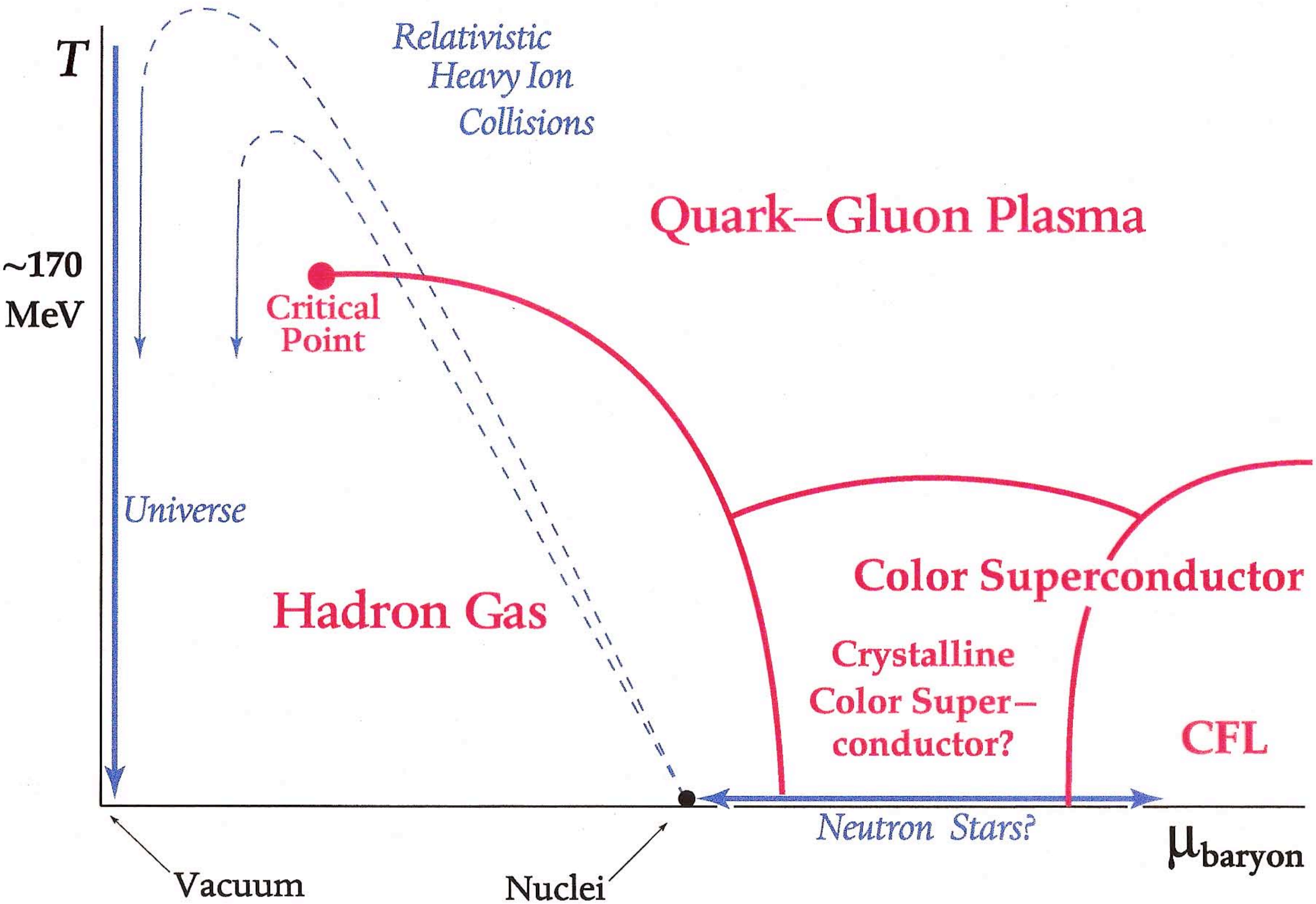
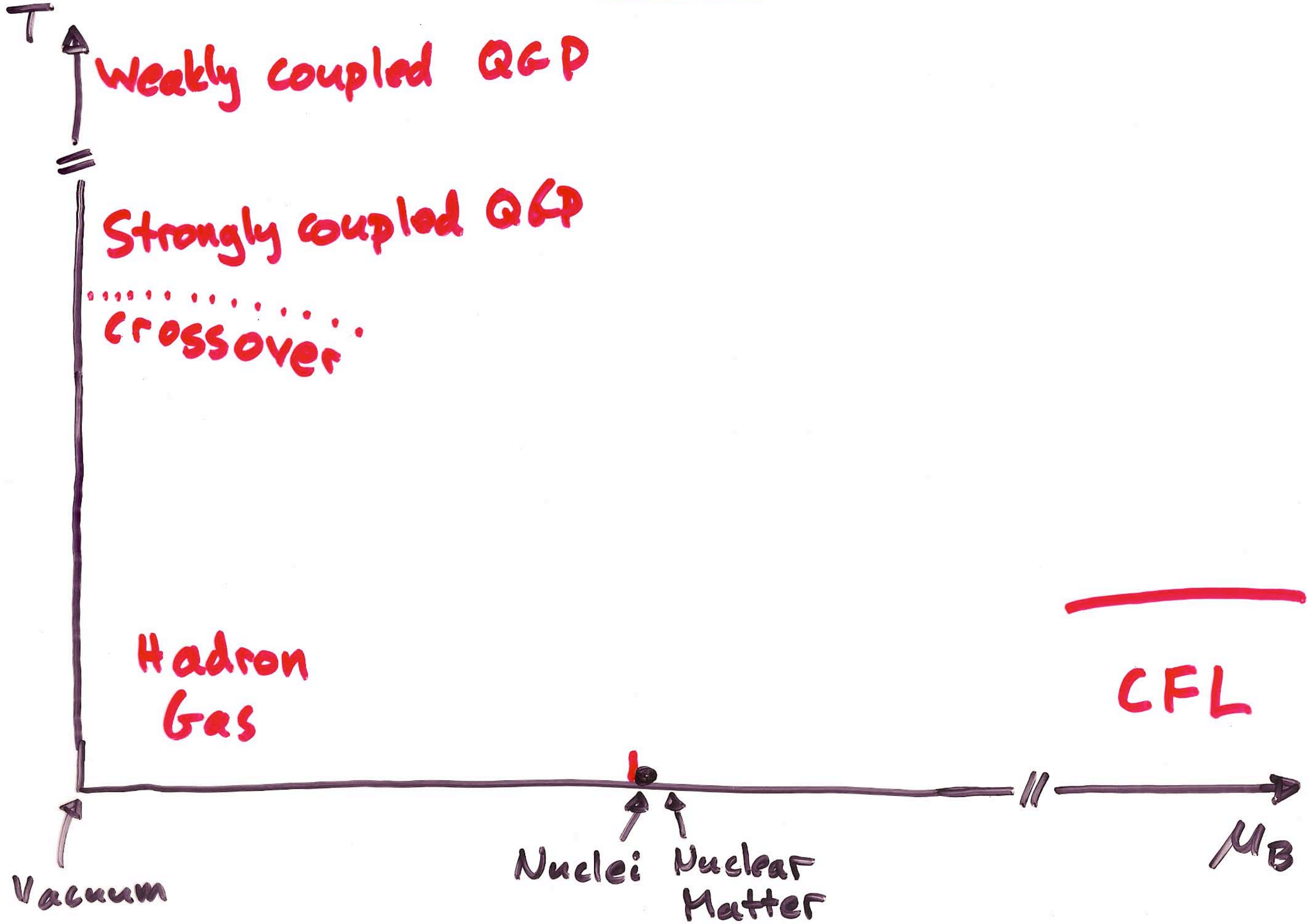
EXPLORING *the* PHASES *of* QCD



Krishna Rajagopal, MIT

KITP AdS/CMT Jul 14, 2009

WHAT WE KNOW, SO FAR



HOW TO FIND THE QCD CRITICAL POINT AT RHIC,

IF IT IS AT $\mu_B \lesssim 400$ MeV

KRISHNA RAJAGOPAL (MIT)

Talks and discussions at:

INT program, August 2008

CPOD conference, June 2009

and reviews:

Koch 0810.2520; Lombardo 0808.3101;

Philipsen 0808.0672; Karsch 0711.0656;

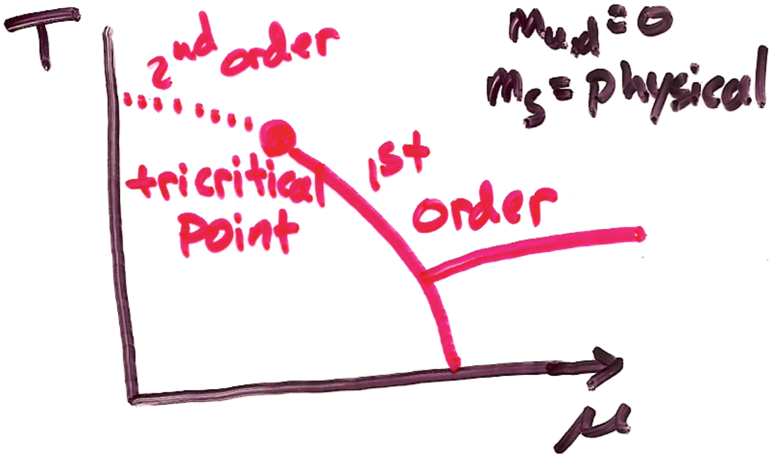
Stephanov hep-lat/0701002

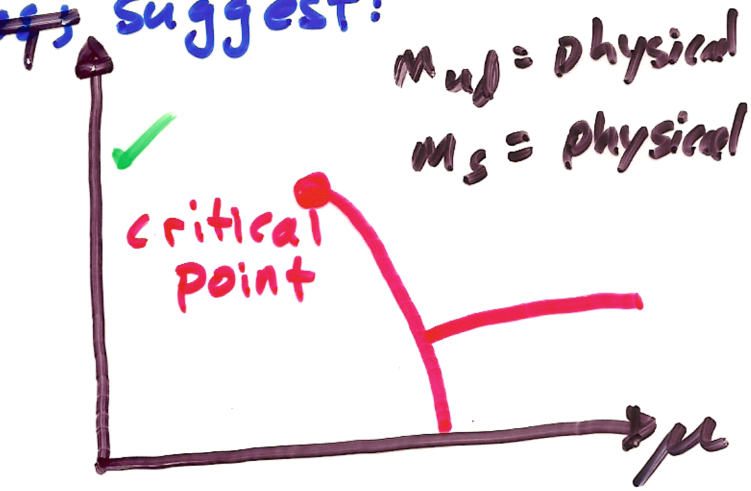
have all been very helpful as I prepared this talk.

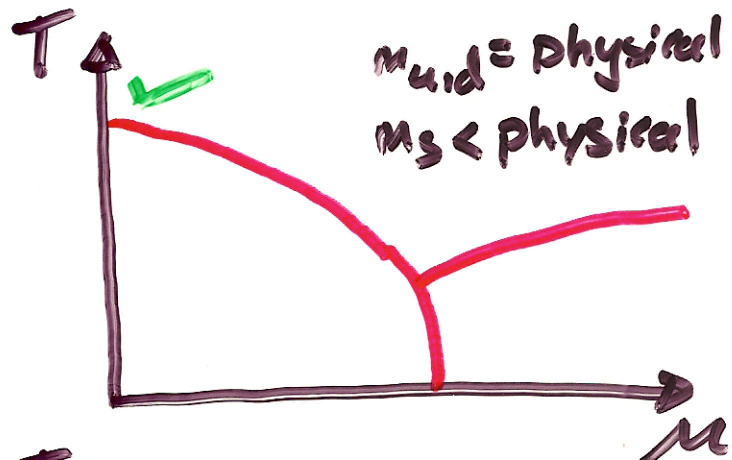
This talk is my attempt to describe a suite of measurements that RHIC can perform that will either find experimental evidence for the QCD critical point or demonstrate convincingly that it is not at $\mu \lesssim 400$ MeV.

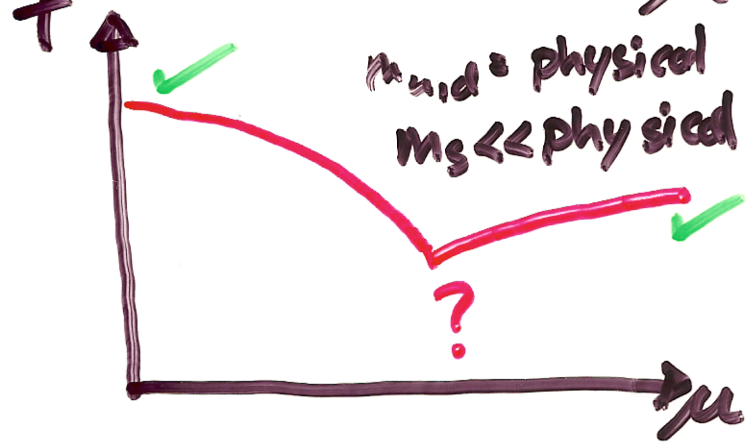
WHY EXPECT A CRITICAL POINT?

- Models; lattice QCD calculations at $\mu = 0$ with varying quark masses, suggest:







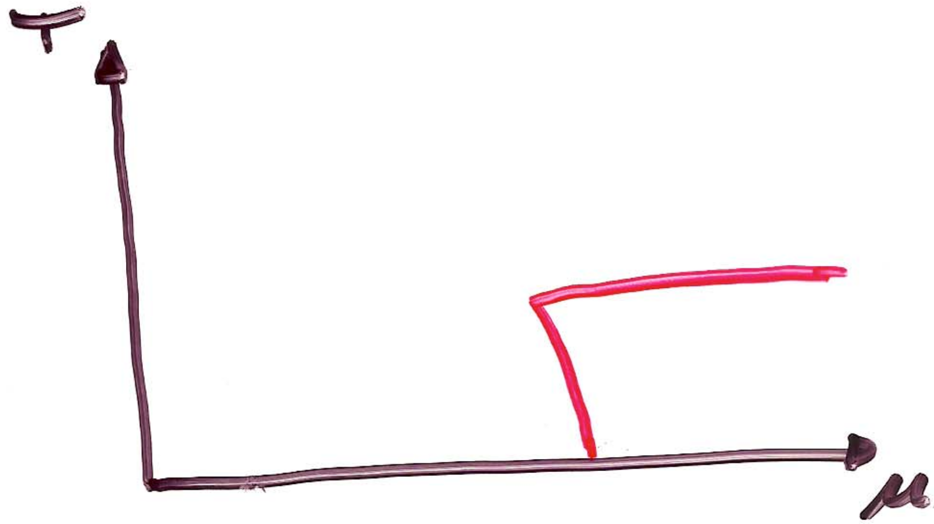


✓ : known

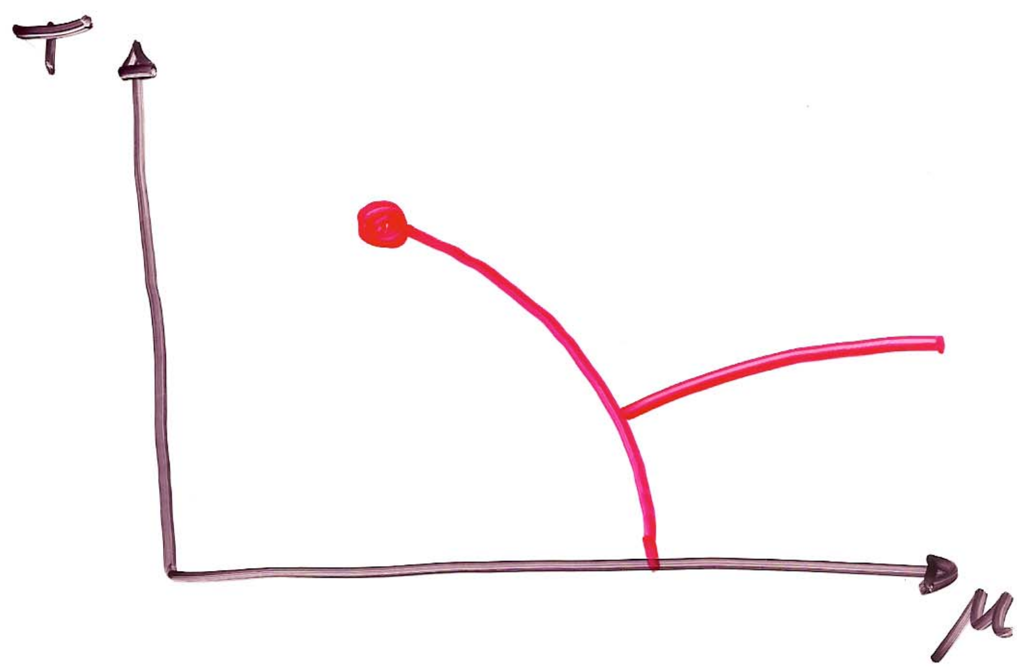
- Universality class of the QCD critical point is known. (ISING)
- Experiments, and lattice calculations with $T \neq 0$, $\mu \neq 0$, needed to locate it.

WARNING

Nothing we **know** precludes pushing the critical point so far to the right that:



although models and **some** lattice calculations favor



LOCATING THE CRITICAL POINT....

... either via lattice calculations,

or via detection of its signatures

in heavy ion collision experiments,

would add a point and a line

to the known QCD phase diagram.

- A qualitative leap in our understanding of QCD in the interior of its phase diagram, currently terra incognita.
- An opportunity for RHIC to write another new chapter in any future book on QCD.

LATTICE QCD WITH $T \neq 0$, $\mu \neq 0$, μ/T NOT LARGE

- $\mu \neq 0$ → complex Euclidean action
  → sign problem
  → difficulty of standard monte carlo ~ $\exp V$
- Several lattice methods now in use rely on smallness of $\mu_q/T = \mu_B/3T$ to control the sign problem:
  - reweighting (Fodor + Katz)
  - continue from imaginary μ (de Forcrand & Philipsen; D'Elia & Lombardo)
  - Taylor expansion of P; radius of convergence (RBC-Bielefeld; Gavai + Gupta)

  Uncertainties still dominated by systematics; (different systematics for different methods, but in all cases includes coarseness of lattice spacing)

  Steady progress; "crawling towards the continuum limit".
- Several groups exploring calculations at fixed n_B, instead of μ. (de Forcrand + Kratochvila; Li, Alexandru & Liu; ...)

LATTICE RESULTS

- via reweighting (Fodor & Katz)

  $\mu_0 = 360 \pm 40\,(\text{stat})$ MeV

- via calculating $dm_c/d\mu$ (de Forcrand + Philipsen)

  $\frac{\mu_0}{T_0} > 3 \rightarrow \mu_0 > 500$ MeV

- via radius of convergence of Taylor expansion

  $\frac{\mu_0}{T_c(\mu=0)} = 1.7 \pm .1\,(\text{stat})$ (Gavai + Gupta)

  $\rightarrow 250 < \mu_0 < 400$ MeV

  (with a "very naive" estimate of systematics)

  $\mu_0 / T_c(\mu=0) > 1.5$ (RBC-Bielefeld)

- STILL SYSTEMATICS DOMINATED

  Nevertheless,

ONE CLEAR LESSON

Lattice calculations provide strong indications, via <u>all</u> algorithms employed to date, that:

$$\boxed{\mu_0 > 200 \text{ MeV}}$$

In the race between lattice calculations and experimental searches to locate the critical point, the lattice team is running strongly but not yet threatening to end the race.

So, lets turn to experimental searches....

HOW EXPERIMENTS CAN LOCATE ●

① At large $\sqrt{s}$, ie small μ, collisions equilibrate well above the crossover....

② Decrease $\sqrt{s}$ in steps, moving freezeout to larger and larger μ.

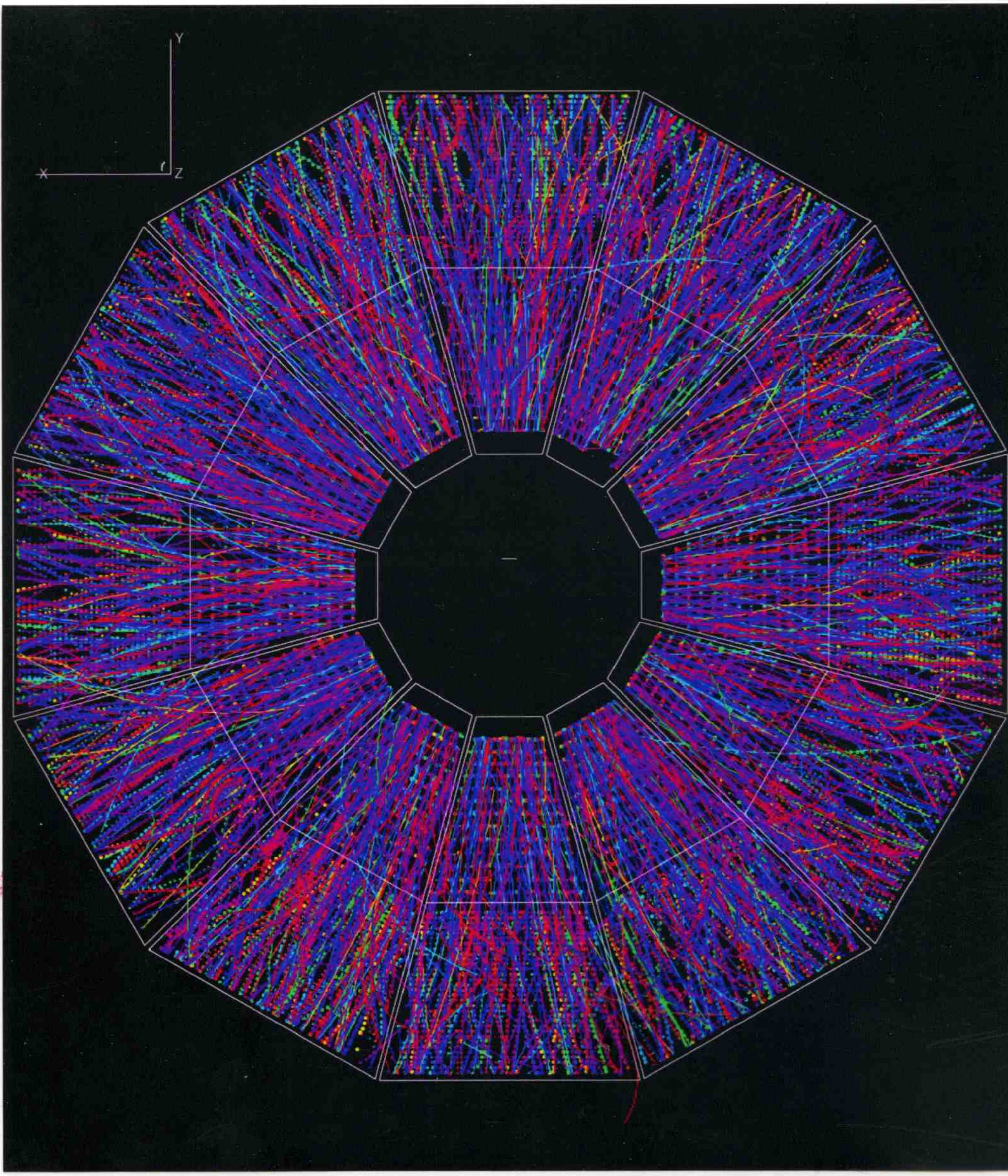
③ Look for signatures:

a) Of the "lumpy" non-equilibrium final state expected after cooling through a first order transition. (Mishustin; Dumitru Paech Stöcker; Randrup; Koch Majumder Randrup;...)

- proton-proton angular correlations ($\Delta\phi$, $\Delta\eta$). [flow turns lumps into clustering in angle] Mocsy & Sorensen
- enhanced event-by-event v_2 fluctuations?

b) Of the critical point itself. I.e. signatures of the long wavelength fluctuations occurring only near ●. Rise and then fall as $\mu\uparrow$, $\sqrt{s}\downarrow$.

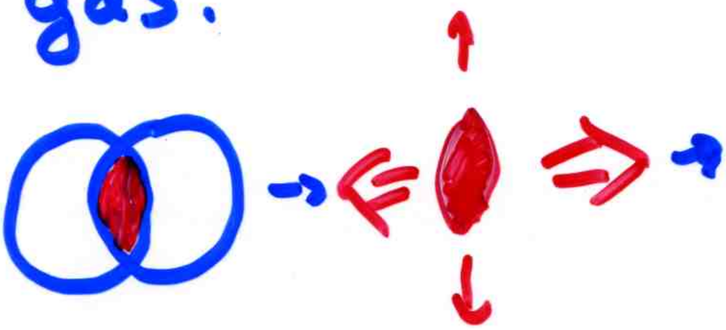
I shall describe several such analogues of critical opalescence.



STAR

QUARK-GLUON LIQUID?

Expts @ RHIC suggest that quark-gluon plasma is so strongly coupled at $T \sim 1.5\,T_c$ accessible at RHIC that it is better thought of as a liquid than a gas.



