Diffusion and the Evolution of Hot Subdwarfs

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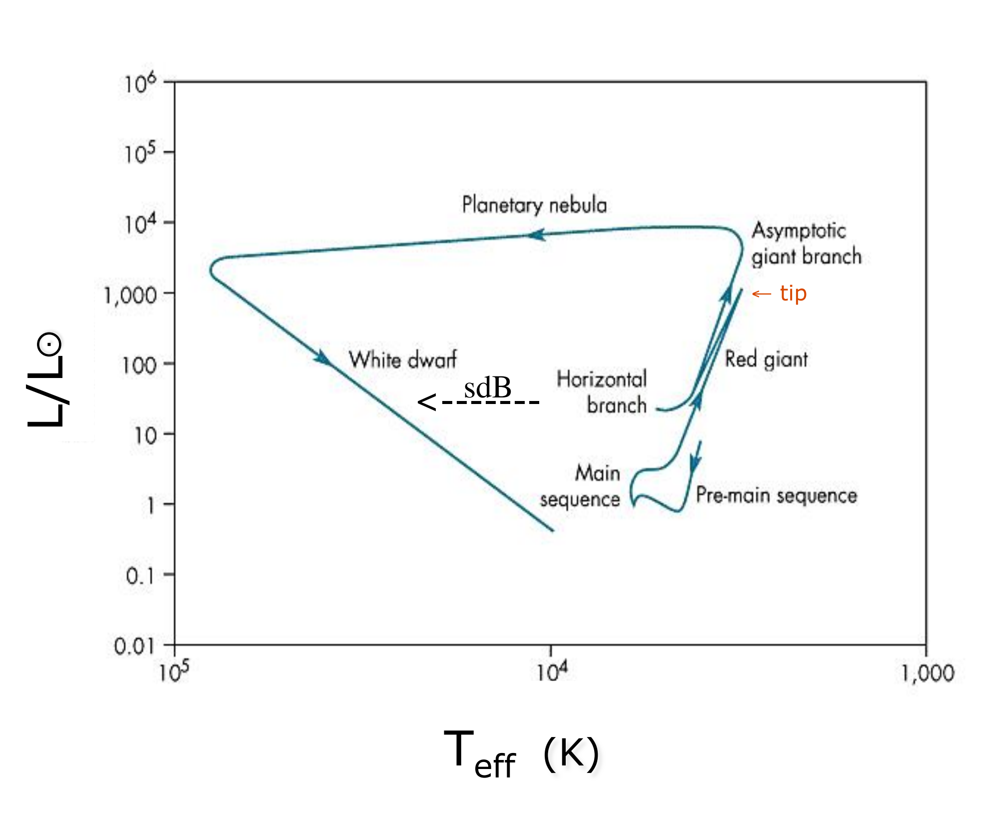
Outline

- Introduction
- Effects of diffusion in sdBs
- Methodology
  - Stellar evolution and pulsations
  - Diffusion formalism
  - Radiative accelerations
- Processes competing with diffusion:
  - mass loss
  - thermohaline mixing

What are sdB stars?

Extreme Horizontal Branch stars (Heber 1986) with:

- He-burning core ~0.5 $M_\odot$
- very thin H-envelope <0.02 $M_\odot$



sdBs/EHBs are found in:

- the field: mostly in binaries
- globular clusters: mostly single
- elliptical galaxies causing UV upturn

All populations can be explained with binary formation channels (Han et al. 2003, 2007, 2008)

Effects of diffusion: pulsations

sdB variables:

- V361 Hya stars (Kilkenny et al. 97):
  - short periods (100-250 s)
  - *p*-mode pulsations
- V1093 Her stars (Green et al. 03):
  - long periods (30 min-2 hr)
  - *g*-mode pulsations
- Both driven by opacity (κ)-mechanism acting in enhanced Iron Opacity Bump caused by atomic diffusion (Charpinet et al. 98, Fontaine et al. 03)

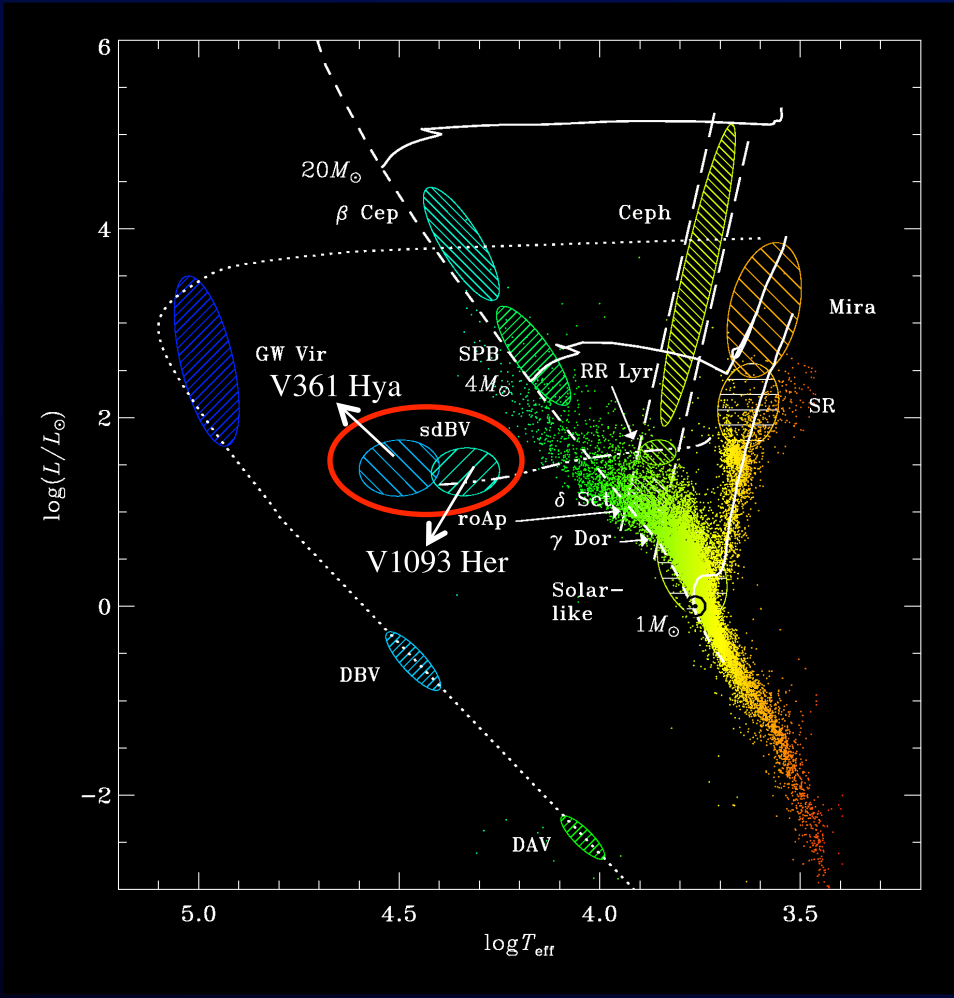


Figure from Christensen-Dalsgaard (2004)

Effects of diffusion: abundance anomalies

- sdB stars are chemically peculiar (e.g. Geier et al 2008):
  - He is subsolar, $n(\mathrm{He})/n(\mathrm{H})=10^{-4}-10^{-1}$, but more abundant than predicted by diffusion theory
  - Fe is near solar for all populations
  - Lighter metals are depleted
  - Heavier metals are enriched

