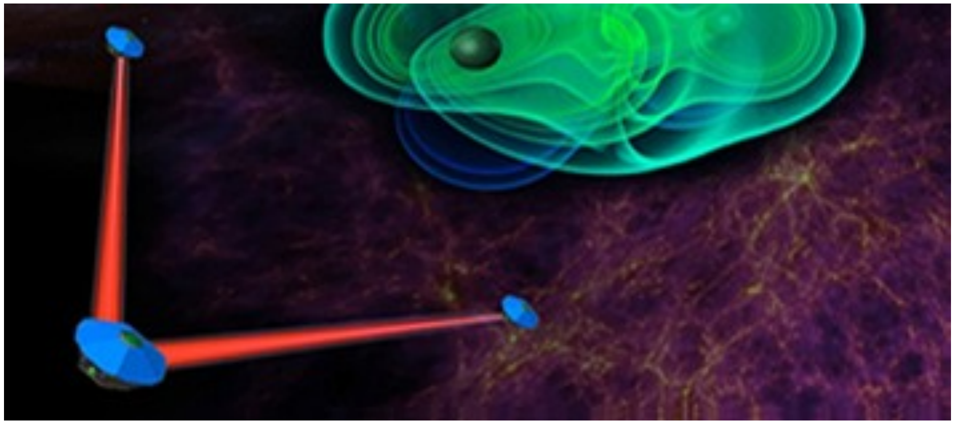
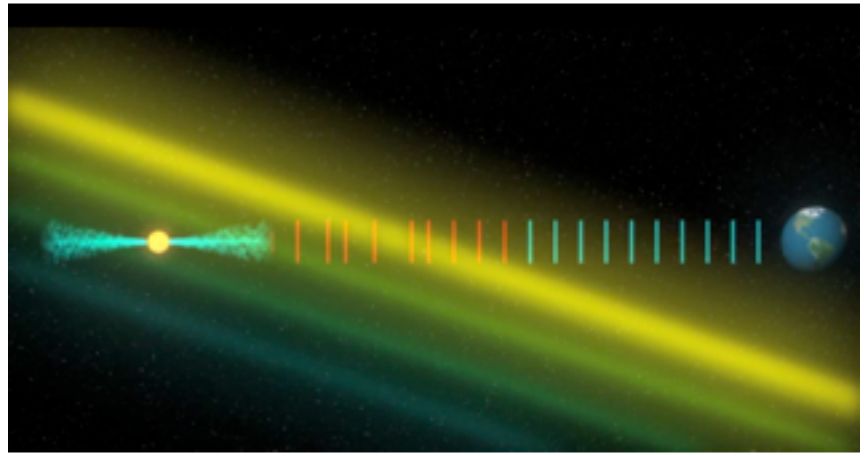
Gravitational waves from massive black hole binaries





Using gravitational-wave observations of black hole mergers to probe the growth of black holes from the early universe

Basic basics

Gravitational radiation *necessary* in any relativistic theory of gravity: Need a mechanism to causally communicate changes in the gravitational field.

In GR, tidal fields ("curvature") play role similar to electric and magnetic fields in E & M ... radiation takes form of tidal gravitational field propagating from source.

Leading radiation *quadrupolar*: monopole violates conservation of energy; dipole violates conservation of momentum.

$$h = \frac{2G}{c^4}\frac{1}{r}\frac{d^2 Q}{dt^2}$$

$$\simeq \frac{2G}{c^4}\frac{1}{r} \times mv^2$$

How to measure

The GW *h* is an oscillation in spacetime, has an impact on propagation of light:

Behavior of light in spacetime with wave:

$$ds^2 = -c^2 dt^2 + \left[1 + h(t,x)\right] dx^2 = 0$$

Solve for the speed of light in this coordinate system:

$$\frac{dx}{dt} = \frac{c}{\sqrt{1 + h(t,x)}}$$

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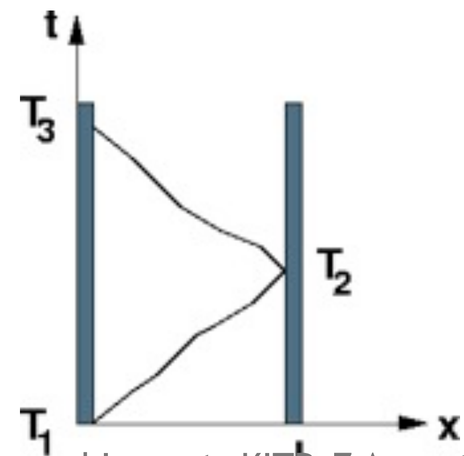
Now imagine that we have mirrors which fall freely in this spacetime. Bounce light between mirrors, record time between bounces.



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Behavior of light in spacetime with wave:

$$ds^2 = -c^2 dt^2 + [1 + h(t,x)]\, dx^2 = 0$$

Time interval between bounces:

$$\Delta T = \int \frac{dx}{dx/dt} \simeq \frac{1}{c} \int \left[1 - \frac{1}{2} h(t,x)\right] dx$$

Gravitational wave enters as an oscillation in interval from bounce to bounce (Bondi 1957).

How to measure

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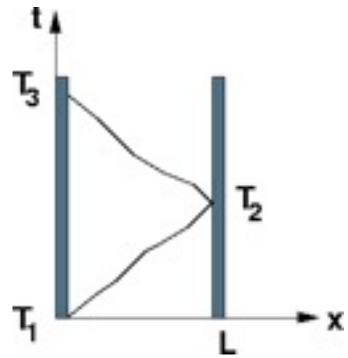
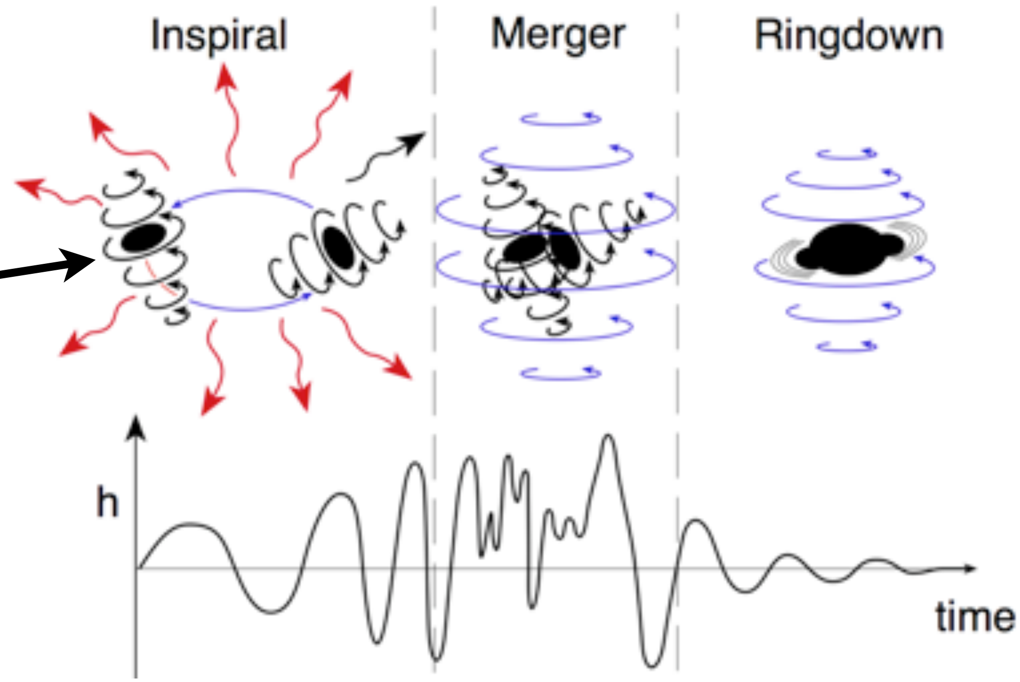
Two big ingredients needed to measure *h*: Good inertial reference frame to define free fall, and good clock to measure time interval.



$$\Delta T = \int \frac{dx}{dx/dt} \simeq \frac{1}{c}\int \left[1 - \frac{1}{2}h(t,x)\right] dx$$

Gross features of BBH GWs

Inspiral: Slow evolution driven by GW loss of orbital energy and angular momentum.



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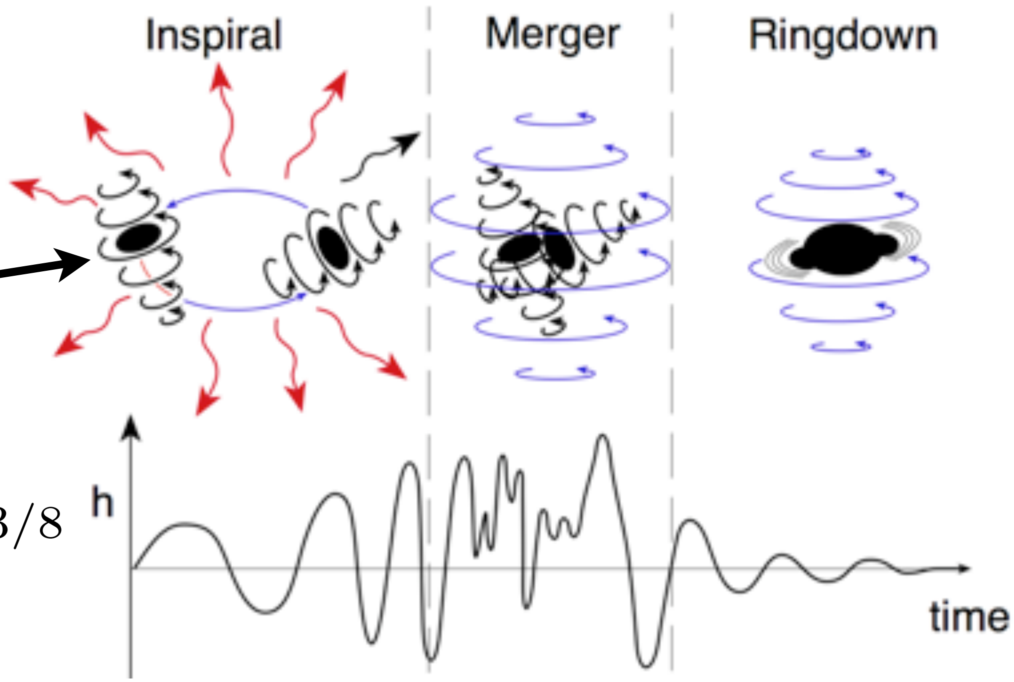
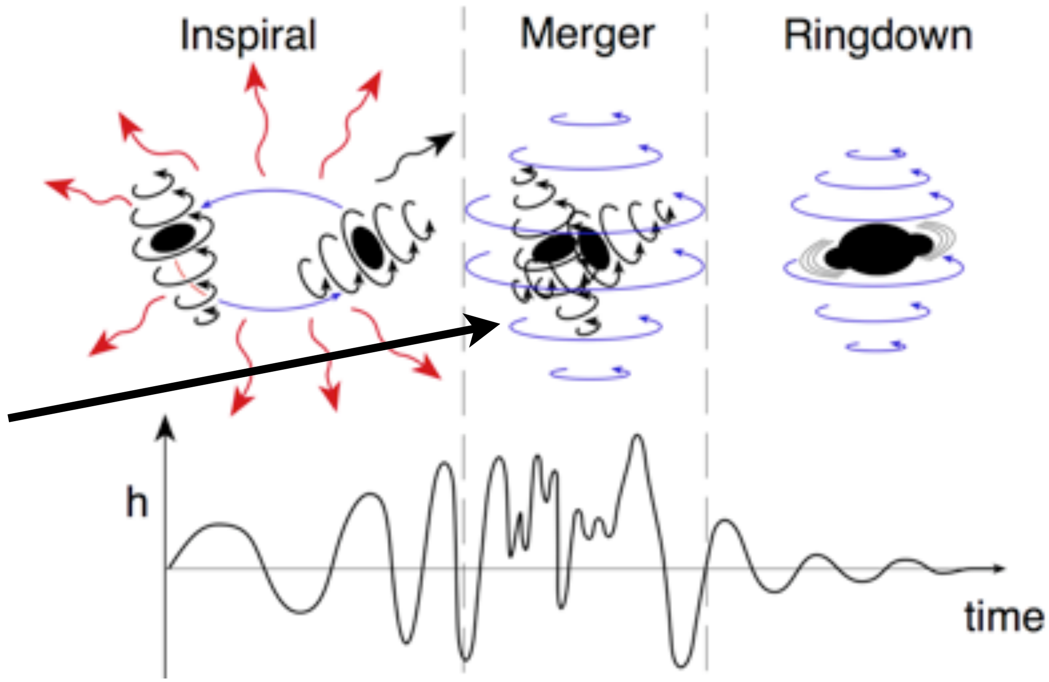


$$f(t) = \frac{1}{\pi}\left[\frac{5c^5}{256(G\mathcal{M})^{5/3}(t_c - t)}\right]^{3/8}$$

Leading solution for rate of change of wave frequency as system evolves ... more careful calculation shows that inspiral encodes a lot of information about members' masses and spins.

Gross features of BBH GWs

Merger: Extremely violent dynamics of spacetime: Two black holes smash together, leaving one behind.



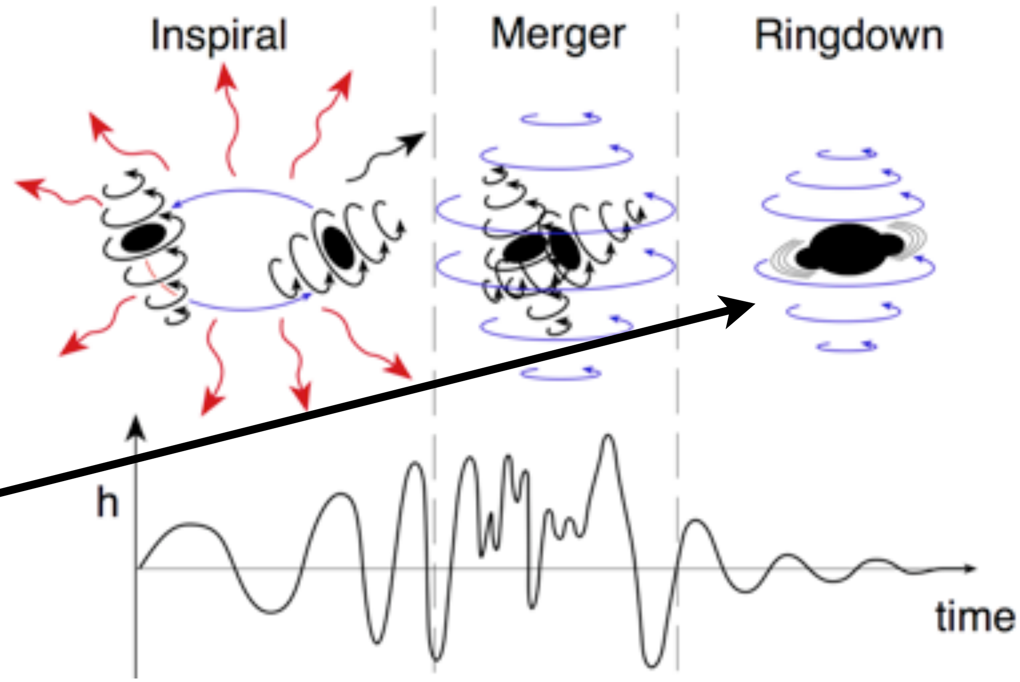
Transition from inspiral to merger happens at

$$f_{\rm merge} \simeq \frac{c^3}{GM_{\rm tot}} \frac{[2-6]^{-3/2}}{\pi} = (0.02 - 0.004)\,{\rm Hz}\left(\frac{10^6\,M_\odot}{M_{\rm tot}}\right)$$

Late inspiral/merger modeled numerically; a lot of binary mass (up to ~10%) comes out in GWs.

