

On Stability and Transport of Cold Holographic Matter

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Credits

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Outline:

- Holographic Matter: Bottom-up
- Holographic Matter: Top-down
- Stability Analysis
- Summary and Outlook

Strongly-coupled
CFT_d

=

Weakly-coupled Gravity
AdS_{d+1}

$$S = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g} (R + d(d-1))$$

$$ds^2 = \frac{dr^2}{r^2} - r^2 dt^2 + r^2 d\vec{x}^2$$

$r \rightarrow \infty$

UV

$r \rightarrow 0$

IR

Strongly-coupled
CFT_d

=

Weakly-coupled Gravity
AdS_{d+1}

Holographic Matter

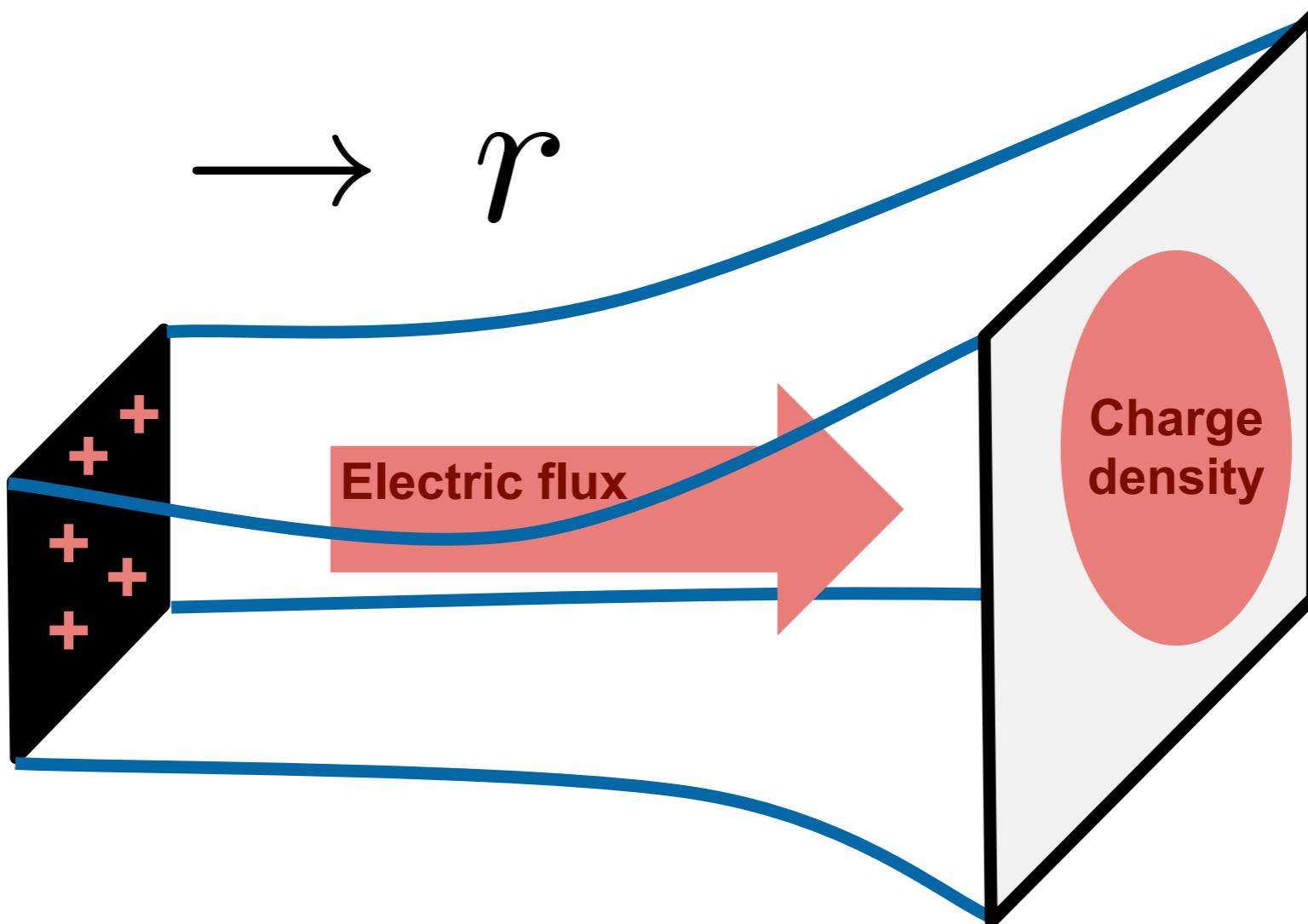
$$\text{global } U(1) = \text{gauged } U(1)$$

$$J^\mu = A_\mu$$

$$\langle J^t \rangle = A_t(r)$$

AdS-Reissner-Nordström

$$S = \int d^{d+1}x \sqrt{-g} \left[\frac{1}{2\kappa^2} (R + d(d-1)) - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} \right]$$



$$F_{rt}(r) = \partial_r A_t(r)$$

AdS-Reissner-Nordström

Thermodynamics

$$T \ll \mu$$

$$s = s_0 \mu^{d-1} + s_1 \mu^{d-2} T + \mathcal{O}\left(\frac{T^2}{\mu^2}\right)$$