  } wide spread
- abundance anomalies caused by:

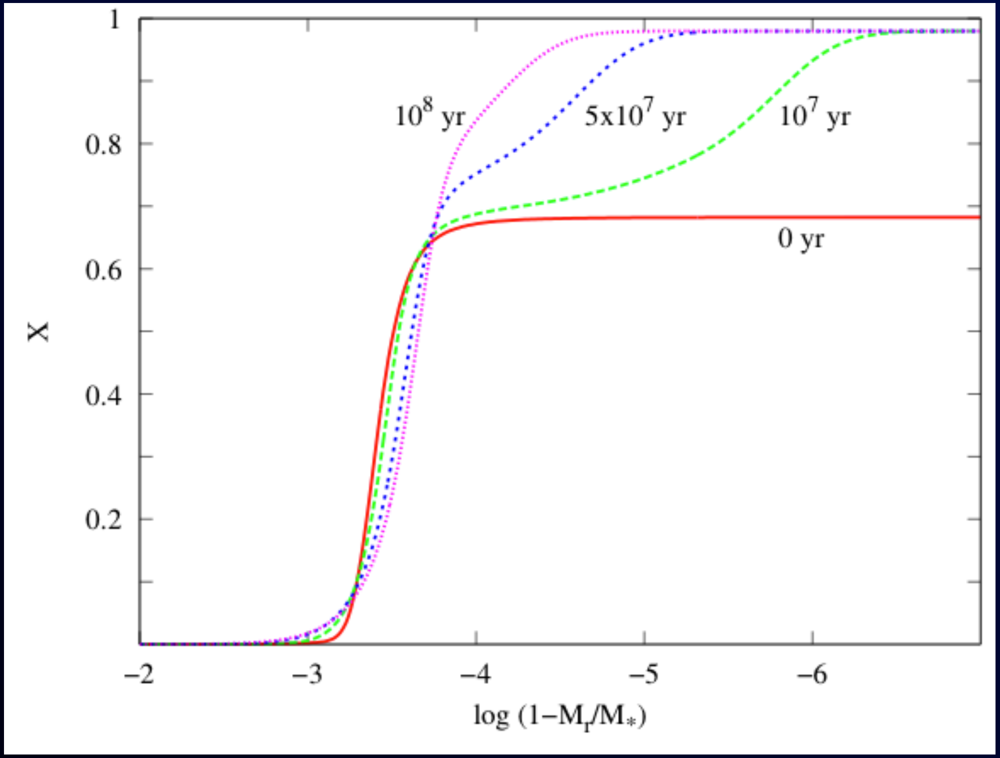
atomic diffusion + mass-loss

or

atomic diffusion + turbulence ?

Effects of diffusion: on chemical gradients

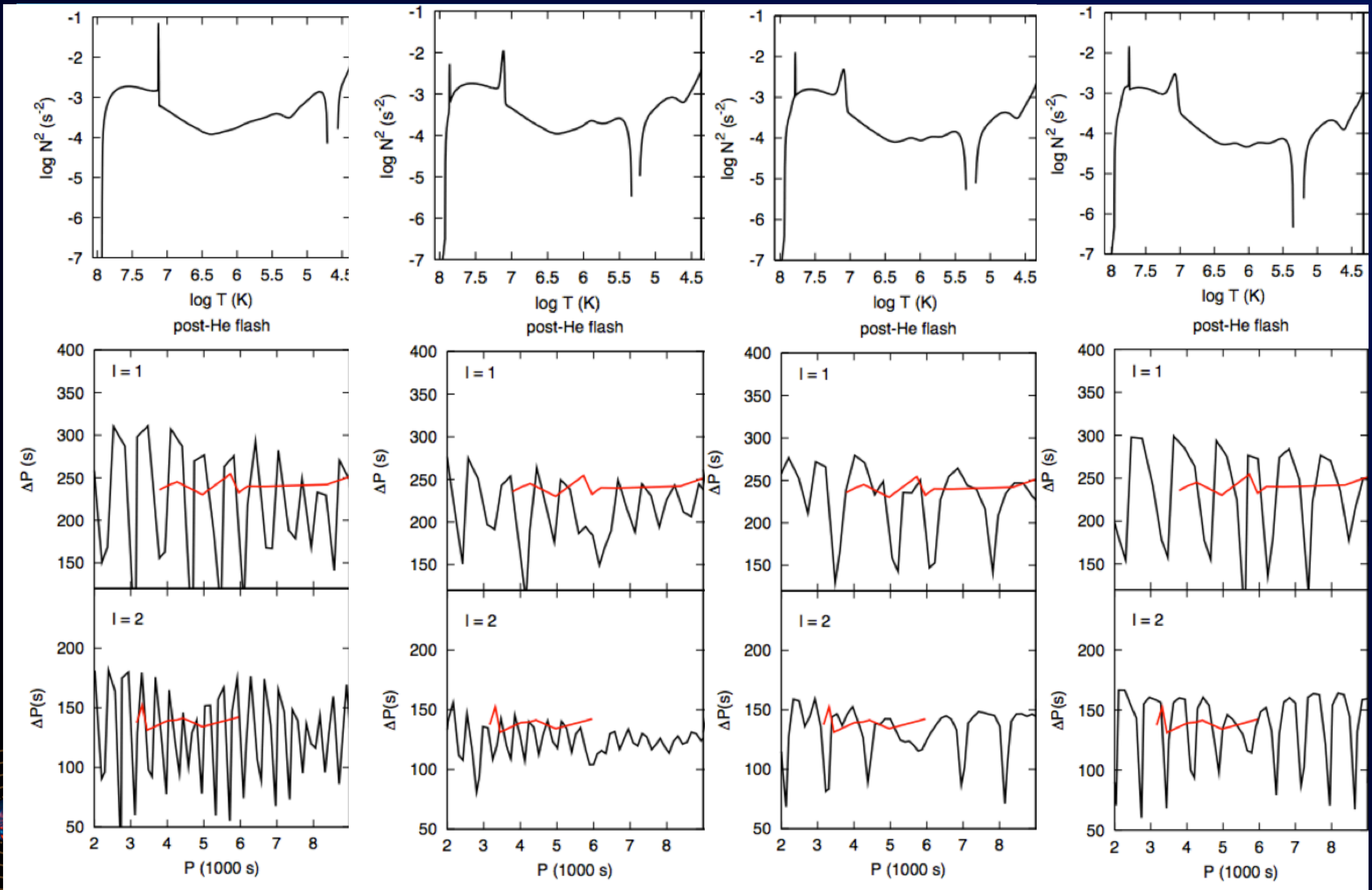
- Diffusion during evolution on EHB



Hu et al. (2010)

Effects of diffusion: on mode trapping

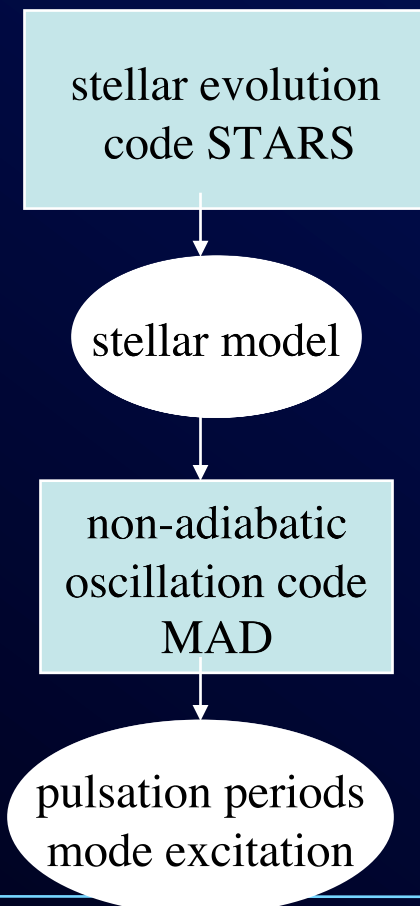
Hu et al. in prep.