Gross features of BBH GWs

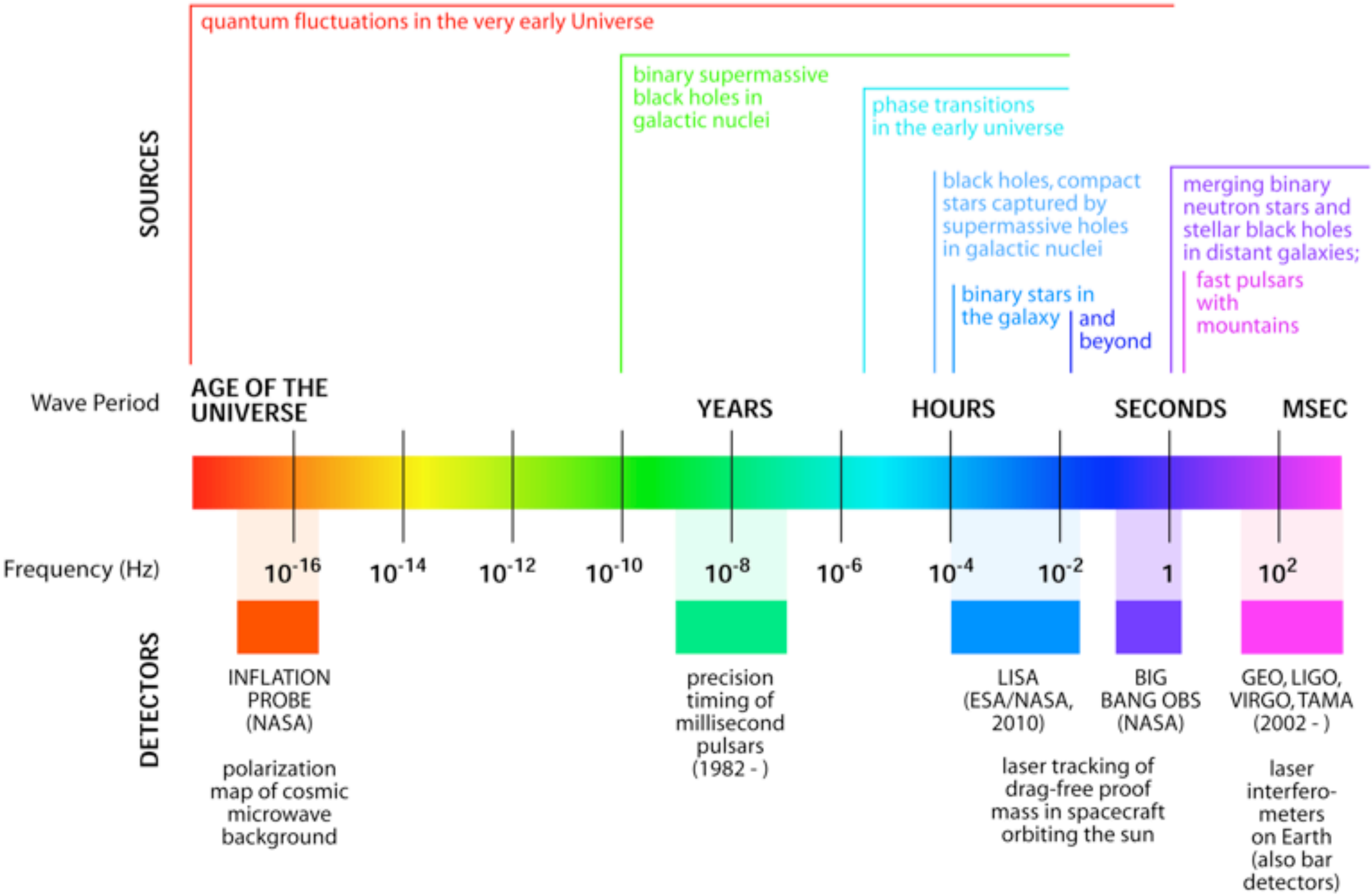
Ringdown: Last wiggles of the merger, enforce the black hole "No Hair" theorems. Kerr solution at end.



Simply described using black hole perturbation theory. Expect mix of modes; mode frequency and damping time set by final mass and spin.

$$f_{22} \approx \frac{c^3}{2\pi G M}\left[1 - 0.63(1 - a/M)^{0.3}\right] \quad \tau_{22} \approx \frac{2}{\pi f_{22}}(1 - a/M)^{-0.45}$$

THE GRAVITATIONAL WAVE SPECTRUM



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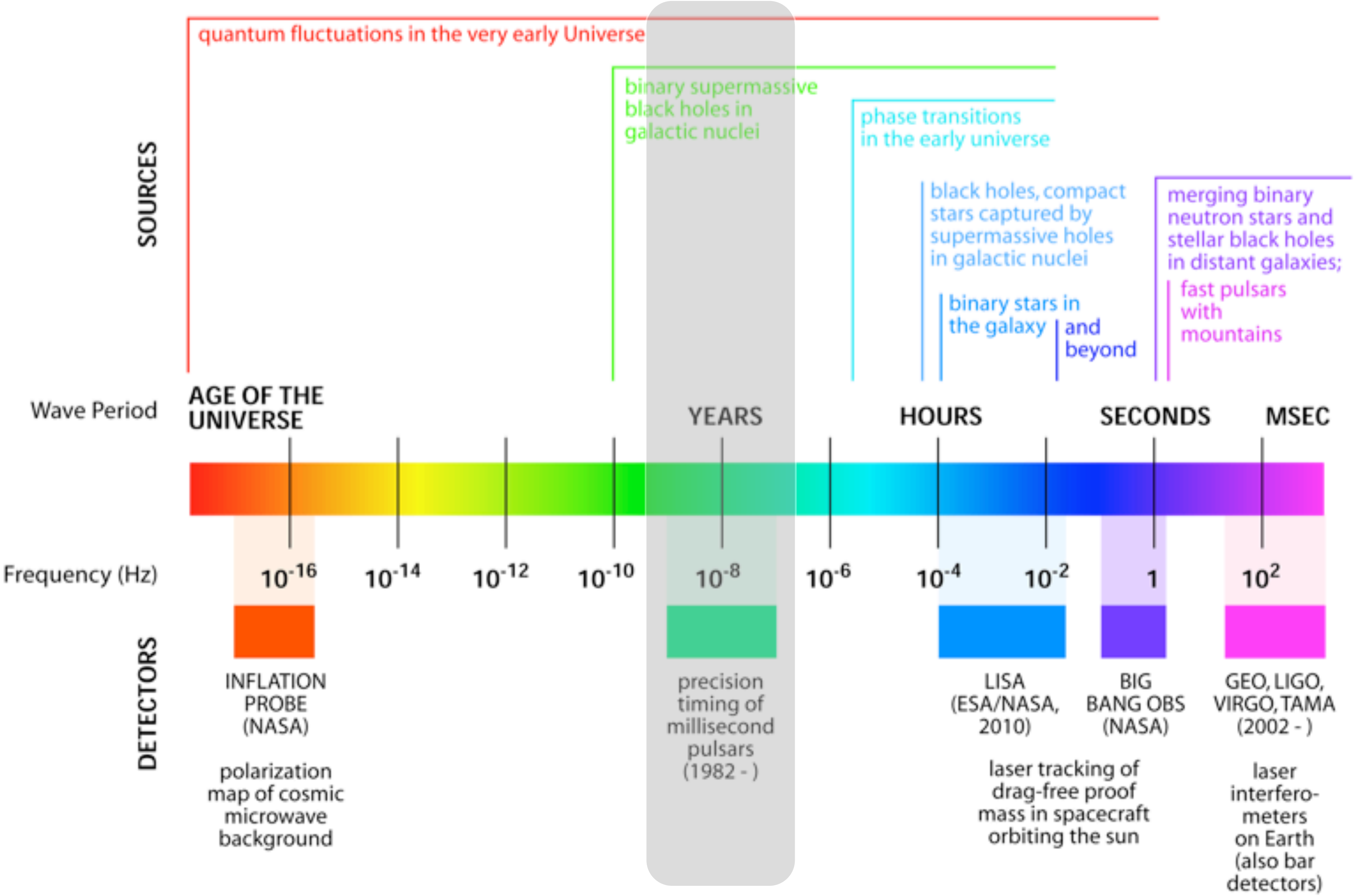
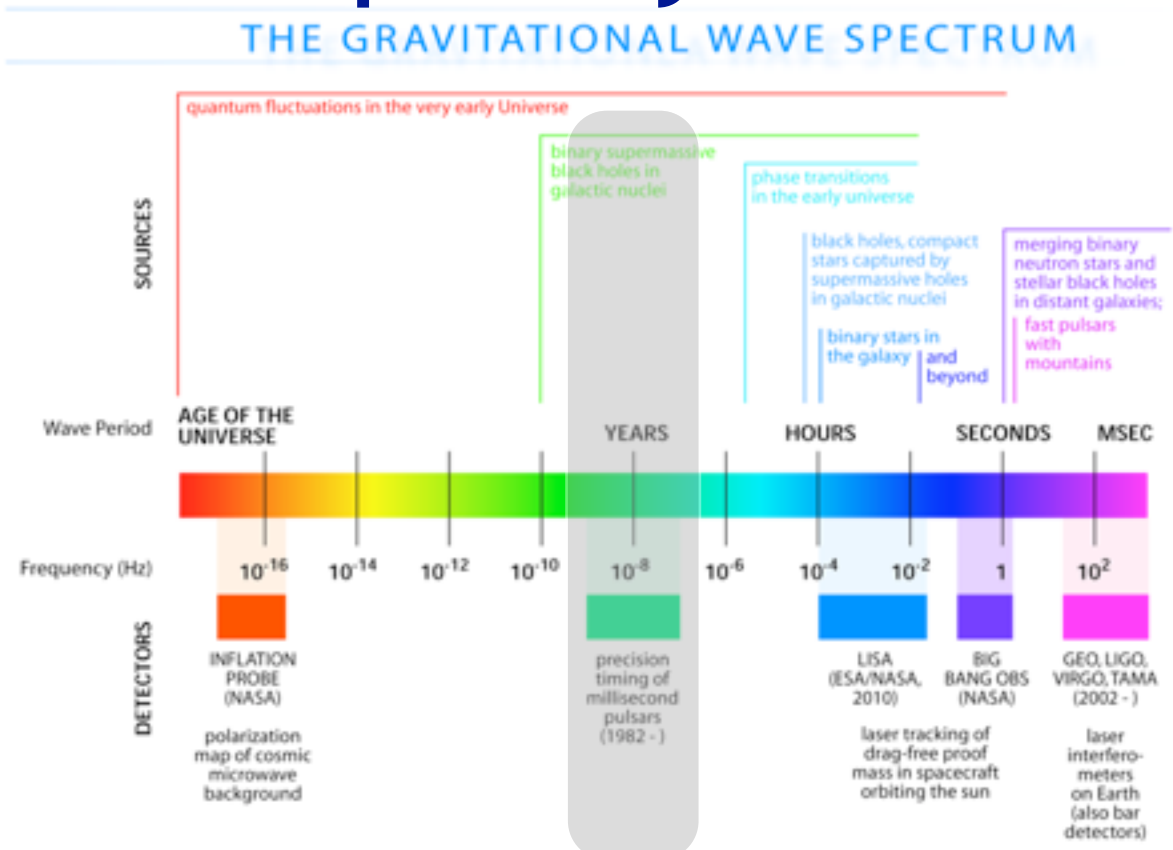


Very low frequency

Frequencies: (years)$^{-1}$ to (months)$^{-1}$

Measure in this band by precision timing of millisecond pulsars: Pulsars are clocks, GWs cause coherent variation in pulse arrival times.



Build a network of pulsars, time them well, look for pulse variations with a particular angular distribution on the sky.

Pulsar timing movie courtesy Penn State Gravitational Wave Astronomy Group, http://gwastro.org

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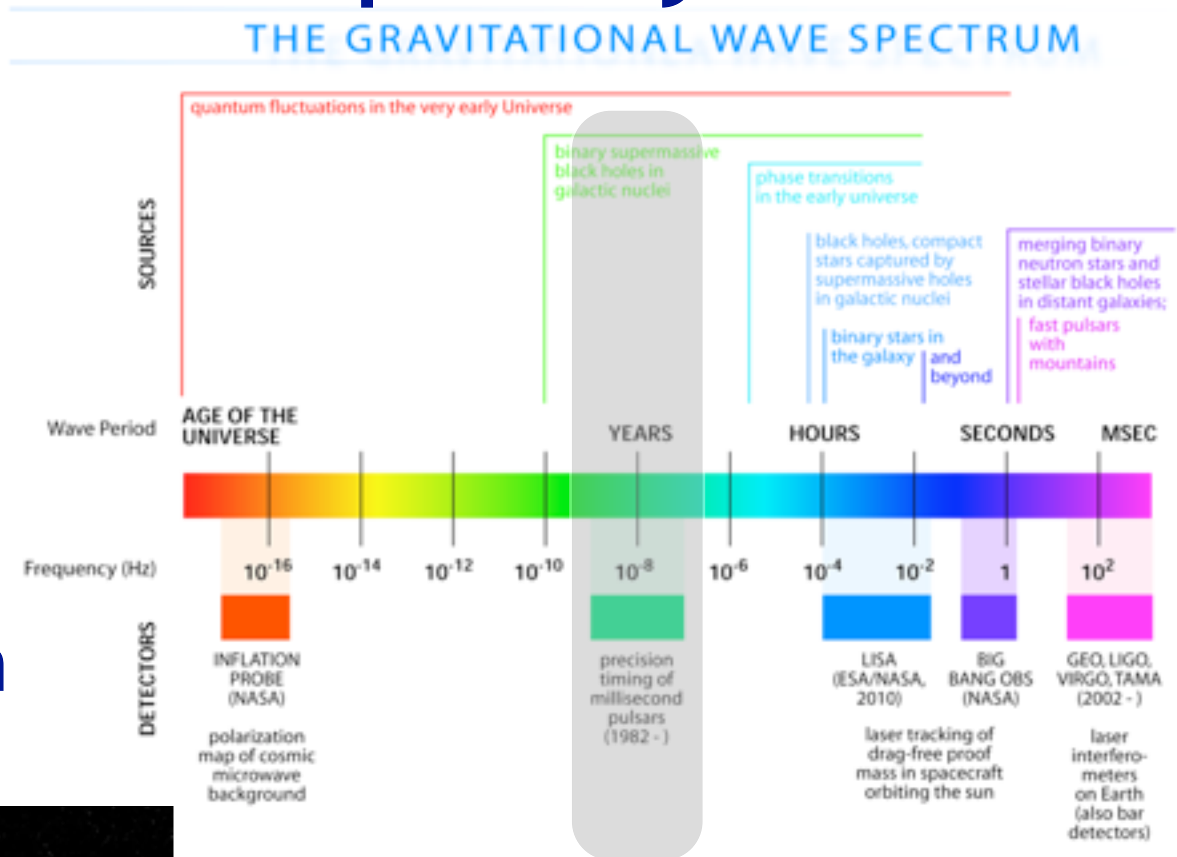

