Diffractive imaging in the presence of noise

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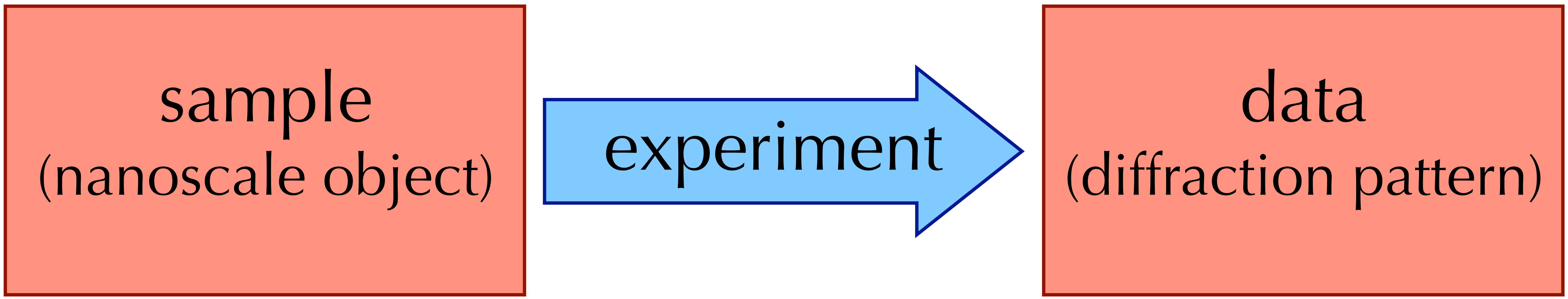
X-ray Science in the 21st Century
KITP, Santa Barbara

Experiment as a noisy communication channel



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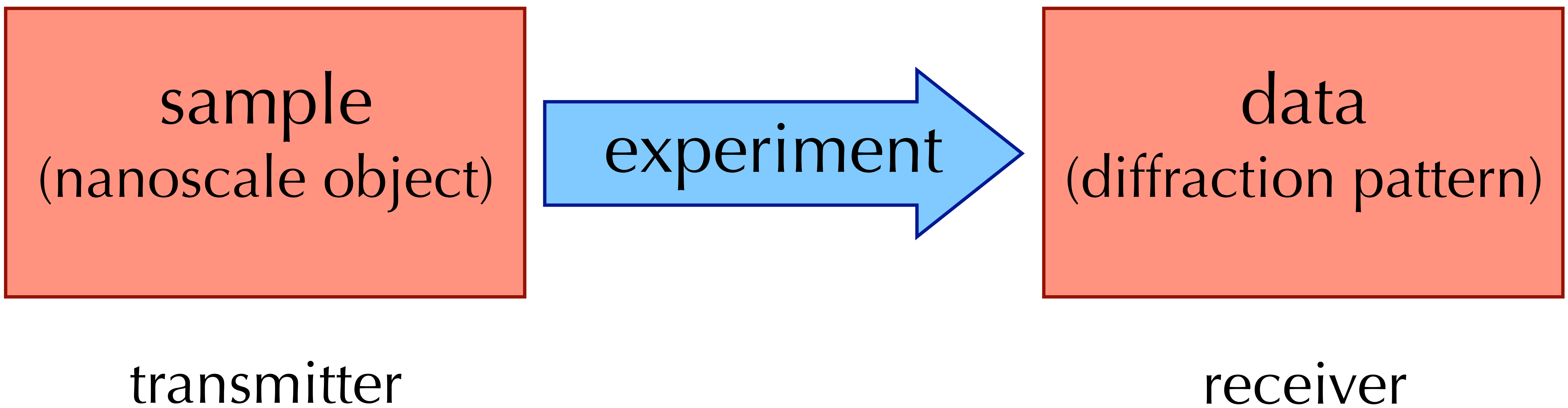

Experiment as a noisy communication channel

sample
(nanoscale object)

experiment

data
(diffraction pattern)

noisy channel

transmitter

receiver

`channel capacity' of experiments

C.E. Shannon

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H(s) = entropy of the sample

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H(s|d) = conditional entropy of the sample given the data

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$H(s)$ = entropy of the sample

$H(s|d)$ = conditional entropy of the sample given the data

C.E. Shannon

$I(s,d)$ = information capacity of the experiment

$= H(s) - H(s|d)$

(mutual information)

experimentalists:

increase the channel capacity

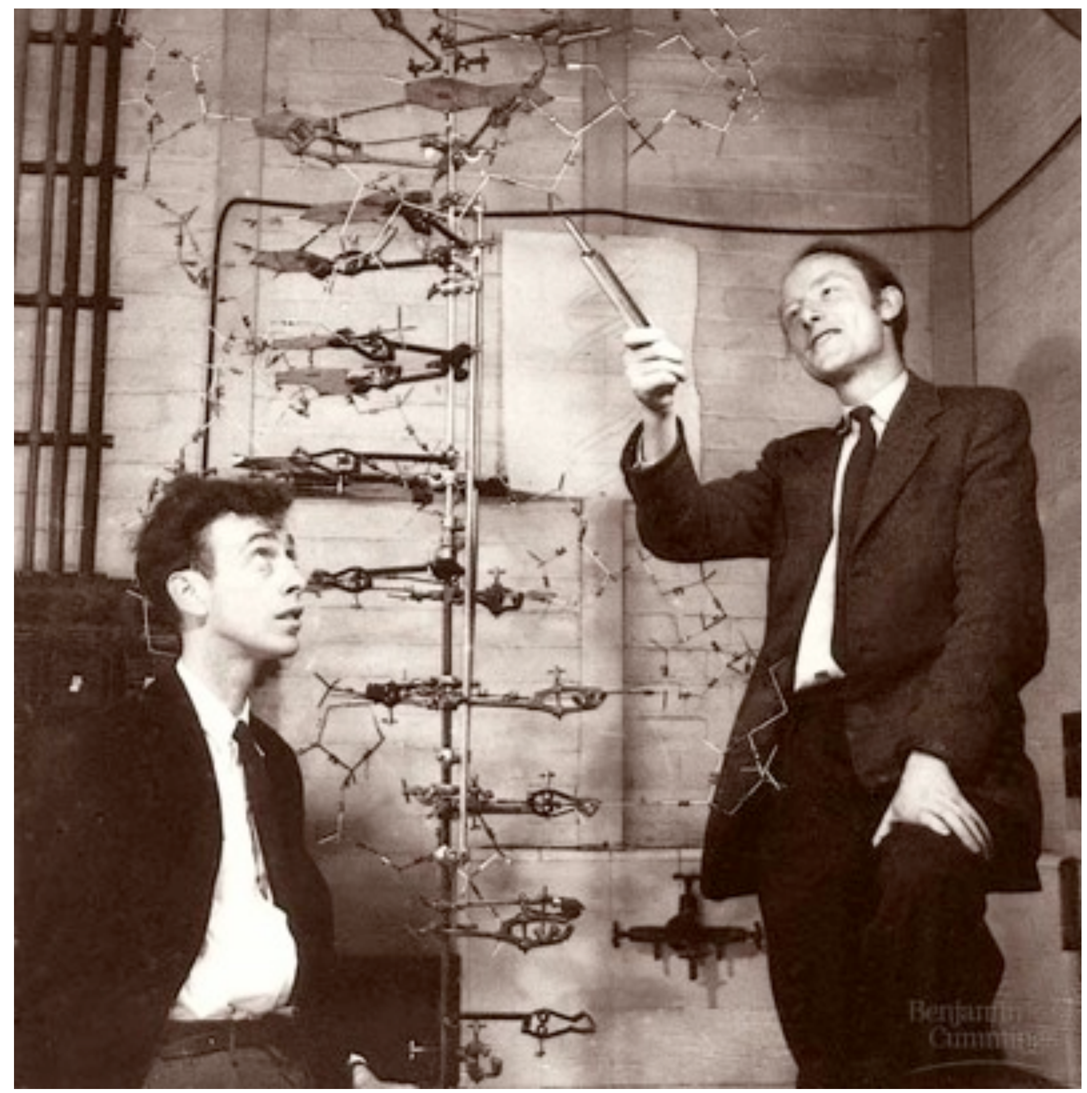
theorists:

decode the data

experimentalists:
increase the channel capacity

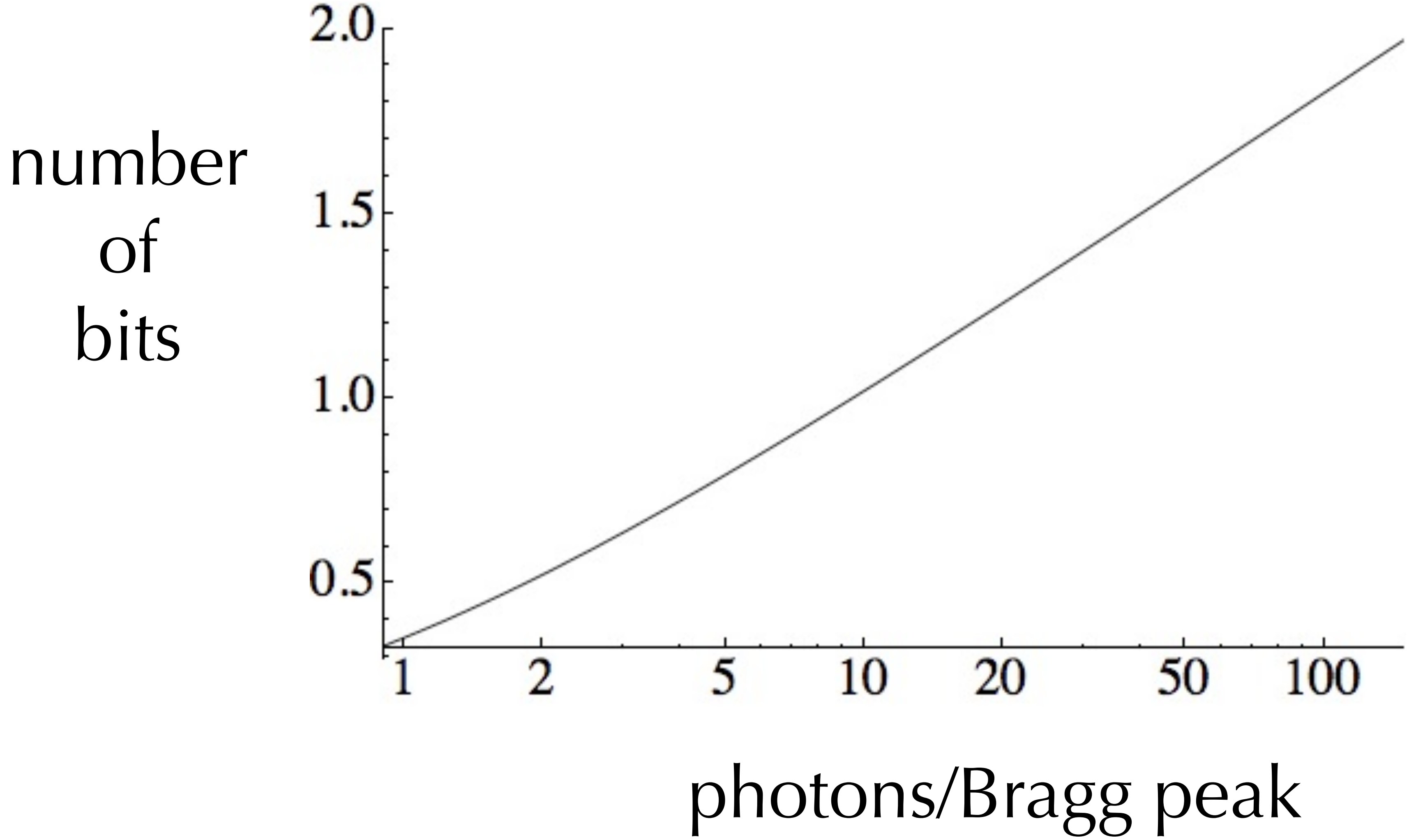
Franklin

theorists:
decode the data

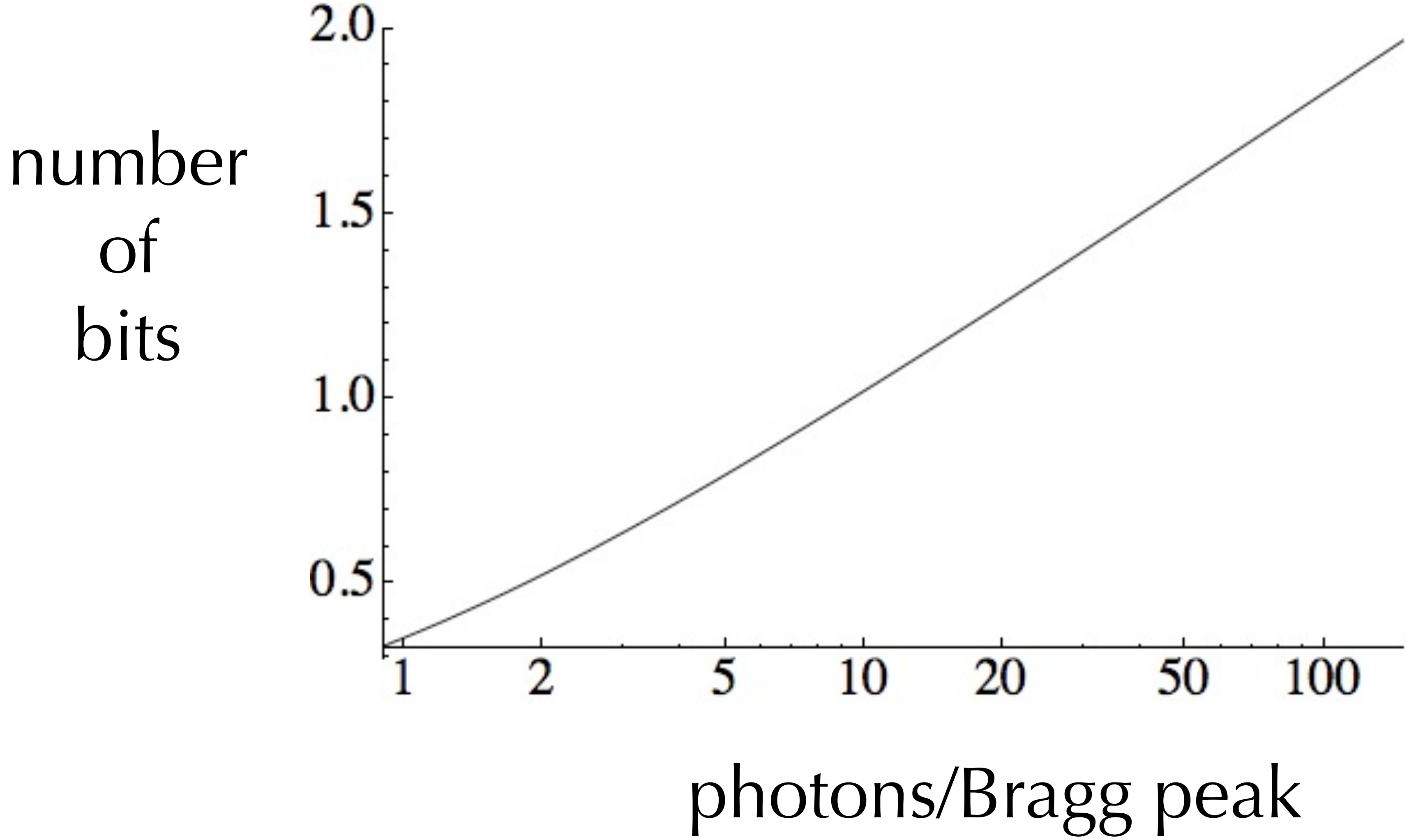


Crick & Watson

example: information in a Bragg peak



example: information in a Bragg peak



$$I(\mu) = (\mu + \tfrac{1}{2}) \log_2 (2\mu + 1) - \frac{\gamma\mu}{\log 2} - \frac{1}{2} \sum_{k=2}^{\infty} \frac{\log_2 k}{(1 + \frac{1}{2\mu})^k}$$

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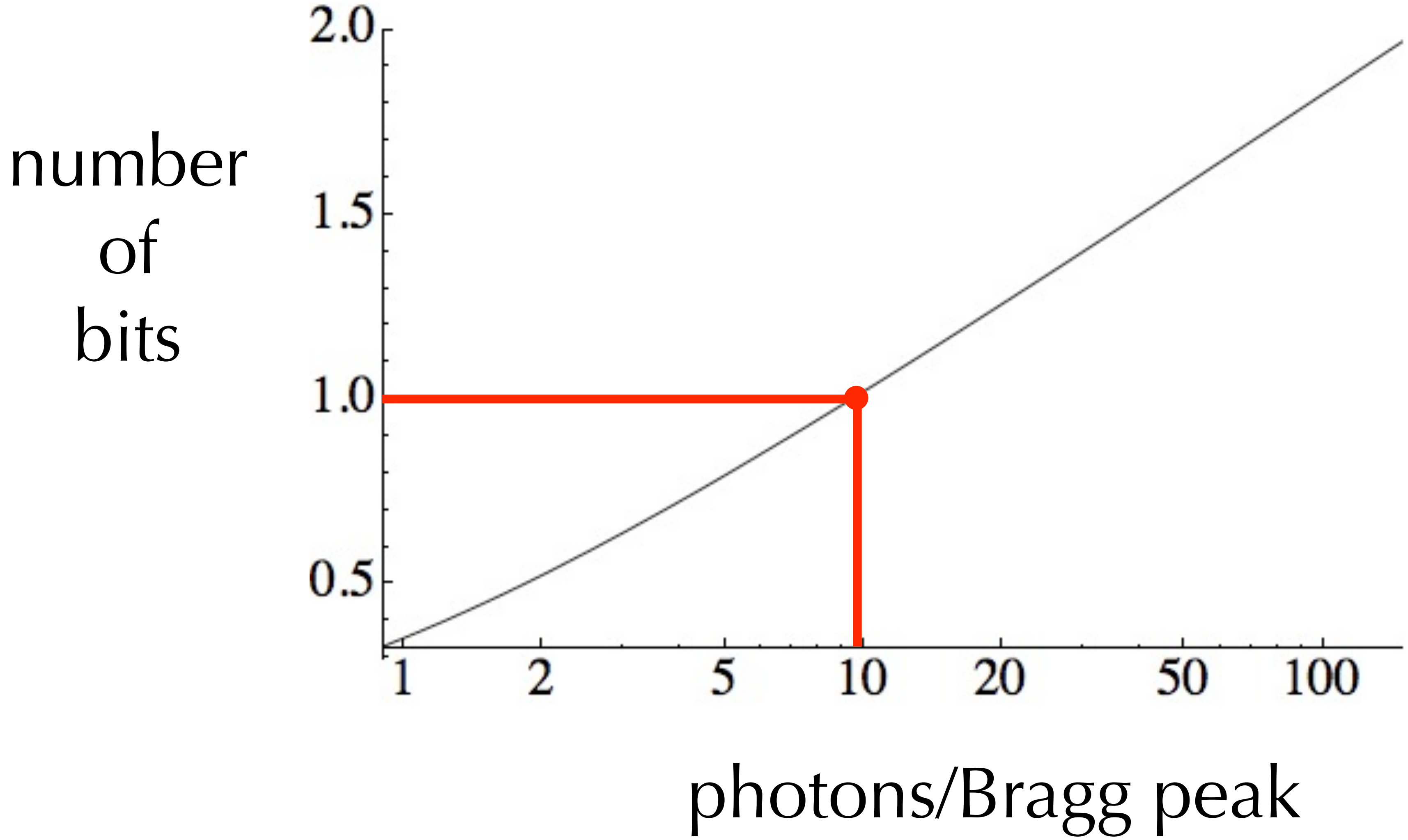

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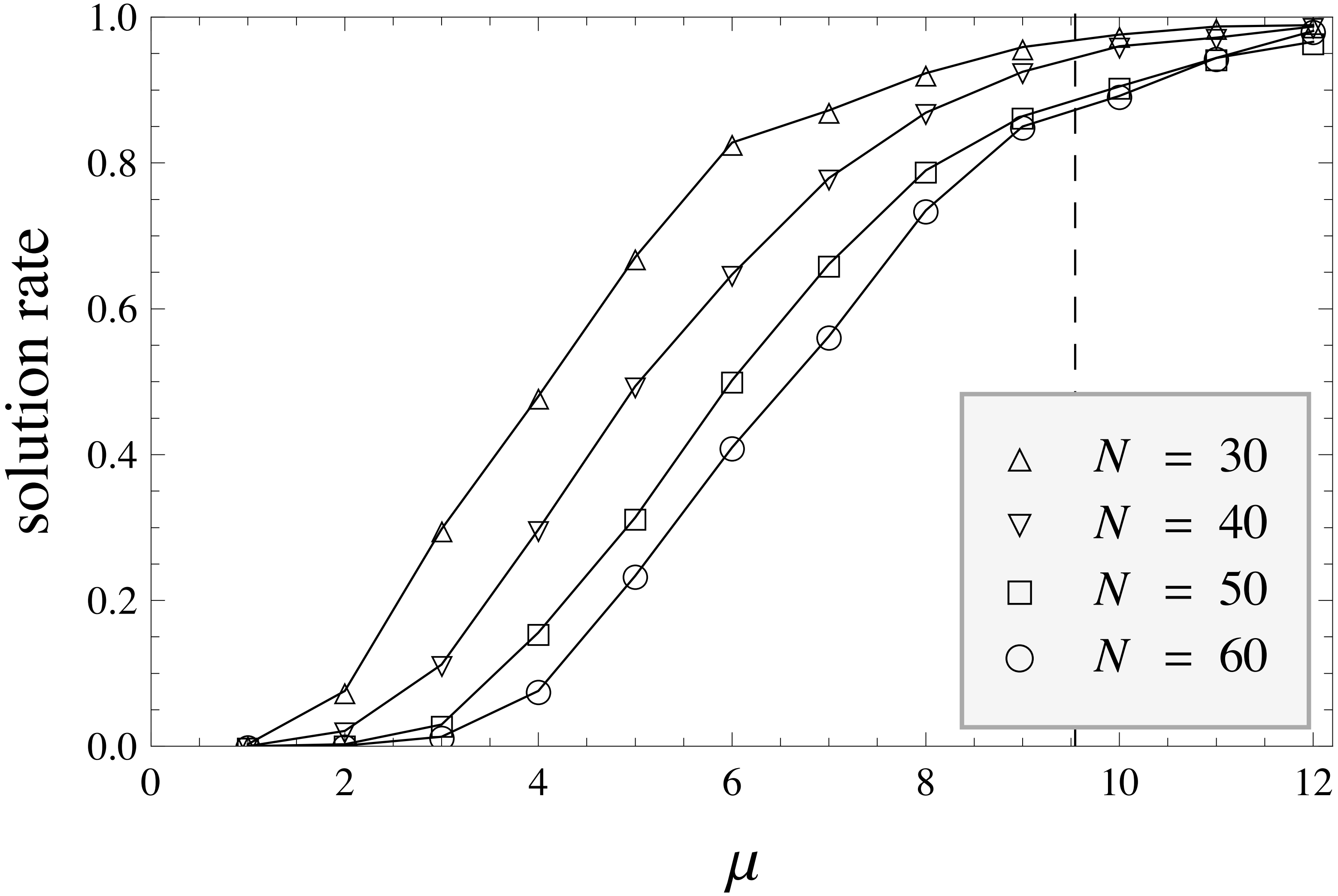
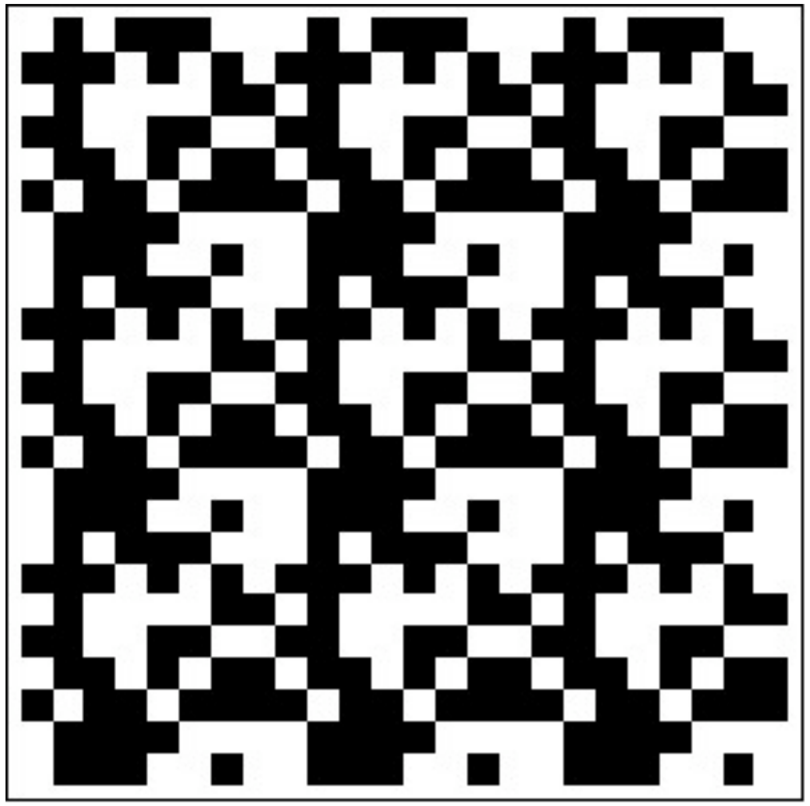


