Imbalanced mixtures and anisotropic hydrodynamics in gauge/gravity duality

Johanna Erdmenger

Max Planck Institute for Physics, Munich

Based on joint work with M. Ammon, V. Grass, M. Kaminski, P. Kerner, H.T. Ngo, A. O'Bannon, H. Zeller

Motivation:

Gauge/gravity duality: New tools for strongly coupled systems

Condensed matter physics: Many timely and interesting questions

How may the two be joined together?

This talk: Some examples

Examples include:

New holographic quantum critical points

Holographic p-wave superfluids/superconductors

Nematic phase (Condensate breaks rotational symmetry)

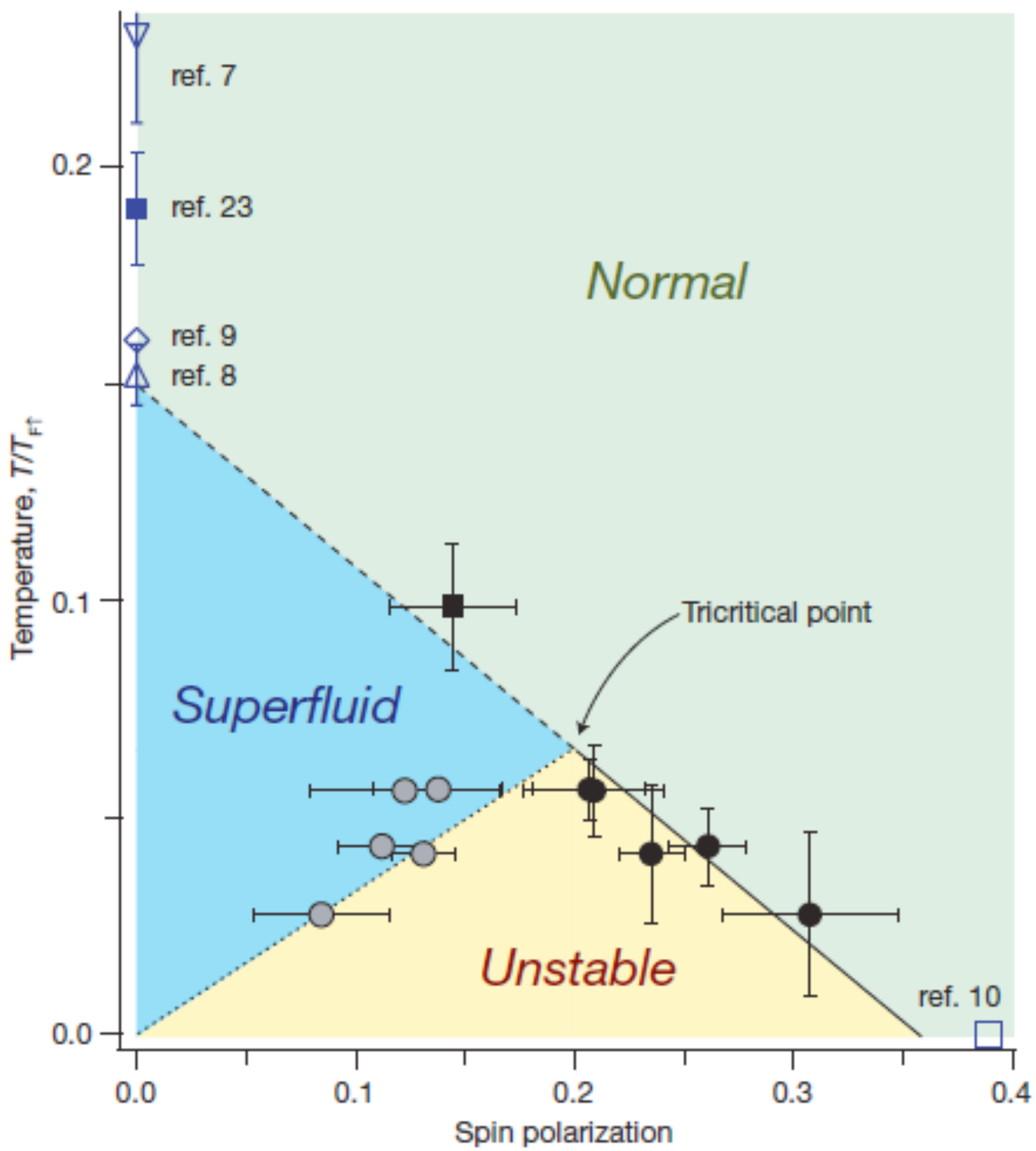
Flexoelectricity (Strain causes polarization)

Imbalanced mixtures

Contain different number of spin up and spin down particles

How does an imbalance in numbers (spin polarization) affect the superfluid phase transition?

Superfluidity in imbalanced mixtures



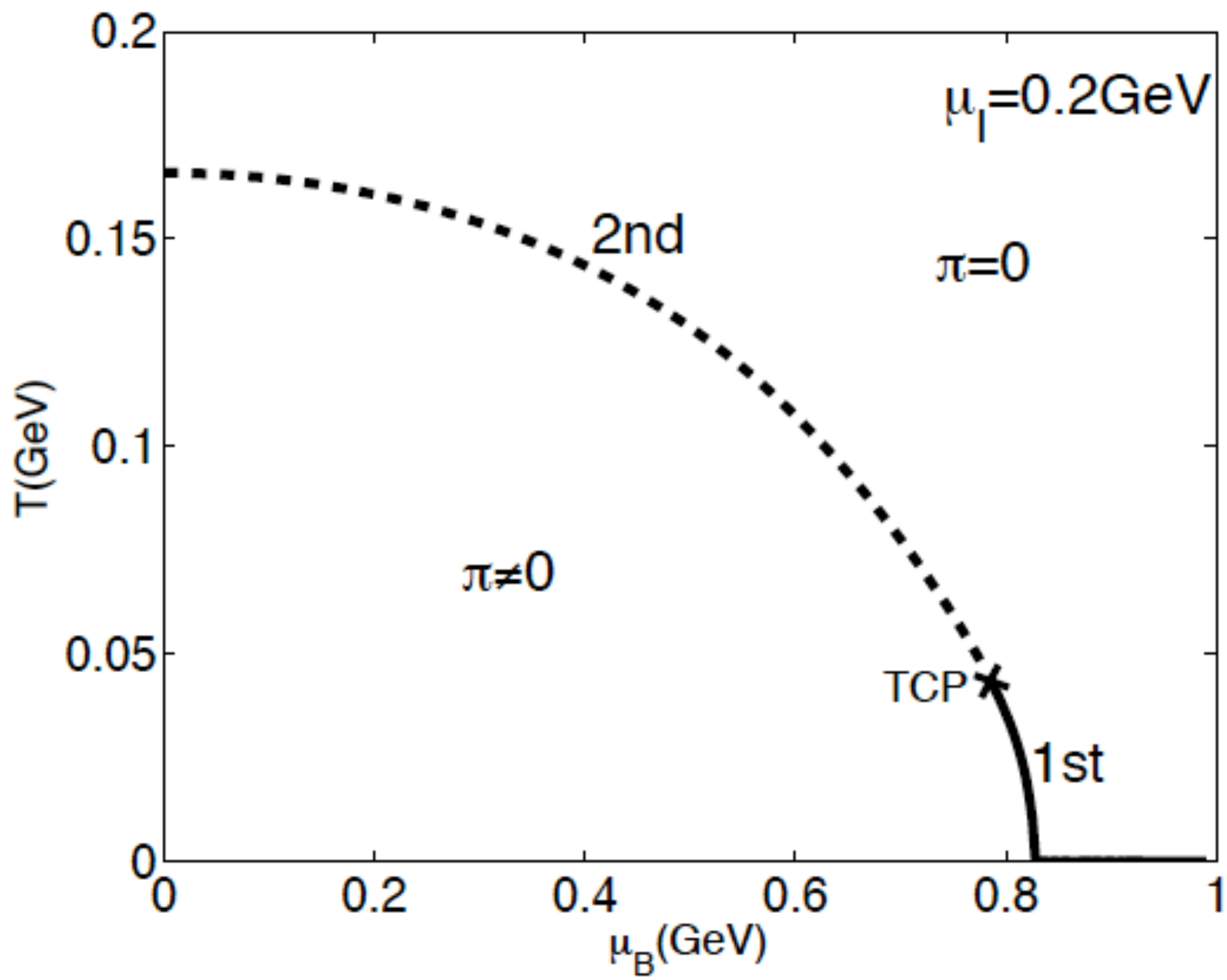
Shin, Schunck, Schirotzek, Ketterle, Nature 2008

