7}{3}},[0,1]_{-\frac{7}{3}}$	0	$2,-2$	$\frac{1}{6}(9+\sqrt{601})$	$\frac{130}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^A\left(\mathcal{Z}^4\right)^2,\mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_A\left(\bar{\mathcal{Z}}_4\right)^2$
	$[1,1]_{\pm1}$	0	±1	5	10	$\mathcal{T}^{(0)}_{\alpha\beta}\left(\mathcal{Z}^A\bar{\mathcal{Z}}_B-\frac{1}{3}\delta^A_B\mathcal{Z}^C\bar{\mathcal{Z}}_C\right)\mathcal{Z}^4$, c.c.
	$[2,0]_{-\frac{1}{3}},[0,2]_{\frac{1}{3}}$	0	$-1,1$	$\frac{1}{6}(9+\sqrt{409})$	$\frac{82}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^{(A}\mathcal{Z}^{B)}\bar{\mathcal{Z}}_4,\mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_{(A}\bar{\mathcal{Z}}_{B)}\mathcal{Z}^4$
	$[2,0]_{\frac{5}{3}},[0,2]_{-\frac{5}{3}}$	0	$1,-1$	$\frac{1}{6}(9+\sqrt{457})$	$\frac{94}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^{(A}\mathcal{Z}^{B)}\mathcal{Z}^4,\mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_{(A}\bar{\mathcal{Z}}_{B)}\bar{\mathcal{Z}}_4$
	$[2,1]_{\frac{1}{3}},[1,2]_{-\frac{1}{3}}$	0	0	$\frac{1}{6}(9+\sqrt{313})$	$\frac{58}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\left(\mathcal{Z}^{(A}\mathcal{Z}^{B)}\bar{\mathcal{Z}}_C-\frac{1}{3}\delta^{(A}_C\mathcal{Z}^{B)}\mathcal{Z}^D\bar{\mathcal{Z}}_D\right)$, c.c.
	$[3,0]_1,[0,3]_{-1}$	0	0	$\frac{1}{2}(3+\sqrt{33})$	6	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^{(A}\mathcal{Z}^B\mathcal{Z}^{C)},\mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_{(A}\bar{\mathcal{Z}}_B\bar{\mathcal{Z}}_{C)}$

Further Directions

- Other examples of AdS_4/CFT_3 dualities with N=1,2,3,... supersymmetry are being studied by many groups.
- Ultimate Physics Goal: to find a `simple' dual of a 3-d strongly interacting fixed point realized in Nature.