Mixing in stars

Macrophysics of stellar interiors

Jean-Paul Zahn
Observatoire de Paris

The Impact of Asteroseismology
across Stellar Astrophysics

KITP, Santa Barbara, Oct. 24 - 28, 2011

Modelling stellar interiors in the 20th century

- 50’s: first realistic models Schwarzschild criterion, MLT
  [Boehm-Vitense, Schwarzschild & coll.]
- 60’s: evolutionary sequences
  [Cameron & Ezer, Hofmeister, Kippenhahn & Weigert, Iben]
- 70’s: refinements - introduction of overshoot
  [Maeder, Roxburgh, etc.]
- 80’s: first attempts to introduce mixing in radiation zones
  [Maeder & Schatzman, Sofia & coll., JPZ, etc.]
- 90’s: implementation of microscopic diffusion
  + gravitational settling + radiative acceleration
  [Michaud, Vauclairs, & coll.]

→ **PRESENT STANDARD MODEL - NO MIXING IN RAD. ZONE**

Evidence of “extra” mixing in radiation zones

He and N excess at surface of massive stars and supergiants

Abundance anomalies at surface of red giants ($^{12}C/^{13}C$)

Li depletion in solar-type stars

Existence of “normal” stars

Reduced abundance anomalies at surface of ‘tepid’ stars

Consequences

Mixing increases life-time of stars

Determines later stages of evolution

Rules chemical evolution of Galaxy / galaxies

How to treat this extra mixing

Numerical approach

Solve directly the 3D hydrodynamic equations

- Extremely time/resources consuming
- Global simulations: cannot grasp all relevant scales
- Applicable to small domains: to study local instabilities
  $\rightarrow$ prescriptions for transport
- Serious problem with shear flows: resolution still insufficient
  to reach threshold for nonlinear instabilities

How to treat this extra mixing

Parametric approach:

Assume that mixing and transport of AM are due to diffusion
Introduce parametrized turbulent diffusivities
Adjust parameters to fit observations

Physical approach:

Strive to implement the physical processes
that are likely to cause mixing:

- meridional circulation induced by torques (wind, accretion, etc.) and structural changes
- turbulence produced by instabilities (shear, magnetic, thermohaline,etc.)

Mixing processes in radiation zones

Meridional circulation

Classical picture: circulation is due to thermal imbalance
caused by perturbing force (centrifugal, magn. field, etc.)

Eddington (1925), Vogt (1925), Sweet (1950), Mestel (1950) etc.

Eddington-Sweet time $$t_{ES} = t_{KH} \frac{GM}{\Omega^2 R^3}, \quad \text{with} \quad t_{KH} = \frac{GM^2}{RL}$$

Revised picture: after a transient phase of about t_{ES},
circulation is driven by the loss (or gain) of angular momentum
and by structural changes due to evolution