→ well-described with ideal hydrodynamics (zero m.f.p.)

$$\rightarrow \frac{\text{shear viscosity}}{\text{entropy density}} = \frac{\eta}{s} < \mathcal{O}(0.1)$$

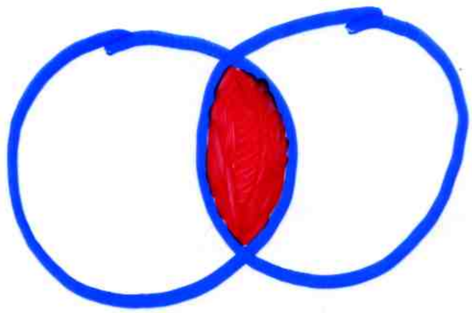
>0.2 ruled out

CF: $\eta/s \sim 1$ according to perturbative QCD calculations

$\eta/s \sim 10$ in water

ELLIPTIC FLOW

Indicates extent of early equilibration:



IF: just lots of p-p collisions, followed by free streaming

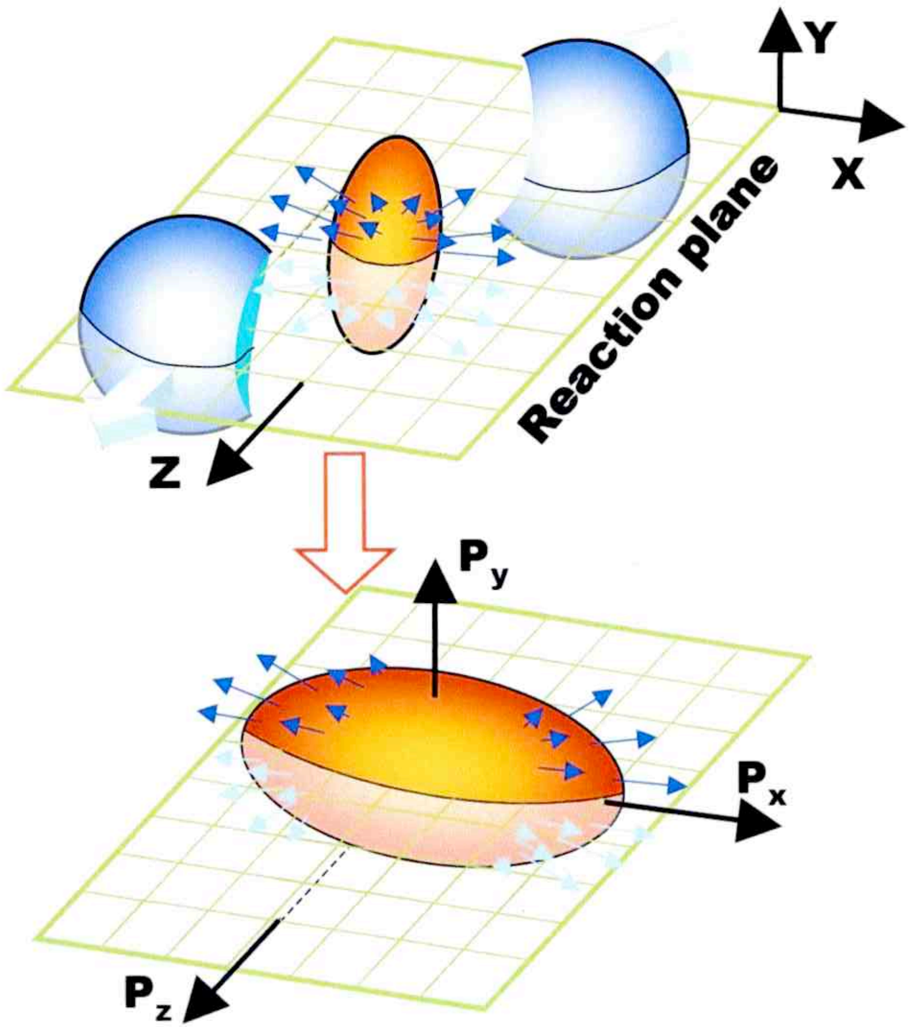
THEN: final state momenta uniformly distributed in azimuthal angle

IF: interaction → equilibration → pressure; pressure gradients → collective flow EARLY, before  circularizes,

THEN: azimuthally asymmetric explosion, final state momenta.

$v_2 \sim \langle \cos 2\phi \rangle \neq 0$

Expansion In Plane



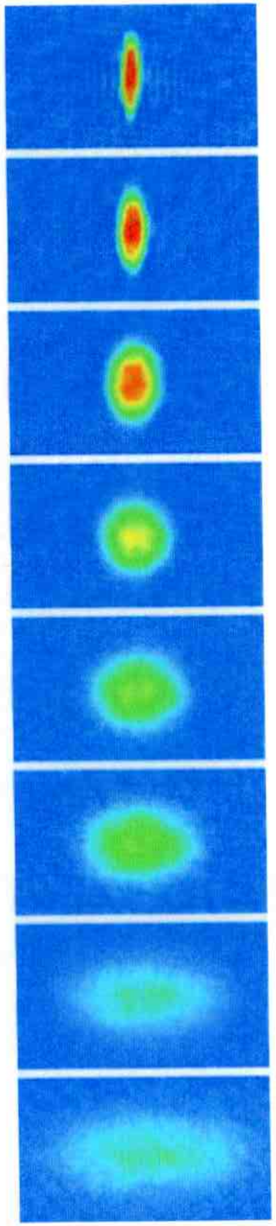
Hiroshi Masui (2008)

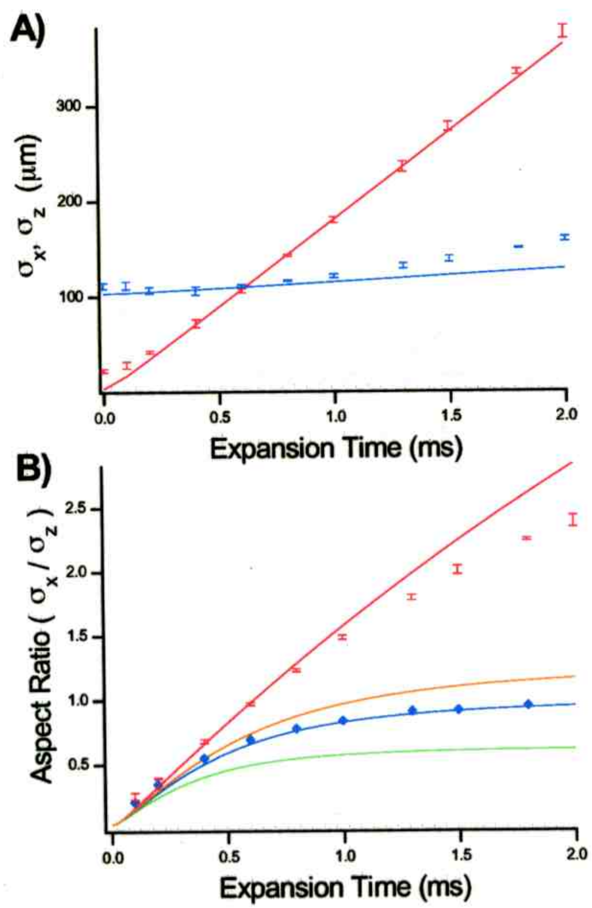
spatial anisotropy ε_2

↓ ↓

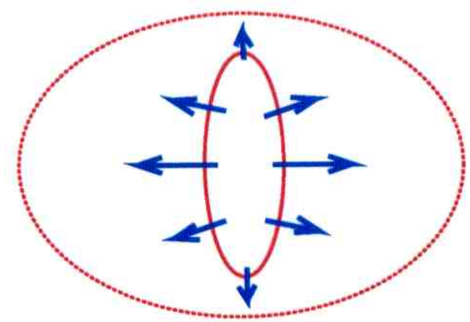
momentum anisotropy v_2

Elliptic Flow of cold fermionic atoms, at unitary point





Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy



Data: Duke group

Transparency: Schaefer

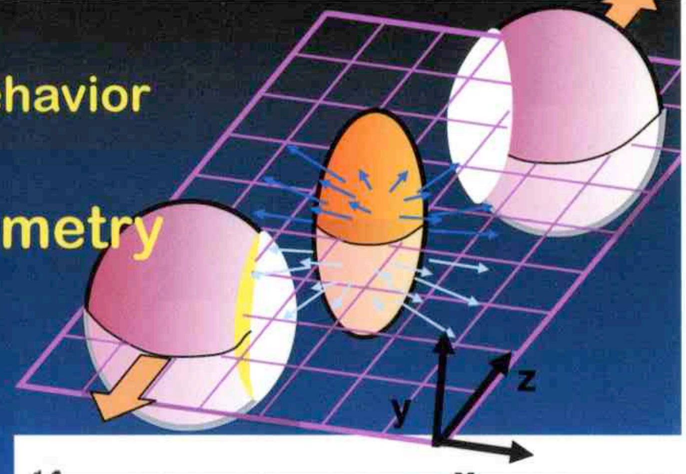
Motion Is Hydrodynamic

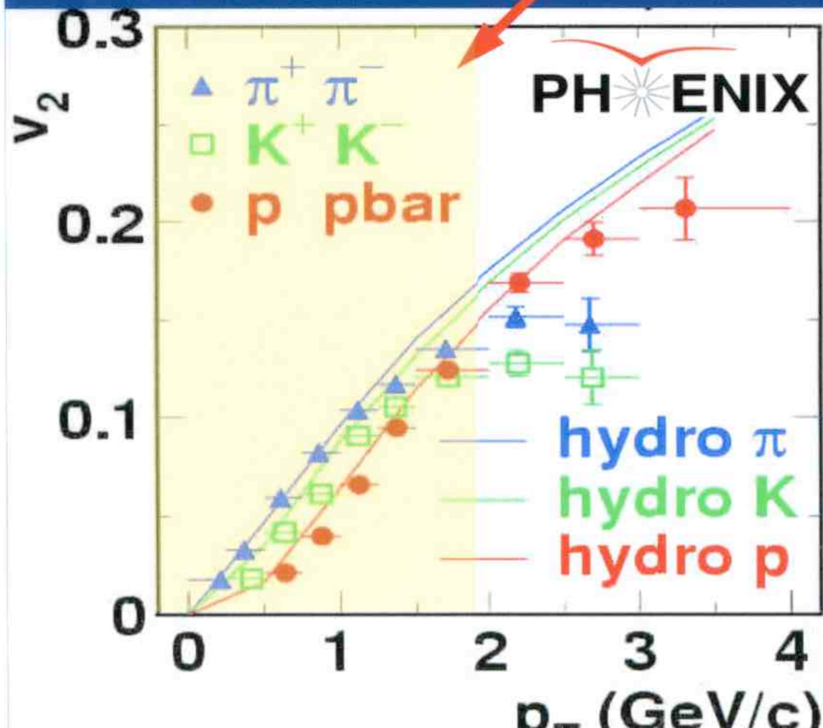
When does thermalization occur?

Strong evidence that final state bulk behavior reflects the initial state geometry

Because the initial azimuthal asymmetry persists in the final state

$dn/d\phi \sim 1 + 2\, v_2(p_T) \cos(2\phi) + \ldots$





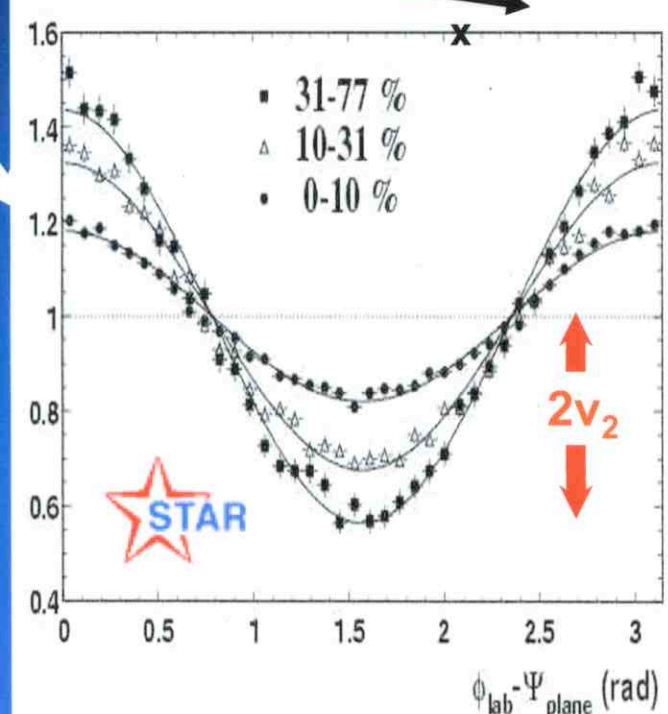


fig: W. Zajc

- Ideal hydrodynamics <u>ASSUMES</u> local equilibrium; zero mean free path; zero dissipation
- Hydro never agreed with v_2 data before RHIC. (At SPS, $v_2^{data} \sim \frac{1}{2} v_2^{hydro}$.)
- At RHIC, hydro does good job of describing v_2, spectra for $p_T < 1$-2 GeV

$\Rightarrow$ "hydro works" by 0.6 - 1 fm after collision

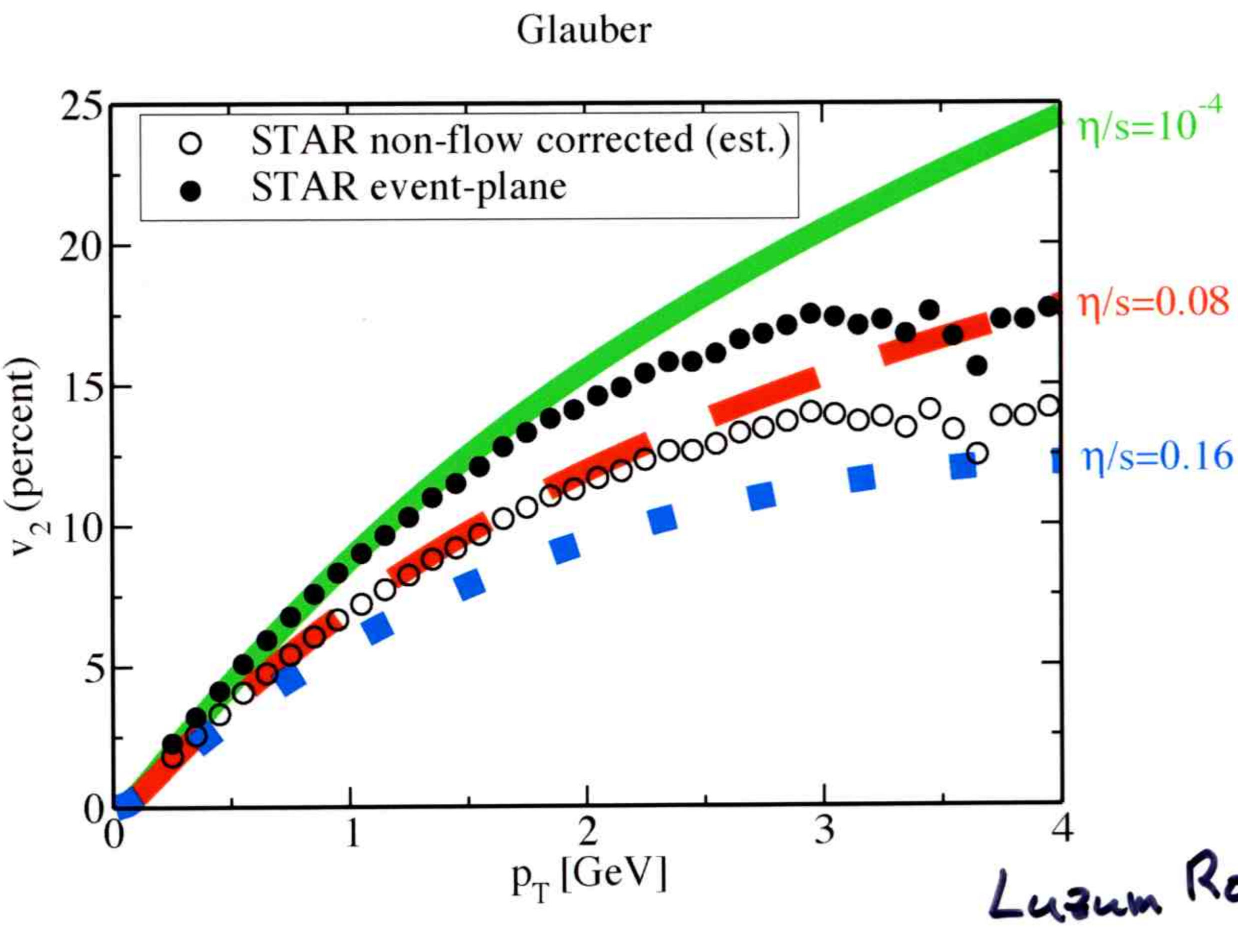
Kolb Heinz

- Challenge to theory: how can equilibration occur so quickly?
- Also, $\Rightarrow$ small shear viscosity

$\frac{\eta}{s} < 0.2$ Teaney

- Challenge: precise extraction of η/s, ie bounding it from below, requires hydro calculations w $\eta \neq 0$; + precise constraints on initial conditions. Muronga; Heinz Song; Romatschke^2; Dusling Teaney; .....

VISCOUS HYDRODYNAMICS

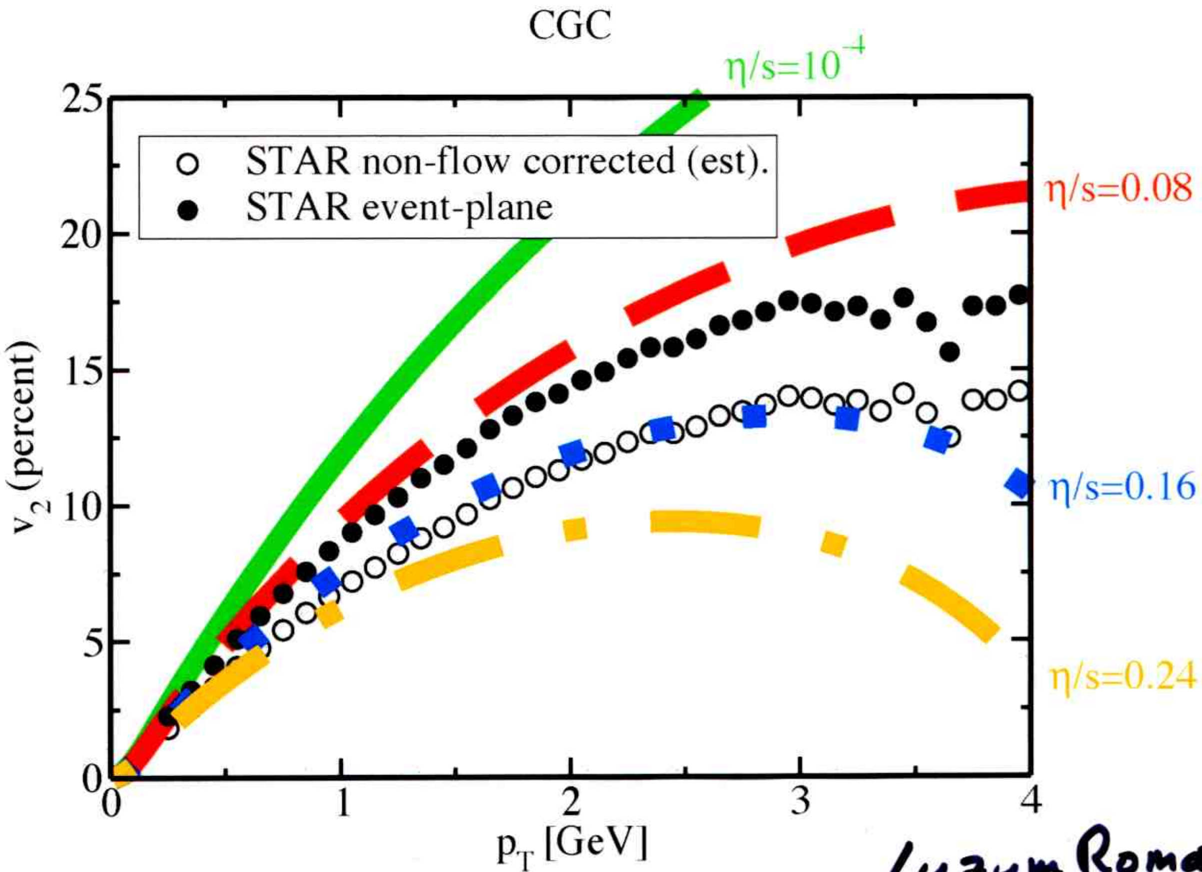


Luzum Romatschke

Data: removing non-flow lowers v_2

Hydro: viscosity lowers v_2

Initial Conditions? →



Luzum Romatschke

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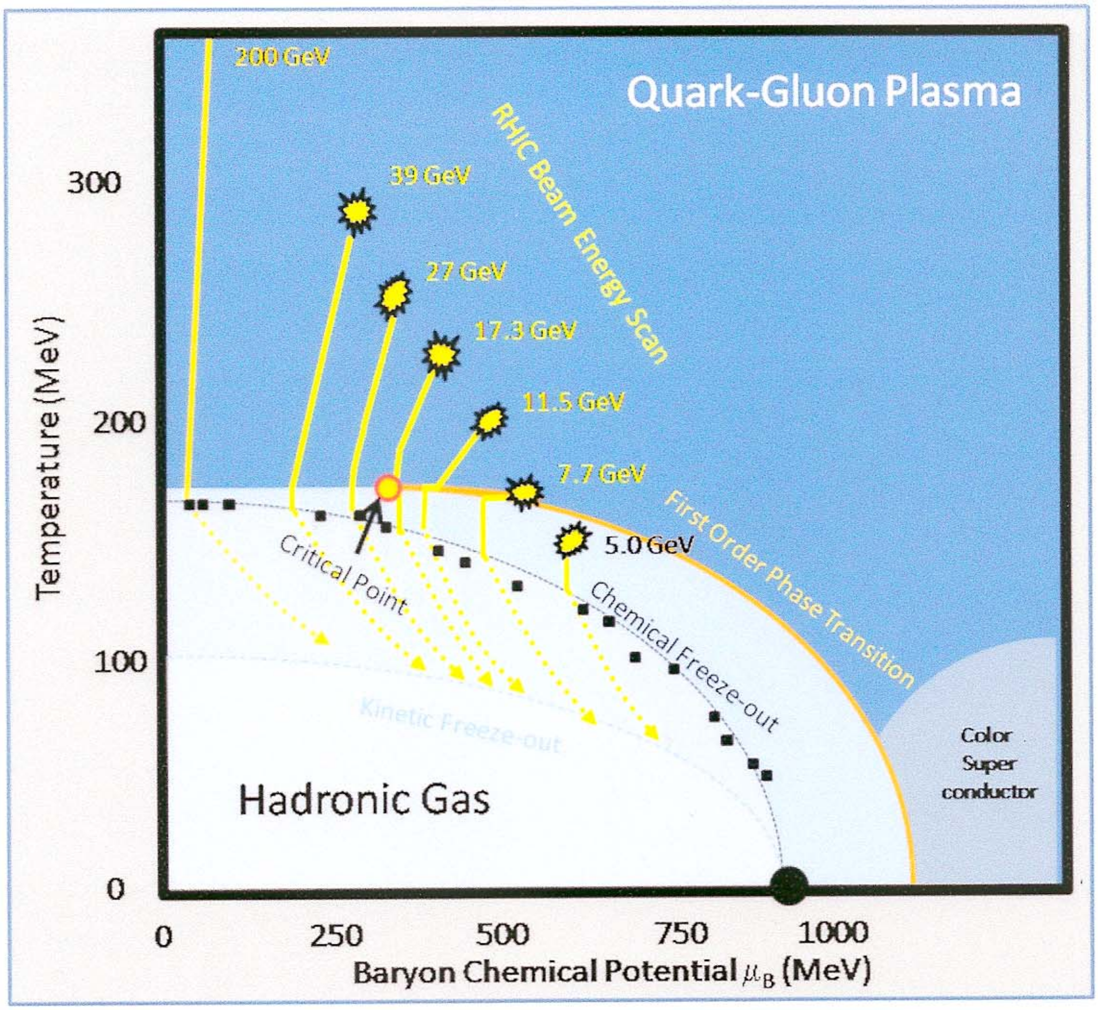
SEARCHING FOR THE CRITICAL POINT

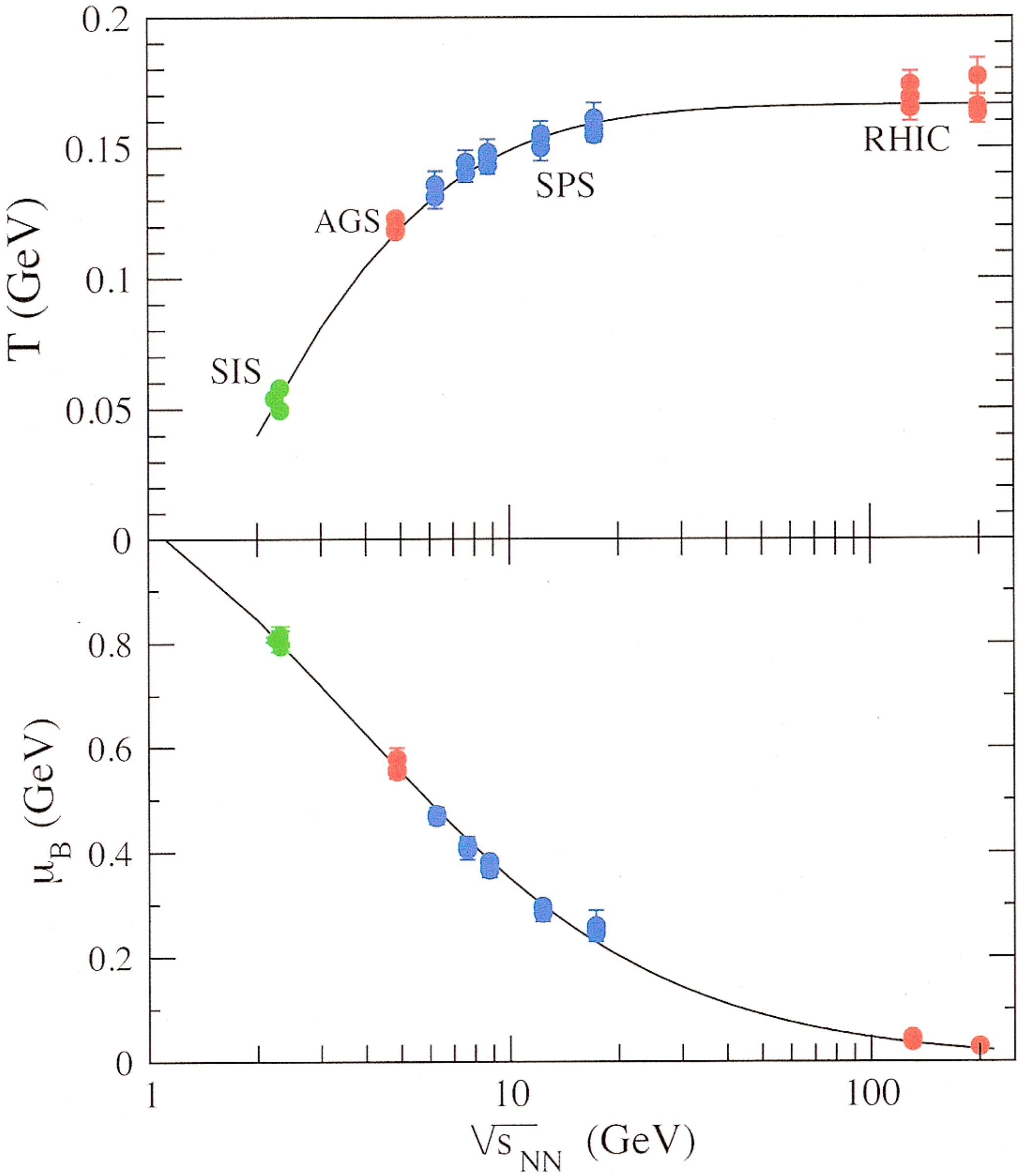


Decreasing $\sqrt{s}$: decreases T and increases μ at which collision equilibrates, "landing on the phase diagram".