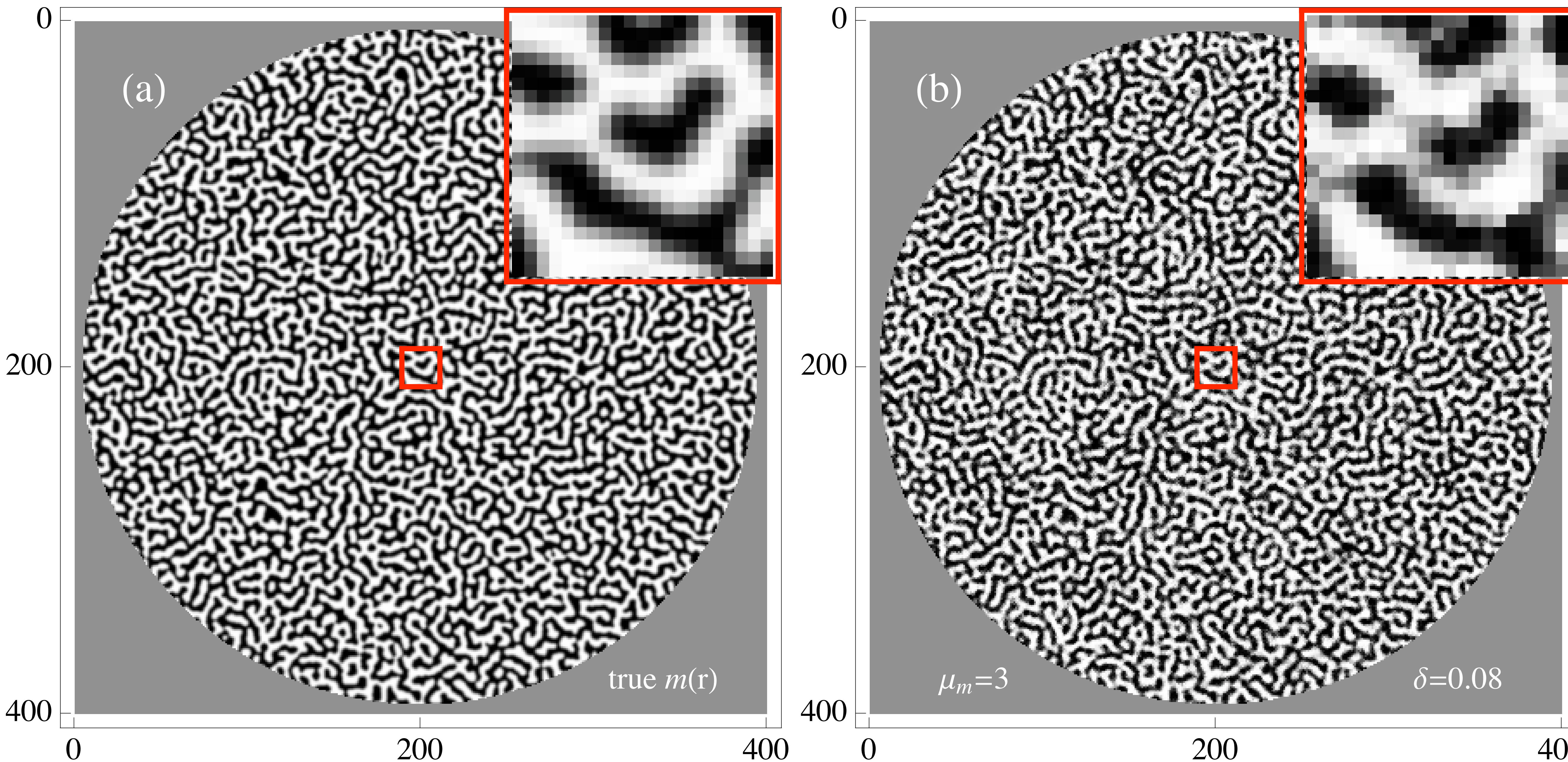
Elser & Eisebitt, NJP (2010)



decoding:
difference map algorithm with binary value constraint

LDPC decoding: Yedidia, Wang & Draper, Physics of Algorithms (2009)

