Higher spin holography and black holes

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Introduction

- holographic duality is powerful but mysterious.
  -- little understanding of how to follow a theory
  from weak to strong coupling
- Typical AdS/CMT scenario: a bulk computation gives an interesting new effect. Is this a prediction for “real” systems, or an exotic strong coupling large N artifact?
- Would be helpful to have some fully worked out toy examples

- D=2+1 CS theory ↔ boundary WZW model (Witten)
  - no black holes, so "too trivial"
- c=1 matrix model:

D=1+1 bulk gravity/scalar theory ↔ QM of fermions in inverted harmonic oscillator potential

- black holes exist, but apparently can't be formed by collapsing matter (singlet vs. nonsinglet sector)

- Recent progress in going from weak to strong coupling based on powerful methods of integrability and localization

- methods rely on susy, hard to apply to finite temp, etc.

D=1+1 CFT

- Prototypes for exactly soluble QFTs are the Virasoro minimal models

$$c = 1 - \frac{6}{m(m+1)} \ , \qquad m = 2, 3, \ldots$$

- small central charge implies that any bulk dual will be highly quantum

- theory that connects to classical bulk gravity should have limit in which $c \to \infty$

D=1+1 CFT

- Minimal models can be constructed as cosets:

m'th minimal model = $\frac{SU(2)_k \oplus SU(2)_1}{SU(2)_{k+1}}$

m = k+2

- A natural generalization:

$$\frac{SU(N)_k \oplus SU(N)_1}{SU(N)_{k+1}}$$

$$c = (N-1)\left[1 - \frac{N(N+1)}{p(p+1)}\right] \qquad p = k + N$$

"$\mathcal{W}_N$ minimal models"

- Besides the stress tensor T(z) and its associated Virasoro algebra, these theories contain a tower of higher spin conserved currents

$$\mathcal{W}_i(z)\ , \quad i = 3, 4, \ldots, N$$

and an enlarged chiral algebra:

$$\mathcal{W}_3(z)\mathcal{W}_3(0) \sim \frac{c}{z^6} + \left(\frac{1}{z^4}T + \frac{1}{z^3}\partial T + \ldots\right) + \left(\frac{1}{z^2}\mathcal{W}_4 + \frac{1}{z}\partial\mathcal{W}_4\right) + \left(\frac{1}{z^2}T^2 + \frac{1}{z}T\partial T\right)$$

$$\mathcal{W}_4(z)\mathcal{W}_4(0) \ \sim \ldots$$

nonlinear

"$\mathcal{W}_N$ algebra"

$\mathcal{W}$ho ordered that?

- Will emerge naturally in AdS

- Gaberdiel and Gopakumar proposed a large N limit:

$$\frac{SU(N)_k \oplus SU(N)_1}{SU(N)_{k+1}} \qquad k, N \to \infty \ , \quad \lambda = \frac{N}{k+N} \ \text{fixed}$$

't Hooft coupling

$$c \sim N(1-\lambda^2)$$

as in vector models

- soluble CFT with large c limit
- infinite number of conserved currents

$$\mathcal{W}_\infty[\lambda]$$ (Garbediel, Hartman)

AdS dual

- boundary spin-j current is dual to a massless spin-j field in the bulk

- bulk theory must be an interacting theory of massless particles of spin 2, 3, 4, … N

- such theories are impossible in Minkowski space (Coleman-Mandula), but do exist in AdS (Vasiliev)

- These theories share much in common with full string theory, but are more tractable

- In any QFT basic question is: “what is the high temperature thermodynamics?”
- In D=1+1 CFT Cardy formula gives high T entropy

$$S = 2\pi\sqrt{\tfrac{c}{6}P_L} + 2\pi\sqrt{\tfrac{c}{6}P_R}$$

$$P_{L,R} = \text{left/right moving Virasoro charges}$$

  - well known to match the Bekenstein-Hawking entropy for a BTZ black hole in AdS_3
- In higher spin case, can consider entropy at specified values of higher spin charges
  - Generalized Cardy formula?
  - Do corresponding black holes exist with correct S?

