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# Plasma Instabilities in Relativistic Shocks

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Nonlinear Processes in Astrophysical Plasmas:  
Particle Acceleration, Magnetic Field Amplification, and Radiation Signatures

September 28, 2009 - October 2, 2009

*Kavli Institute for Theoretical Physics*

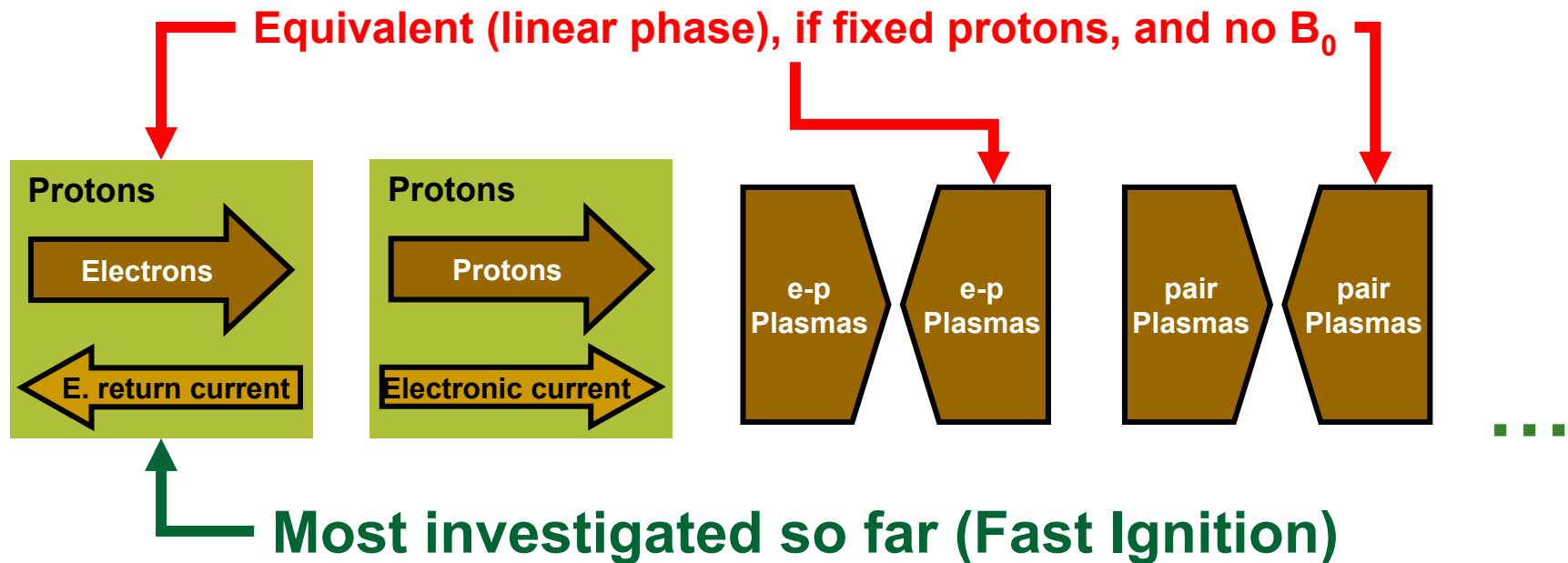
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# Outline

- Kind of systems considered
  - Formalism
  - Electron beam/plasma interaction – No  $B_0$
  - Proton beam,  $B_0 //$
  - Conclusions
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# The kind of system considered

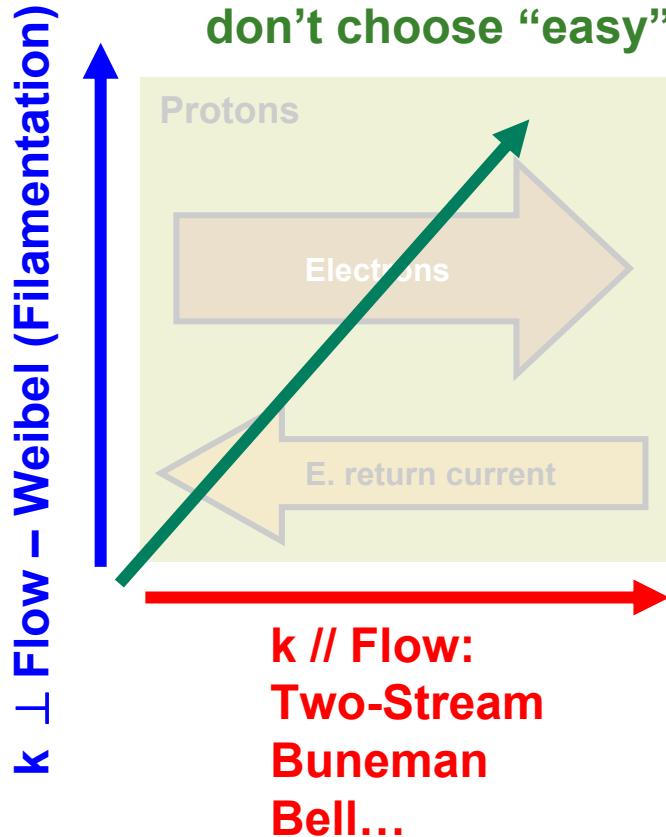
- Infinite beam/plasma system, homogenous, collisionless, current and charge neutral, magnetized or not.
- LINEAR evolution.
- Settings are numerous:



# Formalism

Many ways to perturbate the system:

**k Oblique – because real world perturbations don't choose “easy” alignment -  $\Sigma$  all orientations**



- Vlasov + Maxwell - Linearized
- Perturbation  $\propto \exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)$  with  $\mathbf{k}=(k_{\parallel},k_{\perp})$
- Dielectric tensor  $\mathbf{T}$
- $\text{Det}(\mathbf{T})=0$

$$\mathbf{T} = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{xy}^* & T_{yy} & T_{yz} \\ T_{xz}^* & T_{yz}^* & T_{zz} \end{pmatrix}$$

- Every single unstable mode is recovered

# About the dispersion equation

- General form:

$$\det \left| \frac{\omega^2}{c^2} \varepsilon_{ij} + k_i k_j - k^2 \delta_{ij} \right| = 0$$

