Hall viscosity and electromagnetic response

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joint work with Carlos Hoyos
crucial help from Nicholas Read

Apologies

- This talk will is not directly related to AdS/CFT
  - see Omid Saremi, DTS, arXiv:1103.4851
- Galilean invariance plays important role
  - relativistic case: Fradkin Hughes Leigh 2011
  - relativistic compressible fluids: Nicolis & Son '11
- We are new to the subject, please criticize

What is Hall viscosity?

Standard fluid dynamics: $\partial_t \rho + \partial_i j^i = 0$ continuity eq.

$\partial_t j^i + \partial_j T^{ij} = 0$ Navier-Stokes eq.

$$j^i = \rho v^i$$

$$T^{ij} = \rho v^i v^j + P\delta^{ij} - \eta V_{ij} \qquad V_{ij} = \frac{1}{2}(\partial_i v^j + \partial_j v^i)$$

In 2 spatial dimensions, it is possible to write

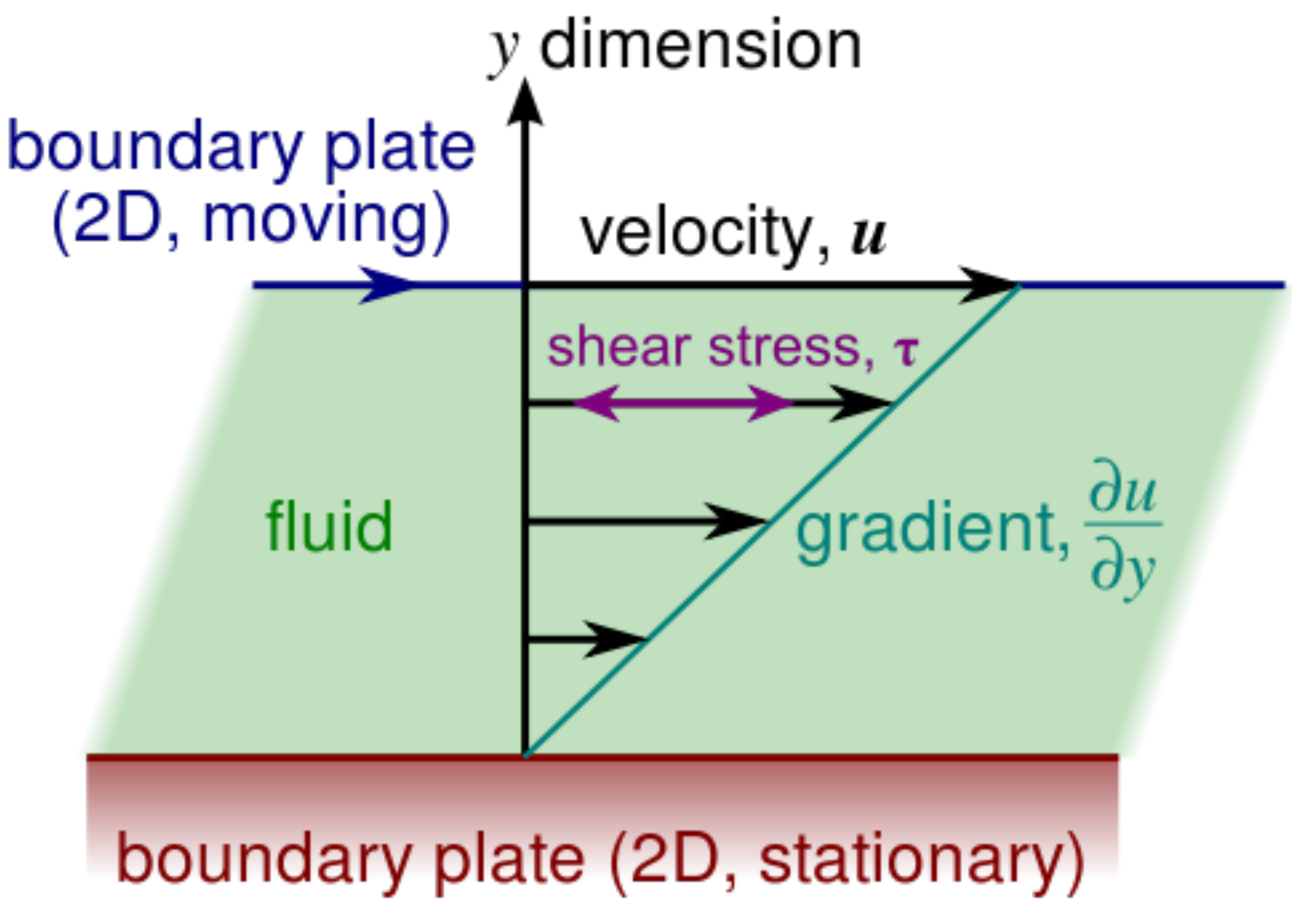
$$T^{ij} = \cdots - \eta_H(\epsilon^{ik}V^{kj} + \epsilon^{jk}V^{ki})$$