⇒ increases μ at which the trajectory followed by the cooling plasma crosses the transition or crossover.

NB: location of ● in fig. is merely illustrative — we don't know where ● is!

CHEMICAL FREEZEOUT T and μ



Parametrization from Cleymans et al, 2005

ISENTROPIC TRAJECTORIES

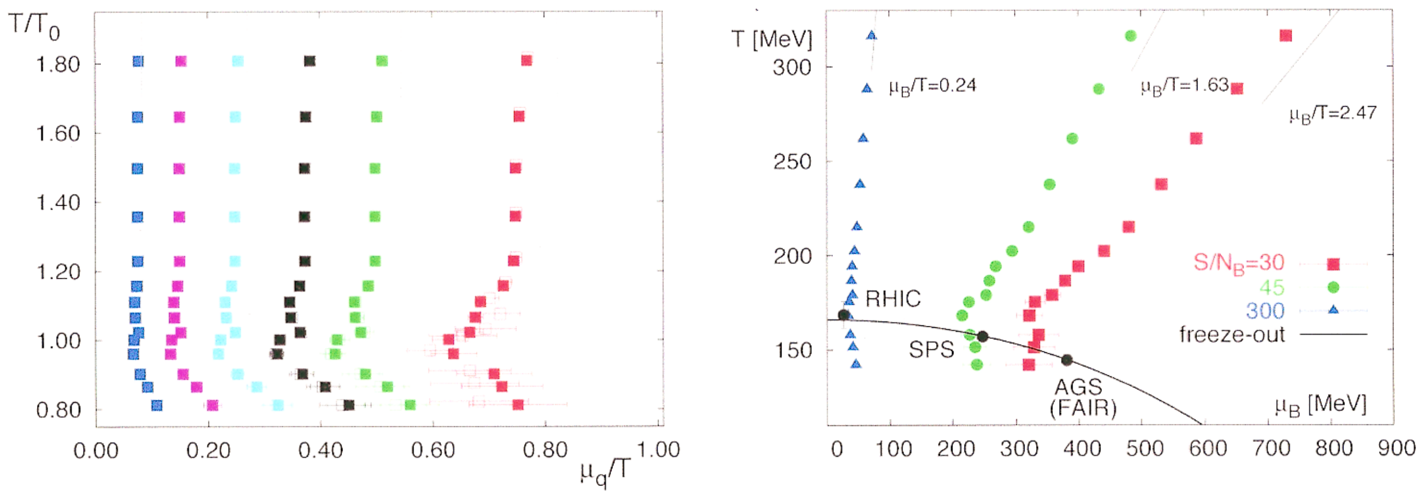


FIG. 3: Lines of constant entropy per quark number versus μ_q/T (left) and in physical units using $T_0 = 175$ MeV to set the scales (right). In the left hand figure we show results obtained using a 4^{th} (full symbols) and 6^{th} (open symbols) order Taylor expansion of the pressure, respectively. Data points correspond to $S/N_B = 300, 150, 90, 60, 45, 30$ (from left to right). The vertical lines indicate the corresponding ideal gas results, $\mu_q/T = 0.08, 0.16, 0.27, 0.41, 0.54$ and 0.82 in decreasing order of values for S/N_B. For a detailed description of the right hand figure see the discussion given in the text.

Ejiri Karsch Laermann Schmidt

- shape of isentropic trajectories in QGP phase and in crossover region is known from lattice calculations
- isentropic trajectories zigzag as they cross first order line

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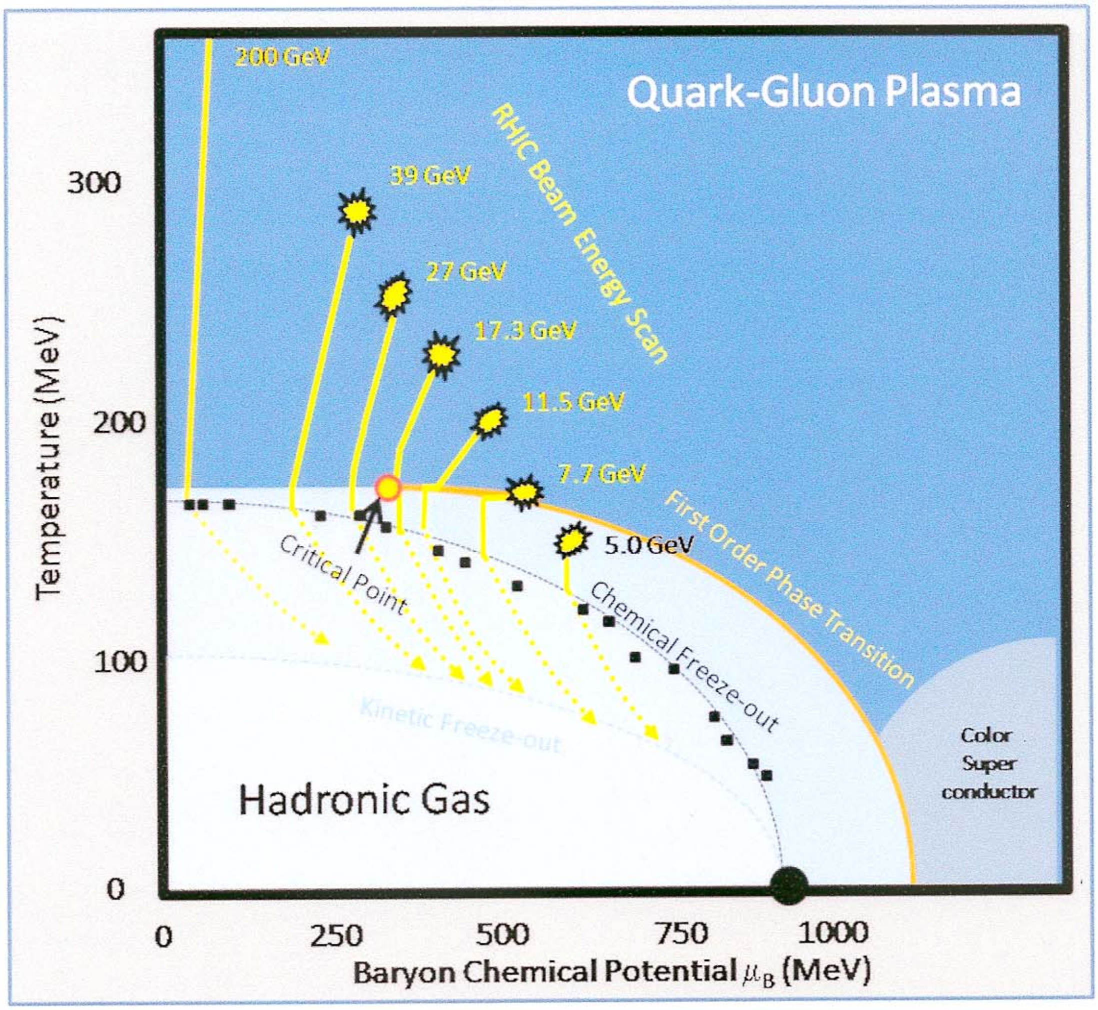

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HOW LOW TO GO?

Down to what $\sqrt{s}$ should we look?

$\Rightarrow$ Up to what μ can we look?

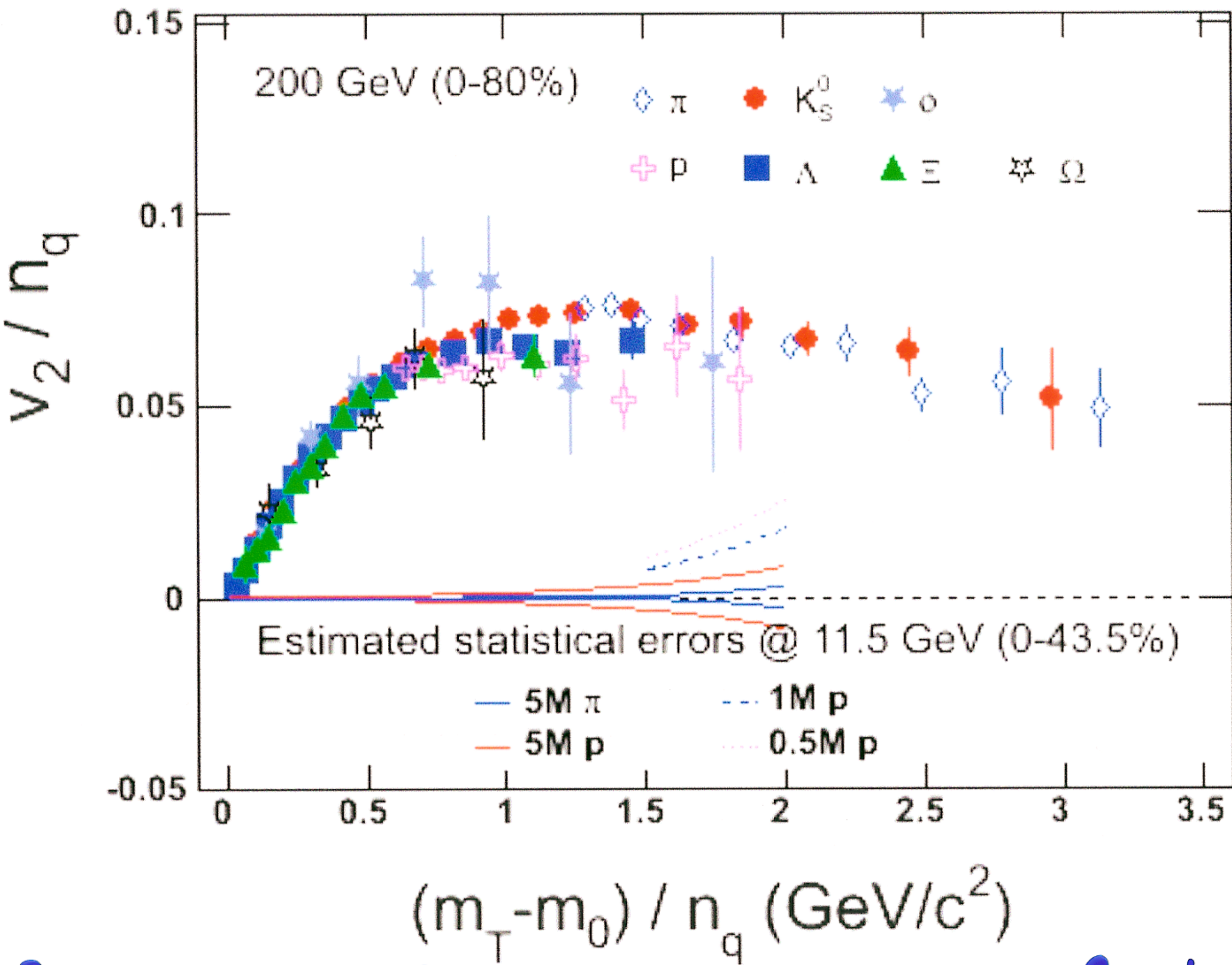
This question should be answered experimentally.

Need an effect that is:

- well-measured at $\sqrt{s} = 200$ GeV
- expected only in collisions that do begin above the crossover/transition
- expected at lower $\sqrt{s}$, as long as collisions do begin above crossover.
  [i.e. jet quenching won't do since that can turn off due to absence of jets]

Here are two suggestions....

n_q - SCALING OF V_2

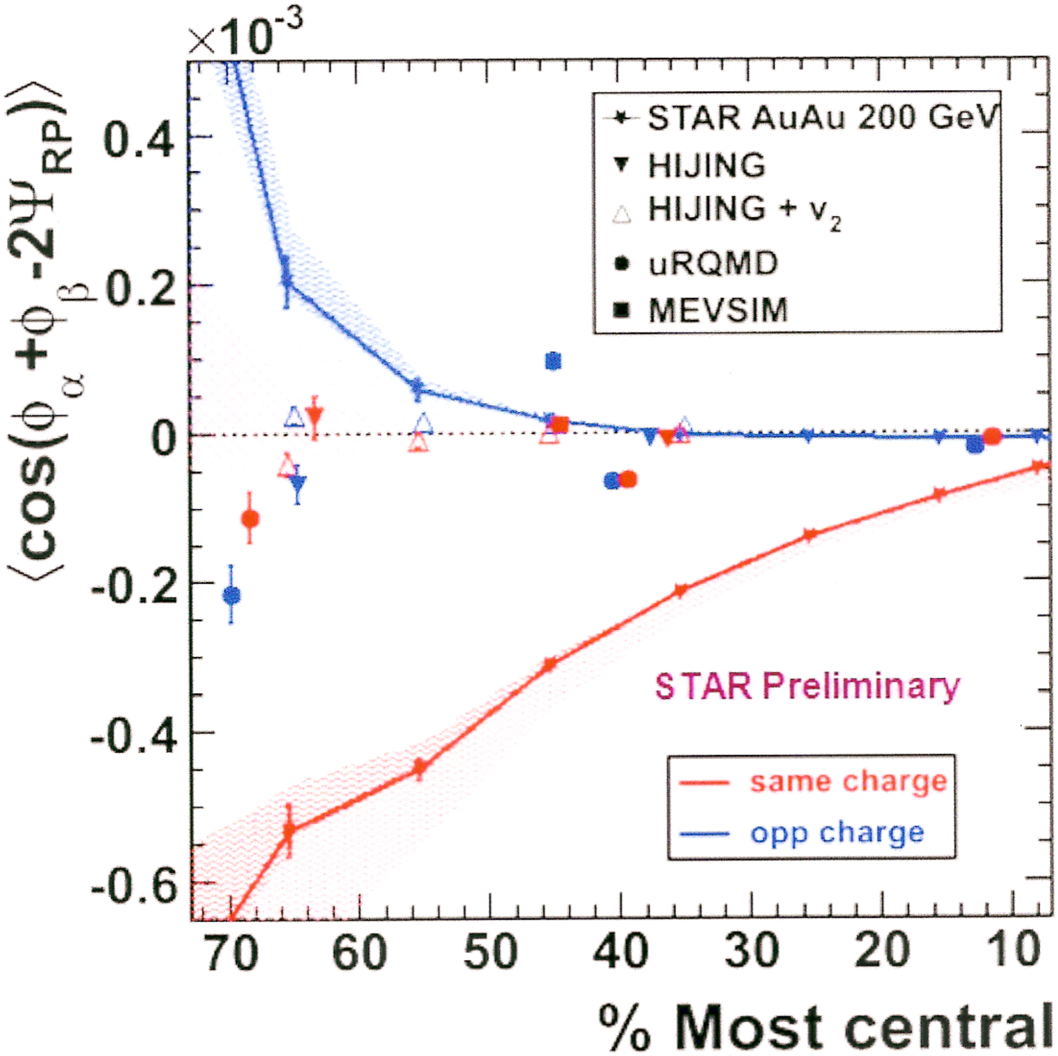


STAR B.U.R.

- V_2 same for mesons of varying mass; same for baryons of varying mass $\Rightarrow$ V_2 developed before hadrons formed
- Measurement can be done for π/K/P/Λ with 5M min. bias events @ $\sqrt{s} = 11.5$ GeV with errors shown

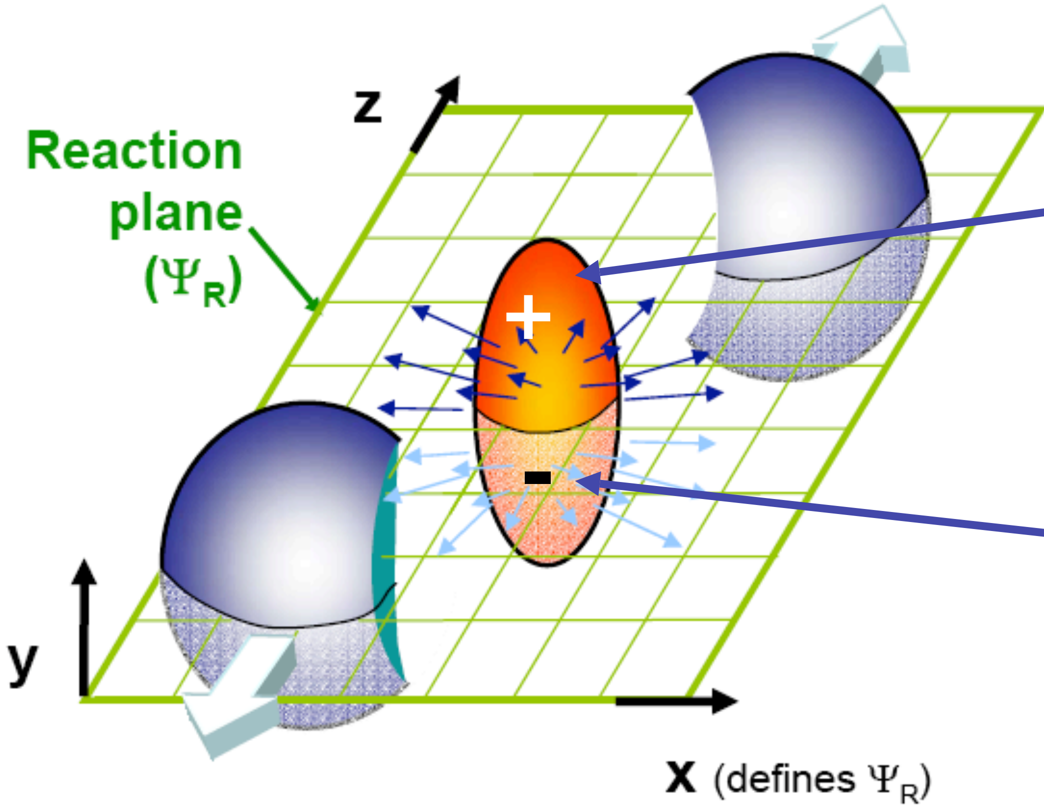
PARITY VIOLATING FLUCTUATIONS

Need 5M min. bias events per $\sqrt{s}$ to measure. (STAR B.U.R.)



- Data ⇒ charge separation !
  ⇒ an electric field ($\perp$ to reaction plane, parallel or antiparallel to $\vec{L}$ & $\vec{B}$) that is coherent over a volume corresponding to many charged particles
- No hadronic explanation of data
- Kharzeev's explanation requires deconfinement

Charge asymmetry w.r.t. reaction plane as a signature of strong P violation



excess of positive charge

excess of negative charge

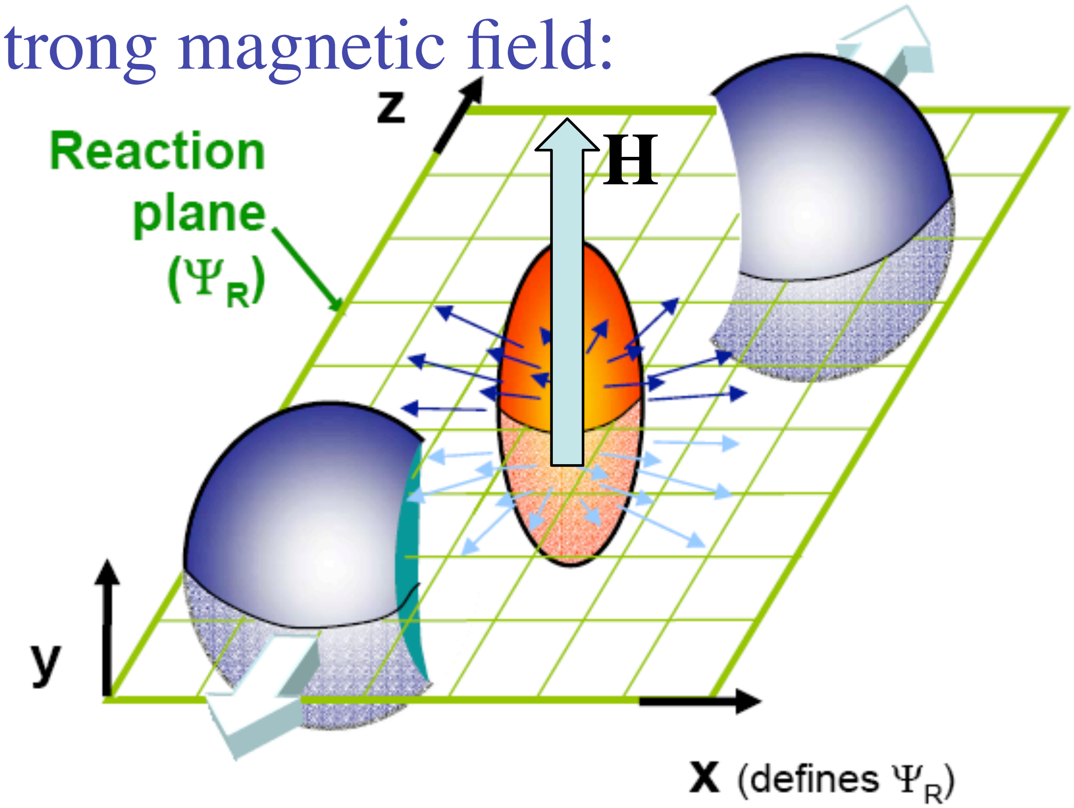
Electric dipole moment of QCD matter!

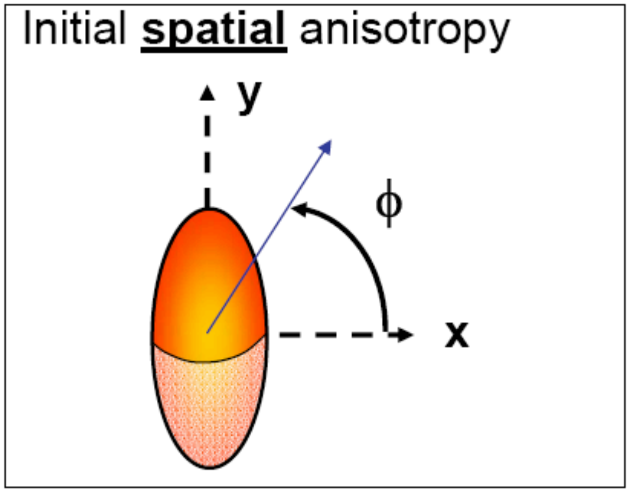
DK, Phys.Lett.B633(2006)260 [hep-ph/0406125]

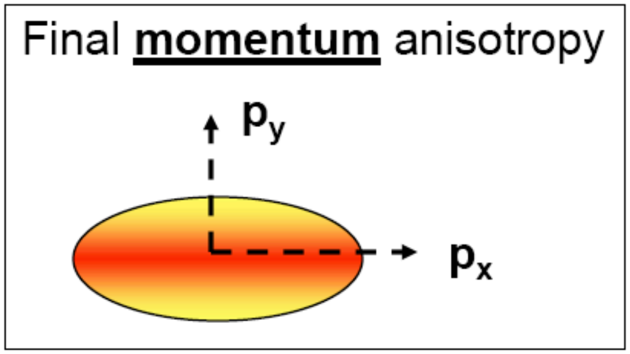
Submitted on 10 June 2004

Is there a way to observe topological charge fluctuations in experiment?