Where,

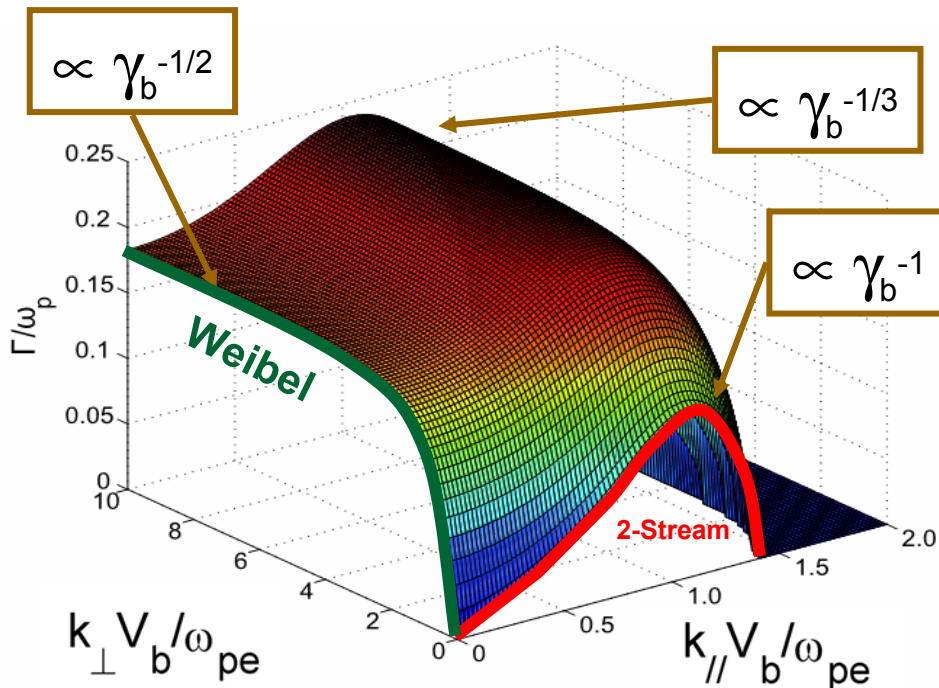
$$\varepsilon_{kl}(\omega, \mathbf{k}) = \delta_{kl} + \sum_{\alpha=b,p} \frac{n_\alpha}{\omega^2} \int \int \int \frac{p_k}{\gamma} \frac{\partial f_\alpha^{(0)}}{\partial p_l} d^3 p + \sum_{\alpha=b,p} \frac{n_\alpha}{\omega^2} \int \int \int \frac{p_k p_l}{\gamma^2} \frac{\mathbf{k} \cdot \partial f_\alpha^{(0)} / \partial \mathbf{p}}{\omega - \mathbf{k} \cdot \mathbf{p} / \gamma} d^3 p.$$

$\gamma(\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z)$  couples de 3 directions,  
can't separate the integral, 2D not equiv. to 3D

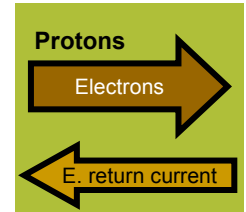
# Cold regime (DF Dirac $\delta$ 's) and no B0

## Spectrum (just 1 branch)

$N_b / N_p = 1/9$ ,  $V_b = 0.9c$   
From Karmakar, PRE, 2009



## Hierarchy



$N_b / N_p$   
If beam // z

$$\mathcal{T} = \begin{pmatrix} T_{xx} & 0 & T_{xz} \\ 0 & T_{yy} & 0 \\ T_{xz}^* & 0 & T_{zz} \end{pmatrix}$$

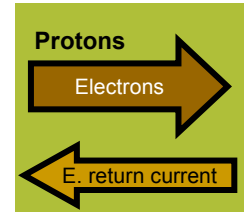
Oblique



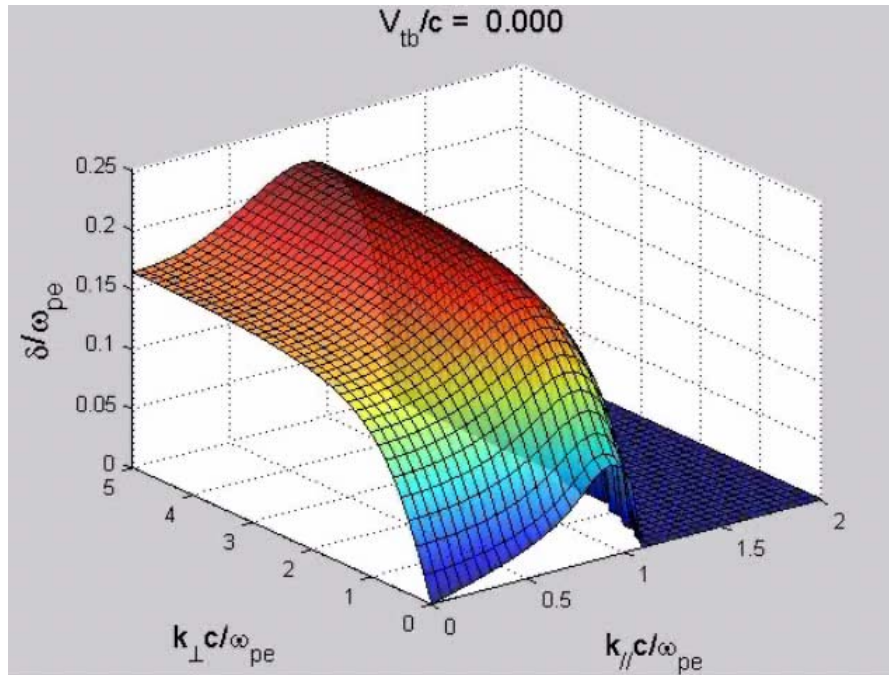
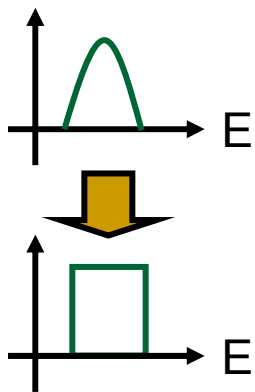
# Kinetic effects (still no $B_0$ )

- **Waterbag** calculation (10 s CPU instead of 3 weeks).