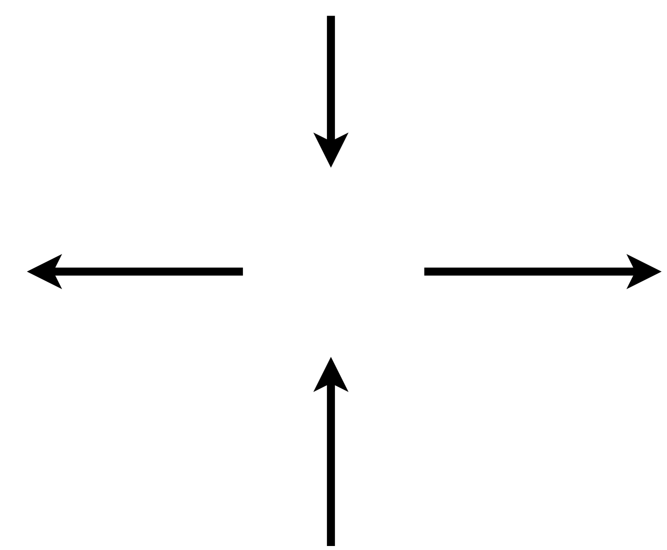
breaks parity

dissipationless

Hall viscosity (Avron Seiler Zograf)

Hall viscosity in picture





Hall shear stress

Quantum Hall state

- simplest example: noninteracting electrons filling n Landau levels
- gapped, no low-energy degree of freedom
- The effective action can be expanded in polynomials of external fields
- To lowest order: Chern-Simons action

$$S = \frac{\nu}{4\pi}\int d^3x\, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

encodes Hall conductivity $\sigma_{xy} = \frac{\nu}{2\pi}\frac{e^2}{\hbar}$

Symmetries of NR theory

DTS, M.Wingate 2006

Microscopic theory

$$S_0 = \int \mathrm{d}t\, \mathrm{d}^2x \,\sqrt{g}\left[\frac{i}{2}\psi^\dagger \overleftrightarrow{D}_t \psi - \frac{g^{ij}}{2m} D_i\psi^\dagger D_j\psi\right] \qquad D_\mu\psi \equiv (\partial_\mu - iA_\mu)\psi$$

Gauge invariance: $\psi \to e^{i\alpha}\psi \qquad A_\mu \to A_\mu + \partial_\mu \alpha$

General coordinate invariance:

$$\delta\psi = -\xi^k \partial_k \psi \equiv \mathcal{L}_\xi \psi$$

$$\delta A_0 = \xi^k \partial_k A_0 \equiv \mathcal{L}_\xi A_0$$

$$\delta A_i = -\xi^k \partial_k A_i - A_k \partial_i \xi^k \equiv \mathcal{L}_\xi A_i$$

$$\delta g_{ij} = -\xi^k \partial_k g_{ij} - g_{kj}\partial_i \xi^k - g_{ik}\partial_j \xi^k \equiv \mathcal{L}_\xi g_{ij}$$

Here ξ is time independent: $\xi=\xi(\mathbf{x})$

NR diffeomorphism

- These transformations can be generalized to be time-dependent: ξ=ξ(t,**x**)

$$\delta\psi = -\mathcal{L}_\xi \psi$$

$$\delta A_0 = -\mathcal{L}_\xi A_0 - A_k \dot{\xi}^k$$

$$\delta A_i = -\mathcal{L}_\xi A_i - m g_{ik} \dot{\xi}^k$$

$$\delta g_{ij} = -\mathcal{L}_\xi g_{ij}$$

Galilean transformations: special case $\xi^i = v^i t$

Where does it come from

Start with complex scalar field

$$S = -\int \mathrm{d}x\, \sqrt{-g}\, (g^{\mu\nu}\partial_\mu \phi^* \partial_\nu \phi + \phi^*\phi)$$