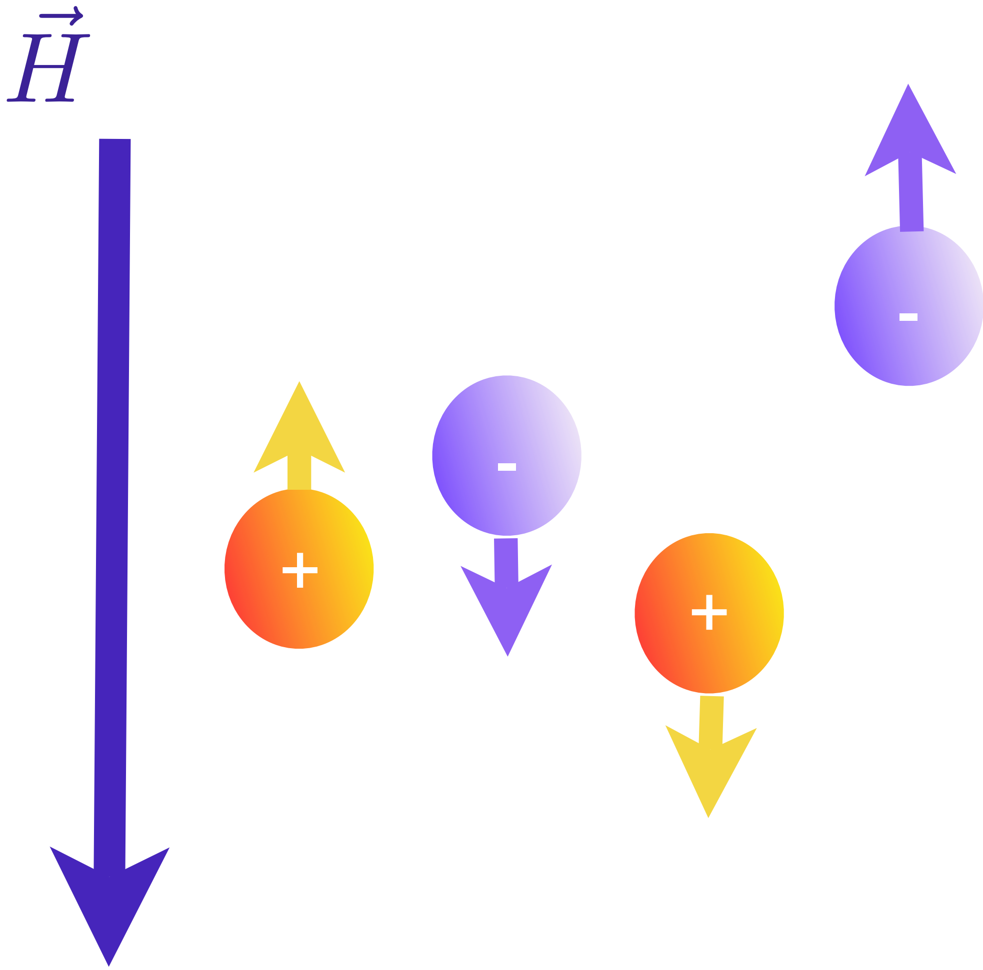
Relativistic ions create a strong magnetic field:







The Chiral Magnetic Effect

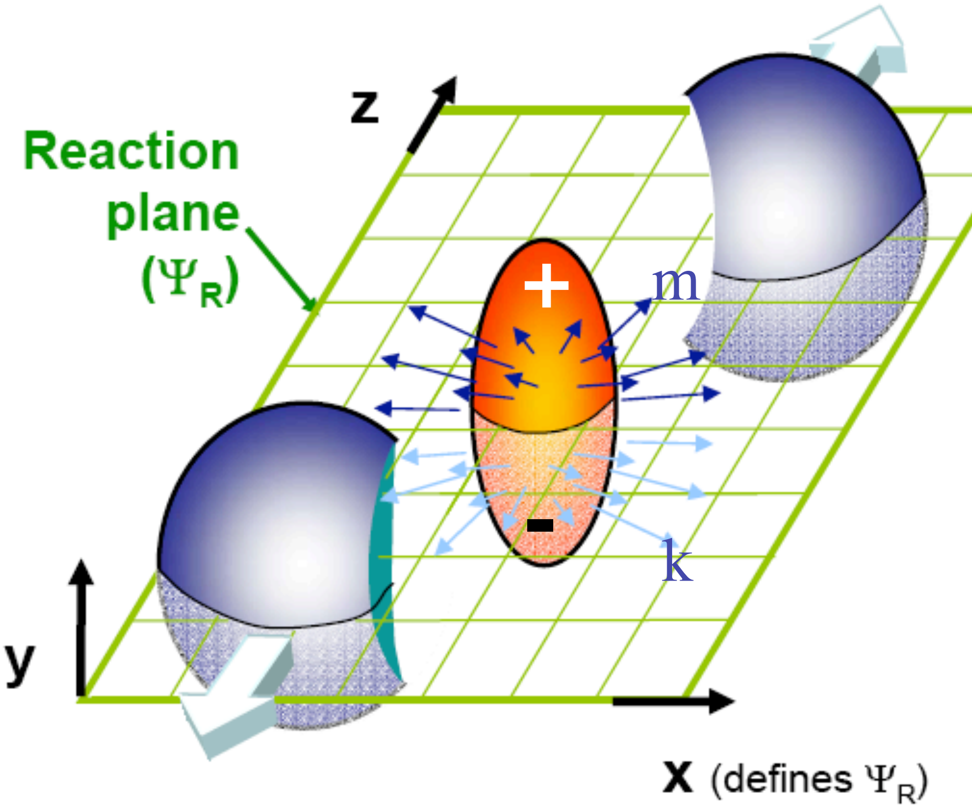


Let all fermions
be right-handed,
$Q = N_R - N_L > 0$

this means the spin
is parallel to momentum.

Magnetic field pins down
the directions of spins
and thus induces
an electric current

Charge asymmetry w.r.t. reaction plane: how to detect it?



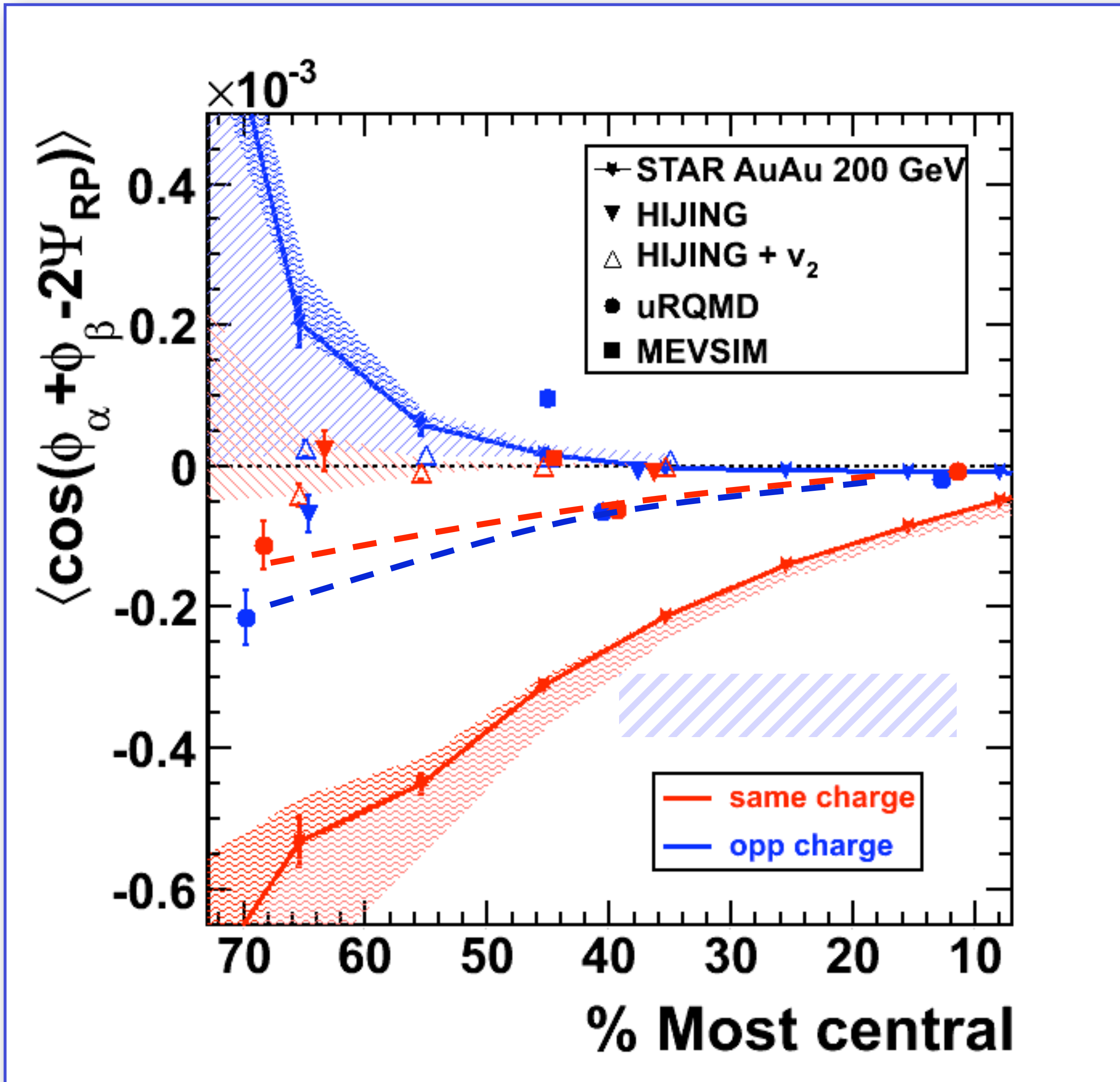
$$\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle =$$
$$= \langle \cos\Delta\phi_\alpha \cos\Delta\phi_\beta \rangle - \langle \sin\Delta\phi_\alpha \sin\Delta\phi_\beta \rangle$$
$$= [\langle v_{1,\alpha} v_{1,\beta} \rangle + B^{in}] - [\boxed{\langle a_\alpha a_\beta \rangle} + B^{out}].$$

S.Voloshin, hep-ph/0406311

A sensitive
(but P-even) measure
of the asymmetry:

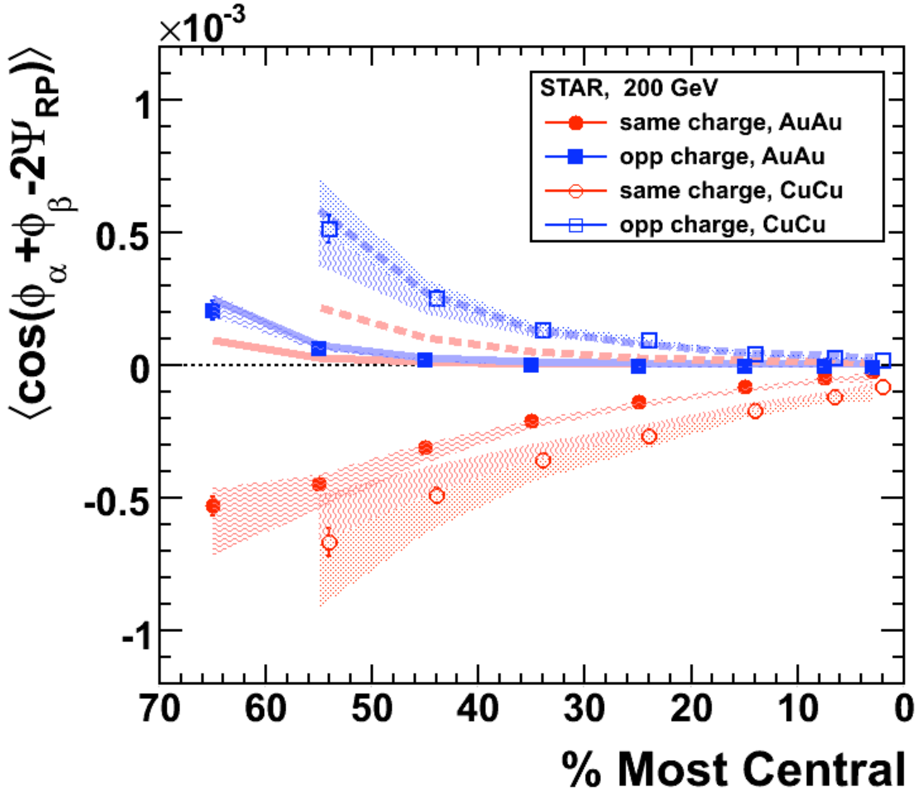
$$a^k a^m = \langle \sum_{ij} \sin(\varphi_i^k - \Psi_R)\ \sin(\varphi_j^m - \Psi_R) \rangle$$

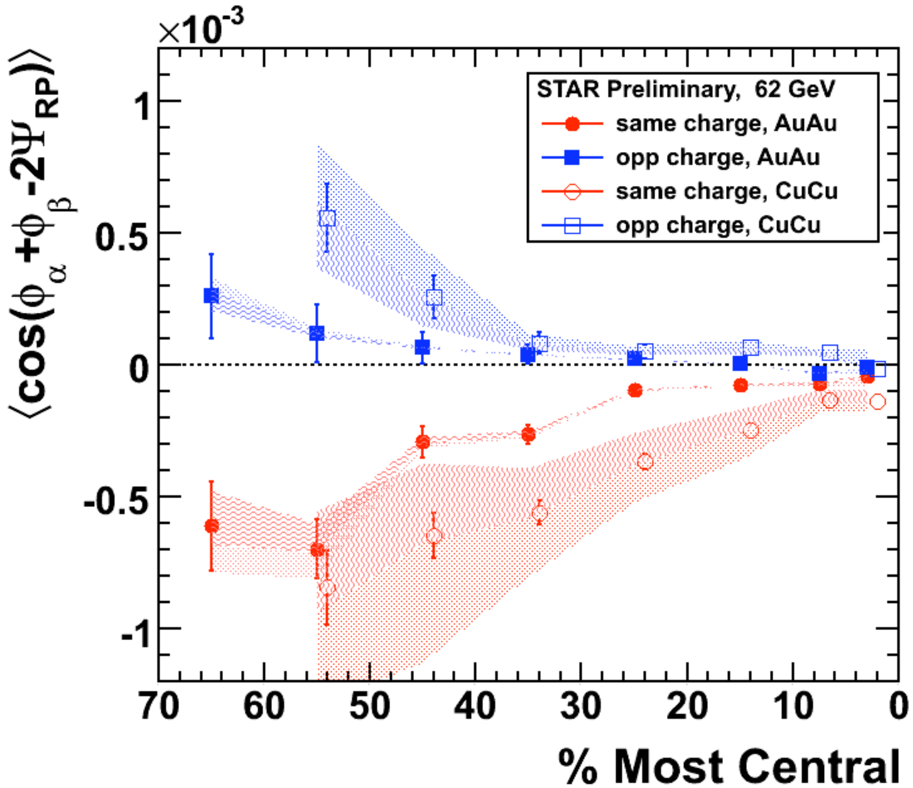
Expect $\quad a^+a^+ = a^-a^- > 0; \quad a^+a^- < 0$



S. Voloshin et al [STAR Coll.]; **Quark Matter '09**

Mass number and energy dependences





Talk by E. Finch;
RHIC/AGS: J. Thomas

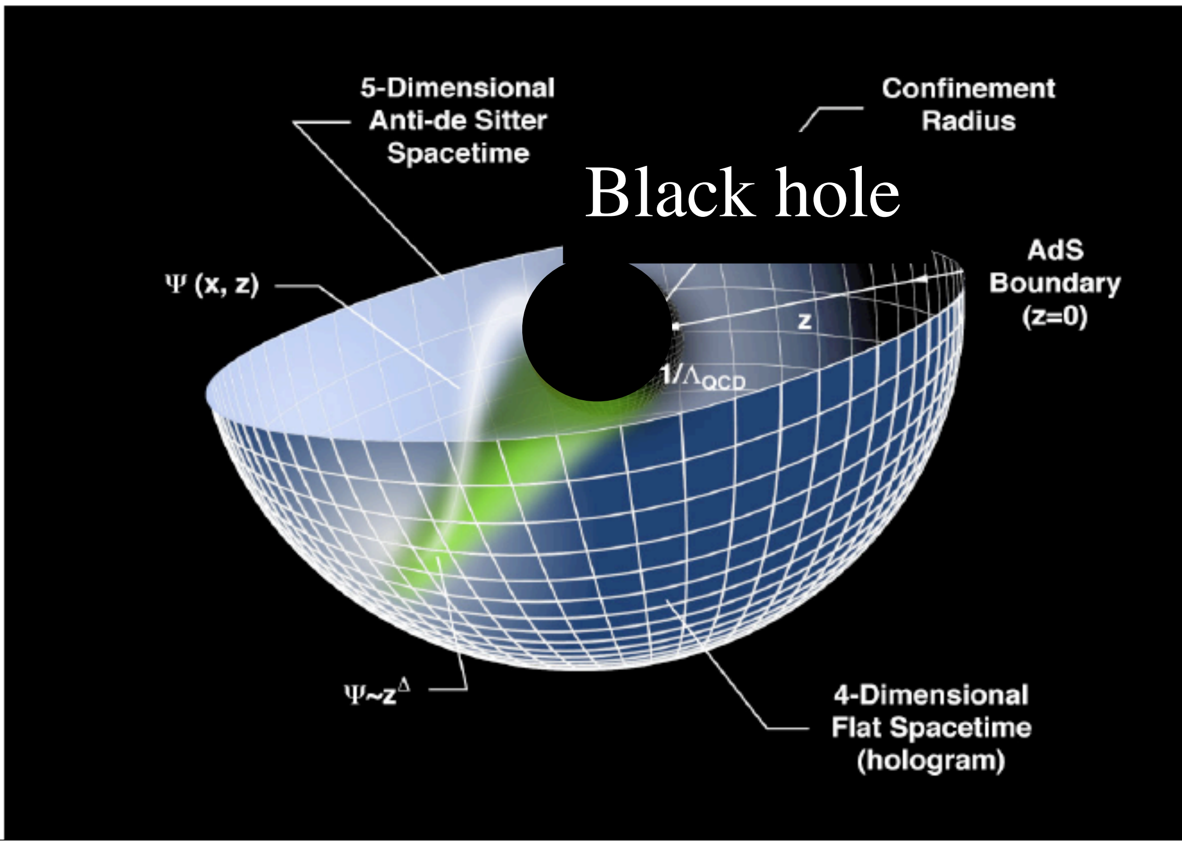
Expectations for the energy dependence: slow growth towards low energies reflecting longer-lived magnetic field, then gradual disappearance (no QGP): there has to be a maximum somewhere

Perfect liquid contains fluctuating topological charge

Chern-Simons number
diffusion rate
at strong coupling

$$\Gamma = \frac{(g^2_{\mathrm{YM}} N)^2}{256\pi^3} T^4$$

D.Son,
A.Starinets
hep-th/
020505



NB: This calculation is completely analogous to the calculation of shear viscosity that led to the "perfect liquid"

So.....

1. Decrease $\sqrt{s}$ in steps
2. Measure the $\sqrt{s}$ at which
   - n_q scaling of v_2
   - parity violating fluctuations (charge separation)

   turn off.
3. You can only look for signatures of down to, or perhaps slightly below, that $\sqrt{s}$.
4. In this way, learn up to what μ heavy ion collision experiments can find .

SIGNATURES OF THE CRITICAL POINT

In those collisions that pass near the critical point as they cool, find long wavelength oscillations of a mode that is a linear combination of σ (ie fluctuations couple to $\pi\pi$ and pp) and baryon number.

Fujii Ohtani; Son Stephanov

The longer the correlation length ξ gets, the bigger the signatures.

Signatures are event-by-event fluctuations of specific observables, calculable in magnitude in terms of ξ. Stephanov KR Shuryak

- Vary μ by varying $\sqrt{s}$
- Search for enhancement of these fluctuations in a window in $\sqrt{s}$, ie μ
- Analogue of critical opalescence
- Long wavelength fluctuations $\Rightarrow$ effects greatest at low $p_\perp$.

Examples ..... But, first :

HOW LARGE CAN ξ GET?

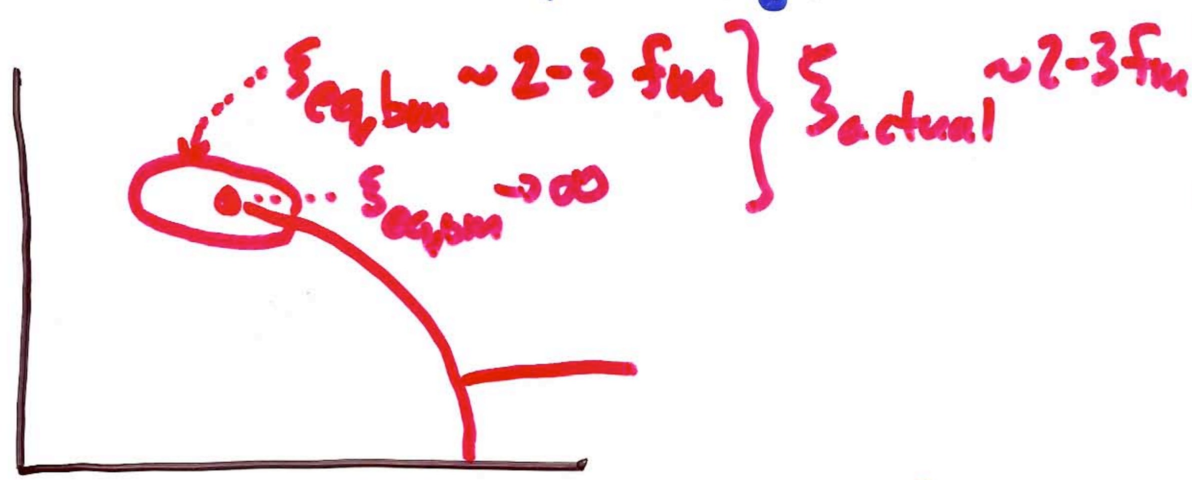
HOW CLOSE TO ● NEED WE BE?

- Obviously ξ limited by finite size of system. But, turns out that finite time is a more severe limitation.

  Berdnikov KR; Asakawa Nonaka

- Finite time spent in critical region means that even if equilibrium value of ξ is much larger, actual ξ won't grow bigger than 2-3 fm.

- Means no need to hit ● precisely.



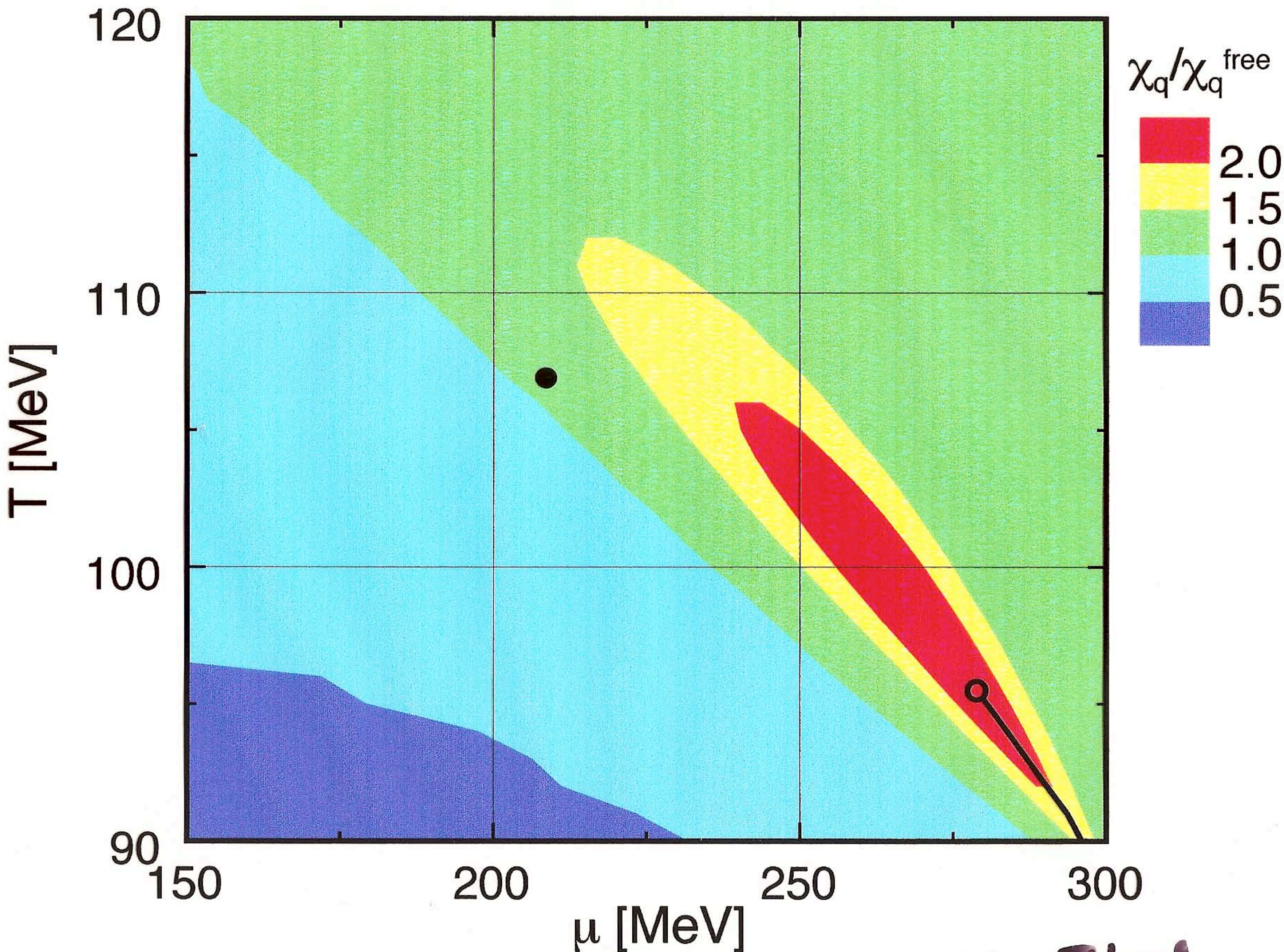
Signatures will be just as big if you pass anywhere in ⬭. No bigger, even if you hit ●.

- Hatta + Ikeda calculated "'s" in a model, but did so with contours of χ_B rather than ξ. → Figs.

  The robust point is that the extent of these 's in μ_B is not small. Width in μ_B is ~ 100 MeV, an estimate that is both crude and uncertain. Can this be obtained on lattice??

- NB also: since ξ cannot be > 2-3 fm, heavy ion collision experiments can never be used to measure the critical exponents of the 2nd order critical point. That's OK: we know it is Ising. What we don't know, and need experiments for, is where it is located.

$m_u = m_d = 5\,MeV$



$= \mu_B/3$

Hatta Ikeda

SIGNATURES OF CRITICAL POINT

- Decreasing $\sqrt{s} \rightarrow$ Increasing μ
- Vary $\sqrt{s}$, and hence μ, and look for nonmonotonic enhancement (rise and then fall) of:

  i) Event-by-event fluctuations of mean p_T of low p_T pions

  ii) Event-by-event fluctuations of net proton number $(N_p - N_{\bar{p}})$

  iii) Event-by-event fluctuations of particle ratios involving pions and/or protons

  iv) Kurtosis of the N_p or $(N_p - N_{\bar{p}})$ event-by-event distribution

  ⋮

In all cases, the enhancement should be greater at lower p_T.