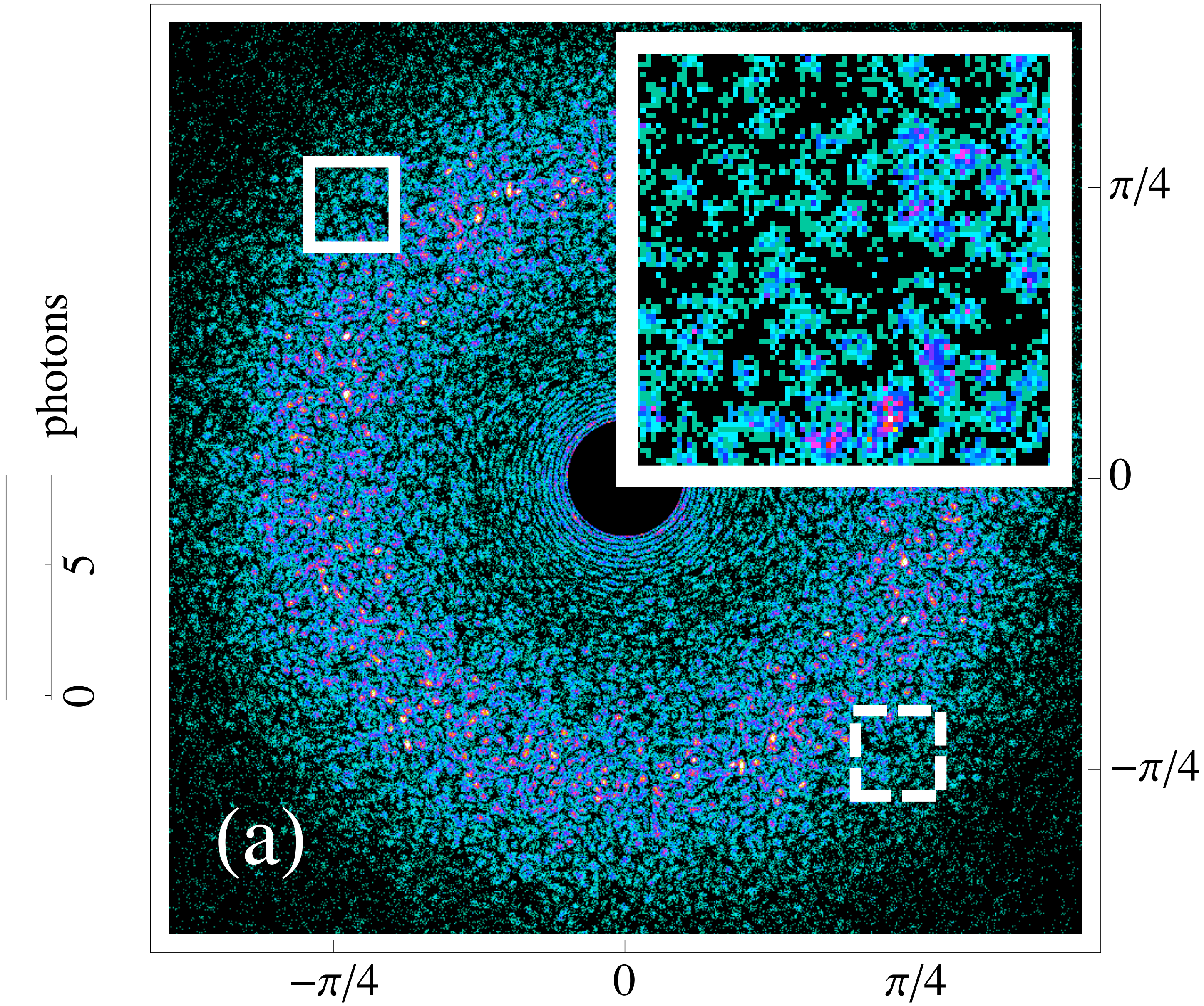
diffractive imaging of magnetic domains



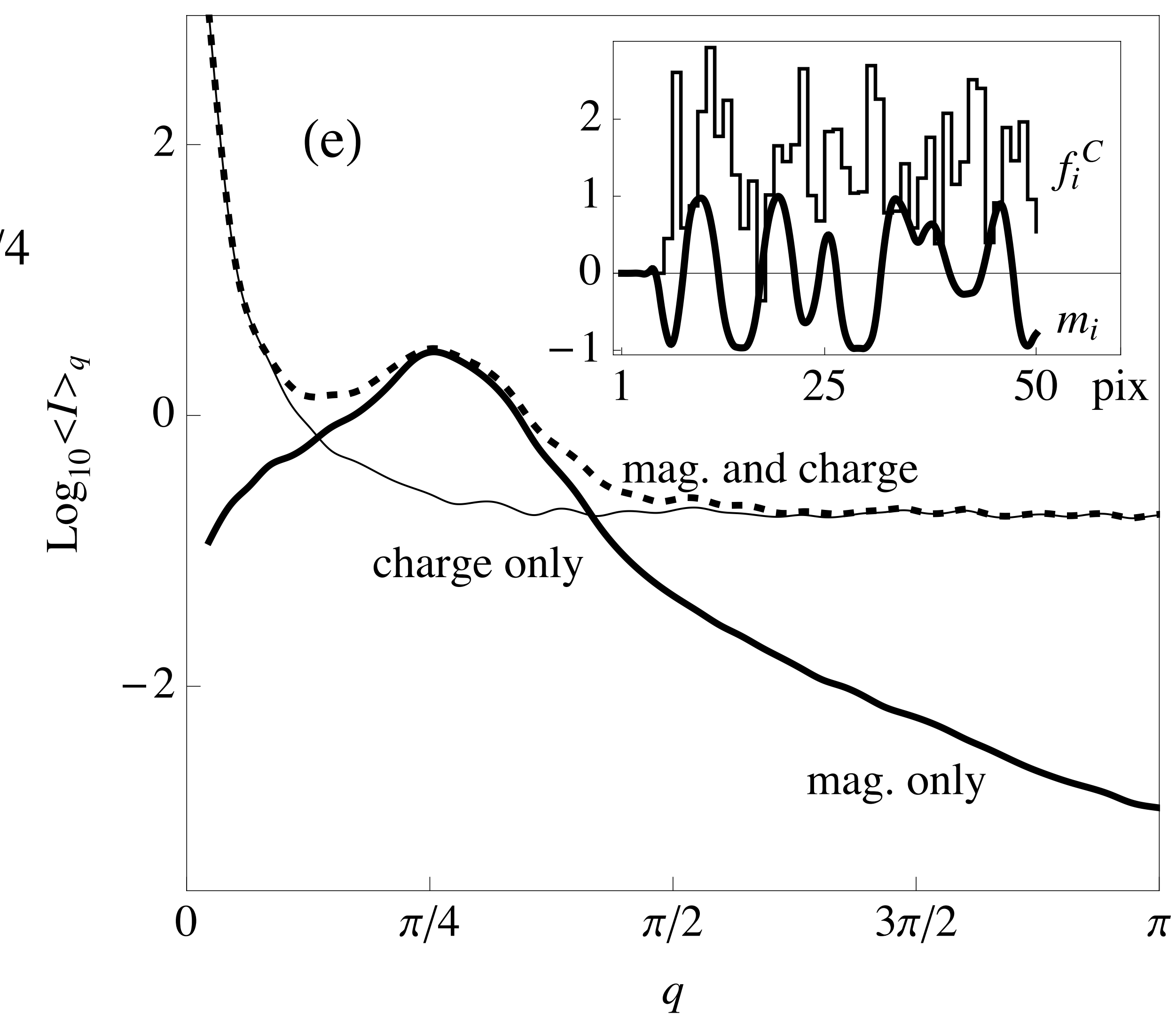
true contrast

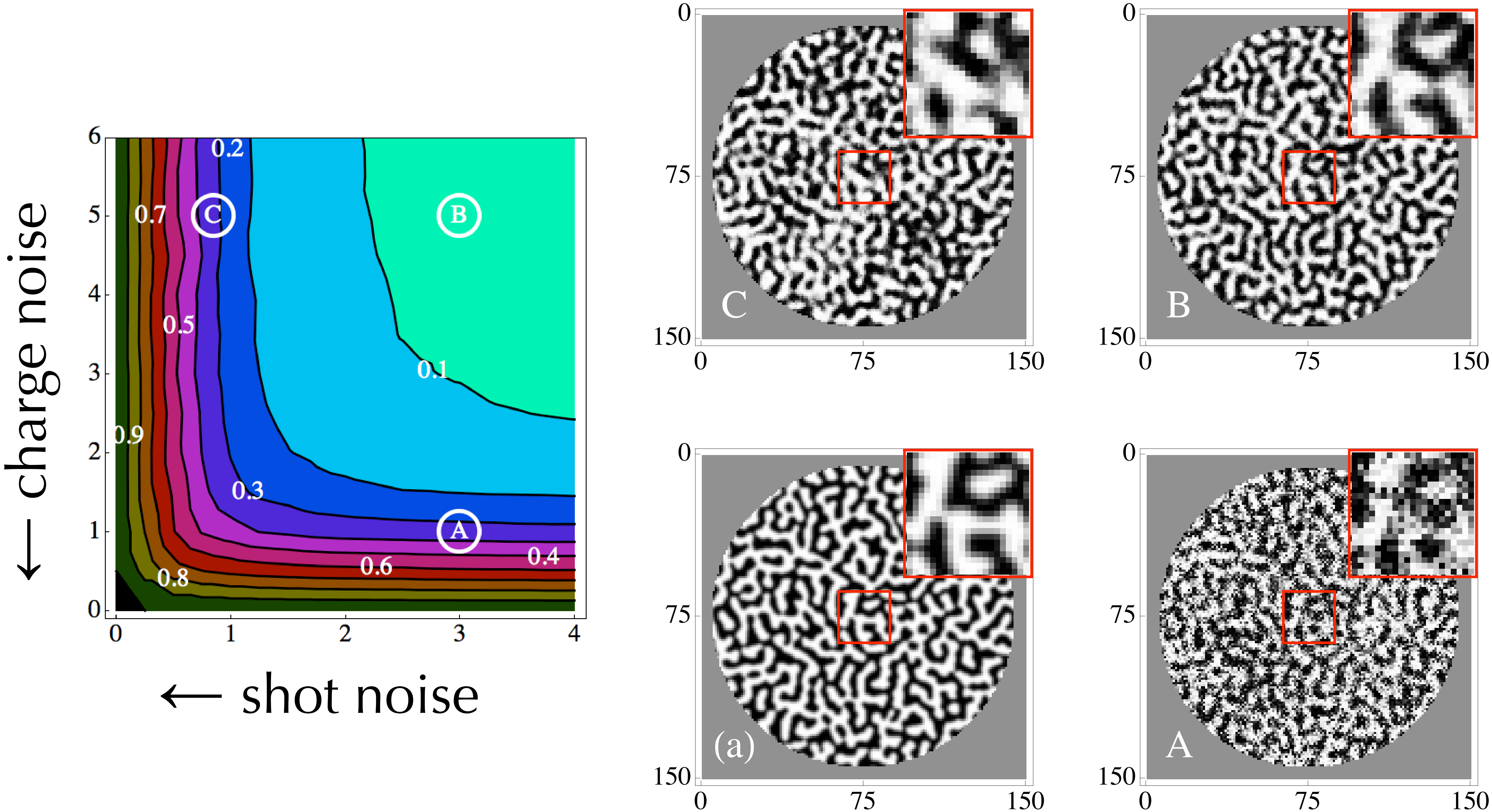
reconstruction

Loh, Eisebitt, Flewett & Elser (2010)



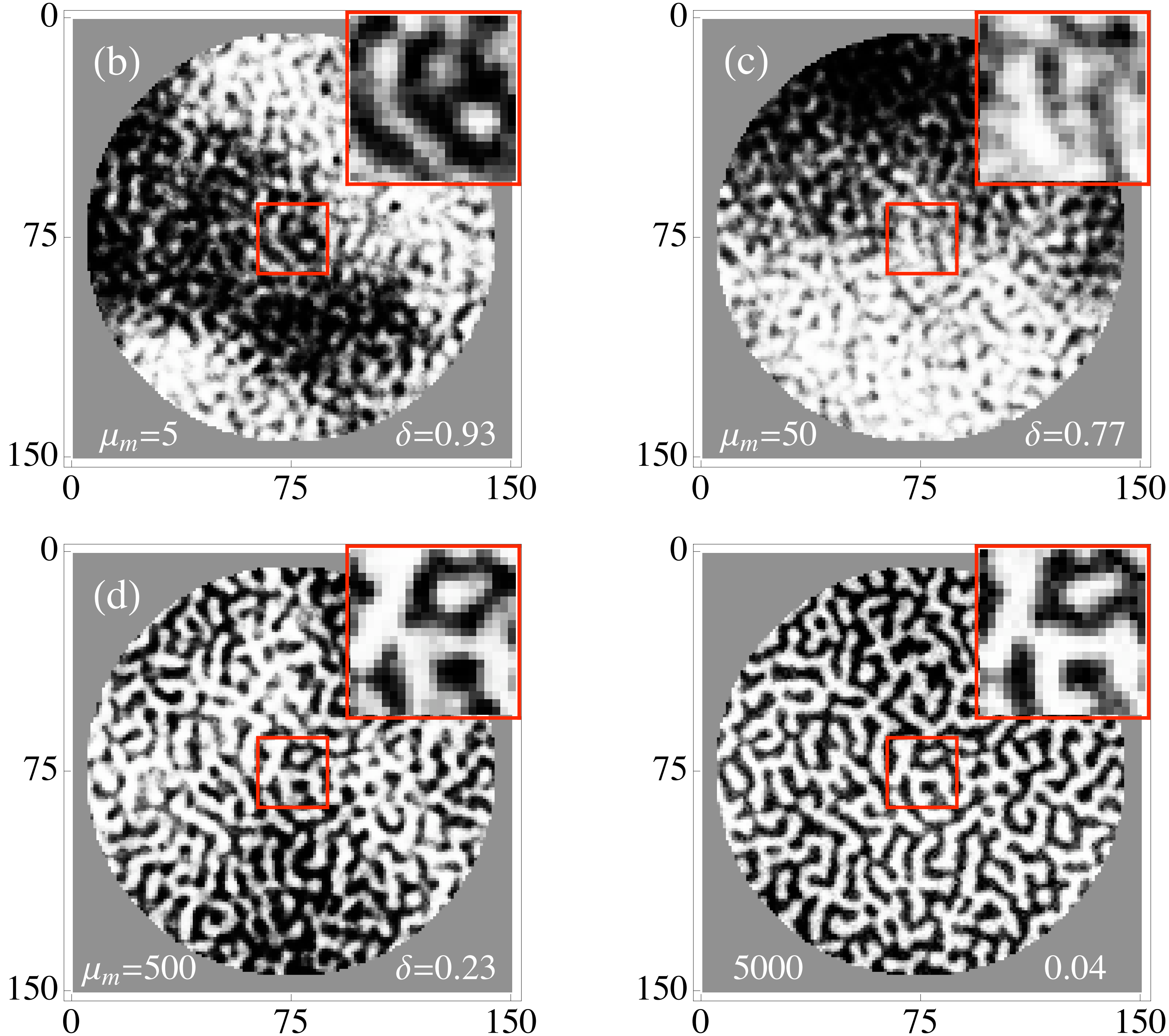
simulated data





← charge noise
← shot noise
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
A
B
C
(a)

Fourier transform holography* simulations



* Eisebitt et al., Nature 432, 885-888 (2004)

decoding tricks:

- histogram constraint on contrast
- speckle-filter intensity
- error-stabilized algorithm