Generic phase diagram in condensed matter

cf. G. Lonzarich's talk

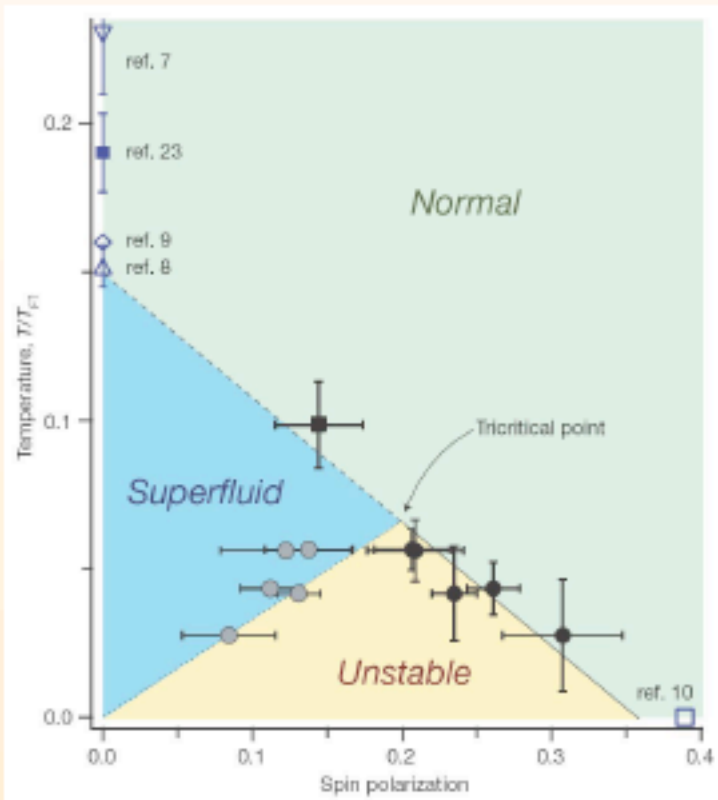
(Sorry for omitting further references...)

QCD at finite isospin chemical potential

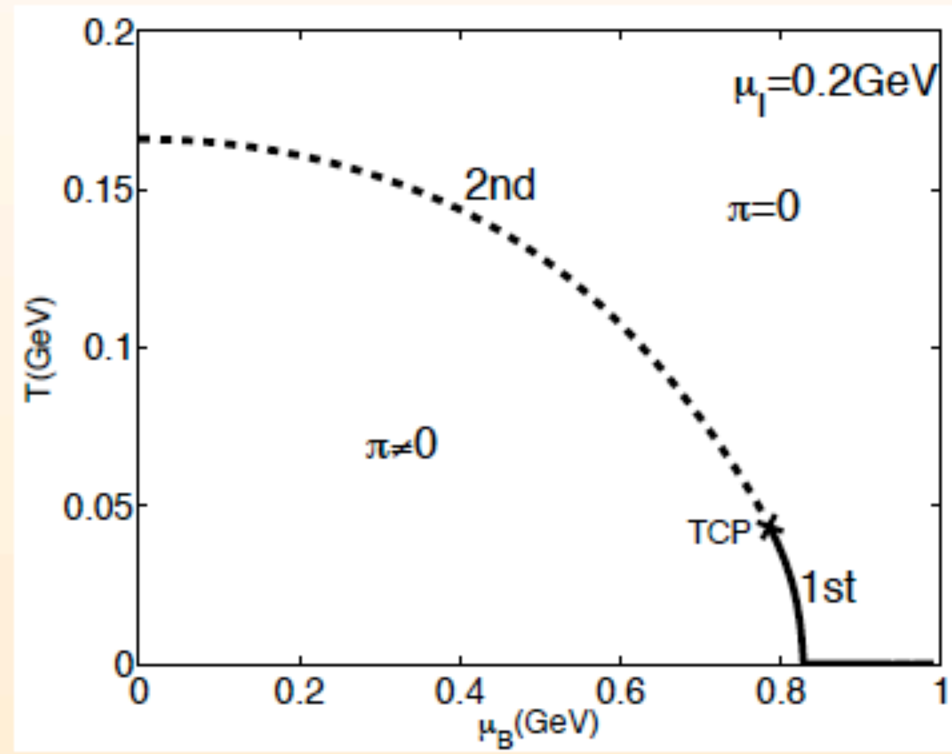


He, Jin, Zhuang, PRD 2005

Inbalanced Mixtures and Quantum Phase Transition



Shin, Schunck, Schirotzek, Ketterle, Nature 2008



He, Jin, Zhuang, PRD 2005

Lithium superfluid

QCD at finite isospin density

There appears to be universal behavior

Can we describe imbalanced mixtures in gauge/gravity duality?

Yes!

Can we obtain a similar phase diagram?

We can, in principle...

We obtain holographic imbalanced mixtures from probe branes in gauge/gravity duality

Additional structure on the gravity side

Lagrangian explicitly known in dual field theory

Holographic superfluid from probe branes

➡ Condensate known explicitly on field theory side

Reminder: Holographic Superfluids/Superconductors

- Holographic Superconductors from charged scalar in Einstein-Maxwell gravity
  (Gubser; Hartnoll, Herzog, Horowitz)
- p-wave superconductor
  current dual to gauge field condensing
  (Gubser, Pufu)
  SU(2) Einstein-Yang-Mills model

s-wave superconductor:

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F^{ab}F_{ab} - V(|\psi|) - |\nabla\psi - iqA\psi|^2$$

Operator $\mathcal{O}$ dual to scalar ψ condensing

Herzog, Hartnoll, Horowitz 2008

p-wave superconductor:

$$S = \frac{1}{2\kappa^2}\int d^4x \left[R - \frac{1}{4}(F^a_{\mu\nu})^2 + \frac{6}{L^2}\right]$$

Current J^1_3 dual to gauge field component A^{1x} condensing

Gubser, Pufu 2008

P-wave superconductor from probe branes

Ammon, J.E., Kaminski, Kerner 0810.2316, 0903.1864

- A holographic superconductor with field theory in 3+1 dimensions for which
- the dual field theory is explicitly known
- there is a qualitative ten-dimensional string theory picture of condensation

This is achieved in the context of
adding flavor to gauge/gravity duality

cf. talks by A. O'Bannon, A. Parnachev

Brane probes added on gravity side $\Rightarrow$
fundamental d.o.f. in the dual field theory (quarks)