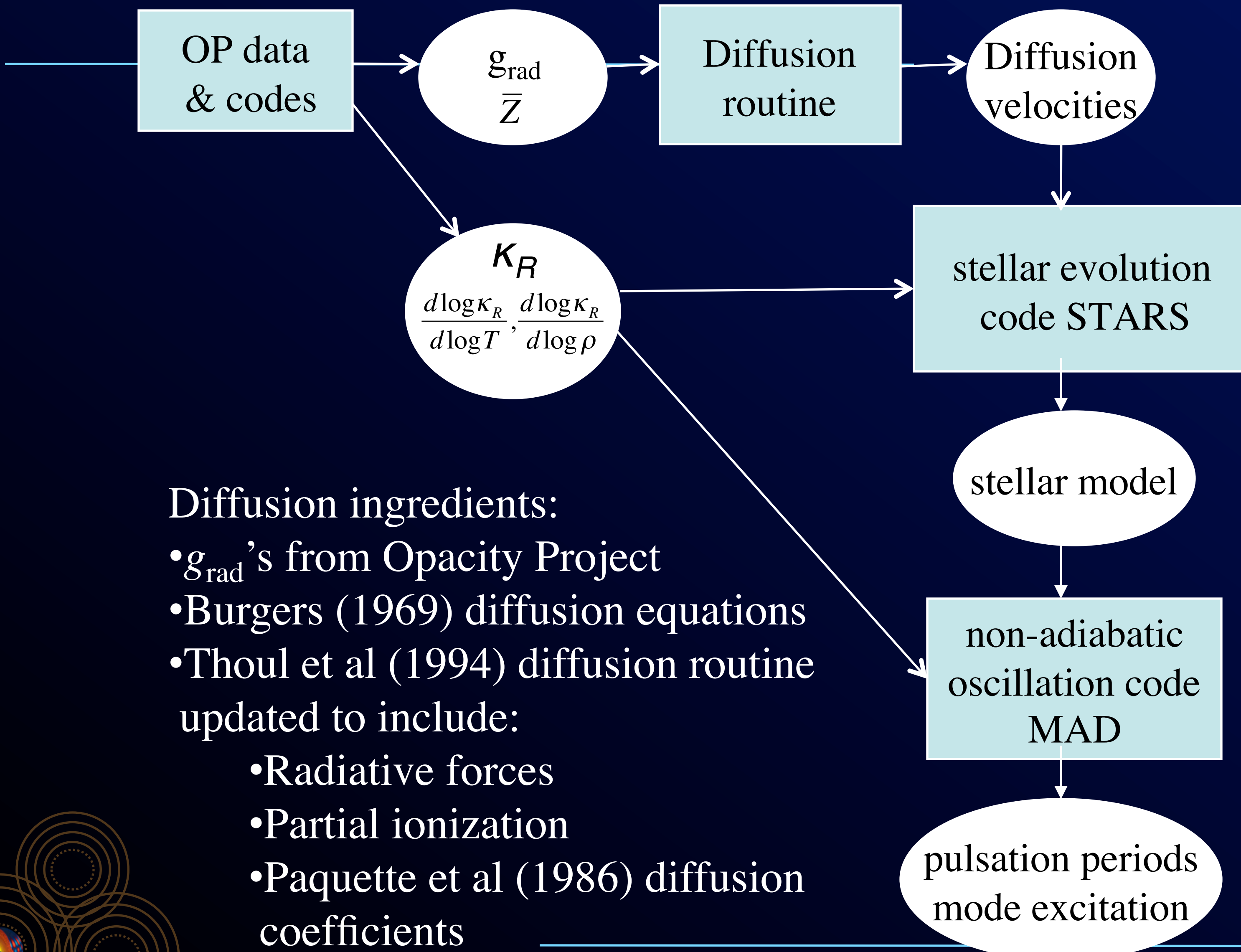
Methodology

Construct evolutionary sdB models with STARS (Eggleton 1971)

Compute non-adiabatic stellar oscillations with MAD (Dupret 2001)



Methodology



Diffusion ingredients:

- g_{rad}'s from Opacity Project
- Burgers (1969) diffusion equations
- Thoul et al (1994) diffusion routine updated to include:
    - Radiative forces
    - Partial ionization
    - Paquette et al (1986) diffusion coefficients

Methodology
diffusion formalism

- Thermal diffusion, concentration diffusion, gravitational settling, and radiative levitation
- Start with Boltzmann transport equation: $\frac{\partial f_i}{\partial t} + \vec{v}_i \cdot \frac{\partial f_i}{\partial \vec{r}_i} + \vec{F}_i \cdot \frac{\partial f_i}{\partial \vec{v}_i} = \left.\frac{\partial f_i}{\partial t}\right|_{collisions}$

  for evolution of distribution function $f_i(\mathbf{r},\mathbf{v},t)$ of species i
- Two formalisms giving approximate solutions of the Boltzmann equation:
  - Chapman-Enskog theory (Chapman&Cowling 1970)
  - Burgers theory (Burgers 1969)

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  - Burgers theory (Burgers 1969) more convenient for multicomponent fluid

Methodology
Burgers (1969)

Burgers' diffusion equations:

Resistance coefficients (K_{ij}, z_{ij}, z'_{ij}, z''_{ij}) derived from a screened Coulomb potential (Paquette et al. 1986)

$$\frac{\mathrm{d}p_i}{\mathrm{d}r} + \rho_i(g - g_{\mathrm{rad},i}) - n_i \bar{Z}_i e E =$$

$$\sum_{j \neq i}^{N} K_{ij}(w_j - w_i) + \sum_{j \neq i}^{N} K_{ij} z_{ij} \frac{m_j r_i - m_i r_j}{m_i + m_j},$$

including the heat flow equations,

$$\frac{5}{2} n_i k_{\mathrm{B}} \nabla T = \frac{5}{2} \sum_{j \neq i}^{N} z_{ij} \frac{m_j}{m_i + m_j}(w_j - w_i) - \frac{2}{5} K_{ii} z''_{ii} r_i$$

$$- \sum_{j \neq i}^{N} \frac{K_{ij}}{(m_i + m_j)^2}(3m_i^2 + m_j^2 z'_{ij} + 0.8 m_i m_j z''_{ij}) r_i$$

$$+ \sum_{j \neq i}^{N} \frac{K_{ij} m_i m_j}{(m_i + m_j)^2}(3 + z'_{ij} - 0.8 z''_{ij}) r_j.$$

