Metals in which the quasiparticle picture fails

Finding confusion by taking theory seriously

KITP – UCSB 2011

I will focus on one widely measured property of solids – the resistivity

It is a theoretically complicated, non-equilibrium property of solids.

Ideally, it is related by the fluctuation dissipation theorem to current density fluctuations.

Somehow, either explicitly or implicitly, there needs to be a mechanism for transfer of momentum to a "heat bath."

At low T especially, ρ is usually dominated by disorder effects:

$(l)_{imp}$ vs $(l)_{int}$

----- **many advances derive from progress in crystal growth.**

I will focus on one widely measured property of solids – the resistivity

It is a theoretically complicated, non-equilibrium property of solids.

Ideally, it is related by the fluctuation dissipation theorem to current density fluctuations.

It probes the properties of the system at low frequency ($\omega = 0$) and long wavelenth (k=0).

It varies enormously from material to material, so it can be used to make clear distinctions in the character of electronic states.

(The room temperature resistivity of Cu is $\rho_{RT} = 1.6\ \mu\Omega$cm while for teflon is as high as $\rho_{RT} \sim 10^{33}\ \mu\Omega$cm)

92.78% of all major discoveries in solid state physics have been made by measuring the resistance!

Dimensional analysis

- Electronic energy scale in solids, $T_F=E_F\sim$ Ry ~ 10^4K.
  - All solids are highly correlated in the sense of the virial theorem that V ~ E_F – this is related to the stability of matter.
  - For the most part, we are always dealing with "ultra-cold electrons." (In dimensionless units, ultracold atoms are orders of magnitude hotter than Cu at room temperature, and always will be.)
- Quantum unit of resistivity:

$$\rho_Q = \frac{h}{e^2}[a_B]^{d-2}$$

$$\rho_{2D} = 25.81 k\Omega \; ; \qquad \rho_{3D} = 136.6 \mu\Omega cm$$

Dimensional analysis

$$\rho = \rho_Q R \; ; \quad \rho_Q = \frac{h}{e^2}[a_B]^{d-2} \; ; \quad R = R\left(\frac{\xi}{a_B}, \frac{T}{T_F}, \frac{Ba_B^2}{\phi_0}, \ldots\right)$$

Naively, we would guess : $R \to \infty$ (insulators)

$R \to 0$ (ideal metal)

$R \sim 1$ everything else

A Fermi liquid in a perfect crystal is a sort of critical state with

$$\xi = \ell \to \infty \text{ as } T \to 0 \quad \text{and } R \sim (\xi/a_B)^{-1}$$

In real crystals, there are at least two emergent lengths : ℓ_{imp} and ℓ_{int}

“Ab initio” theory of the resistivity of liquid and amorphous metals

$$\boxed{\rho = 136.6\mu\Omega cm}$$

$Be_{40}Ti_{50}Zr_{10}$ metallic glass : $\rho(4K) = 280\mu\Omega cm$ $\rho(600K) = 250\mu\Omega cm$

$\rho(\text{Hg at room temperature}) = 98\mu\Omega cm$

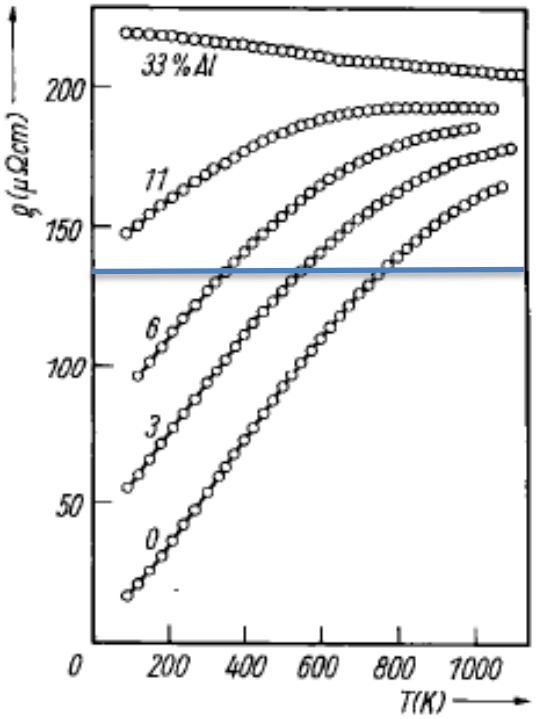


Fig. 3. Resistivity versus temperature for Ti and TiAl alloys containing 0, 3, 6, 11, and 33% Al

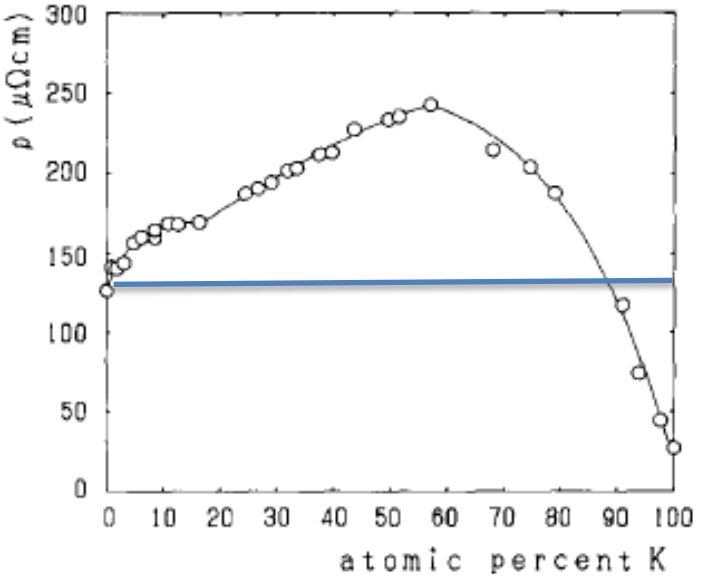


Fig 3. The concentration dependence of the electrical resistivity, ρ, of liquid K–Hg alloys at 573 K.

Theory of nearly ideal metals

$$\text{Drude theory}: \quad \rho = \left(\frac{m}{ne^2}\right)\left(\frac{1}{\tau}\right) \sim \left[\frac{h}{e}^2 (\lambda_F)^{d-2}\right]\left(\frac{\lambda_F}{\ell}\right)$$

1) Bloch's theorem : $\rho \to 0$ as $T \to 0$ and $\ell_{imp} \to \infty$

   a) For good crystals expect $\rho \ll \rho_Q$

   b) ρ increases with increasing T

2) Mattheissen's rule : $(1/\tau) = (1/\tau)_{imp} + (1/\tau)_{ph} + (1/\tau)_{el} + \ldots$

3) Stablity of Fermi liquid : $(1/\tau) - (1/\tau)_{imp} \sim T^x$ with $x \geq 2$.

4) Fermi surface leads $\to$ quantum oscillations at low fields : $R = R(\alpha B/T)$

Linear T resistivity in good metals

If the resistivity is dominated by scattering of electrons from phonons (or any other collective mode) and if $T > T_D$, then by classical equal partition

