Gas, stars and gravitational waves

on the main driver of supermassive black hole binaries path to coalescence

Elisa Bortolas

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A. Gualandris, A. Lupi, M. Dotti,
L. Mayer, P. Capelo, ...

BINARY22 - Kavli Institute for Theoretical Physics
Santa Barbara, April 7th, 2022
Massive black hole binaries

MBHs at the centre of galaxies since early times

Frequent galaxy mergers through the cosmic history

Formation of SMBH binaries in large numbers

Haehnelt+98; Wu+15

White+78; Fakhouri+10

Thorne+76, Begelman+80

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The path to coalescence ~40 years ago...

Begelman, Blandford and Rees 1980

Dynamical friction (against dark matter, gas, stars)

Stellar-driven hardening

Effects of gas

Galaxy merger

Reference mass: $10^6$ Msun

Gravitational waves

Reference timescales:
A few 100 Myr to a few Gyr

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...and the path to coalescence today

Dynamical friction (against dark matter, gas, stars)

Clump scattering

Effect of bars/spirals

Stellar-driven hardening

3rd incoming MBH

Disk-driven migration torques

Circumbinary disk & minidisk torques

Gravitational waves

Galaxy merger

Reference mass: $10^6$ Msun

Reference timescales:
A few 100 Myr to a few Gyr
...and the path to coalescence today

**Dynamical friction** (against dark matter, gas, stars)
- Fiacconi+13, Roskar+15, Tamburello+17, Souza-Lima+17
- Roskar+15, EB+20, 22
- Quinlan96, Sesana+06, Sesana&Khan10, Milosavljević&Merritt03, Khan+11, Vasiliev+15, EB+16, 18
- Bonetti+18, 19, Mannerkoski+21
- Armitage&Natarajan02, Escala+04, Dotti+07, Mayer+07, Lodato+09, Souza-Lima+20[incl.EB]
- Goicovic+17, Moody+19, Munoz+19, 20, Duffel+20, Tiede+20, Zrake+21, Franchini+21, D’orazio&Duffel21, EB+21
- Peters64, Hills&Fullerton80, Mannerkoski+19, Zwick20, 21[incl.EB], Vázquez-Aceves+21[incl.EB]

**Clump scattering**
- Chandrasekhar43, Capelo&Dotti15, Lupi+15, Pfister+17, Tamfal+17, 21; Bonetti+20, 21[incl.EB], Gualandris+ 22[incl.EB]

**Effect of bars/spirals**
- Fiacconi+13, Roskar+15, Tamburello+17, Souza-Lima+17
- Roskar+15, EB+20, 22
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- Peters64, Hills&Fullerton80, Mannerkoski+19, Zwick20, 21[incl.EB], Vázquez-Aceves+21[incl.EB]

**Stellar-driven hardening**
- Bonetti+18, 19, Mannerkoski+21
- Armitage&Natarajan02, Escala+04, Dotti+07, Mayer+07, Lodato+09, Souza-Lima+20[incl.EB]

**3rd incoming MBH**
- Armitage&Natarajan02, Escala+04, Dotti+07, Mayer+07, Lodato+09, Souza-Lima+20[incl.EB]

**Disk-driven migration torques**
- Armitage&Natarajan02, Escala+04, Dotti+07, Mayer+07, Lodato+09, Souza-Lima+20[incl.EB]

**Circumbinary disk & minidisk torques**
- Armitage&Natarajan02, Escala+04, Dotti+07, Mayer+07, Lodato+09, Souza-Lima+20[incl.EB]

**Gravitational waves**
- Armitage&Natarajan02, Escala+04, Dotti+07, Mayer+07, Lodato+09, Souza-Lima+20[incl.EB]

**Galaxy merger**
- Reference mass: $10^6$ Msun
- Reference timescales:
  - A few 100 Myr to a few Gyr

Elisa Bortolas, KITP, April 7 2022
Real galaxies are complex environments

Dynamical friction (against dark matter, gas, stars)
- Chandrasekhar43, Capelo&Dotti15, Lupi+15, Pfister+17, Tamfal+17, 21; Bonetti+20, 21[incl.EB], Gualandris+ 22[incl.EB]
- Fiacconi+13, Roskar+15, Tamburello+17, Souza-Lima+17
- Roskar+15, EB+20, 22
- Quinlan96, Sesana+06, Sesana&Khan10, Milosavljevi&Meritt03, Khan+11, Vasiliev+15, EB+16,18
- Bonetti+18, 19, Mannerkoski+21
- Armitage&Natarajan02, Escala+04, Dotti+07, Mayer+07, Lodato+09, Souza-Lima+20[incl.EB]
- Goicovic+17, Moody+19, Munpz+19,20, Duffel+20, Tiede+20, Zrake+21, Franchini+21, D’orazio&Duffel21, EB+21
- Peters64, Hal, Zwick20,11,12,13

