Strong-field approximation (SFA) vs Coulomb effects

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Direct-electron SFA in brief

$$M_p = -i \int_{-\infty}^{\infty} dt \langle \psi_p^{(Volkov)}(t) \mid V(r) \mid \psi_0(t) \rangle$$

$$\psi_p^{(Volkov)}(r, t) = \frac{1}{(2\pi)^{3/2}} \exp[i(p - eA(t)) \cdot r]$$

$$\times \exp[-\frac{i}{2m} \int^t d\tau (p - eA(\tau))^2]$$

Evaluation by stationary phase:

$$\frac{1}{2m} (p - eA(t))^2 + I_p = 0$$

the (complex) solutions $t = t_s$ determine the tunneling times
Saddle-point approximation to the SFA

\[ M_p \propto \sum_s \sqrt{\frac{2\pi i}{S_p''(t_s)}} \exp[i(I_p t_s + S_p(t_s))] \langle p - eA(t_s) | V | \psi_0 \rangle \]

\[ S_p = \frac{1}{2m} \int d\tau [p - eA(\tau)]^2 \]

\[ t = t_s \text{ (complex) saddle-point solutions} \]

approximately \( p - eA(t_s) = 0 \)
Quantum-orbit expansion of the transition amplitude

\[ M(p) = \sum_{\text{orbits } s} a_s(p) \exp[iS_s(p)] \]

cf. Feynman’s path integral

P. Salières et al., Science 292, 902 (2001)

The quantum orbits are defined by the solutions \((t_s, t'_s, k_s) \ (s = 1, 2, \ldots)\) of the saddle-point equations:

\[
x(t) = \begin{cases} 
(t - t'_s)k_s - \int_{t'_s}^t d\tau eA(\tau), & (\text{Re } t'_s \leq t \leq \text{Re } t_s) \\
(t - t_s)p - \int_{t_s}^t d\tau eA(\tau). & (t \geq \text{Re } t_s)
\end{cases}
\]

\[ x(t = t'_s) = 0, \text{ but } \text{Re } [x(\text{Re } t'_s)] \text{ different from } 0 \]
Examples of direct quantum orbits

One member of a pair of orbits experiences the Coulomb potential more than the other (see later)
Generalized Keldysh theory: Rescattering (cont.)

an alternative expression:

\[
M_{p,E_0} = -i \int_{-\infty}^{\infty} dt \langle \psi_p^{(Vv)}(t) | H_I(t) | \psi_0(t') \rangle
\]

\[
= -i \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \langle \psi_p^{(Vv)}(t) | VU^{(Vv)}(t, t') H_I(t') | \psi_0(t') \rangle
\]

direct electrons
rescattered electrons

in the last line, may replace

\[
V(r) \rightarrow V_{\text{scatt}}(r)
\]
going beyond the SAEA

restored „hard“ Coulomb effects in first-order Born approximation
Hydrogen H(1s) ATI spectra via TDSE and SFA

Problem areas:
very low energies
transition region between $2U_p$ and $5U_p$

solid: TDSE; dashed: SFA

$\omega = 0.056$ a.u.
$E_0 = 0.834$ a.u.
4-cycle sine-square sine pulse

Origin of interferences: short-range potential

Interferences are not an artifact of the SFA

solid: TDSE
dashed: SFA

TDSE:
Coulomb potential
cut at $r_c = 2$ a.u.

SFA:
Yukawa wave function
Interference of the two solutions from within one cycle
(includes focal averaging)

(1.1 x 10^{13} \text{ Wcm}^{-2})
(1.3 x 10^{13} \text{ Wcm}^{-2})

Detachment, no Coulomb potential!

cf. M.V. Frolov, N.M. Manakov, E.A. Pronin, A.F. Starace,
JPB 36, L419 (2003)
The attosecond double slit

one and the same atom can realize the single slit and the double slit at the same time

F. Lindner et al., PRL 95, 040401 (2005)
Single slit vs. double slit by variation of the absolute phase

\[ A(t) = A_0 \ e^x \ \cos^2(\frac{\pi}{nT} t) \ \sin(\omega t - \phi) \]

\[ \phi = 0 \]  

„cosine“ pulse  
one window in either direction

\[ \phi = \frac{\pi}{2} \]

„sine“ pulse  
one window in the positive direction, two windows in the negative direction
Theory vs. experiment:

The Coulomb field IS important

solution of the TDSE including the Coulomb field

F. Lindner et al.
PRL 95, 040401 (2005)

„simple-man“ model ignoring the Coulomb field
Backward-forward asymmetry for a few-cycle field as a function of the absolute phase

$$R = \frac{W(\text{left}) - W(\text{right})}{W(\text{left}) + W(\text{right})}$$

Chelkowski and Bandrauk
PRA 71, 053815 (2005)

SFA predicts $R = 0$ for $\phi = 0$, TDSE for $\phi = -0.3$
Physical consequences of the Coulomb field

If the Coulomb field is ignored, envelope 1 yields backward-forward symmetry.

Due to Coulomb refocusing, the later orbit is preferred, violating b-f symmetry.

The envelope 2 weakens the contribution of the later orbit and restores b-f symmetry.

Argument explains the sign of the symmetry phase \( \phi = -0.3 \)
Electron-electron Coulomb interaction in the final state of nonsequential double ionization

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Two-electron Volkov state:

\[ |\psi_{p_1 p_2}^{(Vv)}(t)\rangle = |\psi_{p_1}^{(Vv)}(t)\rangle \otimes |\psi_{p_2}^{(Vv)}(t)\rangle \]
\[ \times \frac{1}{\Gamma(1+i\gamma)} e^{-\pi\gamma/2} F_1(-i\gamma, 1; i(|p||r| - p \cdot r)) e^{-\pi\gamma/2} \Gamma(1 + i\gamma), \]

\[ p = (p_1 - p_2)/2, \quad r = r_1 - r_2, \quad \gamma = 1/(2|p|) \]

Note: Coulomb repulsion affects \( r_1 - r_2 \), laser field couples to \( r_1 + r_2 \)

without final-state Coulomb repulsion between the two electrons

with final-state Coulomb repulsion between the two electrons
Including Coulomb repulsion in the final state
Small transverse momenta
\[ 0 \leq p_{1\perp}, p_{2\perp} \leq 0.1 \sqrt{U_p} \]

Large transverse momenta
\[ \sqrt{U_p} \leq p_{1\perp}, p_{2\perp} \leq 1.5 \sqrt{U_p} \]