Turbulence, Computers, and Stars

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Mixing Length Theory versus improvements

- no error estimates (Kolmogorov 4/5 law: we get 4/5 ~ 0.85 with boundaries)
- no KE or acoustic Fluxes in MLT
- no deceleration at boundaries (Richardson criterion for mixing, buoyancy braking; alpha overshoot function of velocity, Brunt)
- static (Lorenz strange attractor)

Step 1: 3D simulations of turbulent flow: generate data!

3D simulations by computer

What do we do?

- Oxygen fusion in stars: faster thermal relaxation
- ignore (for now) rotation and magnetic fields
- sectors, not whole stars (convective cells)
- Implicit Large Eddy Simulation (ILES)
- Only a range of Reynolds numbers, but turbulent

ILES

- monotonic methods of shock capture (like PPM) give a sub-grid dissipation ~(dv)^3/dr
- this is consistent with the dissipation in the Richardson-Kolmogorov turbulent cascade
- no further "sub-grid" physics is needed (or desired)

Bethe (1942): shock dissipation is a function of shock width and velocity difference

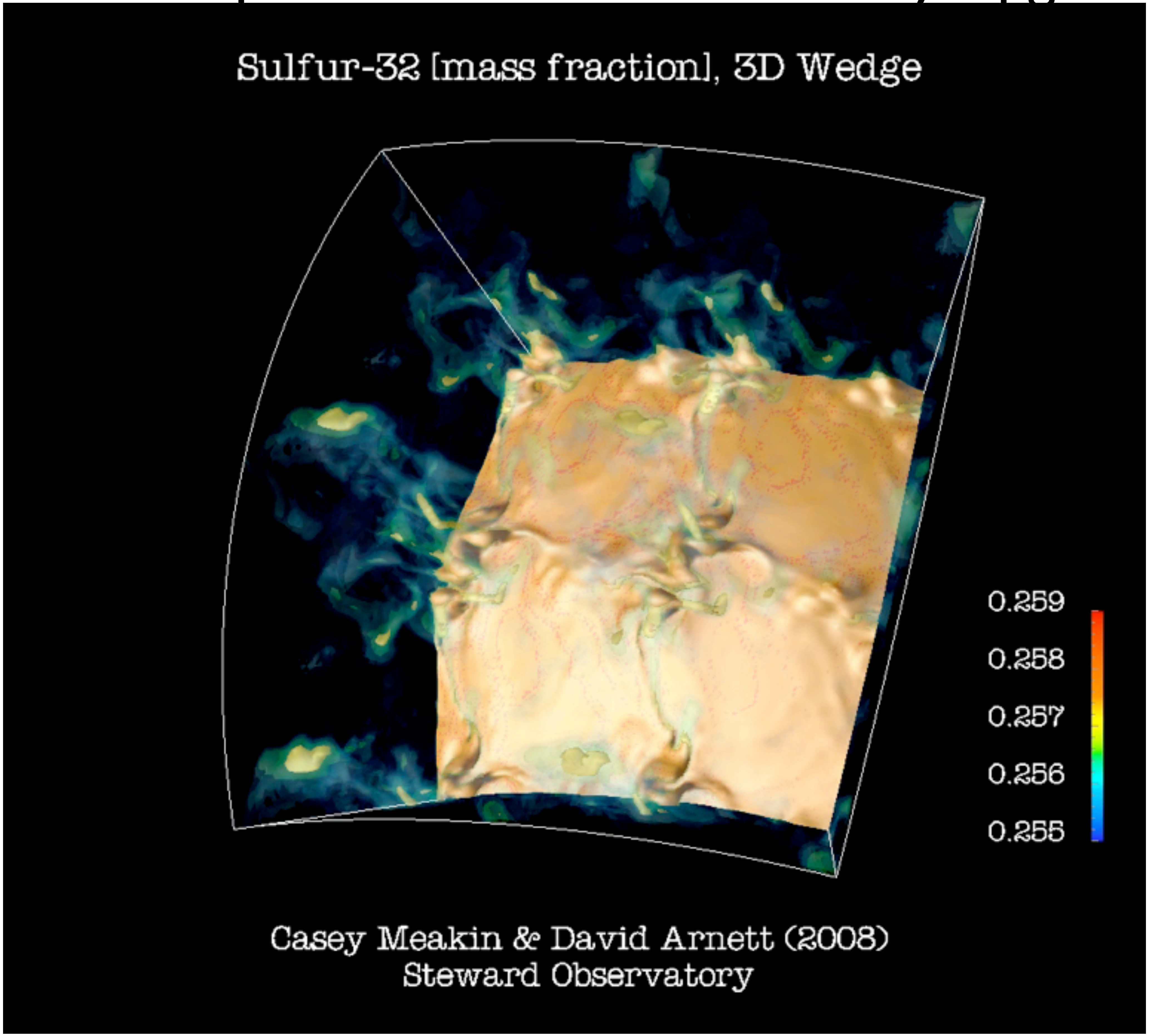
$$\varepsilon \approx \Delta u^3/\Delta r$$