In addition, we have two constraints, current neutrality,

$$\sum_i \bar{Z}_i n_i w_i = 0$$

and local mass conservation,

$$\sum_i m_i n_i w_i = 0.$$

Methodology

computation of g_{rad}'s

$$g_{rad,k} = \frac{L_{rad}(r)\kappa_R}{4\pi r^2 c}\frac{\mu}{\mu_k}\int_0^\infty \frac{\sigma_k(u)}{\sum_j f_j \sigma_j(u)} P(u)du$$

$$u = \frac{h\nu}{kT}$$

and

$$\mathcal{P}(u) = \frac{15}{4\pi^4}u^4\frac{e^u}{(e^u-1)^2}.$$

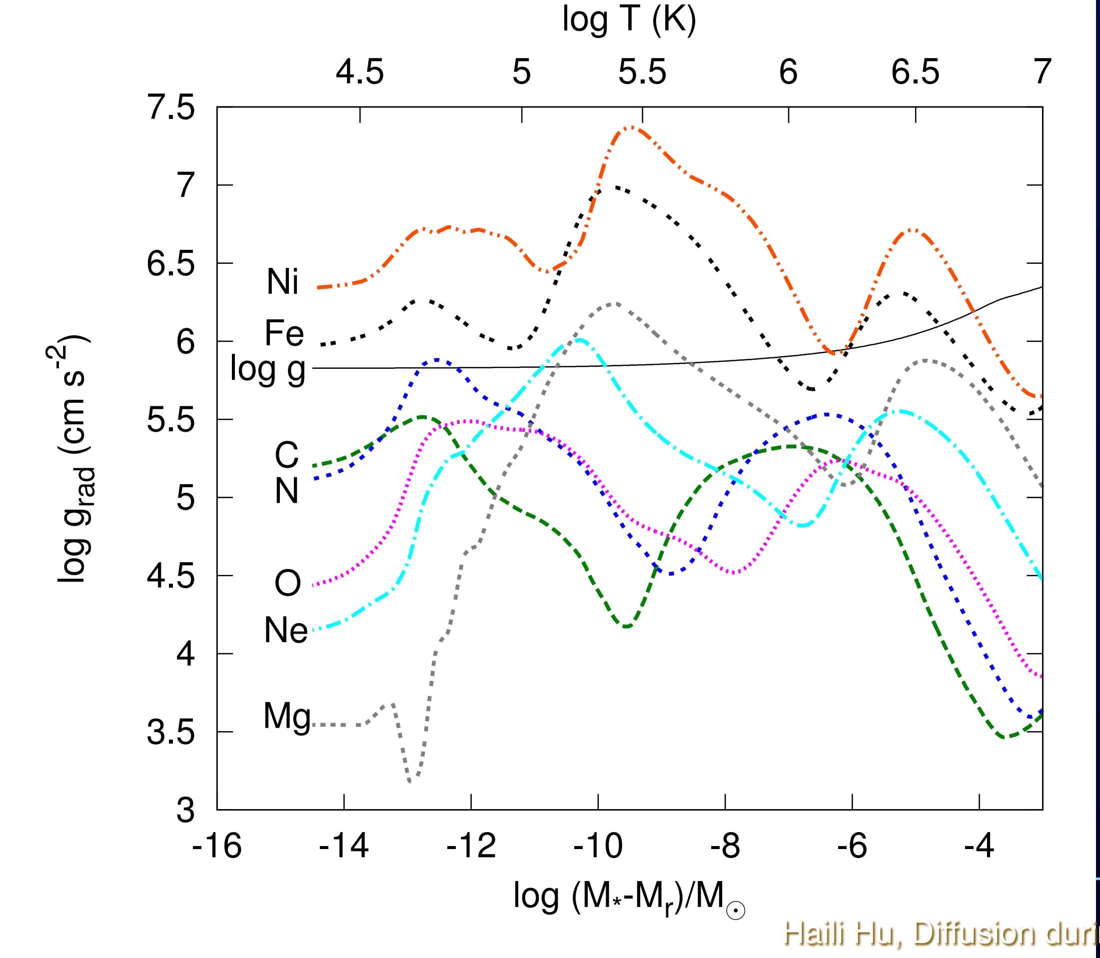
- Atomic data and codes from the Opacity Project

  (Badnell et al. 2005, Seaton 2005)
- Integration over 10 000 frequency points u
- Compute g_{rad}'s and κ_R at each meshpoint of star and each timestep of evolution, thus fully consistent taking into account all composition changes

Results

g_{rad}'s for a typical sdB model

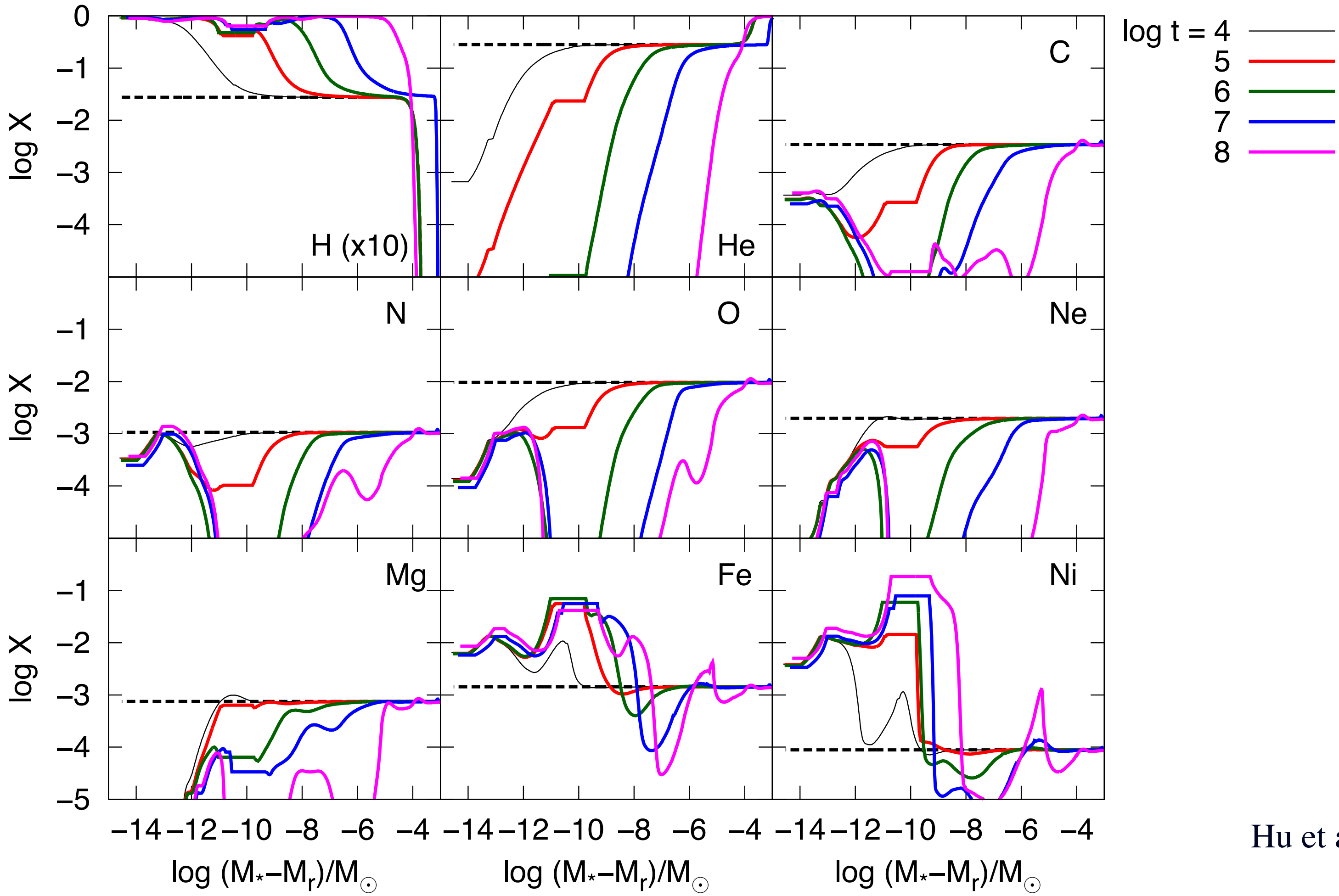
$M_* = 0.46\ M_\odot$, $M_{env} = 10^{-4}\ M_\odot$, $T_{eff} = 30{,}000$ K



