Nematic states of active rods: Ordering and instabilities

Xiaqing Shi

Center for Soft Condensed Matter Physics and Interdisciplinary Research
Soochow University, Suzhou, China

Collaborators:

Yuqiang Ma Nanjing University

Hugues Chate
CEA, Saclay

Hepeng Zhang
Shanghai Jiaotong University

Ming Ji
Nanjing University

Xinchen Jiang
Soochow University

Fu Cheng
Soochow University

Active Processes in Living and Nonliving Matter

KITP Feb 10-Feb 14, 2014

A: Suzhou

B: Santa Barbara



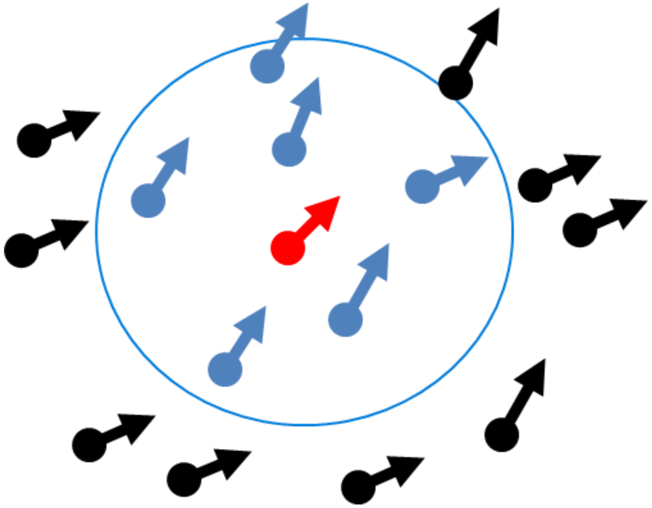

Introduction

VOLUME 75, NUMBER 6 PHYSICAL REVIEW LETTERS 7 AUGUST 1995

Novel Type of Phase Transition in a System of Self-Driven Particles

Tamás Vicsek,[1,2] András Czirók,[1] Eshel Ben-Jacob,[3] Inon Cohen,[3] and Ofer Shochet[3]

'Ferrofishes'



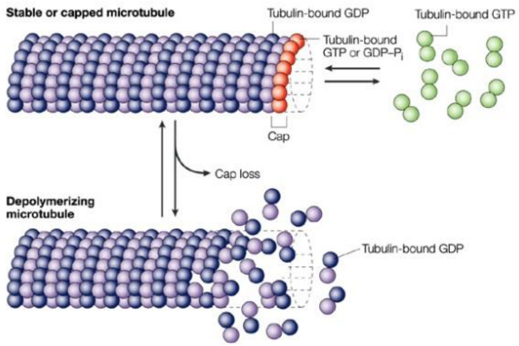
$$\theta_i^{t+1} = \Theta\Big(\sum_{j\in\Phi\{\mathbf{x}_i^t\}} \mathbf{v}_j^t\Big) + \xi_i^t$$

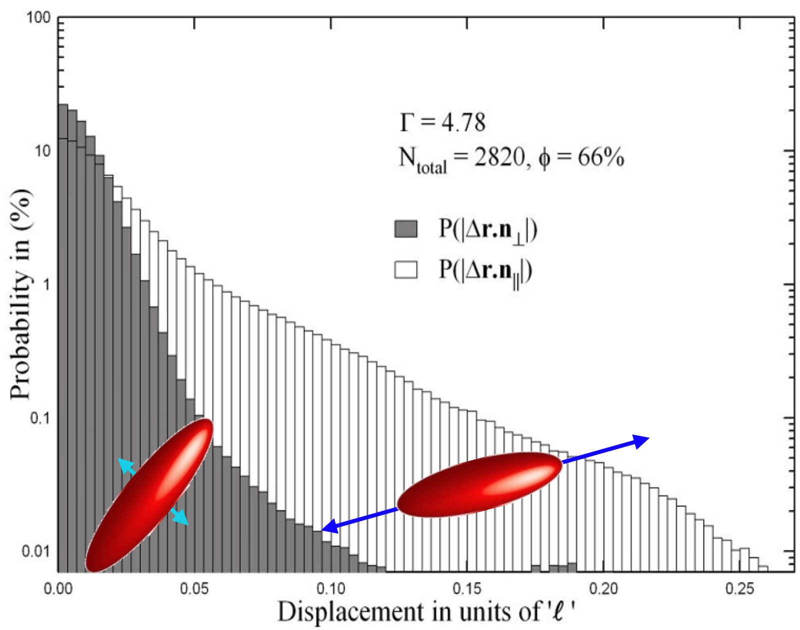
$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + v_0\delta_t\mathbf{v}_i^t$$

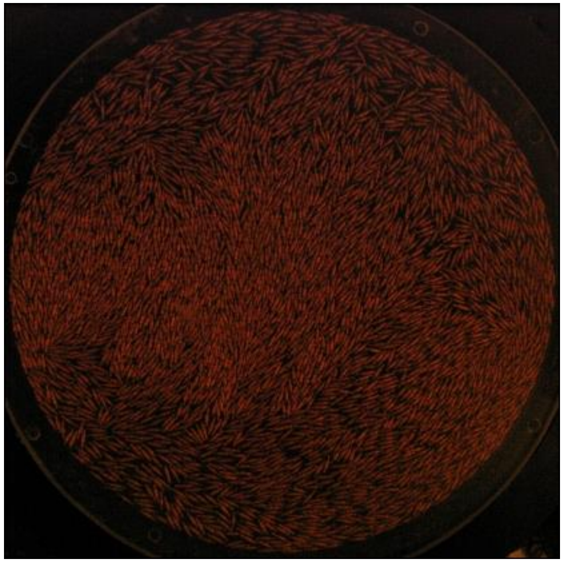
Lessons from Vicsek Model

- Large-scale and longtime behavior of the active system could be interesting.

Two active rod systems

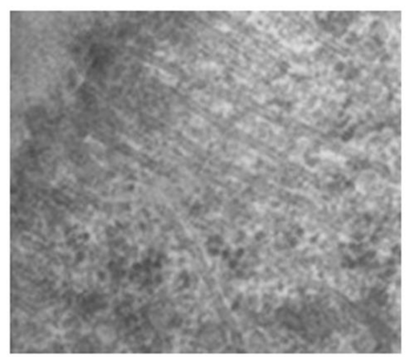






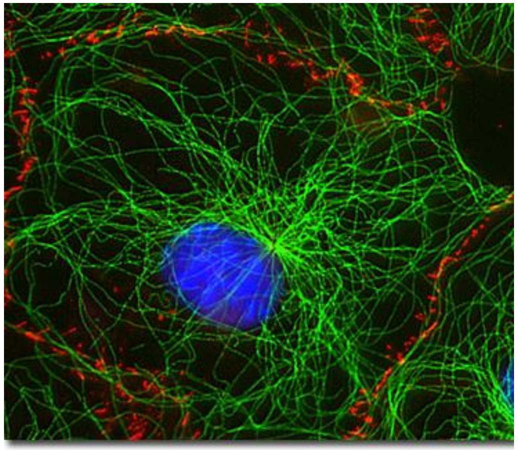
V. Narayan et al, Science, (2007).

Plant Cell Cortical Microtubule Array



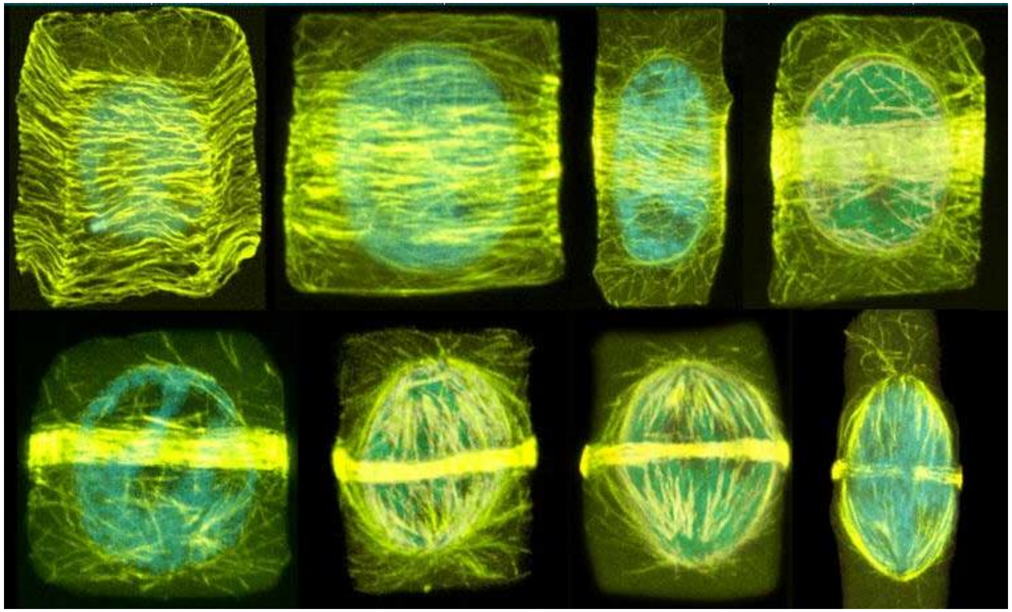
D.B. Slautterback 1963 JCB

Interphase microtubule asters of mice fibroblast cell



http://www.maths.bris.ac.uk/~matbl/research/biophys.html

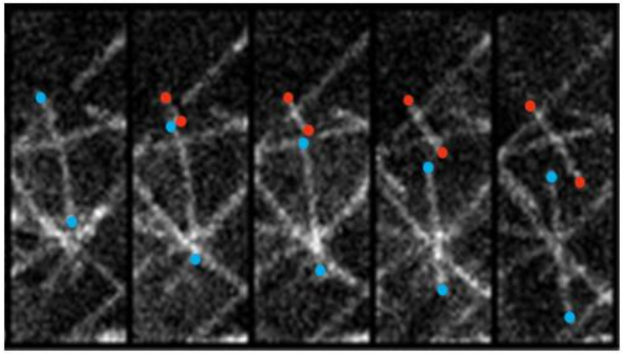
Interphase cortical microtubule array & preprophase band