$$\rho \sim \lambda T + \ldots$$

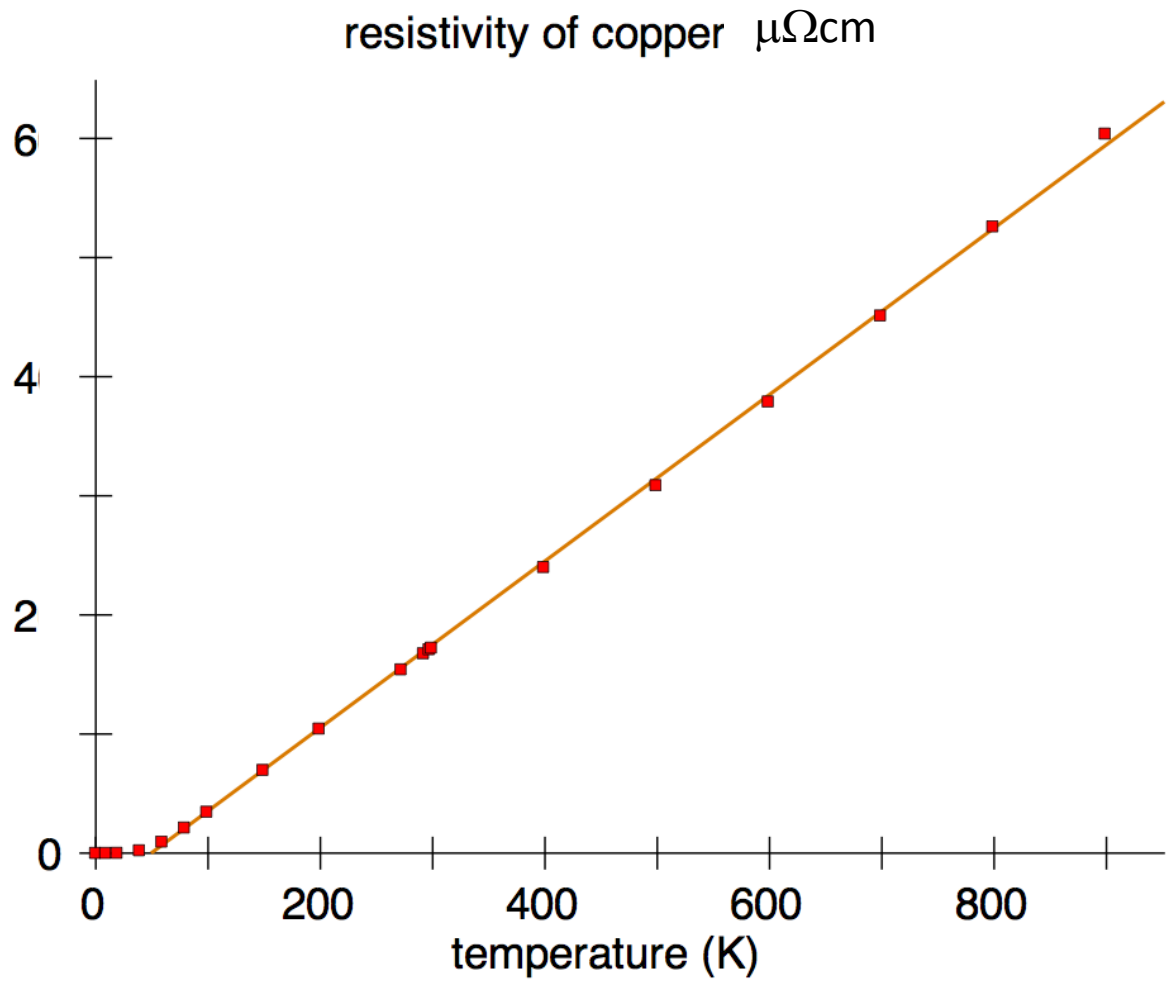
λ is the dimensionless electron-phonon coupling

In the old days, one could argue that resistively challenged materials would make good superconductors:

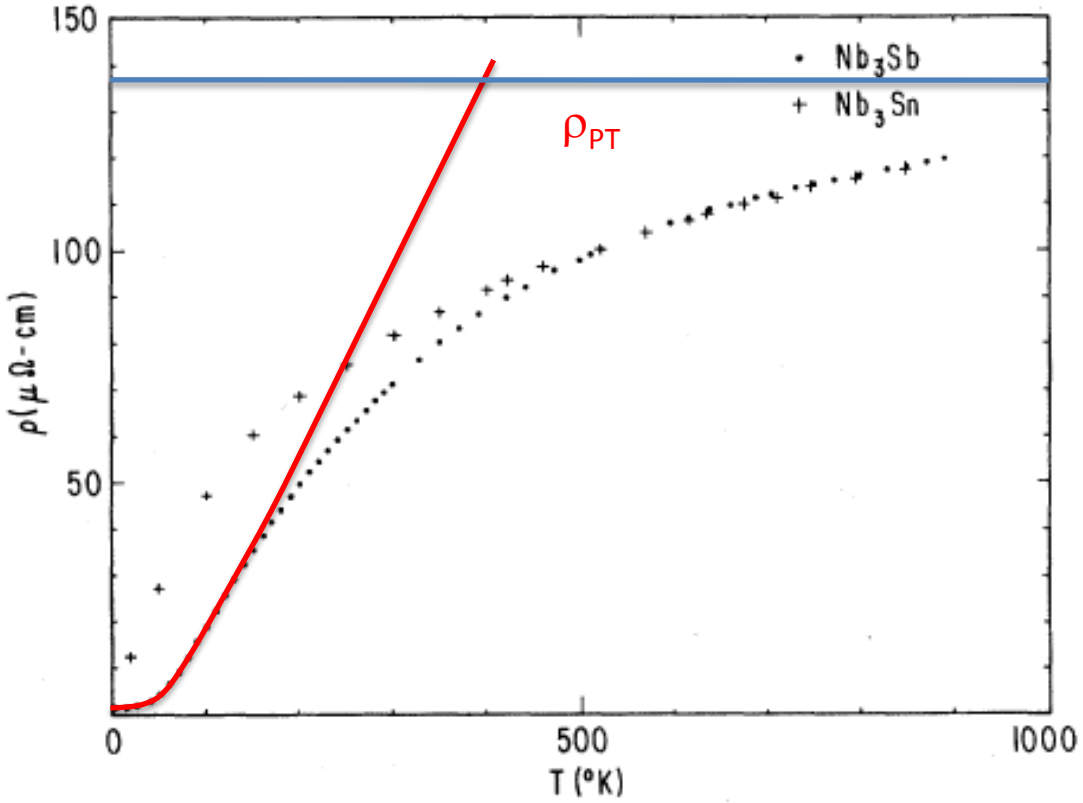
$$T_c \sim \omega_0 \exp[- 1/\lambda]$$

The A15's (another generation's high temperature superconductors) are resistively challenged in just this sense, as we shall see.
(e.g. Nb_3Sn $T_c = 18.3K$)

Linear T resistivity in good metals



Resistivity saturation in good metals



ρ_Q

$$\rho(T) = [1/\rho_{PT}(T) + \sigma_{IR}]^{-1} = \rho_{PT}(T)\,[1 - \sigma_{IR}\;\rho_{PT} + \ldots]$$

$$\sigma_{IR} \sim 1/\rho_Q$$

Heavy Fermions

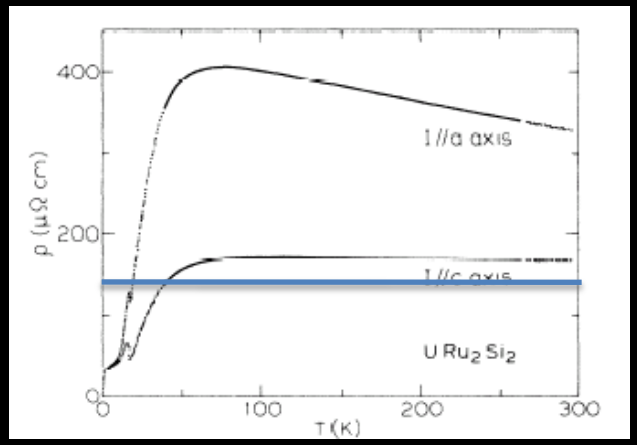
– the most extreme example of a saturating metal

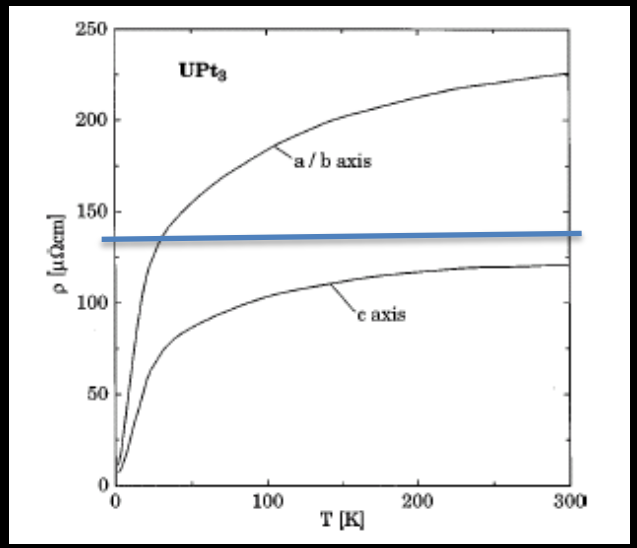
URu_2Si_2

Palstra *et al.*, PRB (86)

