Nonresonant Instability in a Hot Plasma

Based on Work Submitted to ApJ

Ellen Zweibel & John Everett

`zweibel@astro.wisc.edu, everett@physics.wisc.edu`

Departments of Astronomy & Physics

University of Wisconsin, Madison

and

Center for Magnetic Self-Organization

in

Laboratory and Astrophysical Plasmas

Motivation

- How is energy apportioned between magnetic fields, cosmic rays, and thermal plasma?
  - How are magnetic fields generated in cosmic plasma?
  - How do cosmic rays transfer momentum & energy to the thermal background?
  - What limits the energy to which cosmic rays can be accelerated?

We won’t answer all of these questions. We just do linear theory.

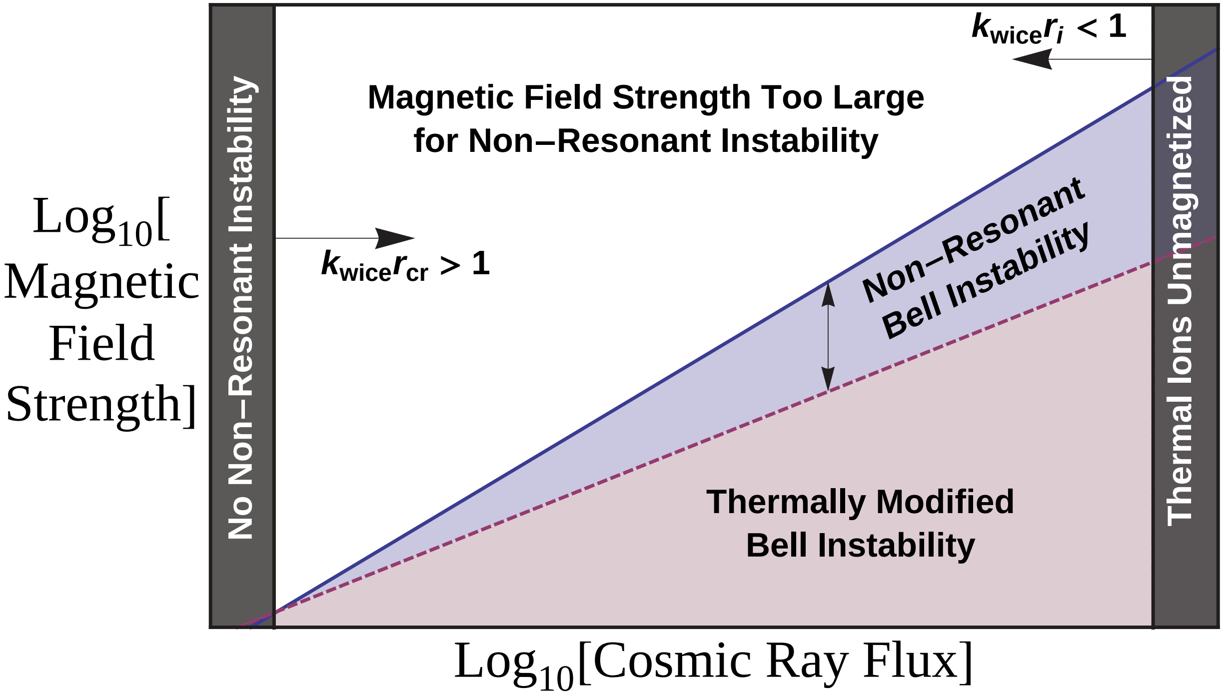
Dynamo Problem

- Theory & simulation suggests that in diffuse plasmas with high $Pm \equiv \nu/\eta$, power builds up at the resistive scale.
- Input of magnetic helicity can drive an inverse cascade to larger scale.
- Nonresonant cosmic ray streaming instabilities amplify right circularly polarized waves & thus inject helicity (on the scale of the background field).

The Plan of This Talk

- Dispersion relation for RCP fluctuations in thermal plasma with streaming cosmic rays (extends Reville et al. 2008).
- Domains of instability on the $(n_{cr}v_D, B)$ plane.
- Application to interstellar medium (ISM) & intracluster (ICM) medium.