$$c_V = s_1 \mu^{d-2} T + \mathcal{O}\left(\frac{T^2}{\mu^2}\right)$$

AdS-Reissner-Nordström

Hydrodynamics

$$T \gg \mu$$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Sound mode

$$\omega(k) = \pm v_s k + i\Gamma k^2 + \dots$$

$$v_s^2 = \frac{1}{d-1} \qquad \qquad \Gamma \propto \frac{1}{T}$$

AdS-Reissner-Nordström

Hydrodynamics

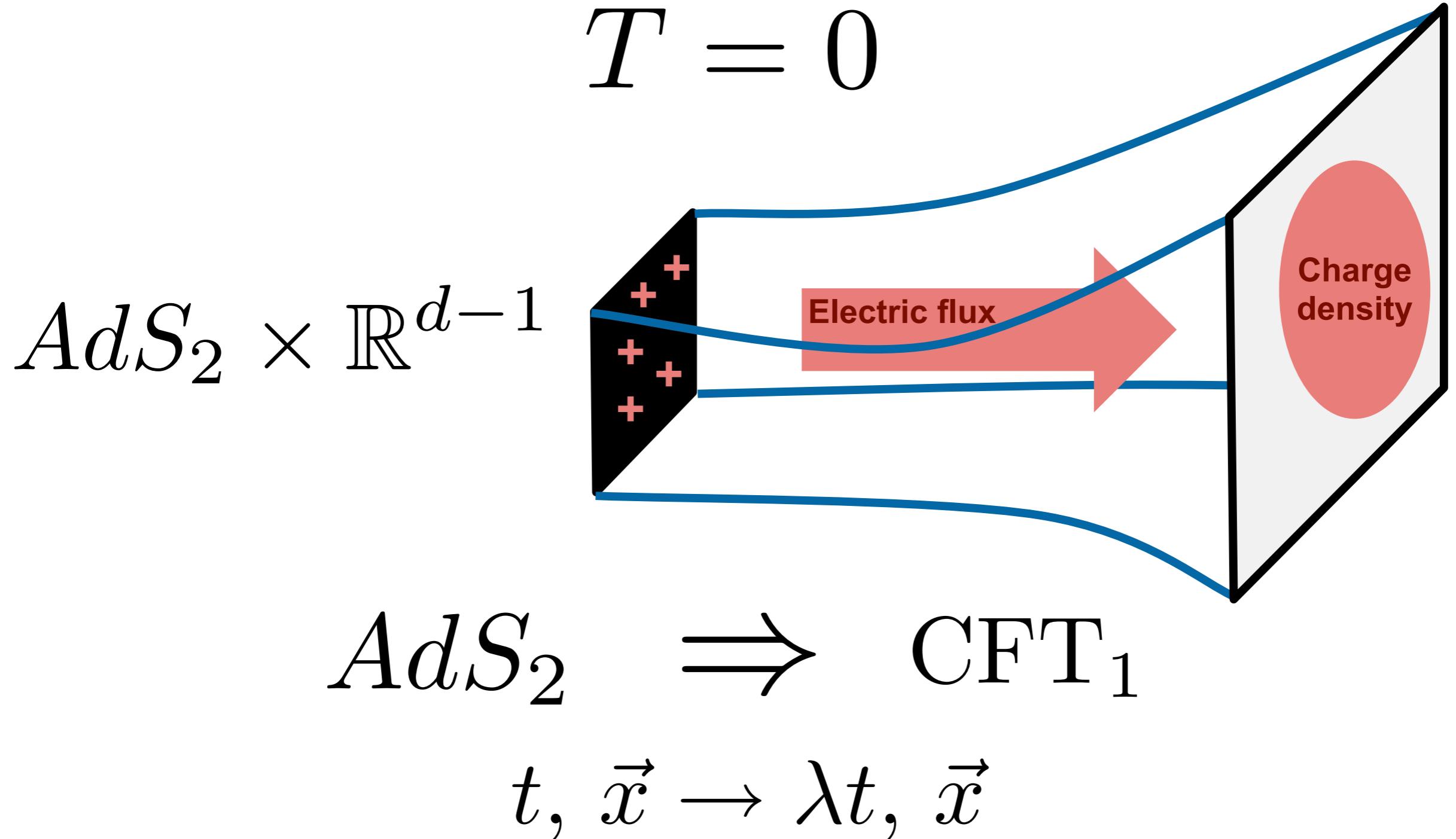
$$T \gg \mu$$

$$\sigma_{DC}(\mu > 0) = \infty$$

NO MOMENTUM DISSIPATION

$$\sigma_{DC}(\mu = 0) \propto T^{d-3}$$

AdS-Reissner-Nordström



“local quantum criticality”

AdS-Reissner-Nordström

$$T = 0$$

$$s = s_0 \mu^{d-1}$$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

$$v_s^2 = \frac{1}{d-1}$$

$$\Gamma \propto \frac{1}{\mu}$$

AdS-Reissner-Nordström

Hydrodynamics

$$T \gg \mu$$

$$\ell_{\text{MFP}} \propto \frac{1}{T} \left(1 + \mathcal{O}\left(\frac{\mu}{T}\right) \right)$$

$$\ell_{\text{MFP}} k \ll 1$$

AdS-Reissner-Nordström

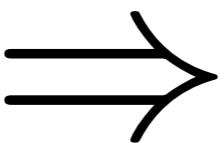
Hydrodynamics

$$T \ll \mu$$

???