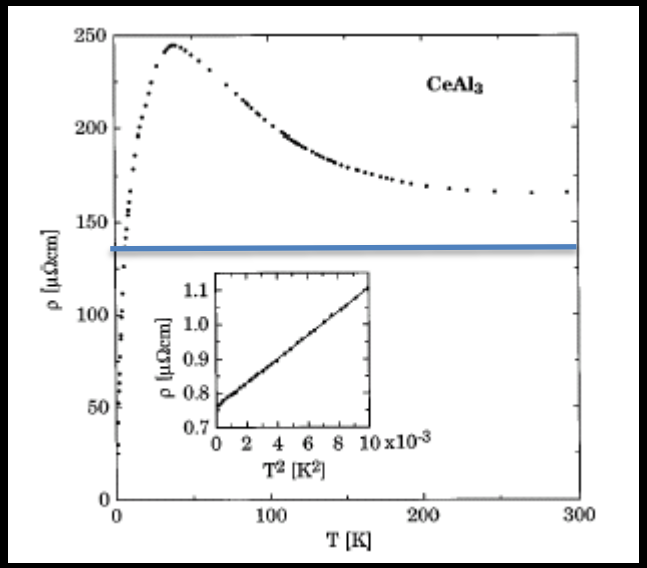
UPt_3

de Visser *et al.*, JMMM (84)





Andres *et al.*, PRL (75)



Slide courtesy of N. Hussey

The Mott-Ioffe-Regel limit

Basic definition: $\ell \geq a$

Other definitions: $k_F \ell \geq 1$

$k_F \ell \geq 2\pi$

Drude theory: $\rho = (e^2/h)\,(k_F/3\pi)\,(k_F l)$

Nigel takes l=a, so $\rho_Q = (e^2/h)\,(k_F/3\pi)\,(k_F a)$ as the canonical quantum of resistivity

Slide courtesy of Nigel Hussey

Actually, there is no generally accepted theory of resistivity saturation

Given that, dimensional analysis may be as good an explanation as any.

At any rate, one would expect a crossover in the behavior of good metals when the quasi-particles cease to be well defined, i.e. when $l \sim \lambda_F$ - and resistivity saturation is such a crossover – this is called the Ioffe – Regel limit.

Note that $(1/\tau)_{qp} \geq (1/\tau)_{tr}$ so $l \sim \lambda_F$ implies that $(1/\tau)_{qp} \geq E_F$

No quasiparticle picture of the high temperature "saturated" state can be consistent.

"Bad Metals"

"Bad metals" are materials in which ρ is an increasing function of T
which does not appear to saturate even though, at high T, $\rho >> \rho_Q$.

Often, but not always, the resistivity of bad metals is a linear function of T.

1) The high T charge transport cannot involve the motion of well-defined quasiparticles because these would necessarily be overdamped.

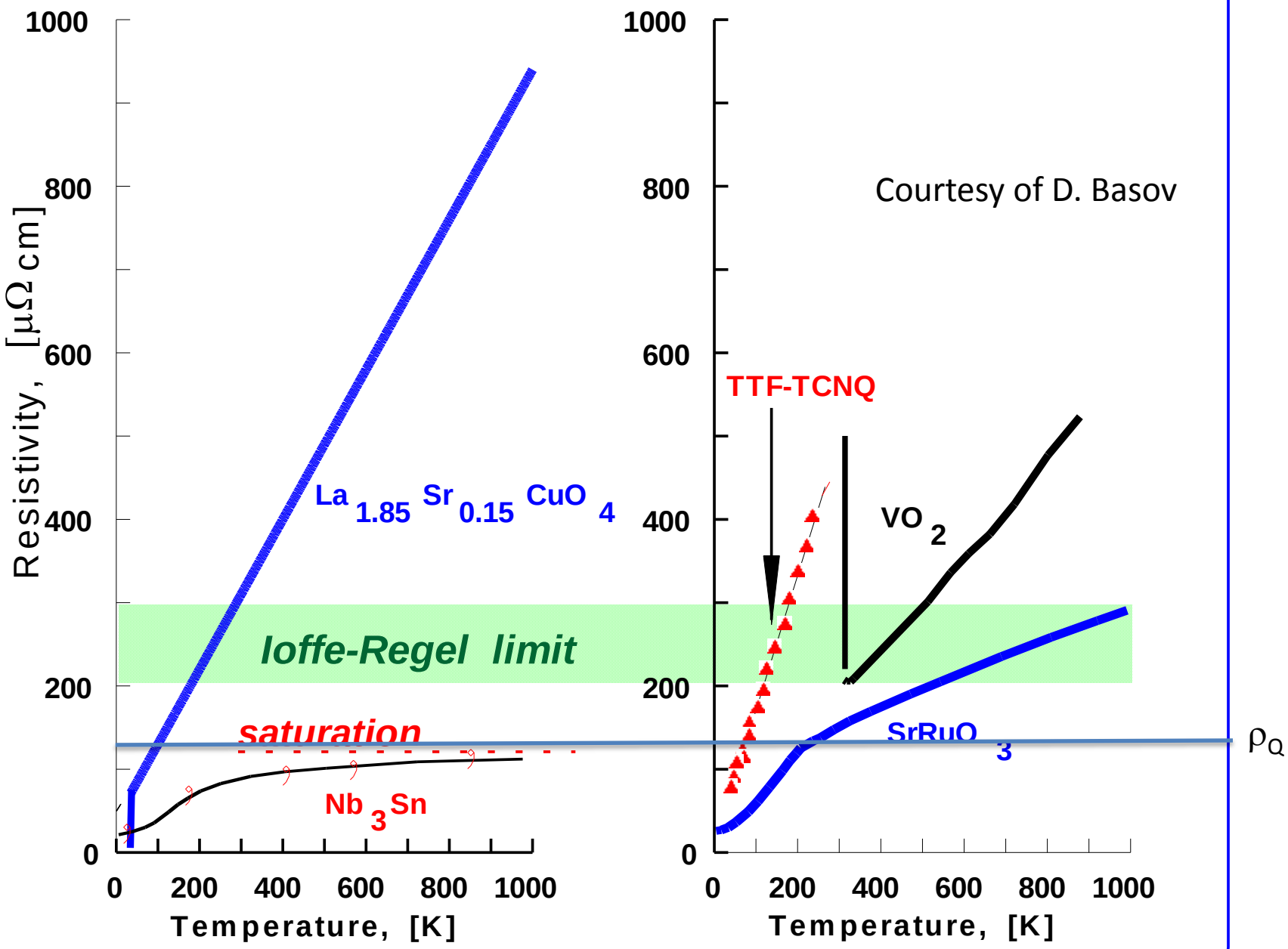
2) At lower T, even when $\rho < \rho_Q$, so long as no crossover is apparent in the T dependence of ρ, there still cannot be any well-defined quasiparticles

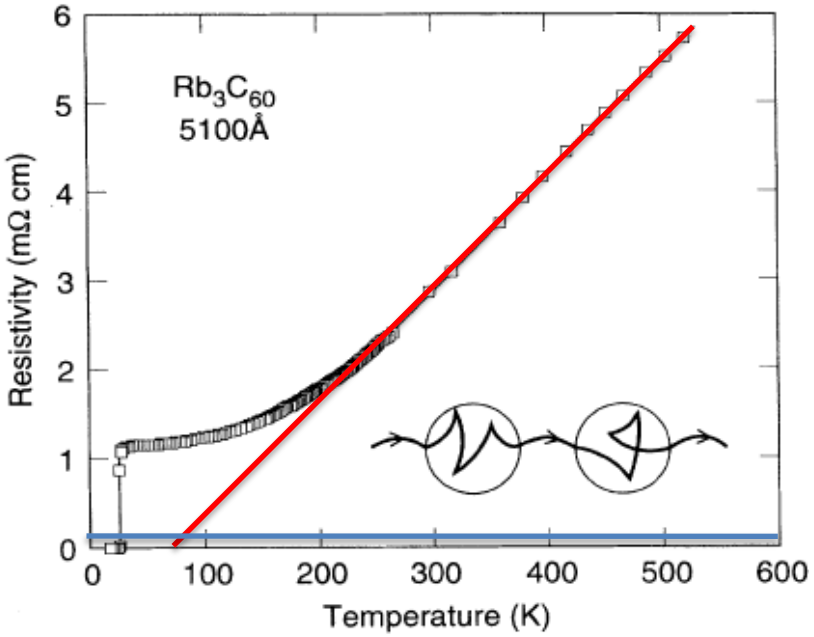
3) If there are no well defined quasiparticles just above T_c, then a Fermi-liquid based (BCS-like) pairing theory is on uncertain theoretical footing.