Schematic Domains of Instability



Schematic domains of nonresonant instability in the $(n_{cr}v_D, B)$ plane, with n_i, T held fixed. The cosmic rays must be unmagnetized, the thermal ions magnetized.

Basic Setup

- Right circularly polarized fluctuations propagating parallel to ambient $\mathbf{B}$
- Cosmic rays with density n_{cr} which are isotropic in a frame moving along $\mathbf{B}$ with speed v_D.
- Thermal plasma of electrons & ions with density n_i & temperature T.
- Population of electrons with density n_{cr} drifting at v_D.

Note on the Electrons

- In a cold plasma, an alternative setup in which $n_e = n_i + n_{cr}$ drifting at speed $v_D \frac{n_{cr}}{n_i+n_{cr}}$ gives same results (Amato & Blasi ́09).
- In a hot plasma the instability is sensitive to electron distribution function f_e (one hump or two?)
- Two-hump distribution in high M shocks can be electrostatically & electromagnetically unstable.
- Self consistent determination of f_e is an important problem for the future.

Dispersion Relation

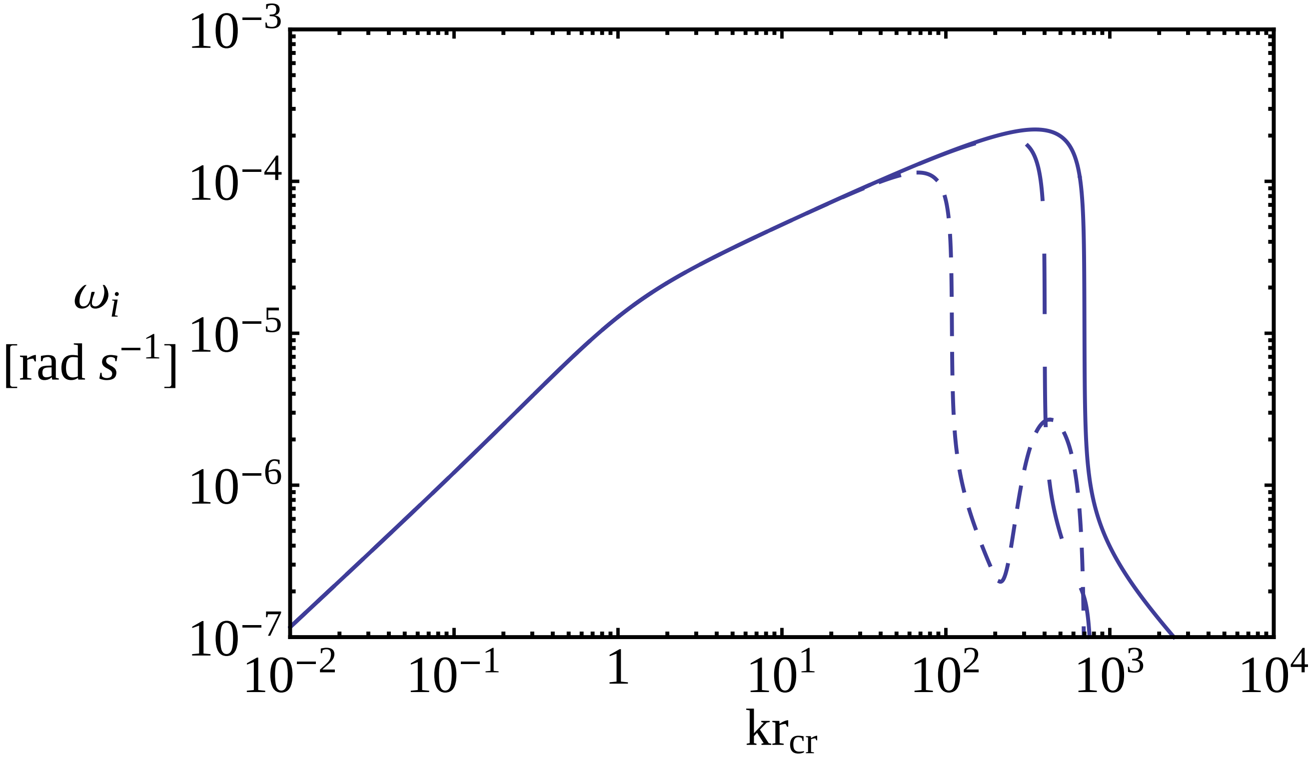
$$\frac{c^2k^2}{\omega^2} - \omega_{ci}\frac{n_{cr}}{n_i}\zeta_r\frac{(\omega - kv_D)}{\omega^2}\frac{c^2}{v_A^2} = \frac{\omega_{pi}^2}{\omega k v_i}Z\left(\frac{\omega_{ci}+\omega}{kv_i}\right) + \frac{\omega_{pe}^2}{\omega k v_e}Z\left(\frac{\omega_{ce}+\omega}{kv_e}\right), \tag{1}$$