Clump scattering

Effect of bars/spirals

Disk-driven

Reference timescales:
- A few 100 Myr to a few Gyr

Gualandris+ 22[incl.EB]

LISA Consortium17

Reference mass: 10

Msun

TianQin

~bound binary

Real galaxies are complex environments

Img credits: Wikipedia
Real galaxies are complex environments

Dynamical friction (against dark matter, gas, stars)

Clump scattering

Effect of bars/spirals

Reference mass: $10^6$ $\text{M}_\odot$

Chandrasekhar43, Capelo&Dotti15, Lupi+15, Pfister+17, Tamfal+17, 21; Bonetti+20, 21 [incl.EB],
Gualandris+ 22 [incl.EB], Fiacconi+13, Roskar+15, Tamburello+17, Souza-Lima+17

Roskar+15, EB+20, 22

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Reference timescales:

A few 100 Myr to a few Gyr

LISA Consortium17

Img credits: Wikipedia
The LARGE scale inspiral is the dynamical friction treatment good enough?
BHs distance from the centre

$R_0 = 1.25 \text{ kpc}$

Bortolas+2020

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Dynamical friction torque modulus
From all components (dark matter, stars, gas)
Semi-analytical calculation in a sphere of 150 pc around each MBH
Dynamical friction torque modulus
From all components (dark matter, stars, gas)
Semi-analytical calculation in a sphere of 150 pc around each MBH

Global torque modulus

Gas
\[
F_{\text{DF}}^{\text{gas}} = -4\pi \ln \left( \frac{b_{\text{max}}}{b_{\text{min}}} \left( \frac{M^2 - 1)^{1/2}}{M} \right) \right) G^2 M_{\text{BH}}^2 \rho_{\text{gas}} \frac{V}{V^3}.
\]

Ostriker99
- \( b_{\text{min}} = GM/v^2 \sim 0.1 \text{ pc (physical)} \)
- \( b_{\text{max}} = 2 \text{ kpc (2 x disc scale radius)} \)

Stars, DM
\[
F_{\text{DF}}^{\text{stars, DM}} = -4\pi \ln \Lambda G^2 M_{\text{BH}}^2 \rho_* \left[ \text{erf} \left( \frac{V}{\sqrt{2}\sigma} \right) - \left( \frac{\sqrt{2} V}{\pi \sigma} \right) \exp \left( -\frac{V^2}{2\sigma^2} \right) \right] \frac{V}{V^3}.
\]

Chandrasekhar43

Bortolas+2020, see also Bortolas+22
Doggy bag #1

The simple dynamical friction treatment for massive black holes inspiral may be poor in realistic galaxies especially:
- At high z
- In irregular/barred galaxies

May I chip in...

5 cents for some discussion?

- How to model this stochasticity in inexpensive semi-analytical models for studying the binary population and merger rates?

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Small scale (bound binary) evolution

Dynamical friction (against dark matter, gas, stars)
Clump scattering
Effect of bars/spirals
Stellar-driven hardening
3\textsuperscript{rd} incoming MBH
Disk-driven migration torques
Circumbinary disk & minidisk torques

Galaxy merger

Reference mass: $10^6$ Msun

100 kpc  1 kpc  100 pc  1 pc  $10^{-2}$ pc  $10^{-6}$ pc  $10^{-7}$ pc

Gravitational waves

PTA LISA

MBHs coalescence

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Small scale (bound binary) evolution

Most galaxies host both GAS and STARS, so that both contribute to the orbital evolution (see Kelley+17,19)

Dynamical friction [against dark matter, gas, stars]

Clump scattering

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Gravitational waves

Reference mass: $10^6$ Msun

Galaxy merger

100 kpc 1 kpc 100 pc 1 pc 10^{-2} pc 10^{-6} pc 10^{-7} pc

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LET’S HAVE THE MOST PESSIMISTIC APPROACH

- **Stellar driven hardening** ➔ Binary shrinking

- **Gas-driven evolution** ➔ Shrinking or **expansion**?

- **Gravitational wave emission** ➔ Efficient small scale shrinking

Artymowicz & Lubow 1994, Moody +19, Munoz +19, 20, Duffel +20, Tiede +20, Heath & Nixon 20, Franchini +21, D’orazio & Duffel 21+.... Almost everyone in the audience 😊

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• Stellar driven hardening

\[ \frac{d a_\star}{d t} \propto -a^2 \]

Binary shrinking

\[ \dot{a}_\star = -\frac{H G \rho}{\sigma} a^2 \]

No nuclear star cluster

M-sigma relation

(Kormendy&Ho13, Merrit+09)

Quinlan96, Sesana+06, Sesana&Khan15
• **Stellar driven hardening**

\[
\frac{da_*}{dt} \propto -a^2
\]

Binary shrinking

• **Gravitational wave emission**

\[
\frac{da_{GW}}{dt} \propto -a^{-3}
\]