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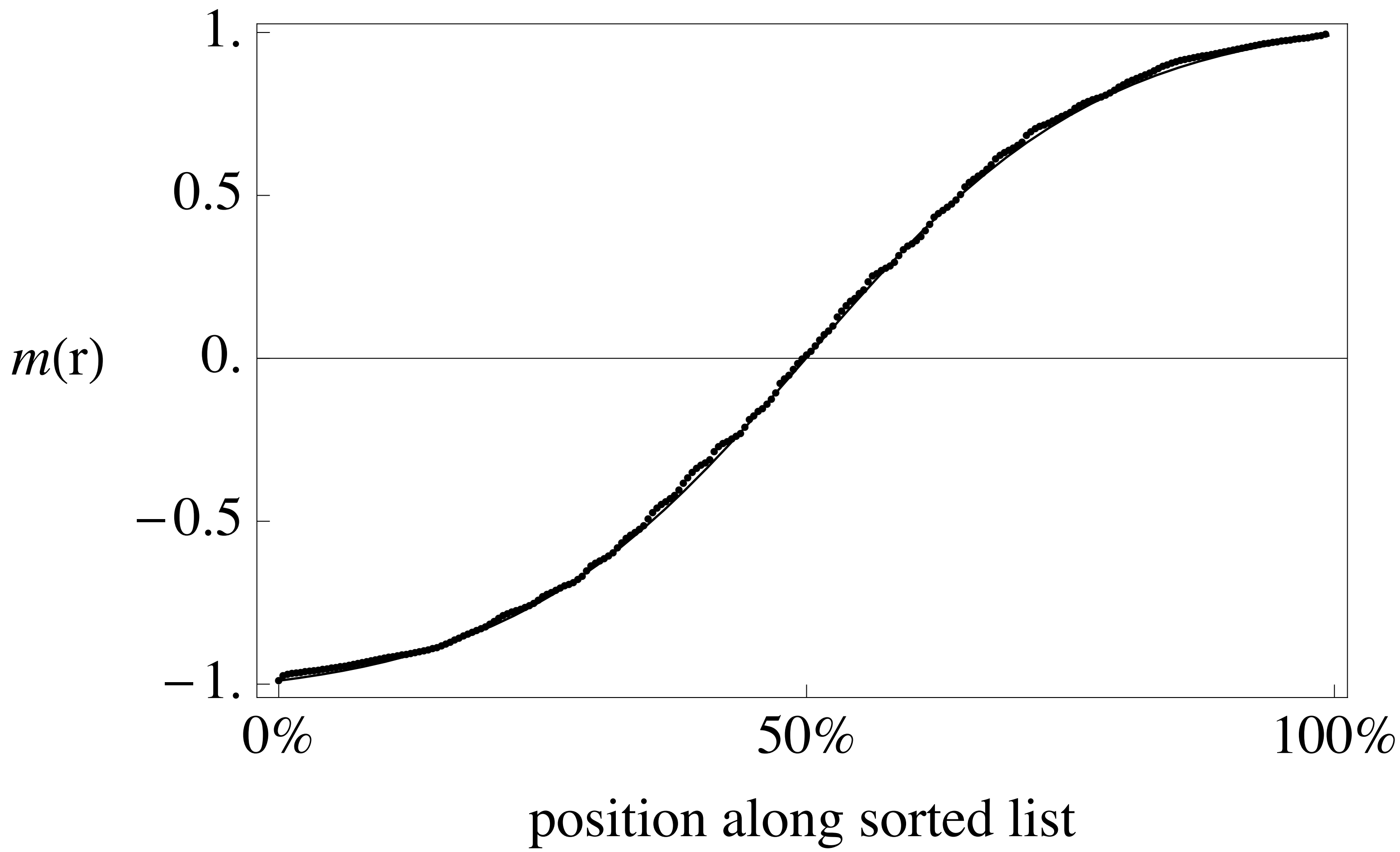

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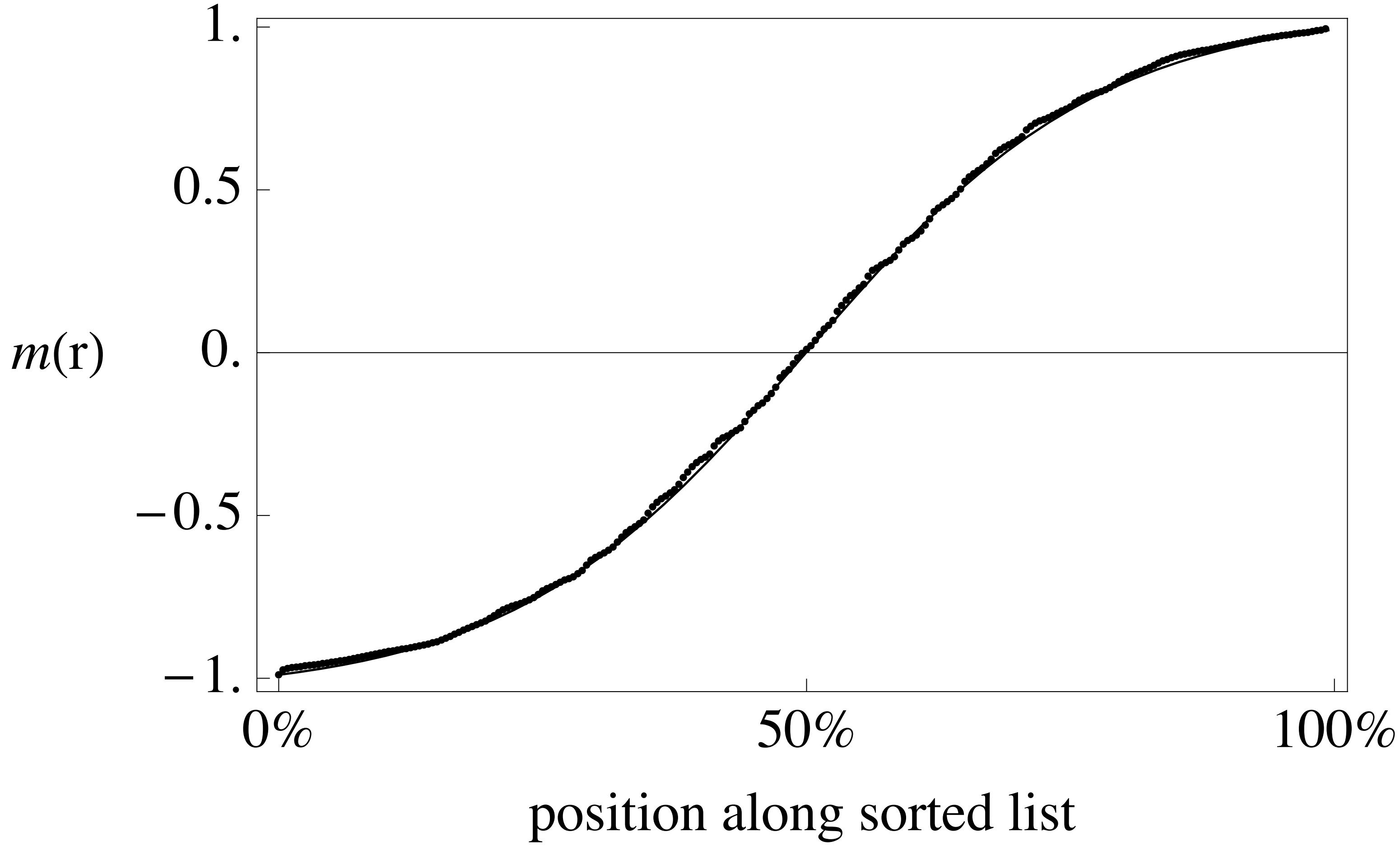