where we have dropped the displacement current, Z is the plasma dispersion function, representing the thermal plasma response

$$Z(z) \equiv \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{e^{-s^2}}{s-z}ds, \tag{2}$$

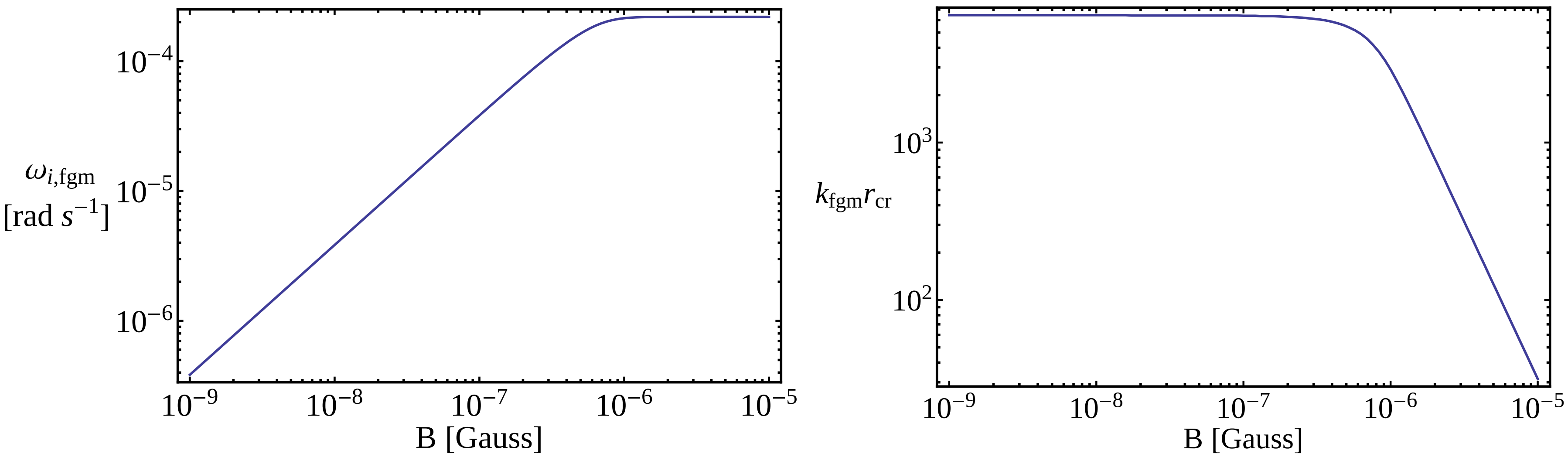
and ζ_r measures the cosmic ray response (Z2003).

Solutions



Growth rate vs scaled wavenumber for $B = 3\mu G$, $n_i = 1$ cm^{-3}, $n_{cr} v_D = 10^4$cm^{-2}s^{-1}, and $T = 10^4 K$ (solid), $T = 10^6 K$ (long dashed), $T = 10^7 K$ (short dashed).

Fastest Growing Mode



Maximum growth rate (left panel) and wavenumber of fastest growing mode (right panel) vs B for $T = 10^4 K$, $n_i = 1$, $n_{cr} v_D = 10^4$.