(Other people had reached this same conclusion in the context of the cuprates even earlier, of course, on the basis of a variety of other arguments.)

V. J. Emery and SAK, PRL (1995).

dc Transport in Synthetic Metals





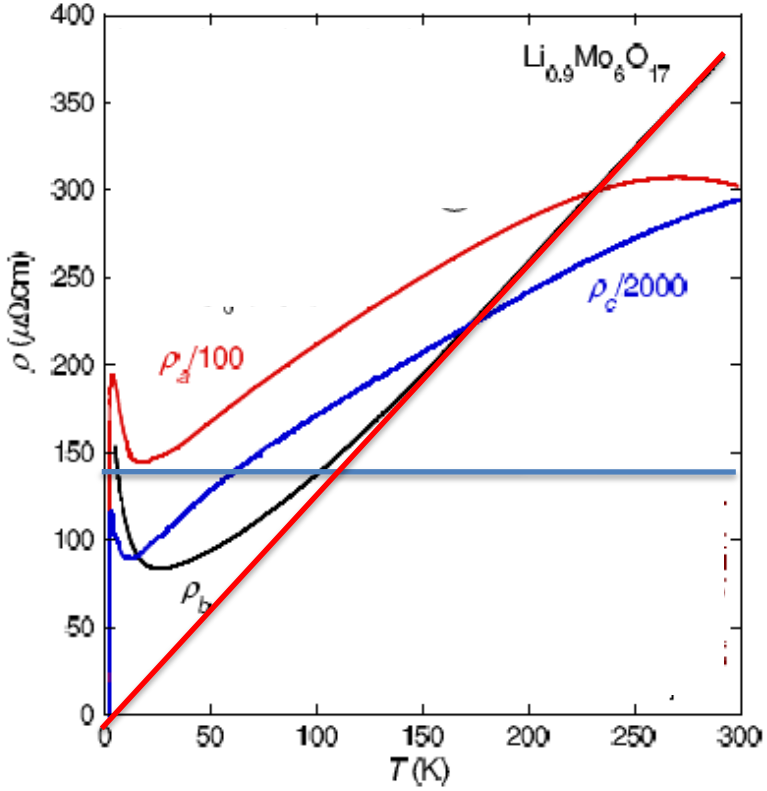
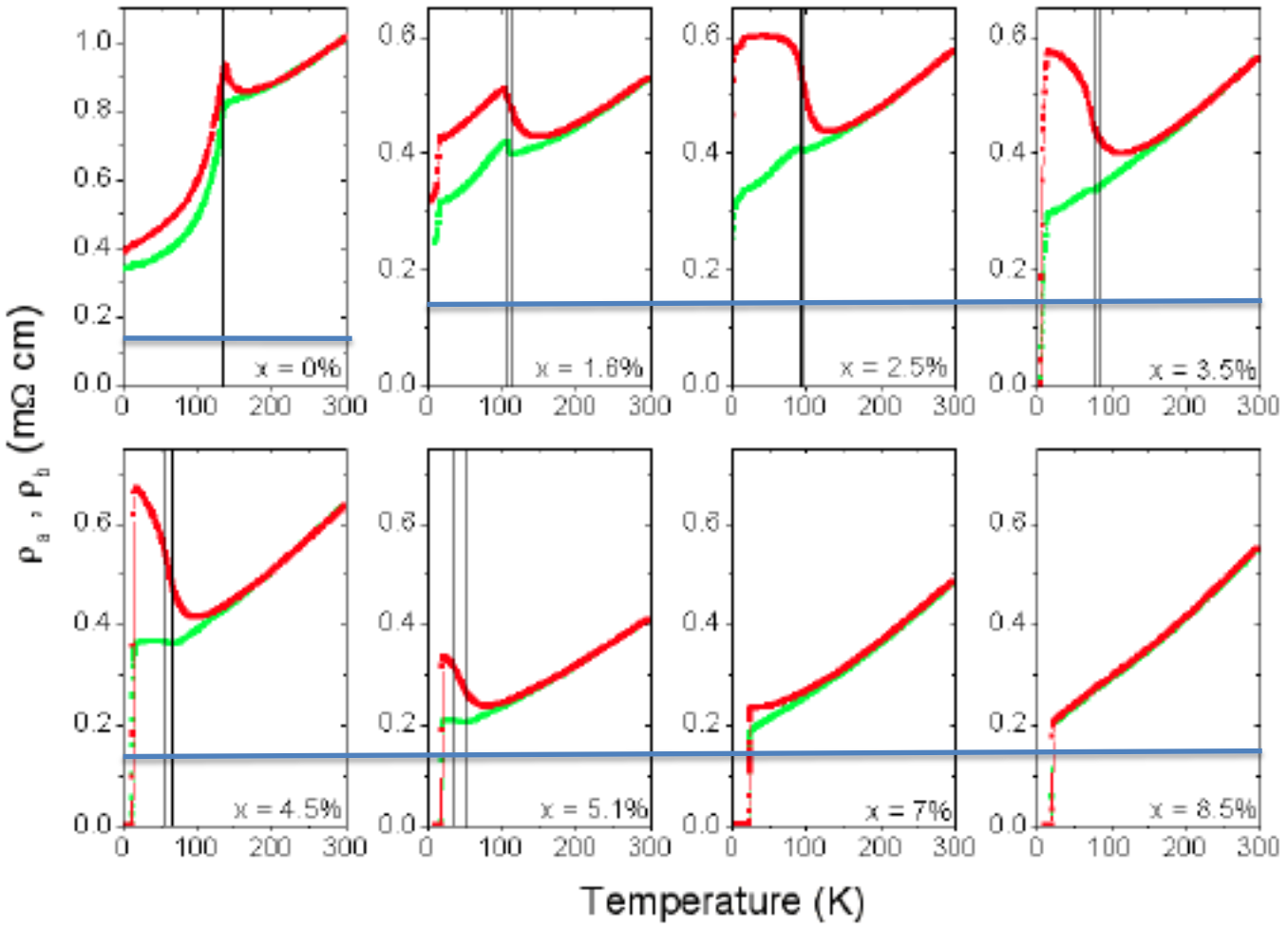


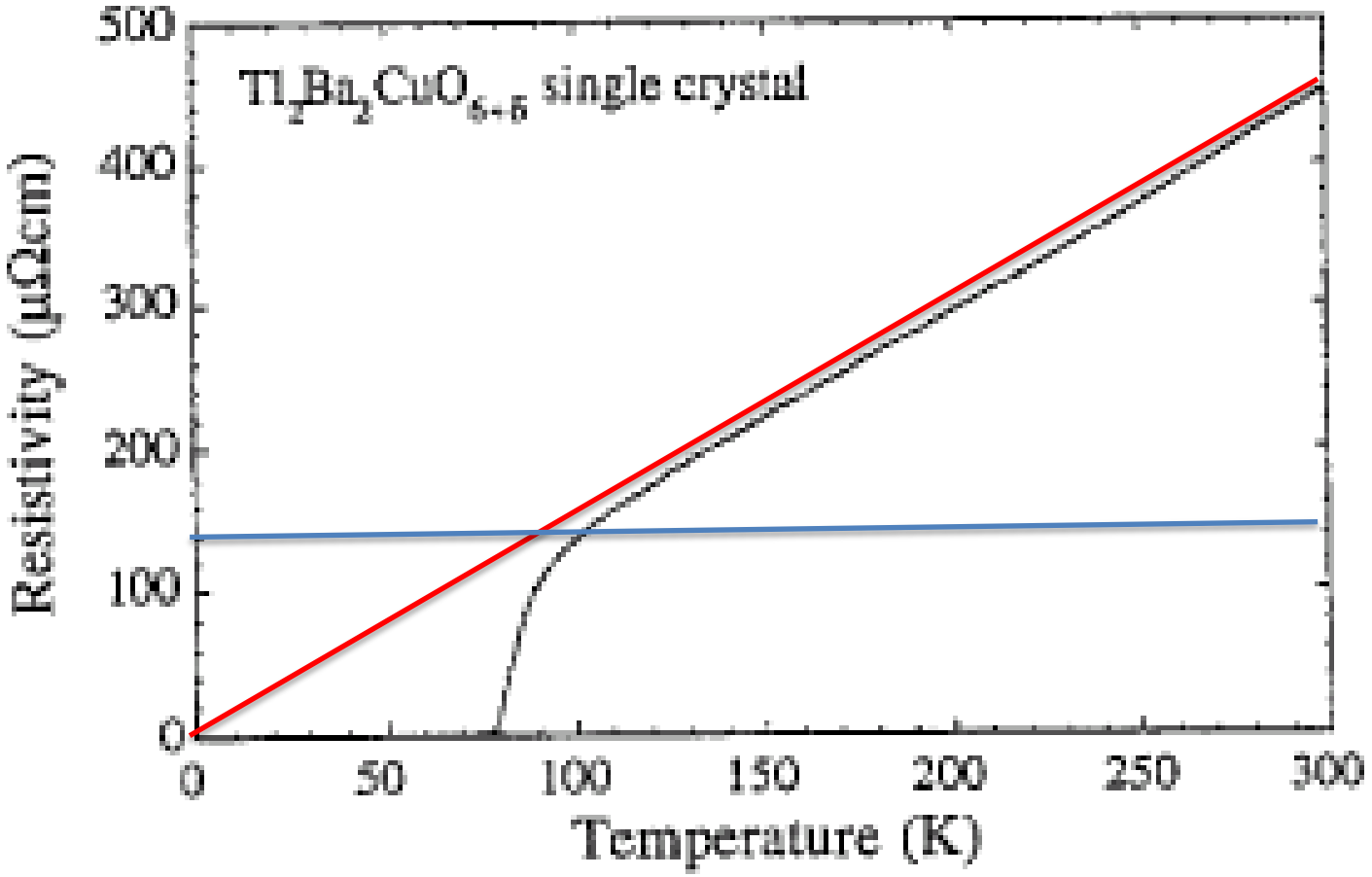
FIG. 1: Zero-field resistivity curves for $Li_{0.9}Mo_6O_{17}$ for $\mathbf{I} \| a, b$ and