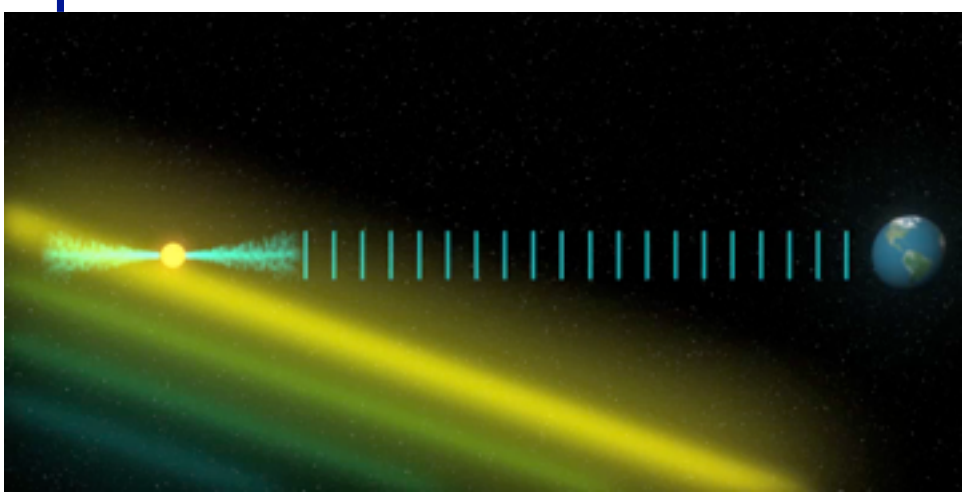

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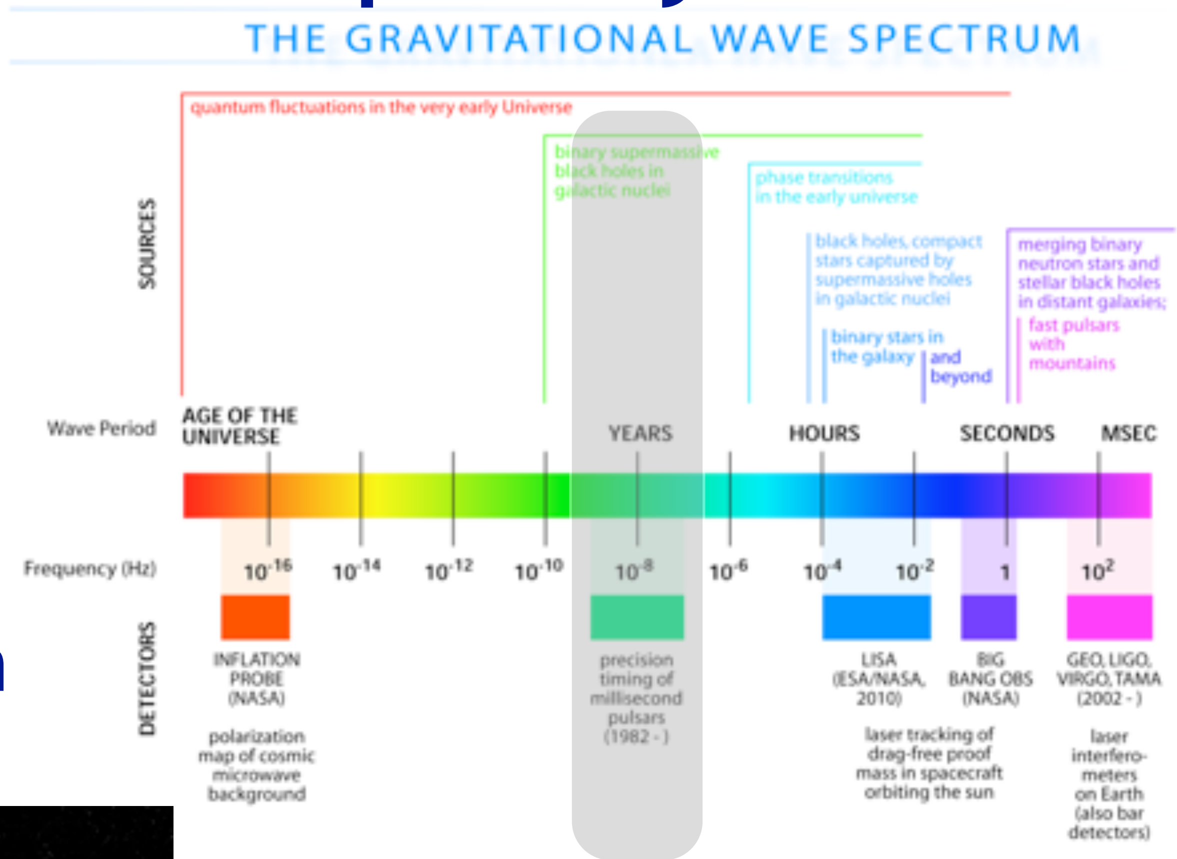

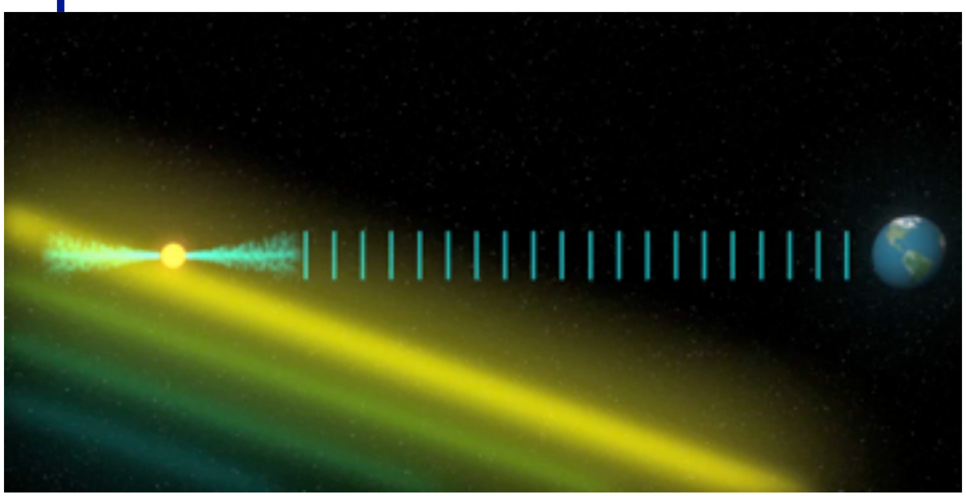

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Network now has almost 40 pulsars, setting impressive limits on backgrounds ... getting close to predictions.

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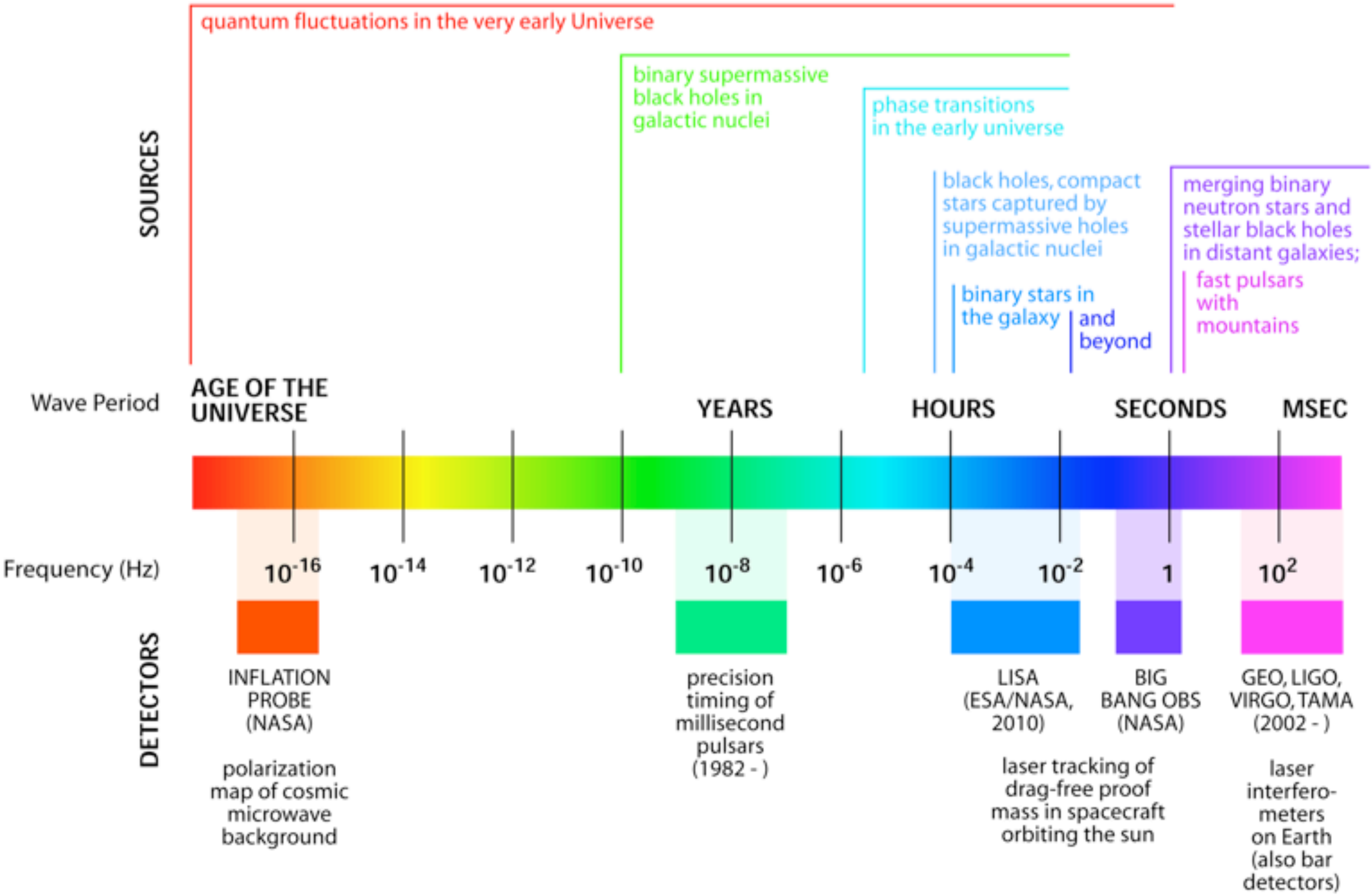

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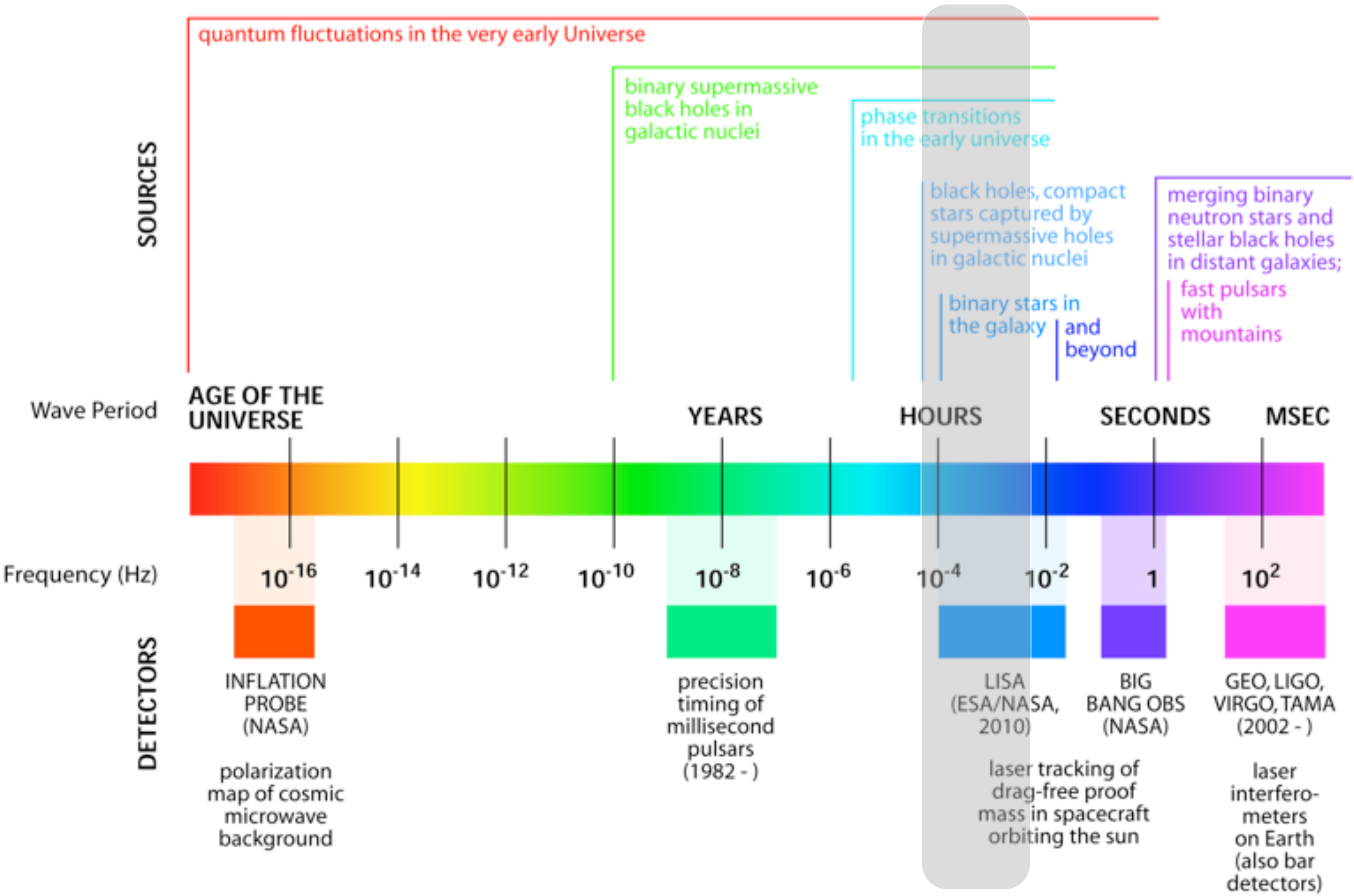
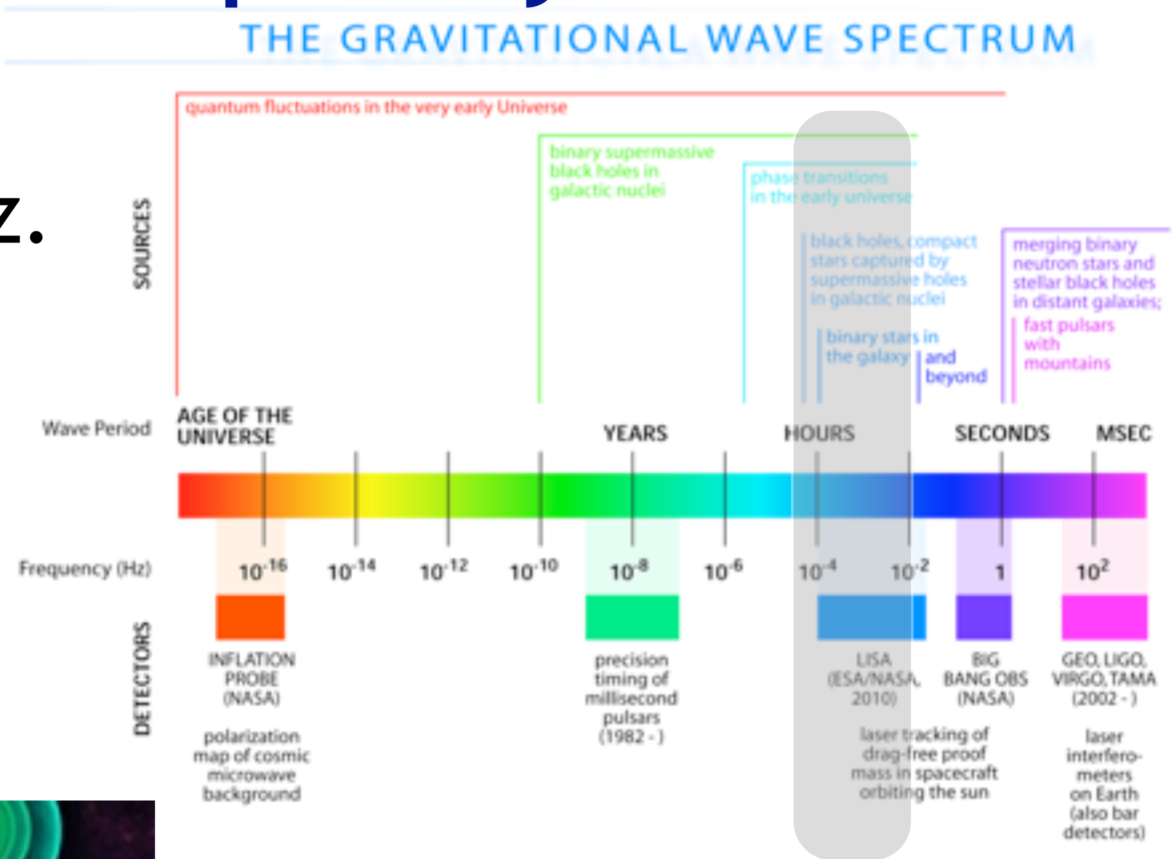
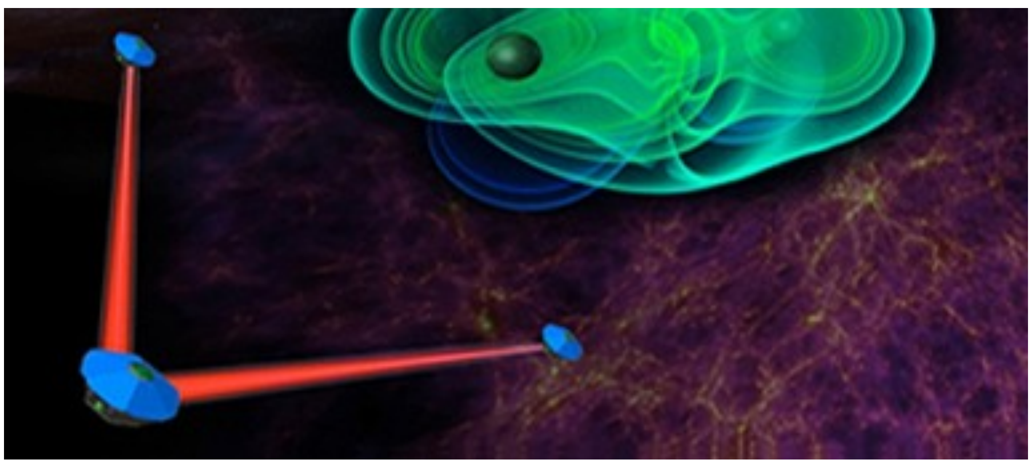