Take nonrelativistic limit:

$$g_{\mu\nu} = \begin{pmatrix} -1 + \dfrac{2A_0}{mc^2} & \dfrac{A_i}{mc} \\ \dfrac{A_i}{mc} & g_{ij} \end{pmatrix}$$

$$\psi = e^{-imcx^0} \frac{\psi}{\sqrt{2mc}}$$

$$S = \int \mathrm{d}t\, \mathrm{d}\mathbf{x}\, \sqrt{g} \left[\frac{i}{2}\psi^\dagger \overleftrightarrow{\partial}_t \psi + A_0 \psi^\dagger \psi - \frac{g^{ij}}{2m}(\partial_i \psi^\dagger + iA_i \psi^\dagger)(\partial_j \psi - iA_j \psi) \right].$$

Relativistic diffeomorphism

$$x^\mu \to x^\mu + \xi^\mu$$

μ =0: gauge transform

$$\psi = e^{-imcx^0}\frac{\psi}{\sqrt{2mc}}$$

μ =i: general coordinate transformations

Interactions

- Interactions can be introduced that preserve nonrelativistic diffeomorphism
  - interactions mediated by fields
- For example, Coulomb interactions

$$S = S_0 + \int \mathrm{d}t\, \mathrm{d}^2x\, \sqrt{g}\, a_0(\psi^\dagger\psi - n_0) + \frac{2\pi\varepsilon}{e^2}\int \mathrm{d}t\, \mathrm{d}^2x\, \mathrm{d}z\, \sqrt{g}\, \left[g^{ij}\partial_i a_0 \partial_j a_0 + (\partial_z a_0)^2\right]$$

$$\delta a_0 = -\xi^k \partial_k a_0$$

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Higher order terms in the action should changed by $-\delta S_{CS}$

But this cannot be achieved by local terms

Resolution

- Higher order terms contain inverse powers of B
- Quantum Hall state with diff. invariance does not exist at zero magnetic field

$$\varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda + \frac{m}{B} g^{ij} E_i E_j + \cdots$$

Let us first discuss power counting

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Slowly varying, nonlinear external fields

$$\delta B \sim B, \quad \delta A_0 \sim \mu, \quad \delta g_{ij} \sim 1$$

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$$E \sim \epsilon \frac{B^{3/2}}{m}$$

Wen-Zee term

- Hall viscosity: described by Wen-Zee term (W.Goldberger & N.Read unpublished; N.Read 2009 KITP talk)
- Introduce spatial vielbein (viel=2) $g_{ij}=e^a{}_i\, e^a{}_j$
- We can now define the spin connection

$$\omega_i = \frac{1}{2}\epsilon^{ab} e^{aj} \nabla_i e^{bj} \qquad \omega_0 = \frac{1}{2}\epsilon^{ab} e^{aj} \partial_0 e^{bj}$$

Vielbein defined up to a local O(2) rotation

$$e_i^a \to e_i^a + \lambda \epsilon^{ab} e_i^b \qquad \omega_\mu \to \omega_\mu - \partial_\mu \lambda$$

like an abelian gauge field

Vielbein and curvature

$$\partial_1\omega_2 - \partial_2\omega_1 = \frac{1}{2}\sqrt{g}\,R$$

Wen-Zee terms

in addition to the Chern-Simons term

$$\frac{1}{2\pi}\epsilon^{\mu\nu\lambda}(\kappa\,\omega_\mu\partial_\nu A_\lambda + \kappa'\omega_\mu\partial_\nu\omega_\lambda)$$

O(ε²)

will be taken into account

O(ε⁴)

will be neglected

The first term gives rise to

- Wen-Zee shift
- Hall viscosity

Wen-Zee shift

- Rewrite S_{WZ} as

$$\frac{\kappa}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu\omega_\lambda = \frac{\kappa}{4\pi}\sqrt{g}\,A_0 R + \cdots$$

Total particle number:

$$Q = \int \mathrm{d}^2x\,\sqrt{g}\,j^0 = \int \mathrm{d}^2x\,\sqrt{g}\left(\frac{\nu}{2\pi}B + \frac{\kappa}{4\pi}R\right) = \nu N_\phi + \kappa\chi$$

On a sphere: $Q = \nu(N_\phi + \mathcal{S}), \quad \mathcal{S} = \frac{2\kappa}{\nu}$

'shift'