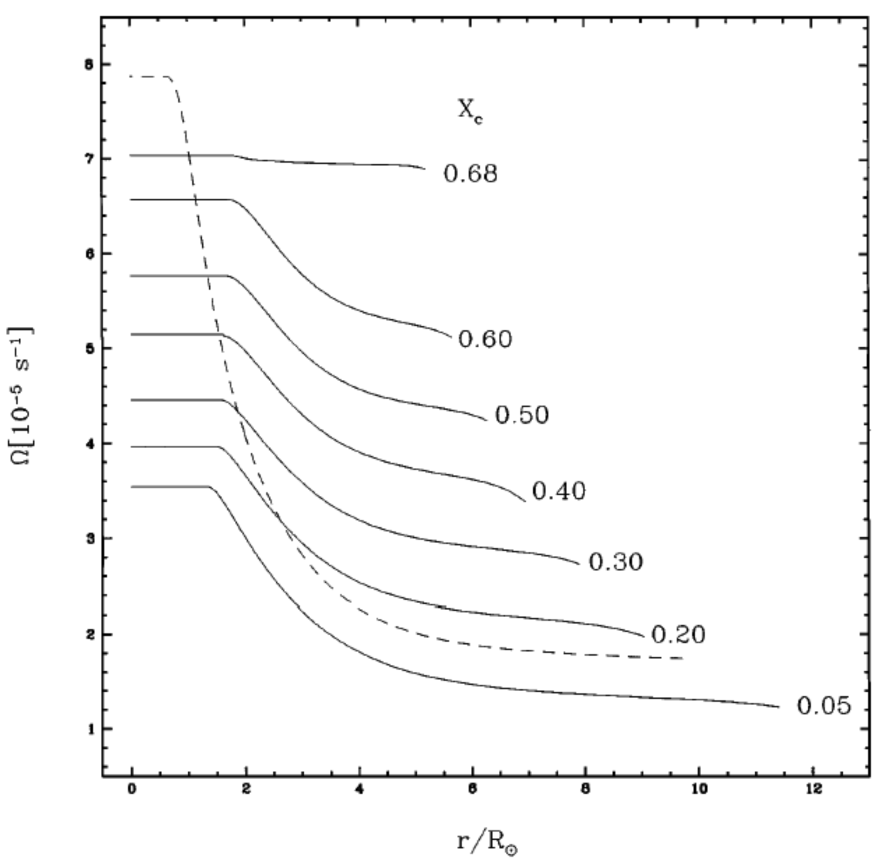
Busse (1981), JPZ (1992),
Maeder & JPZ (1998), JPZ & Mathis (2004)

Meridional circulation - revised treatment

horizontal average
shellular rotation $\Omega(r)$

$$-\frac{8\pi}{15}\rho r^4 \Omega U \quad -\frac{8\pi}{3}\rho \nu_v r^4 \frac{\partial\Omega}{\partial r} \quad = \quad -\frac{\partial J}{\partial t}$$

advection through meridional circulation — turbulent diffusion — angular momentum flux



20 $M_\odot$ star with mass loss

circulation required
to transport AM to surface

Ω profile steepens
as the star evolves

Maeder & Meynet 2000

Rotational mixing in radiation zones

Turbulence caused by vertical shear $\Omega(\mathbf{r})$

(baroclinic instability)

- if maximum of vorticity (inflexion point) : linear instability

- if no maximum of vorticity : finite amplitude instability

- stabilizing effect of stratification reduced by thermal diffusion

turbulence if

$$Ri_c \left(\frac{dV_{hor}}{dr} \right)^2 > N^2 \left(\frac{w\ell}{K} \right)$$

Richardson criterion

from which one deduces the turbulent diffusivity (if μ = cst)

$$D_v = w\ell = Ri_c K \frac{\Omega^2}{N^2} \left(\frac{d \ln \Omega}{d \ln r} \right)^2$$

Townsend 1959
Dudis 1974; JPZ 1974
Lignières et al. 1999

K thermal diffusion; ν viscosity; N buoyancy frequency

Turbulence caused by horizontal shear $\Omega(\theta)$
(barotropic instability)

Assumptions:

- instability acts to suppress its cause, i.e. diff. rotation in latitude $\Omega(\theta)$
- turbulent transport is anisotropic (due to stratification): $D_h >> D_v$

→ "horizontal turbulence" interferes with vertical transport :

- erodes stabilizing effect of stratification ; shear-unstable when

$$Ri_c \left(\frac{d \ln \Omega}{d \ln r} \right)^2 > N_t^2 \left(\frac{wl}{K} \right) + N_\mu^2 \left(\frac{wl}{D_h} \right)$$

Talon & JPZ 1997

- turns advection of chemicals into vertical diffusion

$$D_{eff} = \frac{1}{30} \frac{(rU)^2}{D_h}$$

Chaboyer & JPZ 1992

Main weakness: no firm prescription for D_h

Maeder 2003
Mathis, Palacios & JPZ 2004

Fortunately, one expects $D_v >> D_{eff}$

→ Transport of chemical : diffusion → transport of AM : advection

Rotational mixing - the observational test

Assumption: the processes that cause the mixing of chemical elements
(i.e. circulation and turbulence)
are also responsible for the transport of angular momentum
JPZ 1992, Maeder & JPZ 1998

- quite successful with early-type stars (fast rotators)
  Talon et al. 1997; Maeder & Meynet 2000; Talon & Charbonnel 1999