Painless introduction to higher spin gravity

- higher spin gravity lives “midway” between ordinary low energy gravity and full string theory:
  - infinite tower of fields
  - nonlocal interactions fixed by huge symmetry algebra
  - background independent form of field equations

- Fronsdal formulated free theory of massless symmetric tensor fields in AdS with gauge invariance $\delta\varphi_{\mu_1\mu_2\ldots\mu_n} = \nabla_{(\mu_1}\lambda_{\mu_2\ldots\mu_n)}$
- to include interactions best to work in gauge theory formulation of gravity. Focus on D=2+1

Achucarro, Townsend / Witten:

$$\text{vielbein} \;\; e^a_\mu \;, \quad \text{spin connection} \;\; \omega^a_\mu = \tfrac{1}{2}\epsilon^a{}_{bc}\omega^{bc}_\mu$$

SL(2,R) x SL(2,R) gauge fields $A = (\omega^a + \frac{1}{l}e^a)J_a \;, \quad \overline{A} = (\omega^a - \frac{1}{l}e^a)J_a$

$$[J_a, J_b] = \epsilon_{ab}{}^c J_c \qquad \mathrm{Tr} J_a J_b = \eta_{ab}$$

$$R_{\mu\nu} = \frac{1}{l^2}g_{\mu\nu} \iff \begin{aligned} dA + A\wedge A &= 0 \\ d\overline{A} + \overline{A}\wedge\overline{A} &= 0 \end{aligned}$$

$$S = \frac{k}{4\pi}\int \mathrm{Tr}(AdA + \tfrac{2}{3}A^3) - \frac{k}{4\pi}\int \mathrm{Tr}(\overline{A}d\overline{A} + \tfrac{2}{3}\overline{A}^3) \qquad k = \frac{l}{4G} = \frac{c}{6}$$

Simple solutions

SL(2) generators

$$[L_0, L_{\pm 1}] = \mp L_{\pm 1}\ , \quad [L_1, L_{-1}] = 2L_0$$

$$A = e^{\rho} L_1 dx^+ + L_0 d\rho$$
$$\overline{A} = -e^{\rho} L_{-1} dx^- + L_0 d\rho$$

$\Rightarrow$ $ds^2 = d\rho^2 - e^{2\rho} dx^+ dx^-$ AdS_3

$$A = (e^{\rho} L_1 - \tfrac{2\pi}{k} \mathcal{L} e^{-\rho} L_{-1}) dx^+ + L_0 d\rho$$
$$\overline{A} = -(e^{\rho} L_{-1} - \tfrac{2\pi}{k} \overline{\mathcal{L}} e^{-\rho} L_1) dx^- + L_0 d\rho$$

$\Rightarrow$ BTZ $\quad \mathcal{L} = \frac{M-J}{4\pi} \quad \overline{\mathcal{L}} = \frac{M+J}{4\pi}$

Asymptotic symmetry algebra

- replace: $\mathcal{L} \to \mathcal{L}(x^+)$ still a solution

- solve for general gauge transformation preserving form of A under $\delta A = d\lambda + [A, \lambda]$

- find free function's worth: $\epsilon(x^+)$

$$\delta\mathcal{L} = -\frac{c}{24\pi}\partial_+^3\epsilon - 2\partial_+\epsilon\mathcal{L} - \epsilon\mathcal{L}$$

Virasoro algebra with c = 6k = 3l/2G

- simple way to generate higher spin theory: replace SL(2) by some larger algebra G containing SL(2)

• get an enlarged theory that includes Einstein gravity as a subsector

SL(3)

$$g_{\mu\nu} \sim \mathrm{Tr}(e_\mu e_\nu)$$

$$\varphi_{\alpha\beta\gamma} \sim \mathrm{Tr}(e_\alpha e_\beta e_\gamma)$$

reduces to Fronsdal theory when linearized around AdS

(Campoleoni et. al.)