Anisotropic bad metalicity in the iron-based high temperature superconductors



Phase formation from the bad metal state

Tyler & Mackenzie, Physica C (97)



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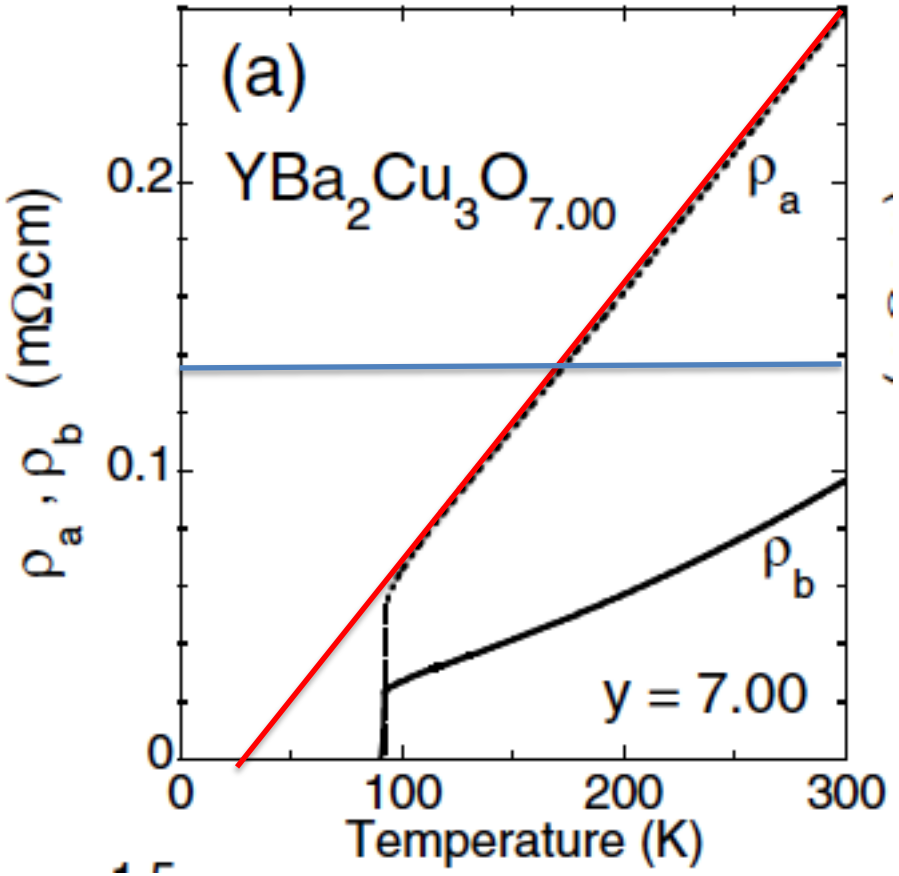
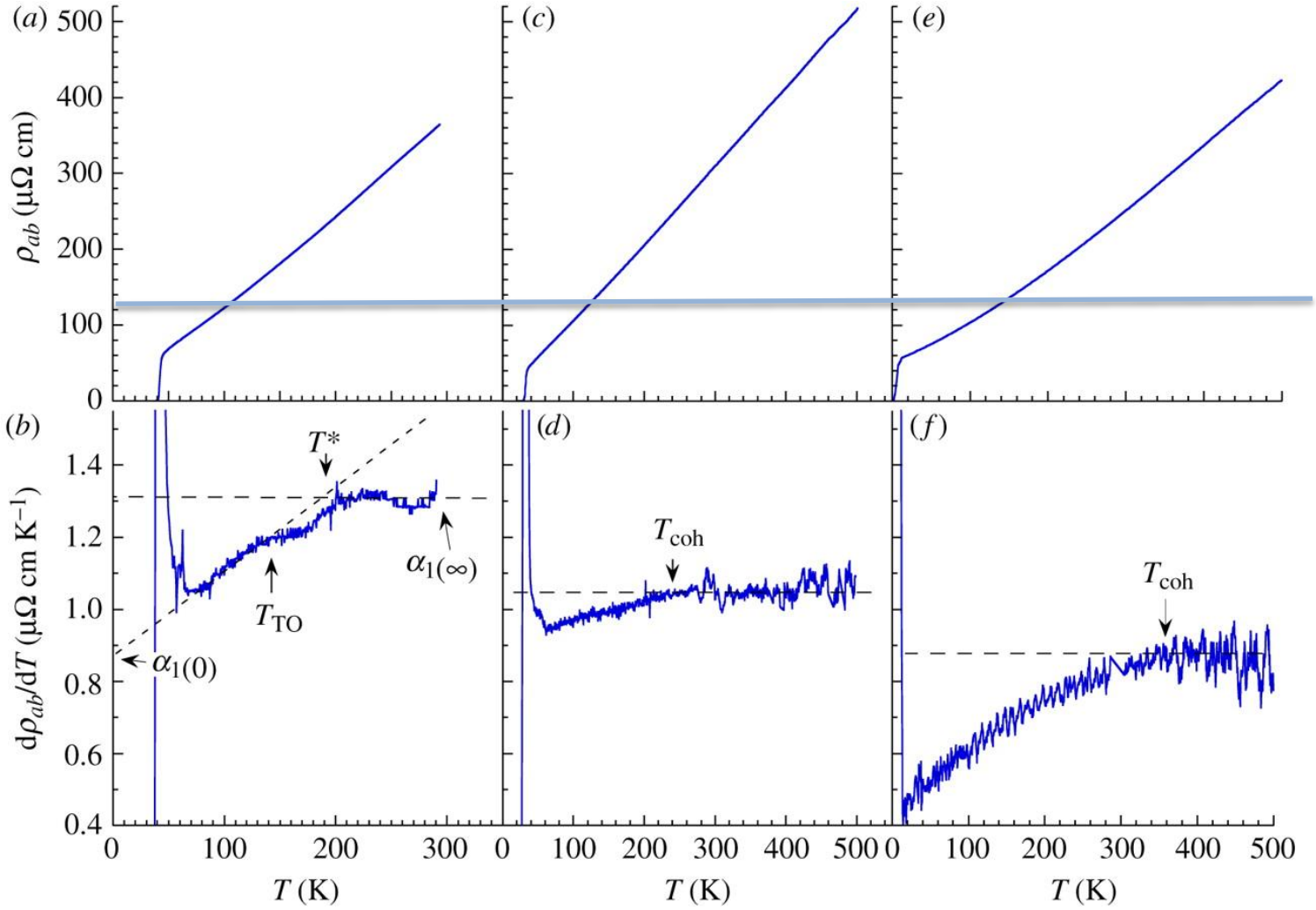


Y. Ando et al, PRL (2002)

Temperature-dependent zero-field resistivity curves for $La_{2-x}Sr_xCuO_4$ (LSCO) with x = (a) 0.17, (c) 0.21 and (e) 0.26.