http://biology.anu.edu.au/CMS/FileUploads/file/gunning/

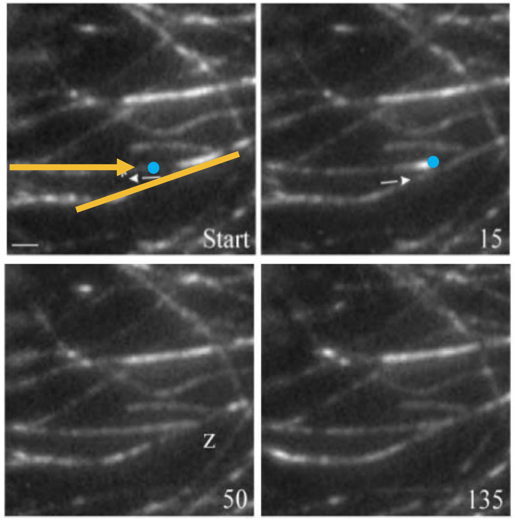
Cytoskeleton Nematics

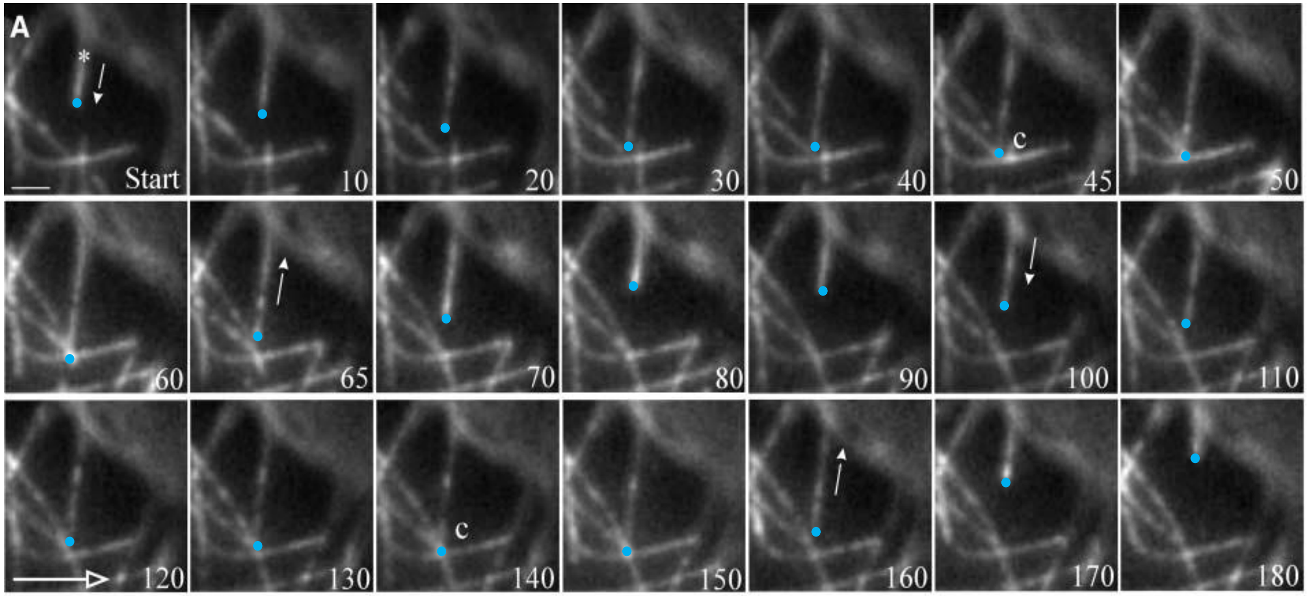
1. Nucleation and treadmilling of microtubules



S.L. Shaw *et al.* 2003 Science

2. Zippering and crossover between interacting microtubules

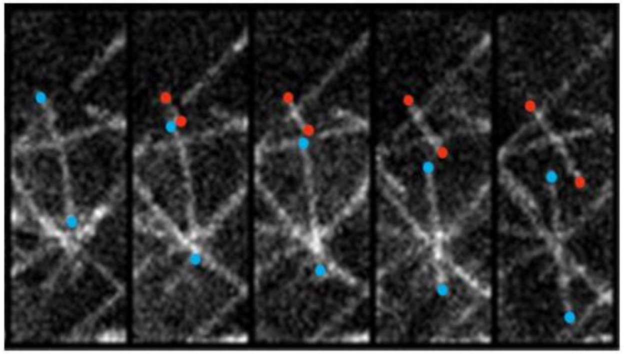




R. Dixit & R. Cyr,
The plant cell, 16, 3274(2004)

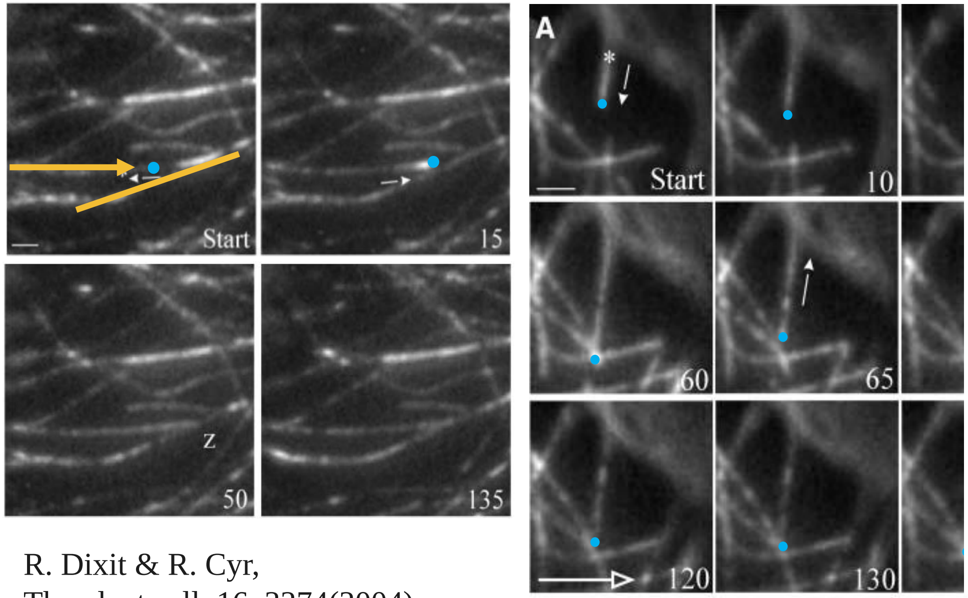
Cytoskeleton Nematics

1. Nucleation and treadmilling of microtubules

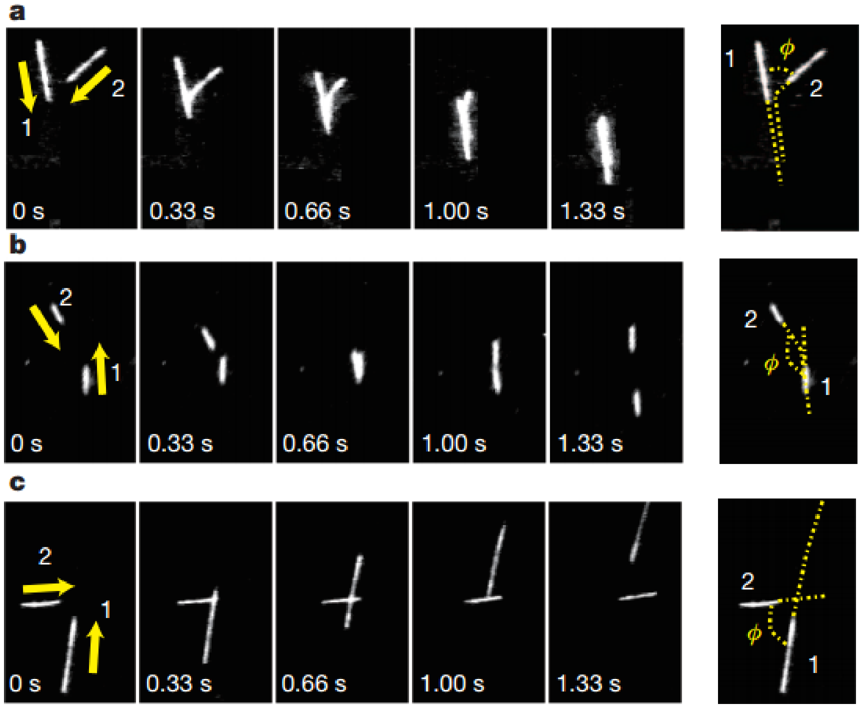


S.L. Shaw *et al.* 2003 Science

2. Zippering and crossover between inter



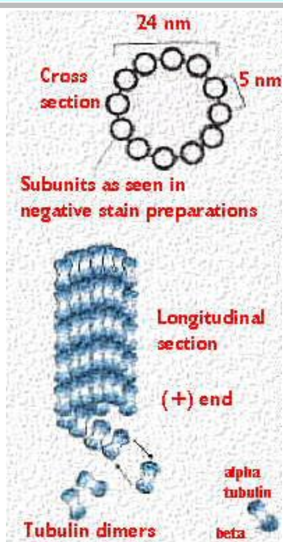
R. Dixit & R. Cyr,
The plant cell, 16, 3274(2004)



Y. Sumino, et al, Nature, 2012

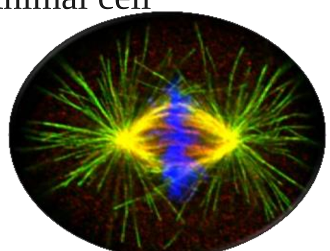
Basics of microtubule

Structures

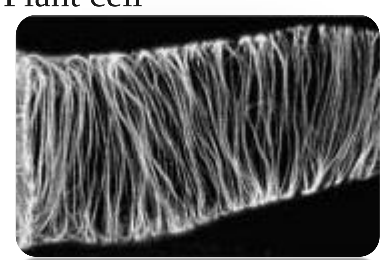


www.cytochemistry.net

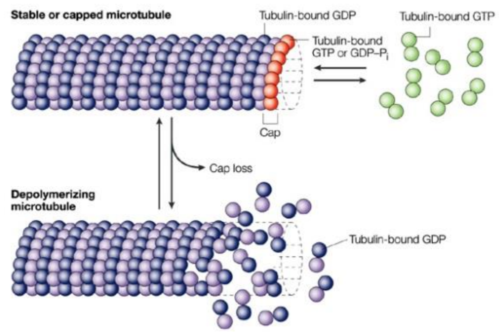
Animal cell



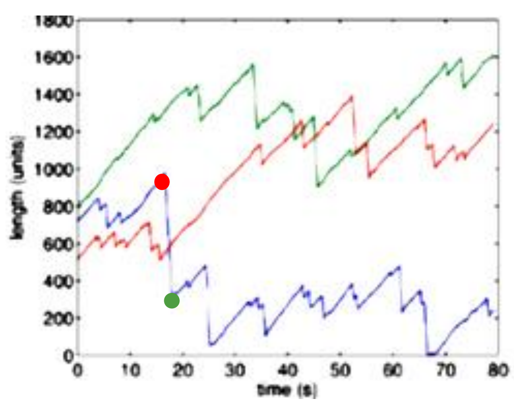
Plant cell



Plus end dynamics



Nature Reviews Cancer
4, 253, (2004)

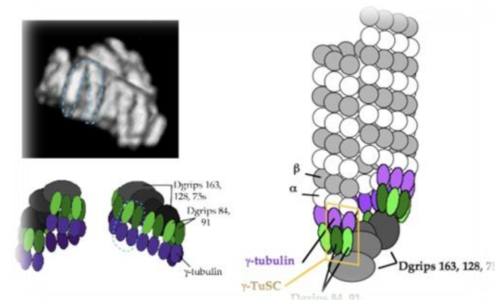


- **Catastrophe point**
- **Rescue point**

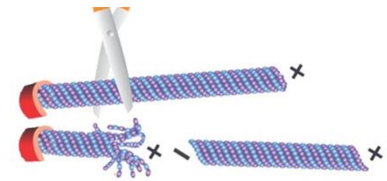
Margolin *et al.* PRE **74**, 041920 2006