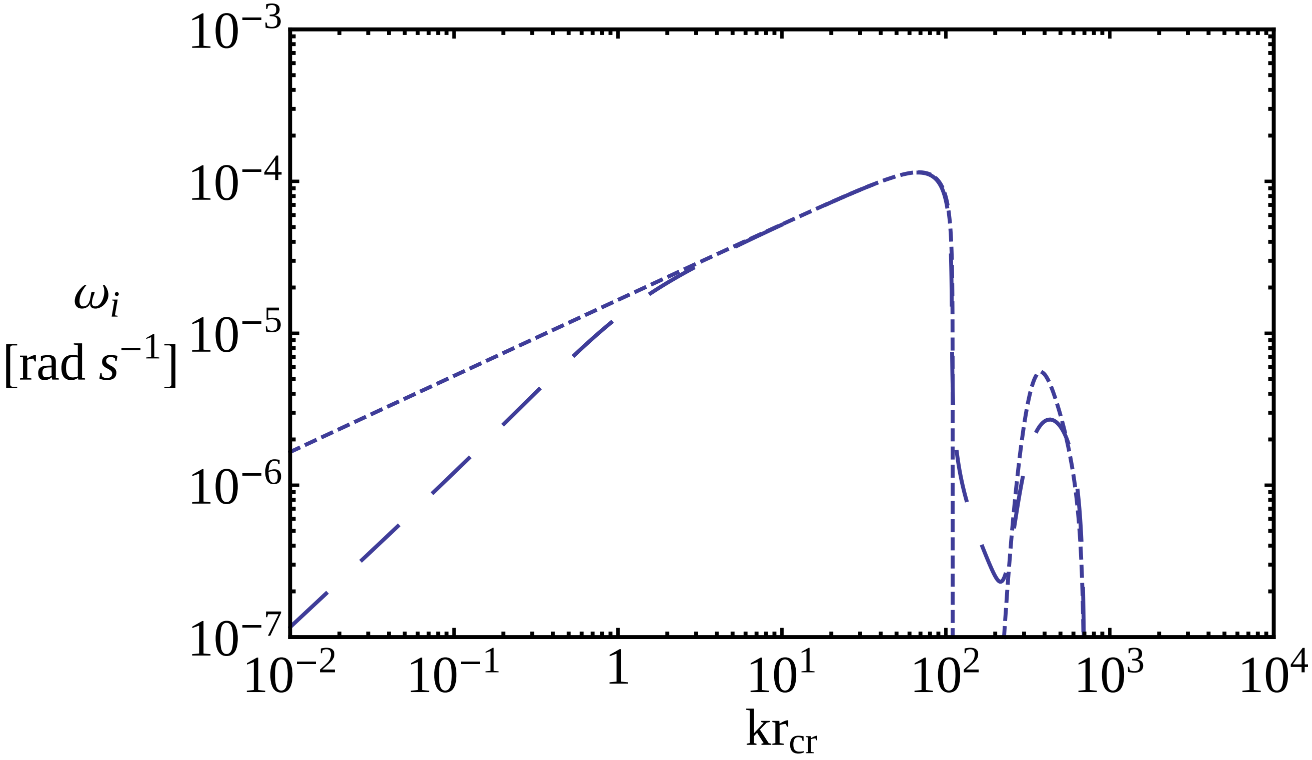
Simplified Dispersion Relation

$$\omega^2 - \omega\left(\frac{k^2 v_i^2}{2\omega_{ci}} + \omega_{ci}\frac{n_{cr}}{n_i} - i\sqrt{\pi}\frac{\omega_{ci}^2}{k v_i} e^{-\frac{\omega_{ci}^2}{k^2 v_i^2}}\right) - k^2 v_A^2 + \omega_{ci}\frac{n_{cr}}{n_i} k v_D = 0.$$

(3)

We've assumed here the electrons are cold ($k v_e/\omega_{ce} \ll 1$), the ions are cold to warm ($k v_i/\omega_{ci} \leq 1$), and the cosmic rays are hot ($\zeta_r \rightarrow -1$).