Hussey N E et al. Phil. Trans. R. Soc. A 2011;369:1626-1639

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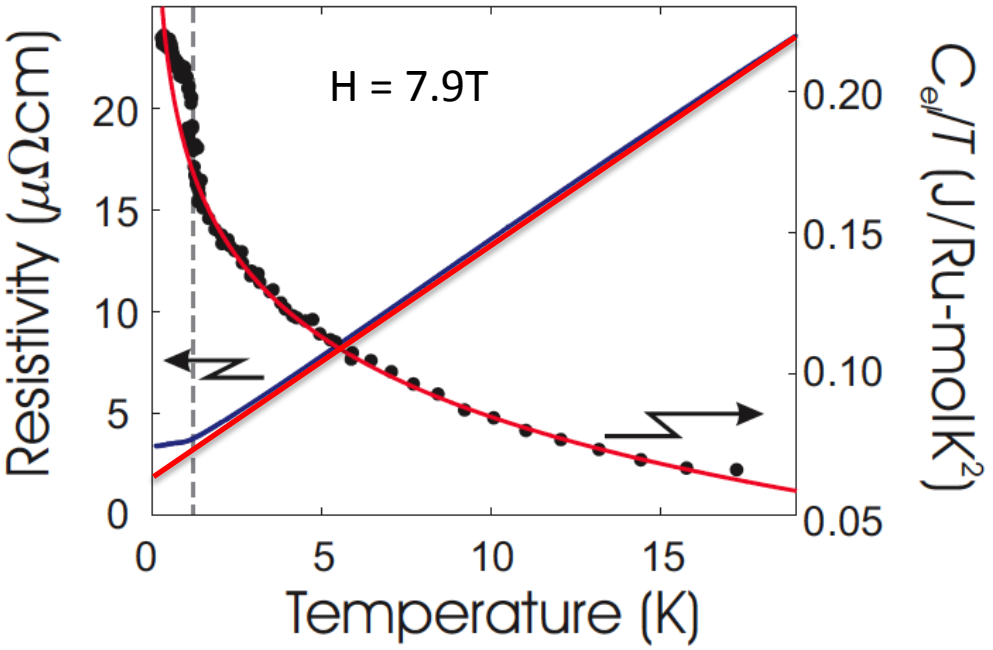
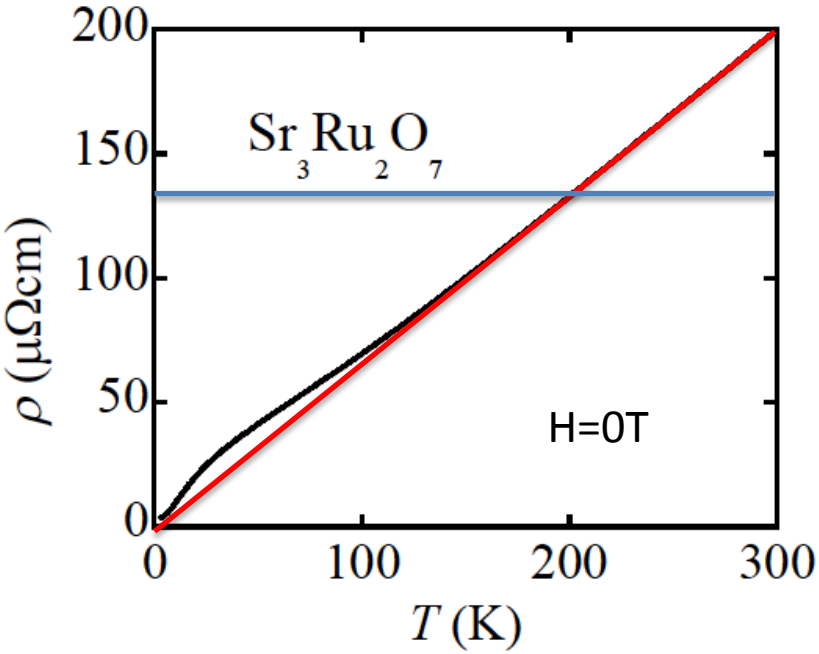
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A

Phase formation and $Sr_3Ru_2O_7$ as a bad metal



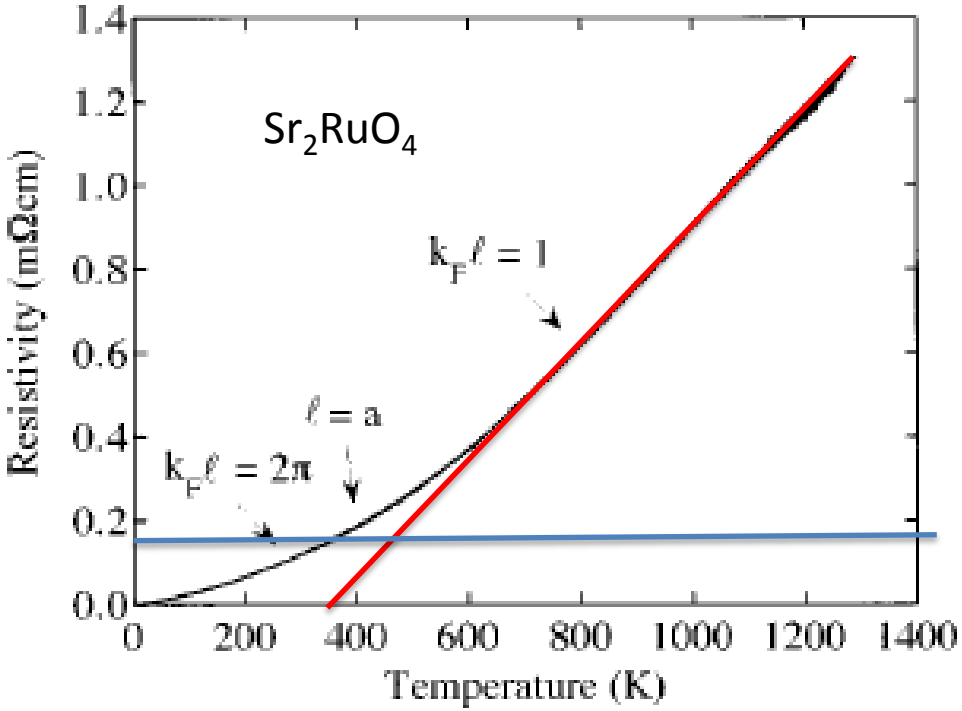


Rost *et al*

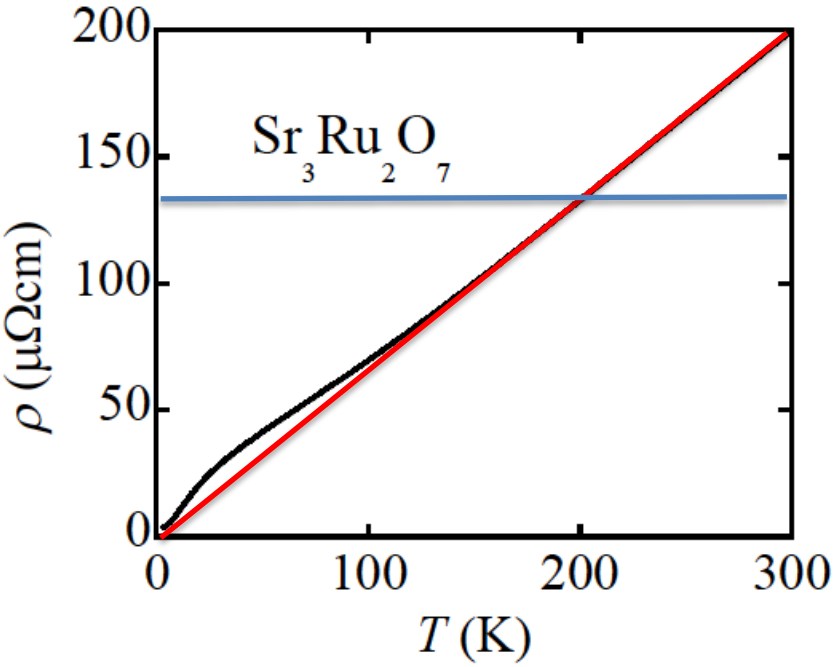
Formation of nematic phase at low T

$$\frac{d\rho(T;H)}{dT} \equiv A(T,H): \quad A(T_{low}, 7.9T) \approx 1\mu\Omega cm \text{ and } A(T_{RT}, 0T) \approx 0.7\mu\Omega cm$$