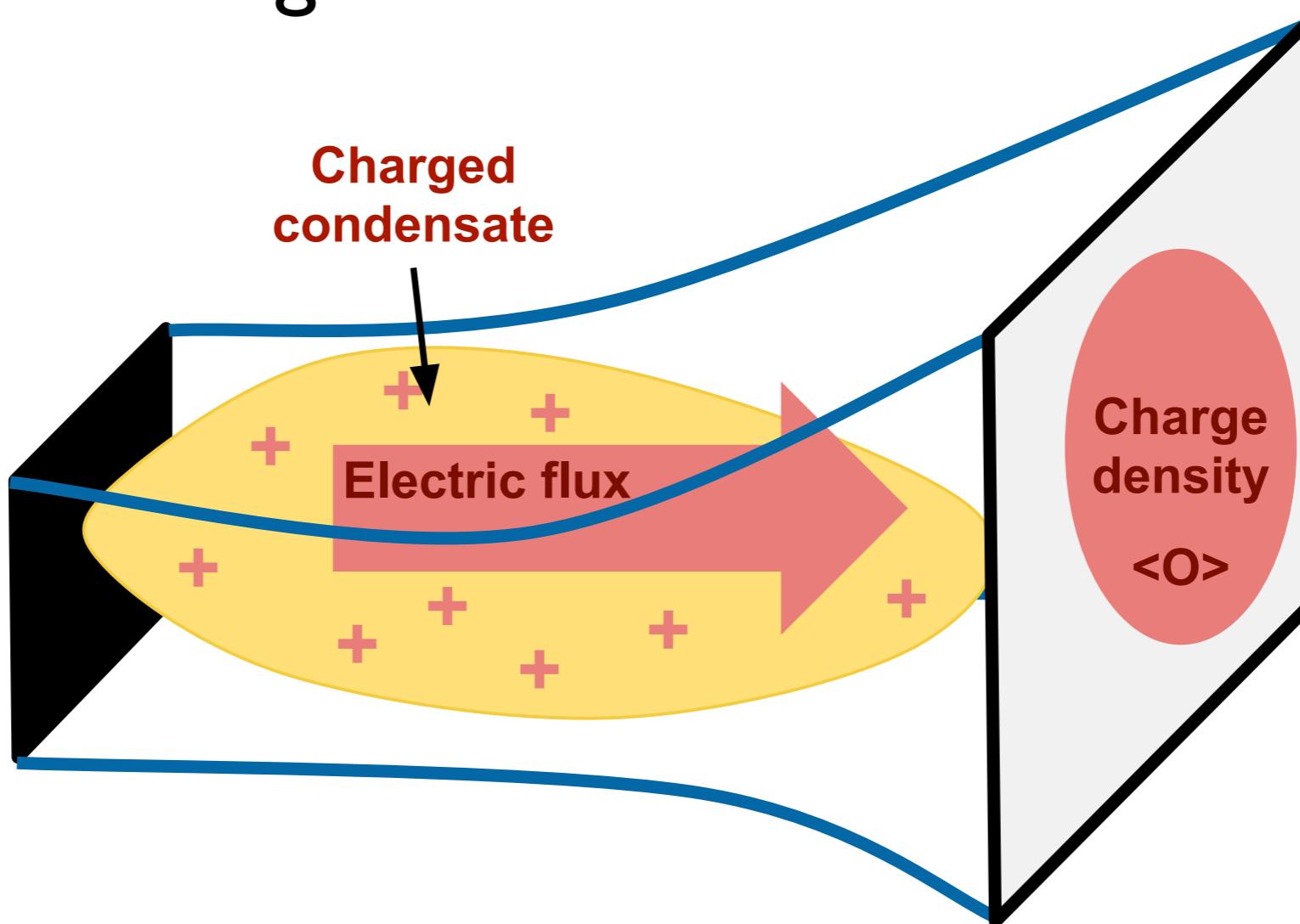
AdS-Reissner-Nordström

Finite entropy



Instabilities

Charged Scalar Condensation



Holographic Superfluid

No Horizon, No Entropy

AdS-Reissner-Nordström

What is the ground state?

Must fix field content: mass, charge, ...

Need UV completion

“Embedding” into string theory

Holographic Matter

Bottom-up:

AdS solution of some *ad hoc* Lagrangian

Top-down:

AdS solution to a string or supergravity theory

Bottom-up:

AdS solution of some *ad hoc* Lagrangian

Good

Simple

Good chance for universality

Bad

Is bulk theory well-defined,
self-consistent?

Does dual field theory exist?

Top-down:

AdS solution to a string or supergravity theory

Good chance dual theory exists

Good

Good chance for
weak-coupling
analysis

Bad

COMPLICATED!

Top-down:

type IIB supergravity bosonic equations of motion

$$\nabla^2 \Phi = e^{2\Phi} \partial_M C \partial^M C - \frac{g e^{-\Phi}}{12} H_{MNP} H^{MNP} + \frac{g e^\Phi}{12} \tilde{F}_{MNP} \tilde{F}^{MNP}$$

$$\nabla^M (e^{2\Phi} \partial_M C) = -\frac{g e^\Phi}{6} H_{MNP} \tilde{F}^{MNP} ,$$

$$d*(e^\Phi \tilde{F}_3) = g F_5 \wedge H_3 ,$$

$$d*(e^{-\Phi} H_3 - C e^\Phi \tilde{F}_3) = -g F_5 \wedge F_3 ,$$

$$d*\tilde{F}_5 = -F_3 \wedge H_3 ,$$

$$\begin{aligned} R_{MN} = & \frac{1}{2} \partial_M \Phi \partial_N \Phi + \frac{e^{2\Phi}}{2} \partial_M C \partial_N C + \frac{g^2}{96} \tilde{F}_{MPQRS} \tilde{F}_N^{PQRS} \\ & + \frac{g}{4} (e^{-\Phi} H_{MPQ} H_N^{PQ} + e^\Phi \tilde{F}_{MPQ} \tilde{F}_N^{PQ}) \\ & - \frac{g}{48} G_{MN} (e^{-\Phi} H_{PQR} H^{PQR} + e^\Phi \tilde{F}_{PQR} \tilde{F}^{PQR}) . \end{aligned}$$

GOAL

Complete stability analysis
in top-down system
of cold holographic matter

Outline:

- Holographic Matter: Bottom-up
- Holographic Matter: Top-down
- Stability Analysis
- Summary and Outlook

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
N_c	D3	×	×	×	×					
N_f	D7	×	×	×	×	×	×	×	×	

D3-branes

$\mathcal{N} = 4$ supersymmetric $SU(N_c)$ Yang-Mills

$$\lambda = g_{YM}^2 N_c$$

$$\beta(\lambda) = 0$$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
N_c	D3	×	×	×	×					
N_f	D7	×	×	×	×	×	×	×	×	

D3-branes

$\mathcal{N} = 4$ supersymmetric $SU(N_c)$ Yang-Mills

$$N_c \rightarrow \infty \quad g_{YM}^2 \rightarrow 0$$

λ fixed

$\lambda \gg 1$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
N_c	D3	×	×	×	×					
N_f	D7	×	×	×	×	×	×	×	×	

D3-branes

$\mathcal{N} = 4$ supersymmetric $SU(N_c)$ Yang-Mills

Type IIB Supergravity

$AdS_5 \times S^5$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
N_c	D3	×	×	×	×					
N_f	D7	×	×	×	×	×	×	×	×	

D7-branes

N_f $\mathcal{N} = 2$ supersymmetric hypermultiplets

in N_c of $SU(N_c)$

“quarks”

ψ

“squarks”

ϕ

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
N_c	D3	×	×	×	×					
N_f	D7	×	×	×	×	×	×	×	×	

D7-branes

N_f $\mathcal{N} = 2$ supersymmetric hypermultiplets

$$U(N_f) = U(1)_B \times SU(N_f)$$

“baryon number”

$$\psi \rightarrow e^{i\alpha} \psi \quad \phi \rightarrow e^{i\alpha} \phi$$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
N_c	D3	×	×	×	×					
N_f	D7	×	×	×	×	×	×	×	×	

D7-branes

N_f $\mathcal{N} = 2$ supersymmetric hypermultiplets

$\mathcal{N} = 2$ supersymmetric mass m

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
N_c	D3	×	×	×	×					
N_f	D7	×	×	×	×	×	×	×	×	

D7-branes

N_f $\mathcal{N} = 2$ supersymmetric hypermultiplets

Probe Limit

$$N_c \rightarrow \infty \quad \lambda \gg 1$$

$$N_f \text{ fixed} \implies N_f/N_c \ll 1$$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
N_c	D3	×	×	×	×					
N_f	D7	×	×	×	×	×	×	×	×	

D7-branes

N_f $\mathcal{N} = 2$ supersymmetric hypermultiplets

neglect QUANTUM EFFECTS of flavor

$$\beta(\lambda) = +\mathcal{O}\left(\lambda^2 \frac{N_f}{N_c}\right) \rightarrow 0$$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
N_c	D3	×	×	×	×					
N_f	D7	×	×	×	×	×	×	×	×	

D7-branes

N_f $\mathcal{N} = 2$ supersymmetric hypermultiplets

N_f probe D7-Branes

$AdS_5 \times S^3$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
N_c	D3	×	×	×	×					
N_f	D7	×	×	×	×	×	×	×	×	

D7-branes

N_f $\mathcal{N} = 2$ supersymmetric hypermultiplets

$$S_{D7} = -N_f T_{D7} \int d^8 \xi \sqrt{-\det(g_{ab} + F_{ab})}$$

Mesons

$$X^8, X^9 = \mathcal{O}_m = \bar{\psi}\psi + \phi^\dagger (m + \Phi) \phi$$

$$A_\mu = J^\mu = \bar{\psi}\gamma^\mu\psi - i\phi^\dagger D^\mu\phi$$

$$\text{fermions} = \bar{\psi}\phi + \phi^\dagger\psi$$

Neutral under $U(1)_b$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
N_c	D3	×	×	×	×					
N_f	D7	×	×	×	×	×	×	×	×	