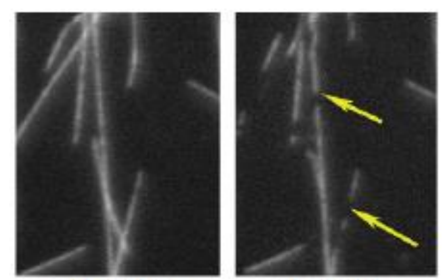
Minus end

γ-tubulin ring complex



Severing proteins



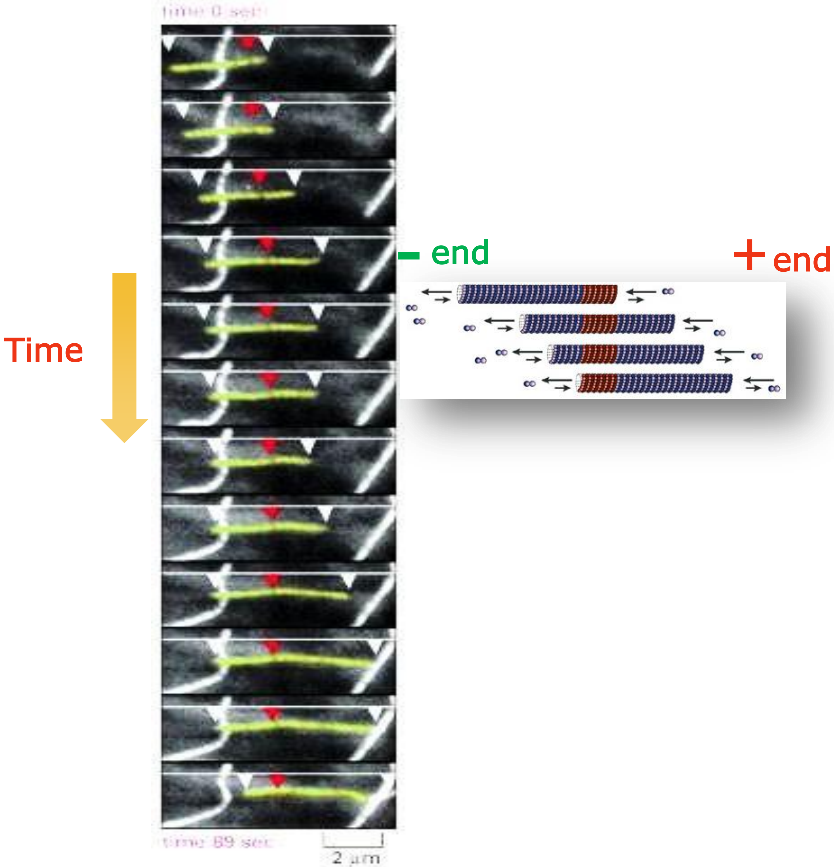


0 min 1 min

A. Roll-Mecak & R.D. Vale,
NATURE 451,363(2008)

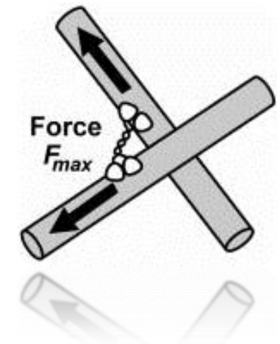
‘Self-propelled’ microtubules

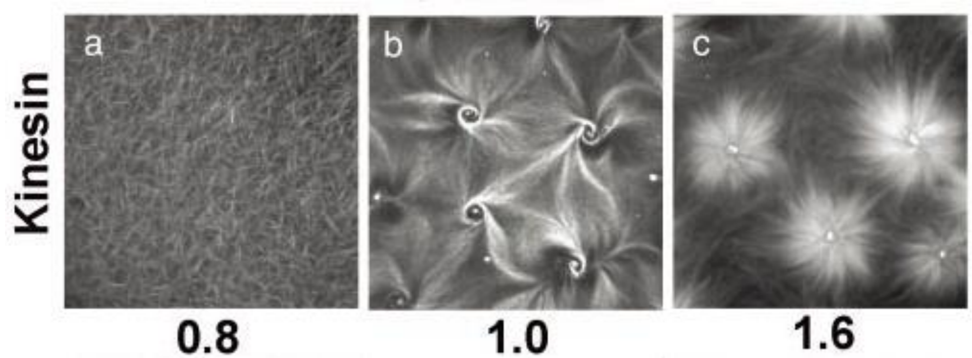
Treadmilling behavior of a microtubule



Molecular Biology of the Cell. 4th ed. Alberts B., et al. New York: Garland Science; 2002.

Active cross-linked microtubules

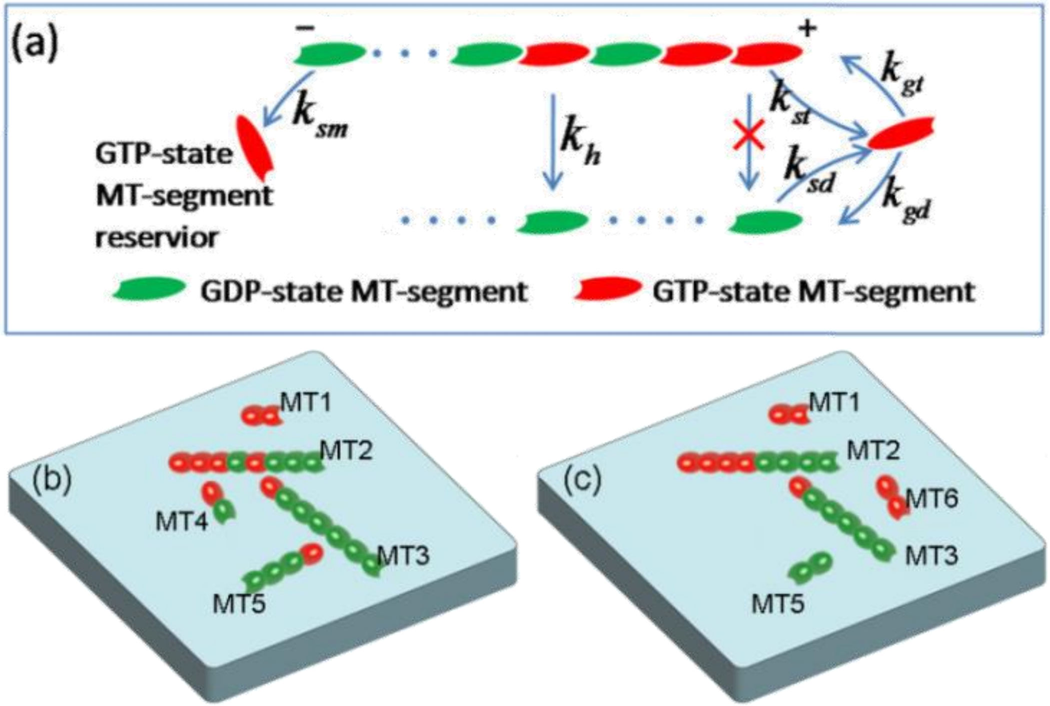




T. Surrey *et al. Science,* **292**, 1167 (2001)

A minimal model for interaction

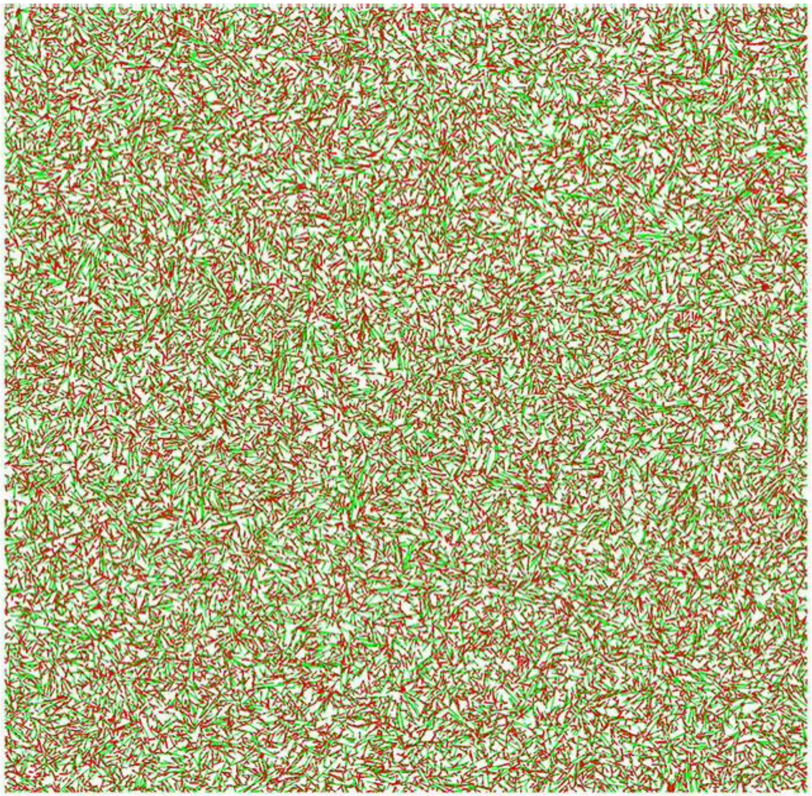
Kinetic Monte-Carlo simulation model



X. Shi & Y. Ma, PNAS, 107, 11709 (2010)

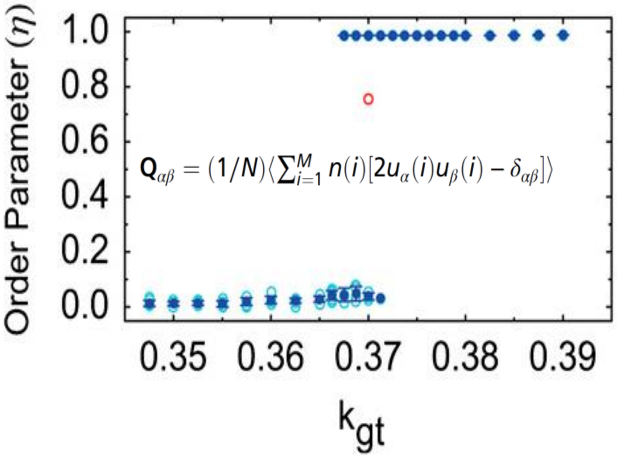
Parameter description (symbol)	Simulated values
Simulated MT segment length (a)	1 (80 nm)
Simulated time step (τ)	1 (0.2 s)
Plus-end GTP-state growing rate (k_{gt})	0.3 (120 nm/s)
Plus-end GTP-state shortening rate (k_{st})	0.005 (2 nm/s)
Plus-end GDP-state growing rate (k_{gd})	0.03 (12 nm/s)
Plus-end GDP-state shortening rate (k_{sd})	0.5 (200 nm/s)
Minus-end GDP-state shortening rate (k_{sm})	0.1 (40 nm/s)
Hydrolysis rate of GTP-state unit (k_h)	0.05 (2.5 segments/s)
Nucleation rate (k_n)	0.005 (~4.0 $\mu m^{-2} s^{-1}$)
Maximum MT length (L_m)	50 (4 μm)

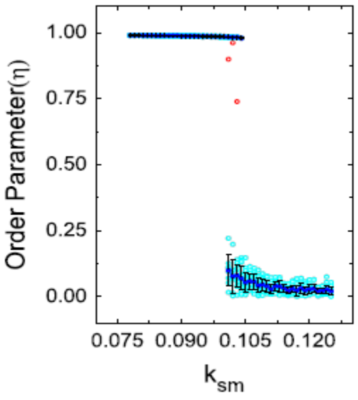
Discontinuous isotropic-nematic transition



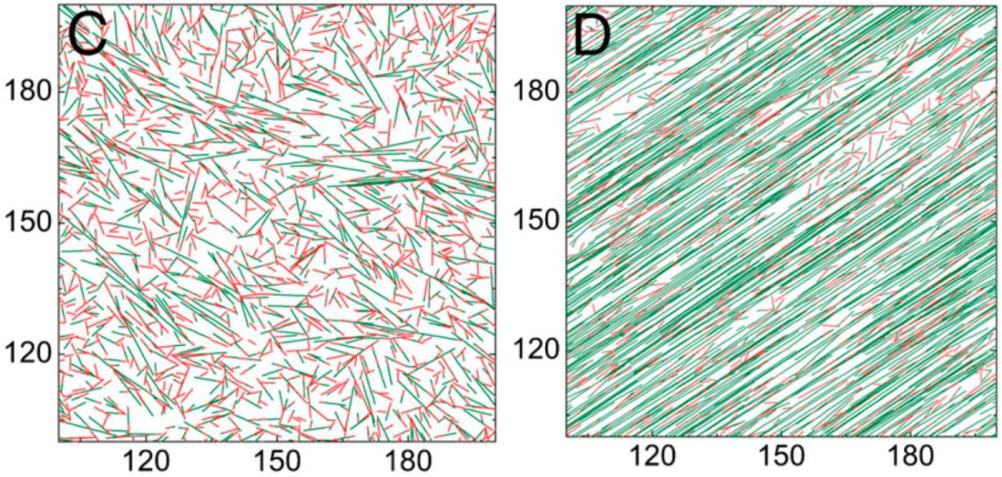
X. Shi & Y. Ma, PNAS, 107, 11709 (2010)