- besides ordinary coord. invariance, have "exotic" spin-3 gauge invariance acting on metric

e.g. Ricci scalar not gauge invariant

Asymptotic symmetry algebra

SL(3) generators: $L_1, L_0, L_{-1} \qquad W_2, W_1, W_0, W_{-1}, W_{-2}$

asymptotic form of connection:

$$A = \left(e^{\rho} L_1 - \frac{2\pi}{k} e^{-\rho} \mathcal{L}(x^+) L_{-1} - \frac{\pi}{2k} e^{-2\rho} \mathcal{W}(x^+) W_{-2}\right) dx^+ + L_0 d\rho$$

- now find two free function’s worth of gauge transformations preserving above form

$$\epsilon_2(x^+)\ , \quad \epsilon_3(x^+)$$

- field variations equivalent to $\mathcal{W}_3$ algebra, with

$\mathcal{L}(x^+)$ stress tensor

$\mathcal{W}(x^+)$ spin-3 current

Vacua and RG flows

- AdS vacua in correspondence with SL(2) subalgebras
- SL(3) has two inequivalent SL(2) subalgebras, so two distinct AdS vacua

- symmetry of new vacuum:

currents: 1 spin-2; 2 spin-3/2, 1 spin-1 $\mathcal{W}_3^{(2)}$ (Polyakov; Bershadsky)

- RG flow from $\mathcal{W}_3^{(2)}$ in UV to $\mathcal{W}_3$ in IR:

$$ds^2 = d\rho^2 - \left(\tfrac{1}{4}e^{4\rho} + \lambda^2 e^{2\rho}\right) dx^+ dx^-$$

$$\varphi_{+++} \sim \varphi_{---} \sim \lambda^2 e^{4\rho}$$

$$c_{IR} = 4c_{UV}$$ non-Lorentz invariant flow

Partition function

$$\mathcal{L}(z) = \sum_n \frac{L_n}{z^{n+2}} \qquad \mathcal{W}(z) = \sum_n \frac{W_n}{z^{n+3}}$$

$$[L_0, W_0] = 0$$

- expect black holes carrying charges (L_0, W_0)
- such black holes can be thought of as contributing to a CFT partition function of form

$$Z(\tau,\alpha) = \mathrm{Tr}\left[e^{4\pi^2 i(\tau L_0 + \alpha W_0)}\right]$$

$\alpha = \text{spin}-3 \text{ chemical potential}$

- all this generalizes in obvious way to spin-N case

- adding a spin-3 potential means adding to the CFT action a term $I = \int d^2x \; \mu(x)\mathcal{W}(x)$
- Following AdS/CFT rules, should incorporate μ in bulk via boundary conditions
- Ward identity analysis establishes:

$$A_- \;\sim\; \mu e^{2\rho} W_2 + \cdots$$

- simple procedure for generating such solutions:

$$A_\rho = L_0$$

$$A_+ = e^\rho L_1 - \frac{2\pi}{k} e^{-\rho} \mathcal{L} L_{-1} - \frac{\pi}{2k} e^{-2\rho} \mathcal{W} W_{-2}$$

$$A_- = \mu A_+^2 \Big|_{\text{traceless}}$$

- now have solutions with free parameters:

$\mathcal{L}, \quad \mathcal{W}$ charges

$\tau \ , \quad x^+ \cong x^+ + 2\pi\tau$ temperature

$\alpha = \overline{\tau}\mu$ spin-3 chemical potential

- on physical grounds, should fix charges in terms of potentials, or vice versa
- ordinary gravity: fix relations by demanding smooth event horizon. Inapplicable here, since not gauge invariant
- similarly, usual methods for computing entropy (area law, Euclidean action, Wald formula) do not directly apply

appeal to first principles

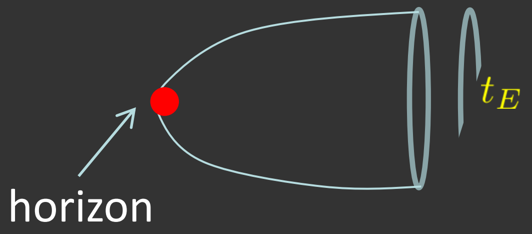
- trying to compute the partition function:

$$Z(\tau,\alpha) = \mathrm{Tr}\left[e^{4\pi^2 i(\tau\mathcal{L}+\alpha\mathcal{W})}\right]$$

- existence requires that we obey integrability condition:

$$\frac{\partial\mathcal{L}}{\partial\alpha} = \frac{\partial\mathcal{W}}{\partial\tau}$$

- want gauge invariant condition for Euclidean time circle to smoothly contract at horizon



holonomy condition

- we propose that holonomy of gauge connection around time circle should take fixed (BTZ) value
- fixes charges and obeys integrability condition

- after dust settles, we get an explicit formula for black hole entropy

$$S = 2\pi\sqrt{2\pi k \mathcal{L}}\; f\left(\frac{27k\mathcal{W}^2}{64\pi\mathcal{L}^3}\right)$$

$$f(x) = \cos\left[\frac{1}{6}\arctan\left(\frac{\sqrt{x(2-x)}}{1-x}\right)\right] = 1 - \frac{1}{36}x - \frac{35}{776}x^2 + \cdots$$

- This is the spin-3 generalization of Cardy's formula.

Should apply to any CFT with $\mathcal{W}_3$ symmetry and c >> 1

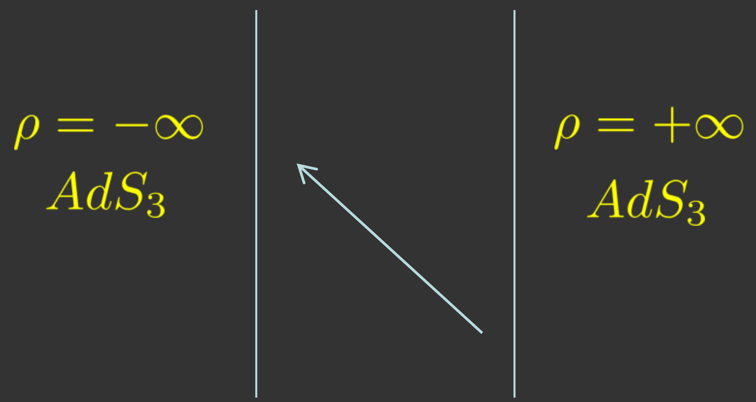
Causal structure

- Metric for non-rotating case takes form

$$ds^2 = d\rho^2 - F(\rho)dt^2 + G(\rho)d\phi^2$$

$F(\rho),\ G(\rho)\ >\ 0$ no event horizon!

traversable wormhole:



- But when holonomy conditions are obeyed, one can find a true black hole metric somewhere on this gauge orbit

Black holes in hs[λ]

- Gaberdiel and Gopakumar conjecture:

$$\frac{SU(N)_k \oplus SU(N)_1}{SU(N)_{k+1}}$$

$$k, N \to \infty \ , \quad \lambda = \frac{N}{k+N} \quad \text{fixed}$$

⟺

higher spin gravity (plus scalars) based on gauge algebra hs[λ]

(Blencowe; Vasiliev)

- hs[λ] is an infinite dimensional Lie algebra based on an associative product

$$V^s_m \star V^t_n = \sum_{u=1}^{s+t-1} g^{st}_u(m,n,\lambda) V^{s+t-u}_{m+n}$$

- hs[N] = SL(N)
- Asymptotic symmetry analysis yields $\mathcal{W}_\infty[\lambda]$