IQH states: $\nu=n$, $\kappa=n^2/2$

Laughlin's states: $\nu=1/n$, $\kappa=1/2$

Hall viscosity from WZ term

$$S_{\mathrm{WZ}} = -\frac{\kappa B}{16\pi}\epsilon^{ij}h_{ik}\partial_t h_{jk} + \cdots$$

$$\eta^a = \frac{\kappa B}{4\pi} = \frac{1}{4}\mathcal{S}n$$

derived by N.Read previously

All diff invariant terms up to O(ε^2)

$$\mathcal{L}_1 = \frac{\nu}{4\pi}\Big(\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda + \frac{m}{B}g^{ij}E_iE_j\Big)$$

$$\mathcal{L}_2 = \frac{\kappa}{2\pi}\Big(\varepsilon^{\mu\nu\lambda}\omega_\mu\partial_\nu A_\lambda + \frac{1}{2B}\,g^{ij}\partial_i B\,E_j\Big)$$

$$\mathcal{L}_3 = -\epsilon(B) - \frac{m}{B}\epsilon''(B)g^{ij}\partial_i B\,E_j$$

$$\mathcal{L}_4 = -\frac{1}{2}K(B)g^{ij}\partial_i B\,\partial_j B$$

$$\mathcal{L}_5 = R\,h(B)$$

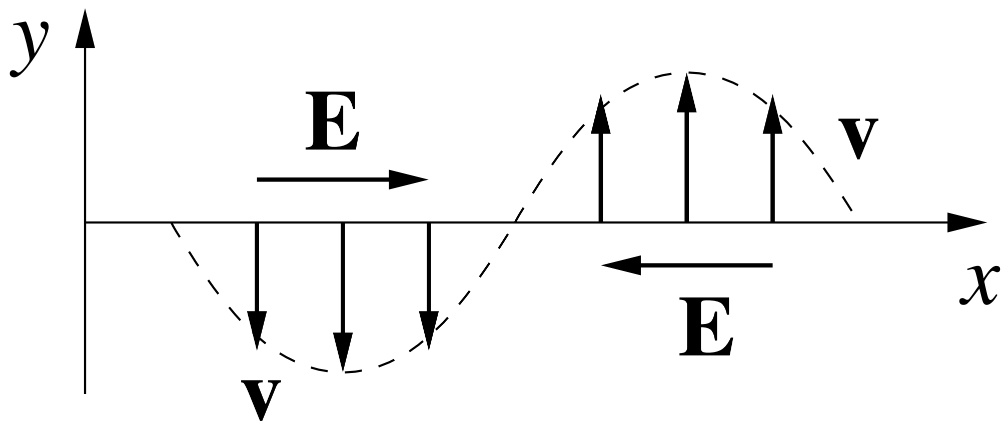
black: O(ε^0) blue: O(ε^2)

Kohn's theorem

- Current-current correlator at q=0, ω≠0 completely fixed: motion of center of mass
- Effective action captures next-to-leading order corrections to conductivities at q=0

$$\sigma_{xy} = \frac{\nu}{2\pi}\frac{\omega_c^2}{\omega_c^2 - \omega^2} \qquad \sigma_{xx} = \frac{\nu}{2\pi}\frac{-i\omega_c\omega}{\omega_c^2 - \omega^2}$$

$\sigma_{xy}(q)$



$$E_x = E\,\mathrm{e}^{iqx}$$

$$j_y = \sigma_{xy}(q) E_x$$

From effective field theory

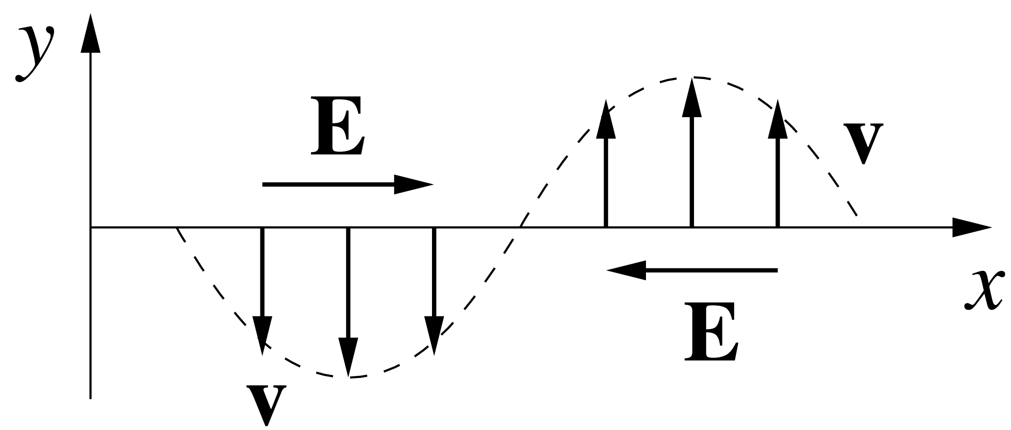
$$\frac{\sigma_{xy}(q)}{\sigma_{xy}(0)} = 1 + C_2 (q\ell)^2 + \mathcal{O}(q^4 \ell^4)$$