1. Isotropic nematic transition



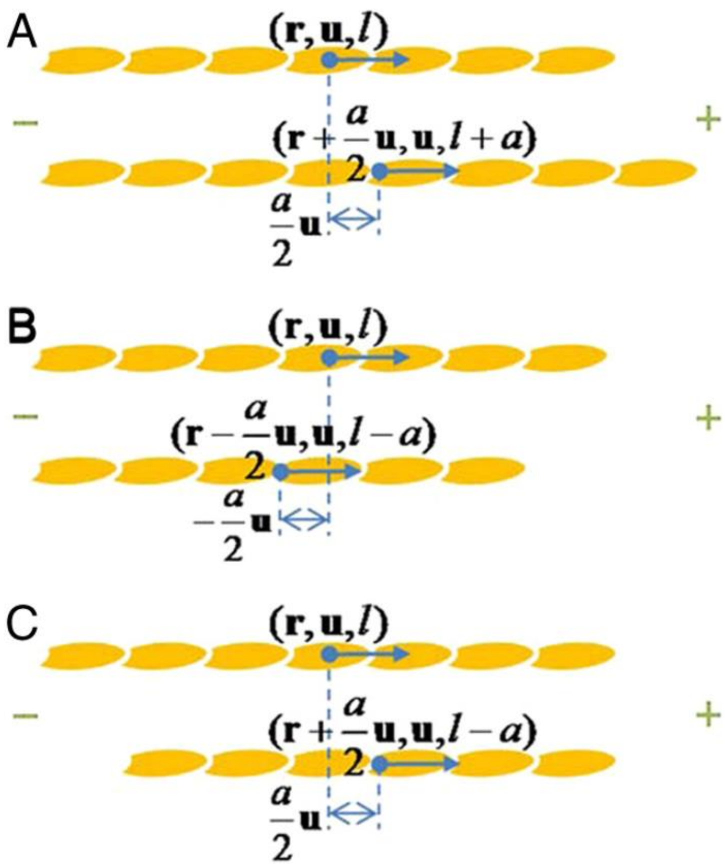


2. Snapshots of disordered and ordered states



Dynamic mean-field theory 1

1. The moving of microtubules' center of mass



2. Discrete rate equation

$$\frac{\partial f(\mathbf{r},\mathbf{u},l,t)}{\partial t} = k_p f\left(\mathbf{r} - \frac{a}{2}\mathbf{u},\mathbf{u},l-a,t\right) - k_p f(\mathbf{r},\mathbf{u},l,t) + k_{df} f\left(\mathbf{r} + \frac{a}{2}\mathbf{u},\mathbf{u},l+a,t\right) - k_{df} f(\mathbf{r},\mathbf{u},l,t) + k_{db} f\left(\mathbf{r} - \frac{a}{2}\mathbf{u},\mathbf{u},l+a,t\right) - k_{db} f(\mathbf{r},\mathbf{u},l,t)$$

3. Spatially homogeneous condition

$$\frac{\partial f(\mathbf{u},l,t)}{\partial t} = k_p f(\mathbf{u},l-a,t) - k_p f(\mathbf{u},l,t) + k_{df} f(\mathbf{u},l+a,t) - k_{df} f(\mathbf{u},l,t) + k_{db} f(\mathbf{u},l+a,t) - k_{db} f(\mathbf{u},l,t).$$

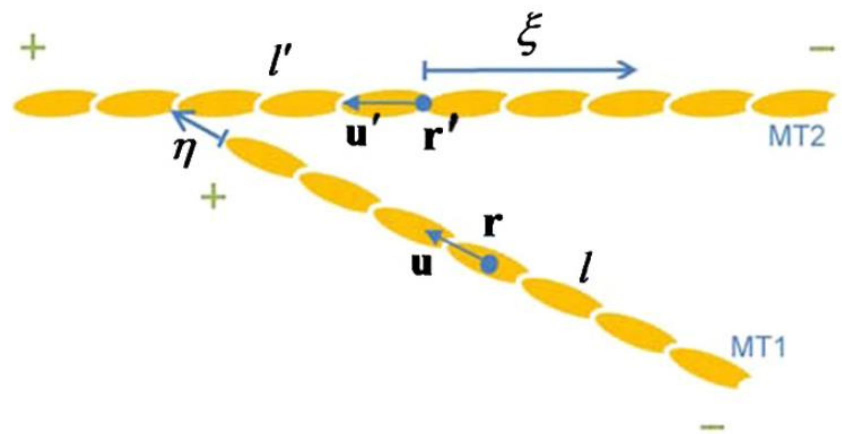
4. Boundary condition for length

$$\frac{\partial f(\mathbf{u},2a,t)}{\partial t} = k_{ne}(\mathbf{u}) - (k_{df} + k_{db}) f(\mathbf{u},2a,t) - k_p(\mathbf{u}) f(\mathbf{u},2a,t) + (k_{df} + k_{db}) f(\mathbf{u},3a,t),$$

$$\frac{\partial f(\mathbf{u},L,t)}{\partial t} = -(k_{df} + k_{db}) f(\mathbf{u},L,t) + k_p(\mathbf{u}) f(\mathbf{u},L-a,t),$$

Dynamic mean-field theory 2

Steric interaction hinders polymerization



Steric interaction kernal of a segment with length a/N

$$W(\mathbf{r},\mathbf{r}',\mathbf{u},\mathbf{u}',l,l') = |\mathbf{u}\times\mathbf{u}'| \int_0^{a/N} d\eta \int_{-\frac{l'}{2}}^{\frac{l'}{2}} d\xi\, \delta\left[\left(\mathbf{r} + \left(\frac{l}{2}+\eta\right)\mathbf{u}\right) - (\mathbf{r}' - \xi\mathbf{u}')\right]$$

Probability of a segment with length a/N intersect with existing microtubule

$$p_r(\mathbf{r},\mathbf{u},l) = \int dl' \int d\mathbf{u}' \int d\mathbf{r}' W(\mathbf{r},\mathbf{r}',\mathbf{u},\mathbf{u}',l,l') f(\mathbf{r}',\mathbf{u}',l')$$

Modified polymerization rate

$$k_p(\mathbf{u}) = k_{p0} \lim_{N\to\infty}\left(1 - p_r(\mathbf{u})\right)^N = k_{p0}\exp\left(-\sum_{l'=2}^{L} l' \int d\mathbf{u}' f(\mathbf{u}',l')|\mathbf{u}\times\mathbf{u}'|\right)$$

Steady state solution

1.Self-consistent integral equation

$$f(\mathbf{u},l) = A\exp\left[\Delta G\cdot l - l\sum_{l'}^{L} l'\int d\mathbf{u}' f(\mathbf{u}',l')|\mathbf{u}\times\mathbf{u}'|\right]$$

where $A = k_n \exp(-2\Delta G)/(k_{df}+k_{db})$

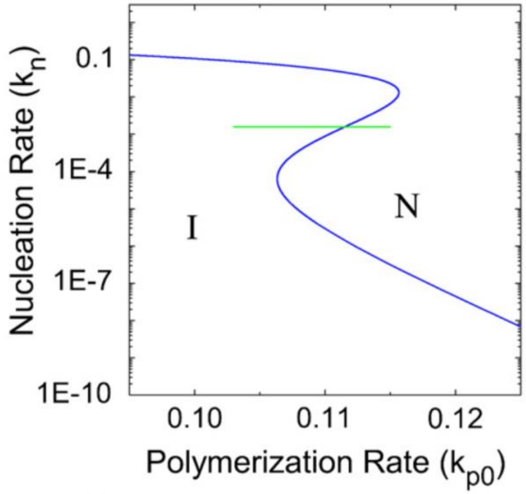
$\Delta G = \ln[k_{p0}/(k_{df}+k_{db})]$

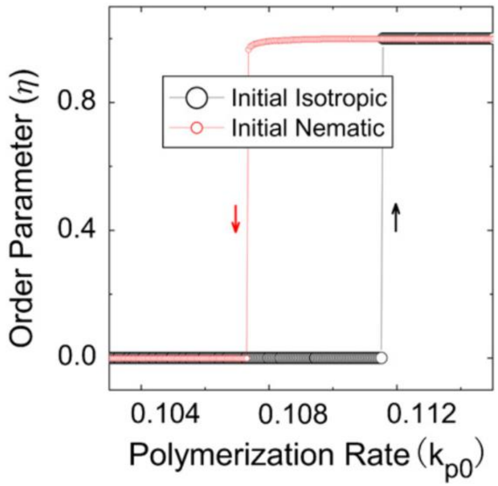
2. Isotropic solution

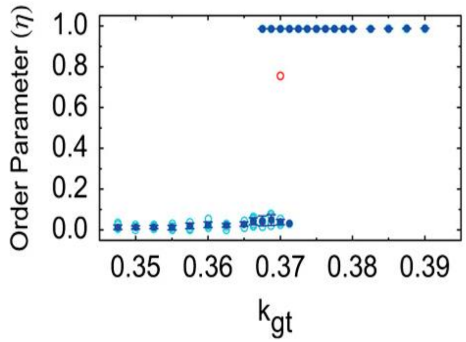
$f(\mathbf{u},l) = f_l = Ae^{\Delta G\cdot l - 2f_0 l\langle l\rangle}$ with $f_0 = \sum_{l=2}^{L} f_l,\ \langle l\rangle = (\sum_{l=2}^{L} l f_l)/f_0$

3. Bifurcation analysis and numerical result

We have $f_0 = \frac{3}{2\langle l^2\rangle}$ at the phase boundary





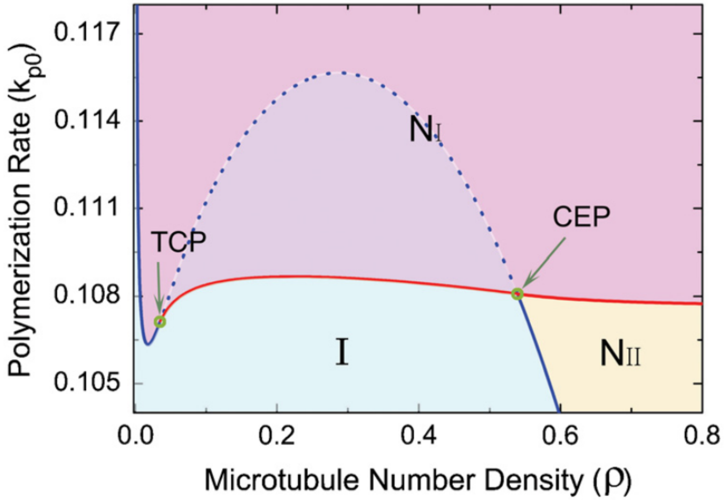


Phase behavior of cortical microtubules

Phase maps for controlled microtubule number system

$$f(\mathbf{u},l) = A\exp\left[\Delta G\cdot l - l\sum_{l'}^{L} l' \int d\mathbf{u}' f(\mathbf{u}',l')|\mathbf{u}\times\mathbf{u}'|\right]$$