D7-branes

N_f $\mathcal{N} = 2$ supersymmetric hypermultiplets

$$S_{D7} = -N_f T_{D7} \int d^8\xi \sqrt{-\det(g_{ab} + F_{ab})}$$

$$\langle J^t \rangle \quad = \quad A_t(r)$$

Thermodynamics

$$T = 0$$

Quantum Phase Transition

$$\mu < m$$

$$\mu > m$$

$$\langle J^t \rangle = 0$$

$$\langle J^t \rangle \propto N_f N_c (\mu^2 - m^2) \mu$$

$$\langle \mathcal{O}_m \rangle = 0$$

$$\langle \mathcal{O}_m \rangle \propto N_f N_c (\mu^2 - m^2) m$$

2nd order, mean-field exponents

NO CHANGE IN SYMMETRY

Thermodynamics

$$T \ll \mu$$

$$m = 0$$

$$s = s_0 \mu^3 + s_1 \mu^{-3} T^6 + \mathcal{O}(\mu^{-4} T^7)$$

$$c_V = s_1 \mu^{-3} T^6 + \mathcal{O}(\mu^{-4} T^7)$$

Hydrodynamics

$$T \ll \mu, m$$

$$\sigma_{DC} \propto \langle J^t \rangle T^{-2}$$

Probe limit MIMICS dissipation

$$\langle T_{\mu\nu} \rangle = \mathcal{O}(N_c^2)_{\mu\nu} + \mathcal{O}(N_f N_c)_{\mu\nu}$$

Hydrodynamics

Karch, Son,
Starinets
0806.3796

$$T = 0$$

Sound mode!

Kulaxizi,
Parnachev
0808.3953

POLE in $\langle J^t J^t \rangle_{\text{ret}}$

$$\omega(k) = \pm v_s k + i\Gamma k^2 + \mathcal{O}(k^3)$$

$$v_s^2 = \frac{\mu^2 - m^2}{3\mu^2 - m^2}$$

Hydrodynamics

Karch, Son,
Starinets
0806.3796

$$T = 0$$

Sound mode!

Kulaxizi,
Parnachev
0808.3953

POLE in $\langle J^t J^t \rangle_{\text{ret}}$

$$\omega(k) = \pm v_s k + i\Gamma k^2 + \mathcal{O}(k^3)$$

$$m = 0 \quad v_s^2 = \frac{1}{3} \quad \propto \frac{1}{\mu}$$

D3/D7 vs. AdS-RN

AdS₅-RN

$$s = s_0 \mu^3$$

$$v_s^2 = \frac{1}{3}$$

$$\Gamma \propto \frac{1}{\mu}$$

$$c_V \propto T$$

$$\sigma_{DC}(\mu = 0) \propto T$$

D3/D7 $m = 0$

$$s = s_0 \mu^3$$

$$v_s^2 = \frac{1}{3}$$

$$\Gamma \propto \frac{1}{\mu}$$

$$c_V \propto T^6$$

$$\sigma_{DC}(\mu > 0) \propto T^{-2}$$

D3/D7 vs. Fermi and Bose

$$\langle \mathcal{O}_m \rangle \propto N_f N_c (\mu^2 - m^2) m$$

Bose

$$\sigma_{DC} \propto T^{-2}$$

Fermi

$$c_V \propto T^6$$

Bose

Fermi

$$c_V \propto T^3$$

$$c_V \propto T$$

D3/D7 vs. Fermi and Bose

$$\omega(k) = \pm v_s k + i\Gamma k^2 + \mathcal{O}(k^3)$$

Bose

Goldstone boson?

No symmetries broken!

Fermi

Zero sound?

Strongly-coupled LFL:
zero and first sound
speeds equal!

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Stability Analysis

Linearized fluctuations of D7-brane fields

$$A_a(r, t, \vec{x}) = A_t(r) \delta_{at} + \delta A_a(r, t, \vec{x})$$

$$\delta A_a(r, t, \vec{x}) = e^{i\omega t - i\vec{k} \cdot \vec{x}} F_a(r)$$

Eigenfrequencies: Quasi-normal Modes (QNM)

Instability = QNM in lower-half of ω plane

Stability Analysis

Linearized fluctuations of D7-brane fields

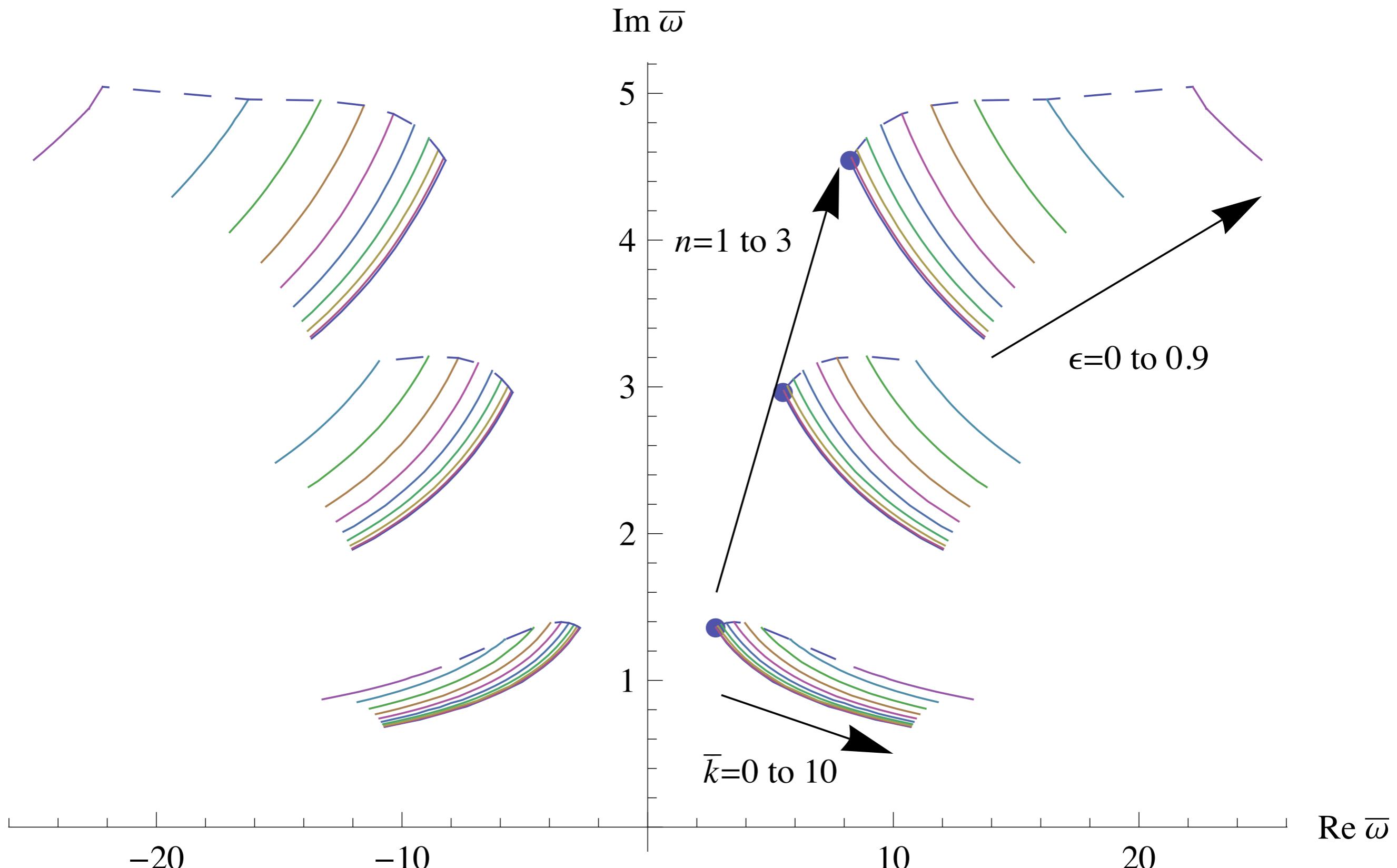
$$A_a(r, t, \vec{x}) = A_t(r) \delta_{at} + \delta A_a(r, t, \vec{x})$$