Low frequency

Frequencies: Inverse hours up to about 1 Hz.

Laser interferometry best tool to measure these waves ... Earth too noisy, detector must go into space.

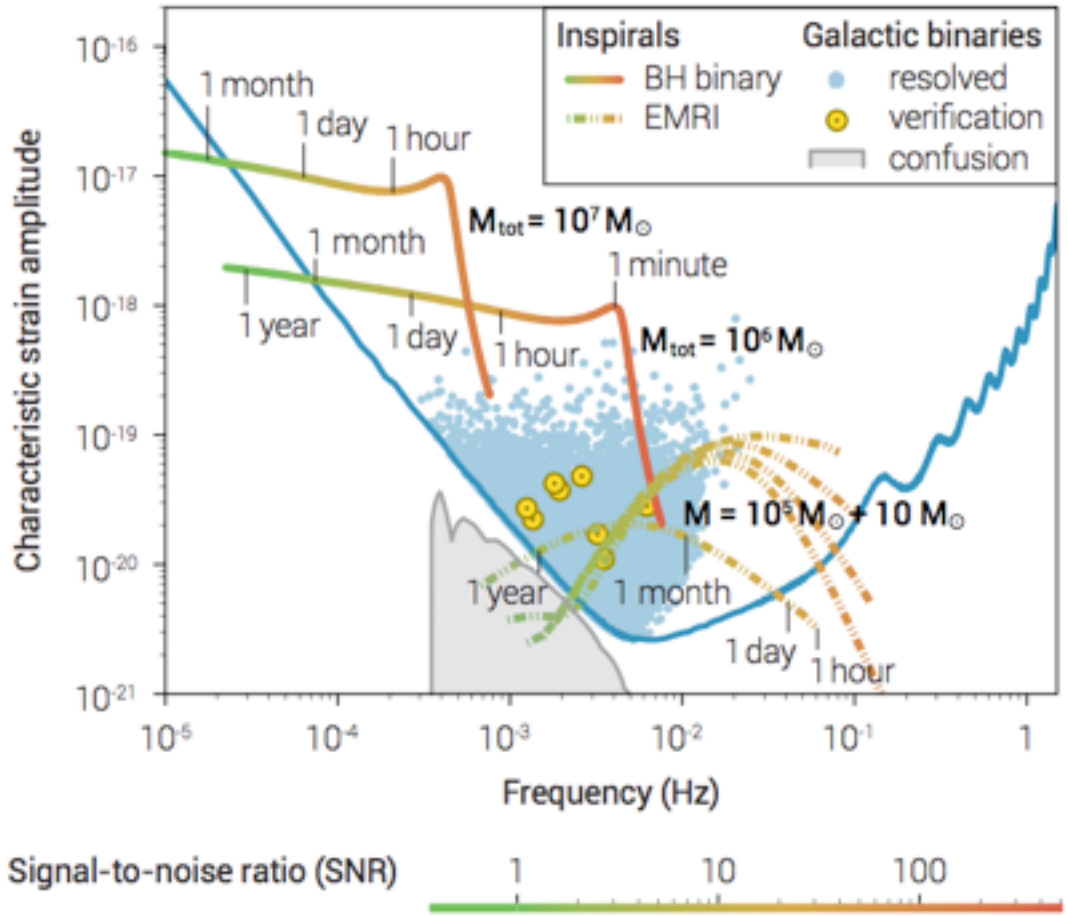




Space antenna like eLISA designed to measure waves in this band.

eLISA sensitivity

Fig 13 of Danzmann et al, arxiv:1305.5720



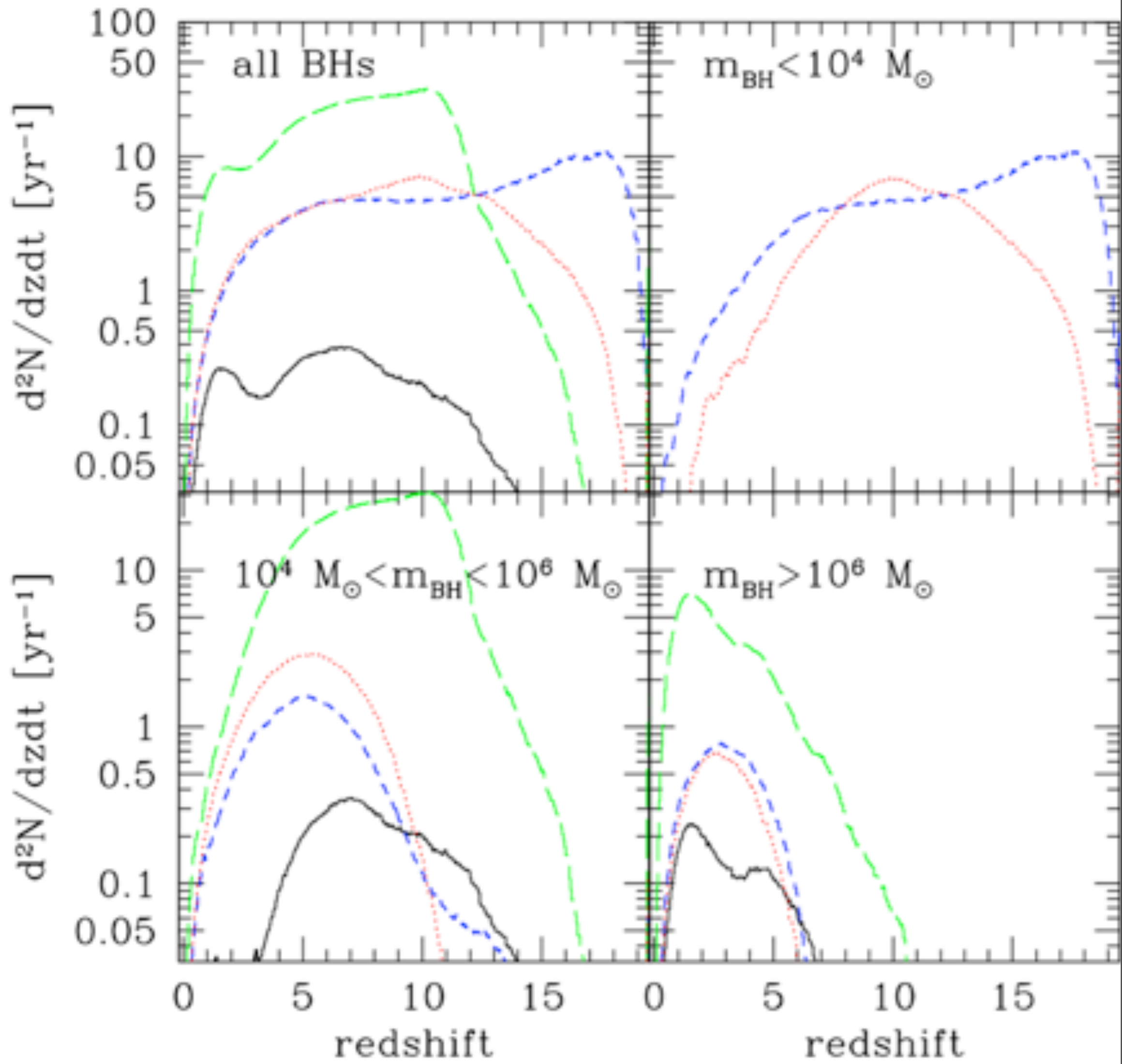
Sensitivity is very good for black holes for late inspiral (last few 10^2 — 10^3 orbits) and merger/ringdown for redshifted masses $(1 + z)M_{\rm tot} \sim 10^5$—$10^7$

Perfectly suited to going after early cosmological seeds in binaries.

Gravitational waves and MBBH

Wide range of MBBH binaries accessible to an instrument like eLISA with decent events rates ...

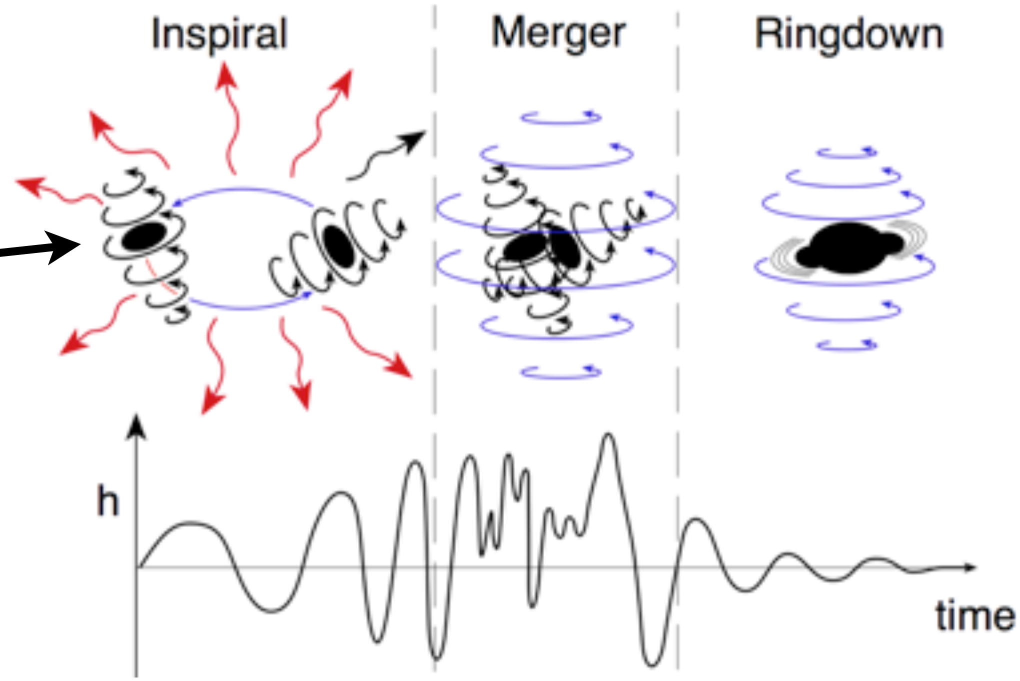
... provided that we can nail down events happening at z ~ 5 — 15.



Sesana, Volonteri, & Haardt 2007 MNRAS **377, 1711**

Inspiral waves

Inspiral: Slow evolution driven by GW loss of orbital energy and angular momentum.



Rather well understood. Waveform described by 17 parameters in general.

2 masses
6 spin components
2 position angles
2 orientation angles
1 distance
1 initial semi-major axis
1 initial orbit anomaly
1 initial eccentricity
1 initial periapsis longitude

Inspiral waveform

$$h_+ = \frac{[G\mathcal{M}/c^2]^{5/3}\,[\pi f(t)/c]^{2/3}}{D_L}\left(1+\cos^2\iota\right)\cos\left[2\pi\int f(t)\,dt\right]$$

$$h_\times = \frac{2\,[G\mathcal{M}/c^2]^{5/3}\,[\pi f(t)/c]^{2/3}}{D_L}\cos\iota\sin\left[2\pi\int f(t)\,dt\right]$$

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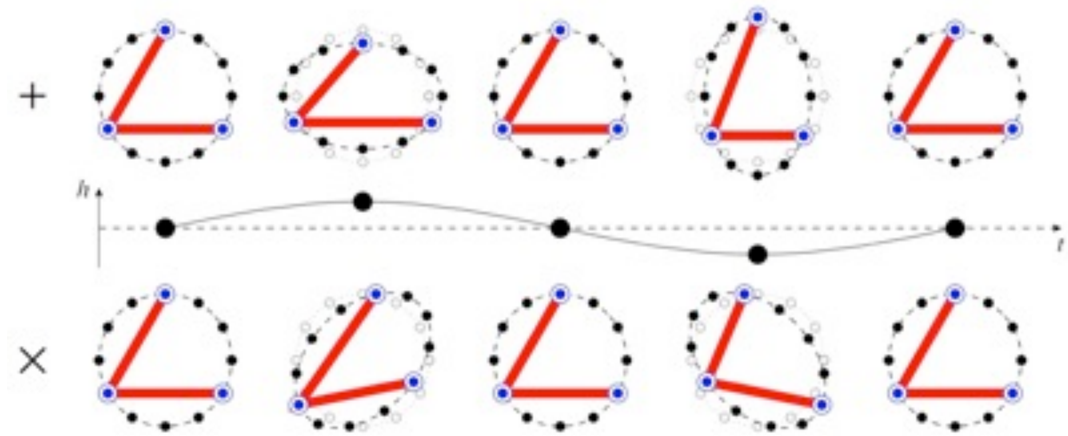
Gravitational waves have two polarizations, named for their tidal action upon a set of test masses:

"+" wave polarization

"x" wave polarization



With two arms (as in the baseline eLISA design), can only measure one polarization at a time.

Pieces of inspiral waveform

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1. Phase. Depends on how rapidly the orbit evolves. Rate is controlled by binary's masses and spins.

Measure the phase, measure masses and spins.