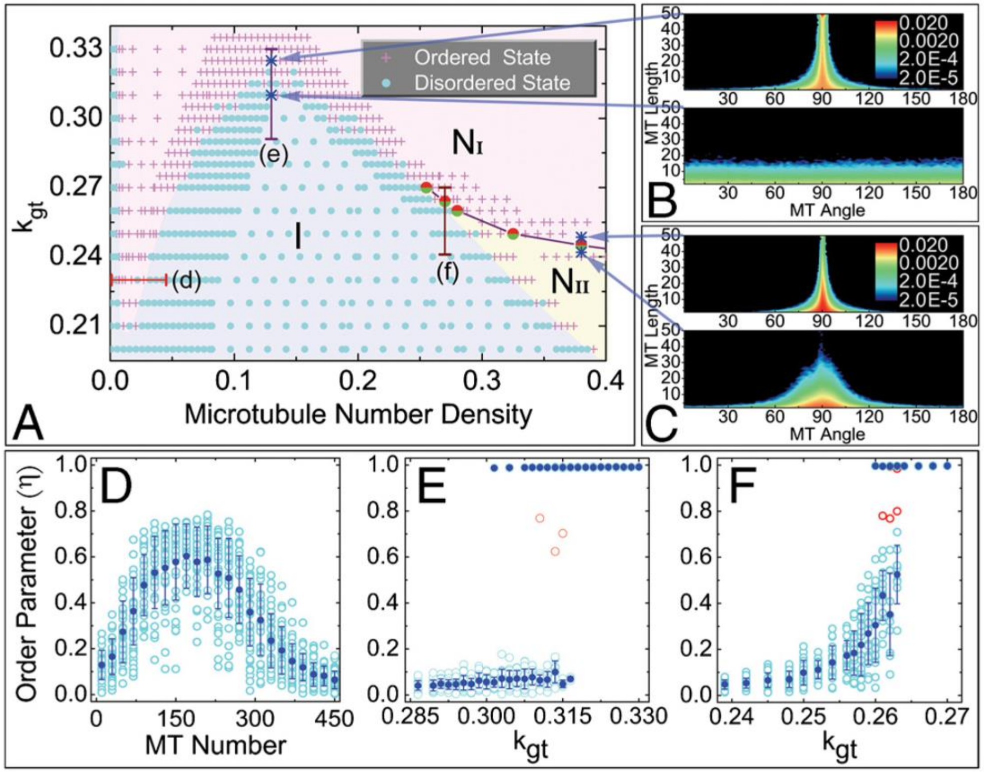
where A is now determined by $\sum_{l=2}^{L}\int_0^{\pi} d\mathbf{u} f(\mathbf{u},l) = \rho$



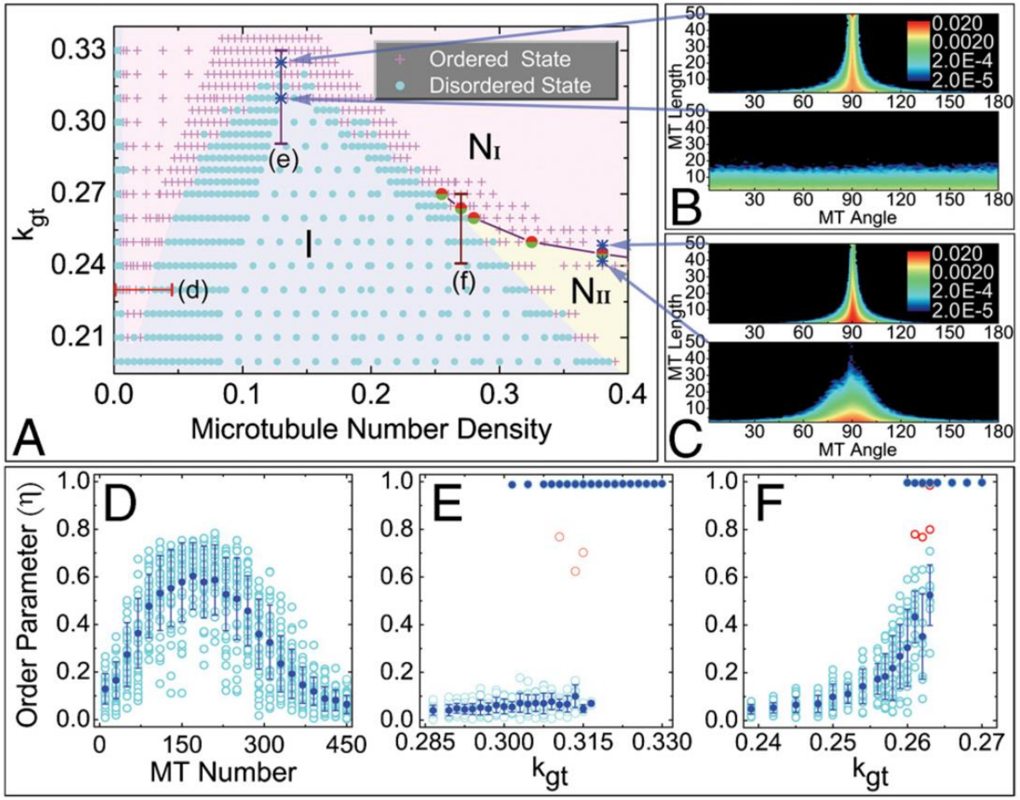
Effective free energy functional for steady state

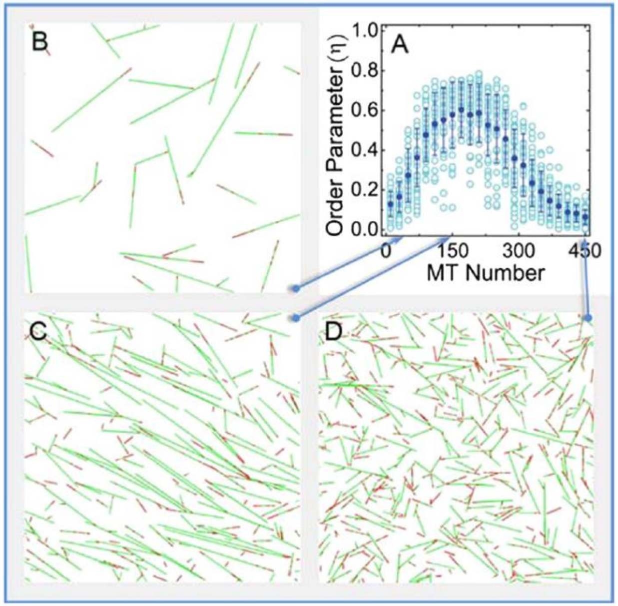
$$F\{f(\theta,l)\} = \sum_{l=2}^{L}\int_0^{\pi} d\mathbf{u} f(\mathbf{u},l)\ln[L\pi f(\mathbf{u},l)]$$
$$+\frac{1}{2}\sum_{l,l'} ll' \int_0^{\pi}\int_0^{\pi} d\mathbf{u}_1 d\mathbf{u}_2 f(\mathbf{u}_1,l) f(\mathbf{u}_2,l')|\mathbf{u}_1\times\mathbf{u}_2|$$
$$-\Delta G\sum_{l=2}^{L} l\int_0^{\pi} d\mathbf{u} f(\mathbf{u},l) + \lambda\left(\sum_{l=2}^{L}\int_0^{\pi} d\mathbf{u} f(\mathbf{u},l) - \rho\right)$$