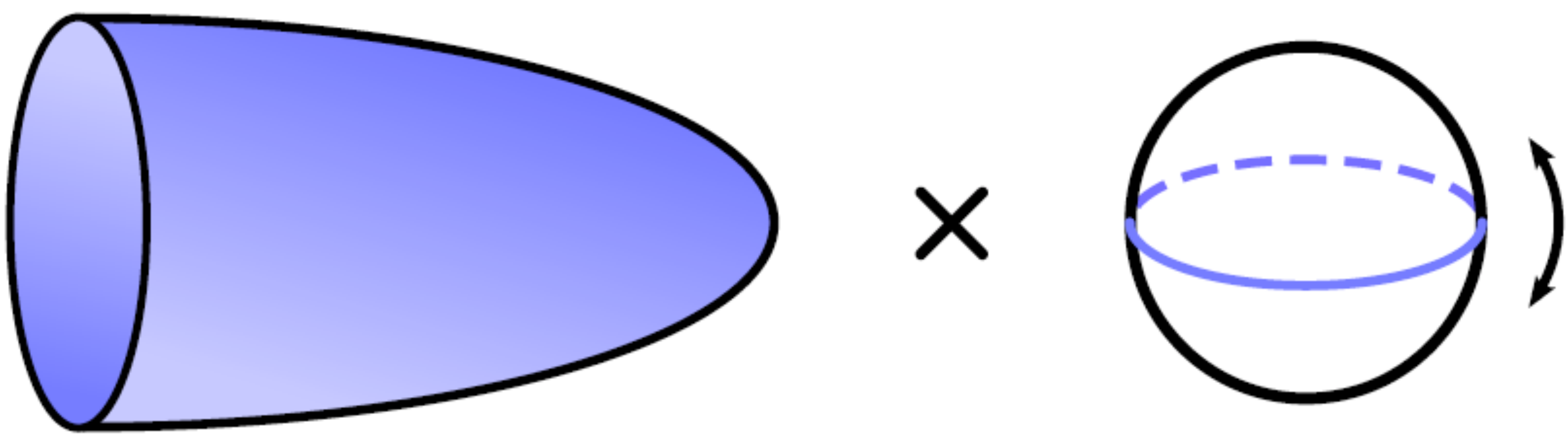
Additional D-branes within $AdS_5 \times S^5$ or deformed version thereof

Quarks within Gauge/Gravity Duality

Adding D7-Brane Probe:

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		

Symmetry: $SO(4) \times SO(2) \sim SU(2) \times SU(2) \times U(1)$



Probe brane fluctuations $\Rightarrow$ Masses of mesons ($\bar{\psi}\psi$ bound states)

On gravity side:
Probe brane fluctuations described by Dirac-Born-Infeld action

$$S_{\mathrm{DBI}} = -T_{D7} \int \mathrm{d}^8\xi \, \mathrm{Str}\sqrt{|\det(G + 2\pi\alpha' F)|}$$

On field theory side: Lagrangian explicitly known

$$\mathcal{L} = \mathcal{L}_{\mathcal{N}=4} + \mathcal{L}({\psi_q}^i, {\phi_q}^i)$$

Fluctuations are representations of SU(2) x SU(2) x U(1)

Turn on finite temperature and isospin chemical potential:

Finite temperature: Embed D7 brane in black hole background

Isospin chemical potential: Probe of two coincident D7 branes

Additional symmetry U(2) = SU(2)$_I$ x U(1)$_B$

$$A_0^3 = \mu - \frac{\tilde{d}_0^3}{2\pi\alpha'}\frac{\rho_H}{\rho^2} + \dots\,, \qquad A_3^1 = -\frac{\tilde{d}_1^3}{2\pi\alpha'}\frac{\rho_H}{\rho^2} + \dots$$

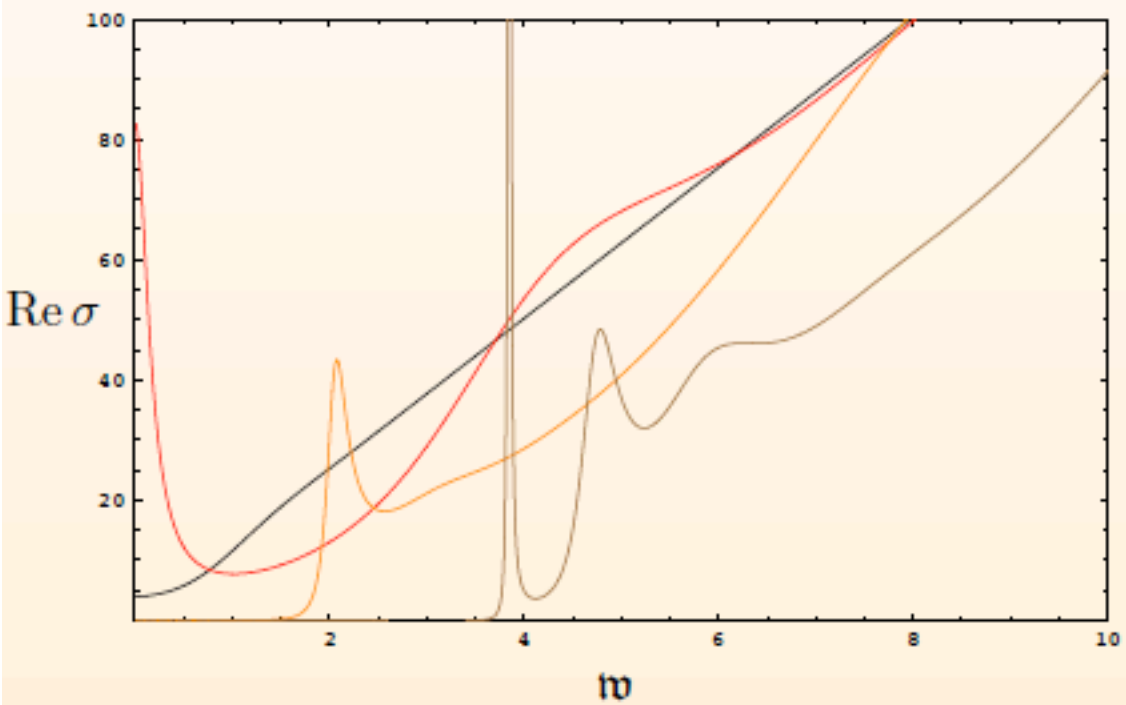
Condensate $\langle J_3 \rangle, \quad J_3 = \bar{\psi}_d \gamma_3 \psi_u + \ bosons$