Hu et al. (2011)

Results

Abundance profiles due to diffusion (no mass-loss/turbulence)



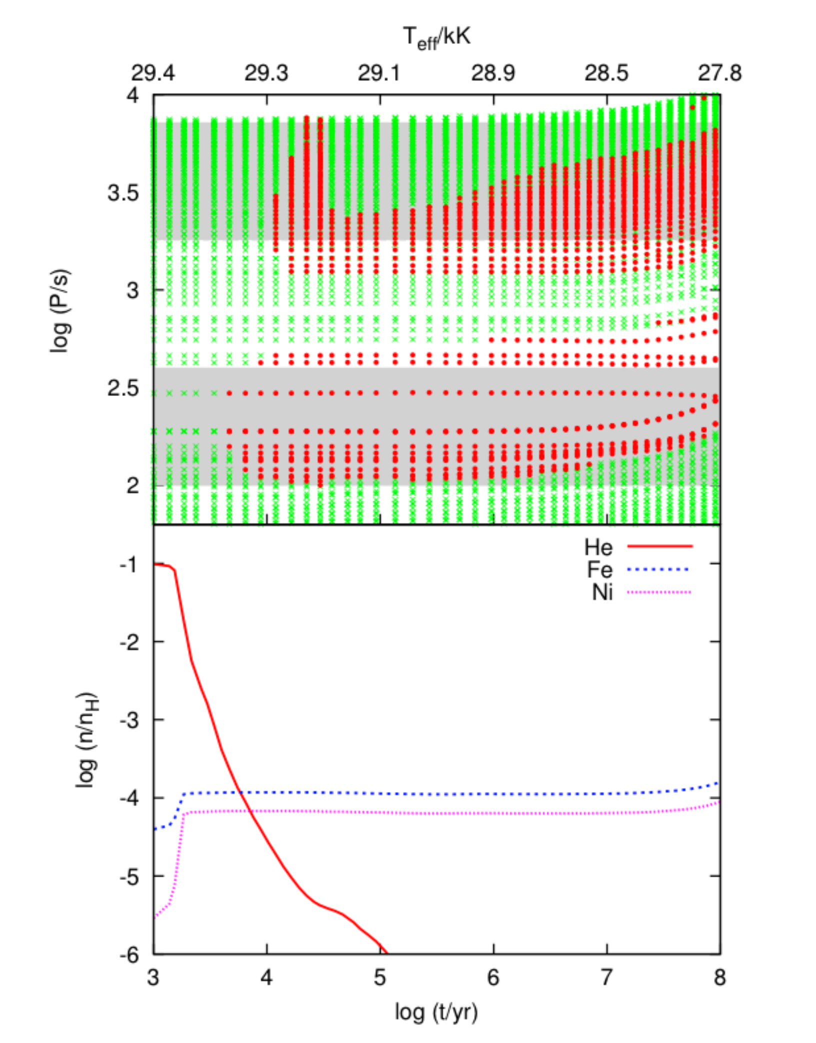
Hu et al. (2011)

Results

no mass-loss/turbulence

Pulsation periods for $l \leq 2$:

Surface abundances:



Results
no mass-loss/turbulence

Pulsation periods for $l \leq 2$:

- mode excitation for $l \leq 2$ up to T_{eff} (blue-edge) ~ 29 kK,
- period ranges of unstable modes ~consistent with observations

→ solves *g*-mode instability problem for sdB stars

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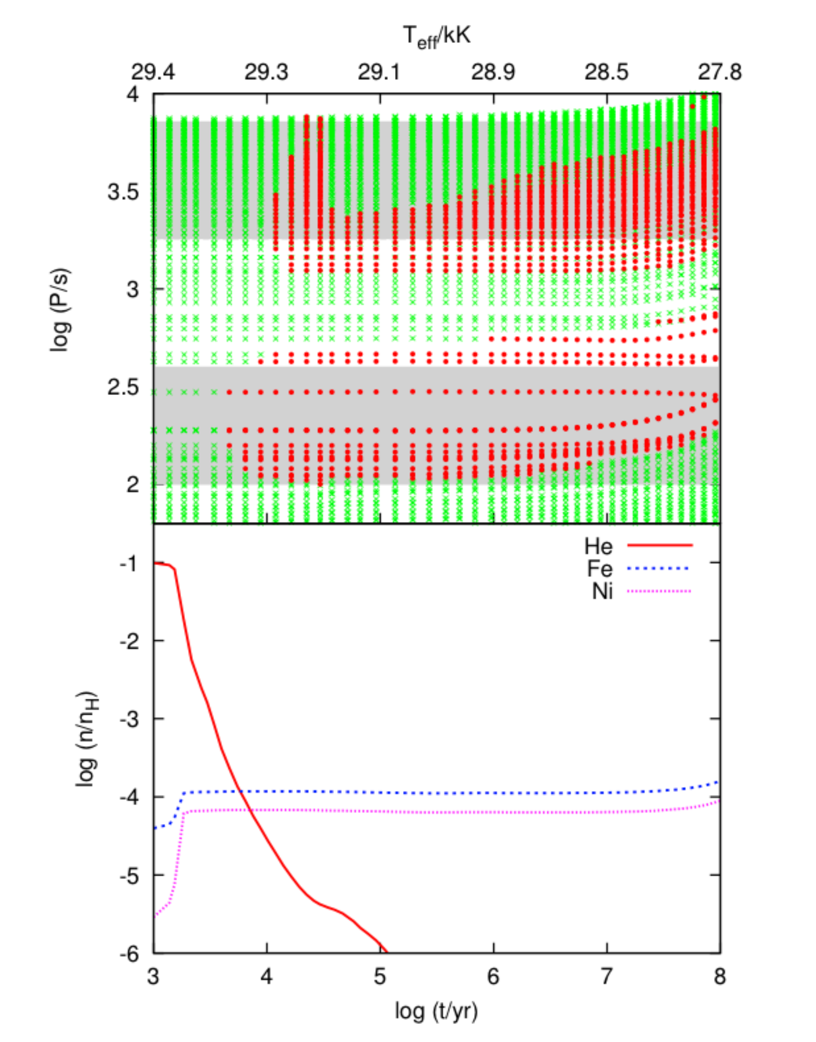