- for late-type stars (which are spun down by wind) predicts
  - fast rotating core — not true: helioseismology
  - strong destruction of Be in Sun — not observed
  - mixing correlated with loss of angular momentum
    not true: Li in tidally locked binaries
    not true: little dispersion in the Spite plateau

$\Rightarrow$ Another, more powerful process is responsible for the transport of angular momentum

Hence 2 types of rotational mixing

In both, circulation and turbulence
are responsible for the mixing of chemical elements

Rotational mixing of type I :

angular momentum too is carried by
circulation and turbulence

Rotational mixing of type II :

another process operates
for the transport of angular momentum;
it has indirect impact on mixing, by shaping $\Omega(\mathbf{r})$

- magnetic field ?
- internal gravity waves ?

Possible effects of magnetic field

(cf. M. Browning, this conf.)

Dynamo field (solar-type stars, or from convective core)

- Likely to have cyclic reversals $\rightarrow$ will not penetrate into RZ

  [Garaud 1999]

Fossil field (such as in Ap/Bp stars)

- Renders the rotation uniform [Mestel and coll.]

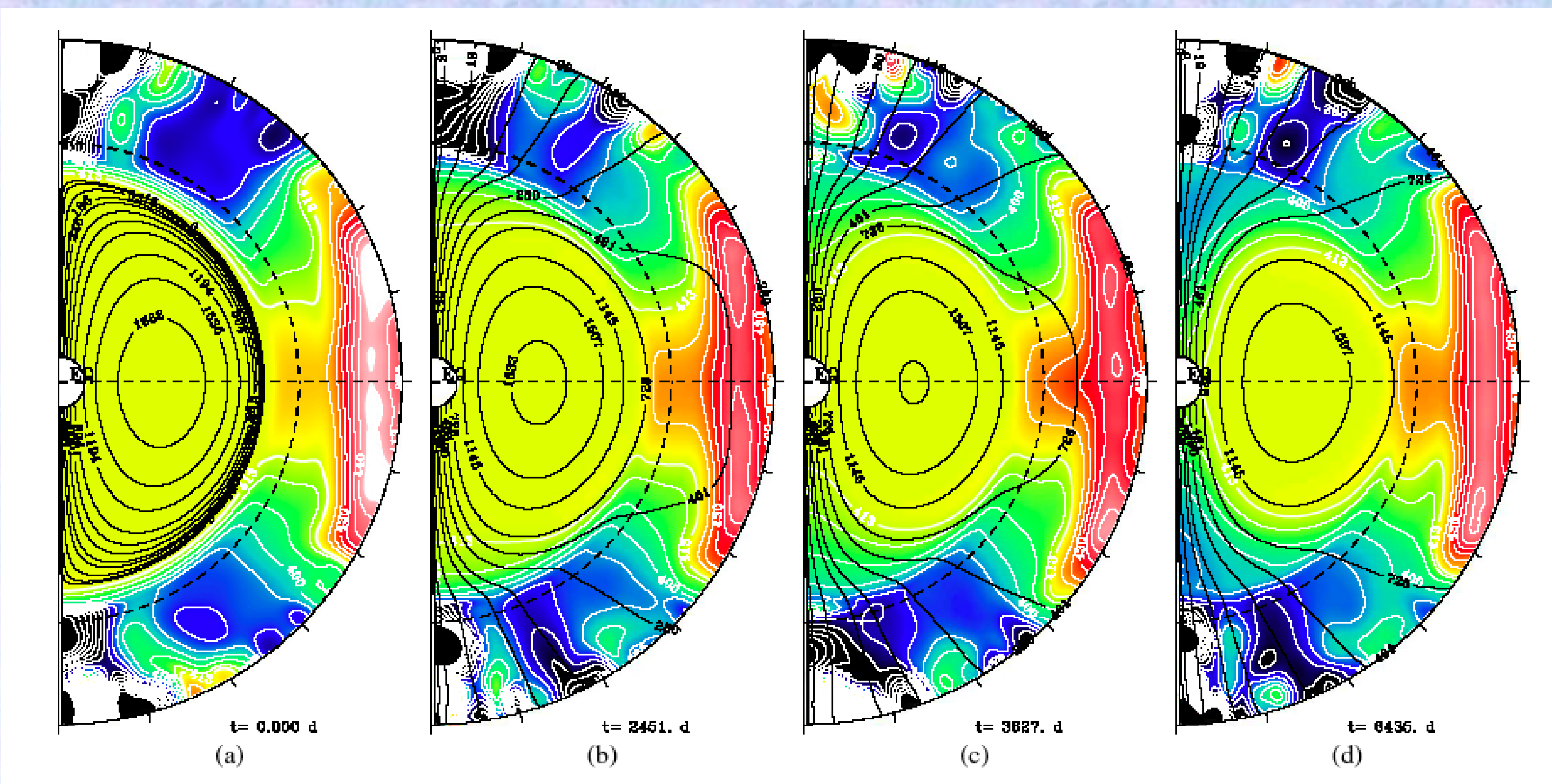
  along field lines if axisymmetric (Ferraro law)