Calculate correlators from fluctuations

Conductivity

Frequency-dependent conductivity $\sigma(\omega) = \frac{i}{\omega} G^R(\omega)$

G^R retarded Green function for fluctuation a_2^3



$\mathfrak{w} = \omega/(2\pi T)$

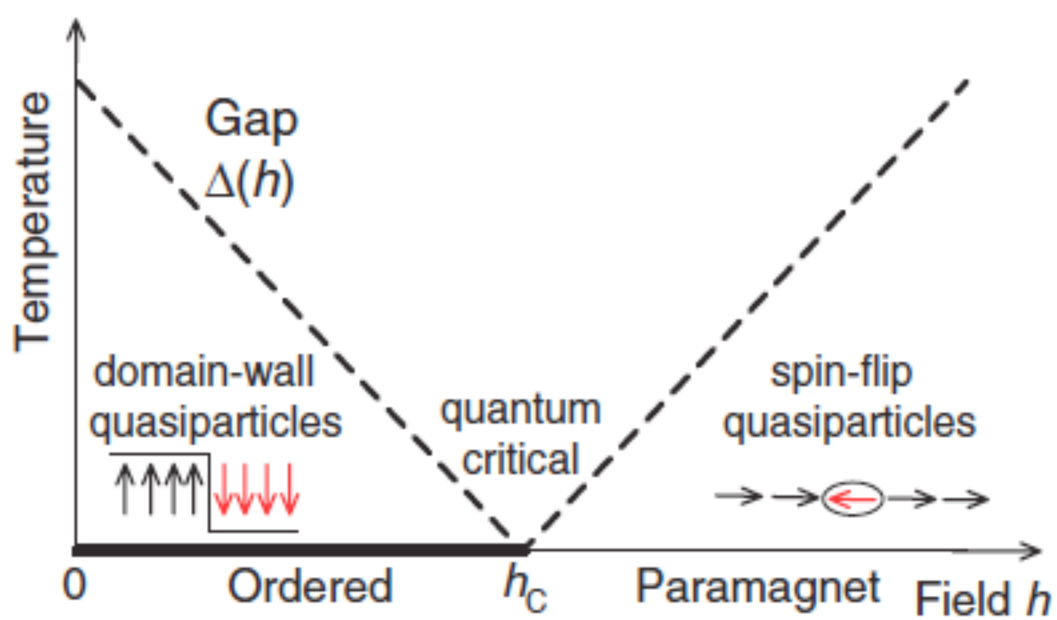
T/T_c: Black: ∞, Red: 1, Orange: 0.5, Brown: 0.28.

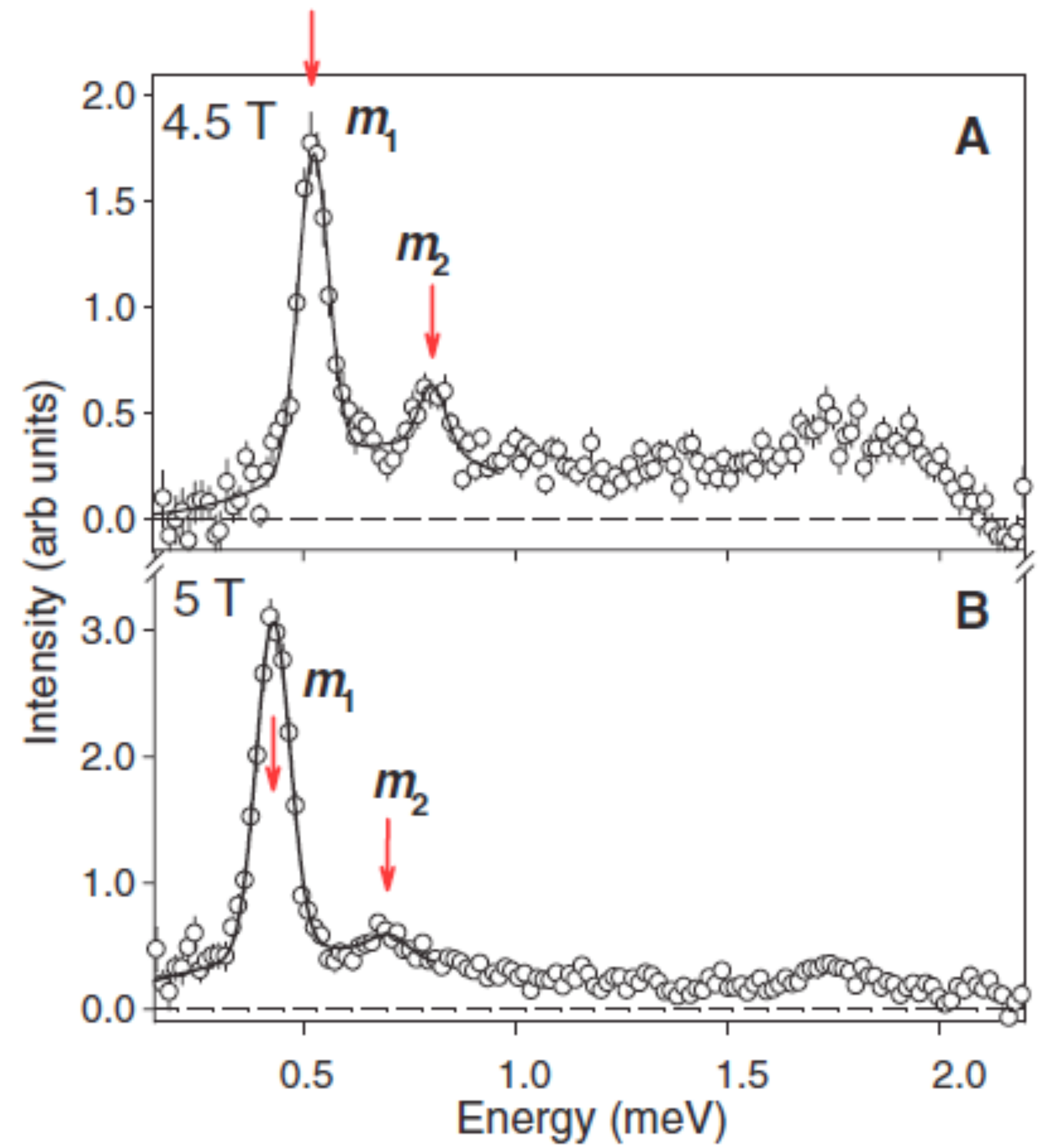
(Vanishing quark mass)

Interpretation: Frictionless motion of mesons through plasma

Aside: Comparison with Ising chain

Quantum Criticality in an Ising Chain: Experimental Evidence for Emergent E_8 Symmetry
R. Coldea, *et al.*
Science **327**, 177 (2010)