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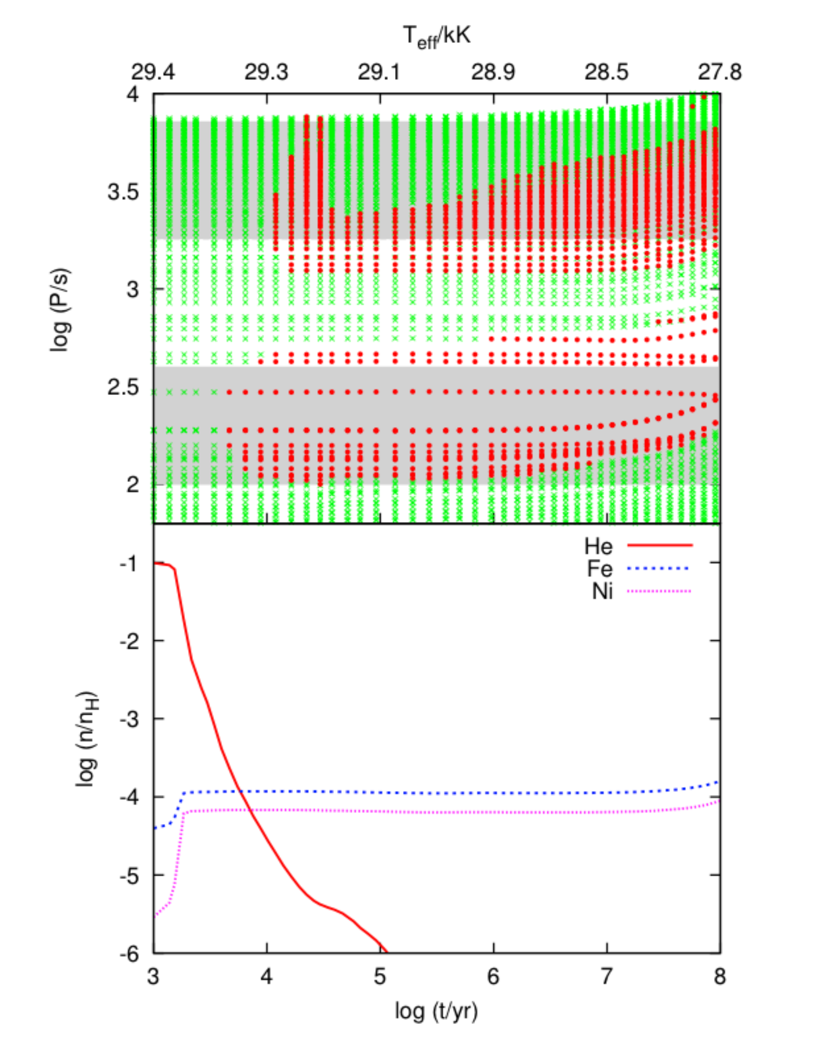
Surface abundances:

- He settles too quickly; below 10^{-4} within 0.01% of EHB lifetime



Processes competing with diffusion:

Mass loss (Fontaine&Chayer 1997, Unglaub&Bues 2001)

or

turbulent mixing (Michaud et al 2001)?

Results
mass-loss

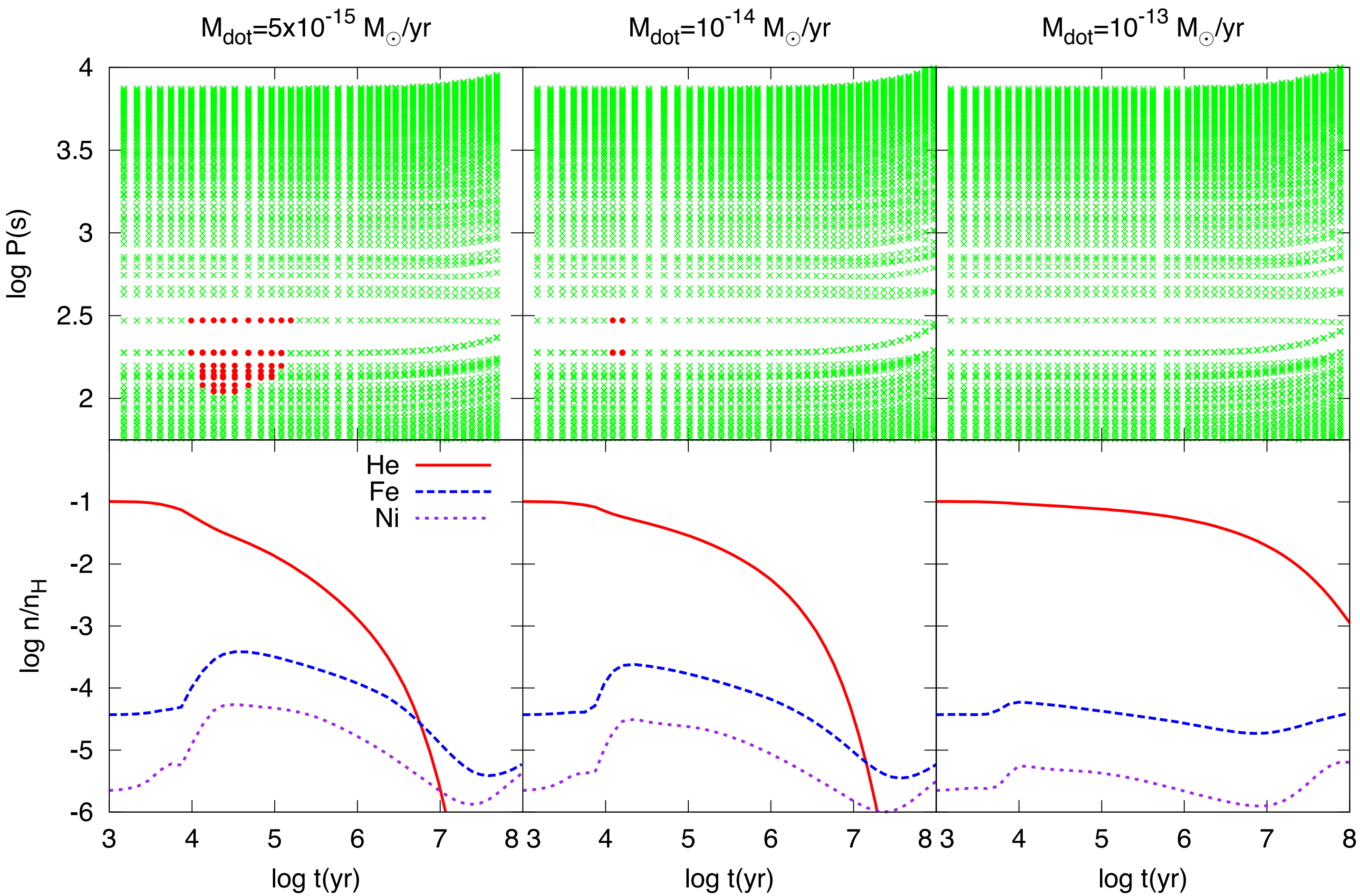
Estimation:

- Takes ~10^4 yr to build up Fe/Ni reservoir
- Reservoir goes down to $M_* - M_r \approx 10^{-10}\,M_\odot$

→ No driving of pulsations if in 10^4 yr more than $10^{-10}\,M_\odot$ is removed, i.e. if $M_{dot} > 10^{-14}\,M_\odot$/yr

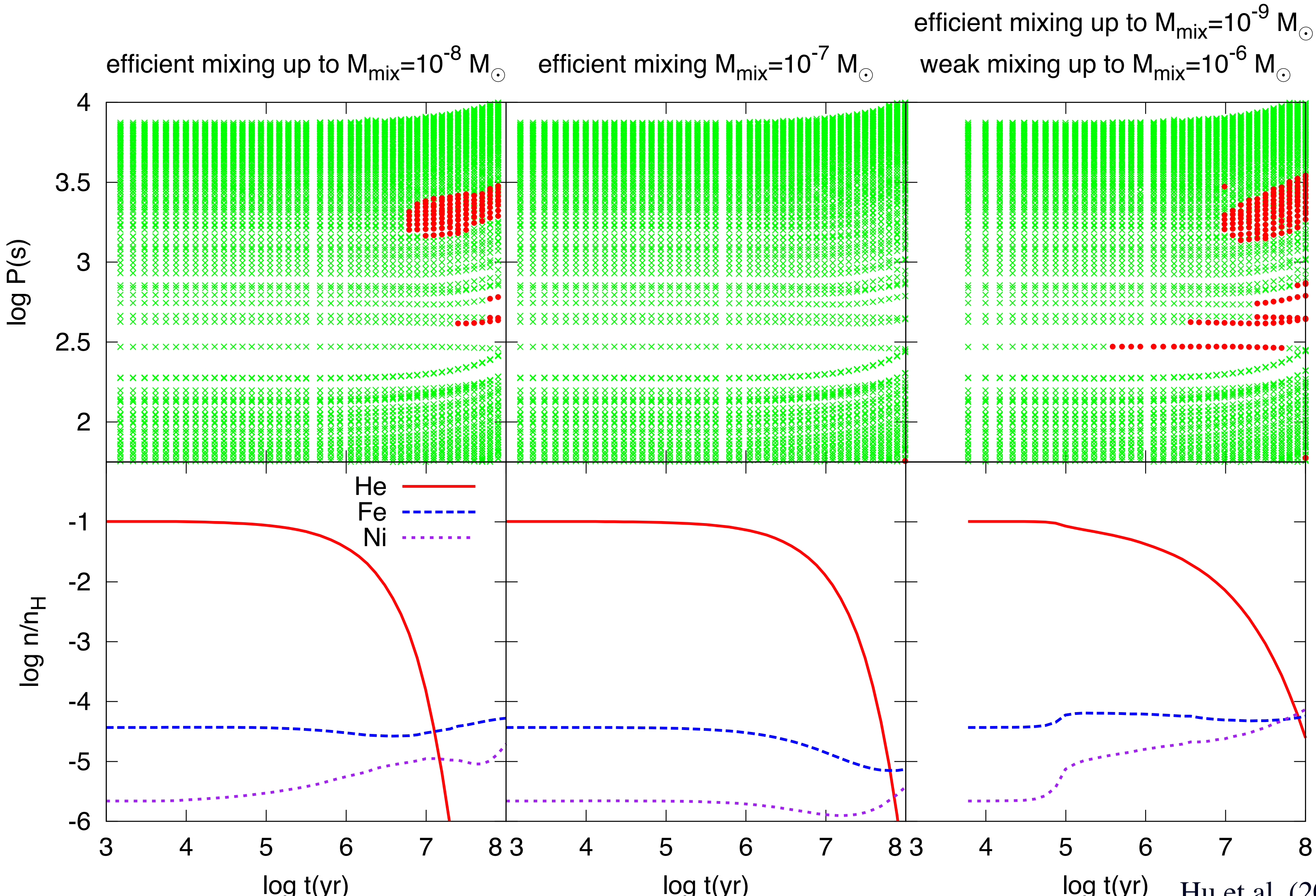
Results

mass-loss



Hu et al. (2011)

Results
turbulence



Hu et al. (2011)

What causes turbulent mixing?

- Thermohaline convection (or rotation, convective overshoot, …)
- In 1D stellar evolution, thermohaline mxing is described as a diffusion process
- What is the correct diffusion coefficient?
  - Ulrich (1972) uses finger geometry
  - Kippenhahn et al (1980) uses blob geometry
  - 3d-hydro simulations (Traxler et al 2011) agree better with Kippenhahn
- We follow Kippenhahn et al (1980) but including g_{rad}'s.

Kippenhahn (1974)

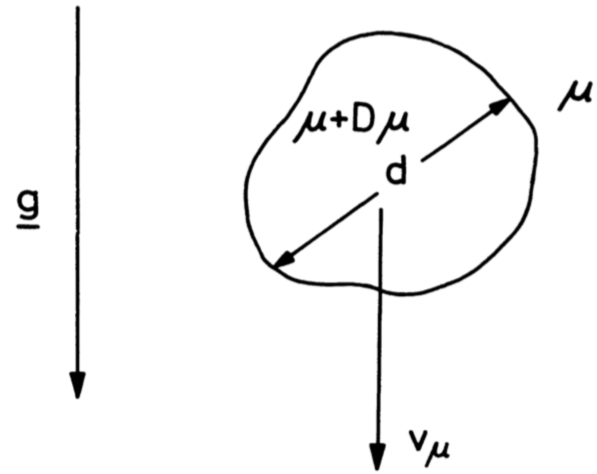


Fig. 1. A blob of molecular weight $\mu + D\mu$ ($D\mu > 0$) in surroundings with molecular weight μ in hydrostatic equilibrium undergoes a slow downward motion which is controlled by the thermal adjustment of the blob.

Assuming hydrostatic equilibrium between the blob and its surroundings, we have $\mathrm{D}P = 0$ and consequently,

$$\frac{\mathrm{D}\rho}{\rho} = -\delta \frac{\mathrm{D}T}{T} + \phi \frac{\mathrm{D}\mu}{\mu}. \tag{A3}$$

If there were no radiative forces, the blob would rise or sink until there is no buoyancy force, i.e. $\mathrm{D}\rho = 0$. However, in the presence of radiative forces, the blob rises or sinks until the net force per unit mass on the blob is the same as the net force per unit mass on the surroundings, i.e.

$$\sum_j \rho_{1,j}(g - g_{\mathrm{rad1},j}) = \sum_j \rho_{0,j}(g - g_{\mathrm{rad0},j})$$

$$D_{g_{\rm rad}} = \frac{4acT^3 H_{\rm P}}{c_{\rm P}\kappa\rho^2(\nabla_{\rm ad}-\nabla)} \left| \frac{\phi}{(\delta+B)} \frac{{\rm d}\mu}{{\rm d}r} \frac{1}{\mu} - \frac{A}{(\delta+B)} \right|$$

with

$$B = \frac{\sum_j X_j \frac{\partial g_{{\rm rad},j}}{\partial \ln T}}{\sum_j X_j (g - g_{{\rm rad},j} - \frac{\partial g_{{\rm rad},j}}{\partial \ln \rho})} \qquad A = \frac{\sum_j \frac{\partial X_j}{\partial r}(g_{{\rm rad},j} + X_j \frac{\partial g_{{\rm rad},j}}{\partial X_j})}{\sum_j X_j (g - g_{{\rm rad},j} - \frac{\partial g_{{\rm rad},j}}{\partial \ln \rho})}$$

Hu et al. in prep.