$$\delta A_a(r, t, \vec{x}) = e^{i\omega t - i\vec{k} \cdot \vec{x}} F_a(r)$$

QNM = pole in retarded Green's function

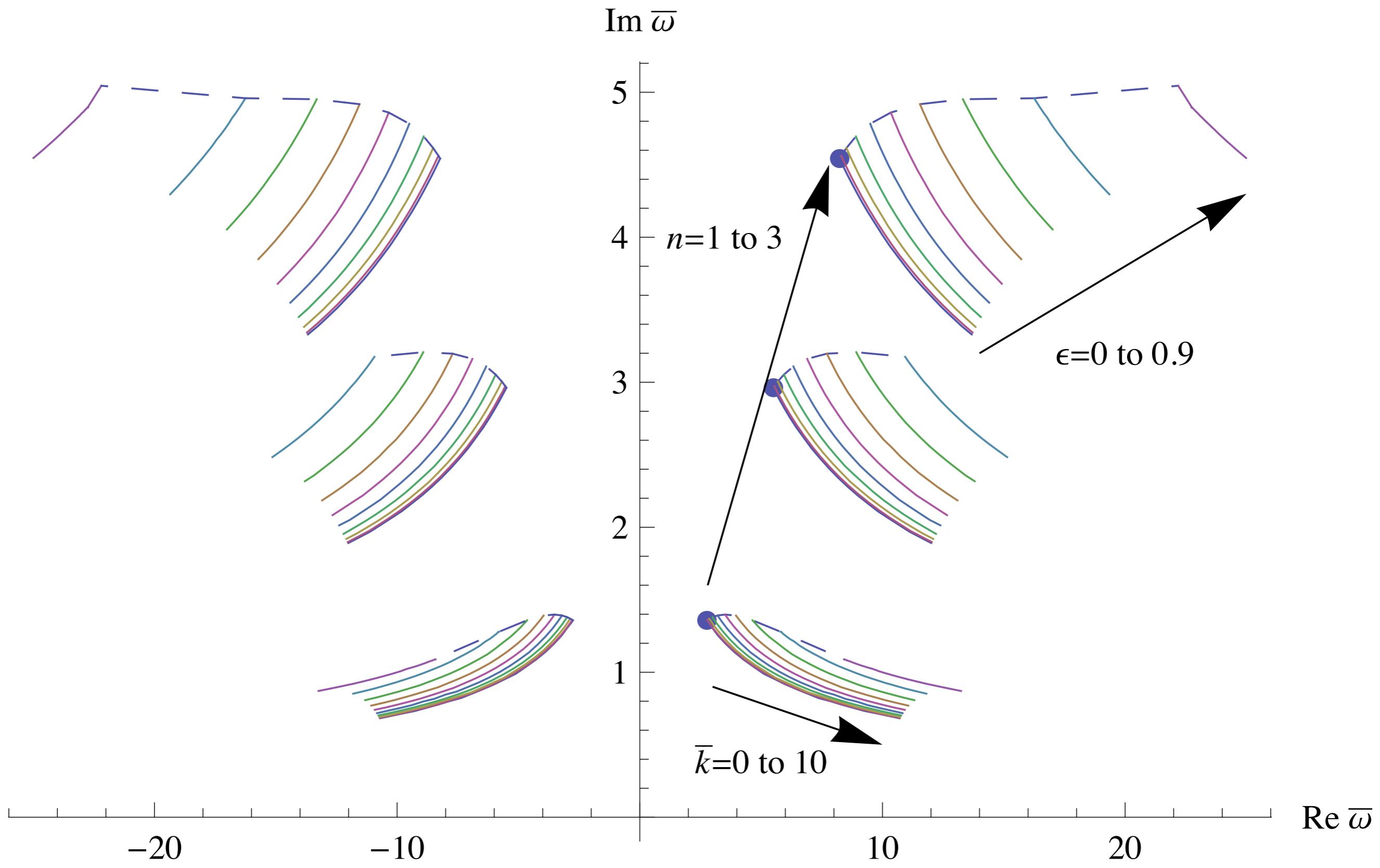
Meson spectrum