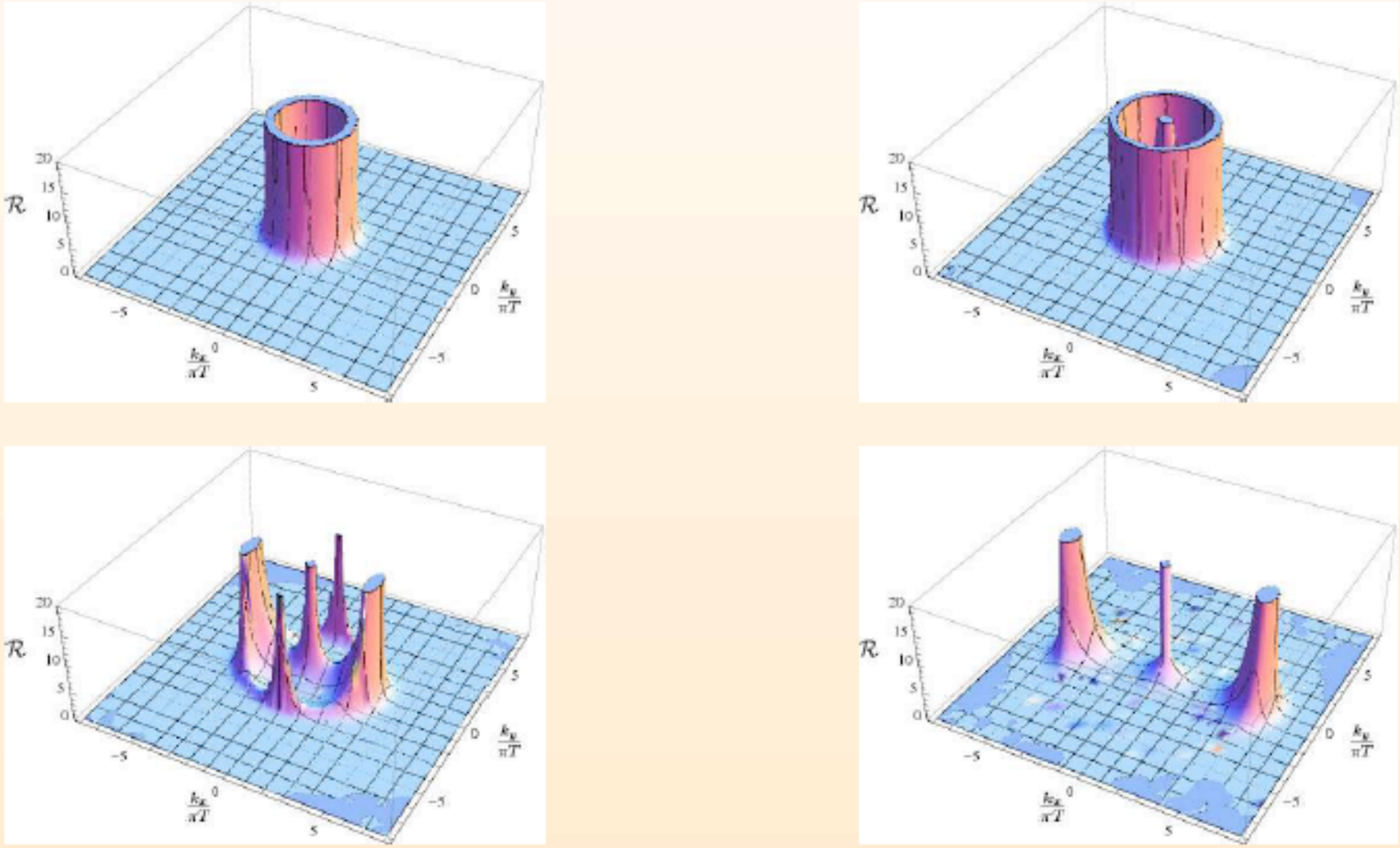


Experimental realization of CFT result
(Zamolodchikov 1989)

Fermions

Ammon, J.E., Kaminski, O'Bannon 1003.1134

Use fermionic part of D7 DBI action to study fermionic fluctuations



Holographic Imbalanced Mixtures

Turn on both isospin and baryon chemical potential

$$U(2) = SU(2)_I \times U(1)_B$$

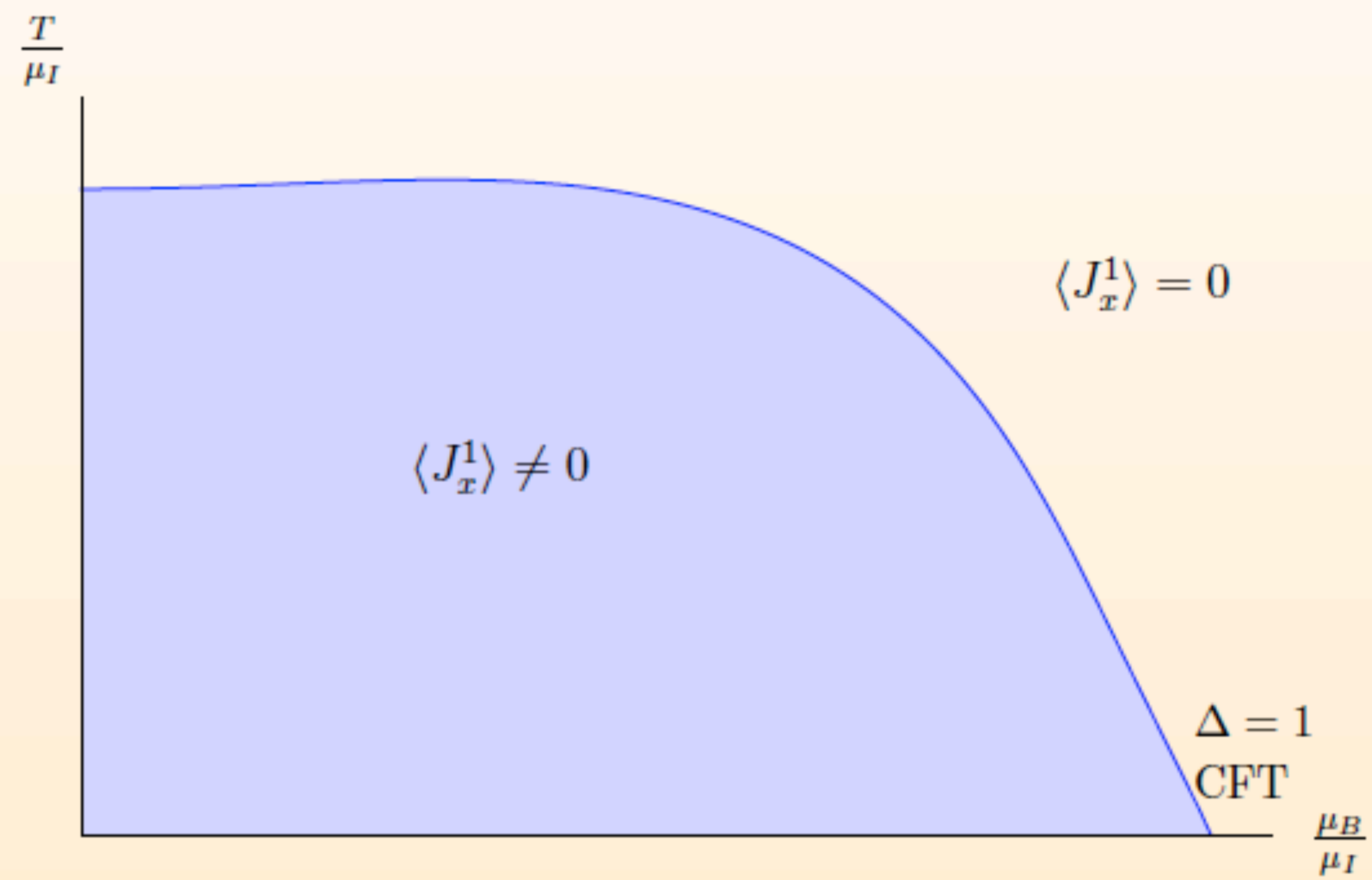
Condensate $\bar{\psi}_d \gamma_3 \psi_u$ (rho meson)