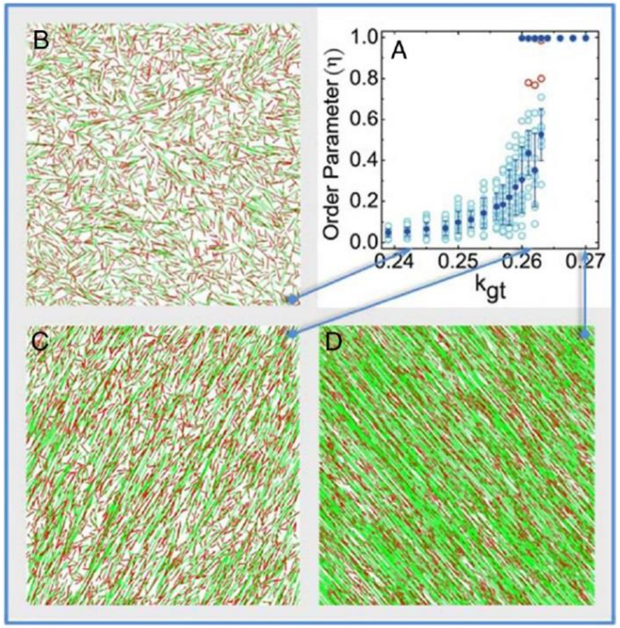
Simulation results

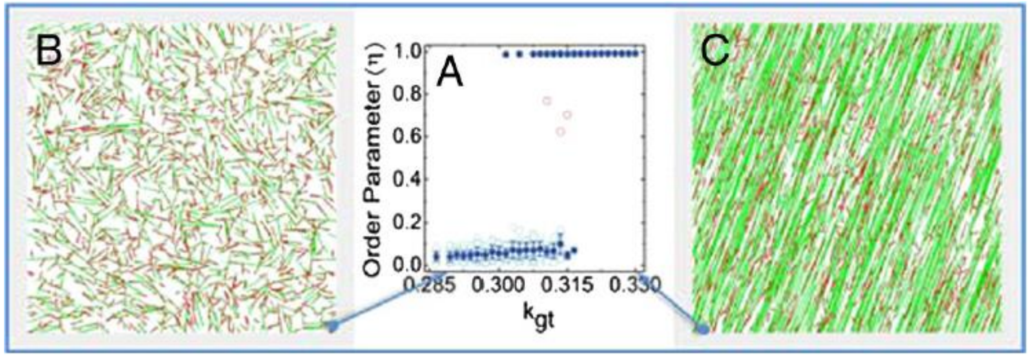


Transition properties across phase boundaries

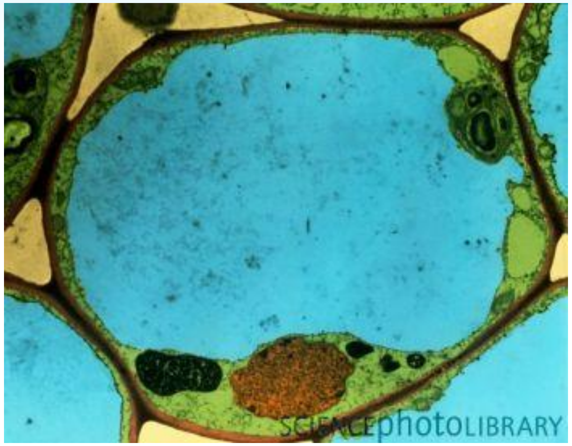


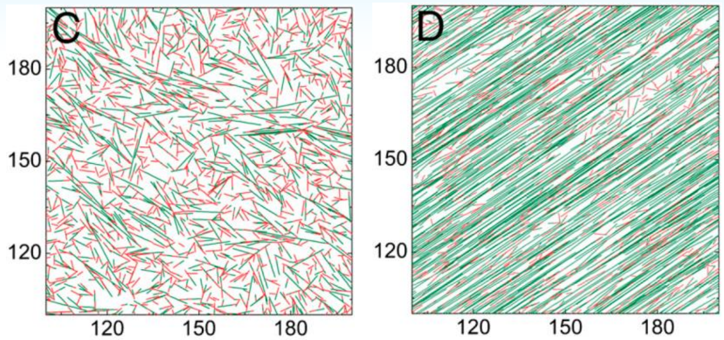


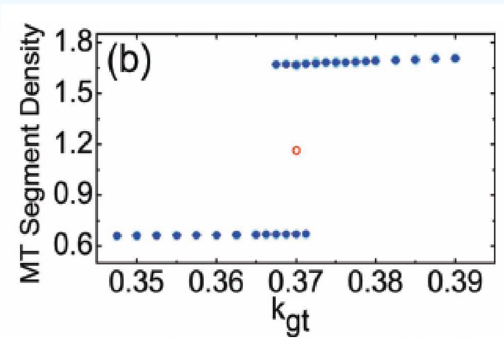




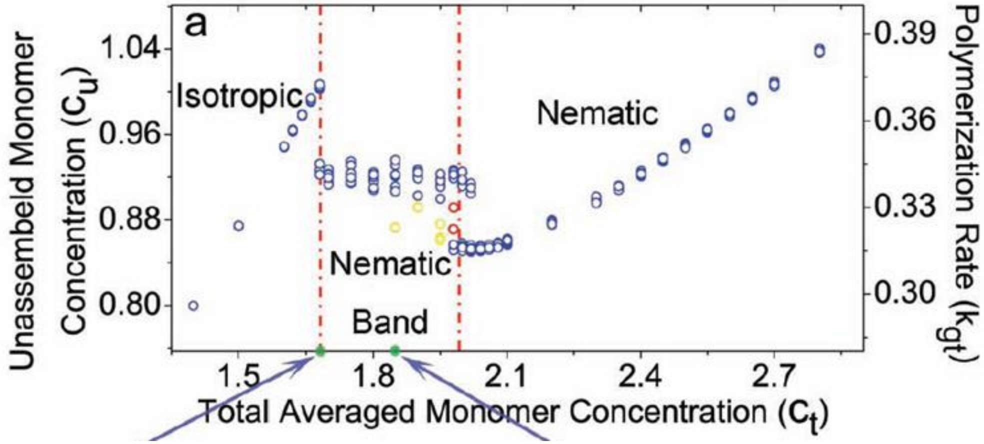
Band formation

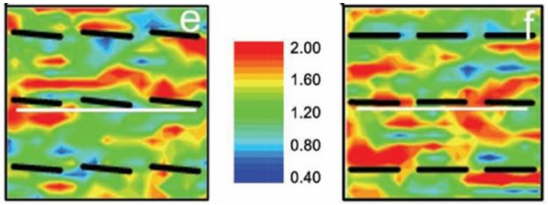




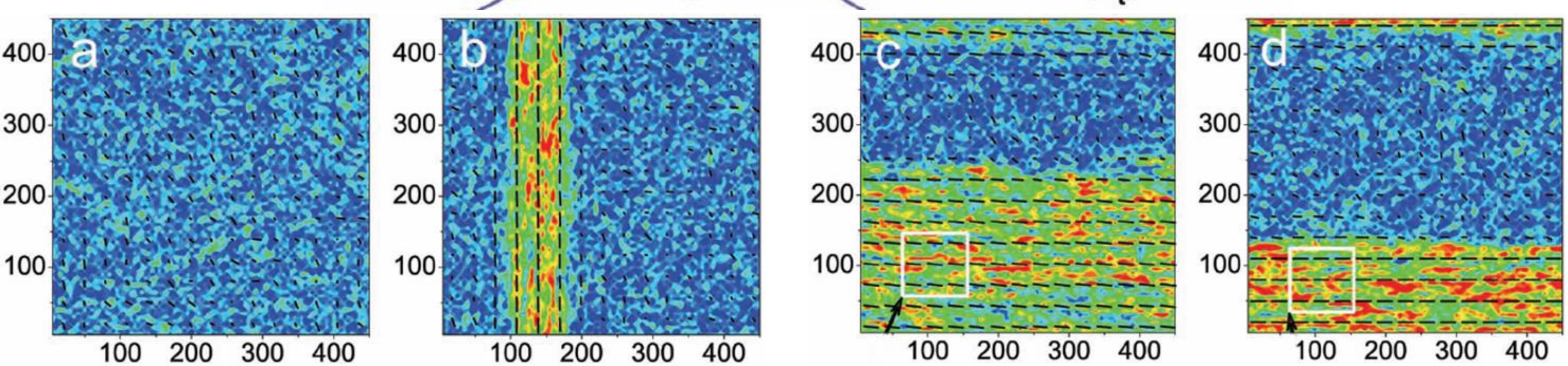


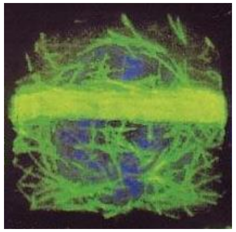
k_{gt} is proportional with unassembled tubulin concentration c_u



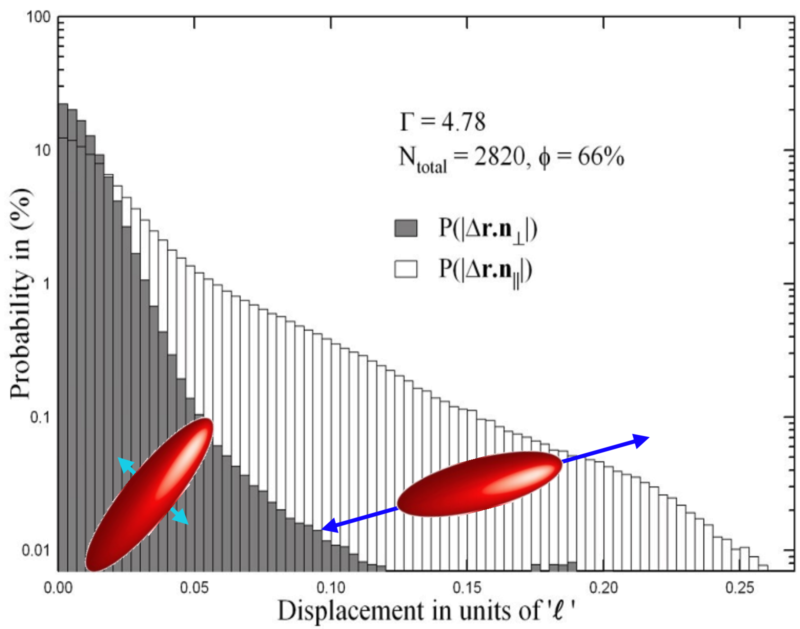


Jiang, Shi, Ma
Unpublished

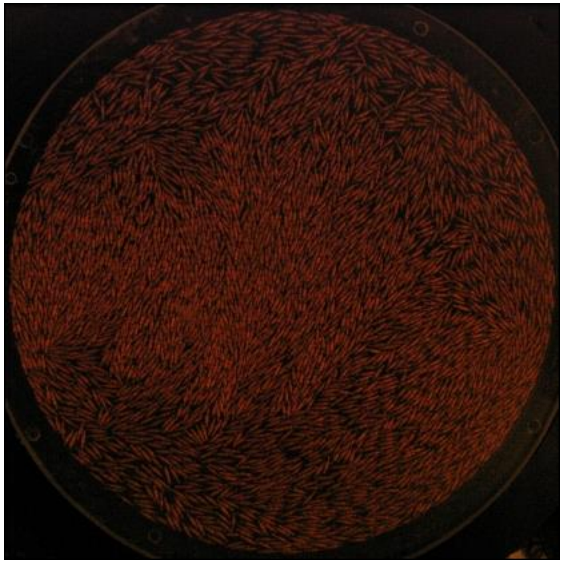




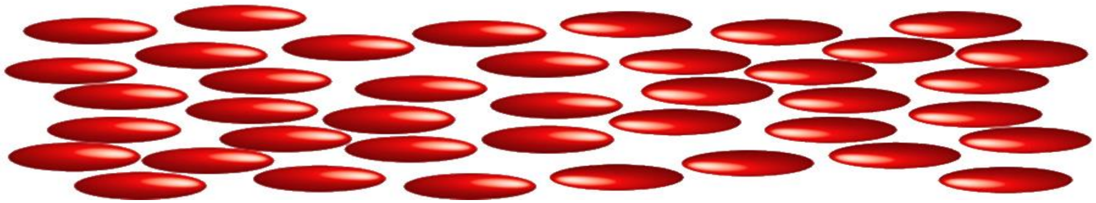
Driven granular rods



V. Narayan et al, Science, (2007).



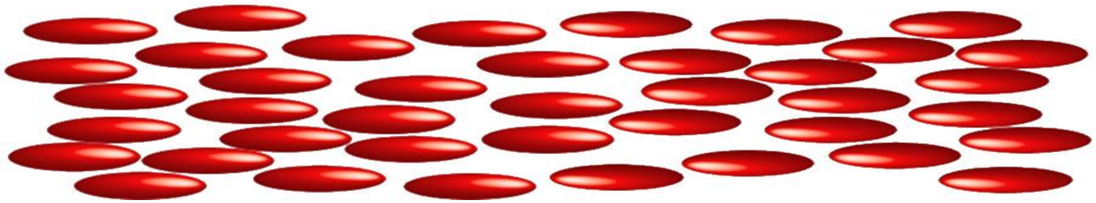
Nematic State



Orientational order
No position order

$$Q_{\alpha\beta} \equiv S(\hat{n}_\alpha \hat{n}_\beta - \delta_{\alpha\beta}/2) = \int \mathrm{d}\mathbf{u}(u_\alpha u_\beta - \delta_{\alpha\beta}/2) f(\mathbf{u})/\rho$$

Nematic State



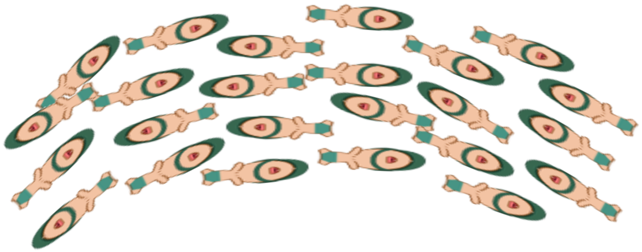
Orientational order
No position order

$$Q_{\alpha\beta} \equiv S(\hat{n}_\alpha \hat{n}_\beta - \delta_{\alpha\beta}/2) = \int \mathrm{d}\mathbf{u}(u_\alpha u_\beta - \delta_{\alpha\beta}/2) f(\mathbf{u})/\rho$$

- Curvature induced particle flows in **active nematics**

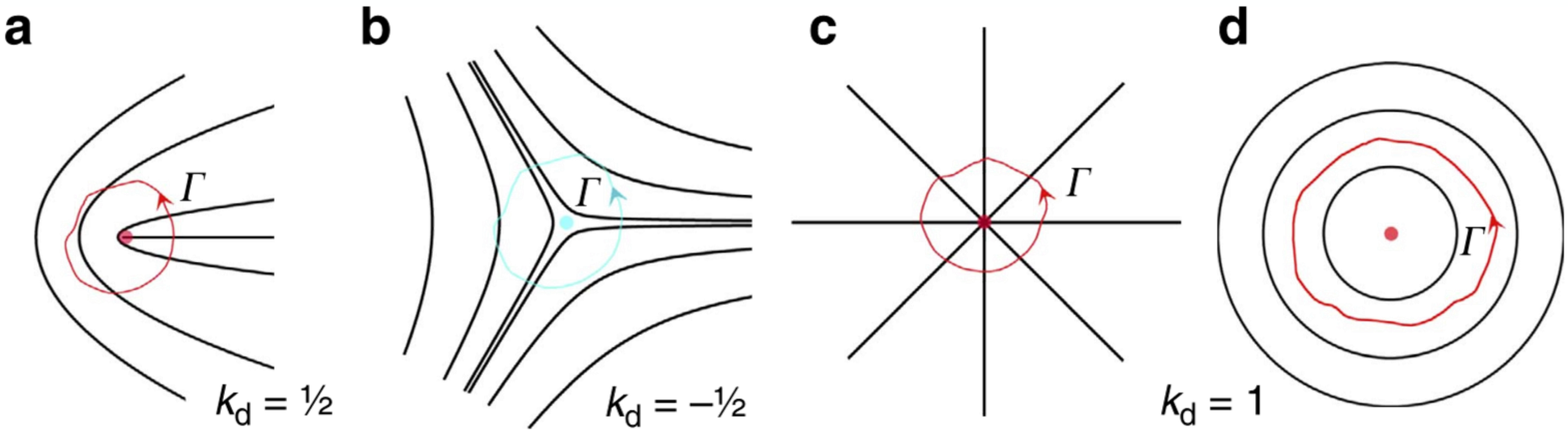
- Active flows:

$$J_i = -\alpha \partial_j \rho(\mathbf{r}) Q_{ij}(\mathbf{r}) \qquad J_y = -\partial_x \delta n_y$$