Phase: Comes from integrating up the (relativistic analog of) Kepler's law

To get that, need relativistic equations of motion. Post-Newtonian expansion of general relativity gives us a good form for inspiral:

$$a_1^i = -\frac{G m_2 n_{12}^i}{r_{12}^2}$$

Lowest order piece: Newtonian gravity

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Lowest order piece: Newtonian gravity

$$+\frac{1}{c^2}\left\{\left[\frac{5G^2 m_1 m_2}{r_{12}^3} + \frac{4G^2 m_2^2}{r_{12}^3} + \frac{Gm_2}{r_{12}^2}\left(\frac{3}{2}(n_{12}v_2)^2 - v_1^2 + 4(v_1 v_2) - 2v_2^2\right)\right] n_{12}^i + \frac{Gm_2}{r_{12}^2}\left(4(n_{12}v_1) - 3(n_{12}v_2)\right) v_{12}^i\right\}$$

Post-Newton gives corrections in *v*/*c*.

... and more corrections ...

... and more corrections ...

$$
\begin{aligned}
&+\frac{1}{c^4}\Bigg\{\Bigg[-\frac{57G^3m_1^2m_2}{4r_{12}^4}-\frac{69G^3m_1m_2^2}{2r_{12}^4}-\frac{9G^3m_2^3}{r_{12}^4}\\
&\quad+\frac{Gm_2}{r_{12}^2}\Big(-\frac{15}{8}(n_{12}v_2)^4+\frac{3}{2}(n_{12}v_2)^2v_1^2-6(n_{12}v_2)^2(v_1v_2)-2(v_1v_2)^2+\frac{9}{2}(n_{12}v_2)^2v_2^2\\
&\qquad+4(v_1v_2)v_2^2-2v_2^4\Big)\\
&\quad+\frac{G^2m_1m_2}{r_{12}^3}\left(\frac{39}{2}(n_{12}v_1)^2-39(n_{12}v_1)(n_{12}v_2)+\frac{17}{2}(n_{12}v_2)^2-\frac{15}{4}v_1^2-\frac{5}{2}(v_1v_2)+\frac{5}{4}v_2^2\right)\\
&\quad+\frac{G^2m_2^2}{r_{12}^3}\left(2(n_{12}v_1)^2-4(n_{12}v_1)(n_{12}v_2)-6(n_{12}v_2)^2-8(v_1v_2)+4v_2^2\right)\Bigg]n_{12}^i\\
&\quad+\Bigg[\frac{G^2m_2^2}{r_{12}^3}\left(-2(n_{12}v_1)-2(n_{12}v_2)\right)+\frac{G^2m_1m_2}{r_{12}^3}\left(-\frac{63}{4}(n_{12}v_1)+\frac{55}{4}(n_{12}v_2)\right)\\
&\qquad+\frac{Gm_2}{r_{12}^2}\Big(-6(n_{12}v_1)(n_{12}v_2)^2+\frac{9}{2}(n_{12}v_2)^3+(n_{12}v_2)v_1^2-4(n_{12}v_1)(v_1v_2)\\
&\qquad\quad+4(n_{12}v_2)(v_1v_2)+4(n_{12}v_1)v_2^2-5(n_{12}v_2)v_2^2\Big)\Bigg]v_{12}^i\Bigg\}\\
&+\frac{1}{c^5}\Bigg\{\left[\frac{208G^3m_1m_2^2}{15r_{12}^4}(n_{12}v_{12})-\frac{24G^3m_1^2m_2}{5r_{12}^4}(n_{12}v_{12})+\frac{12G^2m_1m_2}{5r_{12}^3}(n_{12}v_{12})v_{12}^2\right]n_{12}^i\\
&\quad+\left[\frac{8G^3m_1^2m_2}{5r_{12}^4}-\frac{32G^3m_1m_2^2}{5r_{12}^4}-\frac{4G^2m_1m_2}{5r_{12}^3}v_{12}^2\right]v_{12}^i\Bigg\}
\end{aligned}
$$

... and a few more.

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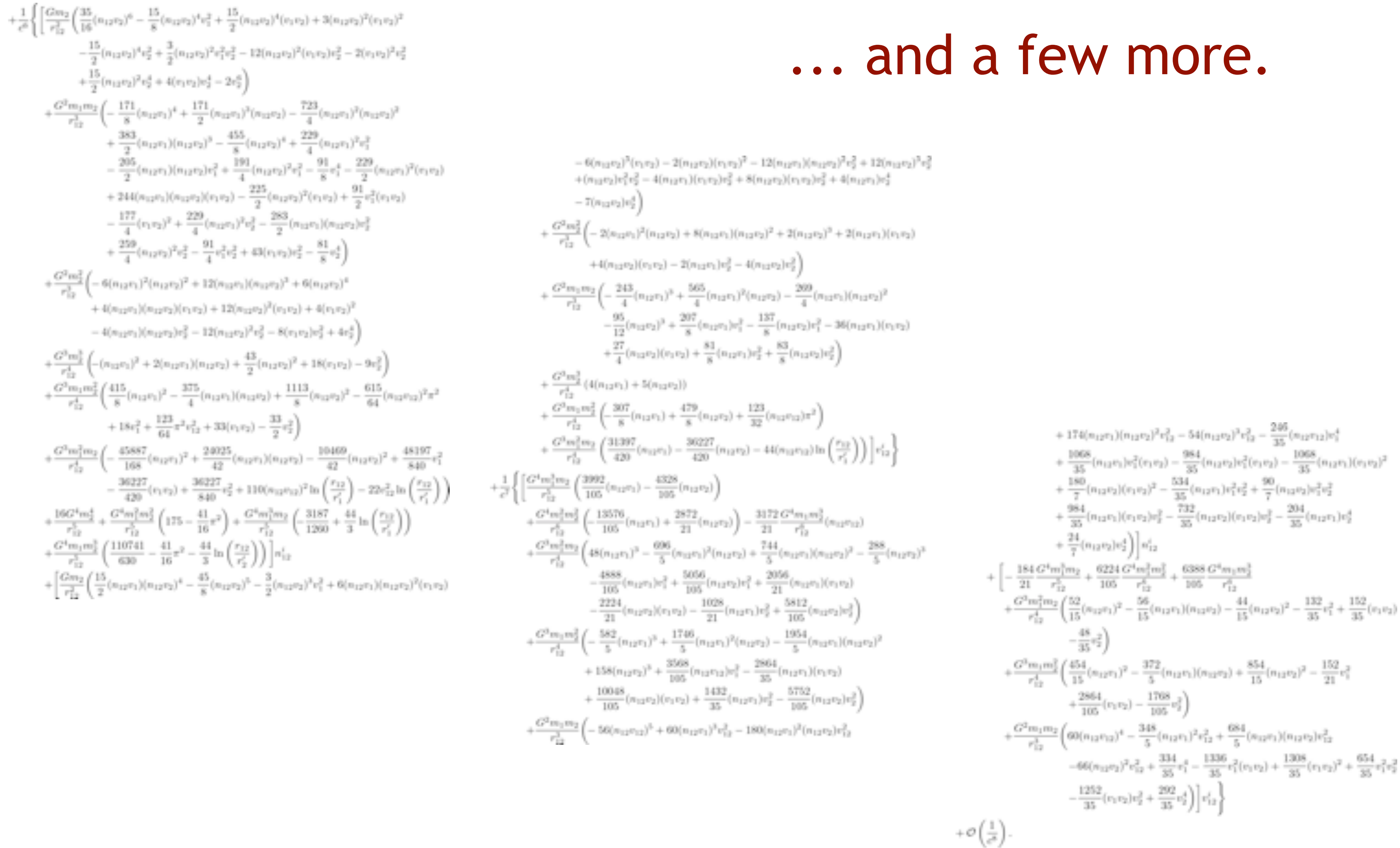

[Blanchet 2006, Liv Rev Rel 9, 4, Eq. (168)]

Dynamical inclination

Relativistic effect: “Magnetic-type” coupling of mass currents to spacetime.

Creates new “forces”, modifying orbit acceleration; also causes spins of binary’s members to precess.

$$\frac{d\mathbf{S}_1}{dt} = \frac{1}{r^3}\left[\left(2+\frac{3}{2}\frac{m_2}{m_1}\right)\mu\sqrt{Mr}\hat{\mathbf{L}}\right]\times\mathbf{S}_1 + \frac{1}{r^3}\left[\frac{1}{2}\mathbf{S}_2 - \frac{3}{2}(\mathbf{S}_2\cdot\hat{\mathbf{L}})\hat{\mathbf{L}}\right]\times\mathbf{S}_1$$

$$\frac{d\mathbf{S}_2}{dt} = \frac{1}{r^3}\left[\left(2+\frac{3}{2}\frac{m_1}{m_2}\right)\mu\sqrt{Mr}\hat{\mathbf{L}}\right]\times\mathbf{S}_2 + \frac{1}{r^3}\left[\frac{1}{2}\mathbf{S}_1 - \frac{3}{2}(\mathbf{S}_1\cdot\hat{\mathbf{L}})\hat{\mathbf{L}}\right]\times\mathbf{S}_2$$

“Gravitomagnetic” field due to orbital motion

“Gravitomagnetic” field due to other body’s spin

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Angular momentum is *globally* conserved:

$$\mathbf{J} = \mathbf{L} + \mathbf{S}_1 + \mathbf{S}_2 = \text{constant}$$

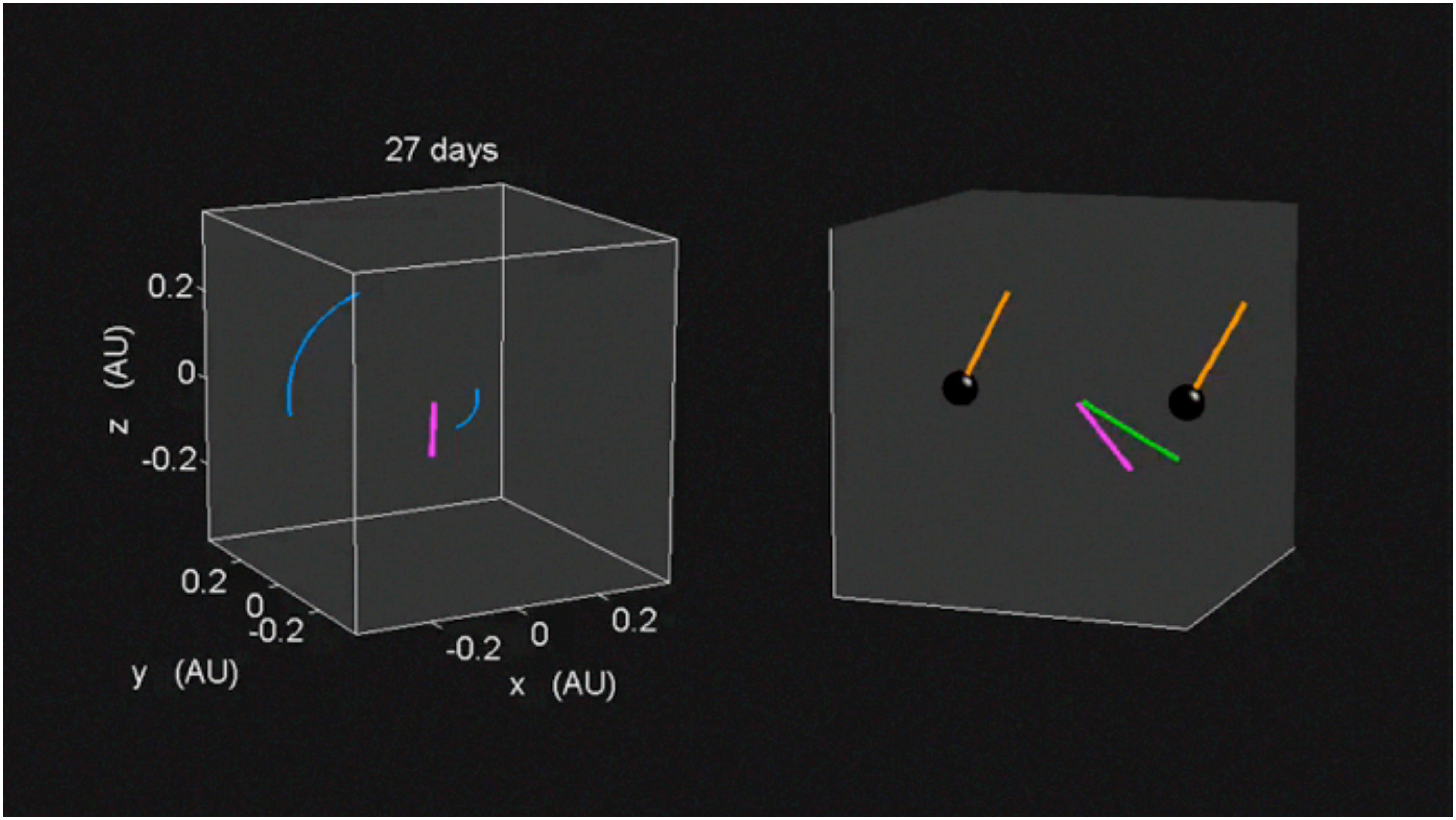
Means that the *orbital plane* precesses to compensate. (Known as Lense-Thirring precession in weak-field.)

Dynamical inclination

Precession of angular momentum vectors in a binary black hole system.

(Animation credit: Peter Reinhardt)

Dynamical inclination



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Pieces of inspiral waveform

$$h_+ = \frac{[G(1+z)\mathcal{M}/c^2]^{5/3}\,[\pi f(t)/c]^{2/3}}{D_L}\mathcal{F}(\text{“angles”})\cos[\Phi(t)]$$

Integrate up motion and precession: 10^3 - 10^5 radians of phase accumulate over measurement.

$$\begin{aligned}
\phi(f) &= \phi_c - \frac{1}{16}(\pi\mathcal{M}f)^{-5/3}\left[1 + \frac{5}{3}\left(\frac{743}{336} + \frac{11}{4}\eta\right)(\pi M f)^{2/3} - \frac{5}{2}(4\pi - \beta)(\pi M f)\right. \\
&+ \left. 5\left(\frac{3058673}{1016064} + \frac{5429}{1008}\eta + \frac{617}{144}\eta^2 - \sigma\right)(\pi M f)^{4/3}\right] \\
\beta &= \frac{1}{12}\sum_{i=1}^{2}\left[113\left(\frac{m_i}{M}\right)^2 + 75\frac{\mu}{M}\right]\frac{\hat{\mathbf{L}}\cdot\mathbf{S}_i}{m_i^2} \\
\sigma &= \frac{\mu}{48M(m_1^2 m_2^2)}[721(\hat{\mathbf{L}}\cdot\mathbf{S}_1)(\hat{\mathbf{L}}\cdot\mathbf{S}_2) - 247(\mathbf{S}_1\cdot\mathbf{S}_2)]
\end{aligned}$$

Key feature: The phase depends on — and thus encodes — masses & spins of the binary's members. Measure phase: Measure masses and spins.