Solutions to Full & Simplified DRs



Solution to full (long dashed line) and simplified (short dashed line) dispersion relations for $T = 10^7 K$, $B = 3\mu G$, $n_i = 1$, $n_{cr} v_D = 10^4$.

Standard Bell Instability

When

$$n_i \frac{v_A^2}{c} < n_{cr} v_D < n_i \frac{v_A^3}{v_i^2}$$

the fastest growing mode has scaled wavenumber

$$k r_{cr} = k_{Bell} \frac{c}{\omega_{ci}} \equiv \frac{1}{2} \frac{n_{cr}}{n_i} \frac{c v_D}{v_A^2} \propto B^{-2}$$

and frequency

$$\omega = \omega_{Bell} = k_{Bell} v_A \propto B^0 .$$

Thermally Modified Instability

When

$$n_{cr} v_D > n_i \frac{v_A^3}{v_i^2},$$

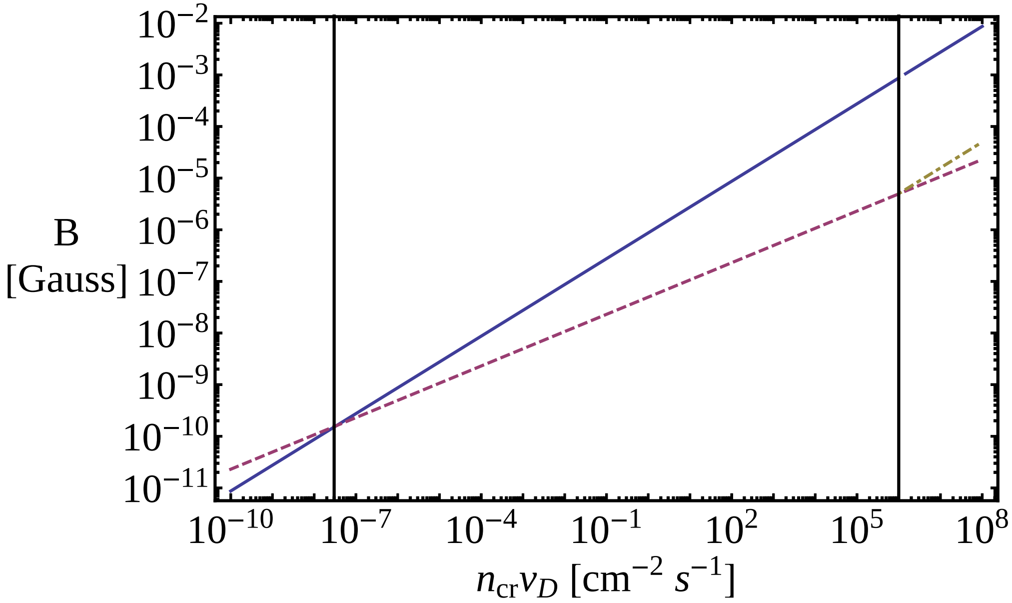
the fastest growing mode is at scaled wavenumber

$$k_{wice} r_{cr} = \frac{c}{v_i} \left(\frac{n_{cr} v_D}{n_i v_i} \right)^{1/3} \propto B^0,$$