Kolmogorov (1942): turbulent kinetic energy dissipates as

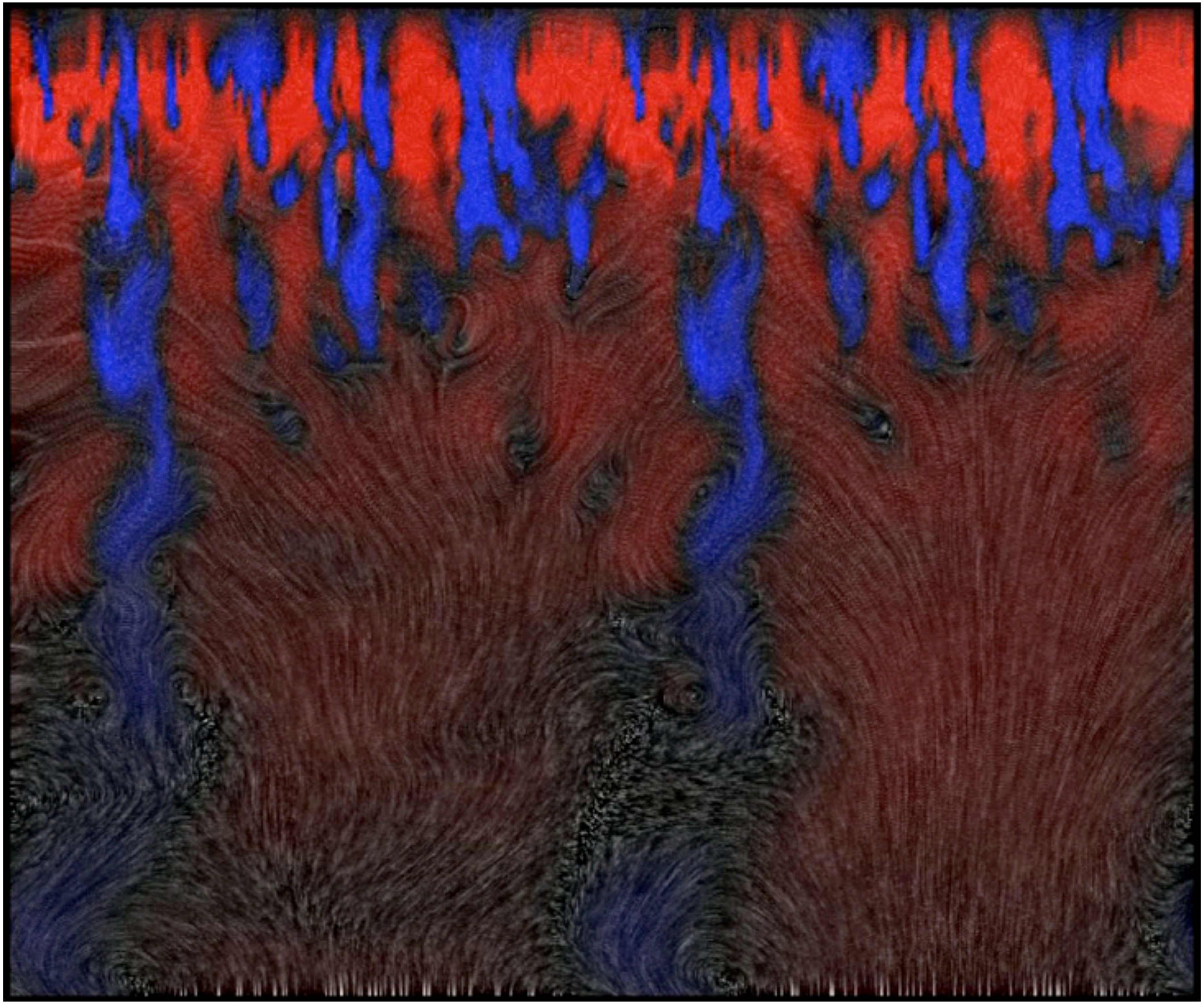
$$\varepsilon \approx u_t^3/\ell$$

in terms of mean turbulent velocity and the linear size of the turbulent region

ob.4hp.l.s32.iso-bottom.boundary.mpg

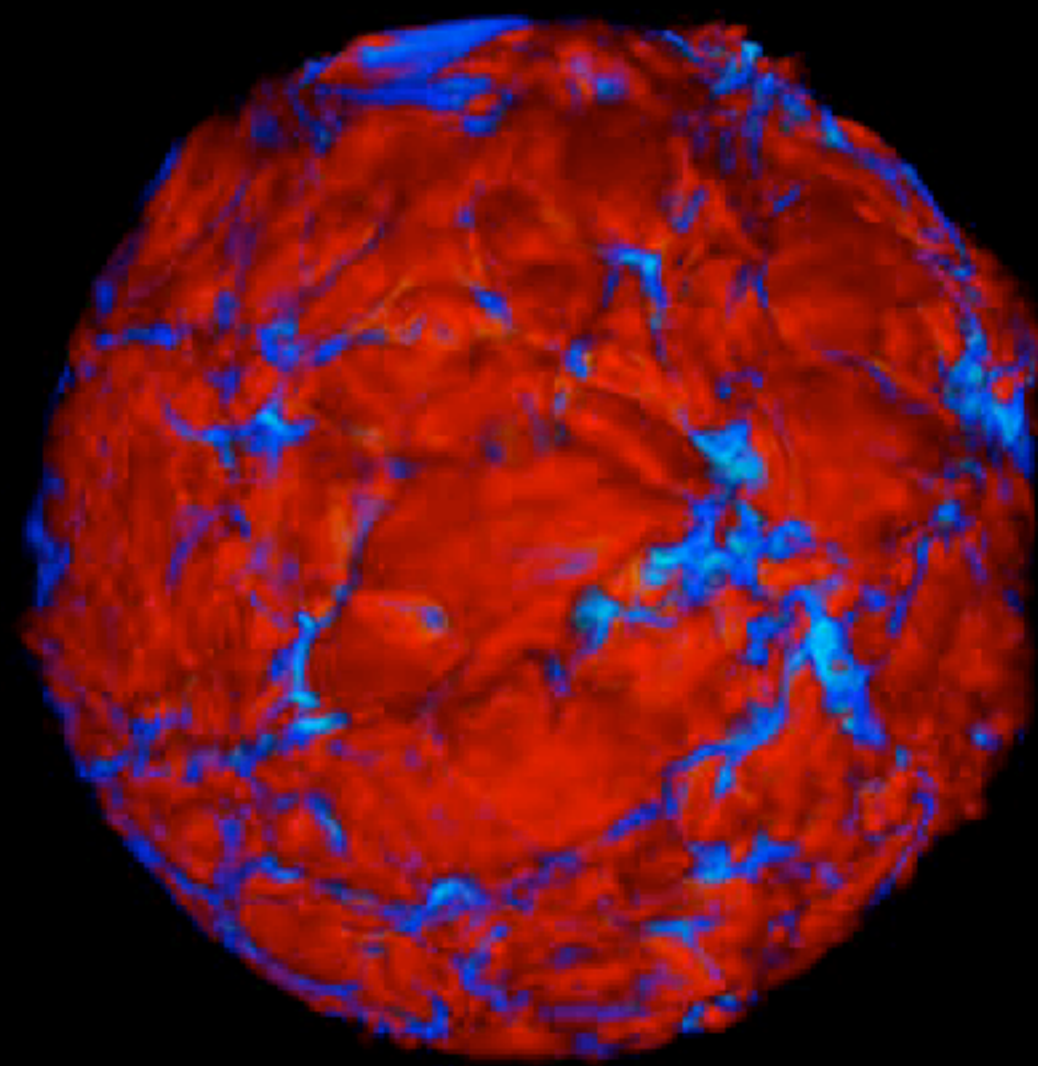


Nordlund & Stein: Solar Atmosphere



Multiple Pressure Scale Heights

Porter and Woodward: red giant
Coupled convection and pulsation



Step 2: detailed quantitative analysis and theory construction

Reynolds decomposition into average and fluctuating parts

$$a = a_0 + a'$$
$$\overline{\langle a \rangle} = a_0$$
$$\overline{\langle a' \rangle} = 0$$

overline is time average
angle bracket is angle average (3D-->1D)