Crossover from a bad metal to a Fermi liquid in $Sr_{1+n}Ru_nO_{3n+1}$



ρ(



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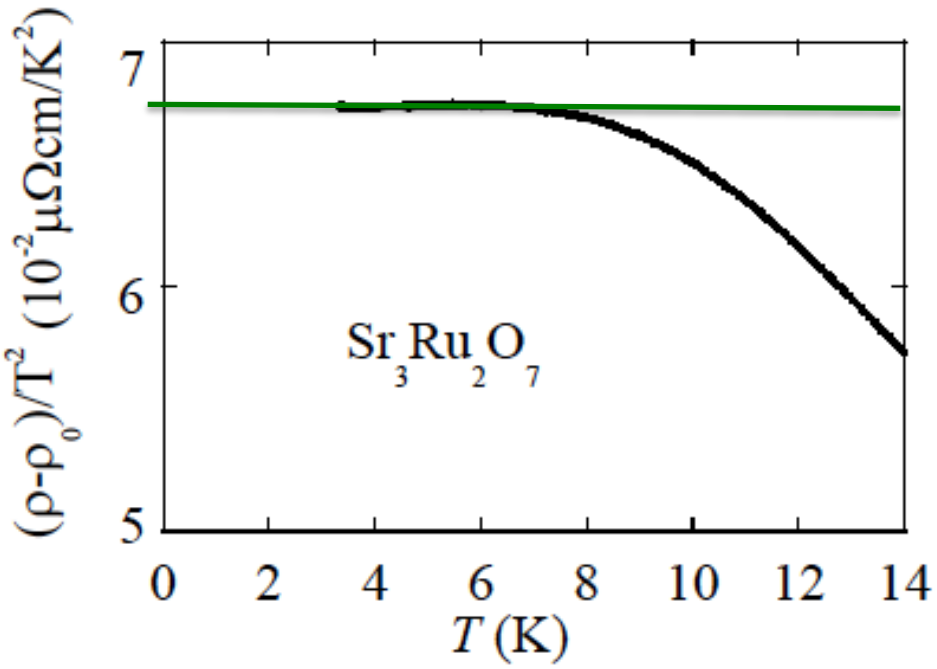
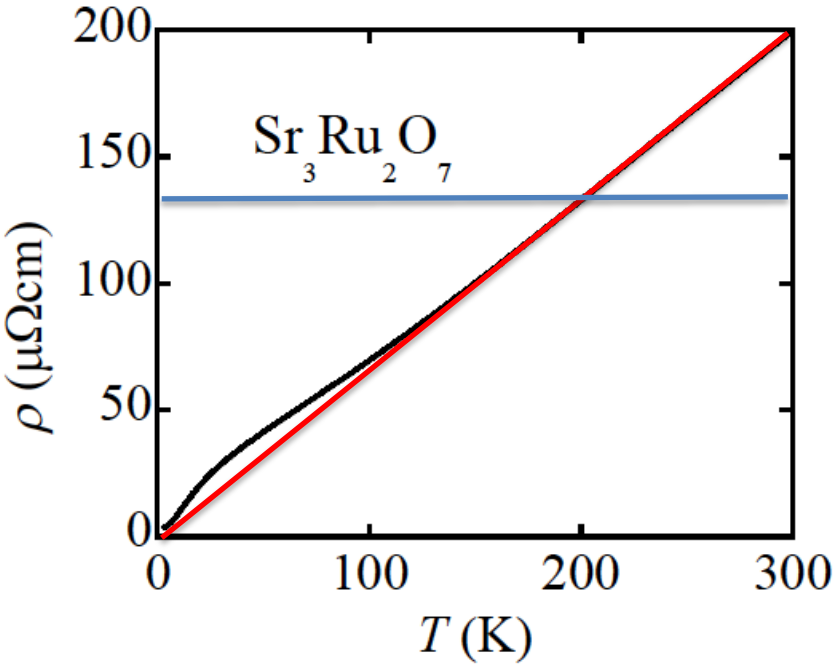




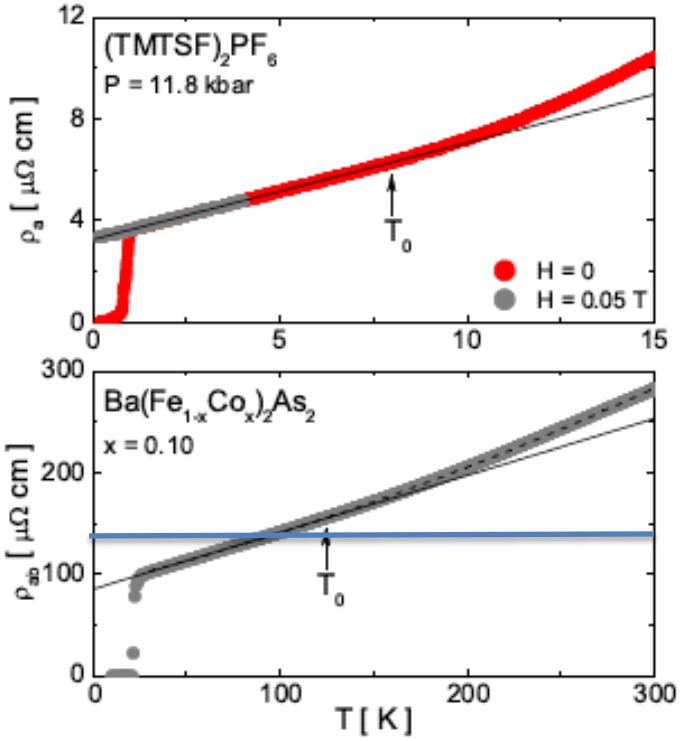
$\rho = \rho_0 + B\,T^2$

Crossover to Fermi liquid appears to violate Mattheisson's rule:

In that sense, it is like resistivity saturation in that the high T resistivity is smaller than extrapolation from low T would suggest.

T linear resistivity as T approaches 0

$$\rho(T) = \rho_0 + AT + BT^2$$



Sometimes can be measured down to low T by suppressing SC with a high B field.

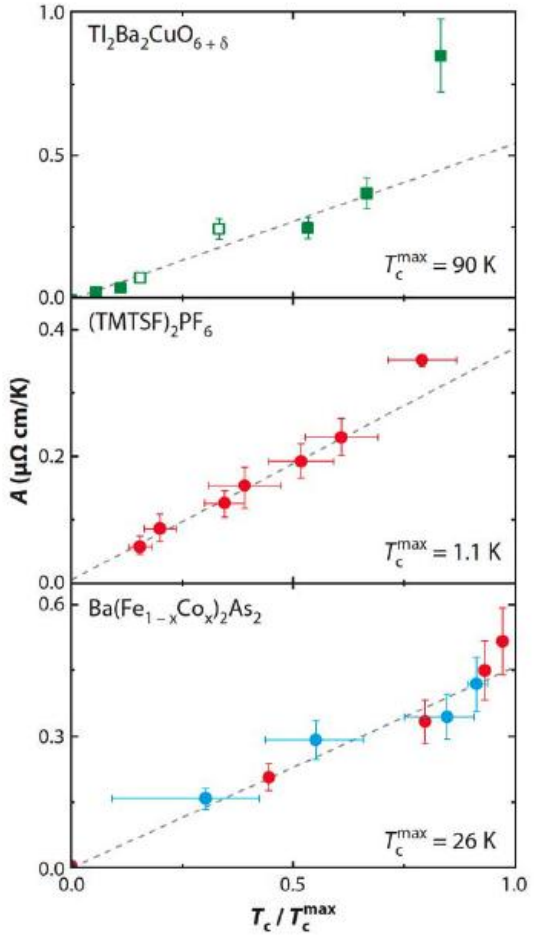
This is more the focus of explicit theories focused on particular quantum critical points.