- Imprints diff. rotation of CZ on RZ [Strugarek, Brun & JPZ 2011]

Fossil field and differential rotation

Fossil field expands into CZ, and prints its differential rotation on RZ

not observed in Sun !



3D simulations - ASH code [Strugarek, Brun & JPZ 2011]

Still debated [Wood, McCaslin & Garaud 2011]

Role of magnetic field

(cf. M. Browning, this conf.)

Dynamo field (solar-type stars, or from convective core)

- Likely to have reversals $\rightarrow$ will not penetrate into RZ

[Garaud 1999]

Fossil field (such as in Ap stars)

- Renders the rotation uniform [Mestel and coll.]

  along field lines if axisymmetric (Ferraro law)

- Imprints diff. rotation of CZ [Strugarek, Brun & JPZ 2011]
- Suppresses hydro instabilities [Charbonnel & JPZ 2008]

Field itself may be unstable [Tayler & coll.; Spruit 1998]

- yes - but instabilities are probably of Alvénic type $\rightarrow$ no mixing
- may these instabilities sustain a dynamo ? Probably not

[Spruit 2002, JPZ, Brun & Mathis 2007]

Angular momentum transport by waves

Press 1981, Garcia-Lopez & Spruit 1991, Schatzman 1993, Zahn et al 1997

Internal gravity waves and gravito-inertial waves
are emitted at the edge of the convection zone

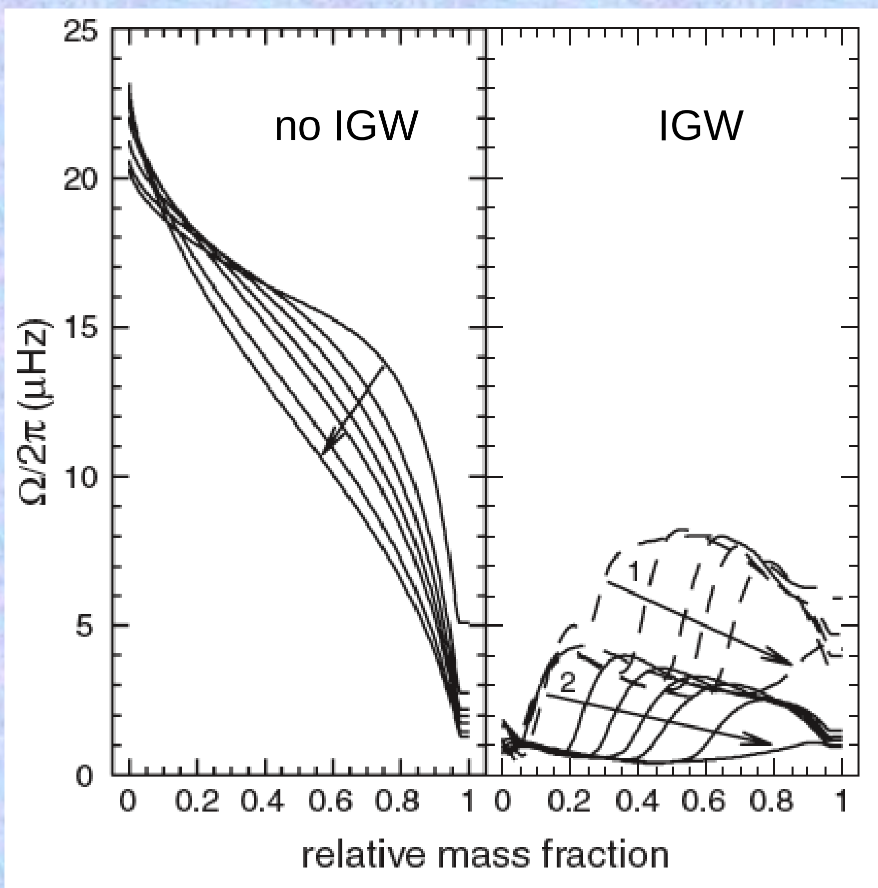
They transport angular momentum into the RZ, which they deposit
when damped through thermal diffusion