Inspiral measurements

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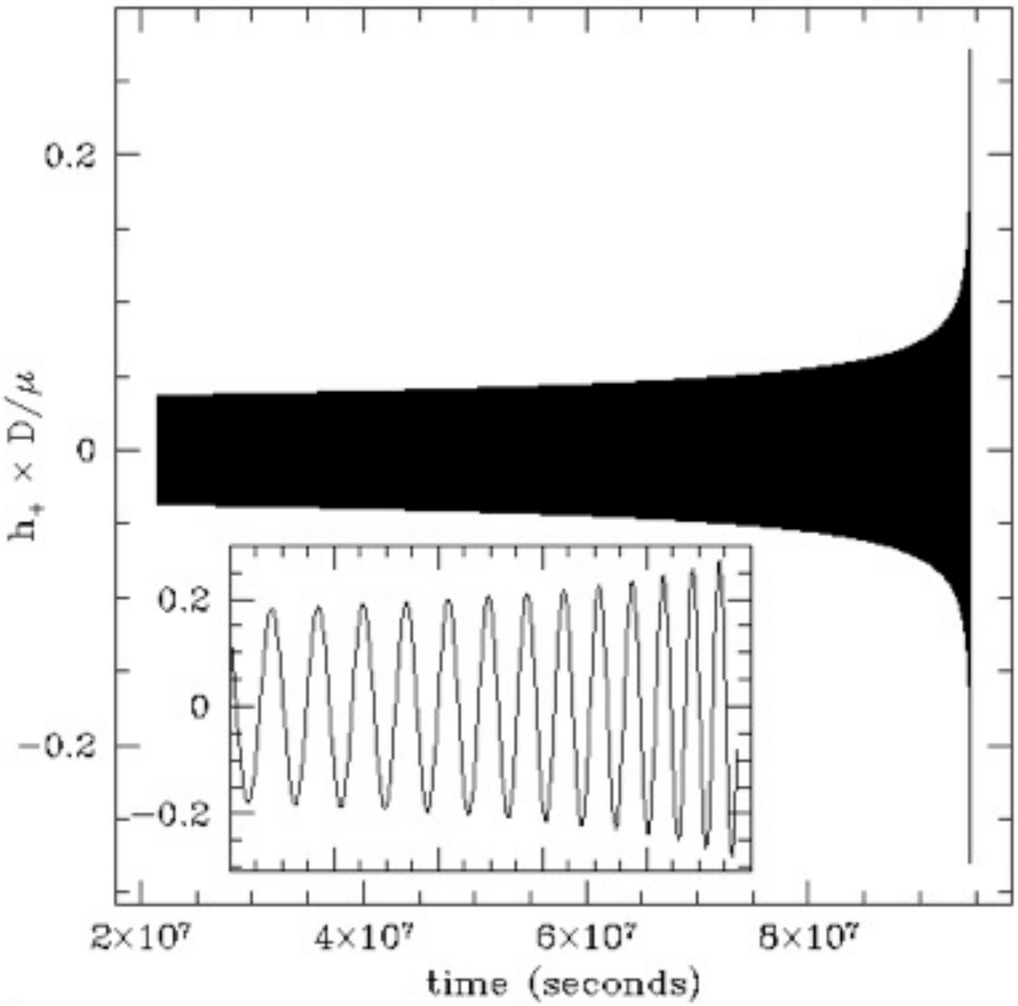

Example waveform: Both black holes non-spinning.

Smooth chirp from low to high frequencies.

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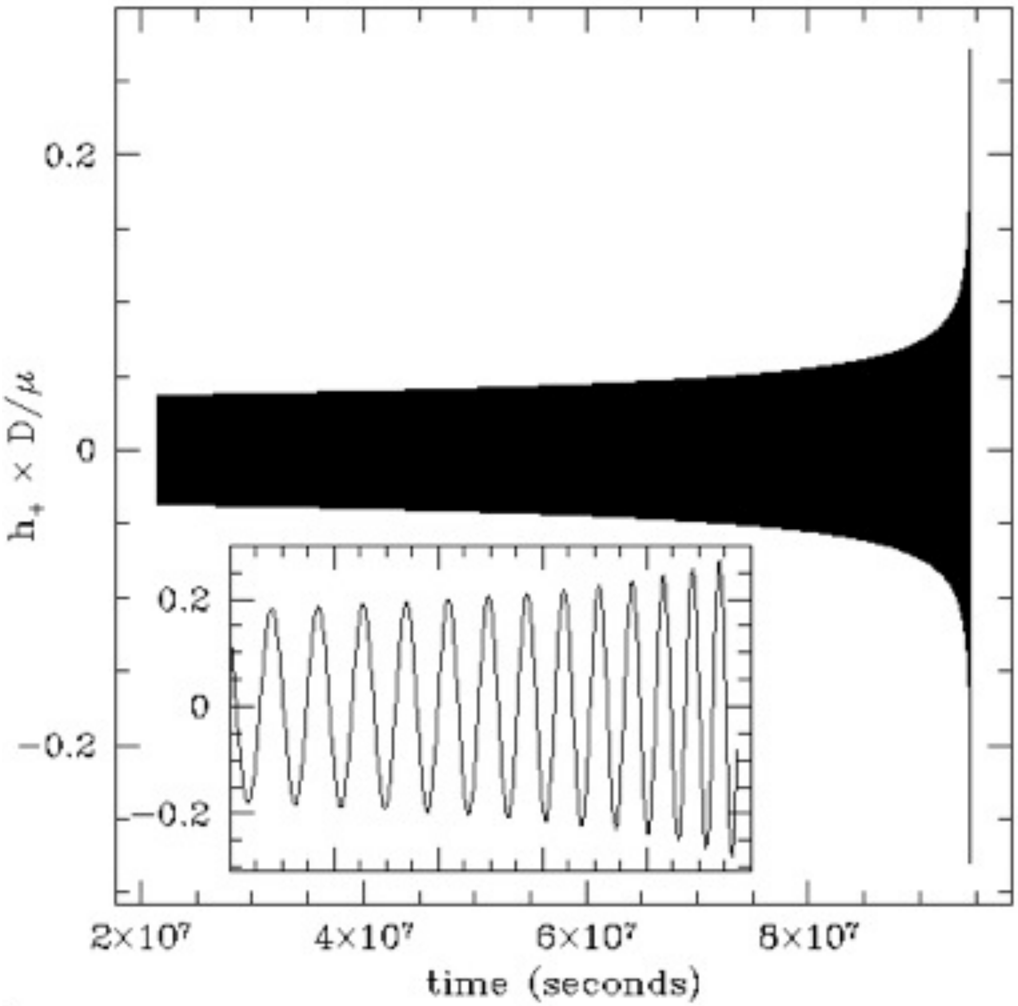

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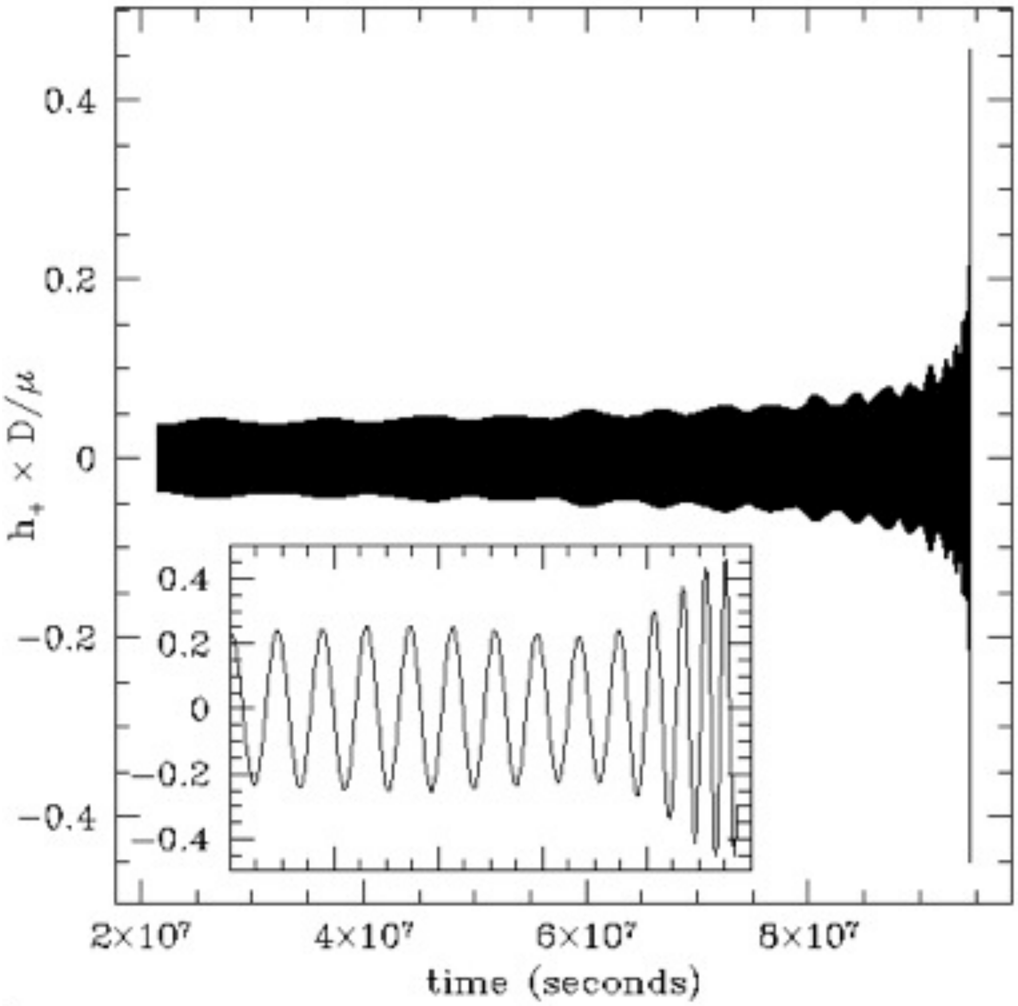
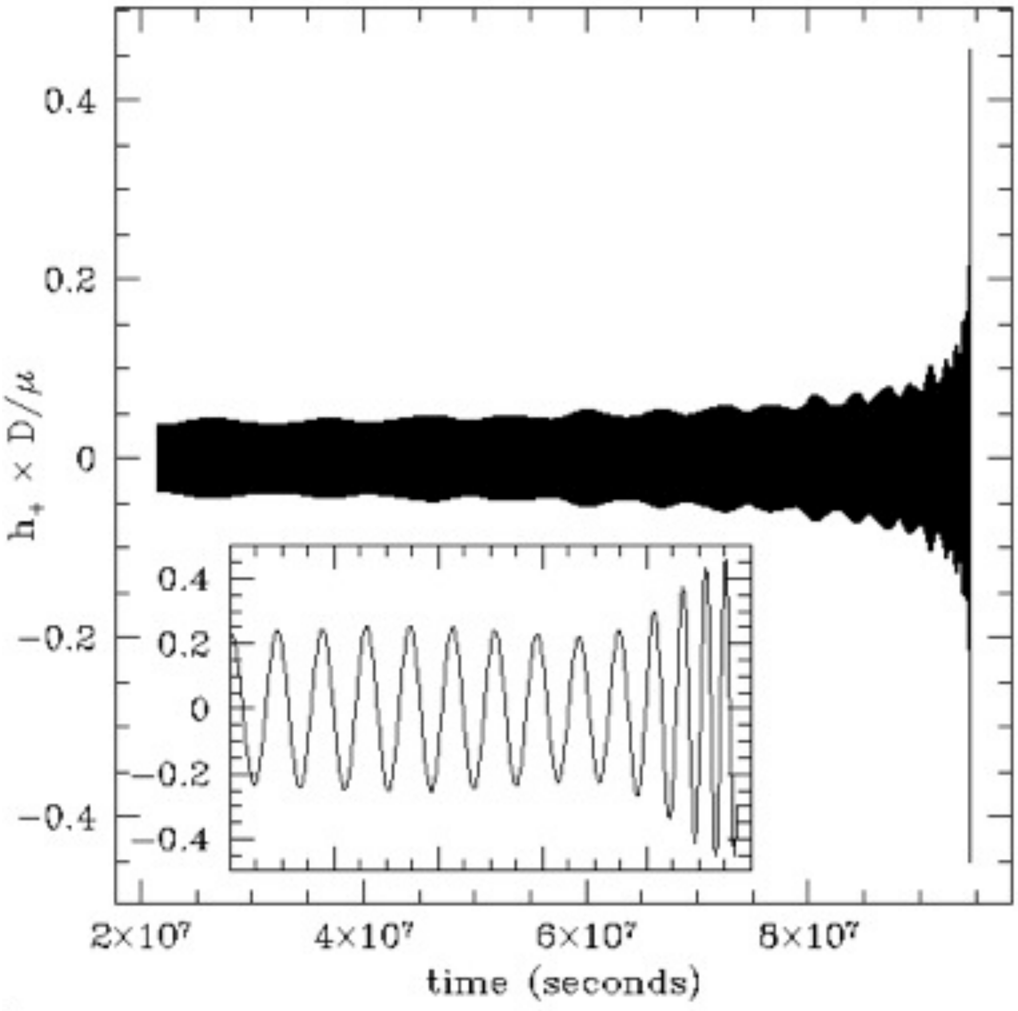


Spins cranked up!
Spin 1 = Spin 2 = 99% maximum

Strong frequency and amplitude modulation gives spin precision.

Inspiral measurements

$$h_+ = \frac{[G(1+z)\mathcal{M}/c^2]^{5/3}\,[\pi f(t)/c]^{2/3}}{D_L}\mathcal{F}(\text{“angles”})\cos[\Phi(t)]$$



Spins cranked up!
Spin 1 = Spin 2 =
99% maximum

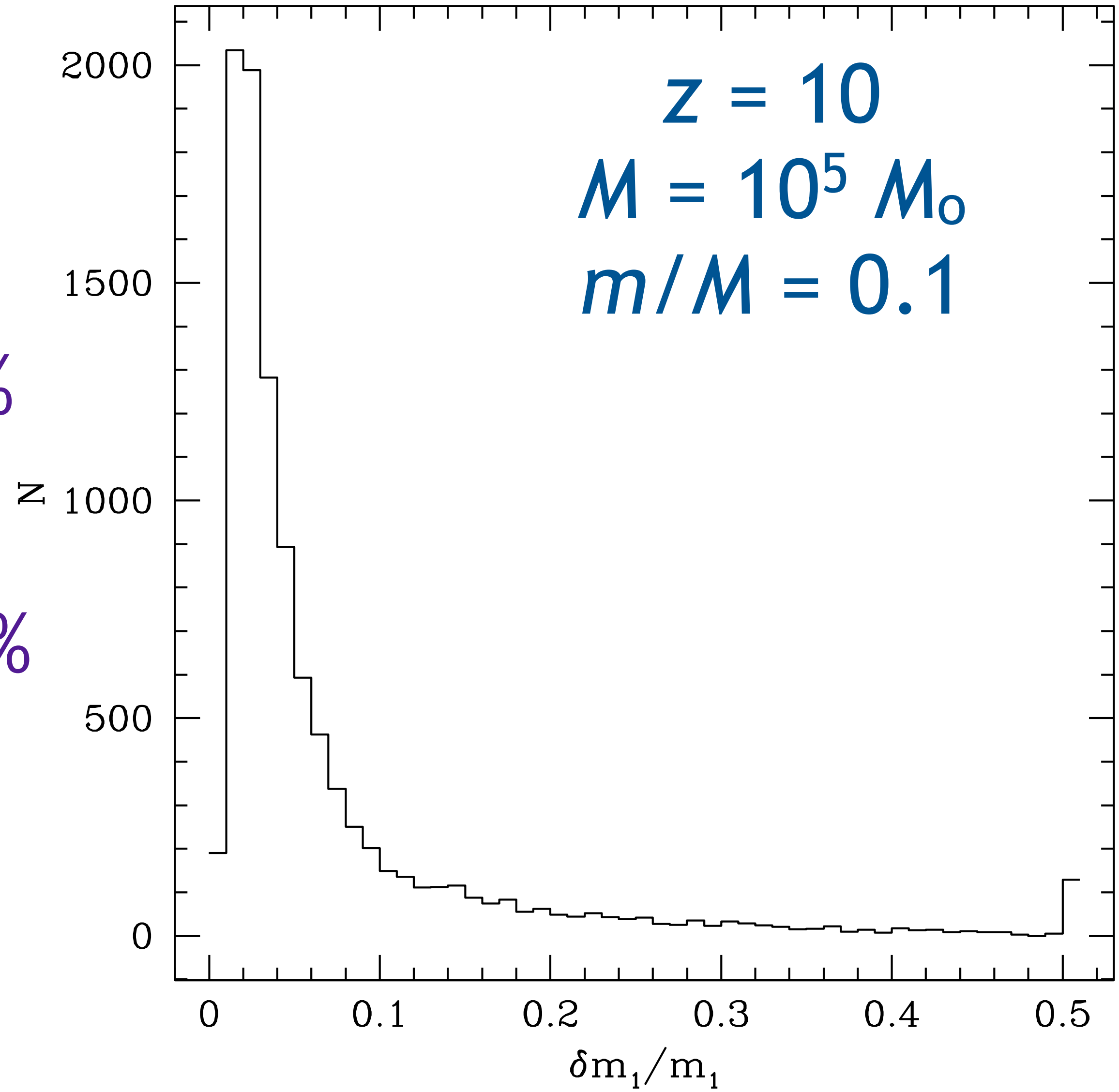
Strong frequency and amplitude modulation gives spin precision.