$$C_2 = \underbrace{\frac{\eta^a}{\hbar n}}_{= \mathcal{S}/4} - \frac{2\pi}{\nu} \frac{\ell^2}{\hbar\omega_c} B^2 \epsilon''(B)$$

also found by B.Bradlyn, M.Goldstein, N.Read

Physical interpretation

- First term: Hall viscosity



$$\partial_x v_y + \cancel{\partial_y v_x} \neq 0$$

$$T_{xx} = T_{xx}(x) \neq 0$$

additional force $F_x \sim \partial_x T_{xx}$
Hall effect: additional contribution to v_y

Physical interpretation (II)

- 2nd term: more complicated interpretation

Fluid has nonzero angular velocity

$$\Omega(x) = \frac{1}{2}\partial_x v_y = -\frac{cE'_x(x)}{2B}$$

$$\delta B = 2mc\Omega/e$$

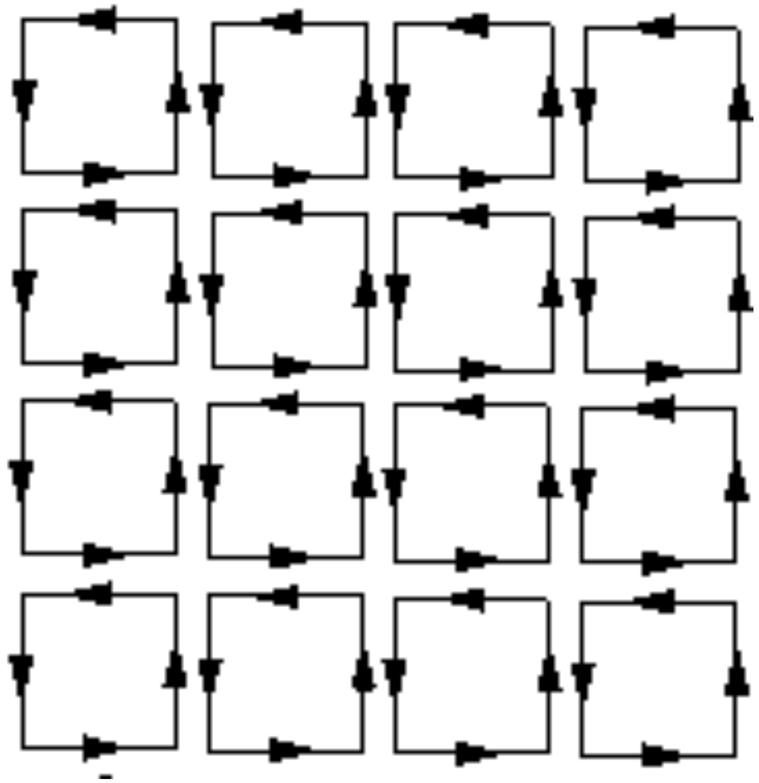
Coriolis=Lorentz

Hall fluid is diamagnetic: $d\epsilon = -M dB$

M is spatially dependent M=M(x)

Extra contribution to current $\mathbf{j} = c\,\hat{\mathbf{z}} \times \nabla M$

Current ~ gradient of magnetization



$$\mathbf{j} = c\,\hat{\mathbf{z}} \times \nabla M$$

High B limit

- In the limit of high magnetic field: ϵ(B) known: free fermions
  - n Landau levels for IQH states
  - first Landau level for FQH states with ν<1
- Wen-Zee shift is known

$$\frac{\sigma_{xy}(q)}{\sigma_{xy}(0)} = 1 - \frac{3n}{4}(q\ell)^2 + \mathcal{O}(q^4\ell^4)$$

IQH, can be checked
for non-interacting electrons
but valid also for interacting case

$$\frac{\sigma_{xy}(q)}{\sigma_{xy}(0)} = 1 + \frac{2n-3}{4}(q\ell)^2 + \mathcal{O}(q^4\ell^4), \quad \nu = \frac{1}{2n+1}$$

exact

Comments on Hall viscosity

- Standard definition of Hall viscosity: stress induced by shear
- Haldane: stress induced by spatially varying electric field
- We found that it gives one contribution to the Hall conductivity at small, finite q
- Can $\sigma_{xy}(q)$ be measured?