MEAN P_T OF LOW P_T PIONS

Stephanov KR Shuryak (1999)

First example of a quantitetive connection between long wavelength fluctuations of the chiral order parameter with correlation length ξ and magnitude of event-by-event fluctuations of an experimental observable.

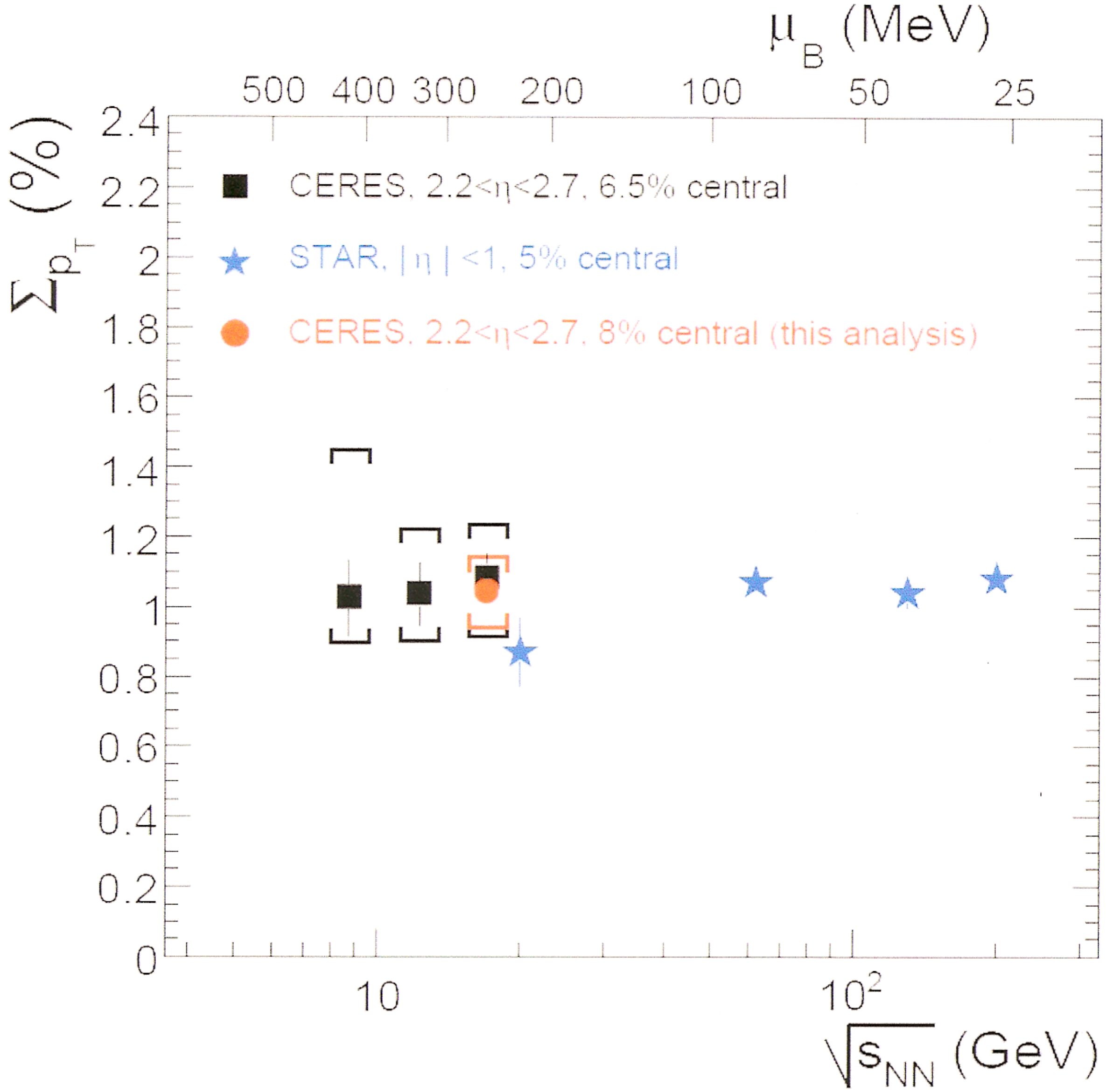
DISADVANTAGES:

- Effect predicted is not large for $\xi = 3$ fm
- Will fluctuations in P_T survive the late time hadronic gas? Will they get washed out between chemical and kinetic freezeout?

RESULT:

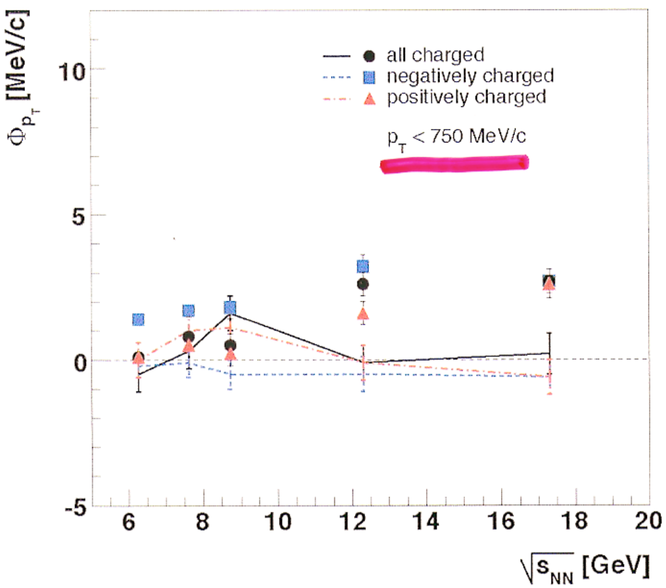
NA49 has done a beautiful analysis and sees no $\sqrt{s}$ dependence....
CERES has done a beautiful analysis and sees no $\sqrt{s}$ dependence....

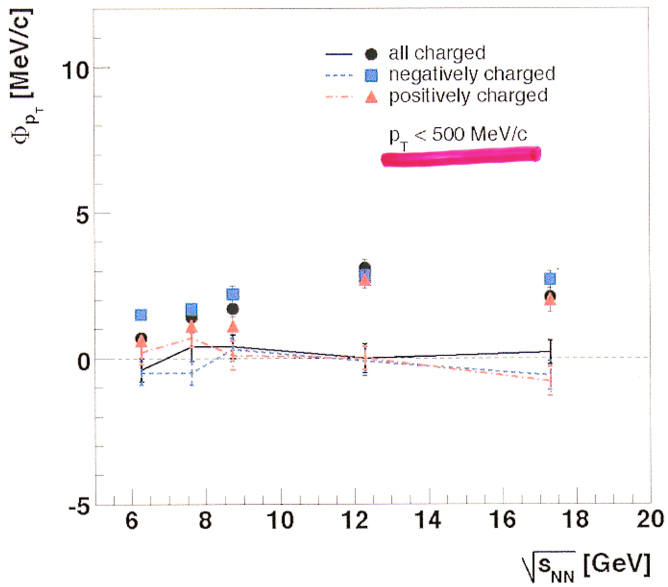
EVENT-BY-EVENT FLUCTUATIONS OF MEAN p_T

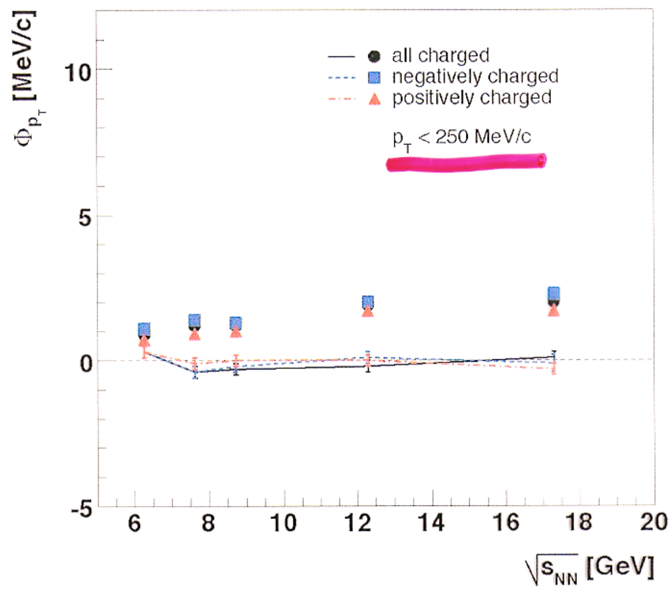


CERES, Nucl Phys A, 2008

EVENT-BY-EVENT FLUCTUATIONS OF MEAN P_T OF PARTICLES WITH P_T BELOW A CUT



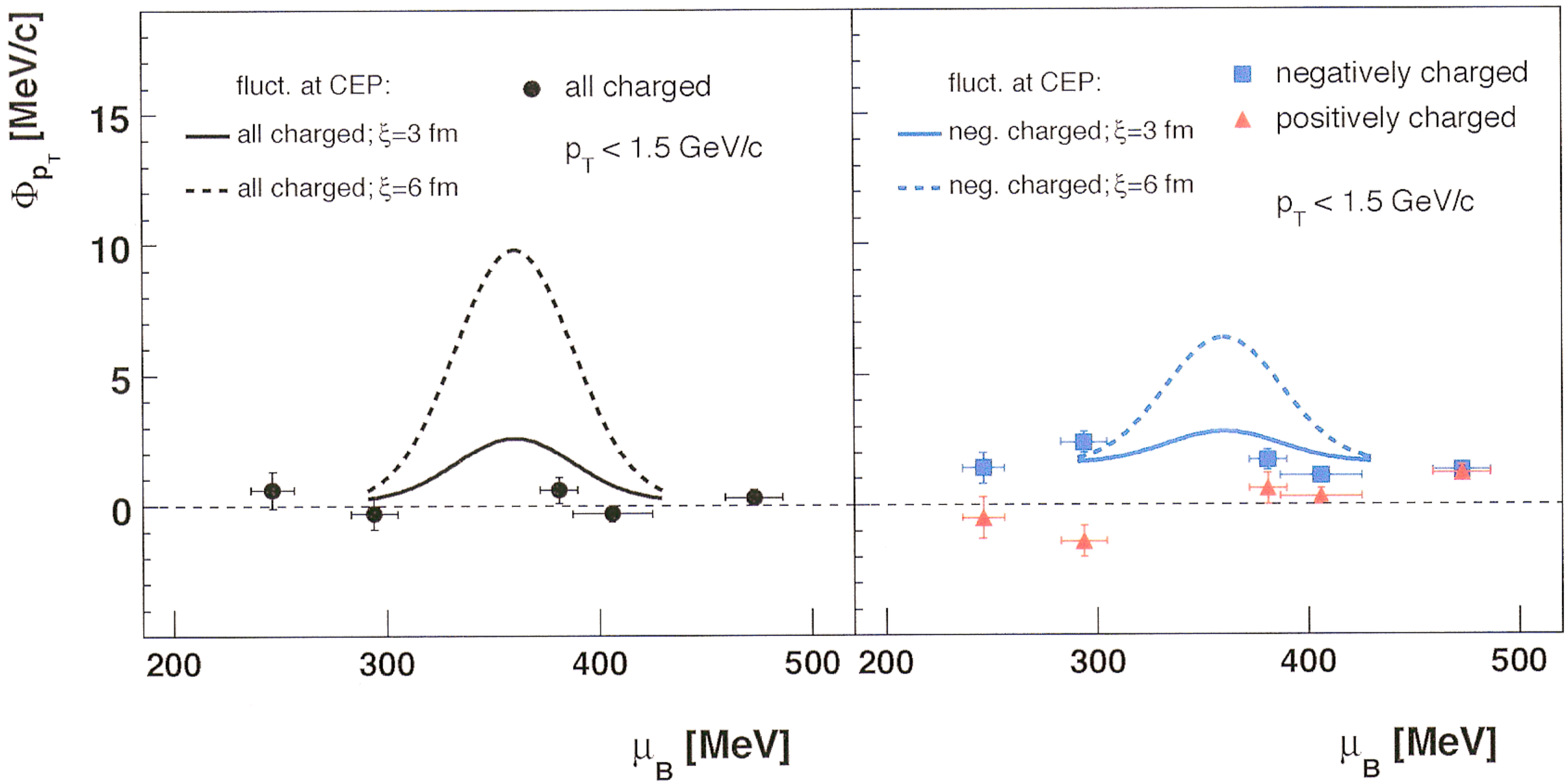




NA49, 2008

No enhancement at low P_T.

MEAN P_T FLUCTUATIONS, vs. μ



NA49, 2008

Height of solid (dashed) curve is magnitude of effect predicted for $\xi = 3$ fm ($\xi = 6$ fm).

POSSIBLE CONCLUSIONS

- $\mu_{\bullet} > 470$ MeV ?
- P_T fluctuations washed out. Predicted effect was not large, and was susceptible to being erased after chemical freezeout.

$\Rrightarrow$ Look for event-by-event fluctuations of other observables

a) for which the predicted effect of proximity to the critical point is larger

b) which cannot be washed out after chemical freezeout

BARYON, AND PROTON, NUMBER FLUCTUATIONS

Hatta Ikeda; Hatta Stephanov

- seen on the lattice → FIG
- should be looked for in experimental data

$\frac{\partial^2 \Omega}{\partial \mu_B^2} \to$ B-fluctuations $\sim \xi^2$ + nonsingular

$\frac{\partial^2 \Omega}{\partial \mu_I^2} \to$ no enhanced (u-d) fluctuations ~ nonsingular

Suggests $\frac{\mu_q}{T} \sim 1$ getting close to .

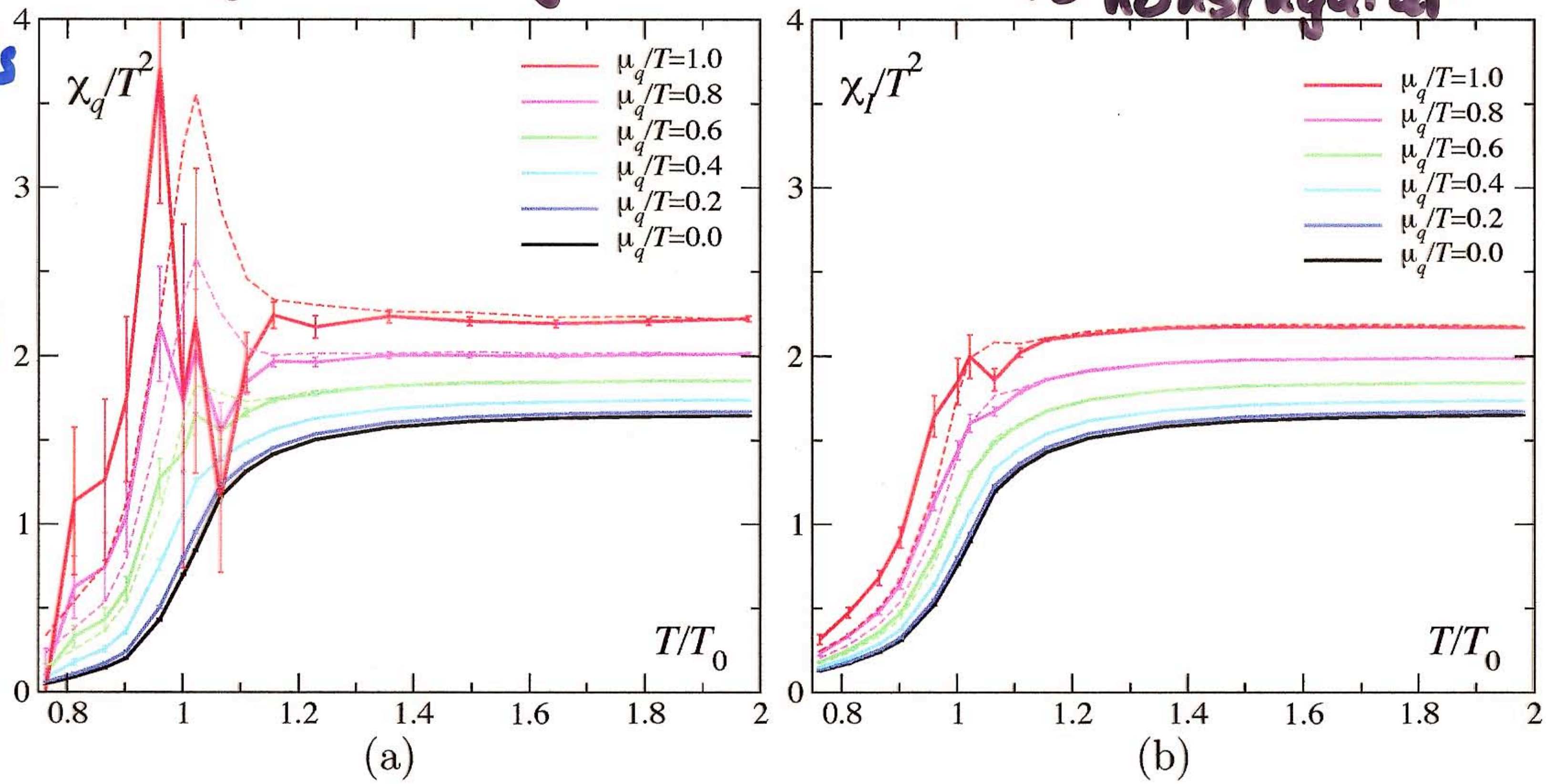


Figure 3.3: The quark number susceptibility χ_q/T^2 (left) and isovector susceptibility χ_I/T^2 (right) as functions of T/T_0 for various μ_q/T ranging from $\mu_q/T = 0$ (lowest curve) rising in steps of 0.2 to $\mu_q/T = 1$, calculated from a Taylor series in 6^{th} order. Also shown as dashed lines are results from a 4^{th} order expansion in μ_q/T.

(Because B fluctuates while isospin does not, proton fluctuations ~ B fluctuations)

Hatta Stephanov

Ejiri et al

PARTICLE RATIOS

NA 49

- Originally motivated by peak in $\langle K\rangle / \langle \pi \rangle$ at $\sqrt{s} = 7.6$ GeV.

  To better understand this, look at fluctuations of K/π ratio.
- Now motivated by observation that these fluctuations will better survive the late time hadron gas.

RESULT:

- Large K/π fluctuations at $\mu_B \sim 400 - 450$ MeV
- Why no P/π fluctuations???

EVENT-BY-EVENT FLUCTUATIONS OF K/π AND p/π

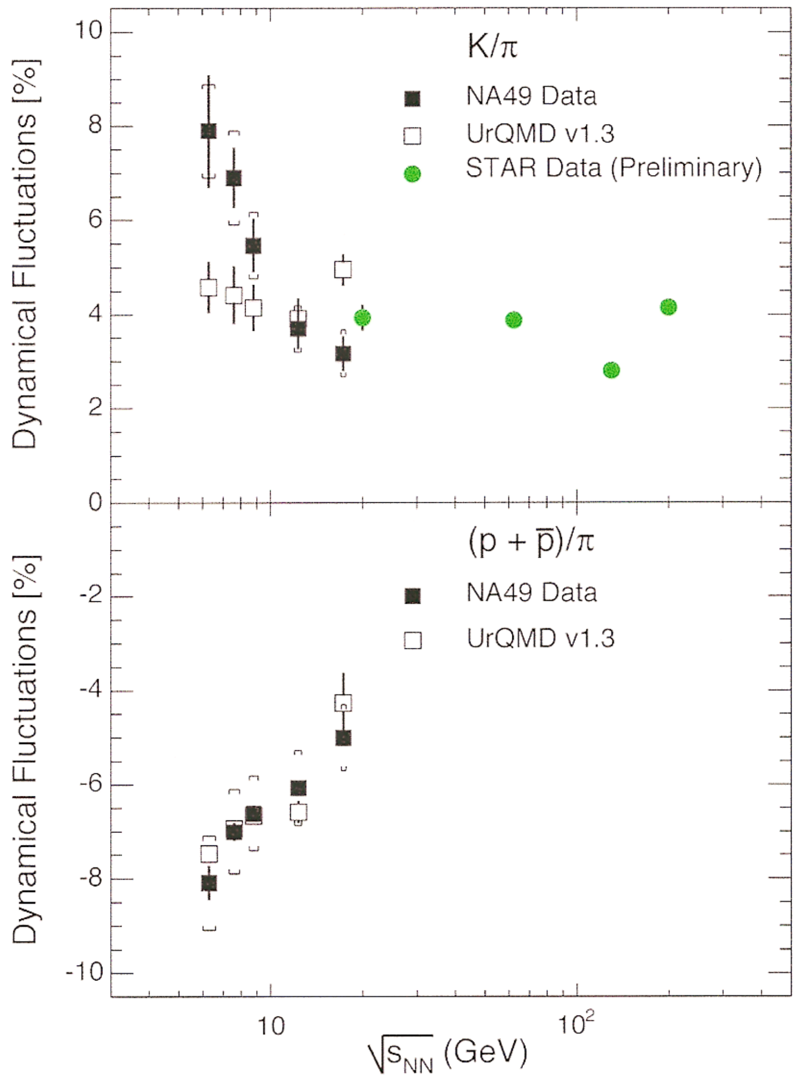


FIG. 8: Energy dependence of the event-by-event non-statistical fluctuations of the K/π ratio (top panel) and the $(p + \bar{p})/\pi$ ratio (bottom panel). Filled symbols show data, open symbols show calculations with the UrQMD transport code, using NA49 acceptance tables. Systematic uncertainties are shown as brackets.

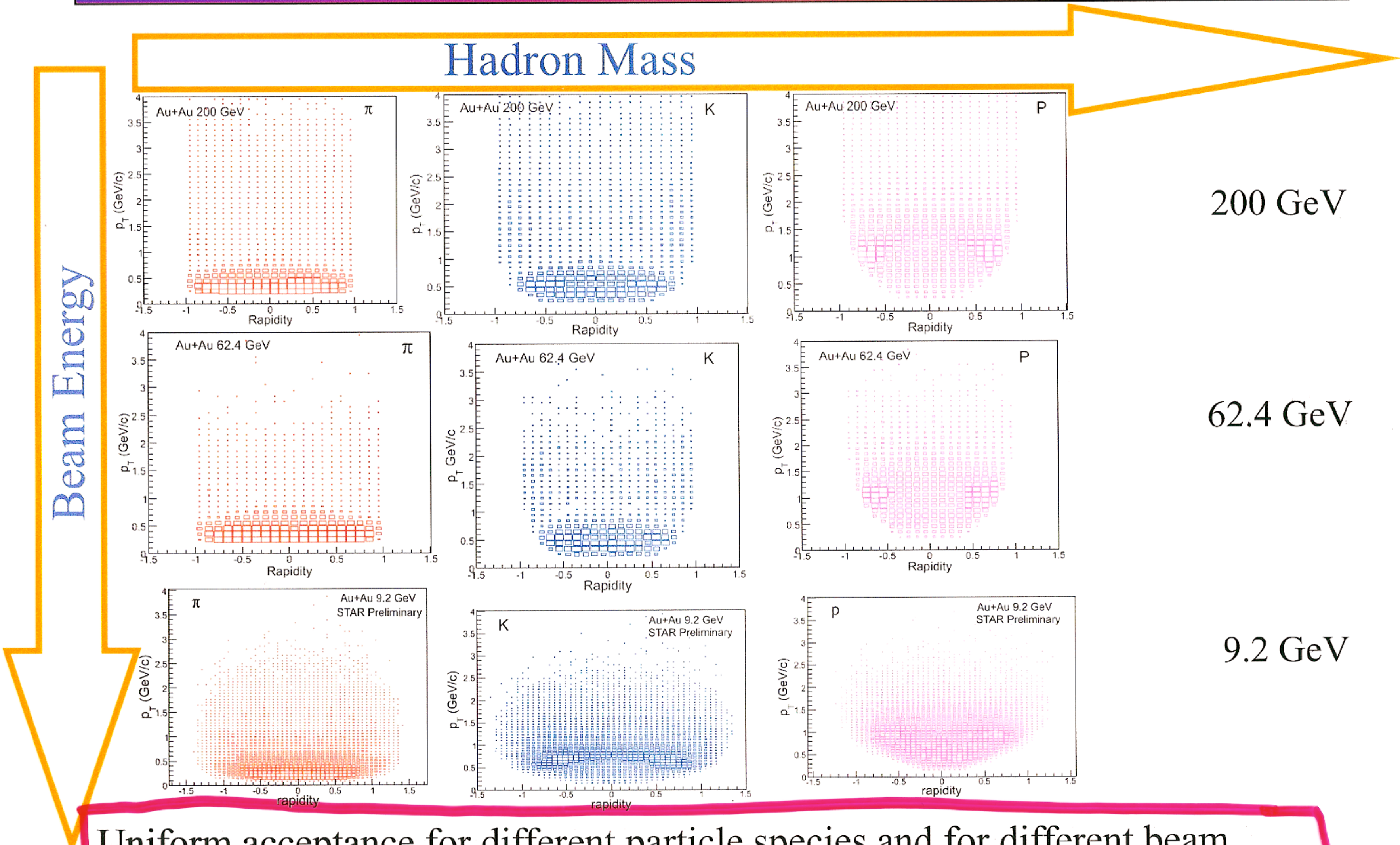
NA49, 2008

- Large event-by-event fluctuations in K/π ratio at $\mu \sim 350-450$ MeV
- Intriguing, but puzzling
  - error bars still substantial
  - interpretation is complicated by change in # of accepted kaons (& pions) as $\sqrt{s}$ changes. Koch Schuster Westfall
  - why no enhancement in p/π fluctuations?
    → K/π fluctuations apparently not driven by low p_T pions

STAR has used this observable as a case study to see how they can improve on this measurement with a beam energy scan at RHIC.

A collider has advantages...