Highly precise masses, moderately precise spins

Example: distribution of errors in mass measurement

Median mass error 3.6%

90% of distribution confined to $\delta m/m < 17\%$

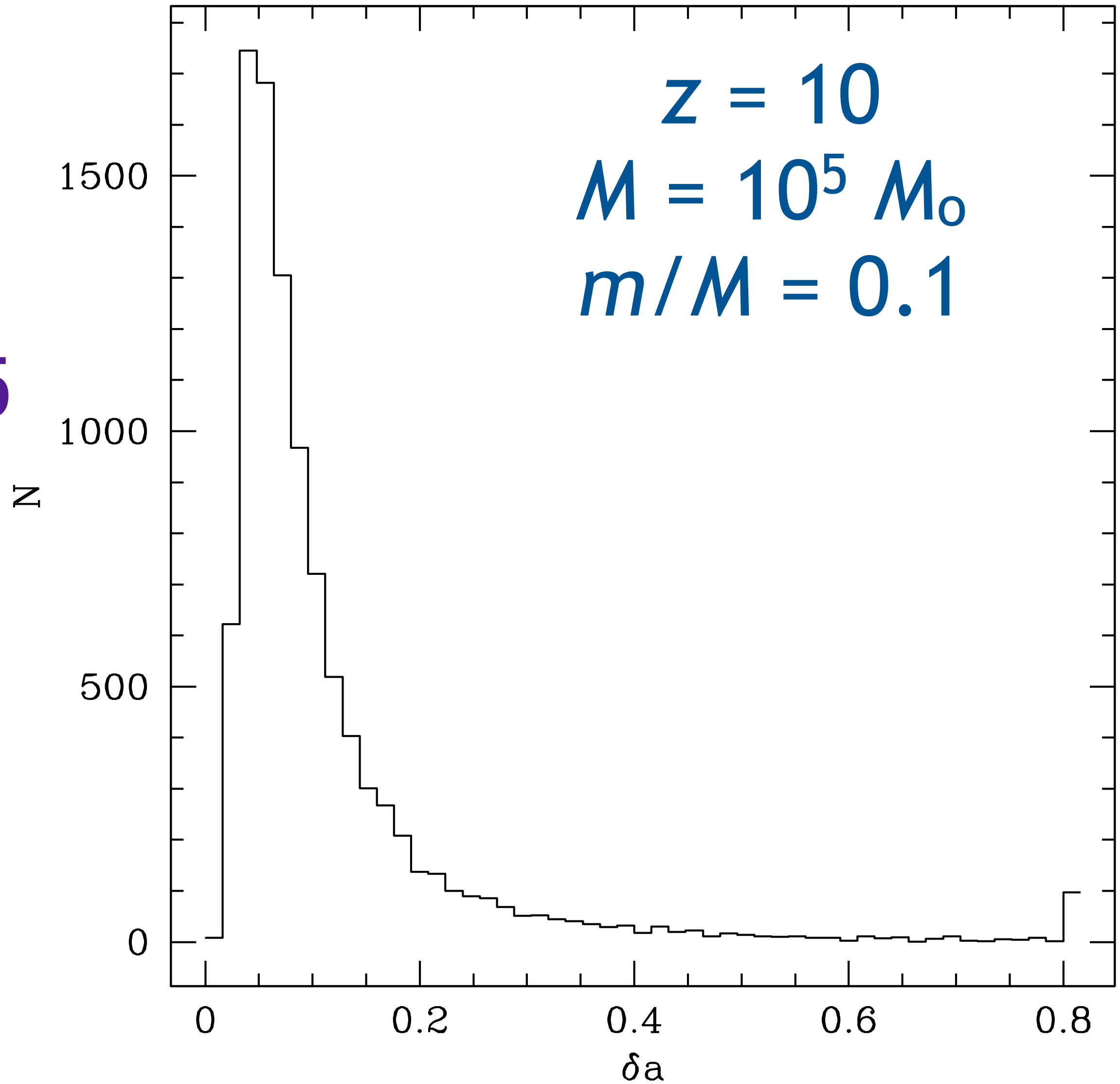


Highly precise masses, moderately precise spins

Example: distribution of errors in spin measurement

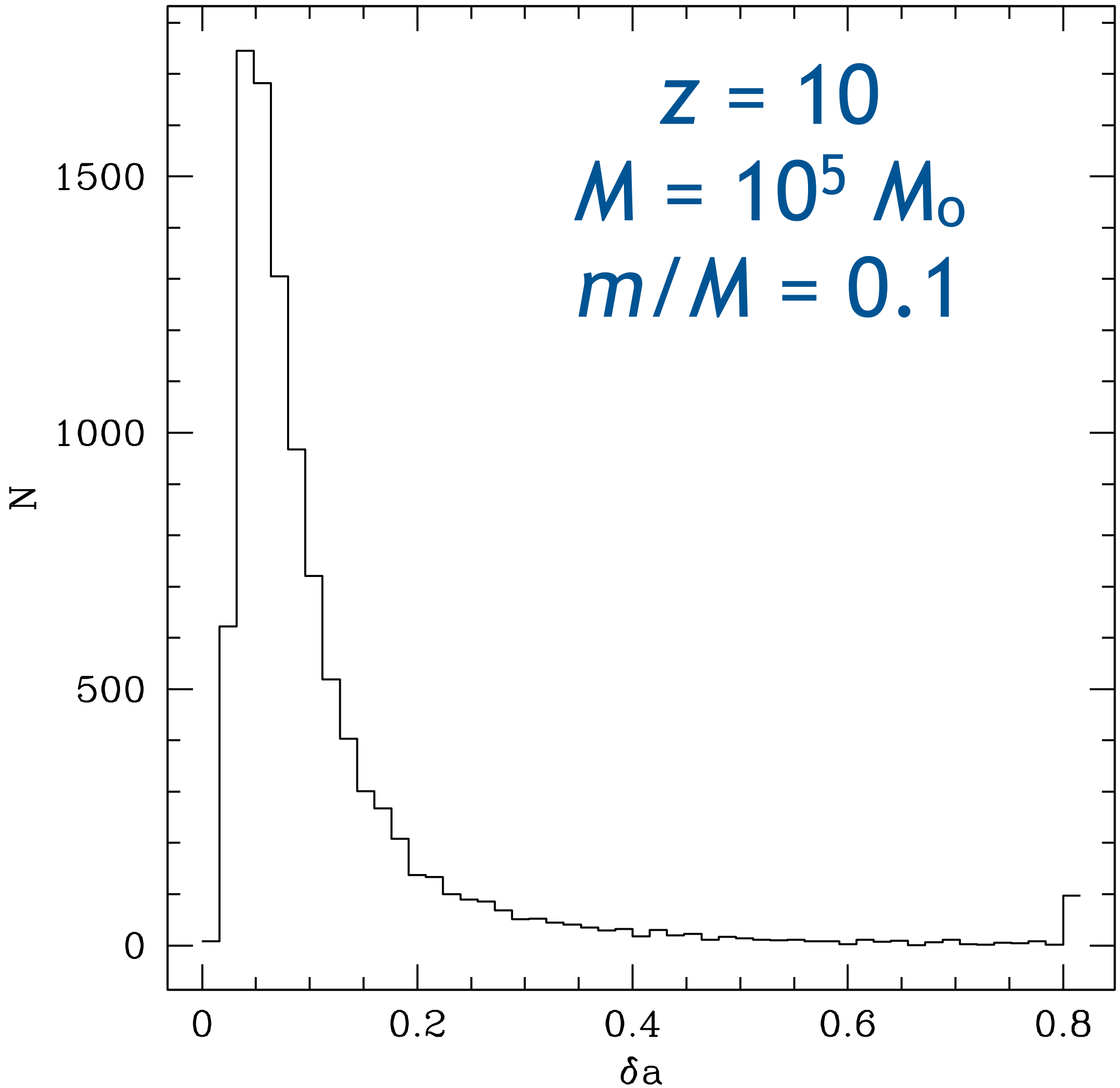
Median spin error 0.075

90% of distribution confined to $\delta a < 0.22$



Highly precise masses, moderately precise spins

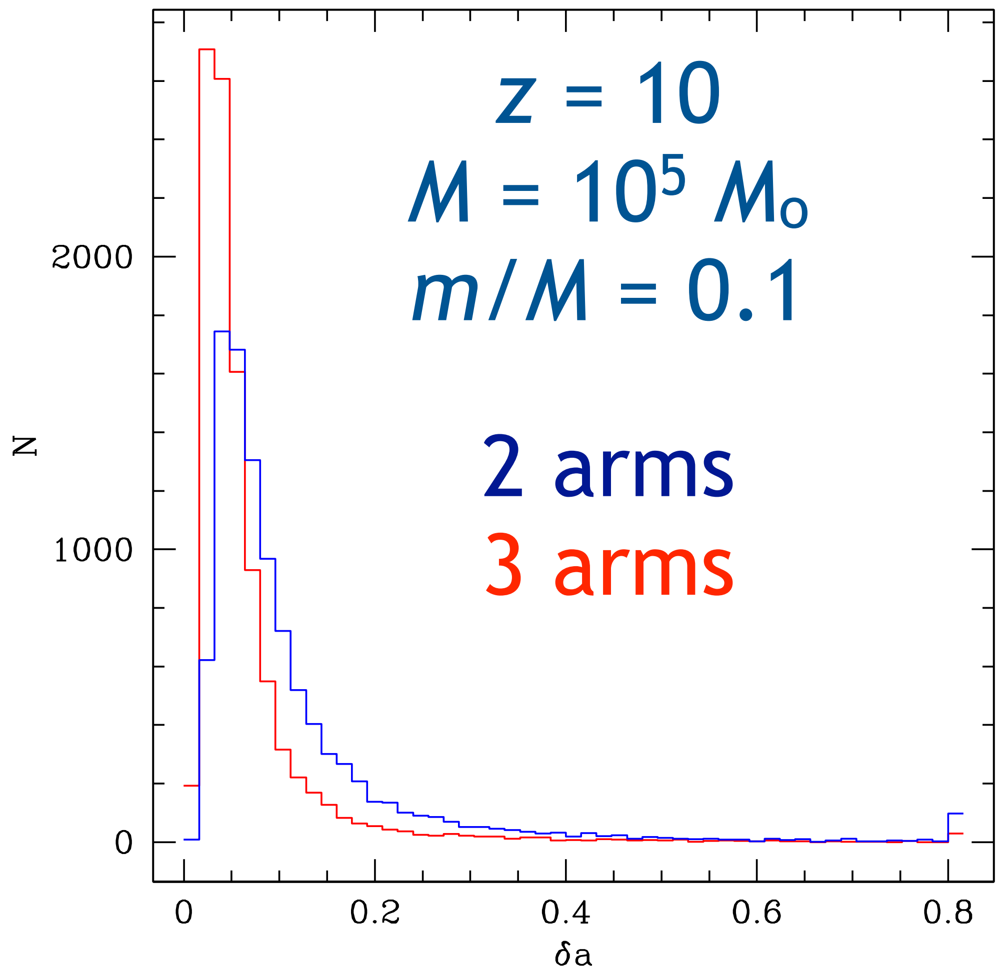
Similar measurement distributions for mass and spin found out to z ~ 15, and for masses of 10^4 - (a few) 10^5 M_o.



Not much improvement on mass and spin if we imagine using a 3-armed variant of eLISA

Example: Precision of spin measurement, comparing eLISA with 2 arms to eLISA with 3 arms.

Improvement entirely due to boost of signal-to-noise ratio.



CAUTION: *Redshifted* masses are precisely measured

Consider nearby source: Phase encodes timescale for orbit change; tells us mass scale:

$$\int f(t)\,dt \to \tau_{\rm orbit} \propto \mathcal{M}$$

Consider cosmological source: Now measure a *redshifted* timescale; infer *redshifted* mass:

$$\int f(t)\,dt \to (1+z)\tau_{\rm orbit} \propto (1+z)\mathcal{M}$$

Redshift is degenerate with masses.

True when taken to higher order as well ...

cannot infer redshift from GW measurables.

Pieces of inspiral waveform

$$h_+ = \frac{[G\mathcal{M}/c^2]^{5/3}\,[\pi f(t)/c]^{2/3}}{D_L}(1+\cos^2\iota)\cos\left[2\pi\int f(t)\,dt\right]$$

$$h_\times = \frac{2\,[G\mathcal{M}/c^2]^{5/3}\,[\pi f(t)/c]^{2/3}}{D_L}\cos\iota\sin\left[2\pi\int f(t)\,dt\right]$$

1. Phase. Depends on how rapidly the orbit evolves. Rate is controlled by binary's masses and spins.

Measure the phase, measure masses and spins.

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Measure both polarizations, you measure this angle.

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1. Phase. Depends on how rapidly the orbit evolves. Rate is controlled by binary's masses and spins.

Measure the phase, measure masses and spins.

2. Inclination of orbital plane to line of sight.

Measure both polarizations, you measure this angle.

3. Luminosity distance. Sets amplitude, once masses and inclination are determined.

Pieces of inspiral waveform

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$$h_\times = \frac{2[G\mathcal{M}/c^2]^{5/3}[\pi f(t)/c]^{2/3}}{D_L}\cos\iota\sin\left[2\pi\int f(t)\,dt\right]$$

Once we know distance, can get *z* by inverting distance-redshift relation, assuming the cosmography ... Lets us break degeneracy and determine *rest frame* parameters of binary.