Acoustic and Turbulent Kinetic Energy Fluxes

$$\mathbf{F_P} = P'\mathbf{u}'$$

$$\mathbf{F_K} = \rho E_K \mathbf{u}'$$

dot u' with Navier-Stokes equation:

Turbulent KE Equation (terms in 1e43 ergs)

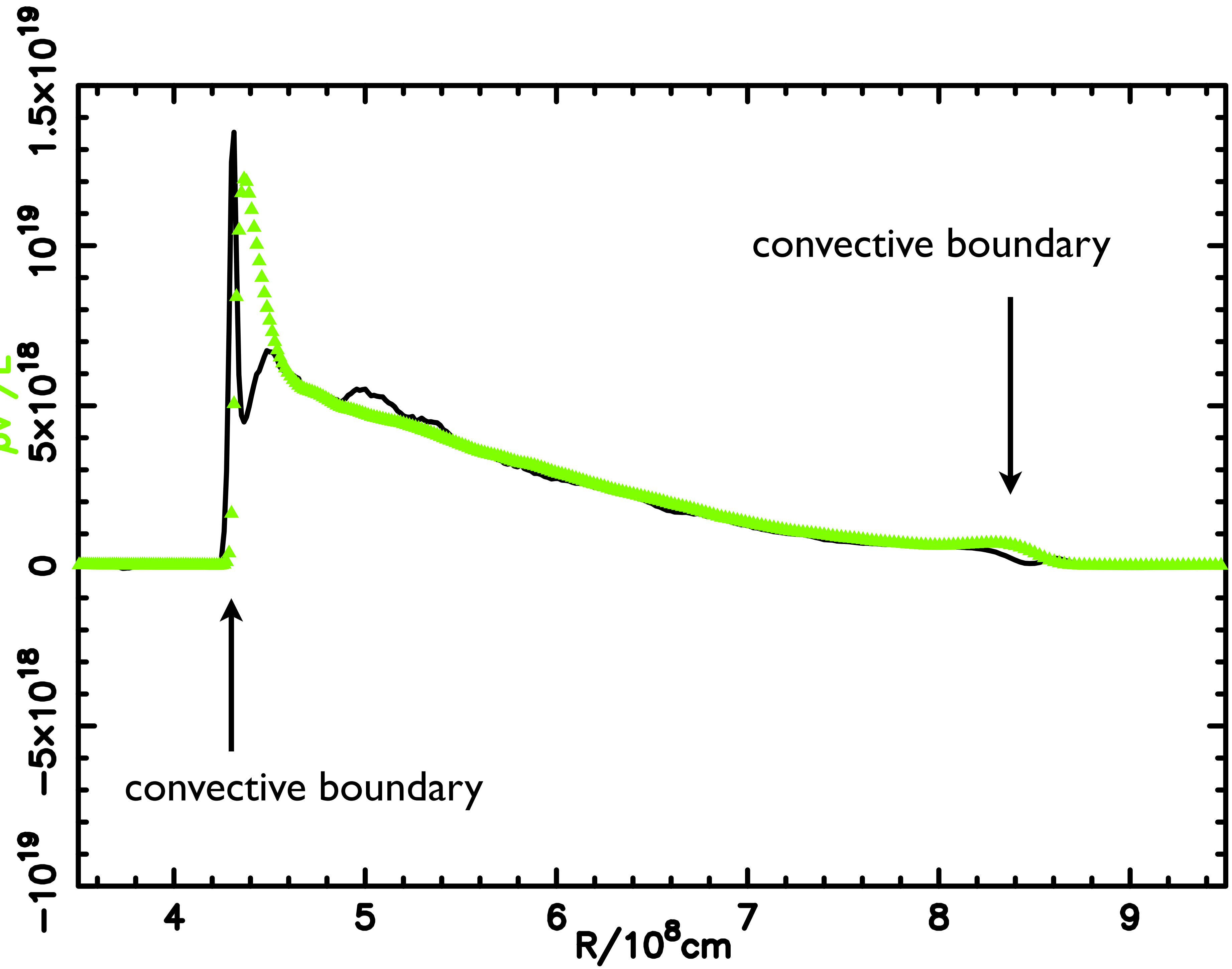
5.79

$$
\begin{aligned}
D_t\overline{\langle\rho E_K\rangle} &= -\nabla\cdot\overline{<\mathbf{F_K}>} && 25.8\\
&- \nabla\cdot\overline{<\mathbf{F_p}>} && -9.92\\
&+ \overline{<P'\nabla\cdot u'>} && 0.0152\\
&+ \overline{<\rho'\mathbf{g}\cdot\mathbf{u}'>} && 457.6\\
&- \varepsilon_K. && -467.7
\end{aligned}
$$