(Henneaux,Rey; Campoleoni. et. al.; Gaberdiel,Hartman)

- we consider CS theory based on this algebra (plus matter), and carry out previous procedure to define black holes

- The spin-3 chemical potential α now sources an infinite number of charges. System can be solved perturbatively in α

Partition function

$$\ln Z(\tau,\alpha) = \frac{i\pi k}{2\tau}\left[1 - \frac{4}{3}\frac{\alpha^2}{\tau^4} + \frac{400}{27}\frac{\lambda^2-7}{\lambda^2-4}\frac{\alpha^4}{\tau^8} - \frac{1600}{27}\frac{5\lambda^4-85\lambda^2+377}{(\lambda^2-4)^2}\frac{\alpha^6}{\tau^{12}} + \cdots\right]$$

valid for: $\tau \to 0, \quad \alpha \to 0, \quad \frac{\alpha}{\tau^2} \quad \text{fixed}$

- should agree with CFT partition function

Comparison with CFT

$$\frac{SU(N)_k \oplus SU(N)_1}{SU(N)_{k+1}} \qquad k, N \to \infty \ , \quad \lambda = \frac{N}{k+N} \ \text{fixed}$$

$$0 \leq \lambda \leq 1$$

- simplifications at $\lambda = 0,1$

$\mathcal{W}_\infty[0]$ free fermion realization

$\mathcal{W}_\infty[1]$ " " boson " "

λ=1: free bosons

D=3k complex bosons: $\partial\overline{\phi}^i(z)\partial\phi_j(0) \sim -\frac{1}{z^2}\delta^i_j$

stress tensor: $T = \partial\overline{\phi}^i\partial\phi_i$

spin-3 current: $\mathcal{W} = ia(\partial^2\overline{\phi}^i\partial\phi_i - \partial\overline{\phi}^i\partial^2\phi_i)$

$$a = \sqrt{\frac{5}{12\pi^2}}$$

- expand in modes and compute partition function in presence of spin-3 chemical potential

$$\ln Z(\tau,\alpha) = -\frac{3ik}{2\pi\tau}\int_0^\infty \left[\ln\left(1 - e^{-x+\frac{2ia\alpha}{\tau^2}x^2}\right) + \ln\left(1 - e^{-x-\frac{2ia\alpha}{\tau^2}x^2}\right)\right]$$

expansion in α matches black hole result at λ=1

λ=1: free fermions

D=6k complex fermions: $\overline{\psi}^i(z)\psi_j(0) \sim -\frac{1}{z}\delta^i_j$

stress tensor: $T = \overline{\psi}^i\partial\psi_i + \psi_i\partial\overline{\psi}^i$

spin-3 current: $\mathcal{W} = ib(\partial^2\overline{\psi}^i\psi_i - 4\partial\overline{\psi}^i\partial\psi_i + \overline{\psi}^i\partial^2\psi_i)$

$$b = \sqrt{\frac{5}{144\pi^2}}$$

- Also have spin-1 current $\overline{\psi}^i\psi_i$ but no bulk spin-1 field, so need to impose vanishing charge
- result now matches black hole for λ=0

- expect to be able to reproduce all free boson/fermion current correlators from bulk

scalar fields

- bulk gauge symmetry fixes allowed masses of scalar fields and their interactions

  - bulk scalars map to scalar boundary operators with

$$\Delta = 1 \pm \lambda$$

$$\lambda = 0: \quad \mathcal{O} = \overline{\psi}^i(z)\tilde{\psi}_i(\overline{z})$$

$$\lambda = 1: \quad \mathcal{O} = \partial\overline{\phi}^i(z)\overline{\partial}\phi_i(\overline{z})$$

Conclusion

- established rules for incorporating black holes in D=2+1 higher spin gravity
- gives predictions for asymptotic growth of states in dual CFT
- agreement with free boson/fermion CFT
- many directions for further development