$$N_b/N_p = 0.1, \gamma_b = 4$$

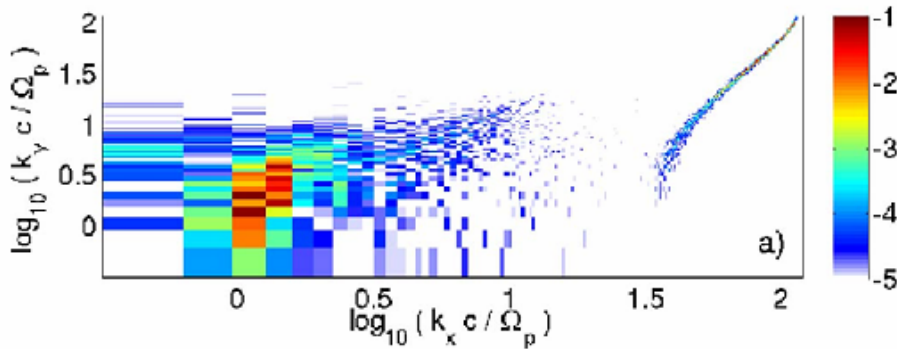


$F(E)$

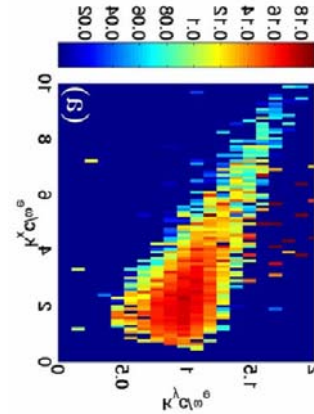


**Beam Temp.**  
first stabilizes Weibel &  
modes with  $k_{\perp} \gg k_{\parallel}$

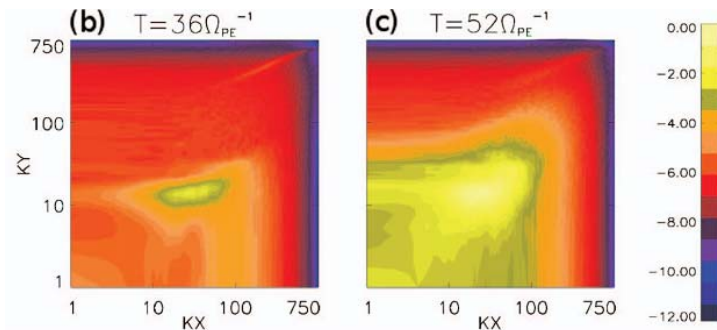
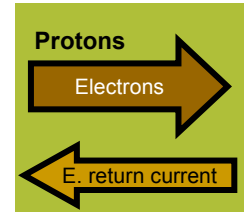
# Waterbag model: PIC's confirmations



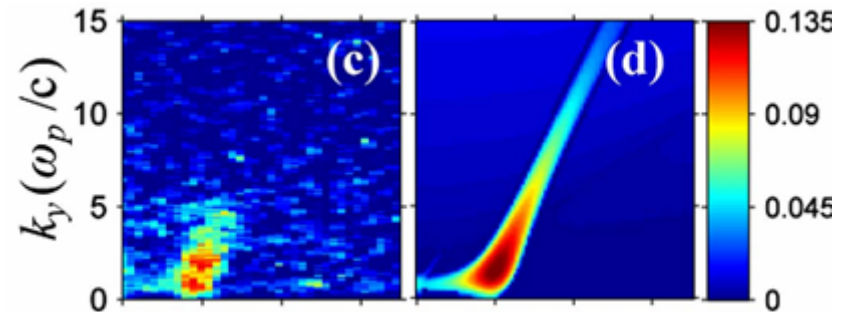
Dieckmann, PoP, 2006



Gremillet, PoP, 2007



Frederiksen, PoP, 2008



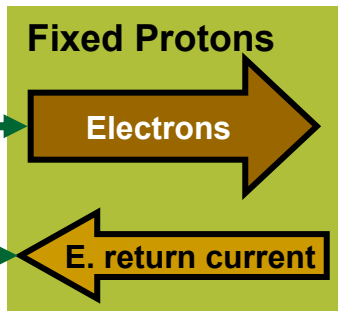
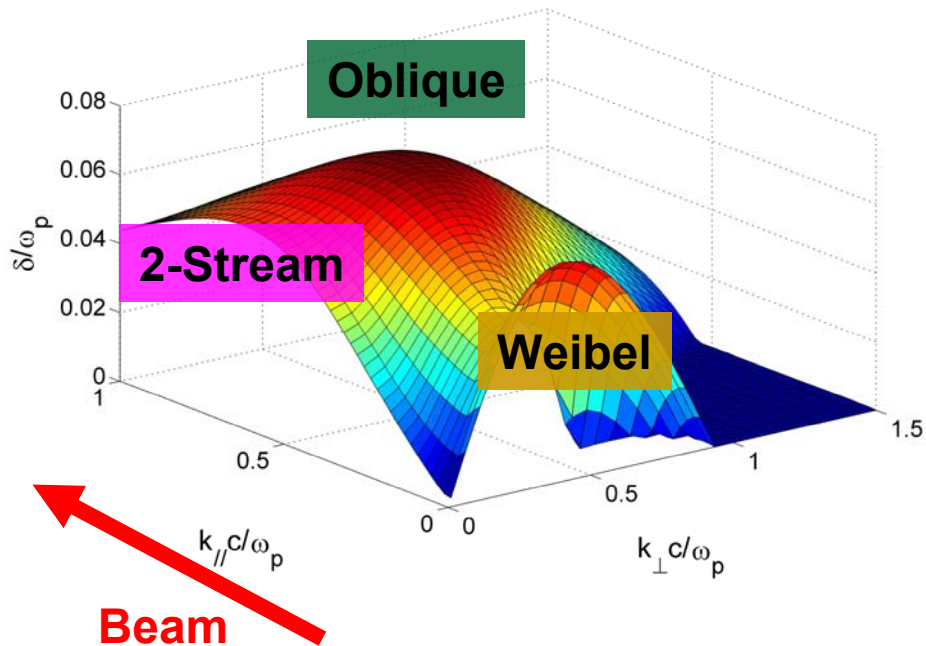
Kong, PoP, 2009



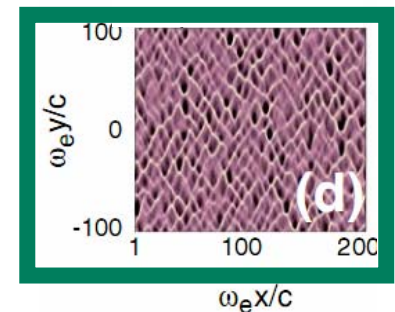
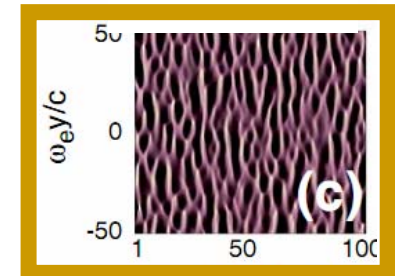
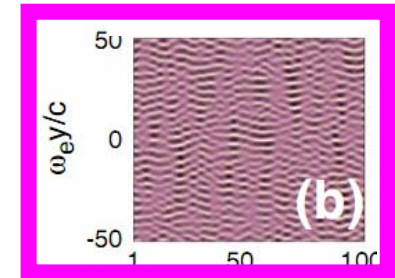
# Kinetic regime (still no $B_0$ )

- Typical spectrum and generated patterns

$$f_{\alpha}^0(\mathbf{p}) = \frac{\mu_{\alpha}}{4\pi\gamma_{\alpha}^2 K_2(\mu_{\alpha}/\gamma_{\alpha})} \exp[-\mu_{\alpha}(\gamma(\mathbf{p}) - \beta_{\alpha}p_y)]$$