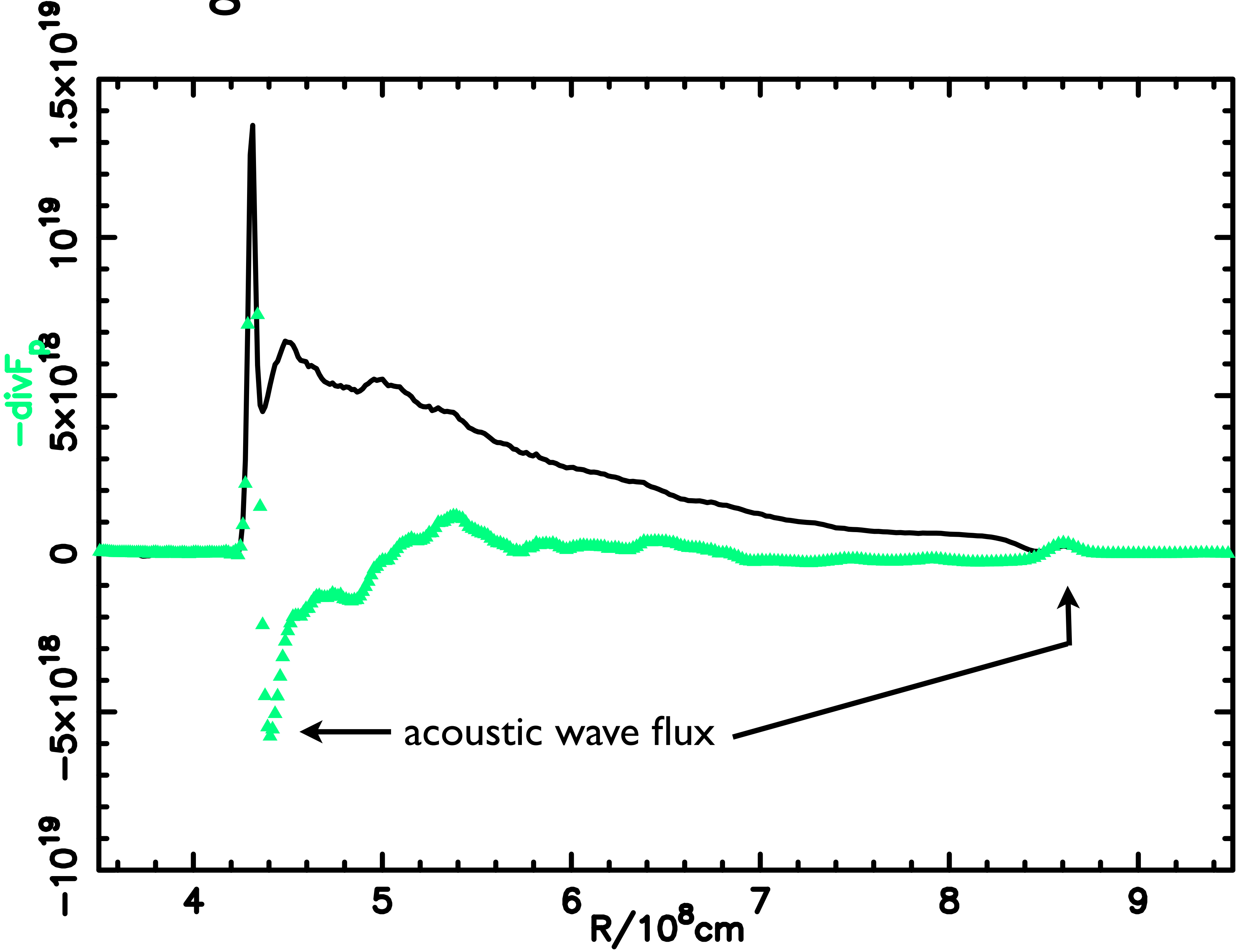
Mach Number ~0.01

Interior Convection zones have low Mach number in general

Subgrid dissipation versus Kolmogorov



Surface waves at convective boundaries generate acoustic wave flux



The role of Pressure

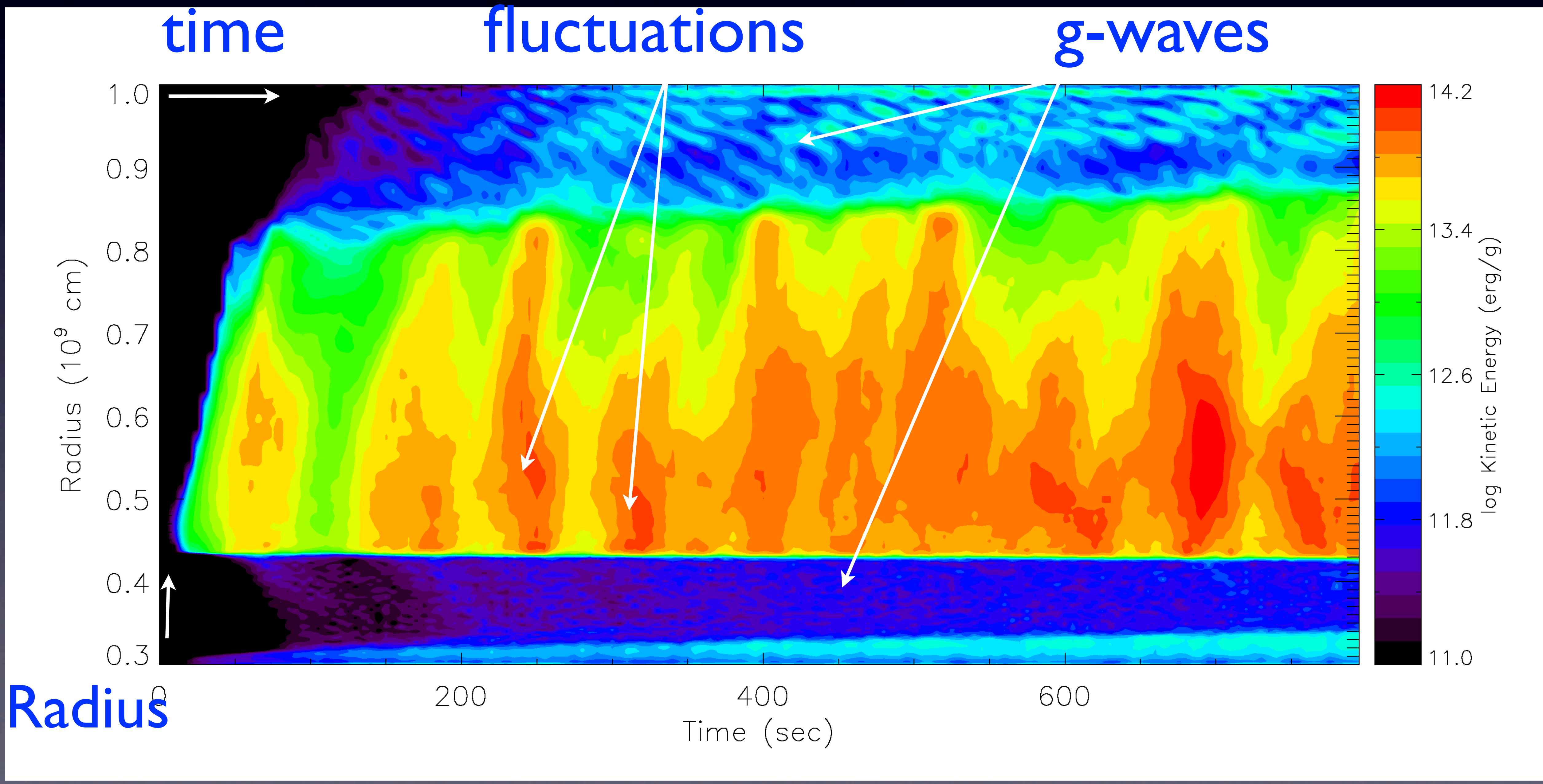
- At low Mach number, the Pressure does not affect the turbulent kinetic energy much
- BUT: it rotates the direction of the flow velocity vectors, and provides coupling of turbulence to wave motion at convective boundaries

Two velocity field approximation

- large, anisotropic, energy bearing, advection, Lorenz model and strange attractor
- small, isotropic, Kolmogorov damping at end of cascade (3D)

KE fluctuations in Oxygen burning in a shell

Meakin & Arnett, 2007, ApJ



Fluctuations in KE

Turbulence is NOT quasi-static, but has large fluctuations in kinetic energy

Simple dynamic systems are known to have chaotic behavior (Lorenz 1963), and resemble 3D simulations of turbulent fluid

(Arnett & Meakin, 2011, ApJ)