damping rate $\propto \sigma^{-4}$

$$\sigma(r,m) = \sigma_c + m[\Omega(r) - \Omega_{zc}]$$

- if there is differential rotation,
  prograde and retrograde waves deposit
  their momentum (of opposite sign)
  at different locations

- waves strengthen the local diff. rotation,
  until the shear becomes unstable
  $\Rightarrow$ turbulence



Talon et al 2002, Talon & Charbonnel 2005
Mathis (this conf.)

Thermohaline mixing

A stable temperature stratification $\nabla_{ad} > \nabla$

can be destabilized by an inverse μ gradient as soon as $\nabla_{\mu} < 0$

because heat diffuses much faster than chemicals (μ)

$\rightarrow$ thermohaline instability

In stars, such inversions occur

- when heavy nuclides are accreted [Vauclair 2004]
- in regions of hydrogen burning, due to $^{3}He + ^{3}He \rightarrow ^{4}He + 2p$
  [Ulrich 1972, Eggleton et al. 2006, Charbonnel & Zahn 2007, Denissenkov & Pinsonneault 2008, Lagarde etal 2011, Théado & Vauclair 2011]

Laboratory analog: stable temperature stratification, unstable salt stratification

mixing efficiency ? fingers ? staircases ?
numerical simulations will tell

[Traxler, Garaud & Stellmach 2010, Denissenkov & Merryfield 2010]

Weak points of present models including mixing in radiation zones

- Description of the turbulence caused by differential rotation
- Power spectrum for IGW emitted at edge of convection zone
- Impact of rotation and magnetic field on IGW - inertial waves
- Particle transport by IGW ?
- Role of instabilities due to magnetic field ?
- Prescription for thermohaline mixing

Necessity of sounding stellar interiors : asteroseismology

Prime targets : Convection Rotation

Some achievements of asteroseismology

- β Cep star HD 129929

  [Aerts, Thoul, Daszynska, Scuflaire, Waelkens, Dupret, Niemcsura, Noels 2003]

  - slow rotator - detection & identification of six modes of $l = 0,1,2$

    $\rightarrow$ extra mixing above core of $\approx 0.1\ H_P$ (convective penetration)

    $\rightarrow$ core rotates 3 to 4 times faster than surface ▪

- β Cep star GX Pegasi

  [Pamyatnykh, Handler & Dziembowski 2004]

  - slow rotator - detection & identification of mixed modes

    $\rightarrow$ core rotates $\approx$ 5 times faster than surface ▪

Space borne asteroseismology

- two pulsating Be stars HD 181231 HD 175869

  [CoRoT; Neiner, Mathis, Saio, Lovekin, Eggenberger, Lee 2011]

  - rapid rotators - detection & identification of g-modes

  $\rightarrow$ extra mixing above core of $\approx$ 0.2 to 0.35 H_P (convective penetration plus rotational mixing) ▪

- red giants observed by Kepler (under embargo)

  [Kepler; Beck, Montalban, Kallinger, De Ridder, Aerts et al 2011]

  - rotational splitting of mixed modes

  $\rightarrow$ cores rotate 10 times faster as surface ▪

Mixing in radiation zones