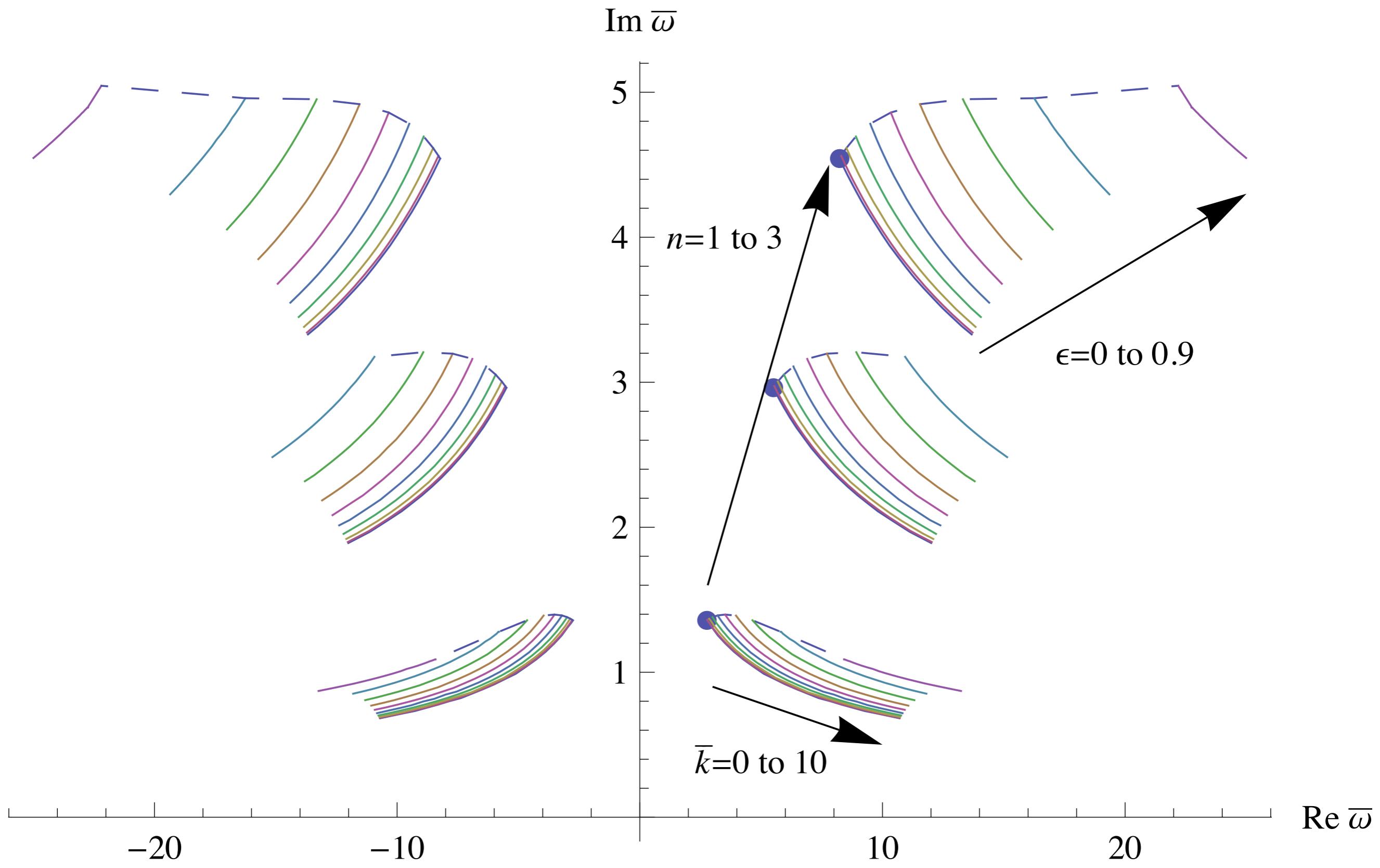
Omit fermionic mesons



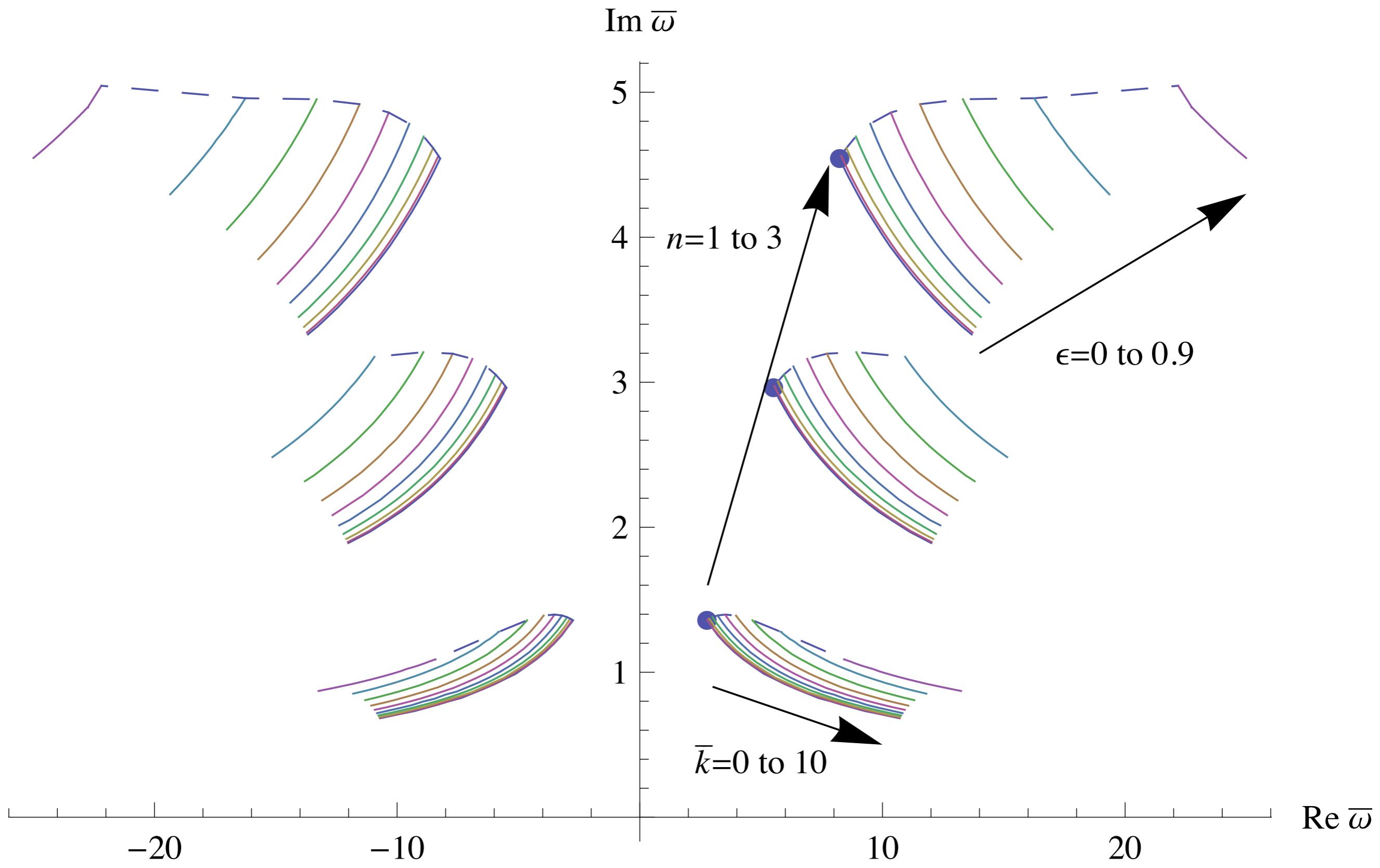
$$\bar{\omega} \propto \frac{\omega}{\sqrt{\mu^2 - m^2}}$$