Lorenz model

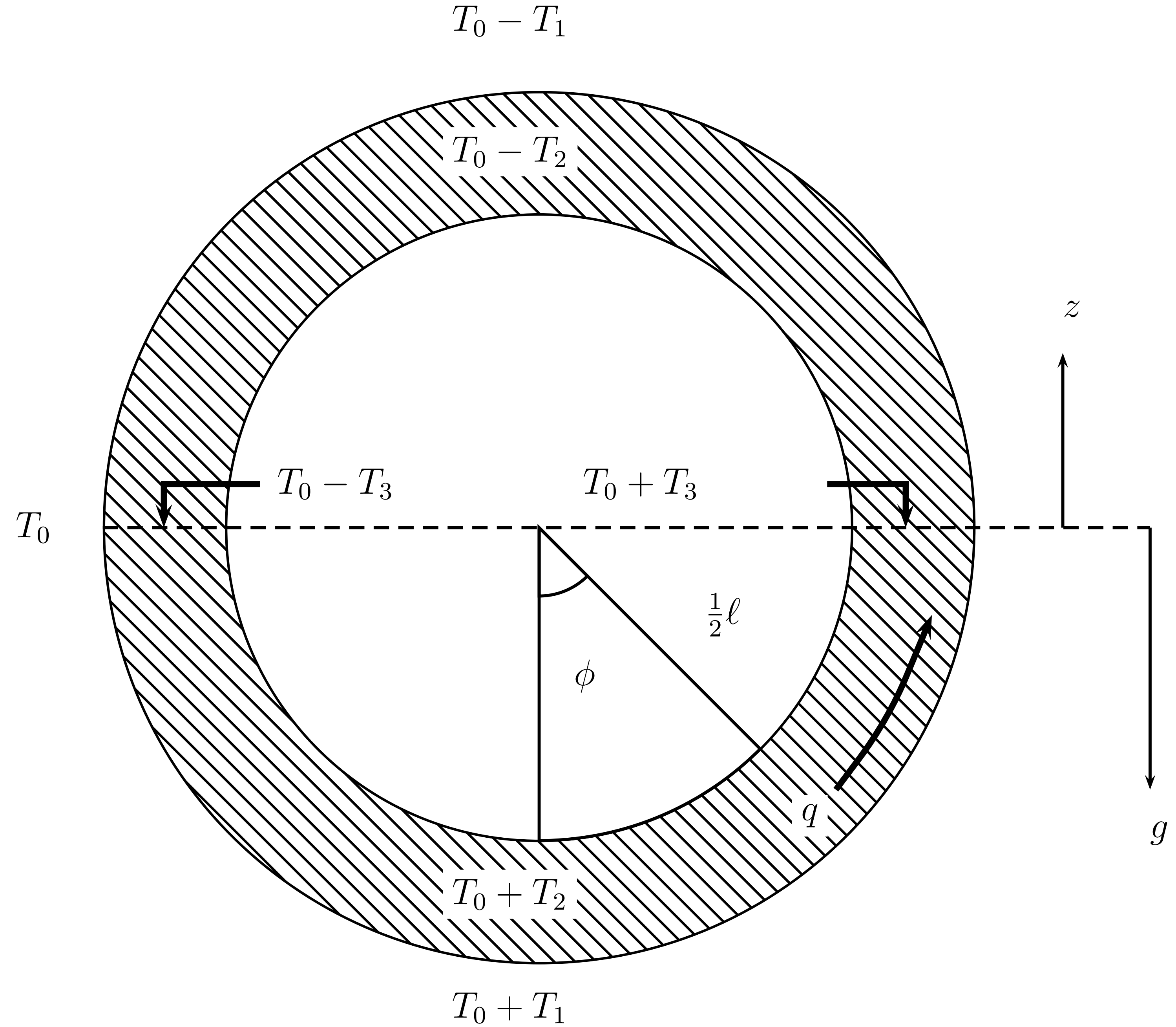


Fig. 3.— The Lorenz Model of Convection: Convection in a Loop.