S. Ramaswamy et al, EPL (2003) & PRL (2003)

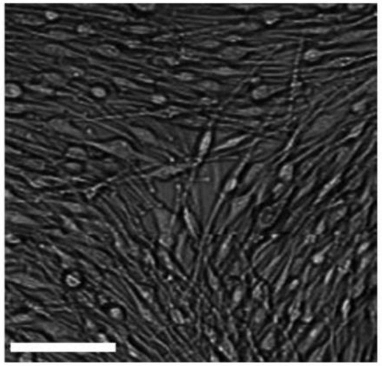
Topological defects



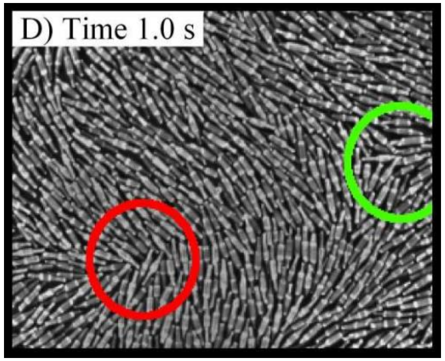
$$\oint_\Gamma \frac{d\theta}{ds} ds = 2\pi k_d$$

P. M. Chaikin & T. C. Lubensky,
Principle of condensed matter

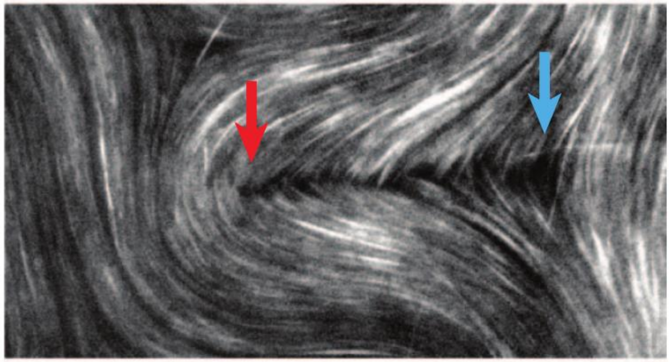
C. Marchetti et al. RMP, 2013



R. Kemkemer, Eur. Phys. J. E 2000

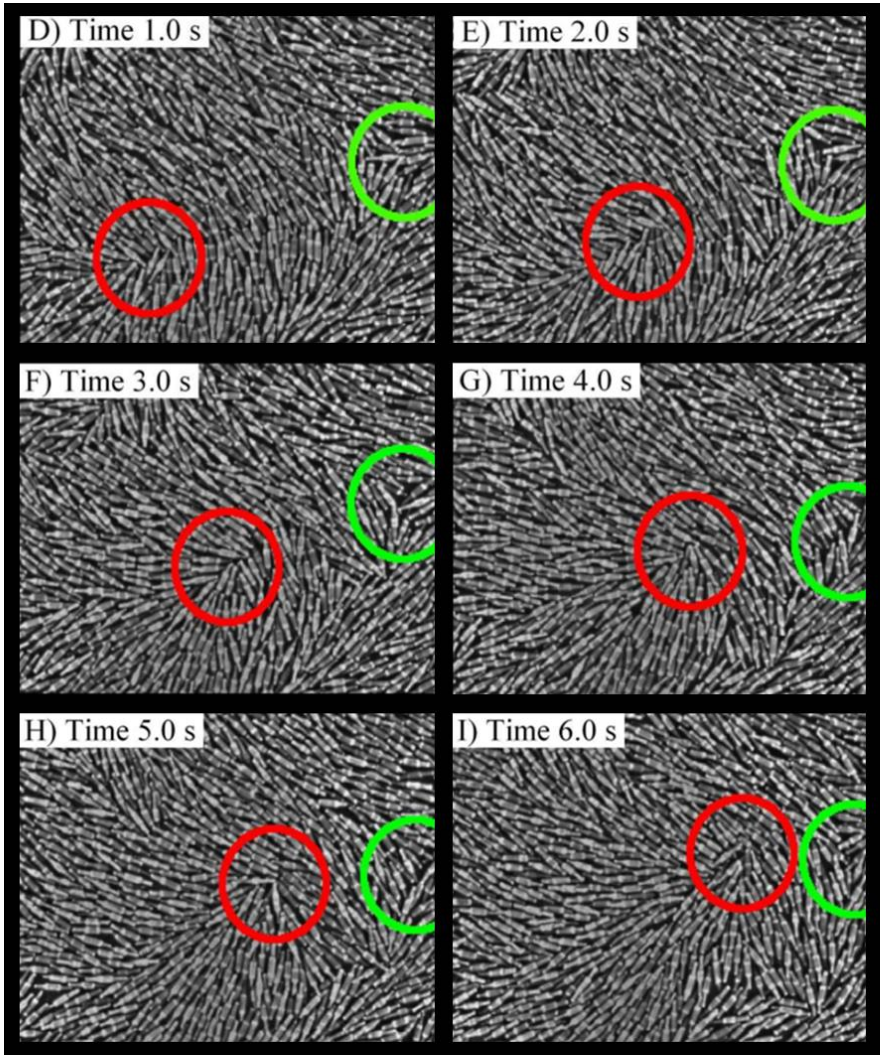


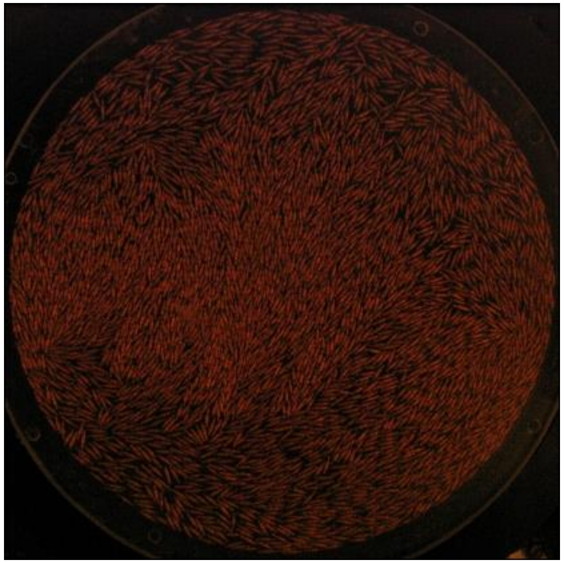
V. Narayan et al, Science 2007



T. Sanchez et al, Nature 2013

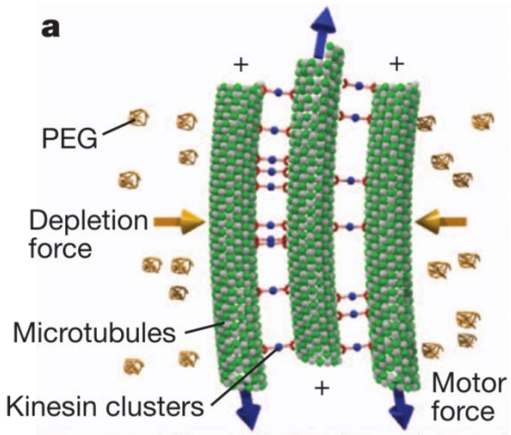
Topological defects

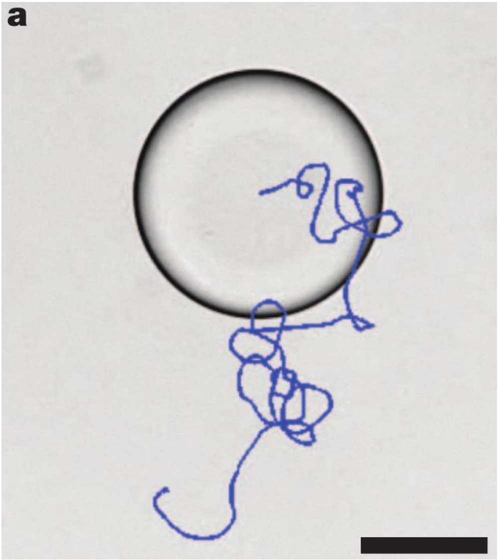


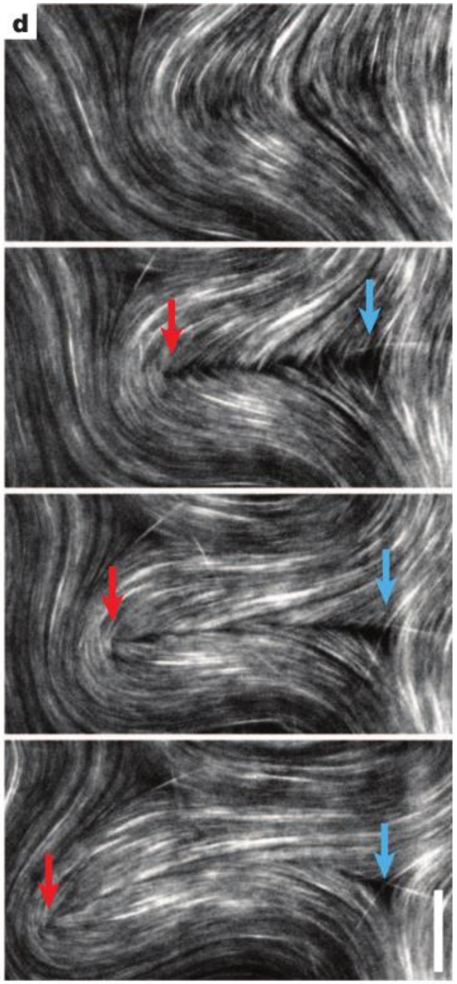


V. Narayan et al, Science 2007

Topological defects

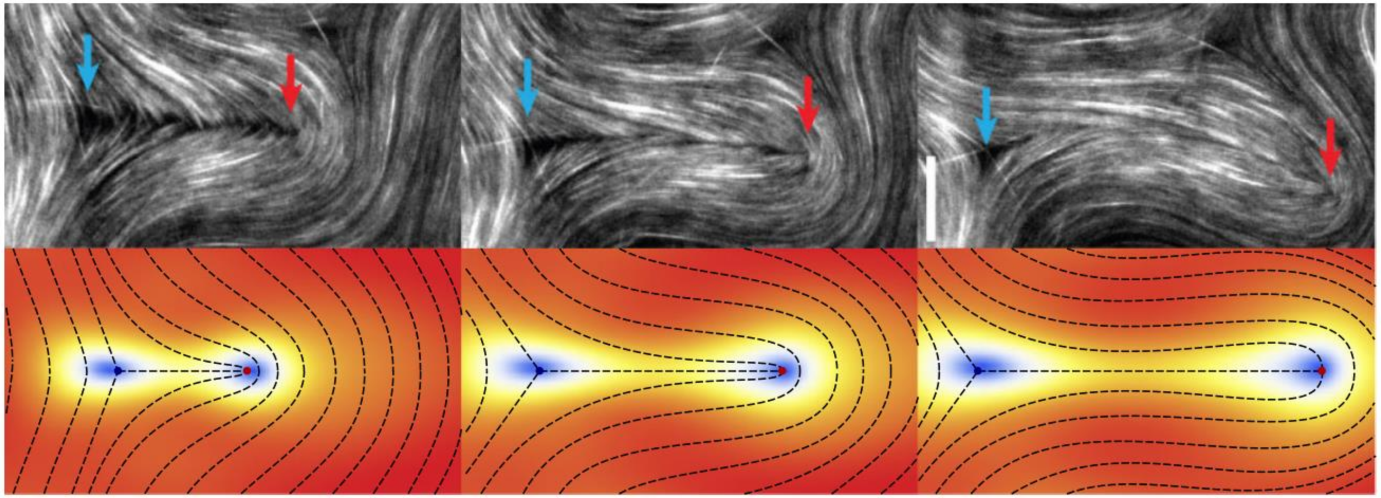






T. Sanchez et al, Nature 2013

Hydrodynamic model with media fluids



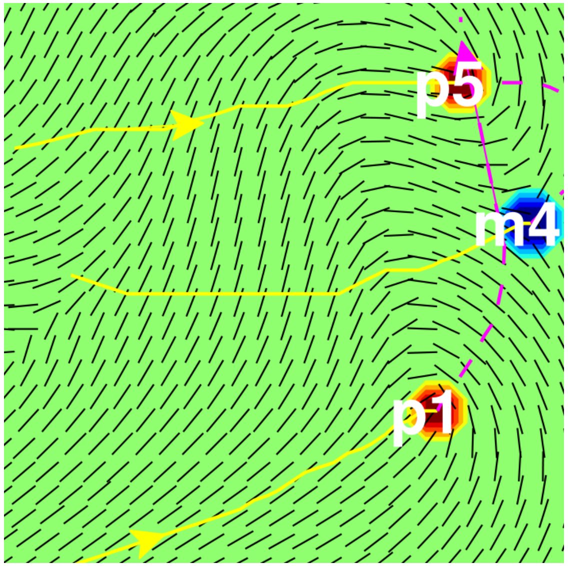
T. Sanchez et al, Nature 2013

L. Giomi et al. PRL, 2013

$$\frac{Dc}{Dt} = \partial_i[D_{ij}\partial_j c + \alpha_1 c^2 \partial_j Q_{ij}],$$

$$\rho\frac{Dv_i}{Dt} = \eta\nabla^2 v_i - \partial_i p + \partial_j \sigma_{ij}, \qquad \boxed{\sigma^a_{ij} = \alpha_2 c^2 Q_{ij}}$$

$$\frac{DQ_{ij}}{Dt} = \lambda S u_{ij} + Q_{ik}\omega_{kj} - \omega_{ik}Q_{kj} + \gamma^{-1}H_{ij}$$