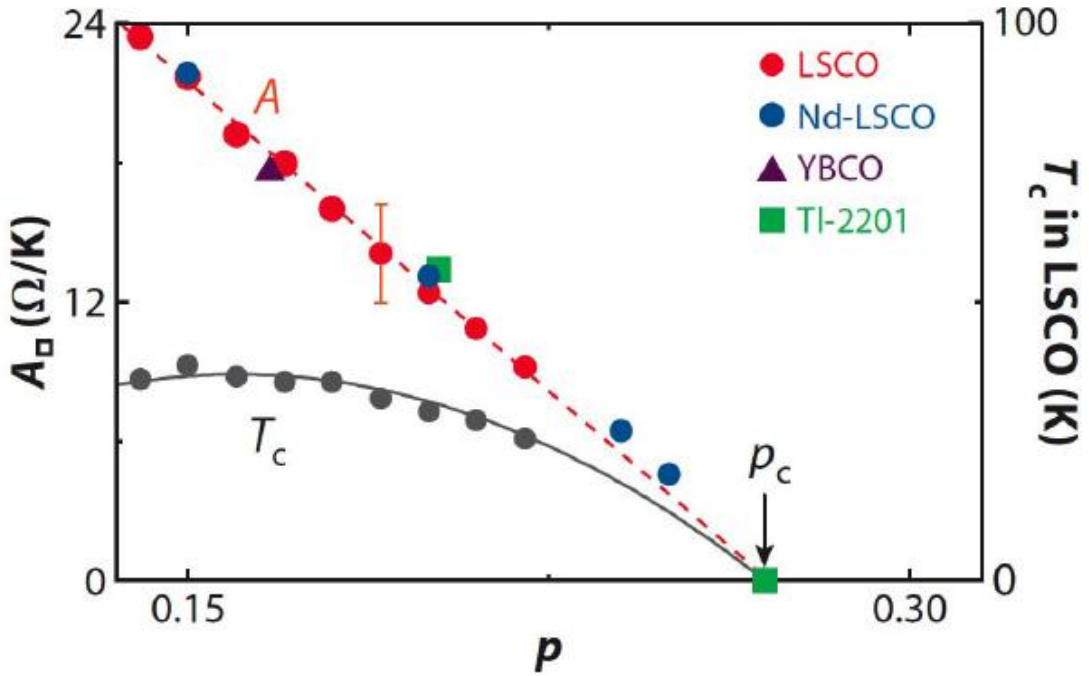
I am worried that the qcp perspective "misses the point."

Taillefer and co.

T linear resistivity as T approaches 0

$$\rho(T) = \rho_0 + AT + BT^2$$





$\rho(T) \sim \rho_0 + AT$ at low T for a range of p !

More like a critical phase than a critical point.

Taillefer and co.

Other signatures of the breakdown of the quasiparticle description of bad metals abound

- $\sigma(\omega,T)$ can exhibit strange power-laws and/or anomalous “mid IR” spectral weight.
- ARPES exhibits broad spectral peaks.
- Neutron scattering often reveals “local” version of various collective modes reminiscent of “nearby” ordered states.
- Etc.

Summary

Many interesting materials are bad metals.

The high resistivity of bad metals is incompatible with the existence of well defined charge carrying quasiparticles. (size matters)

If the T dependence of the resistivity shows no crossover (especially if it is linear) this inference continues to be true to low temperatures. (problems proliferate)

High temperature superconductivity, as well as other interesting phases, often (typically) develop from a bad metal normal state. (this may really matter)

Indeed, there may be a correlation between T linear resistivity and high T_c. (it may really really matter)

Bad metal behavior seems to occur under too broad a range of materials and temperatures to be associated in any conventional way with a particular quantum critical point. (there is a real puzzle)

In my opinion, there exists no convincing theoretical "story," much less a "theory" of bad metals.

Thanks to many colleagues for help and discussions – most particularly -

Nigel Hussey, Phil Allen, Eduardo Fradkin, Ted Geballe, Dimitri Basov,

Andy Mackenzie, and my sorely missed collaborator, Vic Emery

Sorry about the incomplete references – I ran out of time.
Ask me if you want to know who is to blame.

The end

High T persepctive

(Mukerjee, Oganesyan, Huse PRB (2006))

$$\sigma(T,\omega) = \frac{1}{T}\left[\sigma_0(\omega) + \frac{1}{T}\sigma_1(\omega) + \ldots\right]$$

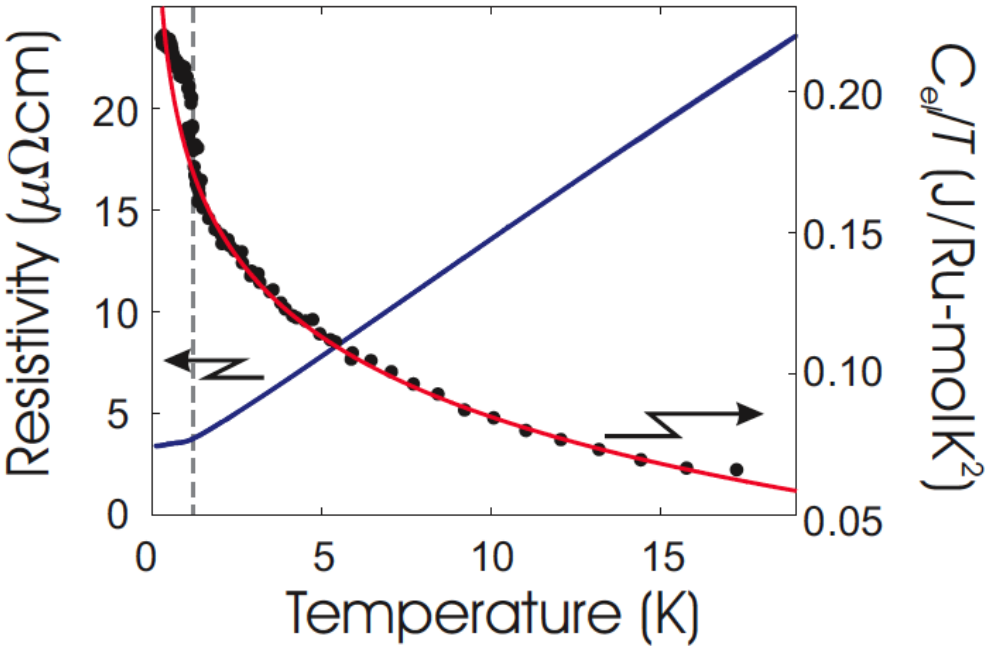
$$\sigma_0(0) = \text{a number}$$

$$\int d\omega \sigma(T,\omega) = \frac{1}{T}\left[K_0 + \frac{1}{T}K_1 + \ldots\right]$$

$$\sigma \sim nD$$

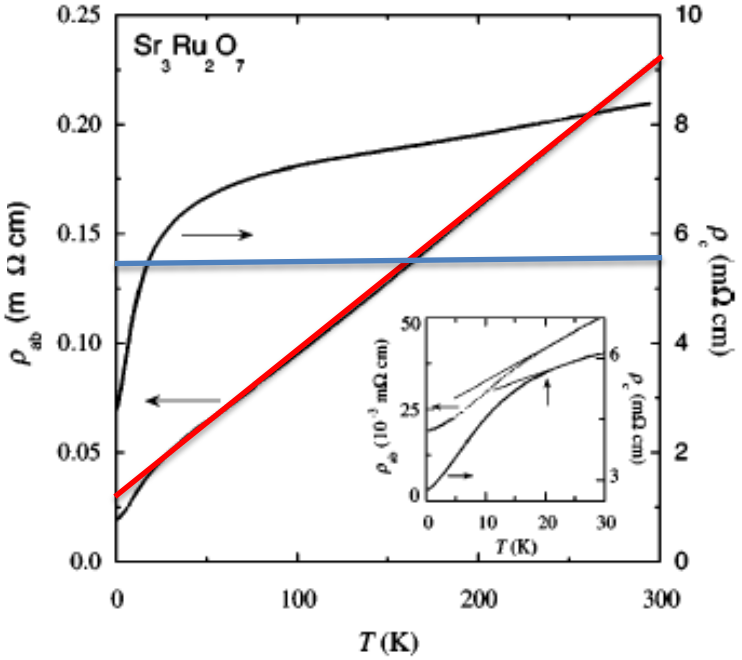
It is as if the diffusion constant approaches a Ioffe-Regel-like limit, while the spectral weight vanishes at high T.

Phase formation and $Sr_3Ru_2O_7$ as a bad metal



Rost *et al* at H = 7.9T?

Formation of nematic phase at low T

Cava and co.

H=0T
(older, not as pure samples)