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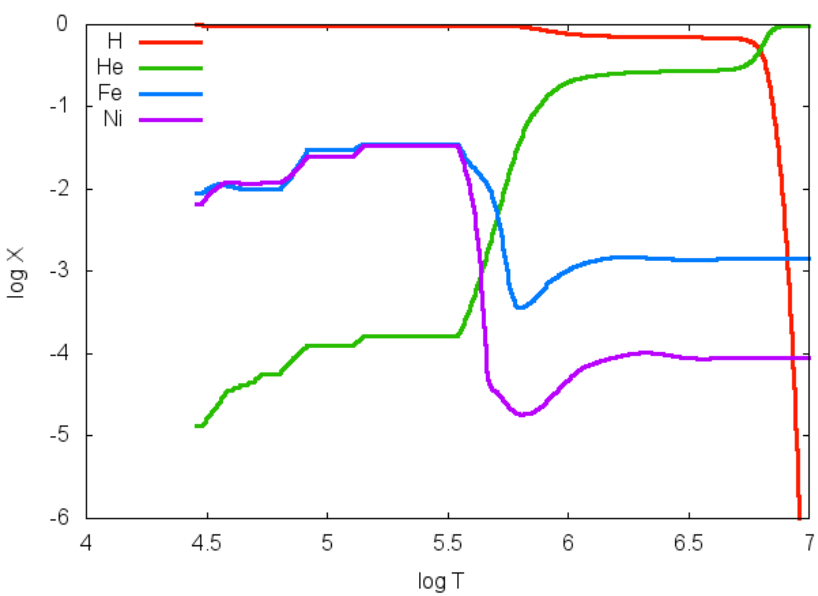
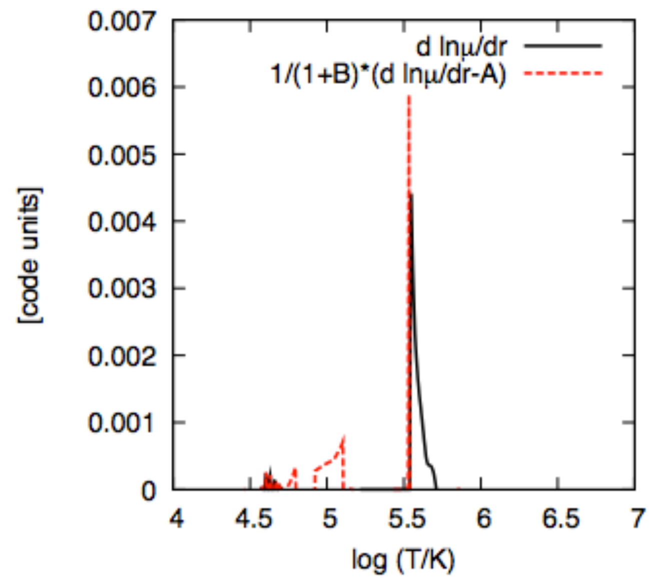
age ≈ 10^6 yr on EHB





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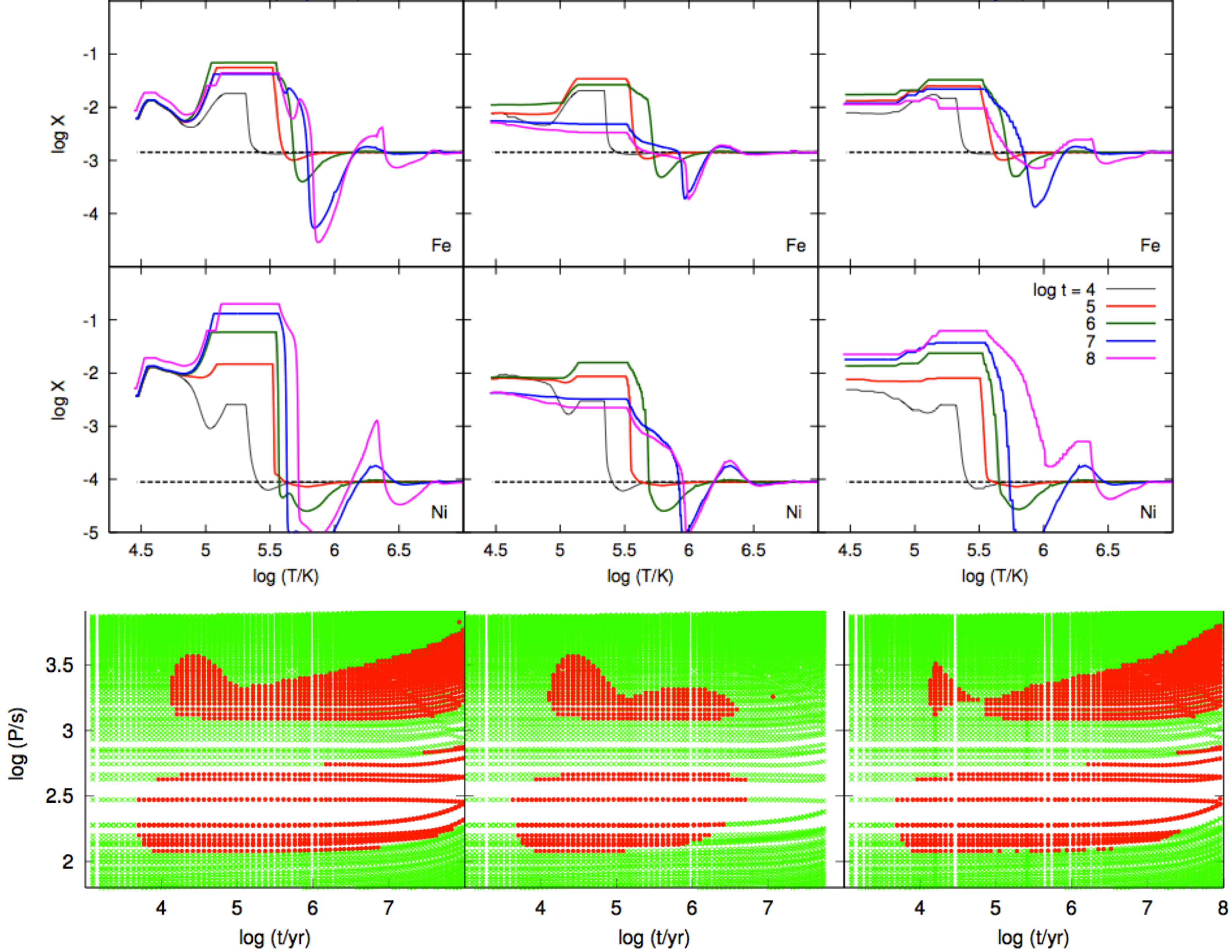
log X
Fe
Ni
log t = 4
5
6
7
8
log (T/K)
log (P/s)
log (t/yr)

Conclusions

- Including diffusion in sdB models is essential
- However, much is unknown about competing processes, i.e. mass loss or turbulence
- Observations of pulsations give strong constraints on these poorly understood processes