$$\epsilon = \frac{m}{\mu} \in [0, 1]$$

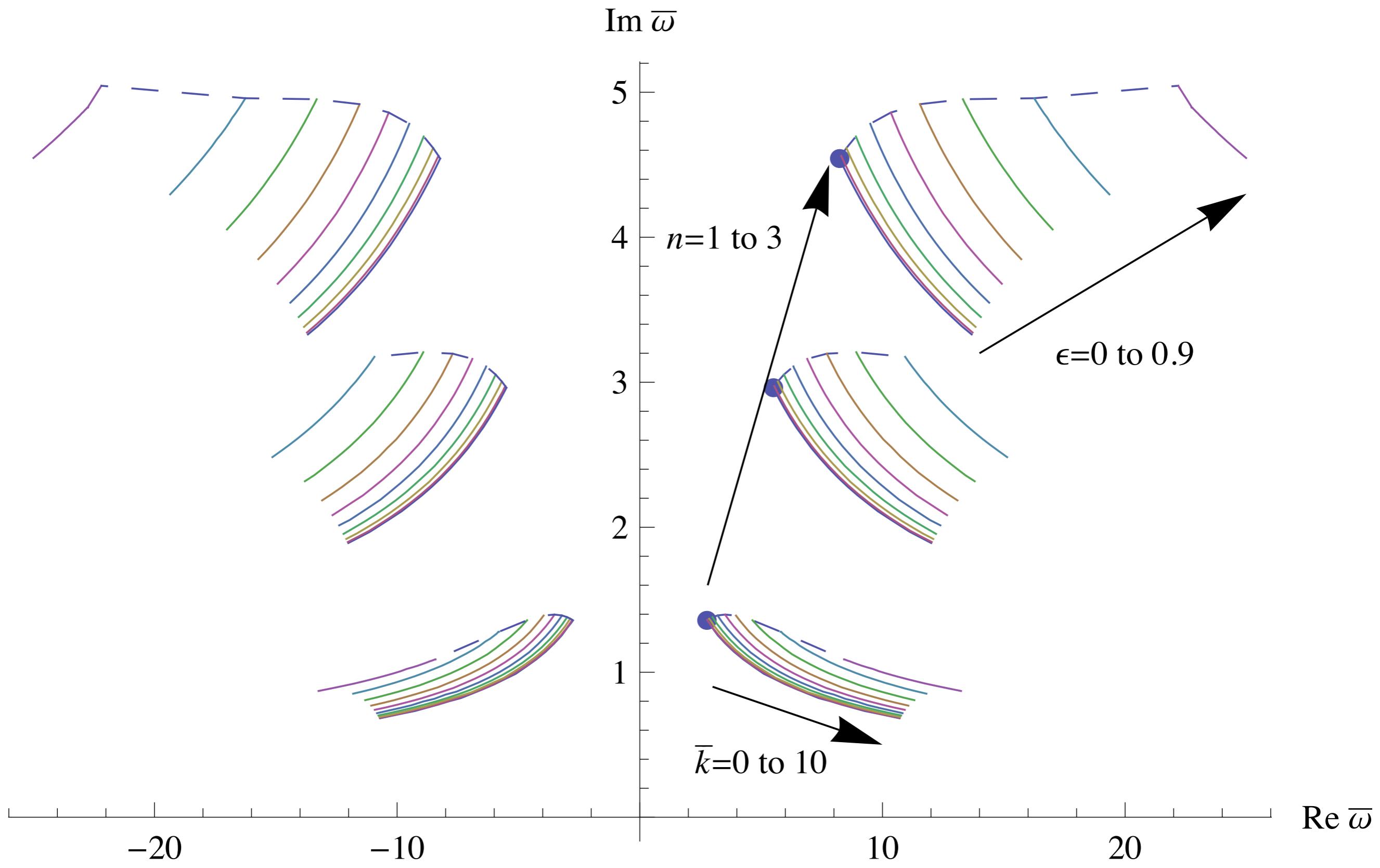

 $\langle \mathcal{O}_m \mathcal{O}_m \rangle_{\text{ret}}$
 $\langle J^t J^t \rangle_{\text{ret}}$
 $\langle J^x J^x \rangle_{\text{ret}}$



Recover sound mode



NO Instabilities!



No $2k_F$ singularities!

Diffusion Mode

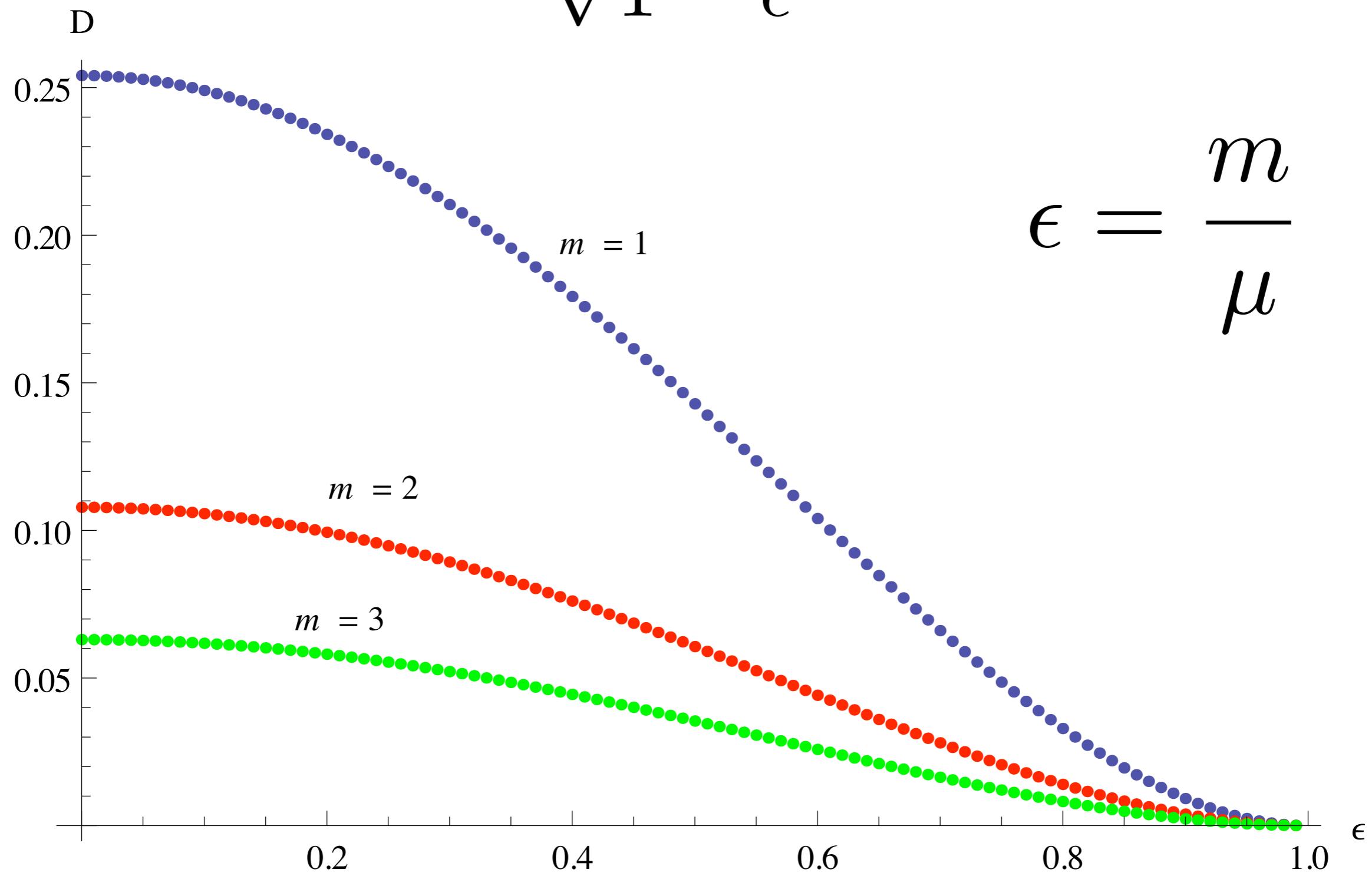
$$\mathcal{O}^I = \phi^\dagger \sigma^I \phi$$