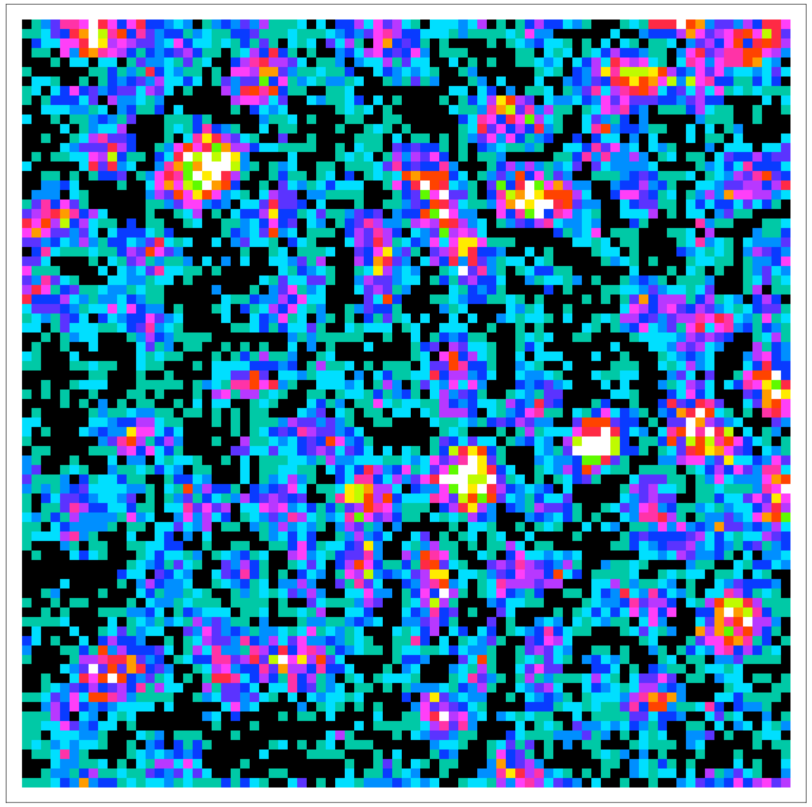

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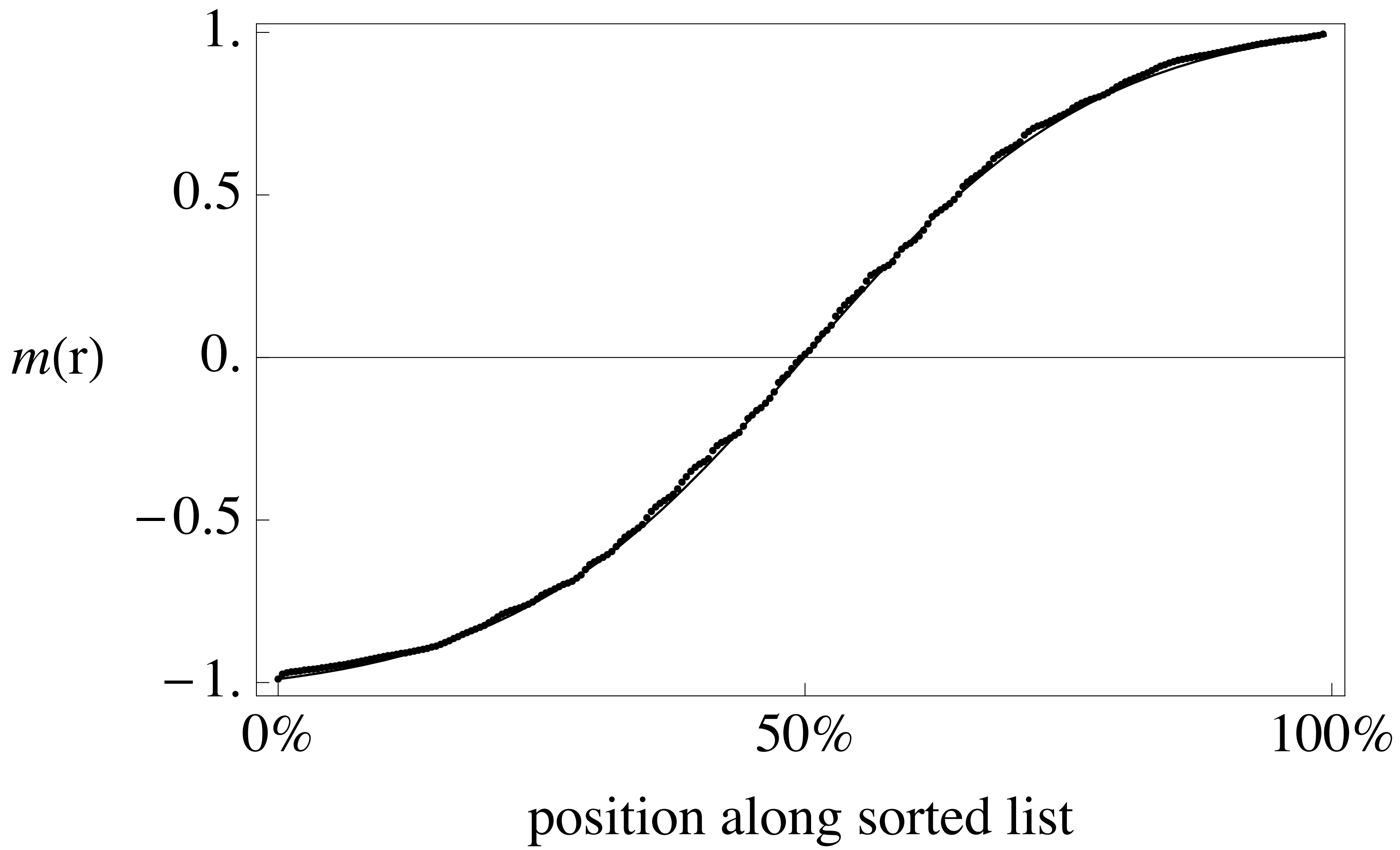
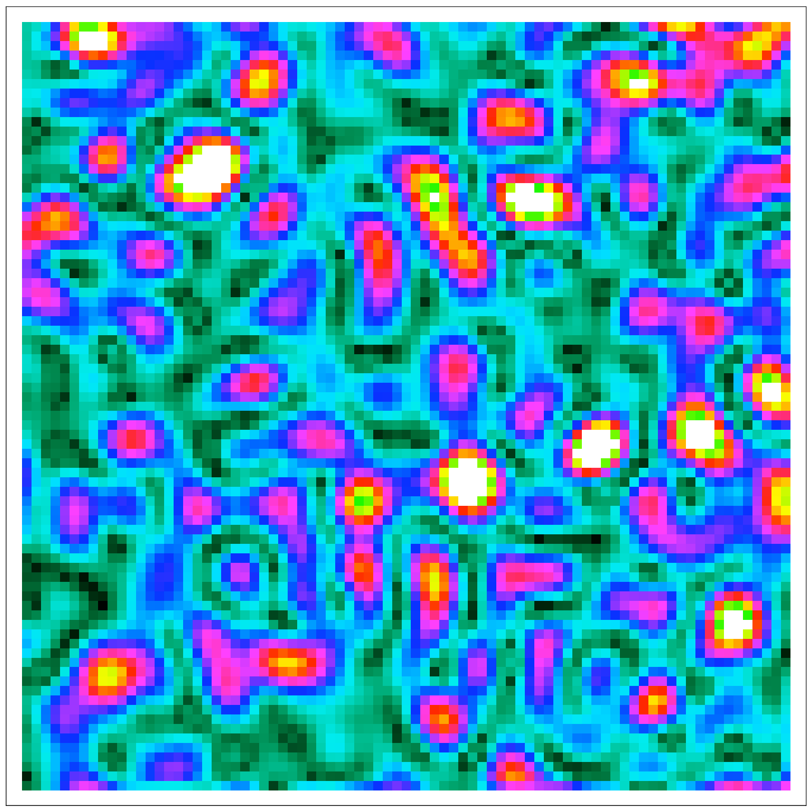
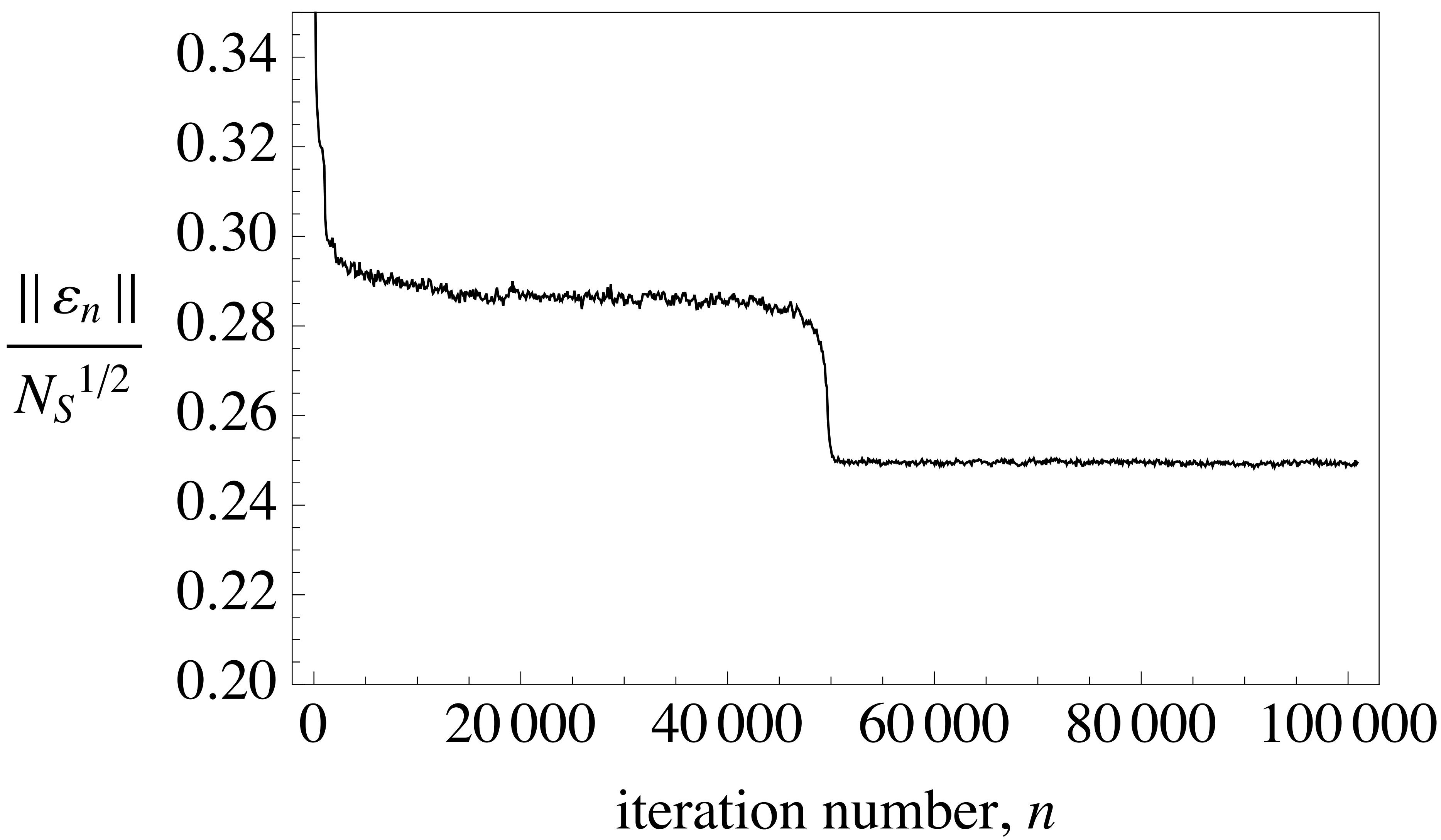




reconstruction (decoding) algorithm



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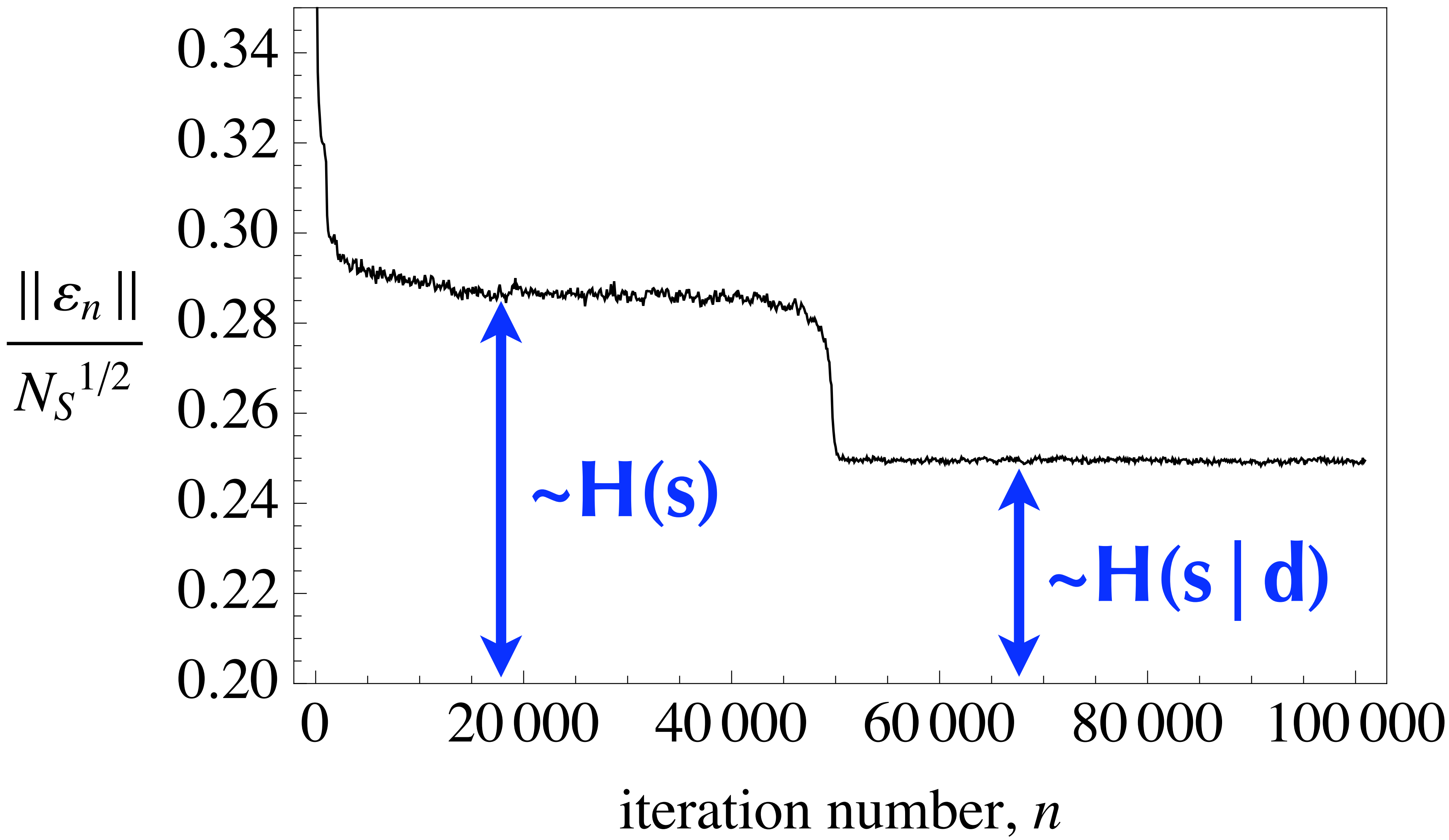
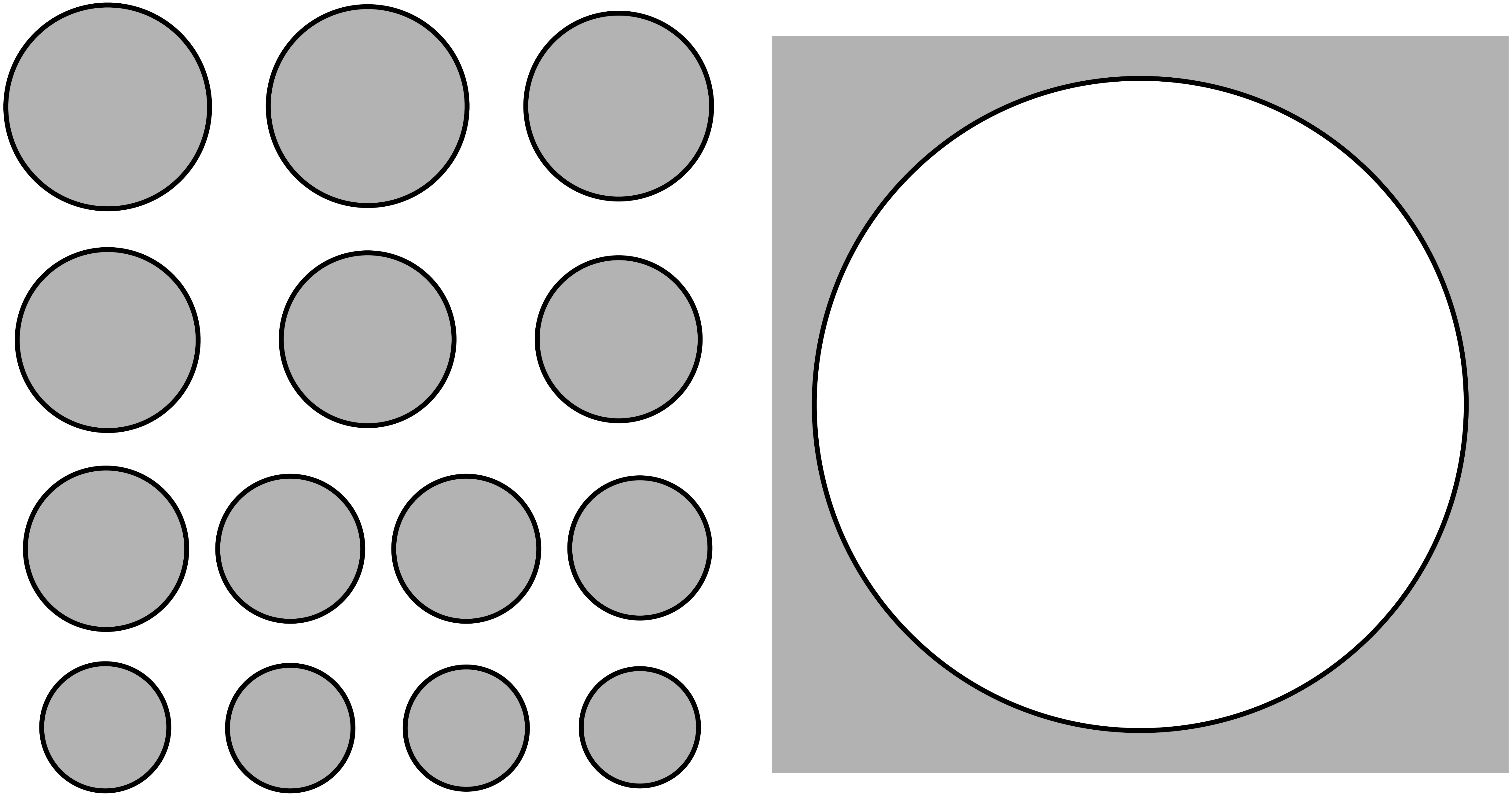