Efficient shrinking at small scale

\[
\dot{a}_{GW} = -\frac{64}{5} \frac{G^3}{c^5} \frac{q}{(1+q)^2} \frac{m^3}{a^3}
\]

Peters64

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• **Stellar driven hardening**

\[
\frac{da^*}{dt} \propto -a^2
\]

Binary shrinking

• **Gas-driven evolution**

\[
\frac{da_{\text{gas}}}{dt} \propto +a
\]

We assume binary expansion

• **Gravitational wave emission**

\[
\frac{da_{\text{GW}}}{dt} \propto -a^{-3}
\]

Efficient shrinking at small scale

\[a_{\text{gas}} = 2.68 \frac{\dot{m}}{m}\]

Munoz+20

Shrinking direction

GW emission phase

Stellar hardening

Gas driven expansion

Efficient shrinking at small scale

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When is the gas effective?

\[
\begin{align*}
\dot{a}_* &= -\frac{HG_\rho}{\sigma} a^2 \\
\dot{a}_{\text{gas}} &= 2.68 \frac{\dot{m}}{m} a \\
\dot{a}_{\text{GW}} &= -\frac{64 G^3}{5} \frac{q}{c^5} \frac{m^3}{(1+q)^2} \frac{1}{a^3}
\end{align*}
\]

Some more assumptions:
Equal mass binary;
Binary eccentricity = 0 at all times

How do we model accretion?

- **Fixed eddington ratio** \( f_{\text{Edd}} \) so that \( \dot{m} = f_{\text{Edd}} \dot{m}_{\text{Edd}} \propto m \)
  
  (the accretion rate grows linearly with the binary mass, and remains a fixed fraction of the Eddington accretion rate)

**NOTE THAT DIFFERENT ASSUMPTIONS (FIXED MDOT) FOR THE MASS ACCRETION RATE RESULT IN AN EVEN LESS EFFICIENT GAS-DRIVEN EXPANSION**
Fixed eddington ratio $f_{\text{Edd}}$ so that $\dot{m} = f_{\text{Edd}} \dot{m}_{\text{Edd}} \propto m$ (the accretion rate grows linearly with the binary mass, and remains a fixed fraction of the Eddington accretion rate)

**NOTE THAT DIFFERENT ASSUMPTIONS (FIXED MDOT) FOR THE MASS ACCRETION RATE RESULT IN AN EVEN LESS EFFICIENT GAS-DRIVEN EXPANSION**
Can the binary expand indefinitely?

NO!

THE SELF GRAVITATING RADIUS

Franchini+21
Can the binary expand indefinitely?

NO!

THE SELF GRAVITATING RADIUS

\[ R_{sg} \propto f_{\text{Edd}}^{-22/45} m^{-7/45} \]

\[ R_{sg} \sim 10^{-2} \text{ pc for a } 10^6 \text{ solar mass binary accreting at Eddington} \]
Can the binary expand indefinitely?

NO!

THE SELF GRAVITATING RADIUS

\[ R_{\text{sg}} \propto f_{\text{Edd}}^{-22/45} m^{-7/45} \]

\[ R_{\text{sg}} \sim 10^{-2} \text{ pc} \] for a \( 10^6 \) solar mass binary accreting at Eddington

Here the disk is self gravitating down to the scales at which GW always dominate

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Evolution for fixed $f_{\text{Edd}}$

Equal mass binary; Binary eccentricity = 0 at all times

Elisa Bortolas, KITP, April 7 2022
Evolution for fixed $f_{\text{Edd}}$

The binary efficient shrinking even when gas expansion is active has to be attributed to the fact that the binary mass grows by orders of magnitude → this may shift binaries from LISA to PTA band.

Equal mass binary; Binary eccentricity = 0 at all times.
Different initial binary mass

\[ f_{\text{Edd}} = 1, \ q = 1 \]

Fixed \( f_{\text{Edd}} \)

\[ m = m_0 \times 10^3, \ m_0 = 10^4, \ m_0 = 10^5, \ m_0 = 10^6, \ m_0 = 10^7, \ m_0 = 10^8 \]

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To be compared with the timescale of large scale inspiral, which can be much longer!
Inspiral timescale

This can imply that accreting, expanding binaries can be observable for a longer time via electromagnetic surveys, but they anyways eventually coalesce!
Doggy bag #2

- **GAS AND STARS GENERALLY COEXIST!!**
- Gas driven expansion does not dramatically impact the coalescence time of massive binaries

→ Expansion necessarily reverts into shrinking when the binary mass gets large enough owing to accretion: at that point, gravitational wave emission becomes dominant.

May I chip in...

*another 5 cents for some discussions/ideas?*

- This would imply expanding binaries would be observed at lower frequencies (PTAs)
- We should think of better simulations/works accounting for concurrent effects of stars and gas
- Would it be possible to use a similar approach in the framework of stellar binaries? [e.g. tides instead of stellar hardening and so on...]

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