RHIC Critical Point Search Program - Advantage



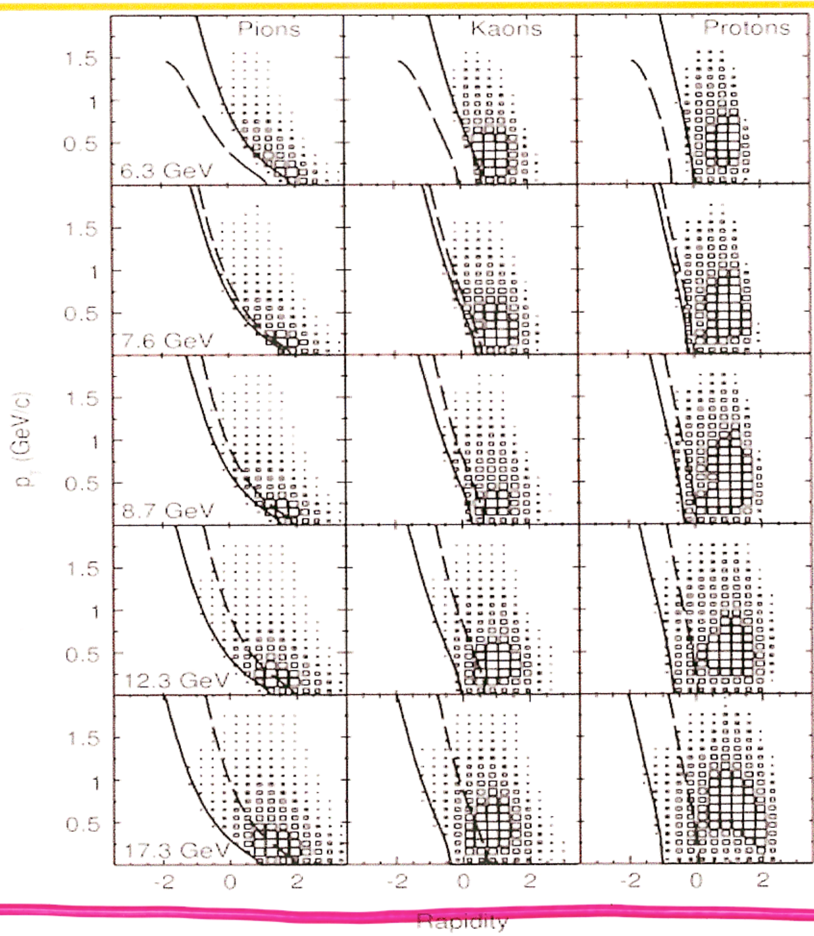
Uniform acceptance for different particle species and for different beam energies in the same experimental setup (advantage over fixed traget expt.)

Mohanty, QM 09

Old Program : Fixed Target

Hadron Mass

Beam Energy



NA49 : arXiv : 0808.1237

6.3 GeV

7.6 GeV

8.7 GeV

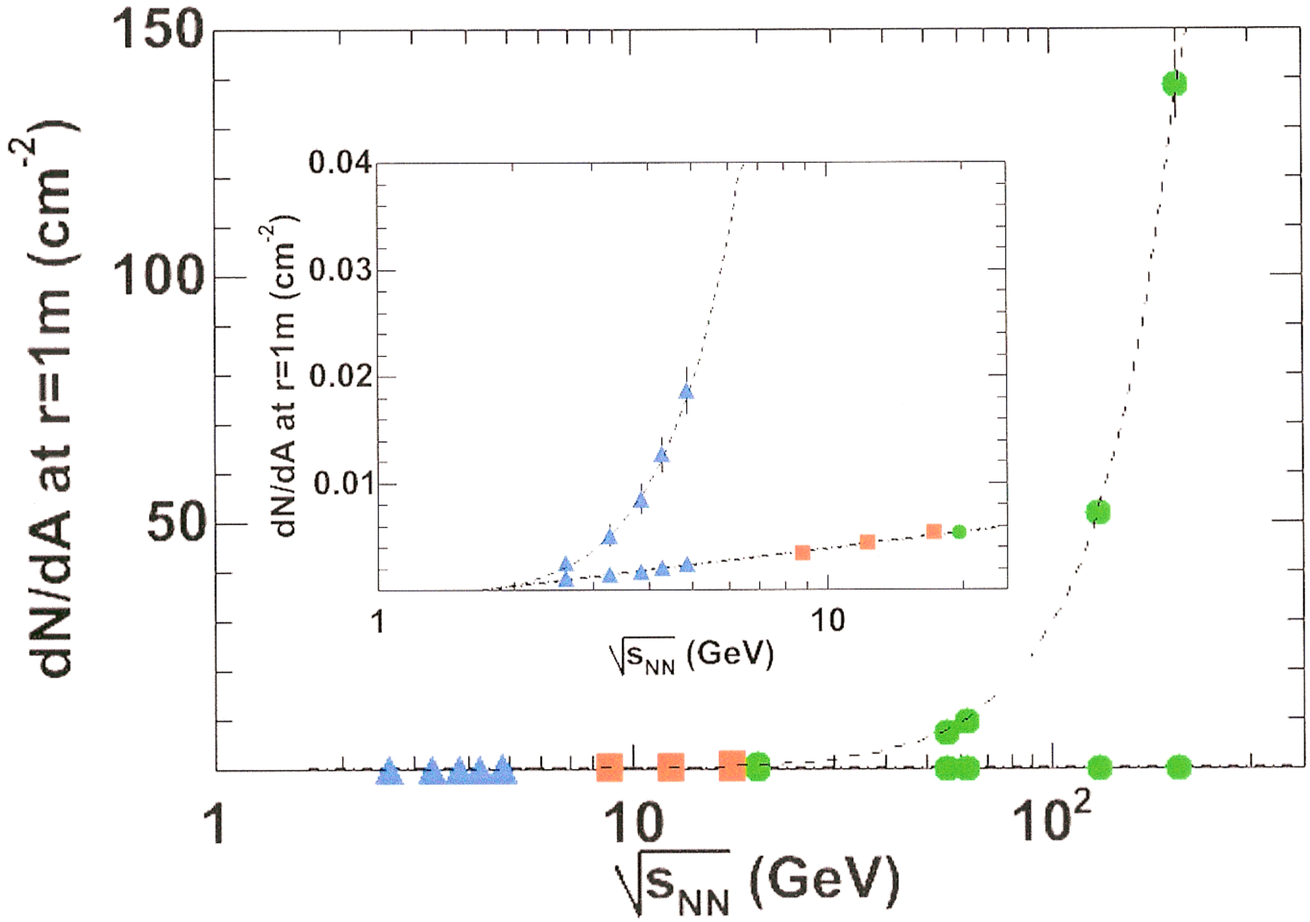
12.3 GeV

17.3 GeV

Non-Uniform acceptance for different particle species
and for different beam energies in the same experimental setup

Mohanty, QM09

RHIC Critical Point Search Program - Advantage



G. Roland

Collider experiment : Variation of particle density with beam energy slower. Occupancy in detectors reasonable compared to fixed target experiments at similar collision energy

Mohanty, QM09

K/π FLUCTUATIONS WITH A RHIC BEAM ENERGY SCAN

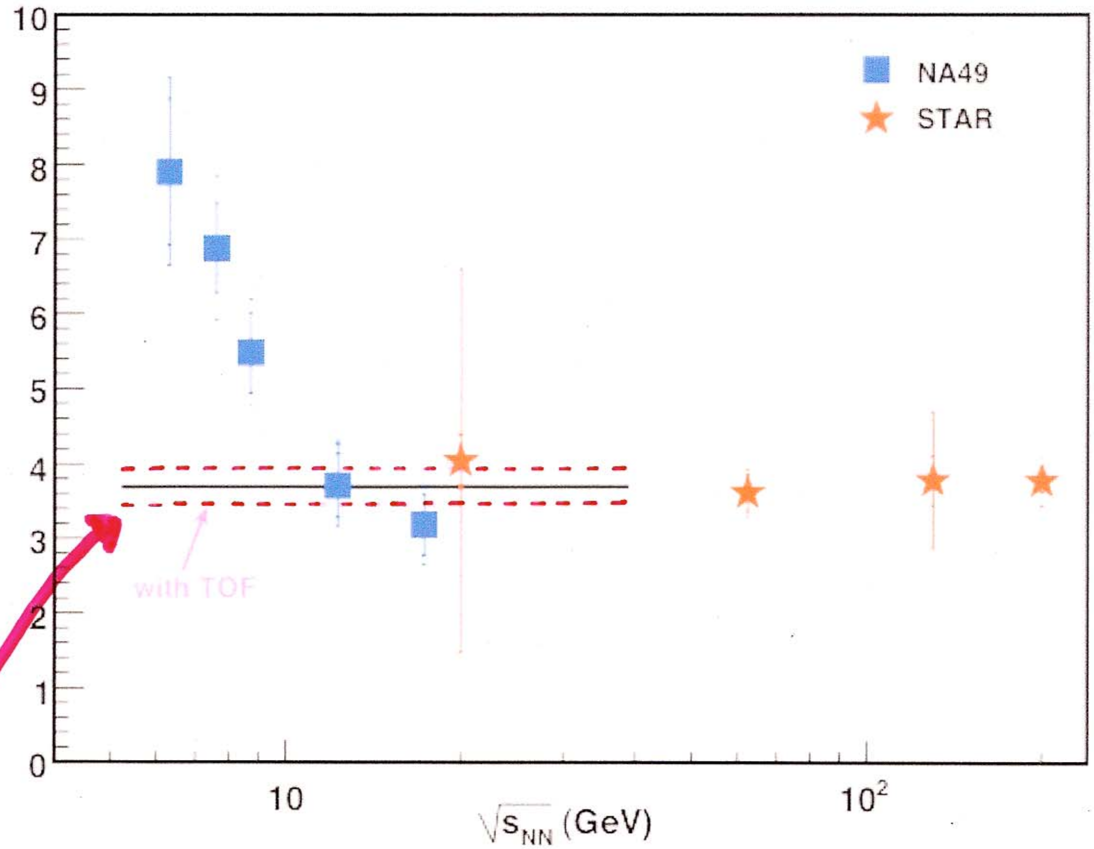


Fig. 3-7: Estimate of the error in σ_{dyn} for charge-integrated K/π fluctuations, based on 100K central events analyzed in the STAR detector (with the newly completed ToF). Shown for comparison are the current measurements from NA49 and STAR.

STAR B.U.R.

Error bars with 1M min bias events per energy

KURTOSIS OF EVENT-BY-EVENT DISTRIBUTION OF (NET) PROTON NUMBER

Stephanov; "a direct consequence of discussions" at Aug 2008 INT workshop

Critical fluctuations couple to $\pi\pi$, pp

→ event-by-event fluctuations in their multiplicities, multiplicity ratios, p_T, that are $\propto \xi^2$

Higher moments of the event-by-event distributions receive effects that are more sensitive to ξ.

Skewness $\propto \xi^{4.5}$

Kurtosis $\propto \xi^7$!!!

The prefactors work out particularly nicely for kurtosis of proton distribution, but Stephanov makes predictions for π & p, skewness & kurtosis.

DEFINITIONS

N = # of protons (or π; or $p - \bar{p}$) in 1 event

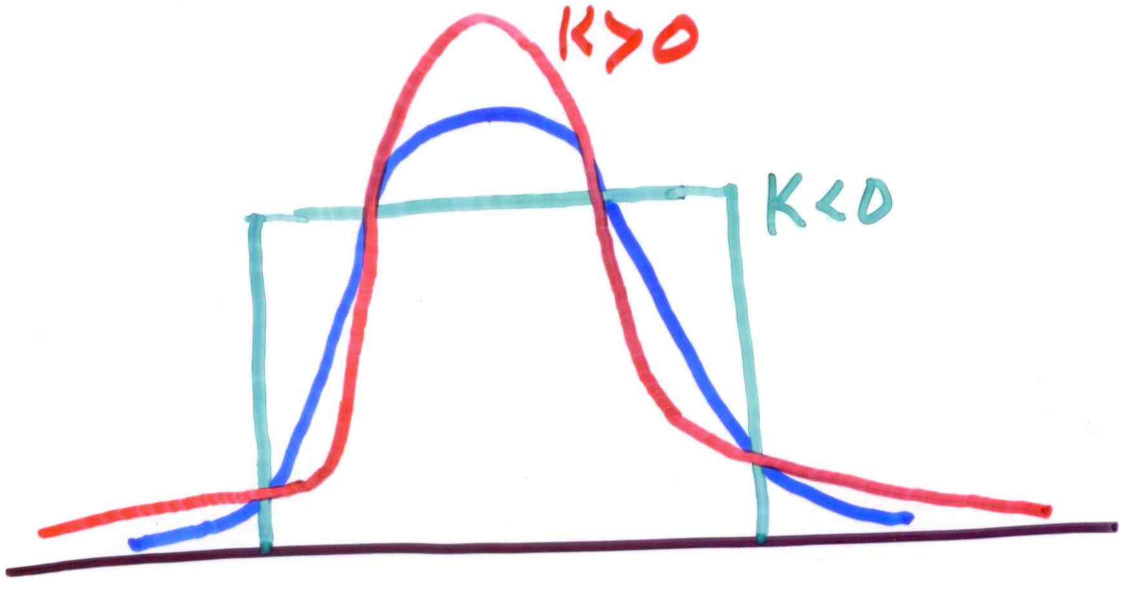
$\bar{N} \equiv \langle N \rangle$ = mean

$\delta N \equiv N - \bar{N}$ in one event

$\langle (\delta N)^2 \rangle$ = variance

$$K^{eff} \equiv \underbrace{K}_{\text{Kurtosis} \sim \frac{1}{\bar{N}}} \underbrace{\langle (\delta N)^2 \rangle}_{\text{Variance} \sim \bar{N}} \equiv \frac{\langle (\delta N)^4 \rangle - 3 \langle (\delta N)^2 \rangle^2}{\langle (\delta N)^2 \rangle}$$

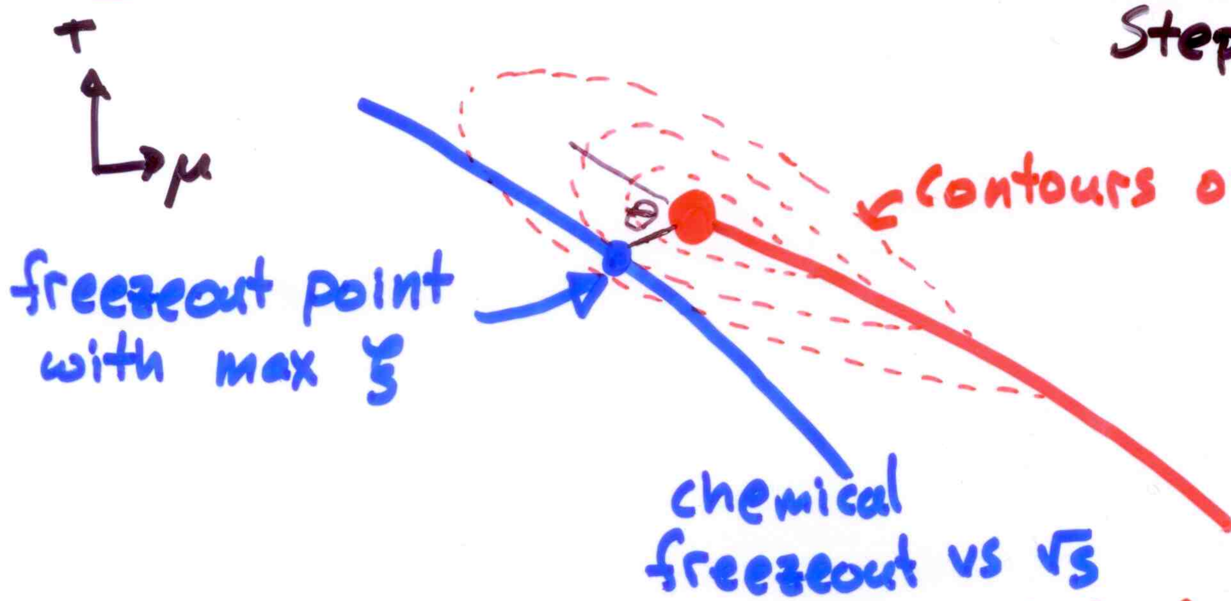
[In lattice QCD literature, K^{eff} is called κ_4/κ_2 or χ_4/χ_2. It is calculated for N = baryon number, & N = charge.]



Gaussian: $K=0$

EFFECT OF CRITICAL FLUCTUATIONS

Stephanov



$$K^{eff} \underbrace{\left(\frac{\bar{N}}{\langle(\delta N)^2\rangle}\right)}_{\simeq 1} = \underbrace{23}_{!} \underbrace{\left(\frac{2\tilde{\lambda}_3^2 - \tilde{\lambda}_4}{50}\right)\left(\frac{g}{10}\right)^4}_{\Theta(1)} \underbrace{\left(\frac{\xi}{1\,fm}\right)^7}_{!!!!}$$

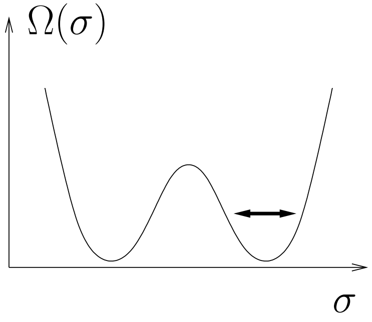
$\tilde{\lambda}_3, \tilde{\lambda}_4$: Obscure but universal constants that depend on θ. Known for Ising ●.

g: σpp coupling. $g \sim m_p/f_\pi \sim 10$

So, how big is the background? Theory suggests $\Theta(1)$, but better to determine it experimentally....

Critical mode and equilibrium fluctuations

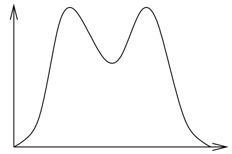
- $\boxed{1}$ $\mu > \mu_E$



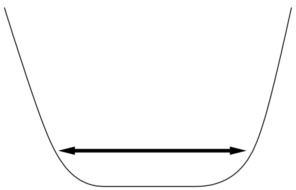
$$\boxed{\bar\psi\psi - \langle\bar\psi\psi\rangle \equiv \sigma}$$

$\langle\sigma^2\rangle \sim (\Omega'')^{-1}$

think $e^{-\Omega}$:

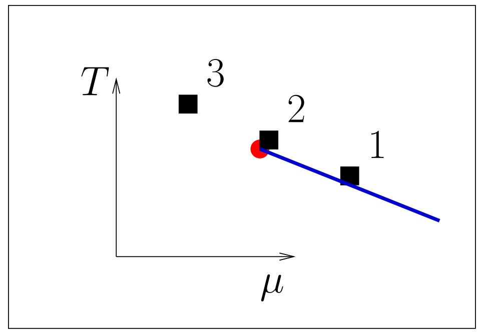


$\boxed{2}$ $\mu = \mu_E$

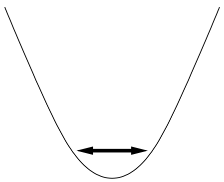


$(\Omega'')^{-1} \to \infty$

large equilibrium fluctuations



$\boxed{3}$ $\mu < \mu_E$



- Magnitude of fluctuation and correlation length:

$$\langle\sigma(\boldsymbol{x})\sigma(\boldsymbol{0})\rangle \sim \begin{cases} e^{-|\boldsymbol{x}|/\xi} & \text{for} \quad |\boldsymbol{x}| \gg \xi \\ 1/|\boldsymbol{x}|^{1+\eta} & \text{for} \quad |\boldsymbol{x}| \ll \xi \end{cases}$$

$$\langle\sigma_{\boldsymbol{0}}^2\rangle = \int d^3\boldsymbol{x}\langle\sigma(\boldsymbol{x})\sigma(\boldsymbol{0})\rangle \sim \xi^{2-\eta}$$

critical singularity is a *collective* phenomenon

- σ or n_B or T^{00}? Because they mix, only *one* linear combination is critical.

Relation between σ fluctuations and observables

Consider example: fluctuations of multiplicity of pions (or protons).

- Free gas: $n^0_{\boldsymbol{p}}$ – fluctuating occupation number of momentum mode $\boldsymbol{p}$. Ensemble (event) average $\langle n^0_{\boldsymbol{p}} \rangle = f_{\boldsymbol{p}}$ and

$$n^0_{\boldsymbol{p}} = f_{\boldsymbol{p}} + \delta n^0_{\boldsymbol{p}}; \quad \langle \delta n^0_p \delta n^0_{\boldsymbol{k}} \rangle = f'_{\boldsymbol{p}} \delta_{\boldsymbol{pk}}; \qquad f_{\boldsymbol{p}} = (e^{\omega_{\boldsymbol{p}}/T} \mp 1)^{-1};\ f'_{\boldsymbol{p}} \equiv f_{\boldsymbol{p}}(1 \pm f_{\boldsymbol{p}}).$$

- Couple these particles to σ field: $\quad G\sigma\pi\pi$ (or $g\sigma\bar{N}N$). Think of $m^2 \equiv m_0^2 + 2G\sigma$ as "fluctuating mass". Then

$$\delta n_{\boldsymbol{p}} = \delta n^0_p + \frac{\partial f_{\boldsymbol{p}}}{\partial m^2} 2G\sigma = \delta n^0_p + \frac{f'_{\boldsymbol{p}}}{\omega_{\boldsymbol{p}}} \frac{G}{T} \sigma$$

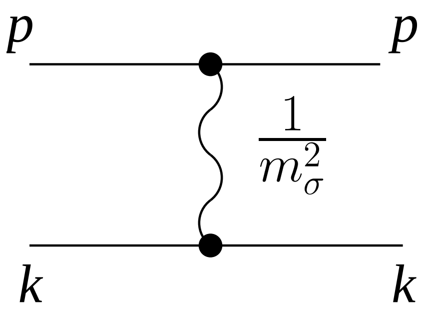
- Using $\langle \delta n^0_p \sigma \rangle = 0$ and $\langle \sigma^2 \rangle = (T/V)\xi^2$.

$$\langle \delta n_p \delta n_{\boldsymbol{k}} \rangle = f'_{\boldsymbol{p}} \delta_{\boldsymbol{pk}} + \frac{1}{VT} \frac{f'_{\boldsymbol{p}}}{\omega_{\boldsymbol{p}}} \frac{f'_{\boldsymbol{k}}}{\omega_{\boldsymbol{k}}} G^2 \xi^2.$$

More formal derivation: PRD65:096008,2002

4-point function

- The 2-particle correlator measures 4-point function at $q = 0$ (for $p \neq k$). Singularity appears at $q = 0$ due to vanishing σ screening mass $m_\sigma \to 0$. (i.e., $\xi = 1/m_\sigma \to \infty$).



$$\langle \delta n_p \delta n_k \rangle_\sigma = \frac{1}{T} \frac{f_p(1+f_p)}{\omega_p} \frac{f_k(1+f_k)}{\omega_k} \frac{G^2}{m_\sigma^2} .$$

Check: $\langle \delta n_p \delta n_k \rangle = \langle n_p n_k \rangle - \langle n_p \rangle \langle n_k \rangle > 0$ — as in attraction.
Attraction lowers the energy of a pair (making it more likely)
by $\langle H_{\text{interaction}} \rangle \sim$ forward scattering amplitude.

- Consider baryon number susceptibility, which should diverge: $\chi_B \sim \xi^{2-\eta}$

$$\chi_B \sim \langle \delta B \delta B \rangle_\sigma = \langle (\delta N_p - \delta N_{\bar{p}} + \delta N_n - \delta N_{\bar{n}})^2 \rangle_\sigma = \langle \delta N_p \delta N_p \rangle_\sigma + \ldots$$

Each term on r.h.s. is $\sim \dfrac{1}{m_\sigma^2}$, $\qquad \Rightarrow \qquad \langle \delta B \delta B \rangle \sim 1/m_\sigma^2 = \xi^2$.

- - It is enough to measure protons $\langle \delta N_p \delta N_p \rangle$ (Hatta, MS, PRL91:102003,2003)

Higher moments (cumulants) of fluctuations

- Consider probability distribution for the order-parameter field:

$$P[\sigma] \sim \exp\left\{-\Omega[\sigma]/T\right\},$$

Ω – effective potential:

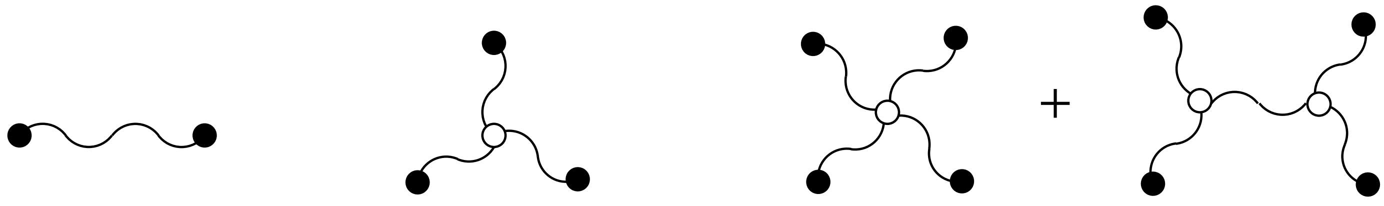
$$\Omega = \int d^3x \left[\frac{1}{2}(\boldsymbol{\nabla}\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \ldots\right]. \qquad \Rightarrow \qquad \xi = m_\sigma^{-1}$$

- Moments of zero-momentum mode $\sigma_0 \equiv \int d^3x\, \sigma(x)/V$.

$$\kappa_2 = \langle\sigma_0^2\rangle = \frac{T}{V}\,\xi^2\,; \qquad \kappa_3 = \langle\sigma_0^3\rangle = \frac{2\lambda_3 T^2}{V^2}\,\xi^6\,;$$

$$\kappa_4 = \langle\sigma_0^4\rangle_c \equiv \langle\sigma_0^4\rangle - \langle\sigma_0^2\rangle^2 = \frac{6T^3}{V^3}\left[2(\lambda_3\xi)^2 - \lambda_4\right]\xi^8\,.$$

- Tree graphs. Each zero-momentum propagator gives m_σ^{-2}, i.e., ξ^2.