information measures for single particle imaging

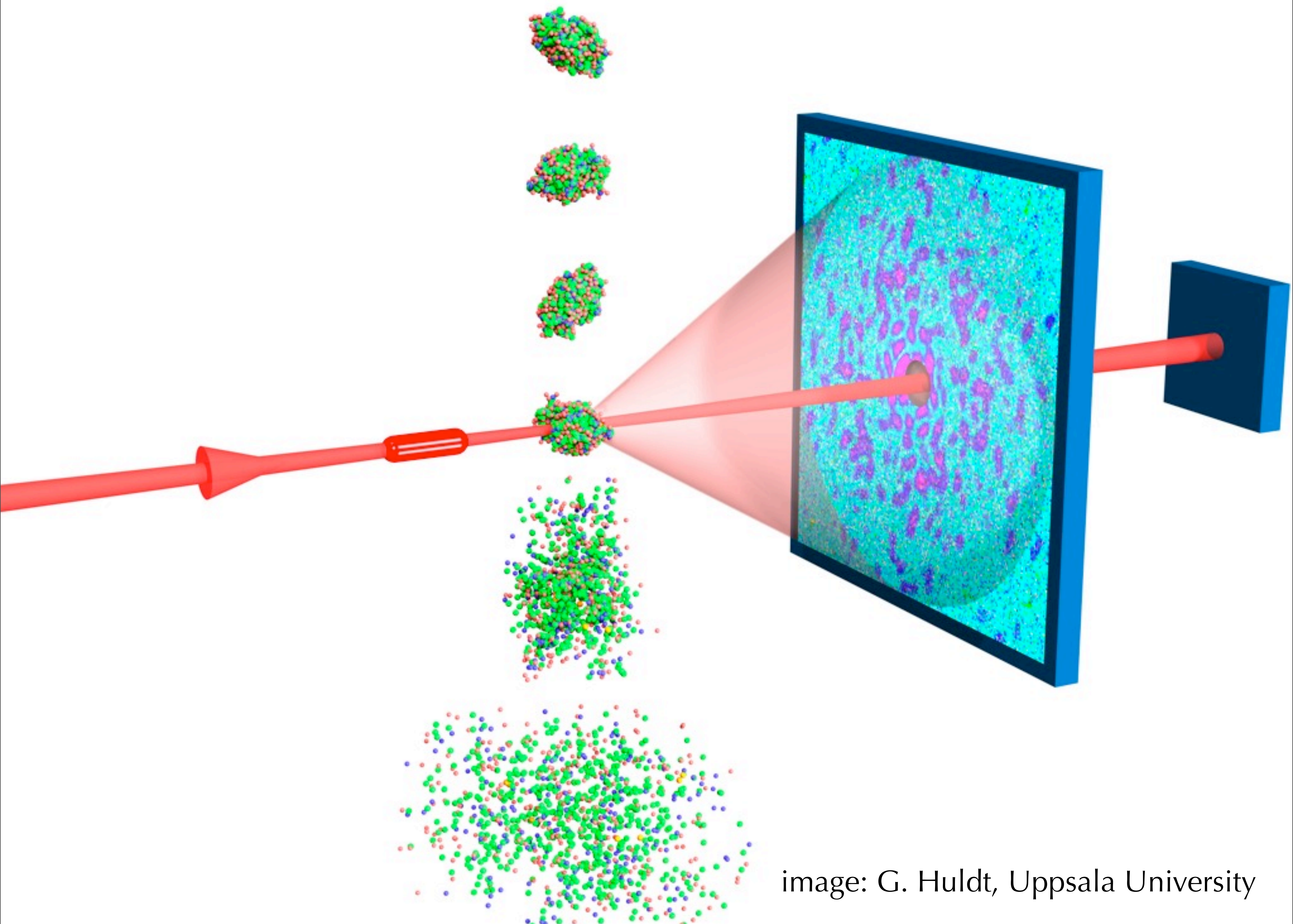
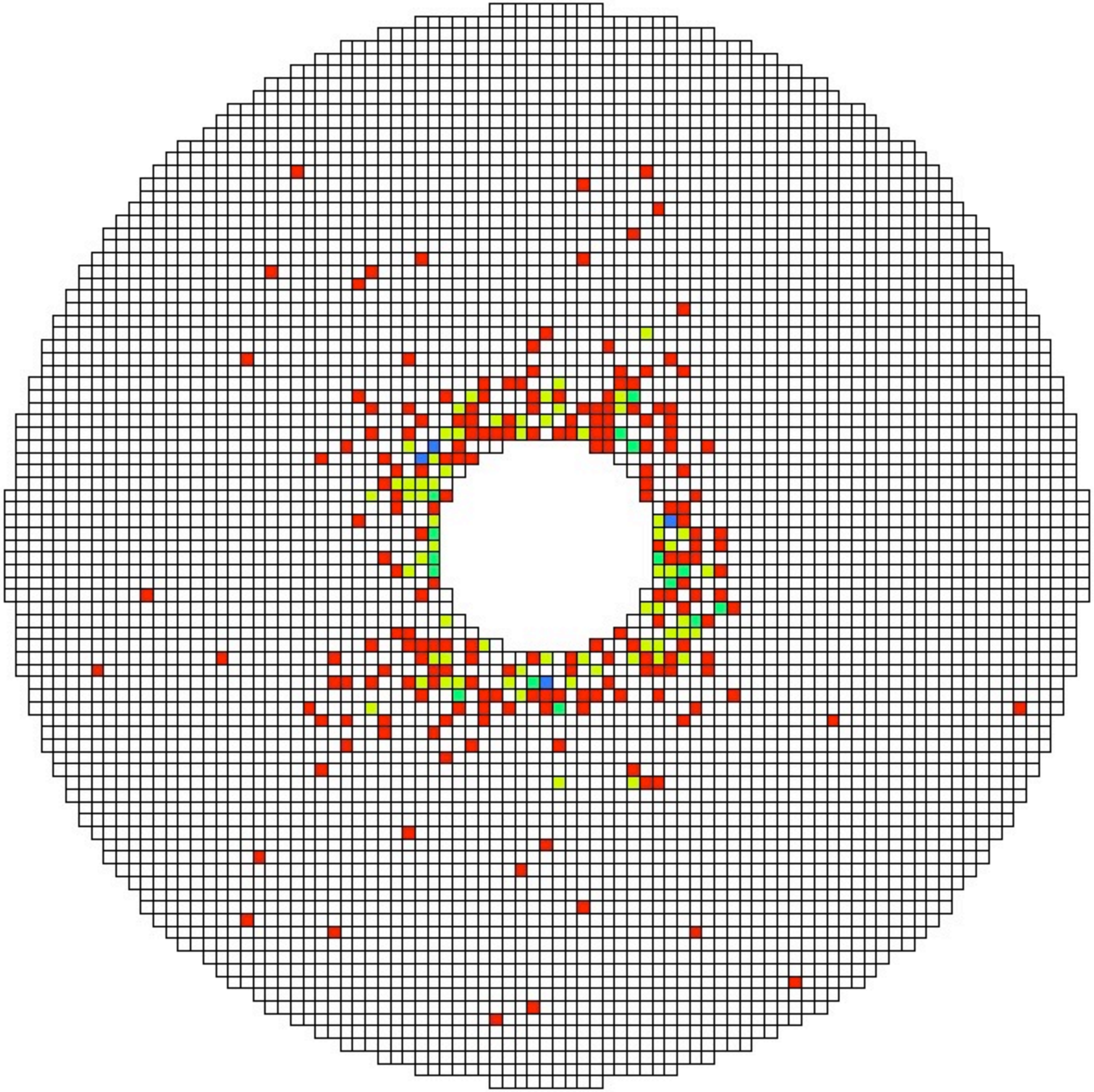
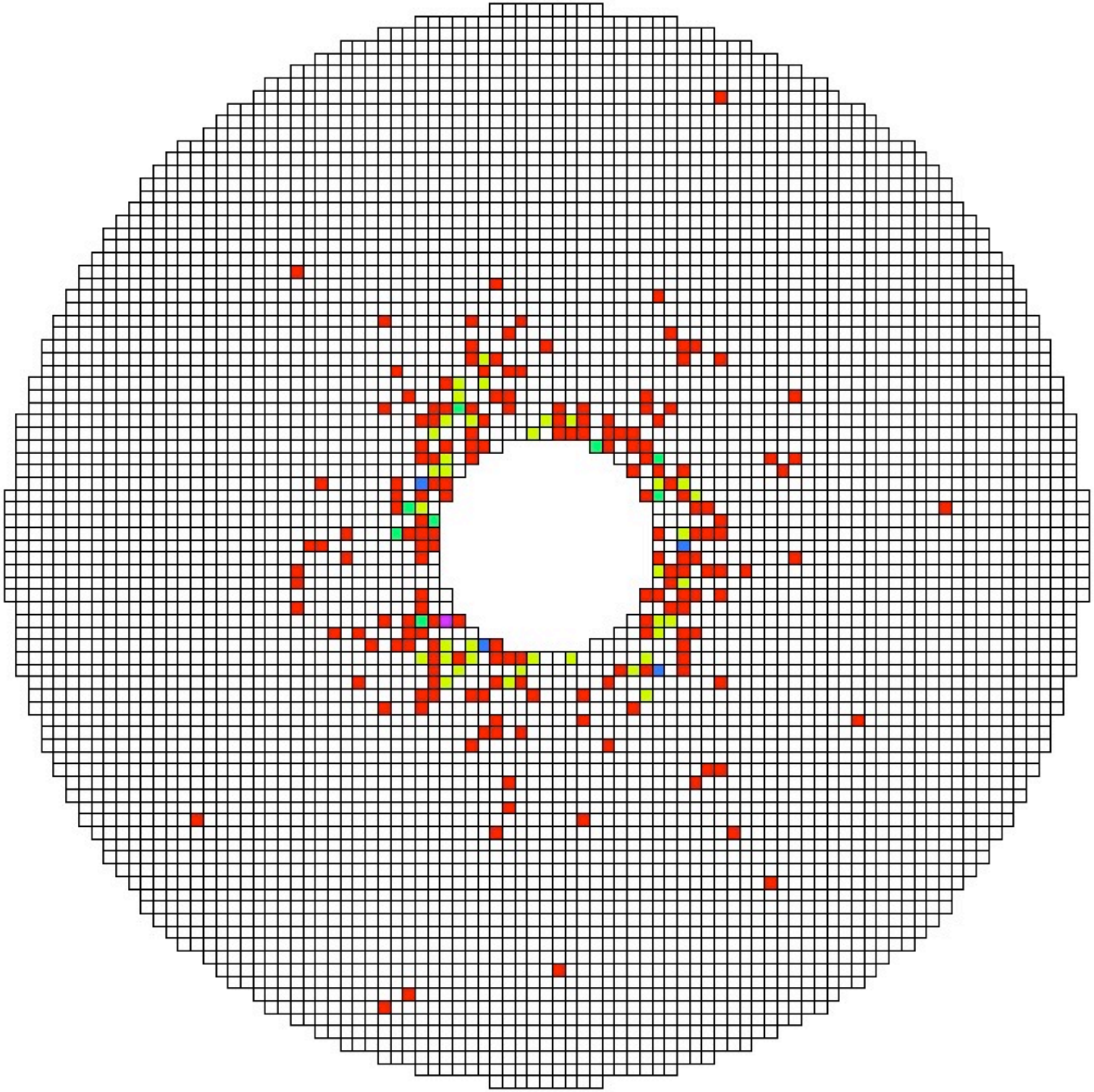