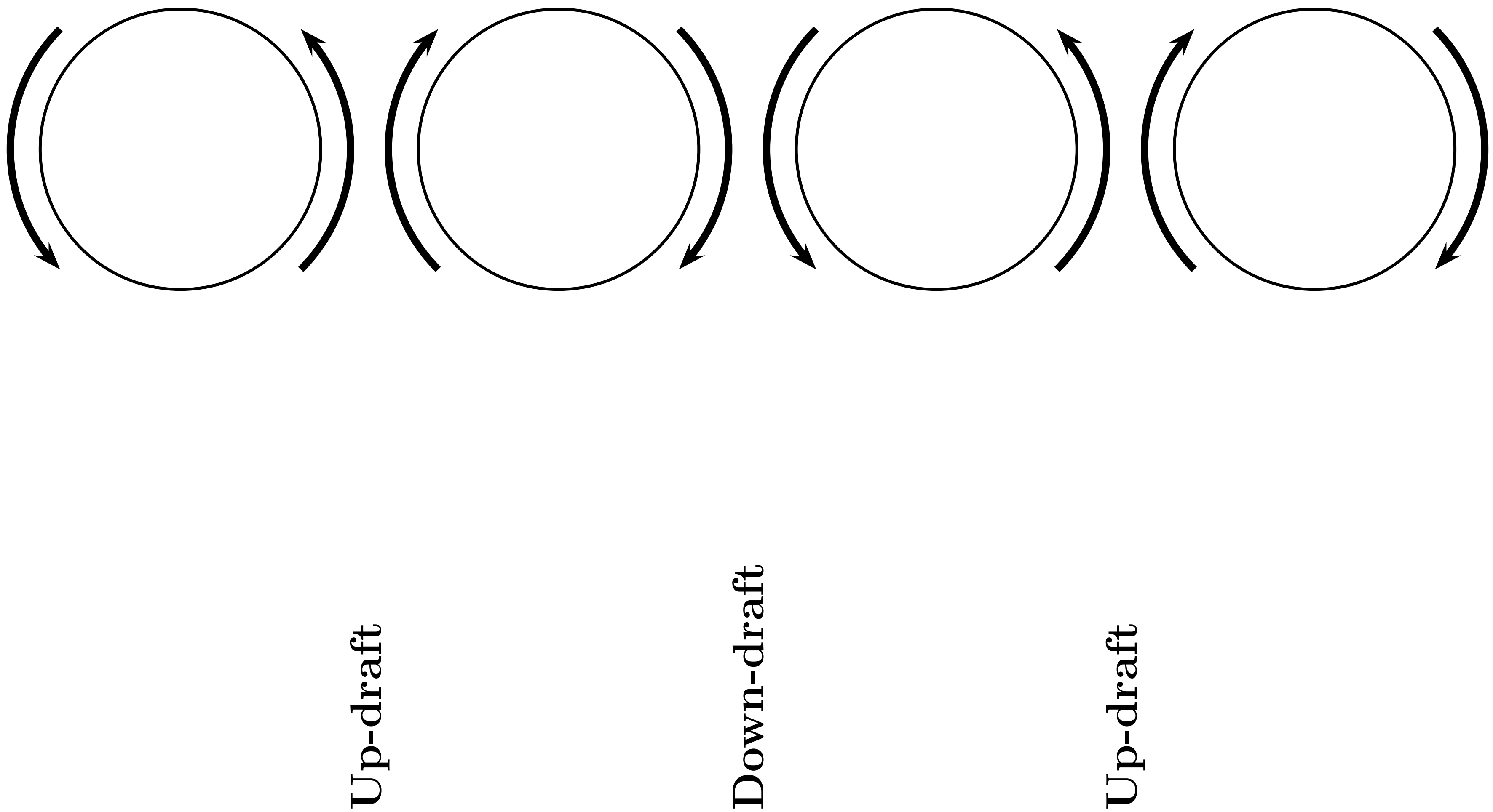


Fig. 4.— The Lorenz Model extended: Convection in a shell composed of cells. Notice the alternation of the sign of rotation. This may be thought of as a cross sectional view of infinitely long cylindrical rolls, or of a set of toroidal cells, with pairwise alternating vorticity. Each cell can exhibit random fluctuations in time.

Lorenz equations:

$$
\begin{aligned}
dX/d\tau &= -\sigma X + \sigma Y \\
dY/d\tau &= -XZ + rX - Y \\
dZ/d\tau &= XY - bZ,
\end{aligned}
$$

X: dimensionless speed

Y: dimensionless temperature difference (horizontal)

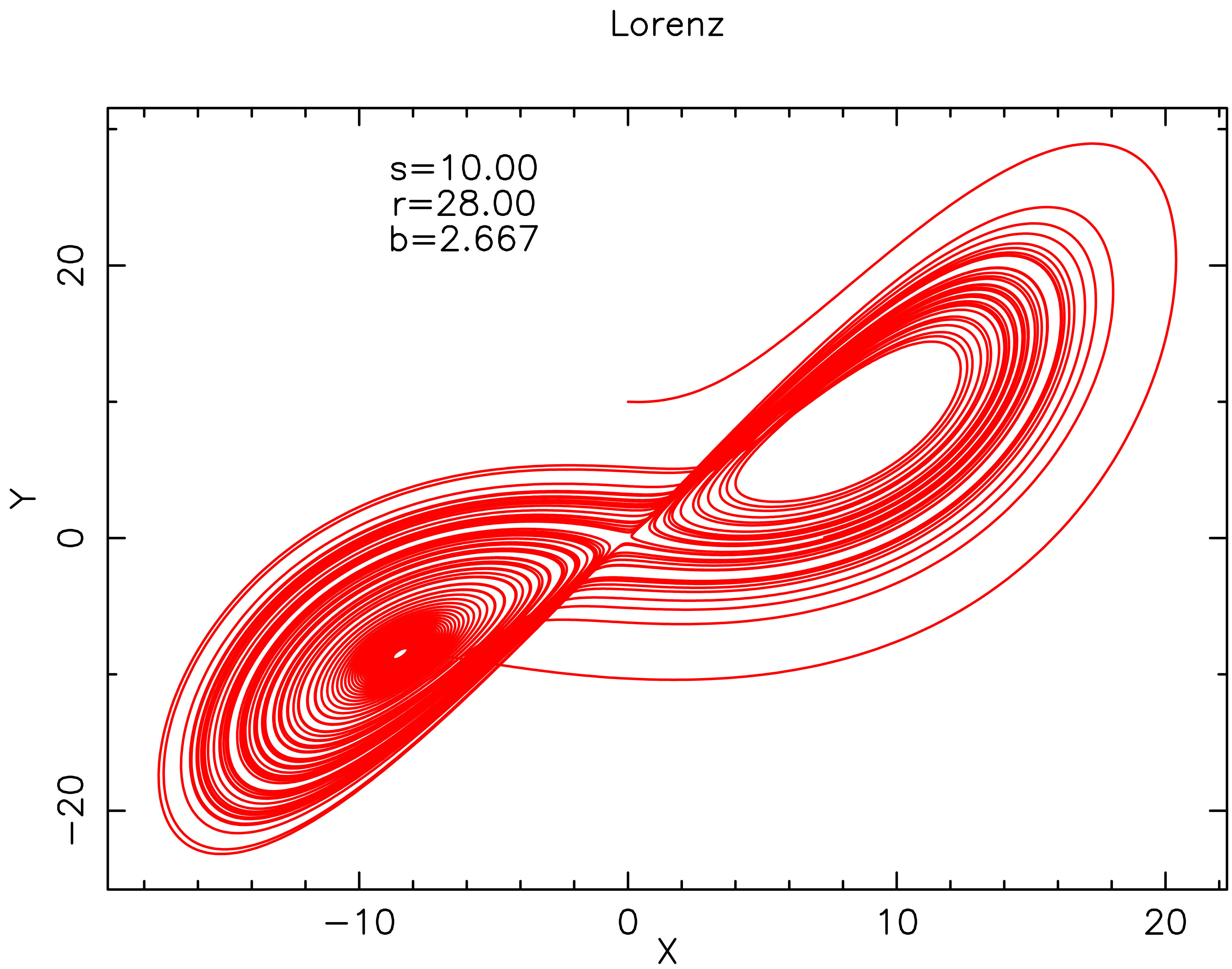
Z: dimensionless temperature difference (vertical)