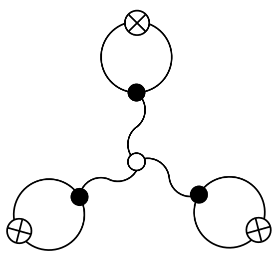
Moments of *observables*

- Use multiplicity for an example. Since multiplicity is just the sum of all occupation numbers, and thus

$$\delta N = \sum\nolimits_{\boldsymbol{p}} \delta n_{\boldsymbol{p}},$$

the cubic moment (skewness) of the pion multiplicity distribution is given by

$$\langle (\delta N)^3 \rangle = \sum\nolimits_{\boldsymbol{p}_1} \sum\nolimits_{\boldsymbol{p}_2} \sum\nolimits_{\boldsymbol{p}_3} \langle \delta n_{\boldsymbol{p}_1} \delta n_{\boldsymbol{p}_2} \delta n_{\boldsymbol{p}_3} \rangle \,, \qquad \text{where } \sum\nolimits_{\boldsymbol{p}} = V \int d^3\boldsymbol{p}/(2\pi)^3.$$

- 

$$\langle \delta n_{\boldsymbol{p}_1} \delta n_{\boldsymbol{p}_2} \delta n_{\boldsymbol{p}_3} \rangle_\sigma = \frac{2\lambda_3}{V^2 T} \left(\frac{G}{m_\sigma^2} \right)^3 \frac{v_{\boldsymbol{p}_1}^2}{\omega_{\boldsymbol{p}_1}} \frac{v_{\boldsymbol{p}_2}^2}{\omega_{\boldsymbol{p}_2}} \frac{v_{\boldsymbol{p}_3}^2}{\omega_{\boldsymbol{p}_3}}$$

$$v_{\boldsymbol{p}}^2 = \bar{n}_{\boldsymbol{p}}(1 \pm \bar{n}_{\boldsymbol{p}})$$

Similarly for $\langle (\delta N)^4 \rangle_c$.

- Since $\langle (\delta N)^3 \rangle$ scales as V^1 it is convenient to normalize it by the mean total multiplicity $\bar{N}$ which scales similarly. Thus we define

$$\omega_3(N) \equiv \frac{\langle (\delta N)^3 \rangle}{\bar{N}}$$

Moments of observables contd.

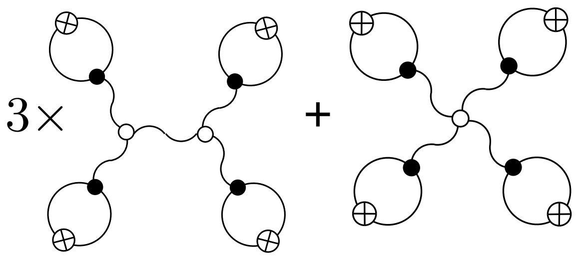
... and find

$$\omega_3(N)_\sigma = \frac{2\lambda_3}{T}\frac{G^3}{m_\sigma^6}\left(\int_{\boldsymbol{p}}\frac{v_{\boldsymbol{p}}^2}{\omega_{\boldsymbol{p}}}\right)^3\left(\int_{\boldsymbol{p}}\bar{n}_{\boldsymbol{p}}\right)^{-1}.$$

- Similarly, for

$$\omega_4(N) \equiv \frac{\langle(\delta N)^4\rangle_c}{\bar{N}}$$

from



we find

$$\omega_4(N)_\sigma = \frac{6}{T}\left[2\frac{\lambda_3^2}{m_\sigma^2} - \lambda_4\right]\frac{G^4}{m_\sigma^8}\left(\int_{\boldsymbol{p}}\frac{v_{\boldsymbol{p}}^2}{\omega_{\boldsymbol{p}}}\right)^4\left(\int_{\boldsymbol{p}}\bar{n}_{\boldsymbol{p}}\right)^{-1}.$$

Scaling, λ_n

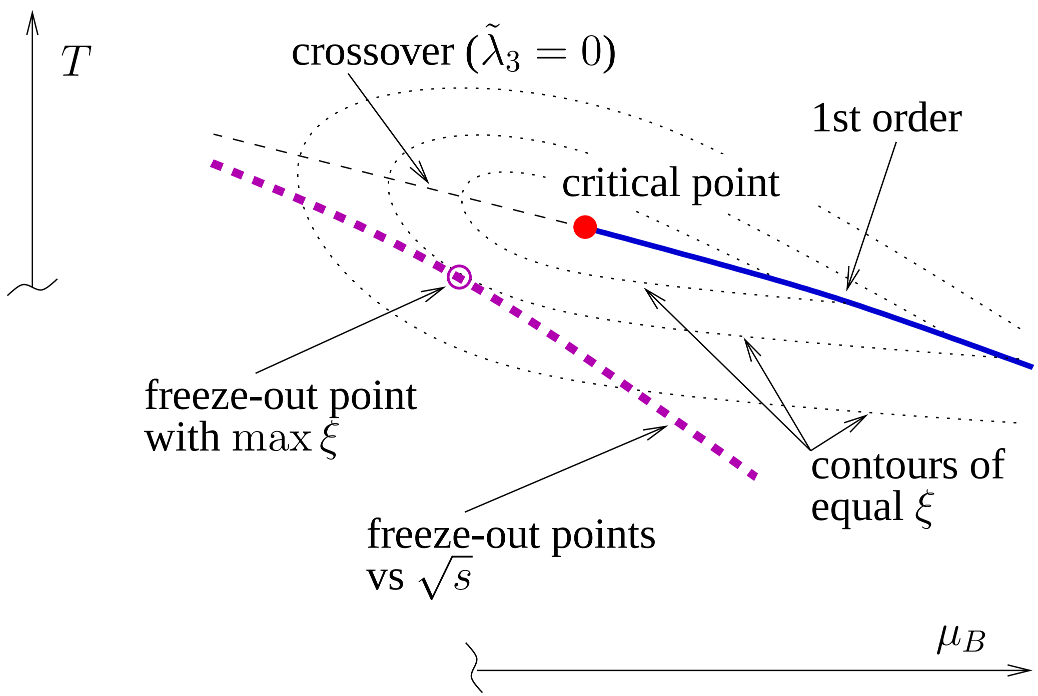
- Scaling requires that both λ_3 and λ_4 vanish with a power of ξ given by:

$$\lambda_3 = \tilde\lambda_3 T \cdot (T\xi)^{-3/2}, \qquad \text{and} \quad \lambda_4 = \tilde\lambda_4 \cdot (T\xi)^{-1}, \qquad\qquad (\eta \ll 1)$$

(because $[(\nabla\sigma)^2] = 3 \;\Rightarrow\; [\sigma] = 1/2$ and $\Rightarrow\; [\lambda_n] = 3 - n/2$)

Dimensionless couplings $\tilde\lambda_3$ and $\tilde\lambda_4$ are universal, and for the Ising universality class they have been measured on the lattice.

- λ_3 is nonzero:



Estimates

Pions (top SPS):

$$\omega_3(N_\pi)_\sigma \equiv \frac{\langle(\delta N_\pi)^3\rangle}{\bar{N}_\pi} \approx 1.\left(\frac{\tilde{\lambda}_3}{4.}\right)\left(\frac{G}{\text{300 MeV}}\right)^3\left(\frac{\xi}{\text{3 fm}}\right)^{9/2}$$

$$\omega_4(N_\pi)_\sigma \equiv \frac{\langle(\delta N_\pi)^4\rangle_c}{\bar{N}_\pi} \approx 12.\left(\frac{2\tilde{\lambda}_3^2-\tilde{\lambda}_4}{50.}\right)\left(\frac{G}{\text{300 MeV}}\right)^4\left(\frac{\xi}{\text{3 fm}}\right)^{7}$$

Protons (top SPS):

$$\omega_3(N_p)_\sigma \equiv \frac{\langle(\delta N_p)^3\rangle}{\bar{N}_p} \approx 3.\left(\frac{\tilde{\lambda}_3}{4.}\right)\left(\frac{g}{10.}\right)^3\left(\frac{\xi}{\text{1 fm}}\right)^{9/2}$$

$$\omega_4(N_p)_\sigma \equiv \frac{\langle(\delta N_p)^4\rangle_c}{\bar{N}_p} \approx 23.\left(\frac{2\tilde{\lambda}_3^2-\tilde{\lambda}_4}{50.}\right)\left(\frac{g}{10.}\right)^4\left(\frac{\xi}{\text{1 fm}}\right)^{7}$$

Notes:

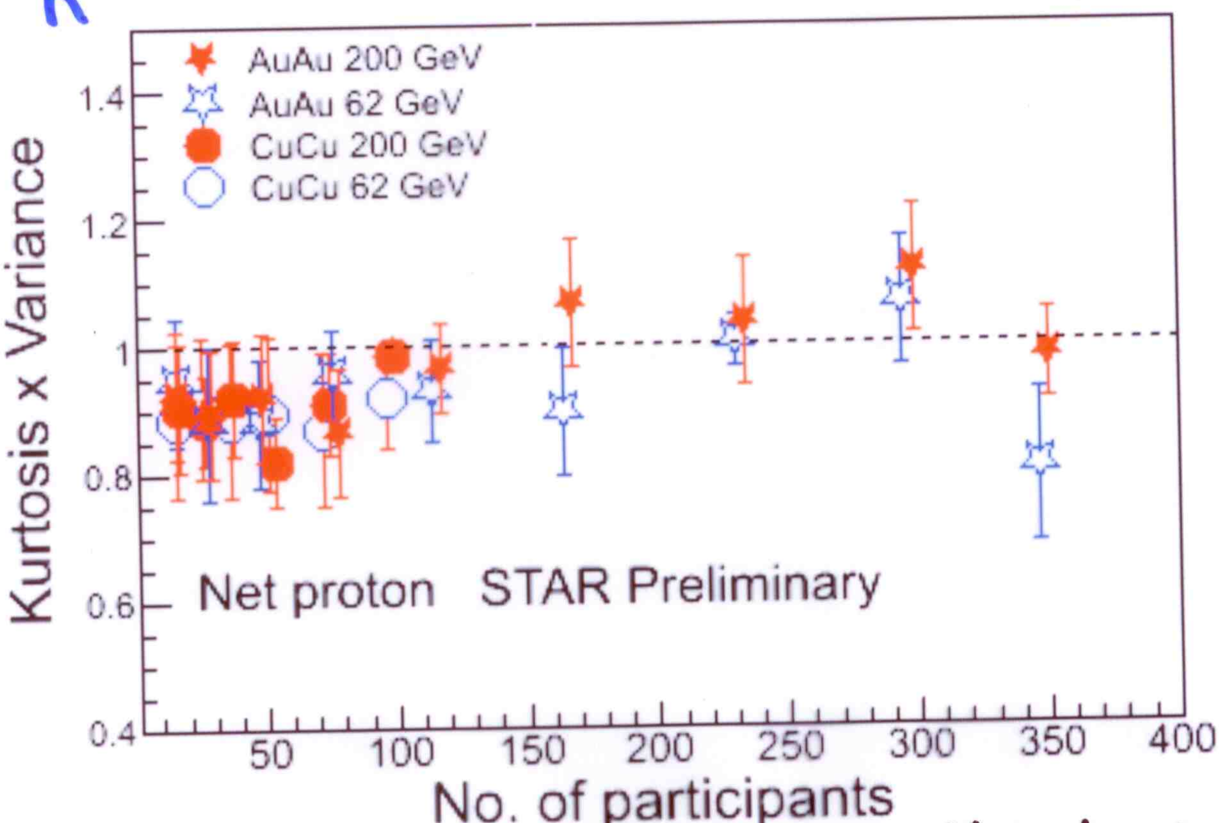
- Strong dependence on ξ, compared to $\omega_2 \sim \xi^2$.
- Significant uncertainty due to G, g.
- Crosscheck: same exponents as baryon number cumulants from scaling/universality:

$$\langle(\delta N_B)^k\rangle_c = VT^{k-1}\frac{\partial^k P(T,\mu_B)}{\partial \mu_B^k} \sim \xi^{k(5-\eta)/2-3}. \qquad (\eta \ll 1)$$

K^{eff} AT $\sqrt{s}$ = 200 GeV AND 62 GeV

ie at very low μ, far from •

K^{eff}



Nayak, QM 09

- Background K^{eff} measured, $\mathcal{O}(1)$.
- Error bars $\simeq \pm .1$.

PROTON NUMBER KURTOSIS

- Predicted effect of proximity to is:
  - large
  - strongly ξ-dependent, $\therefore$ $\sqrt{s}$-dependent
  - larger at lower p_T

  How you know its if you see it.
- Cannot be washed out between chemical & kinetic freezeout
- Background measured, and small, at $\sqrt{s} = 62$ & 200.
- Error bars $\sim \pm .1$ with 5M events STAR B.U.
- Calculations done for $\bar{N} \to \infty$. Finite $\bar{N}$ and acceptance corrections need to be assessed. (Latter minimal for STAR)
- Analogous analyses can be done for skewness, and for pions. Many signatures will be in play if/when found.

A BEAM ENERGY SCAN TO SEARCH FOR •

$\sqrt{s}/A$ (GeV)	μ (MeV)*	8 hr days per 5 M events †
5	550	
6.2	485	
7.7	420	56
9.8	355	30
12.7	290	13
17.3	230	5
27	155	2
39	110	1
200	25	
		107 days

*: from Cleymans et al's 2005 empirical fit to compilation of data

†: from STAR B.U.R.

WHAT NEED BE MEASURED AT EACH $\sqrt{s}$

- Enough <particle ratios> to evaluate μ.
  So you know where on phase diagram
  you are freezing out.
- n_q scaling of V_2 for $\pi/K/p/\Lambda$
  parity violating fluctuations
  So you know whether collision got
  above the crossover/transition
- Event-by-event fluctuations of K/π and
  p/π ratios with significantly smaller
  error bars than in NA49 data
- Variance, skewness & kurtosis of
  event-by-event distributions of
  N_p, $N_p - N_{\bar{p}}$, N_π.
- **All can be done with 5M min bias events per $\sqrt{s}$.** STAR B.U.R.

CAN WE DISCOVER THE QCD CRITICAL POINT IN RUN 10 AT RHIC?

YES, IF:

Nature is kind, and puts $\mu_{\bullet} \lesssim 420$ MeV

IF YES:

- The landmark discovered. Our map of the QCD phase diagram then anchored by experiment.
- Assuming reasonable progress in lattice QCD, quantitative comparison between theory & experiment for $\mu_{\bullet}$ will come.
- RHIC (and FAIR) can study the first order phase transition. RHIC can further probe the region around $\bullet$.

IF NO:

- We learn that $\mu_{\bullet} > 420$ MeV
- We will be able to make a data-driven decision about whether to run at $\sqrt{s} = 6.2$ and 5 in a future year.

$T \neq 0$; $\mu \neq 0$; μ/T NOT LARGE

- a regime explored by heavy ion collisions
- a regime explored by lattice calculations that rely on smallness of μ/T to keep fermion sign problem under control. [$\mu \neq 0 \to$ complex Euclidean action $\to$ sign problem that makes difficulty of standard Monte Carlo $\sim \exp V$.]
- Either method may be used to locate the CRITICAL POINT, a 2nd order point where a line of 1st order transitions ends, if it is located at a μ/T that is not too large....

SEVERAL LATTICE METHODS

① Reweighting   Fodor + Katz

Want physics at $a \equiv (\mu, T_a)$

Simulate using an ensemble of configurations at $b \equiv (0, T_b)$, and "reweight": lump difference between physics at b and a into observables.

Difficulty $\sim \exp\left[\frac{|F_b - F_a| V}{T}\right]$

F+K: choose T_b to minimize ☡

BUT: still cannot use method at large volumes....

The endpoint is at $T_E = 162 \pm 2$ MeV, $\mu_E = 360 \pm 40$ MeV. As expected, μ_E decreased as we decreased the light quark masses down to their physical values (at approximately three-times larger $m_{u,d}$ the critical point was at μ_E=720 MeV; see [8]).

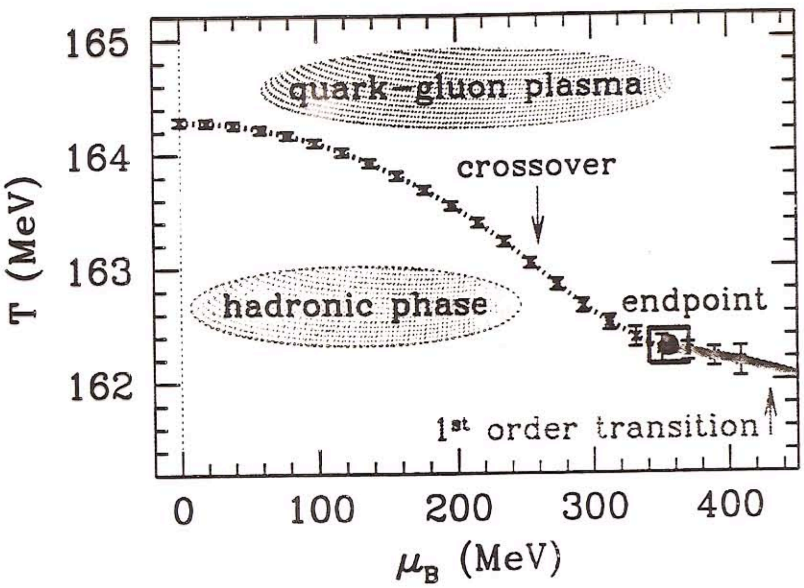


Figure 2: The phase diagram in physical units. Dotted line illustrates the crossover, solid line the first order phase transition. The small square shows the endpoint. The depicted errors originate from the reweighting procedure. Note, that an overall additional error of 1.3% comes from the error of the scale determination at T=0. Combining the two sources of uncertainties one obtains $T_E = 162 \pm 2$ MeV and $\mu_E = 360 \pm 40$ MeV.

The above result is a significant improvement on our previous analysis [8] by two means. We increased the physical volume by a factor of three and decreased the light quark masses by a factor of three. Increasing the volumes did not influence the results, which indicates the reliability of the finite volume analysis. Clearly, more work is needed to get the final values. Most importantly one has to extrapolate to the continuum limit.

Fodor, Katz
2004

$$\left.\begin{aligned} \mu_E &= 360 \pm 40 \text{ MeV} \\ \frac{\mu_E}{T_E} &= 2.22 \pm .25 \end{aligned}\right\} \text{statistical errors only}$$

CONCERNS, aka "SYSTEMATIC ISSUES"

- $N_\tau = 4$ (no continuum limit)
- $V = 12^3$, and method must break down for $V \to \infty$
- $\frac{\mu_E}{3} \simeq \frac{m_\pi}{2}$. This was also the case in older F+K calculation at larger m_π. If this is not a coincidence, it is a problem. Splittorf

  [$\mu_q = m_\pi/2$ is where phase quenched QCD has onset of pion condensation.]
- $\frac{m}{T}$ held fixed during reweighting, not m.

ALL these, except for $V \to \infty$, are IMPROVABLE.

(2) Continue from imaginary μ.

deForcrand + Philipsen

D'Elia + Lombardo et al

Simulate at $\mu = i\mu_I$; calculate

$T_c(\mu_I)$; Taylor expand:

$$= C_0 + C_2 \mu_I^2 + C_4 \mu_I^4 + \dots$$

- valid for $\frac{\mu_I}{T} < \frac{\pi}{3}$
- Good luck ... $C_4, C_6, \dots$ terms all small over this range.
- So, boldly continue:

$$T_c(\mu) = C_0 - C_2\mu^2 + C_4\mu^4 \dots$$

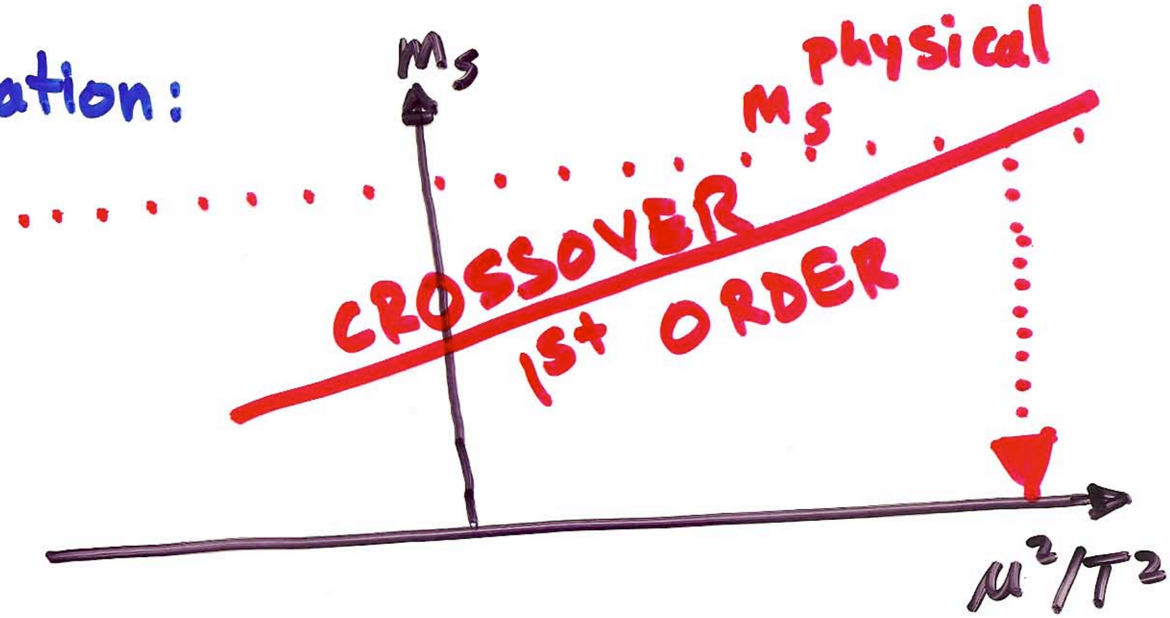
Curvature of crossover line on phase diagram

CRITICAL POINT ??

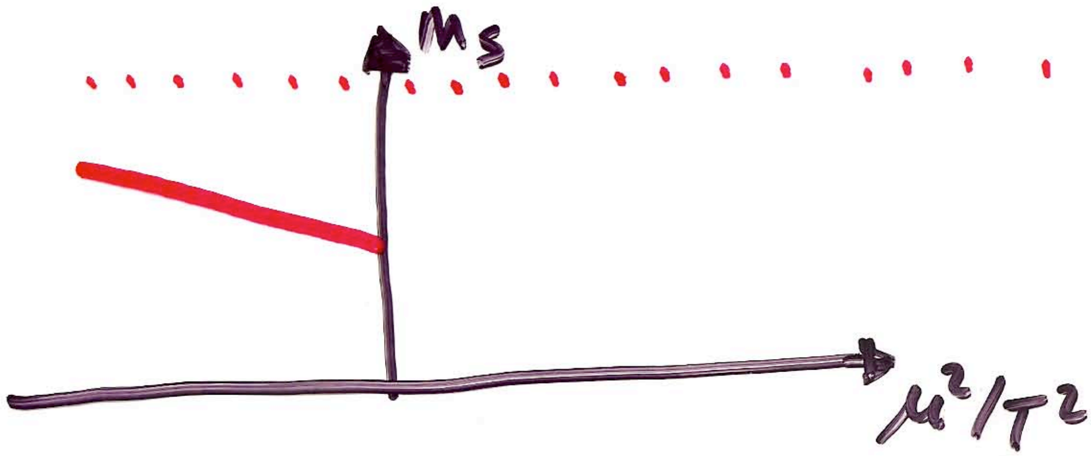
- Calculate

$$\frac{\partial}{\partial \mu^2}\left[\begin{array}{l} m_q \text{ at which transition goes} \\ \text{from } 1^{st} \text{ order to crossover} \end{array}\right]$$

- Expectation:



- deForcrand + Philipsen find:



- $\Rightarrow$ NO CRITICAL POINT with $\frac{\mu}{T} < \mathcal{O}(1)$.

CONCERNS, aka "SYSTEMATIC ISSUES"

Lets defer their discussion to after Philippe's talk, but here are two:

- $N_\tau = 4$
- Staggered fermions with $N_f = 3$ or $2+1$ .....
  - $Det^{3/4}$ or $Det^{1/2}\, Det^{1/4}$
  - First order phase transition at small m_s originates from 't Hooft $uds\bar{u}\bar{d}\bar{s}$ interaction, in low energy effective theory. Pisarski Wilczek
  - do staggered fermions describe this adequately ?? Fukushima, Stephanov

also an issue for F+K

③ Taylor Expansion of the Pressure.

Bielefeld-Swansea; Gavai Gupta

Calculate the coefficients in:

$$\frac{P}{T^4} = b_0(T) + b_2(T)\mu^2 + b_4(T)\mu^4 + b_6(T)\mu^6 + \ldots$$

and hence in:

$$\chi_B \equiv \frac{\partial^2 P}{\partial \mu^2} = c_0(T) + c_2(T)\mu^2 + c_4(T)\mu^4 + c_6(T)\mu^6 + \ldots$$

which should diverge at critical point.

Several ways to look for critical point:

- Look for μ at which χ_B peaks
- Do Taylor expansion at varying m_q and evaluate

$$\frac{\partial}{\partial \mu^2}\left[\begin{array}{l} m_q \text{ at which crossover at} \\ \mu = 0 \text{ becomes } 1^{st} \text{ order} \end{array}\right]$$

[Defer discussion of these to Karsch.]

- And ...