image: G. Huldt, Uppsala University





diffraction data: $D = \{K_1, K_2, \ldots, K_N\}$ (N hits)

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information capacity of experiment: $I(\{W, R\}, D)$

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but $N \sim 10^6 - 10^7$ ...

need more practical information measures

data from one hit: K

particle orientation in one hit: Ω

3D intensity: W

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particle orientation in one hit: Ω

3D intensity: W

I(K,W) = information rate in experiment with unknown orientation

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particle orientation in one hit: Ω

3D intensity: W

$I(K,W)$ = information rate in experiment with unknown orientation

$I(K,W)|_\Omega$ = information rate in experiment with known orientation

reduced information rate

$$r = \frac{I(K,W)}{I(K,W)|_{\Omega}}$$ = ratio of information rates without/with orientation

V. Elser, IEEE Trans. Information Theory 55, 4715-22 (2009)

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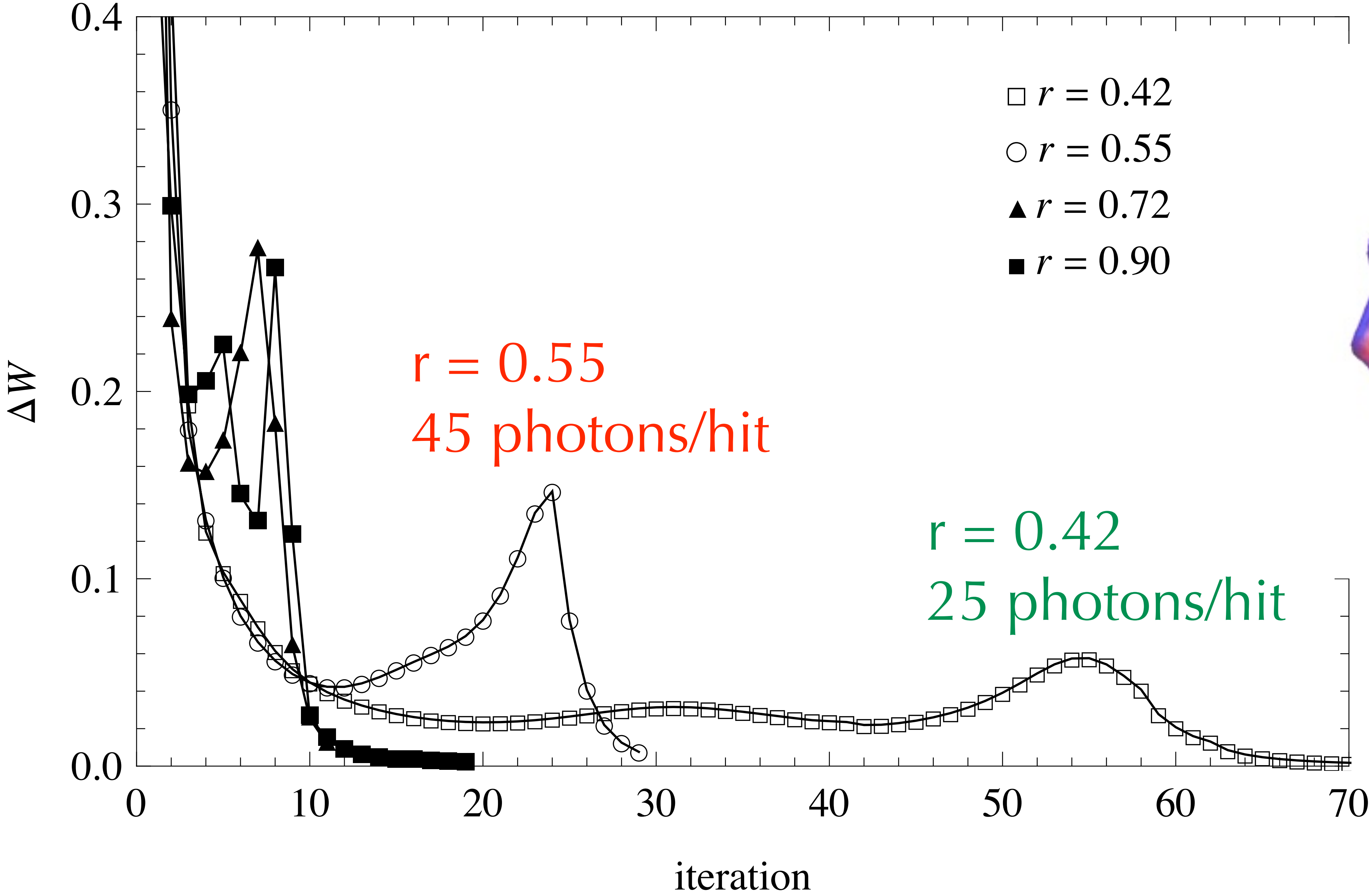
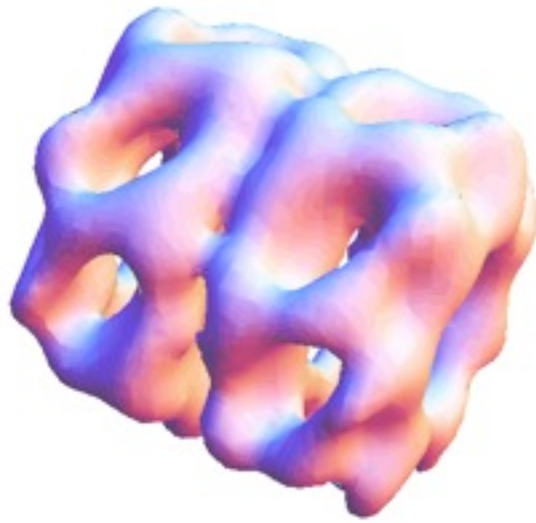
r = 1/2 : information in one hit = information obtained about **orientation**

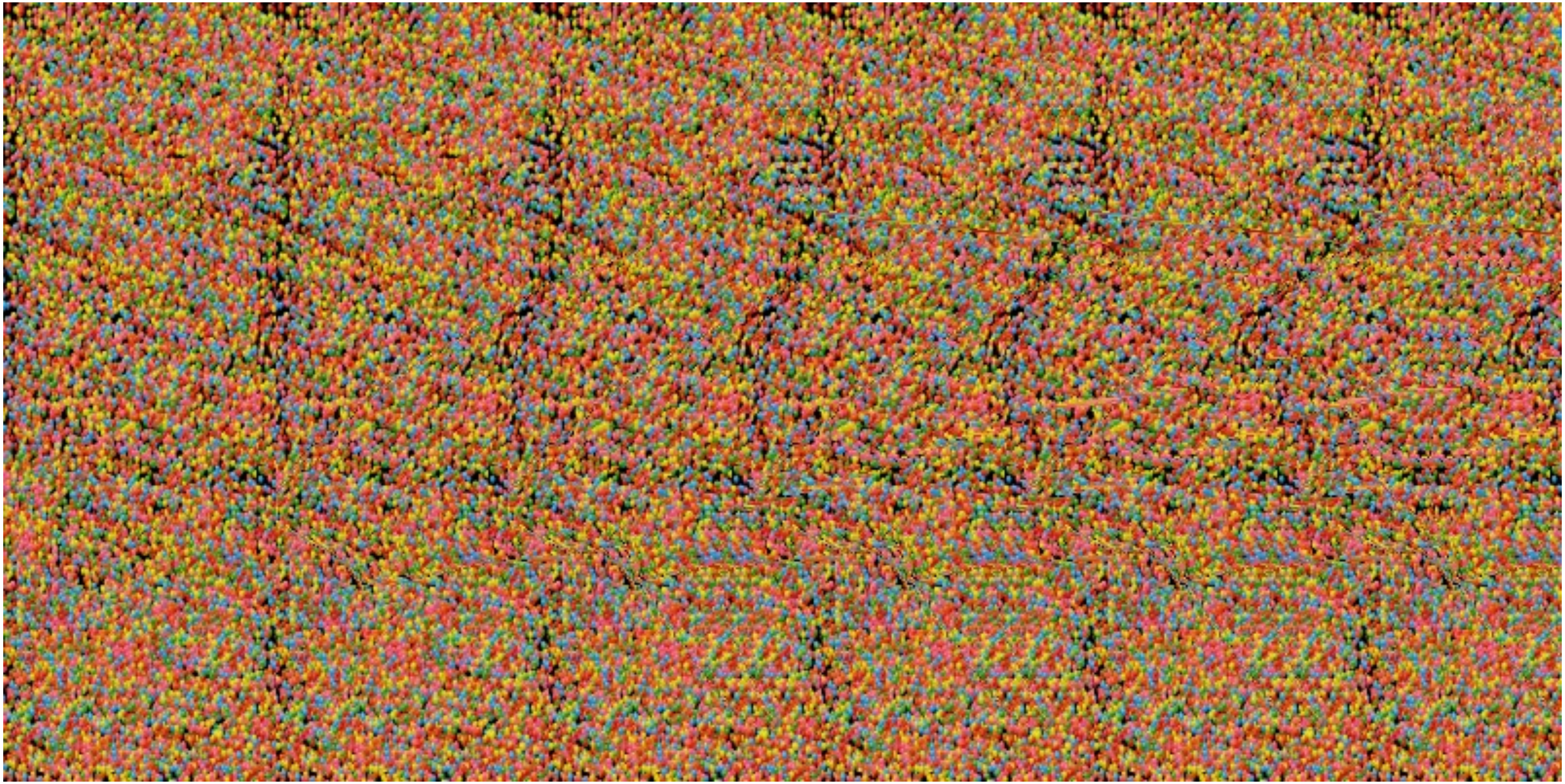
V. Elser, IEEE Trans. Information Theory 55, 4715-22 (2009)

EMC algorithm convergence





Loh & Elser, Phys. Rev. E 80, 026705 (2009)



Thank you for your attention