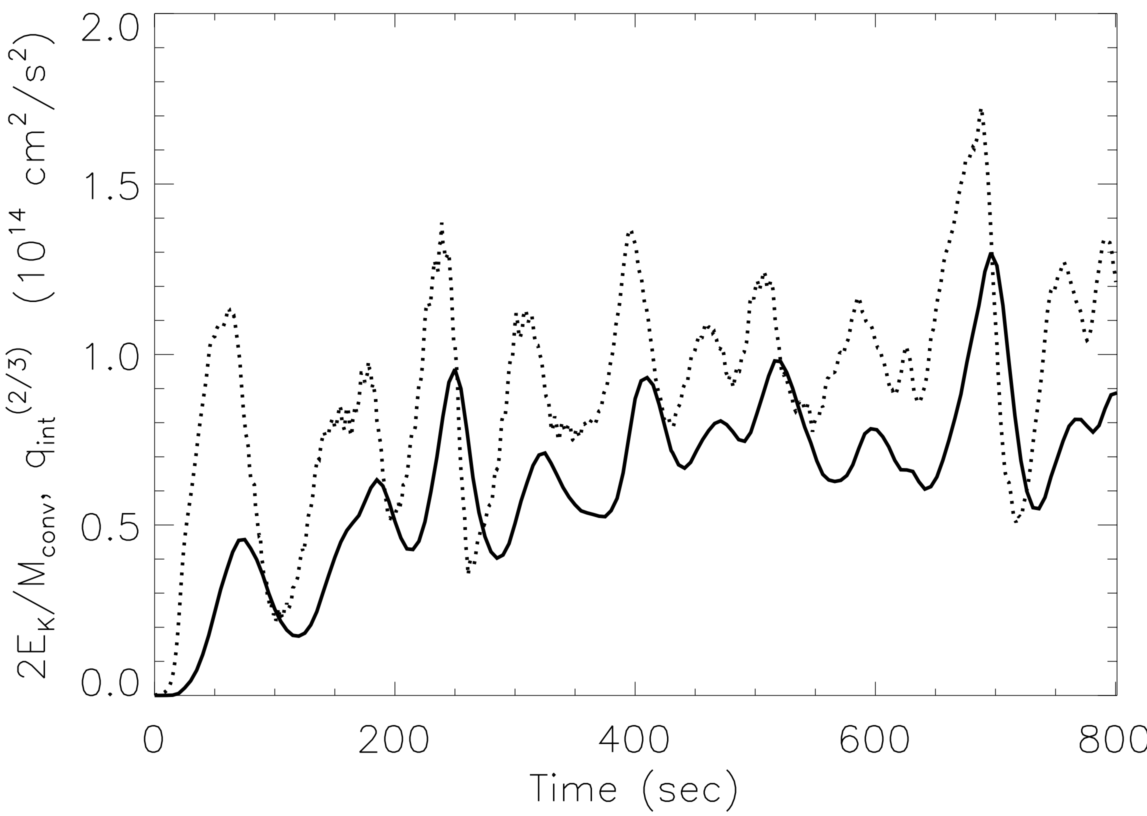
r:Rayleigh No./critical, sigma: Prandtl No.

Time in units of radiative cooling time

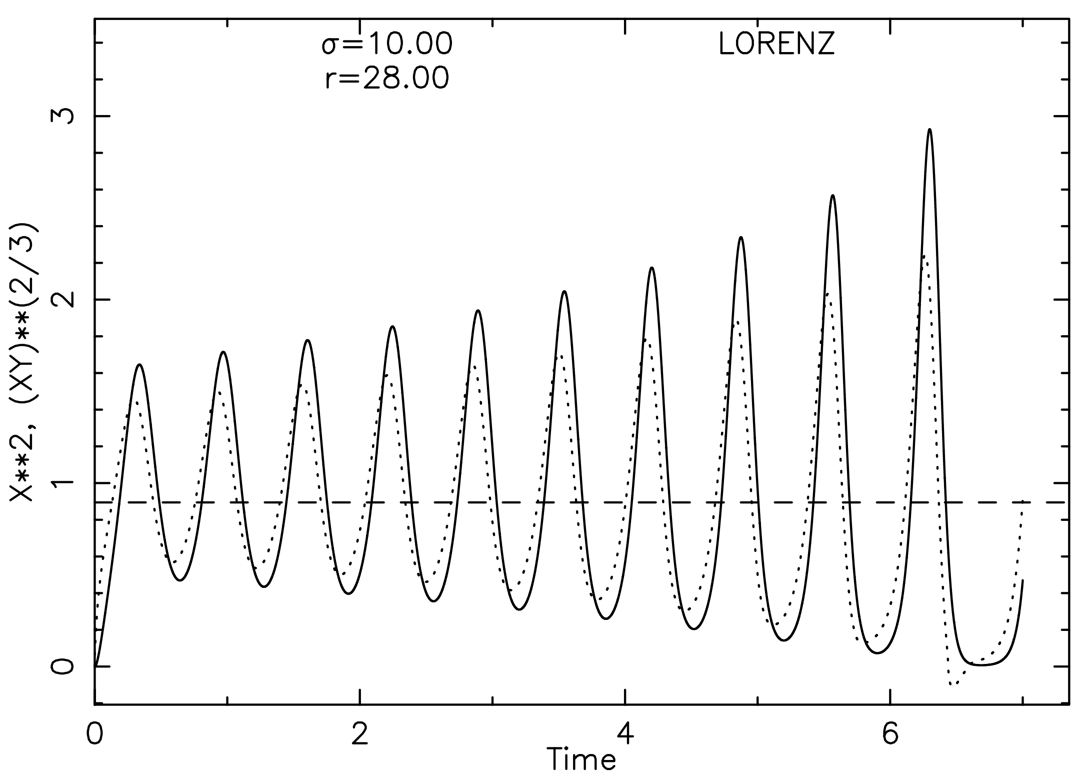
The Lorenz equations are non-linear.

Linear analysis of stars with convection will miss the nonlinear effects of turbulence.