**IF dominant...**



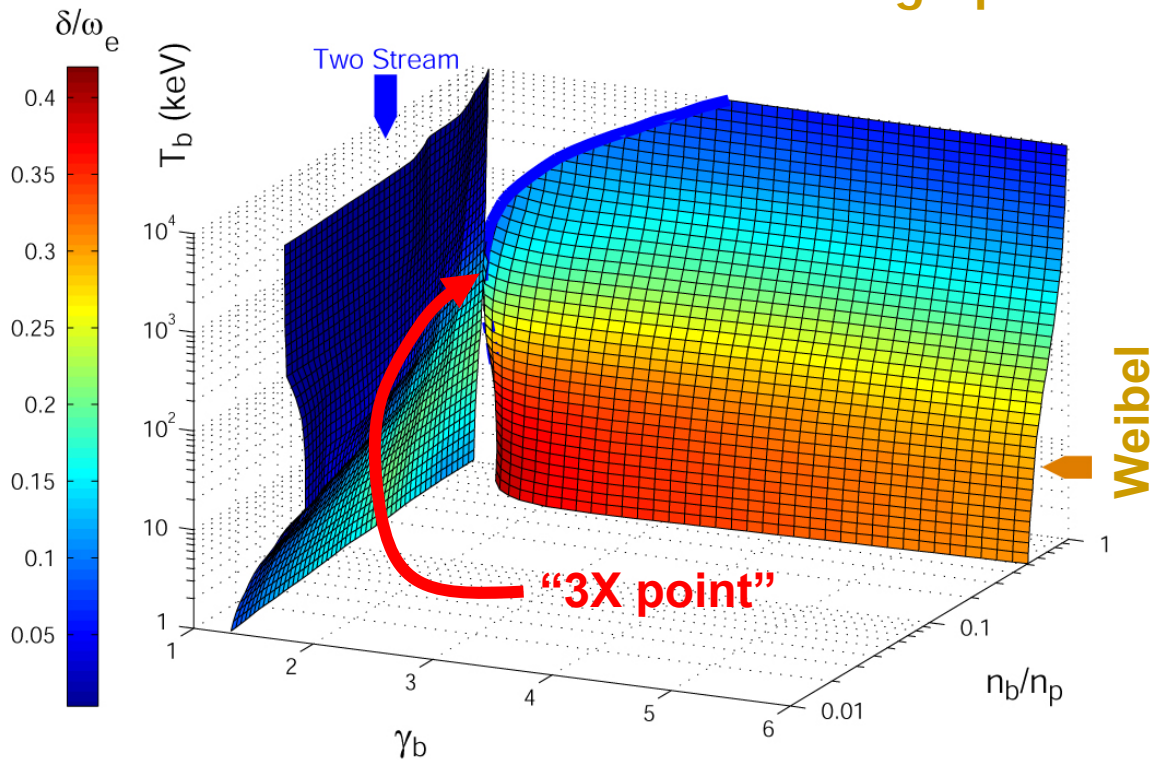
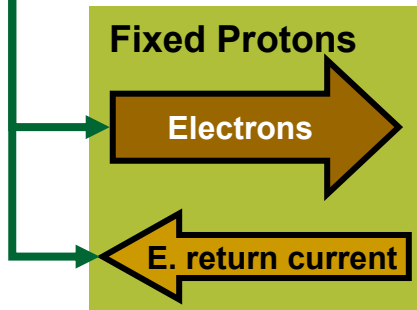
**Beam flow** ↑

# Kinetic regime (still no $B_0$ ) – Modes hierarchy

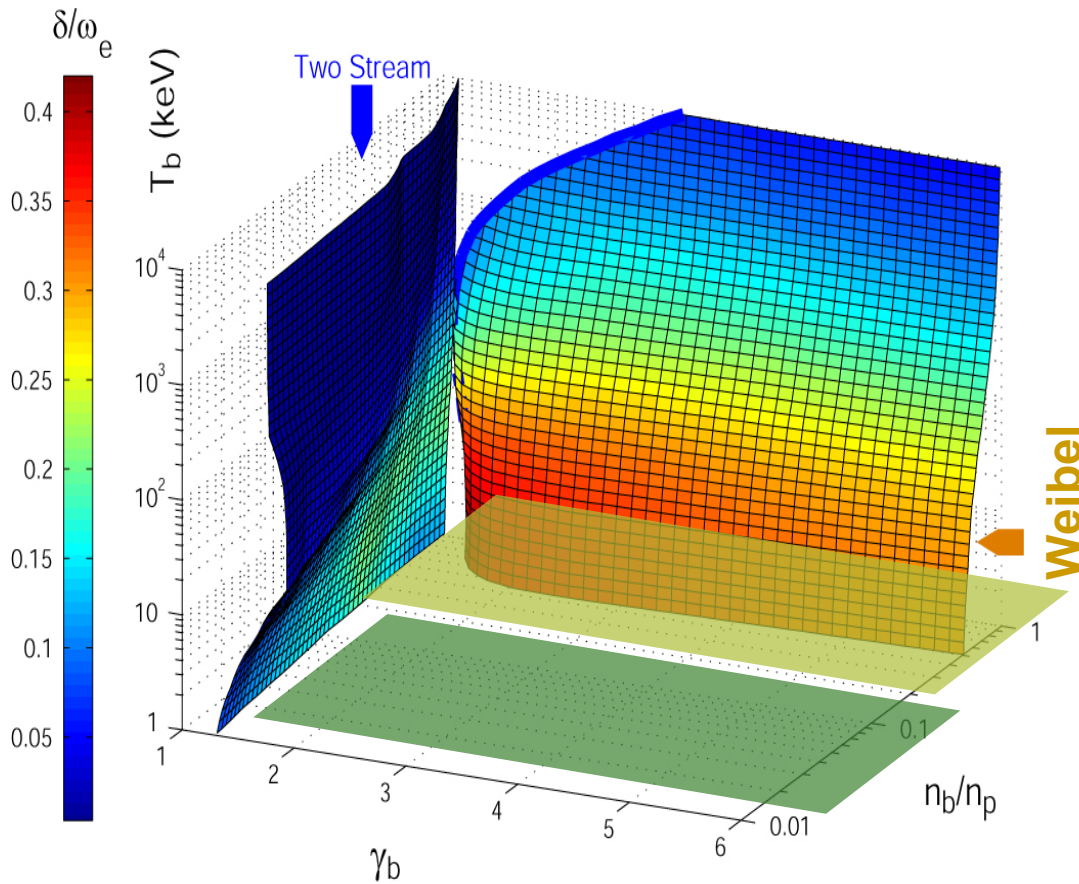
$$f_{\alpha}^0(\mathbf{p}) = \frac{\mu_{\alpha}}{4\pi\gamma_{\alpha}^2 K_2(\mu_{\alpha}/\gamma_{\alpha})} \exp[-\mu_{\alpha}(\gamma(\mathbf{p}) - \beta_{\alpha} p_y)]$$

$T_{\text{plasma}}: 5 \text{ keV}$

Universal graph



# Shock physics context



- Shock formation
  - Weibel
  - Oblique

- Particle acceleration
  - Oblique (Sironi, Silva...)