and has growth rate

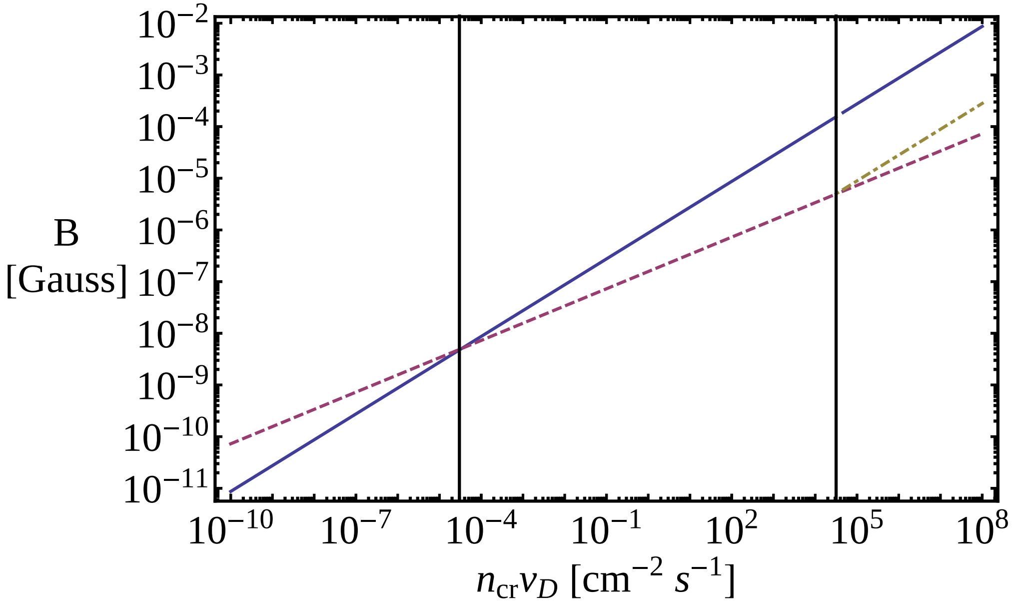
$$\omega_{wice} \sim \omega_{ci} \left(\frac{n_{cr} v_D}{n_i v_i} \right)^{2/3} \propto B.$$

Domains of Instability – ISM



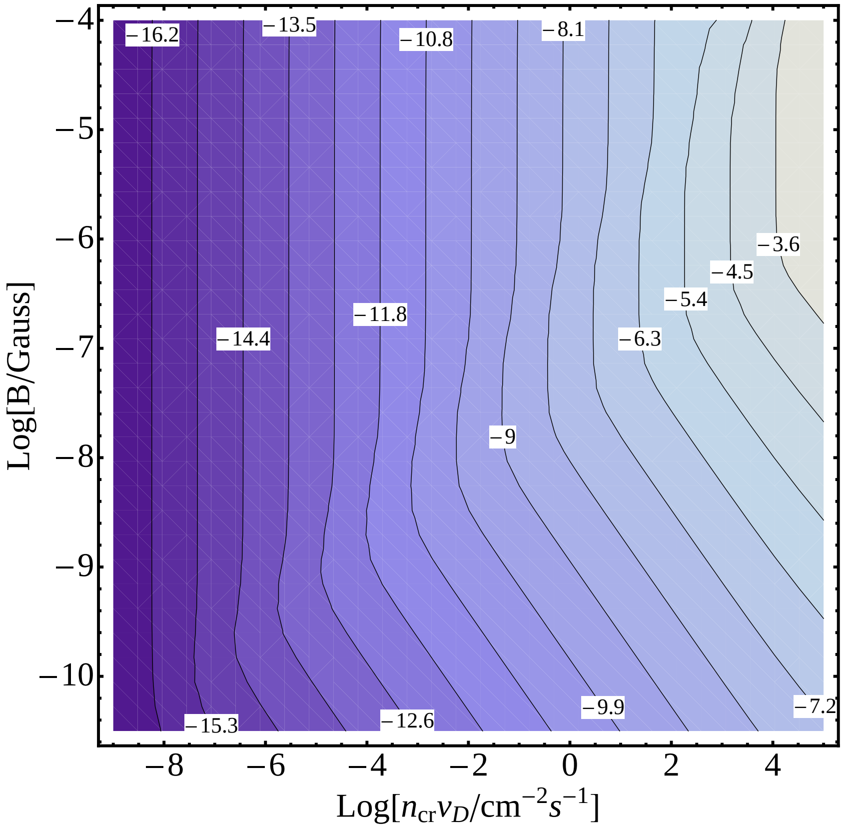
Instability domains for $n_i = 1$ cm^{-3}, $T = 10^4 K$. Above the blue line & outside the vertical bars only resonant instabilities exist. Between the red & blue lines the nonresonant instability is standard Bell, and below the red line, modified Bell.