Lorenz
s=10.00
r=28.00
b=2.667
20
0
−20
Y
−10
0
10
20
X

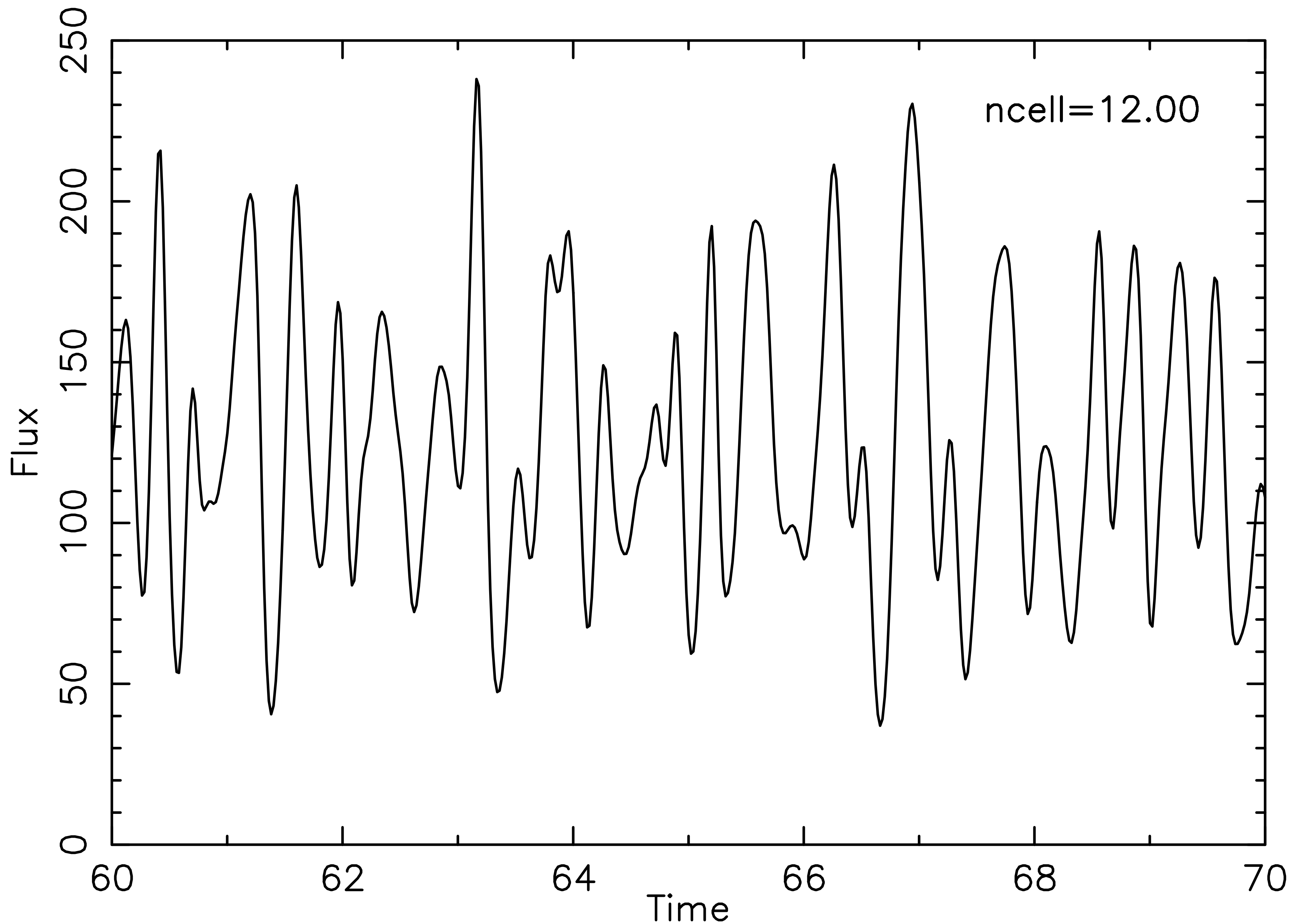


3D simulation

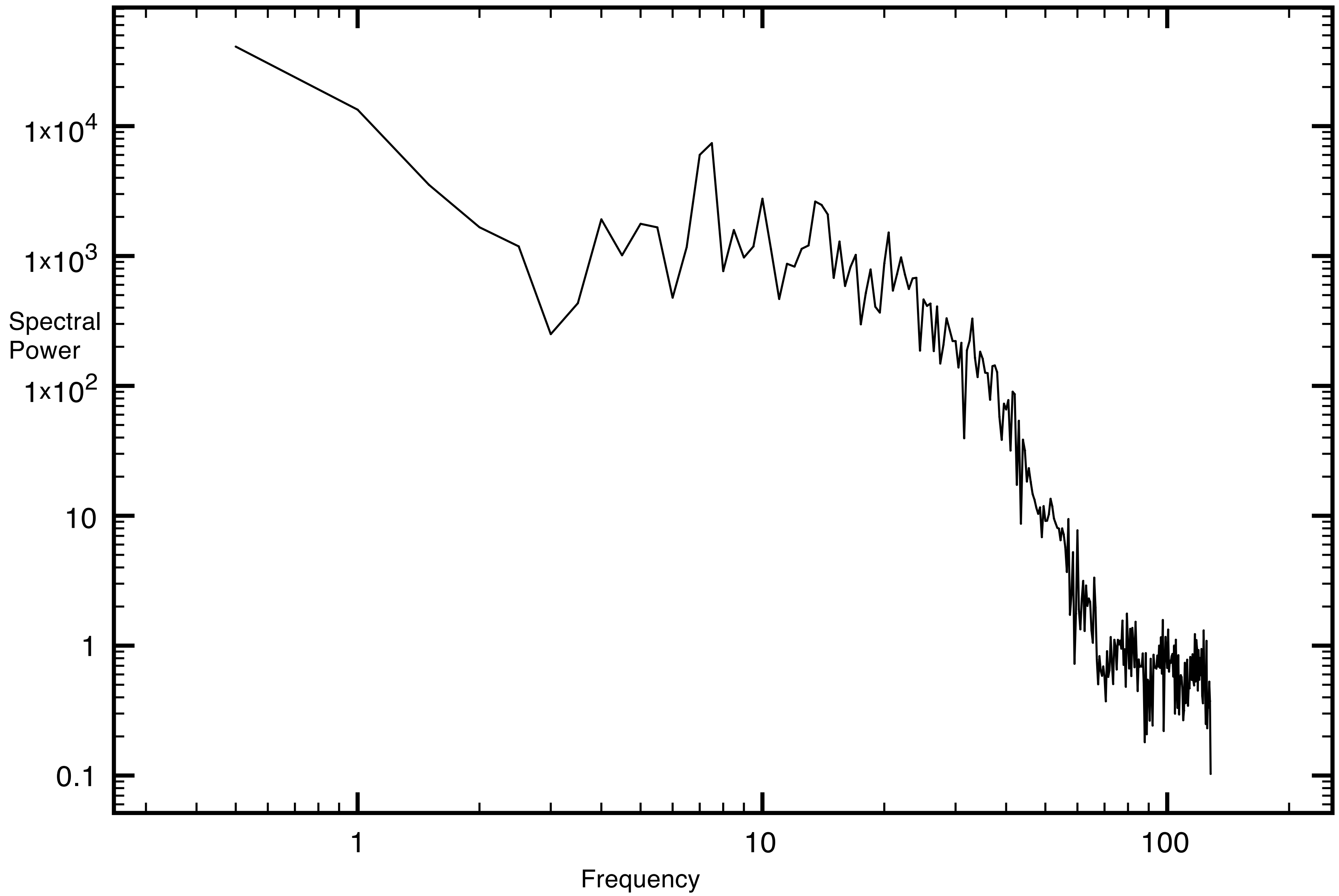


Lorenz model

Schwarzschild model using Lorenz : a fake Betelgeuse



Spectral power of Schwarzschild-Lorenz Fluctuations



we can simply modify MLT to get the correct velocity (from balance of buoyant driving and Kolmogorov damping)

MLT velocity - entropy excess relation

$$v^2 = g H_P \beta_T (\nabla - \nabla_e) \alpha^2 / 8$$

Buoyancy-Damping balance has
no free parameter alpha

$$m_{CZ} v^3 / \ell_d = \int_{CZ} g \overline{\langle u' \rho' \rangle} 4 \pi r^2 dr$$

Buoyant driving is proportional to
convective enthalpy flux

$$\overline{\langle u' \rho' / \rho \rangle} \propto \overline{\langle u' T' / T \rangle}$$