# Oblique modes: Growth rate scalings

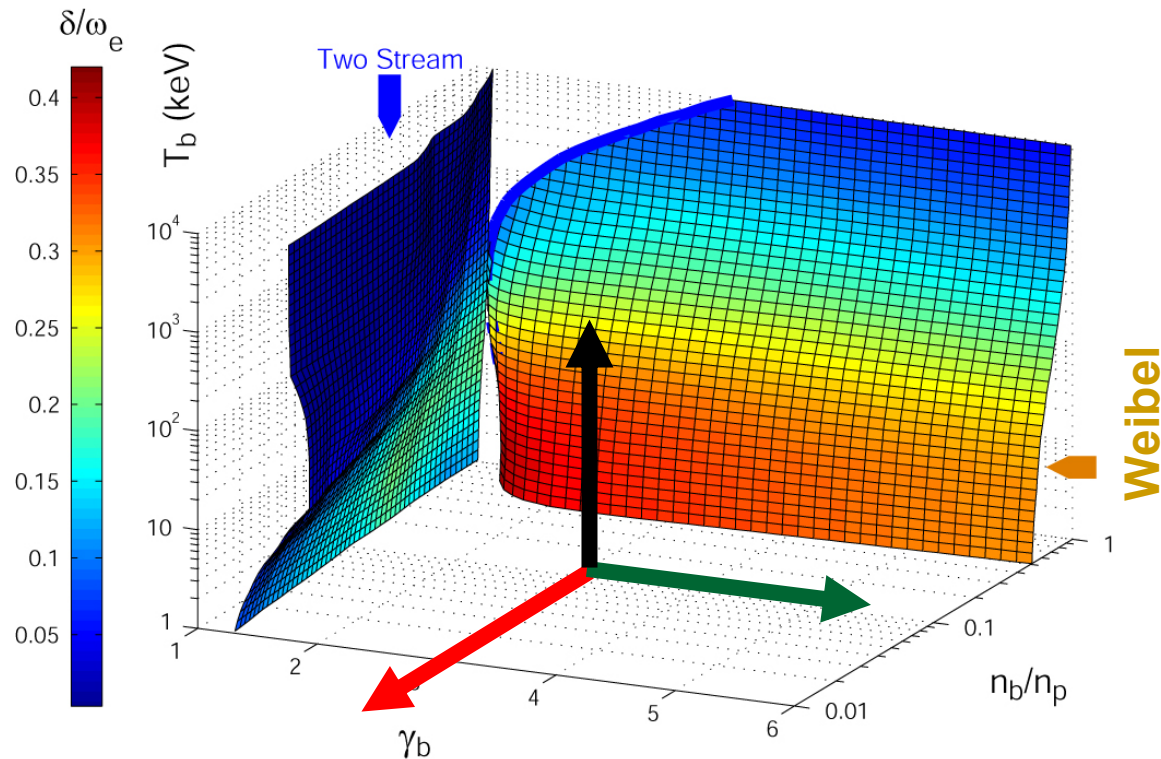
- Oblique modes govern in the following limits:

- $N_b/N_p \rightarrow 0$

- $T_b \rightarrow \infty$

- $\gamma_b \rightarrow \infty$

Limits are relevant as  
Oblique modes  
keep governing



# Oblique modes: Growth rate scalings

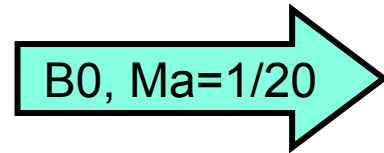
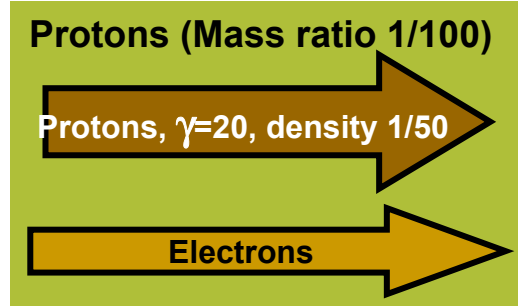
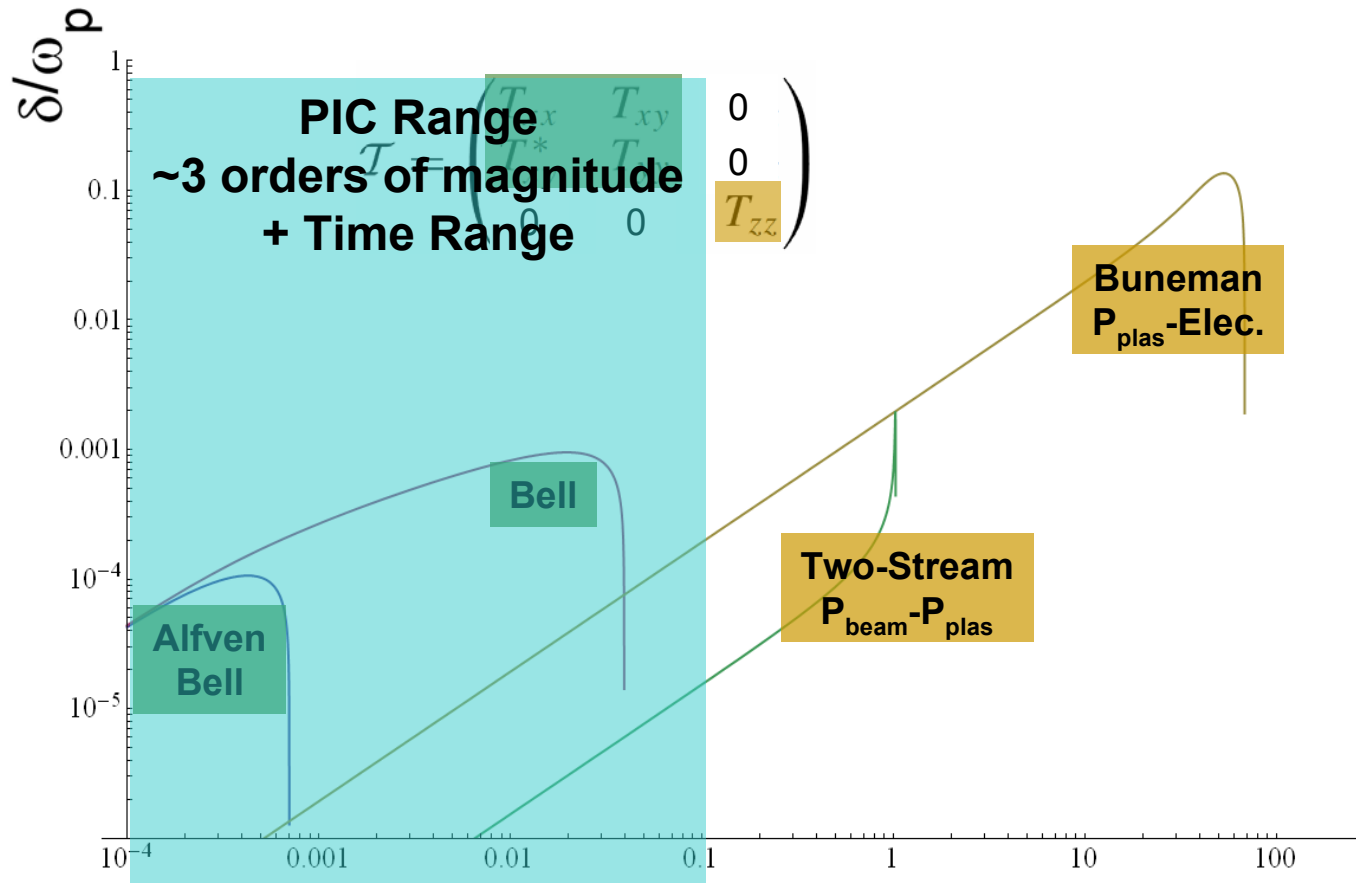
- Oblique modes similar to Two-stream ones:

Parameters	Two-stream	Oblique	
$T_b$	$T_b^{-1}$	$T_b^{-1}$	
$\gamma_b$	$\gamma_b^{-1}$	$\gamma_b^{-1/3}$	→ Only difference
$n_b/n_p$ , low $T_b$	$(n_b/n_p)^{1/3}$	$(n_b/n_p)^{1/3}$	→ Fluid
$n_b/n_p$ , large $T_b$	$n_b/n_p$	$n_b/n_p$	→ Kinetic

$$\frac{T_b}{m_e c^2} > \gamma_b \left( \frac{n_b}{n_p} \right)^{2/3}$$

# Proton beam – Magnetized, cold

- Flow aligned  $\mathbf{k}$ 's, //  $z$

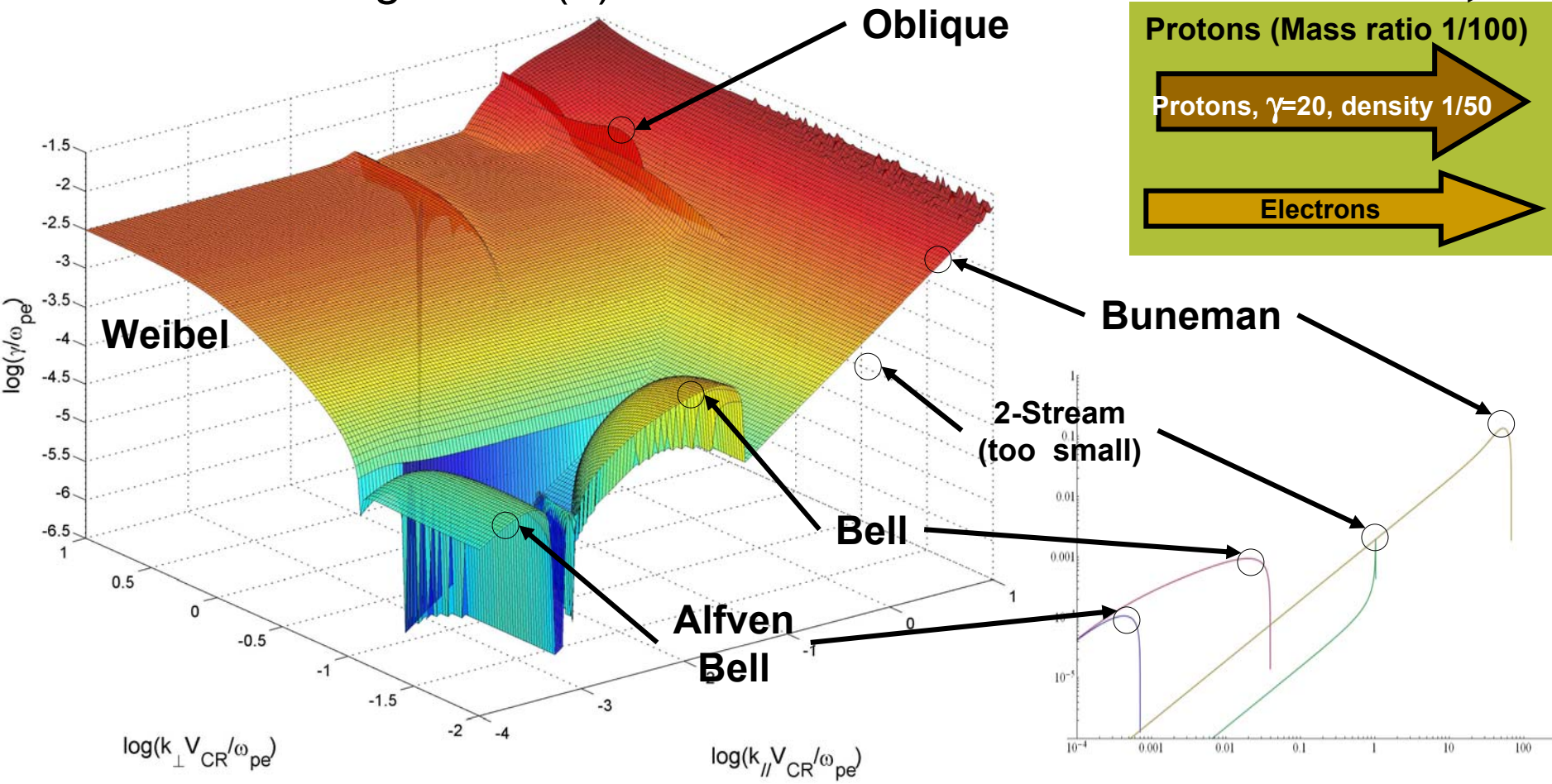
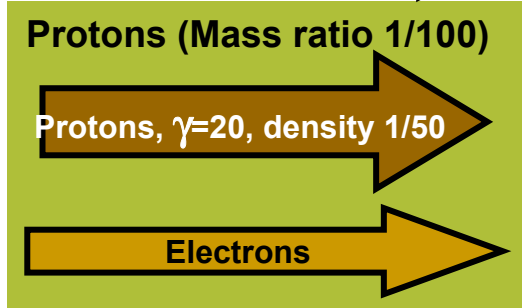


- Nothing more
- Multi-scale (PICs !)
- If you want your insta...