Domains of Instability – ICM



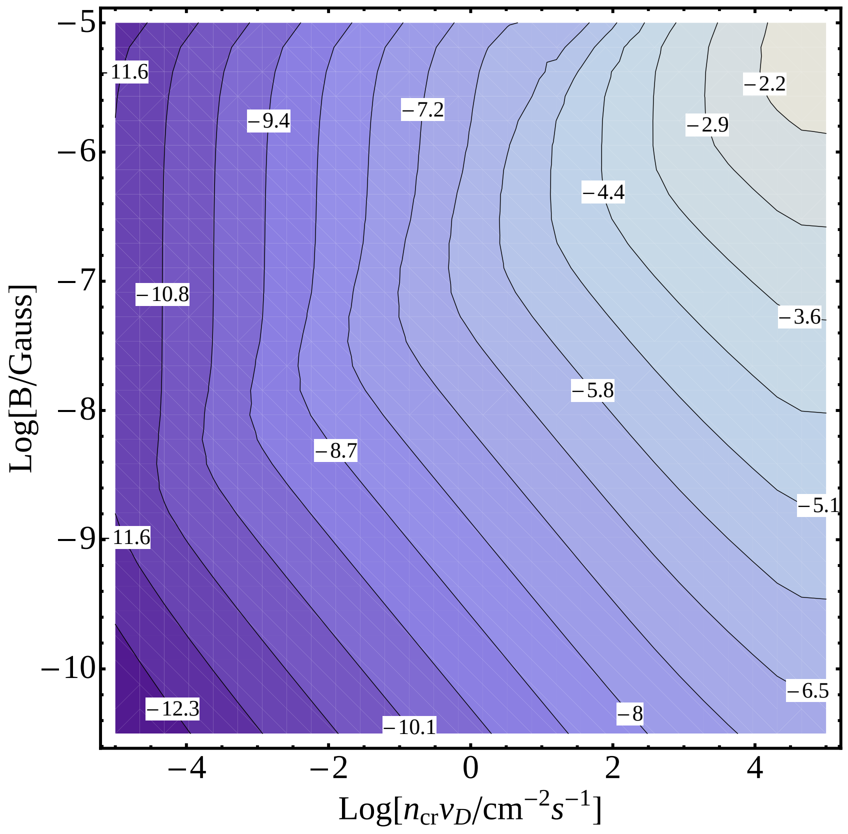
Instability domains for $n_i = 10^{-3}$ cm^{-3}, $T = 10^7 K$. Above the blue line & outside the vertical bars only resonant instabilities exist. Between the red & blue lines the nonresonant instability is standard Bell, and below the red line, modified Bell.

Growth Rates – ISM



Contour plot of maximum growth rate for ISM parameters, $n_i = 1$ cm^{-3}, $T = 10^4 K$.

Growth Rates – ICM



Contour plot of maximum growth rate for ICM parameters, $n_i = 10^{-3}$ cm^{-3}, $T = 10^7 K$.

Examples – ISM

- An estimated young SNR cosmic ray flux of 10^4 cm^{-2} s^{-1} excites the Bell instability for $1.1\mu G < B < 87\mu G$ if $n_i = 1$, $T = 10^4 K$.
- The Galactic flux ($n_{cr} v_D \sim 10^{-2}$) excites the standard Bell instability for $1.1 \times 10^{-8} G < B < 8.7 \times 10^{-8} G$ and the modified Bell instability for $B < 1.1 \times 10^{-8} G$.
- In low density ISM ($n_i = 3 \times 10^{-3}$), if $n_{cr}/n_i \sim 10^{-5}$, $\omega_{Bell} > \omega_{ci} v_S^2 / c^2$ requires $B < 1.3\mu G$.

Examples – ICM

- For $n = 10^{-3}$, $T = 10^7 K$, $\frac{n_{cr}}{n_i} = 10^{-5}$, the standard Bell instability operates for $.16\mu G < B < .87\mu G$, and the modified Bell instability for $B < .16\mu G$.
- Stronger B sometimes observed, but nonresonant instability is plausible.

Summary

- We explored nonresonant instability in $n_{cr}v_D$, B, T space.
- If the ions are hot, any instability depends on the thermal electron distribution function, which must be determined self consistently.
- Nonresonant instability exists for a wide range of fluxes & could be an important source of helical magnetic fluctuations.