Increasing μ_B turns u into $\bar{u}$ quarks

Inbalanced Mixtures and Quantum Phase Transition

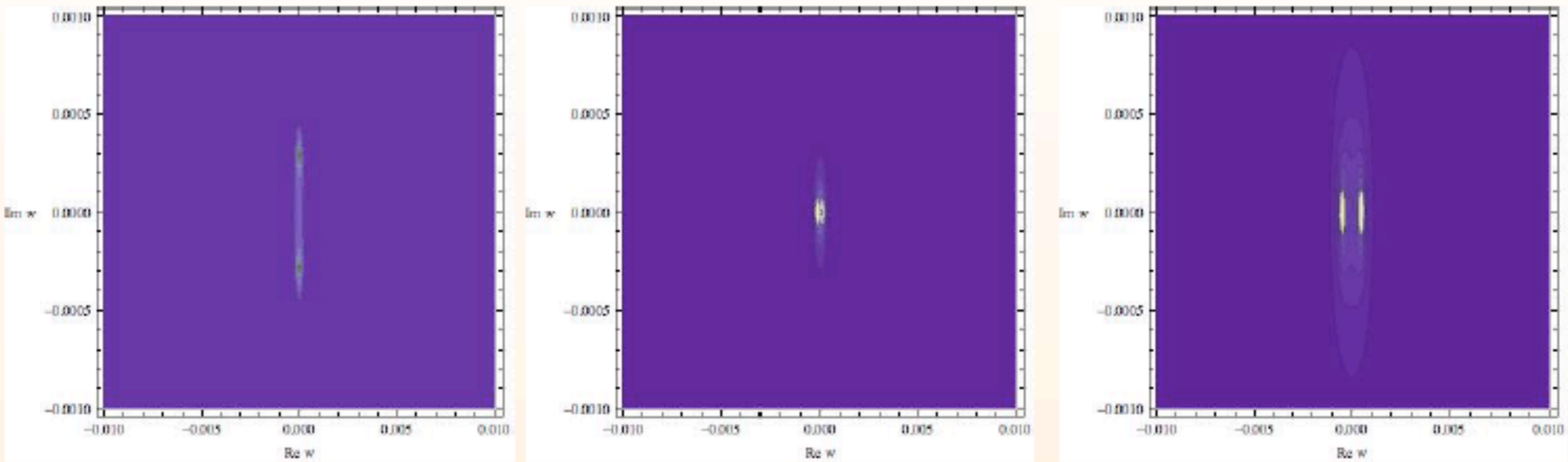
J.E., Graß, Kerner, Ngo 1103.4145

Turn on both isospin and baryon chemical potential in D3/D7 setup



Phase transition second order

Quantum Phase Transition



Quantum phase transition

Figure by Patrick Kerner

2. Imbalanced Mixtures

Bottom-up: Including the backreaction

Ammon, J.E., Graß, Kerner, O'Bannon 0912.3515

Einstein-Yang-Mills-Theory with $SU(2)$ gauge group

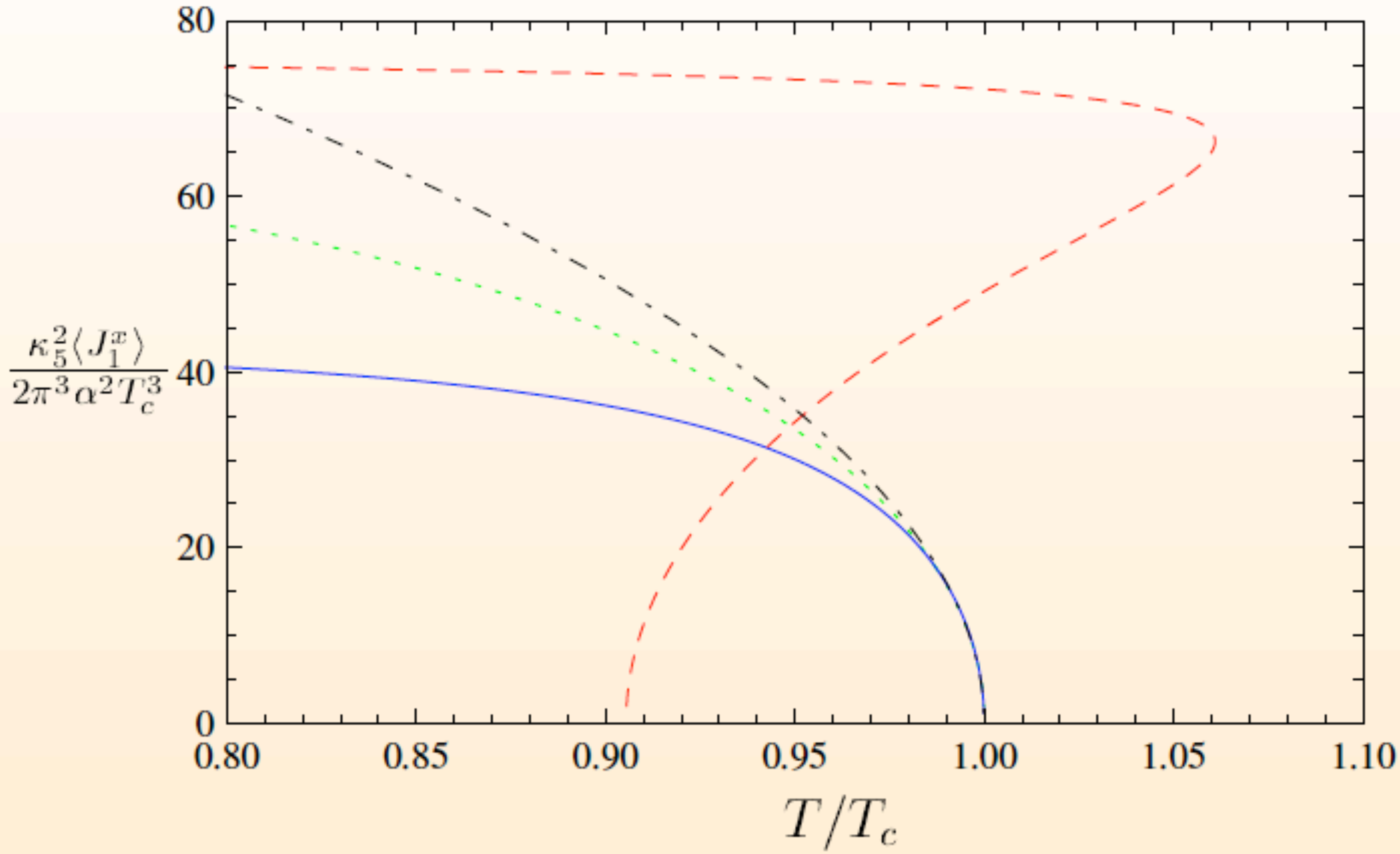
$$S = \int d^5x \sqrt{-g} \left[\frac{1}{2\kappa^2} (R - \Lambda) - \frac{1}{4\hat{g}^2} F^a_{\mu\nu} F^{a\mu\nu} \right]$$

$$\alpha = \frac{\kappa_5}{\hat{g}}$$

$\alpha^2 \propto$ number of charged d.o.f./all d.o.f.