# Proton beam – Magnetized, cold

- All  $k$ 's – largest  $GR(k)$

$B_0, Ma=1/20$



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# Conclusions

- Beam plasma instabilities are relevant for:
  - Shock Formation (the trigger),
  - Particle acceleration.
- The simplest system (no  $B_0$ , fixed ions, e-beam+RC) has been analyzed thoroughly, including in the kinetic regime
- Shock Formation: comparable density shells. Weibel/Oblique dominant.
- Particle Acceleration: Diluted beam regime, Oblique modes.
- System with CR,  $B_0$ : rich, multi-scale spectrum.

**THANK YOU**

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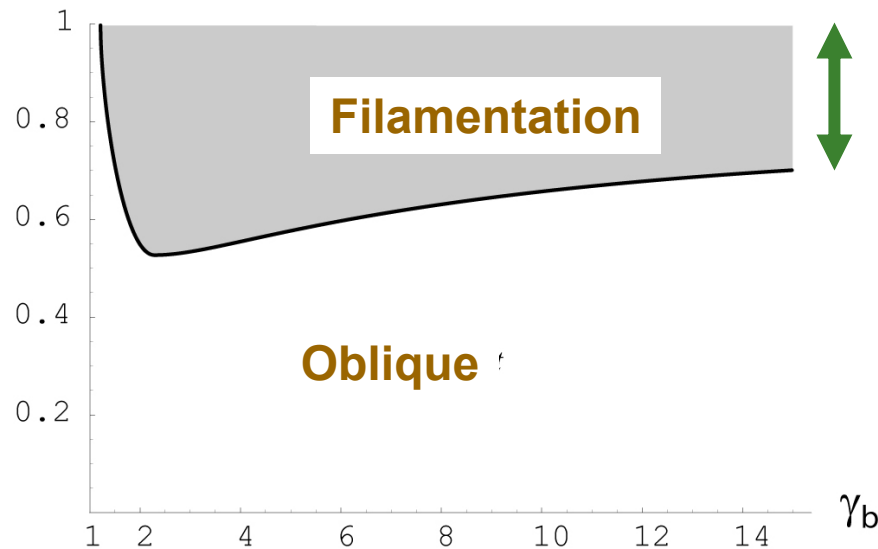


# Shock Physics: Formation

- Typical scenario: colliding shells.
- PIC scheme: Plasma interact with itself  
-> equal density shells (Fila/Weibel)
- What if different density shells?
- If un-magnetized, pair plasmas, or e-protons:



Nb/Np

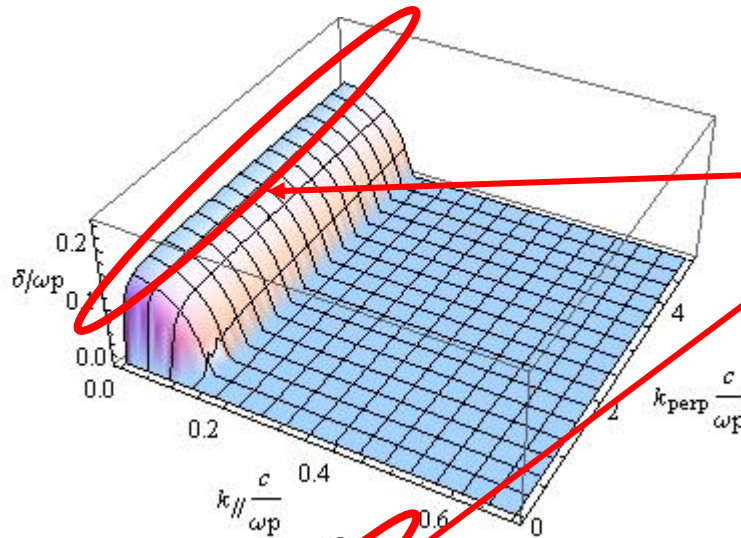


$$\propto \gamma_b^{-1/3}$$

**Fila. may not govern high  $\gamma_b$  collisions of slightly  $\neq$  density shells.  
Shock Formation?**

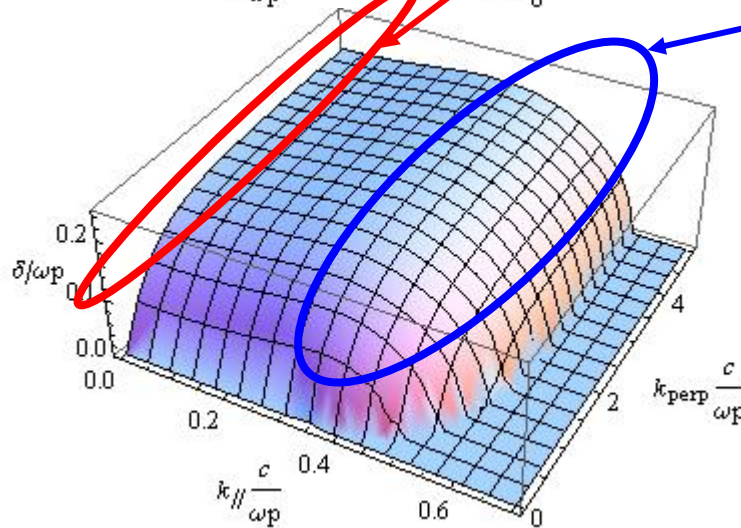
# Shock Physics: Formation

$\gamma_b = 100$   
 $N_b/N_p = 1$



Electromagnetic  
Good **B** generators  
 $V_{\phi} = 0$

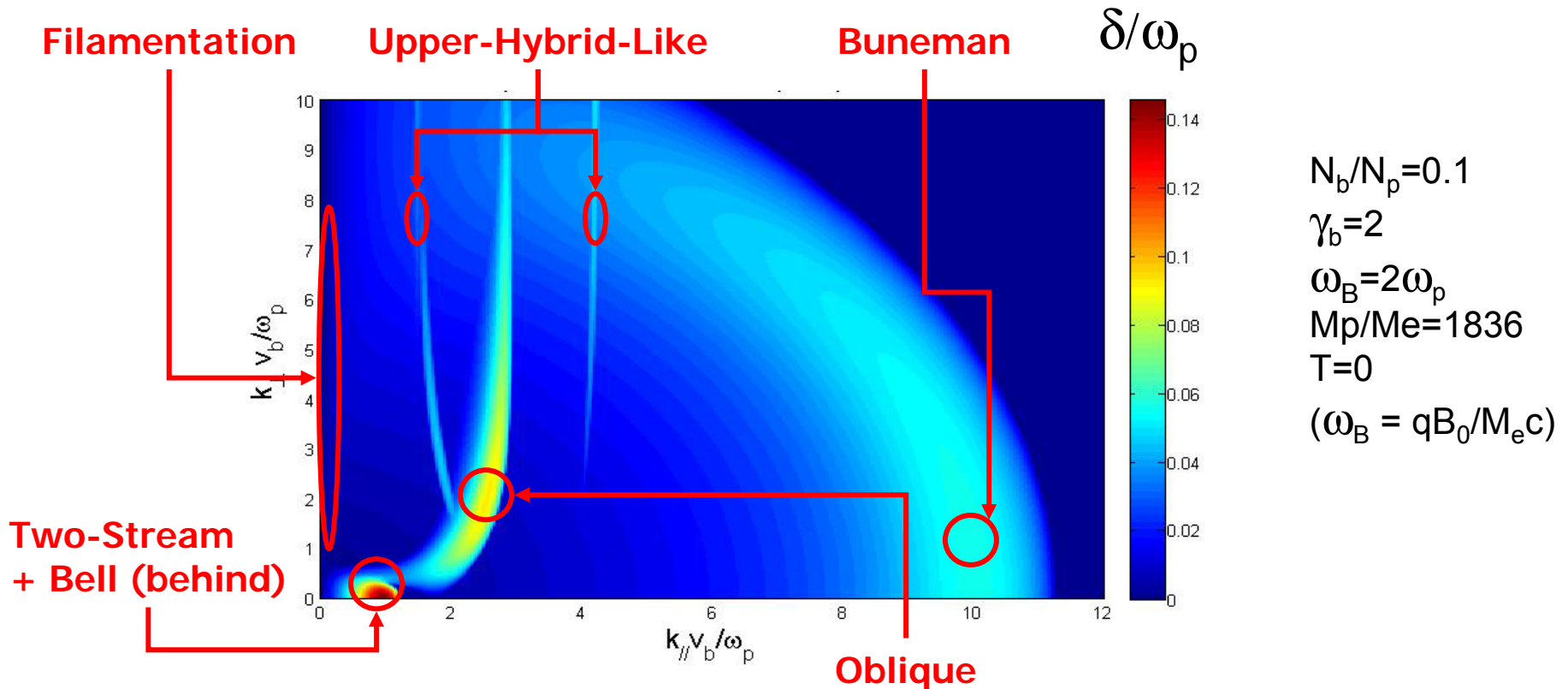
$\gamma_b = 100$   
 $N_b/N_p = 0.7$



Quasi-electrostatic  
Poor **B** generators  
 $V_{\phi} \neq 0$

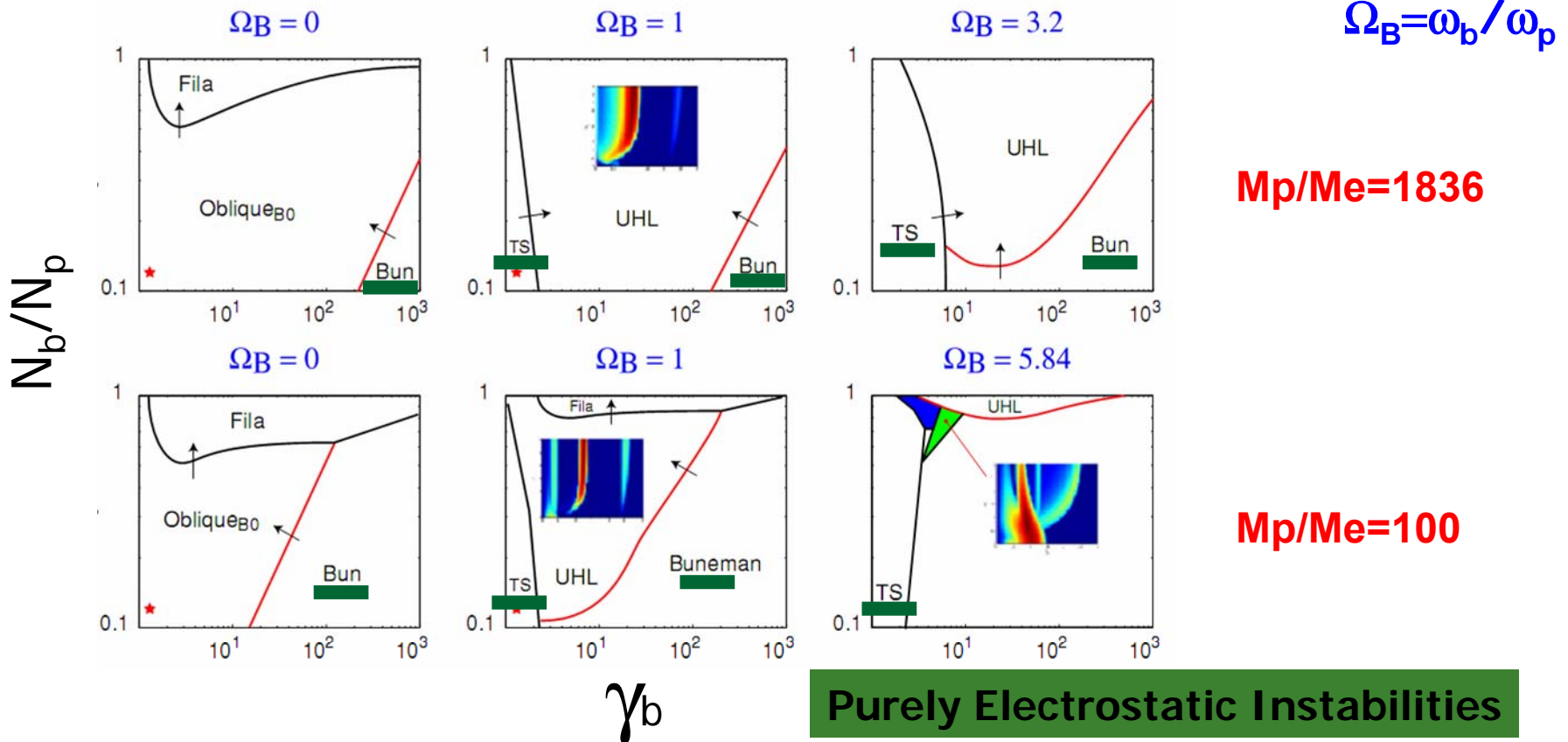
# Shock Physics: Particle acceleration

- Relativistic, diluted, e-beam/plasma interaction,  $\mathbf{B}_0 \parallel \text{Flow}$ .
- Typical spectrum:



# Shock Physics: Particle acceleration

- Consider a relativistic, diluted, e-beam/plasma interaction.
- Dominant mode:



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# Outline of the Talk

- System considered
  - Plasma instabilities and relativistic shocks
    - Shock Formation
    - Particle acceleration
  - Oblique Modes – Cold and Waterbag results
  - Kinetic theory of Oblique Modes
    - Scaling laws for the Growth Rate
  - Non-parallel flows
  - Conclusions
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