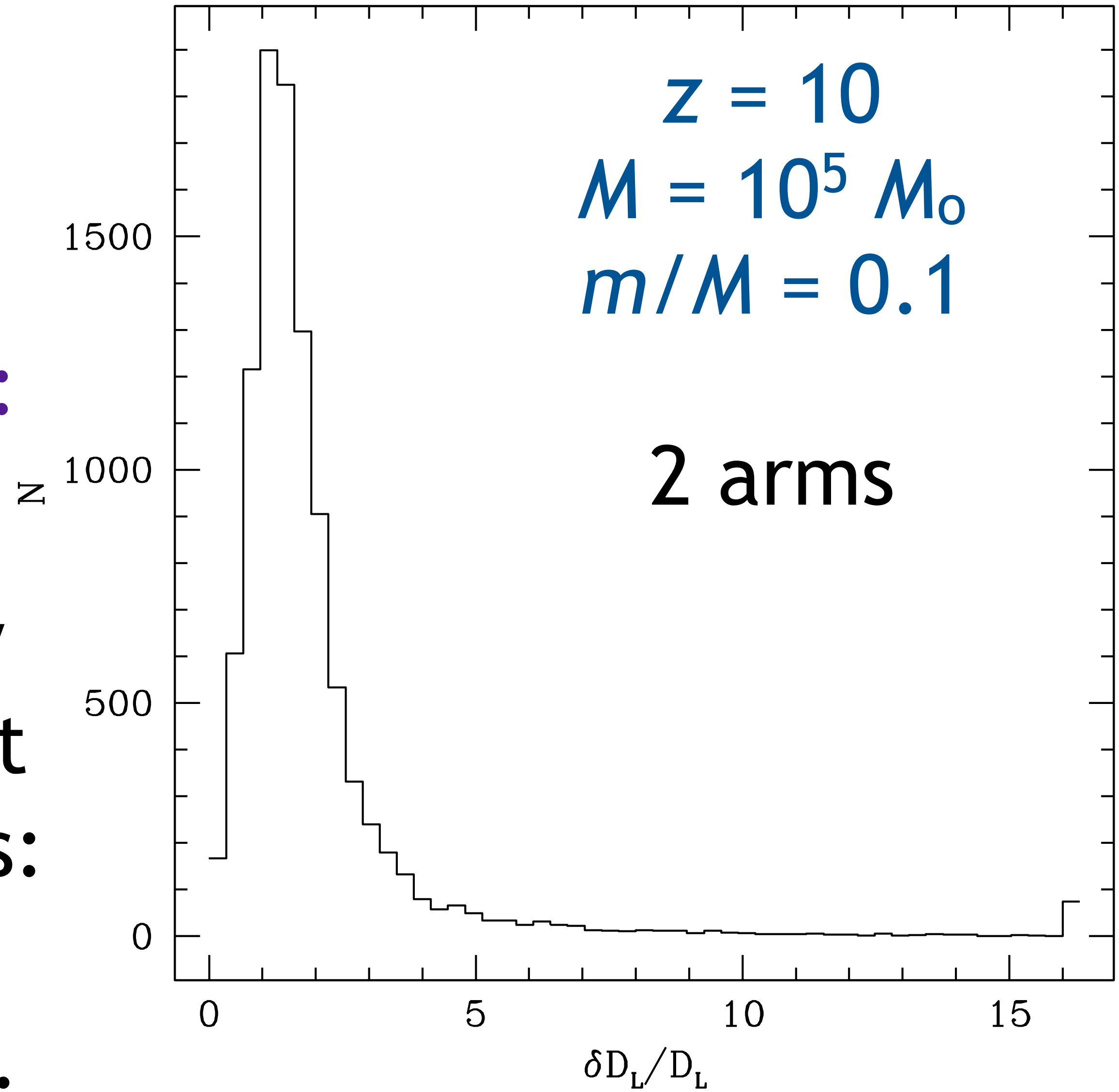
Both polarizations and directly measuring distance requires 3 arms

Example: distribution of errors in distance for 2-armed eLISA

Median distance error:

$\delta D/D = 1.5$

Distance is essentially unconstrained for most of these measurements: Not good enough to break *m*-*z* degeneracy.



Both polarizations and directly measuring distance requires 3 arms

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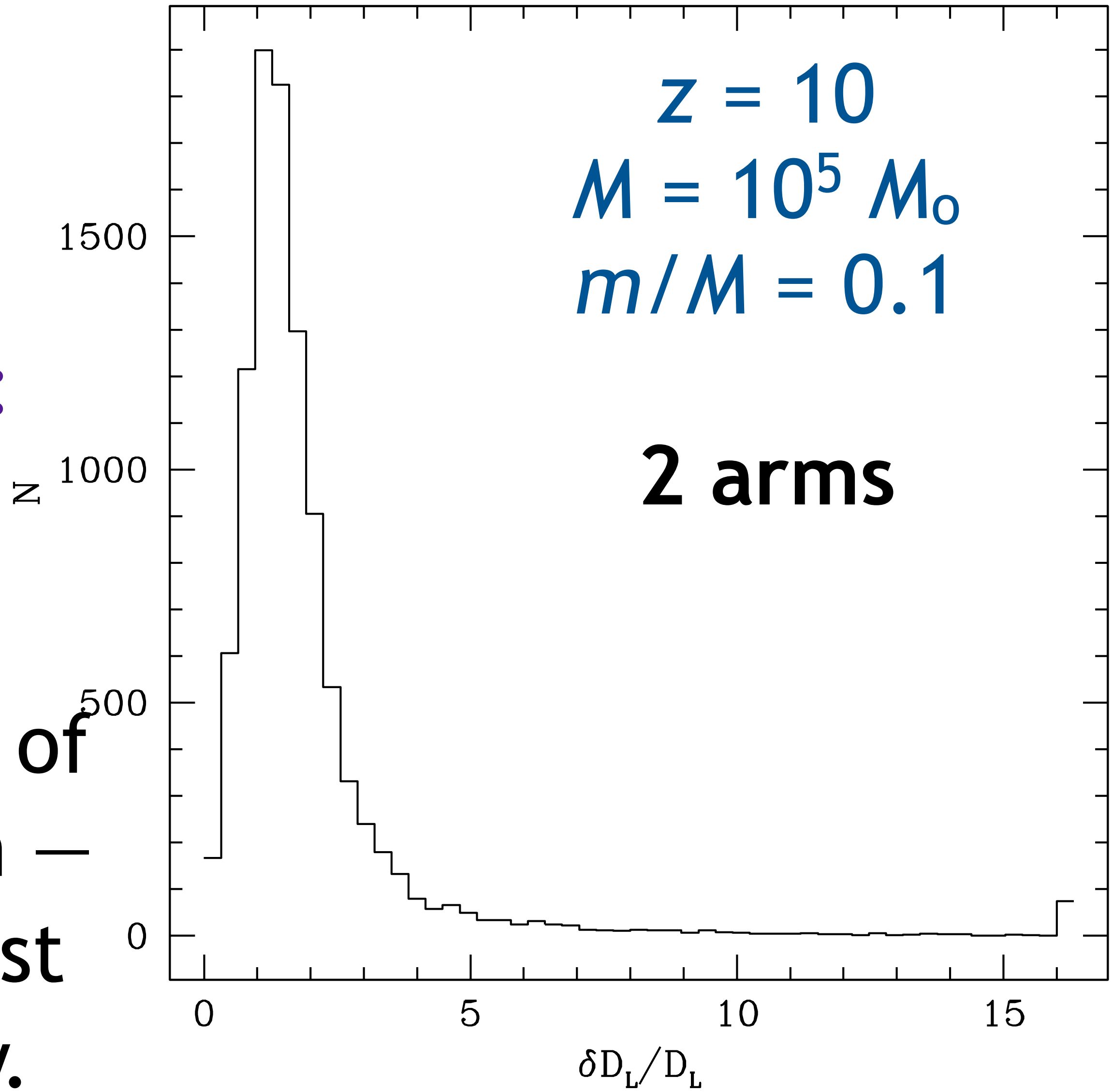
Median distance error:

$\delta D/D = 1.5$

Many events make it possible to test models of early black hole growth — but not as well as if rest frame masses directly.

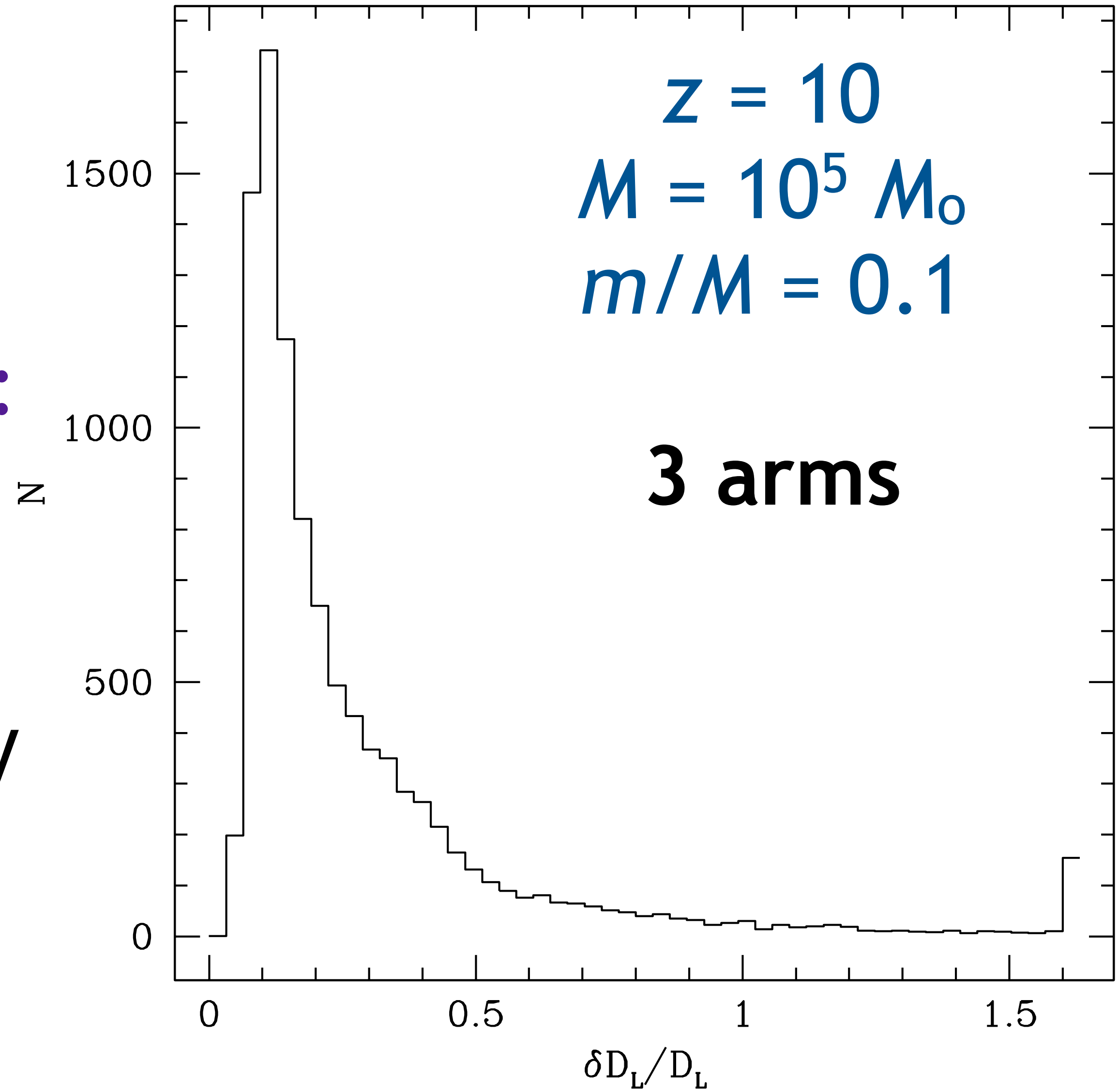


Both polarizations and directly measuring distance requires 3 arms

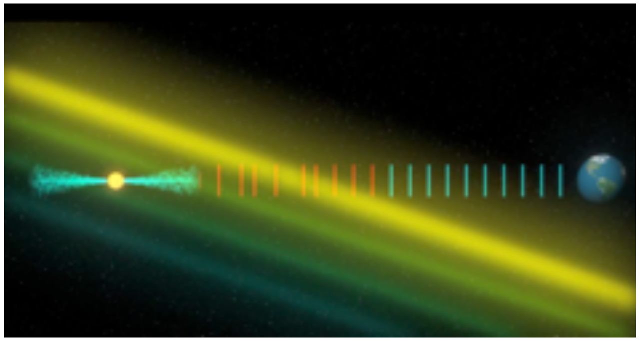
Example: distribution of errors in distance for 3-armed eLISA

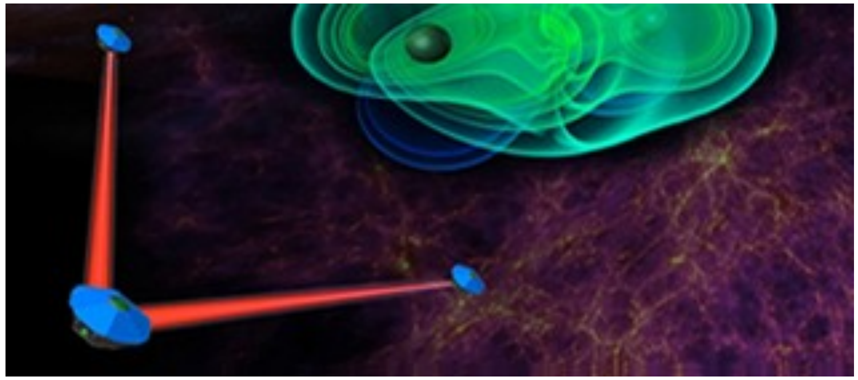
Median distance error: $\delta D/D = 0.17$

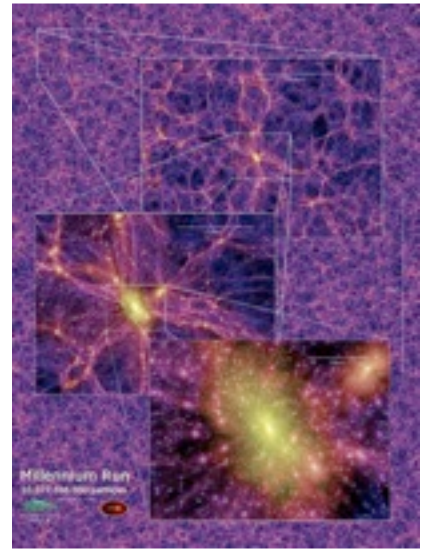
Accurate enough to break *m*-*z* and directly track rest frame evolution of masses.



Conclusion





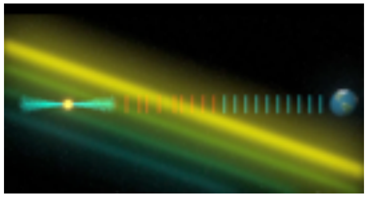


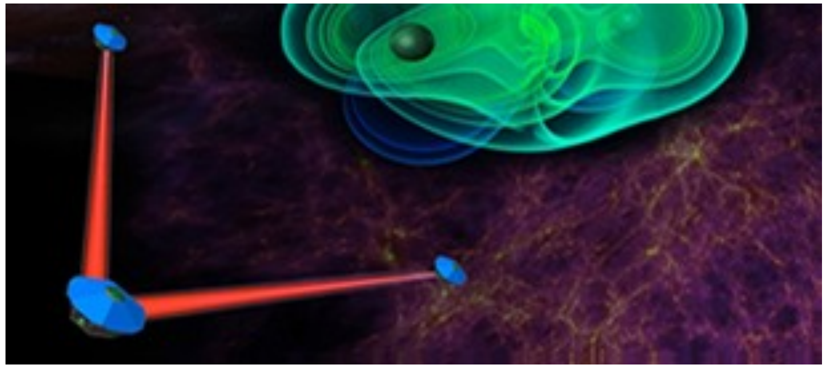
GWs from MBH binaries information rich:

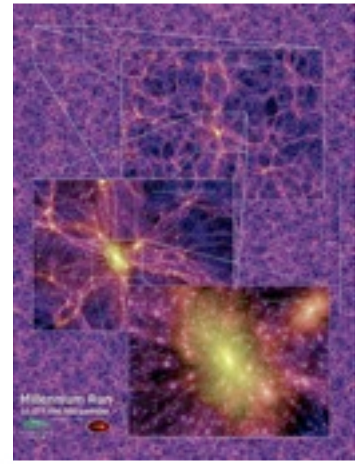
* Precise (redshifted) masses
* Good information about spin parameter
* *Possibly* distances accurately enough to break mass-redshift degeneracy.

Nature appears to be giving us the binaries ...
"just" need to start measuring these waves.

Conclusion







Let's grab some babies!!

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