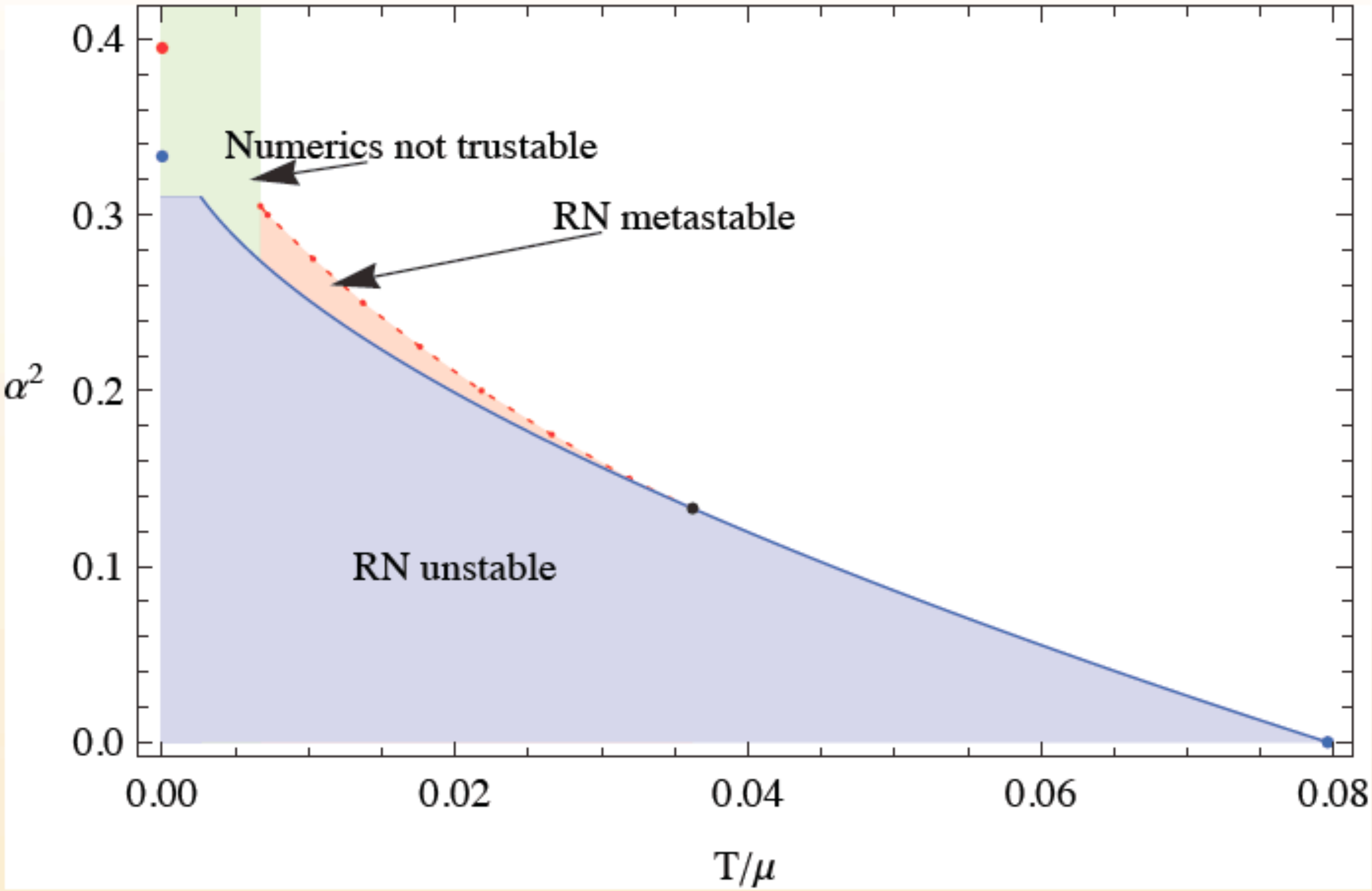
In presence of $SU(2)$ chemical potential, same condensation process as before

Phase transition



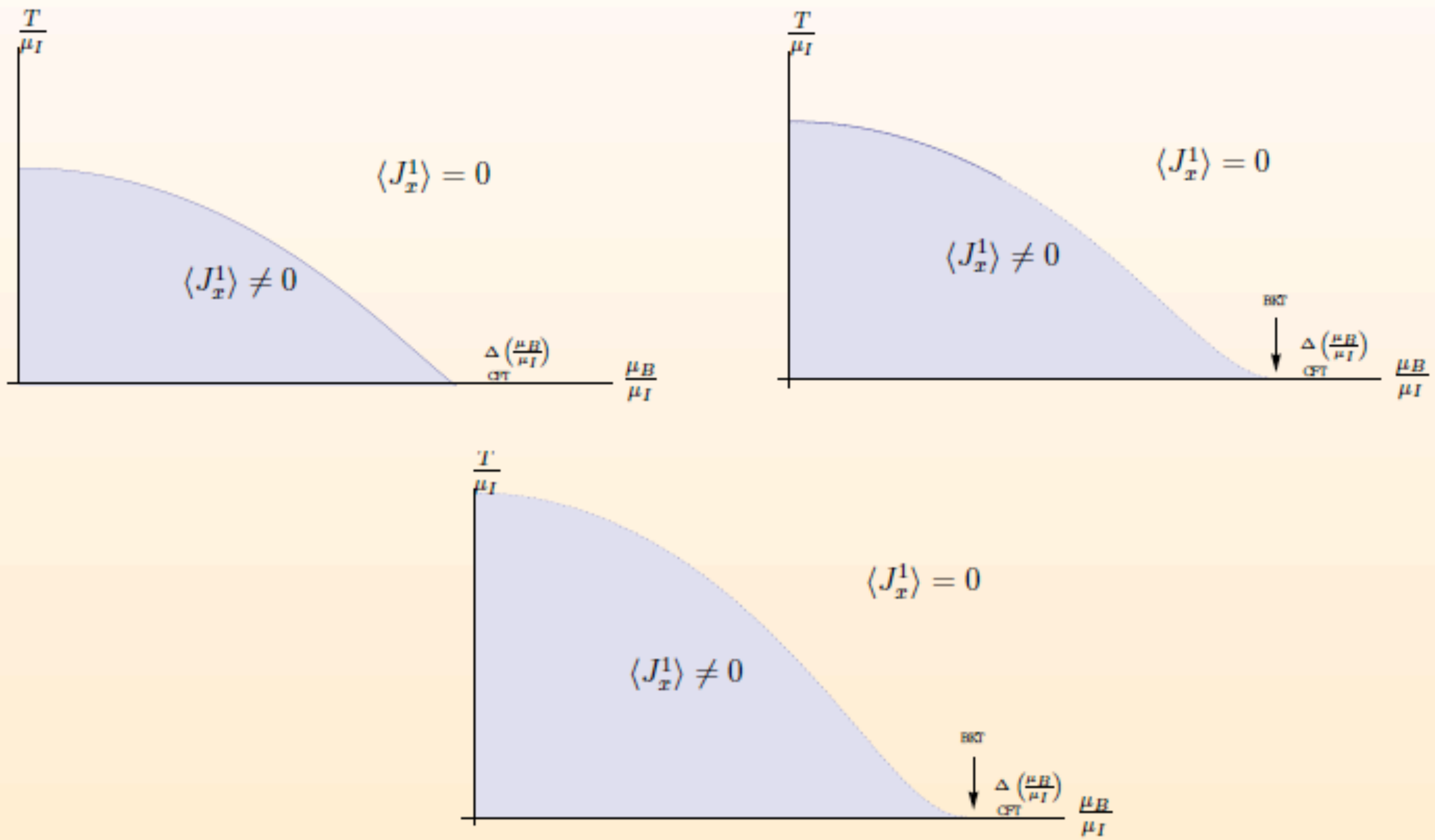
Phase transition becomes first order above α_{crit}

Phase diagram



Inbalanced Mixtures and Quantum Phase Transition

Example with backreaction: $SU(2)$ Einstein-Yang-Mills Model



J.E., Grass, Kerner, Ngo 1103.4145

BKT transition in gauge/gravity duality

Jensen, Karch, Son, Thompson 2010
Evans, Gebauer, Kim, Magou 2010

Order parameter scales as $\exp(-c/\sqrt{T_c - T})$

Gravity side: violation of the BF bound in the IR

IR $AdS_2 \times S^2$ region

Only possible when the two parameters have the same dimension

D3/D7 vs. backreacted model

D3/D7:

Effective IR mass of A_x^1/r vanishes,
independently of μ_B
BF bound violated along flow, but not in IR
Flavor fields directly interact with each other

Einstein-Yang-Mills:

Effective IR mass depends on μ_B/μ_I
BF bound violated in IR
$AdS_2 \times S^2$ region in IR
Flavor fields interact with gluon fields

3. Anisotropic shear viscosity

J.E., Kerner, Zeller 1011.5912

Universal result of AdS/CFT:

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Shear viscosity/Entropy density

Calculated from Kubo formula involving stress tensor two-point function

Kovtun, Policastro, Son, Starinets

Proof of Universality relies on:

Metric fluctuations = helicity two states

Anisotropic shear viscosity

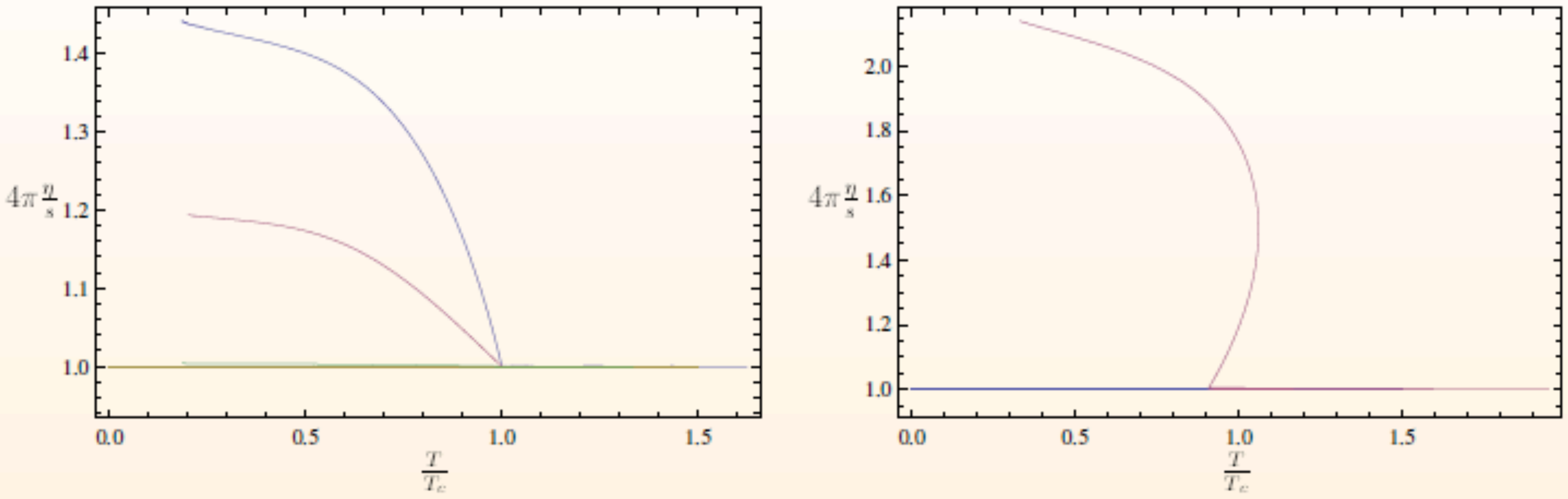
J.E., Kerner, Zeller 1011.5912

p-wave superconductor:

Fluctuations characterized by their transformation properties under unbroken $SO(2)$:

$$\begin{aligned} \text{helicity } 2: &\quad h_{yz}, h_{yy} - h_{zz} \\ \text{helicity } 1: &\quad h_{yt}, h_{xy}, h_{yr}; a^a_y \\ &\quad h_{zy}, h_{xz}, h_{zr}; a^a_z \\ \text{helicity } 0: &\quad h_{tt}, h_{yy} + h_{zz}, h_{xx}, h_{xt}, h_{xr}, h_{tr}, h_{rr}; \\ &\quad a^a_t, a^a_x, a^a_r \,. \end{aligned}$$

Anisotropic shear viscosity



$\eta_{yz}/s = 1/4\pi$; η_{xy}/s dependent on T and on α

Critical behaviour: $1-4\pi\frac{\eta_{xy}}{s} \propto \left(1-\frac{T}{T_c}\right)^{\beta}$ with $\beta = 1.00 \pm 3\%$, α-independent

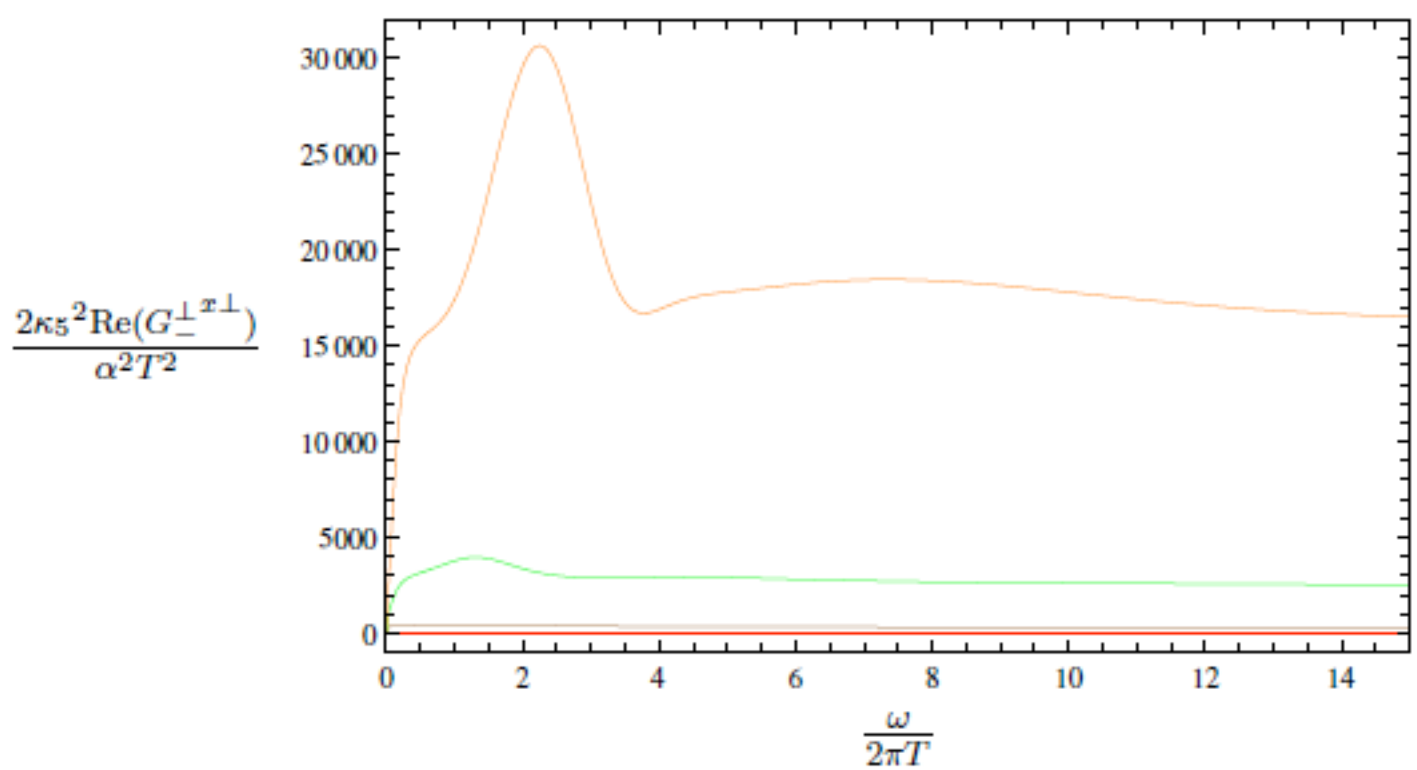
Non-universal behaviour at leading order in λ and N

Critical exponent confirmed analytically in Basu, Oh 1109.4592

Flexoelectric Effect

Nematic crystals:
A strain introduces spontaneous electrical polarization

In our case:
A strain $h_{x\perp}$ introduces an inhomogeneity in the current $\mathcal{J}_1^x$ which introduces a current $\mathcal{J}_\pm^\perp$



J.E., Kerner, Zeller
1110.0007

Conclusion

- D3/D7 with finite isospin: Holographic p-wave superconductor with known dual field theory
- Add baryon chemical potential: Imbalanced mixtures
- 1st order transitions possible in backreacted system
- Quantum critical point arising from AdS_2 in IR
- Anisotropic shear viscosity:
  Non-universal contribution at leading order in N and λ
- Flexoelectric effect