RADIUS OF CONVERGENCE METHOD

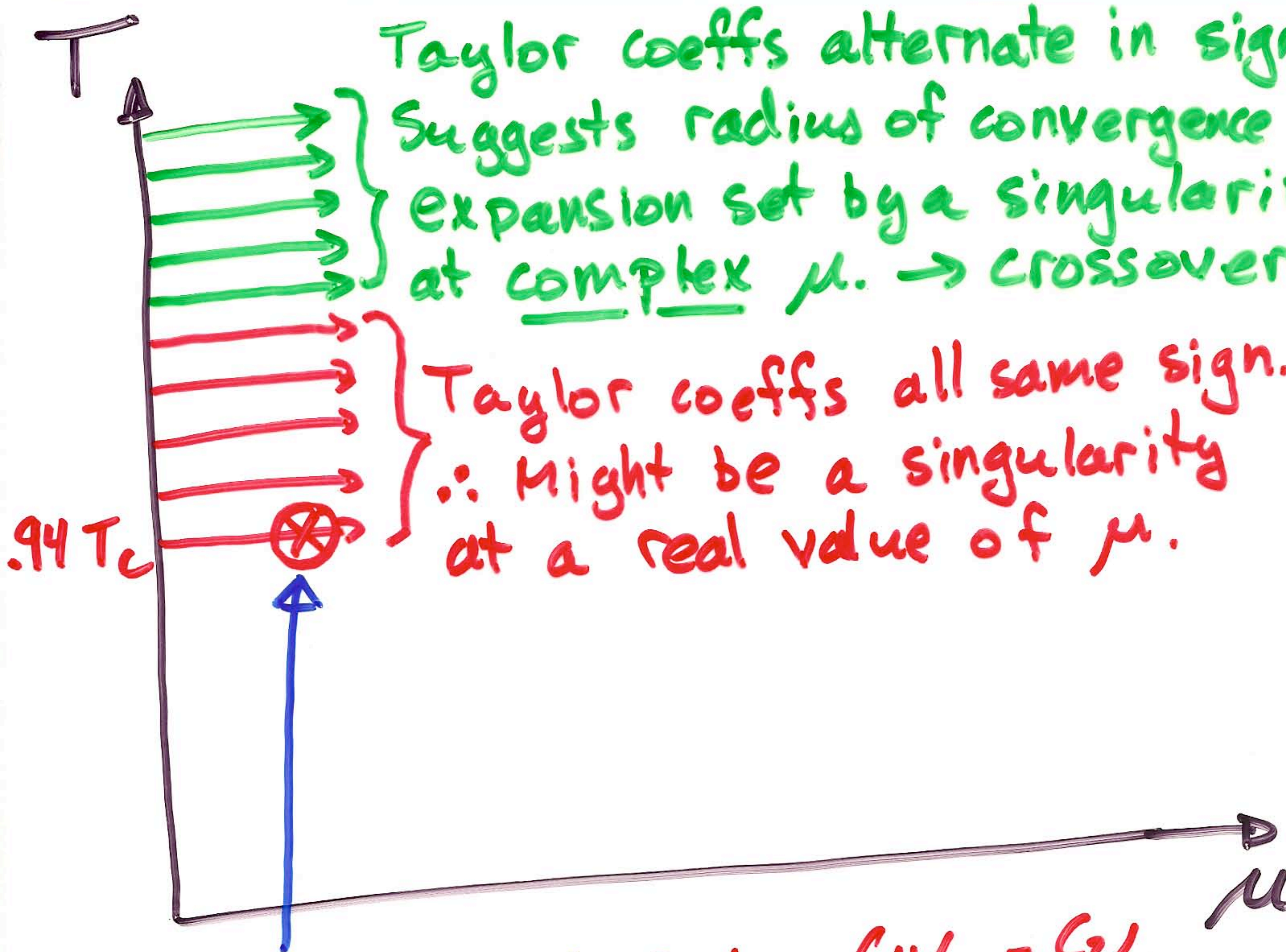
Use fact that Taylor expansion must break down at critical point.

Bielefeld Swansea ; Gavai Gupta

New results from Gavai + Gupta, June 2008 & earlier this workshop:

- $N_\tau = \underline{\underline{6}}$; $V = 24^3$
- $N_f = 2$; $m_\pi = 230$ MeV
- staggered fermions, so maybe not a bad thing that $N_f = 2$.
- Taylor coefficients $c_0(T)$, $c_2(T)$, $c_4(T)$, $c_6(T)$.

  [ie up to μ^8 term in P]



Taylor coeffs alternate in sign. Suggests radius of convergence of expansion set by a singularity at complex μ. → crossover

Taylor coeffs all same sign. ∴ Might be a singularity at a real value of μ.

At this T, find $c_6/c_4 = c_4/c_2 = c_2/c_0$, as would be the case for a pole at real μ. And, as yields a consistent estimate of radius of conv.

Also, at the same T, coeffs have expected finite size scaling (upon comparing LT = 2 and 4).

I dentify this ~~Te~~ T as T_E, and this radius of convergence as μ_E.

Gavai and Gupta find :

$$\frac{T^E}{T_c} = 0.94 \pm 0.01$$

$$\frac{\mu_E}{T_E} = 1.8 \pm 0.1$$

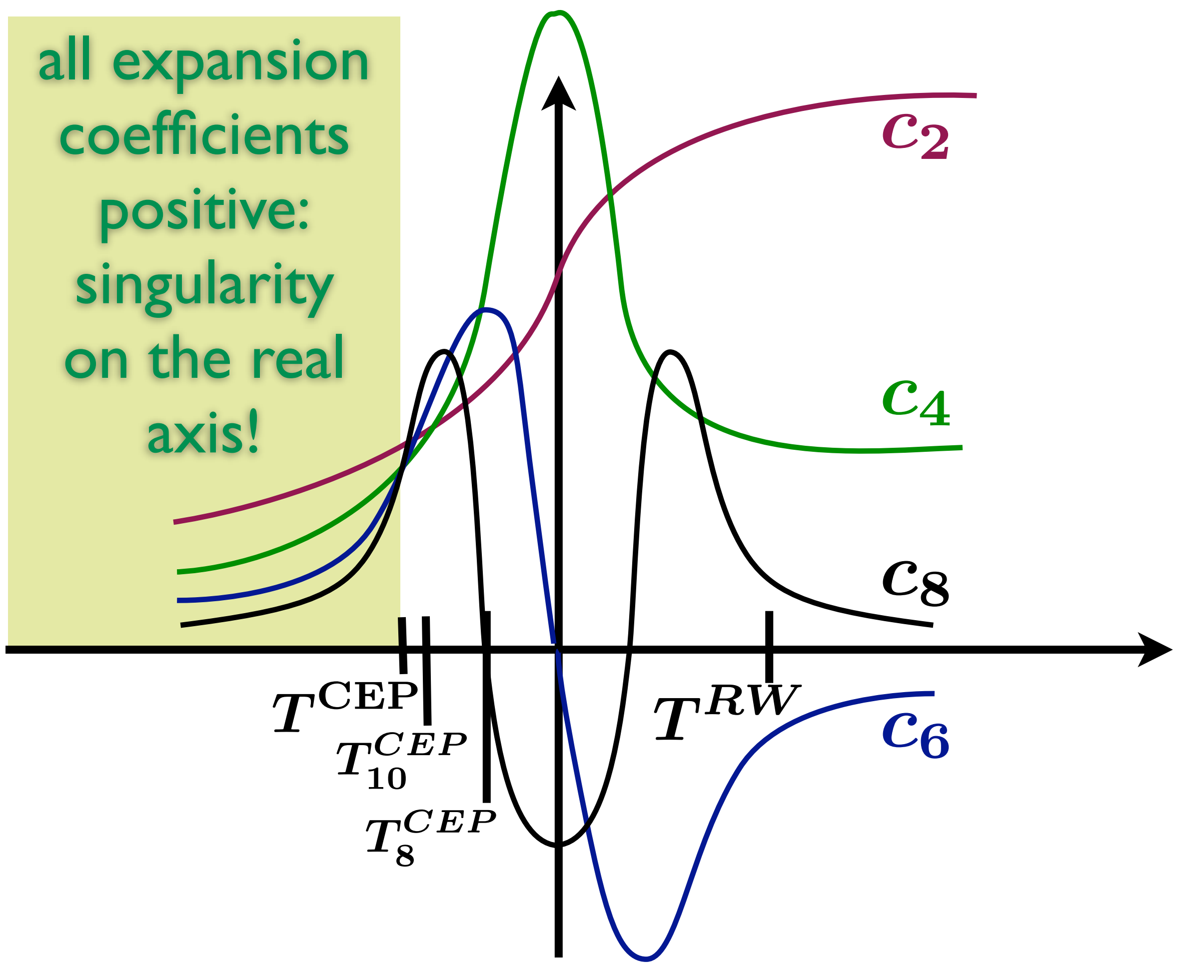
Issues :

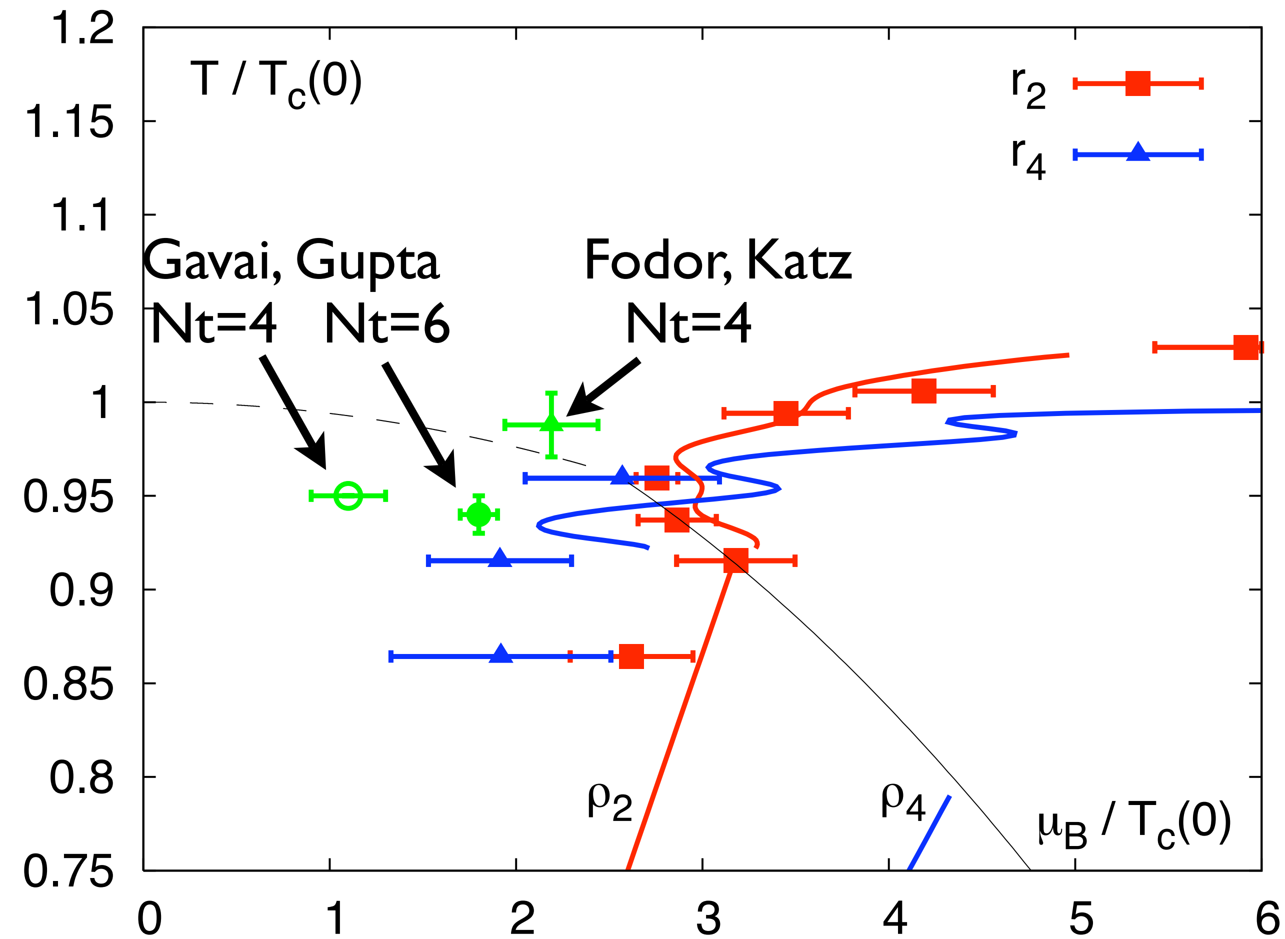
- $N_\tau = 6$. "Crawling towards the continuum limit." Gupta
- $N_f = 2 \rightarrow N_f = 2+1$
- What is the best estimator of T^E, ie what combination of criteria, given $C_0(T)$, $C_2(T)$, $C_4(T)$, $C_6(T)$?

Radius of Convergence

Method to determine the CEP:

- find largest temperature were all expansion coefficients are positive → T^{CEP}
- determine the radius of convergence at that temperature → μ^{CEP}





$$\mu^{\mathrm{CEP}} = \lim_{n\to\infty} \mu_n^{\mathrm{CEP}}$$

$$\mu_n^{\mathrm{CEP}} = T^{\mathrm{CEP}} \sqrt{c_n/c_{n+2}}$$

→ first non-trivial estimate of T^{CEP} from c_8
second non-trivial estimate of T^{CEP} from c_{10}

The Resonance gas limit:

$$\frac{p}{T^4} = G(T) + F(T)\cosh\left(\frac{\mu_B}{T}\right)$$

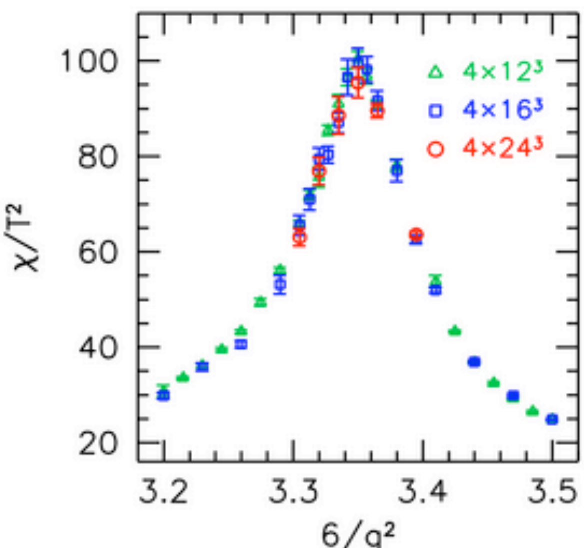
$$\longrightarrow \rho_n = \sqrt{(n+2)(n+1)}$$

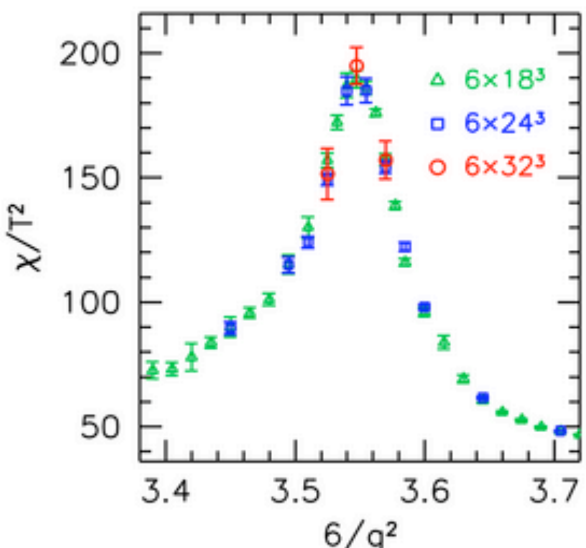
Order of Phase Transition for $\mu_B \sim 0$

Physical quark masses
Continuum limit
Simulations along Lines of Constant Physics
$m_K/m_\pi = 3.689$; $f_K/m_\pi = 1.185$
Staggered fermionic action

$$\chi(N_s, N_t) = \partial^2/(\partial m_{ud}^2)(T/V)\cdot \log Z$$

Y. Aoki et al., Nature443:675-678,2006





T grows with $6/g^2$, g : gauge coupling

1[st] order :
Peak height ~ V
Peak width ~ 1/V
Cross over :
Peak height ~ const.
Peak width ~ const.
2[nd] order :
Peak height ~ V^α

No significant volume dependence (8 times difference in volumes)
Phase transition at high T and $\mu_B = 0$ is a cross over

Relevant to LHC and current RHIC regimes

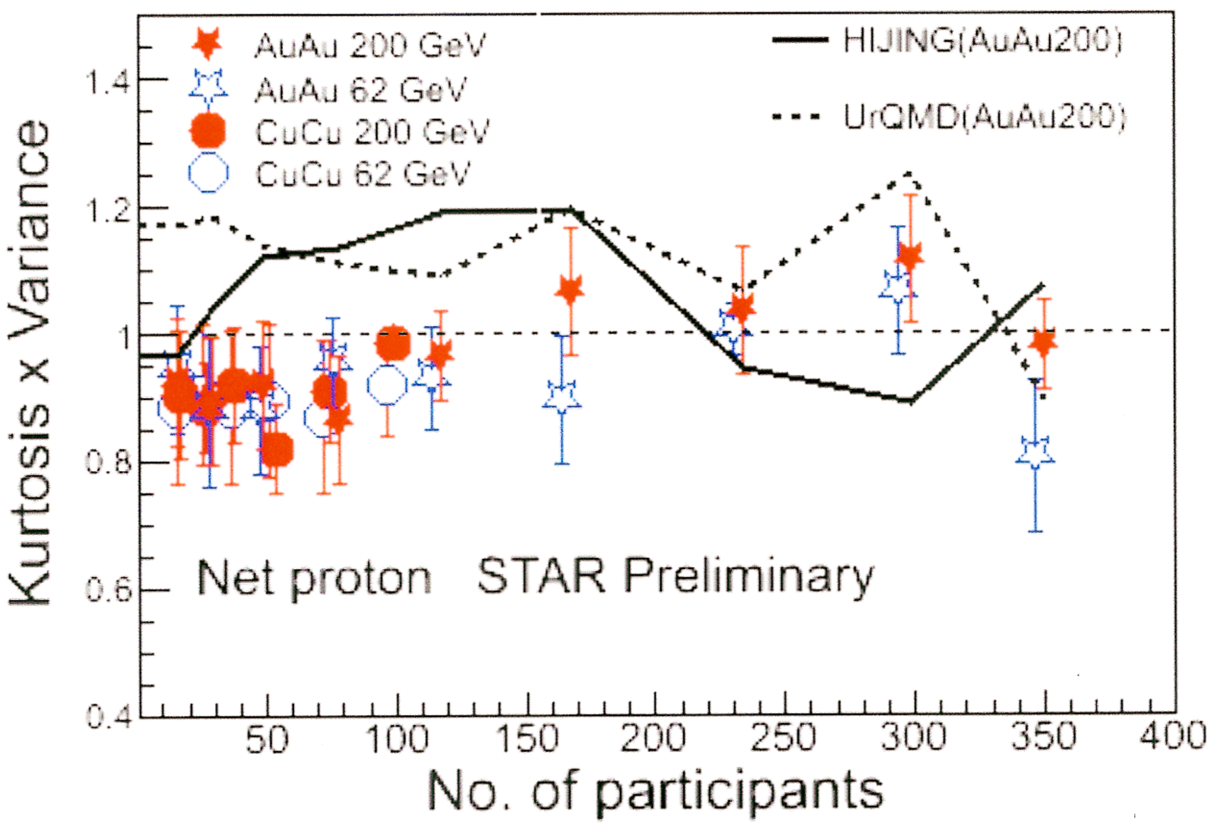
Lattice results on electroweak transition in standard model is an analytic cross-over for large Higgs mass

K. Kajantie et al., PRL 77, 2887-2890,2006

	Collision Energies (GeV)	**5**	**7.7**	**11.5**	**17.3**	**27**	**39**
Section	Observables	Millions of Events Needed					
A1	v_2 (up to ~1.5 GeV/c)	**0.3**	**0.2**	**0.1**	**0.1**	**0.1**	**0.1**
A1	v_1	0.5	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**
A2	Azimuthally sensitive HBT	4	**4**	**3.5**	**3.5**	**3**	**3**
A3	PID fluctuations (K/π)	1	**1**	**1**	**1**	**1**	**1**
A3	net-proton kurtosis	5	**5**	**5**	**5**	**5**	**5**
A3	differential corr & fluct vs. centrality	4	**5**	**5**	**5**	**5**	**5**
A3	integrated p_T fluct (T fluct)						
B1	n_q scaling $\pi/K/p/\Lambda$ $(m_T\text{-}m_0)/n$<2GeV		**6**	**5**	**5**	**4.5**	**4.5**
B1	ϕ/Ω up to p_T/n_q=2 GeV/c		56	25	**18**	**13**	**12**
B2	R_{CP} up to p_T ~4.5 GeV/c (at 17.3) 5.5 (at 27) & 6 GeV/c (at 39)				**15**	**33**	**24**
B3	untriggered ridge correlations		27	13	**8**	**6**	**6**
B4	parity violation		**5**	**5**	**5**	**5**	**5**
See[1]:	charge-photon fluctuations (DCC)	1	**1**	**1**	**1**	**1**	**1**
	kink/step/horn	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**
	v_2 fluctuations	0.5	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**
	HBT (R_l, R_o/R_s)	0.8	**0.8**	**0.5**	**0.5**	**0.5**	**0.5**
	Jet/ridge 2<trig<4, 1<assoc<trig				30	**8.8**	**4.5**
	Jet/ridge 3<trig<6, 1.5<assoc<trig					53	**24**
	Baryon-Strangeness cor (hypernuc)						50
	Forward π^- yield (rapidity scaling)						
	Forw. $\gamma(\pi^0)$ yield (rapidity scaling)						
	Long-range forward-backward corr.						
	Other PID fluctuations (esp. K/p)						
	Particle ratios (many examples)						
	p_T spectra						
	Prod. of light nuclei & antinuclei						
	Yields of species & stat model fits						

Table 3-2: Observables and statistics needed for the first BES run. The observables in the yellow-shaded area relate to the search for a phase transition or critical point (see section A), while observables in the blue-shaded area search for turn-off of new phenomena already established at higher RHIC energies (see section B). The numbers listed in boldface above are all within reach (nominally require no more than 1.5 times the proposed statistics) in the first BES run plan as set out in Table 3-1. The remaining numbers (not boldface) will need to wait for higher

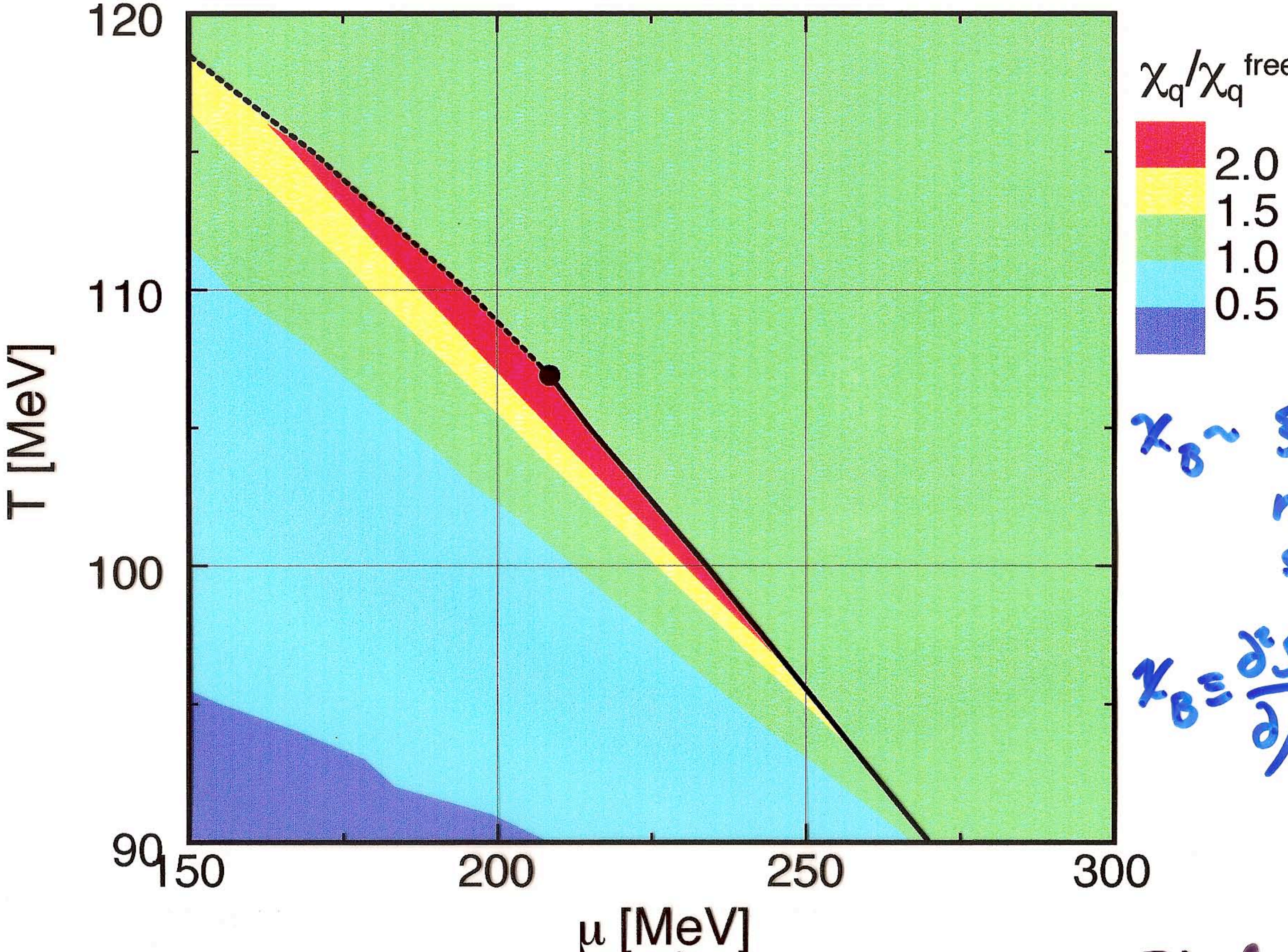
K^{eff}



Nayak, QM09

MODEL ANALYSIS OF EXTENT OF CRITICAL REGION

$m_u = m_d = 0$



$= \mu_B/3$

$\chi_B \sim \xi^2 +$ non-singular

$\chi_B = \frac{\partial^2 \Omega}{\partial \mu_B^2}$

Hatta Ikeda