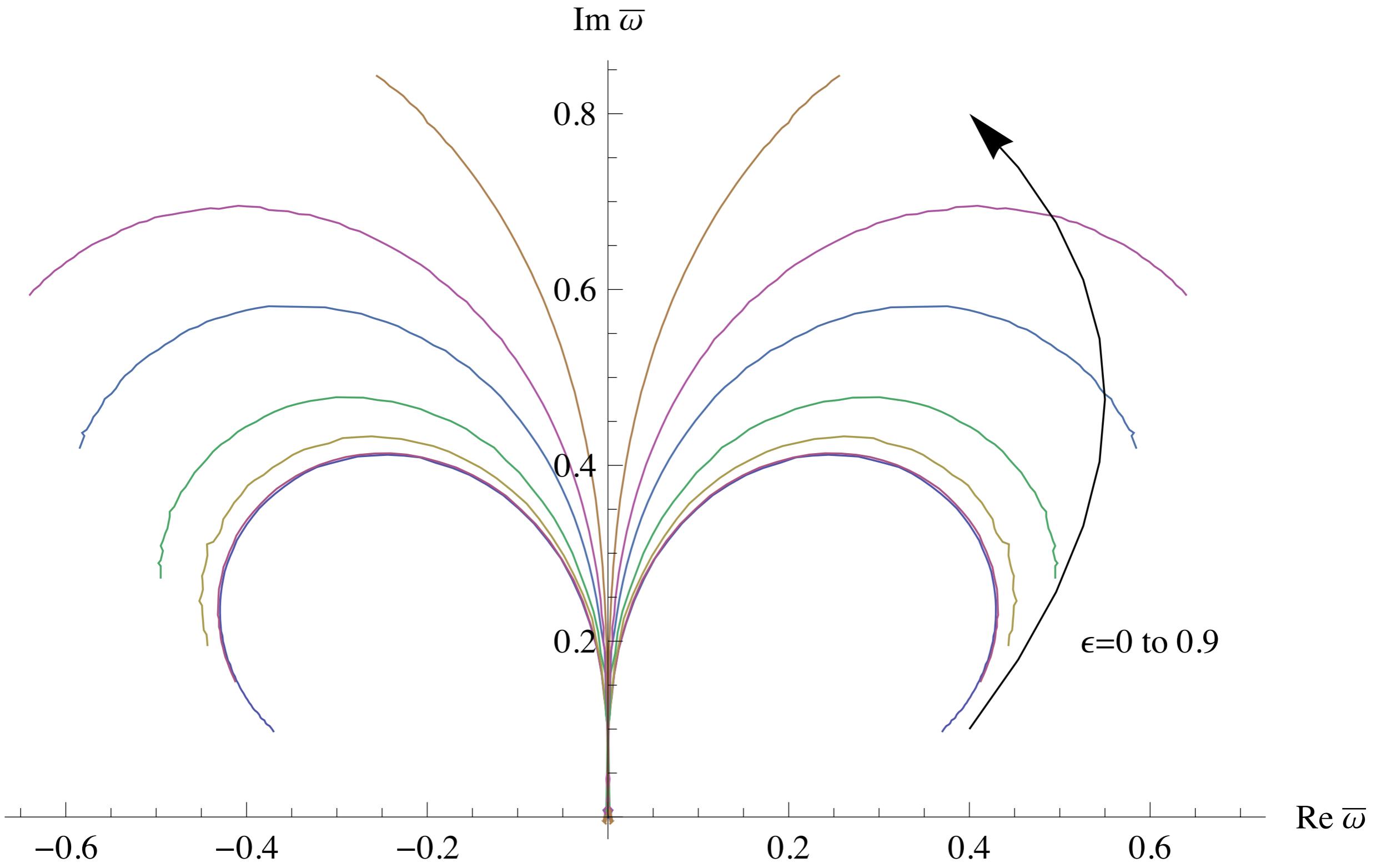
3 of $SU(2)_R$

$$\langle \mathcal{O}^I \mathcal{O}^I \rangle_{\text{ret}}$$

$$\bar{\omega}(\bar{k})=i\frac{D(\epsilon)}{\sqrt{1-\epsilon^2}}\bar{k}^2+\mathcal{O}\left(\bar{k}^3\right)$$

$$\bar{\omega}(\bar{k}) = i \frac{D(\epsilon)}{\sqrt{1 - \epsilon^2}} \bar{k}^2 + \mathcal{O}(\bar{k}^3)$$





$\langle \mathcal{O}^I \mathcal{O}^I \rangle_{\text{ret}}$

AdS₂!

$$r \rightarrow 0$$

ALL linearized equations reduce to

$$\Phi'' + \frac{2}{\bar{r}}\Phi' + \frac{\bar{\omega}^2}{\bar{r}^4}\Phi = 0$$

Massless scalar in AdS2!

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Summary

$\mathcal{N} = 4$ SYM + $\mathcal{N} = 2$ hypermultiplet

T=0, finite baryon density

Finite T=0 entropy

Near-horizon AdS₂

(Mostly) complete (perturbative) stability analysis

STABLE!

Sound mode

Diffusion mode

No $2k_F$ singularities

Outlook

- Fermions?
- Back-reaction?
- What happens at weak coupling?

Probe Branes are Great!

Superfluids

s-wave, p-wave, ...

Fermi surfaces

Marginal Fermi liquid, superfluid Fermi surfaces ...

Topological Insulators

Quantum Hall Effect, T-invariant, ...

Nonlinear Transport

MUCH MORE!

Thank You.