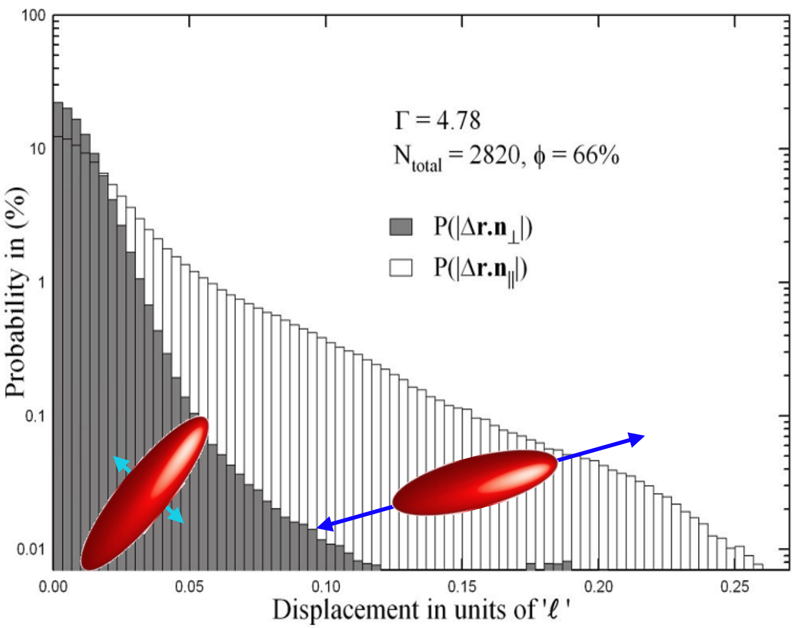
S. P. Thampi et al. PRL, 2013
S. P. Thampi et al. arXiv:1312.4836

Simulation model for granular rods

- Kinetic Monte Carlo model of driven hard ellipse

$$\Delta \mathbf{r}'^{n}_{j} = 2 v_0 h^n_j \mathbf{u}^n_j \eta^n_j$$

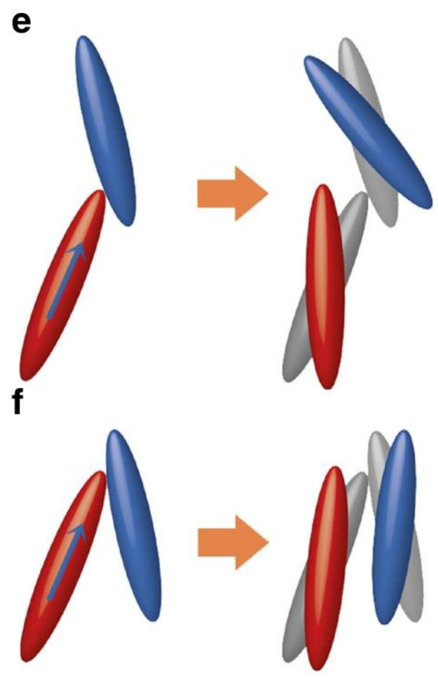
h^n_j ={+1,+1,+1,-1,+1,+1,-1,-1-1-1,+1,+1,+1,+1……}



V. Narayan et al, Science, (2007).

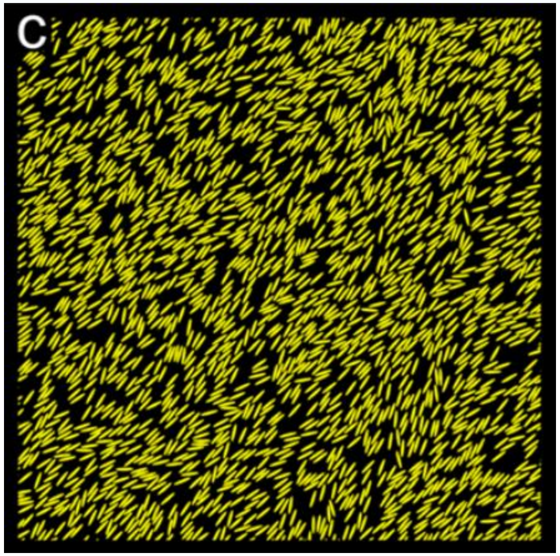
$$\Delta \mathbf{r}''^{n}_{j} = \sigma_l g^n_j \zeta^n_j$$

$$\Delta \theta'^{n}_{j} = \sigma_r g^n_j \xi^n_j$$

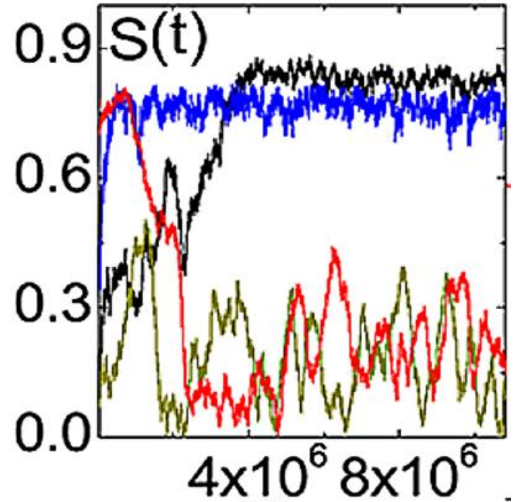


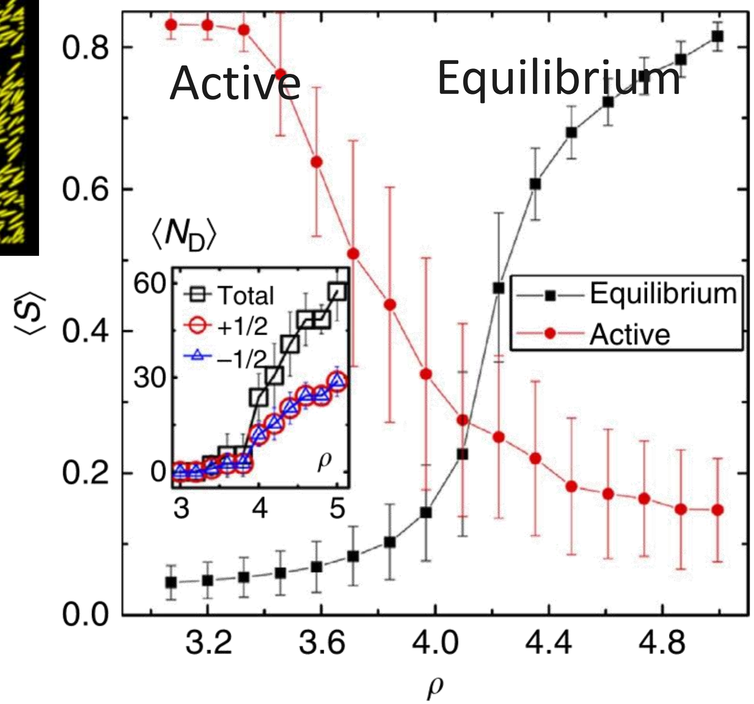
Shi & Ma, Nat. Commun. 2013, 4:3013

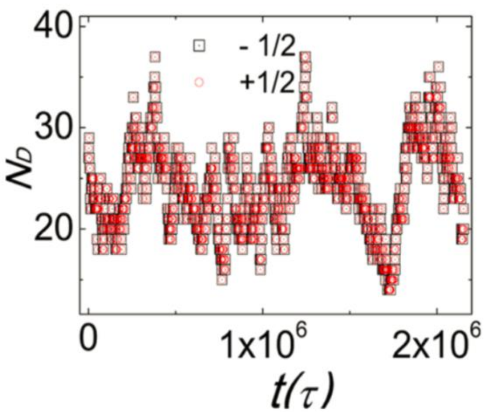
Breakdown of nematic order



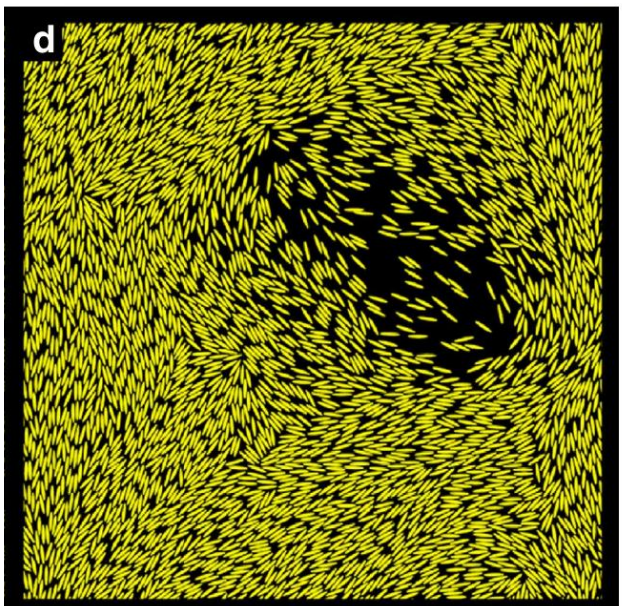
$$S(\hat{n}_\alpha \hat{n}_\beta - \delta_{\alpha\beta}/2) =$$
$$N^{-1}\langle \sum_{i=1}^{N} [2u_\alpha(i)u_\beta(i) - \delta_{\alpha\beta}] \rangle$$



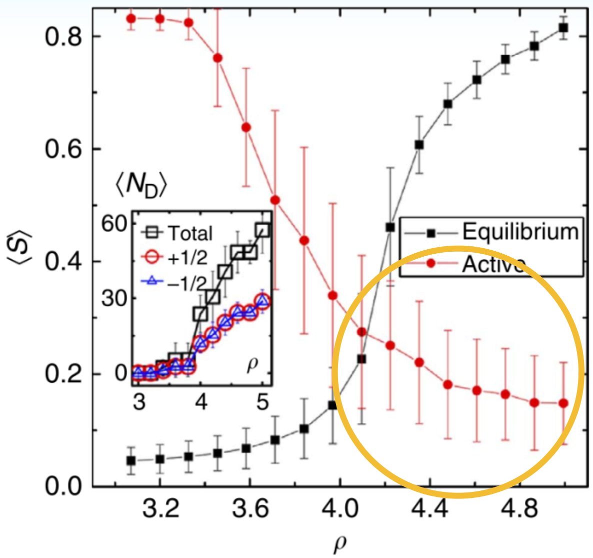


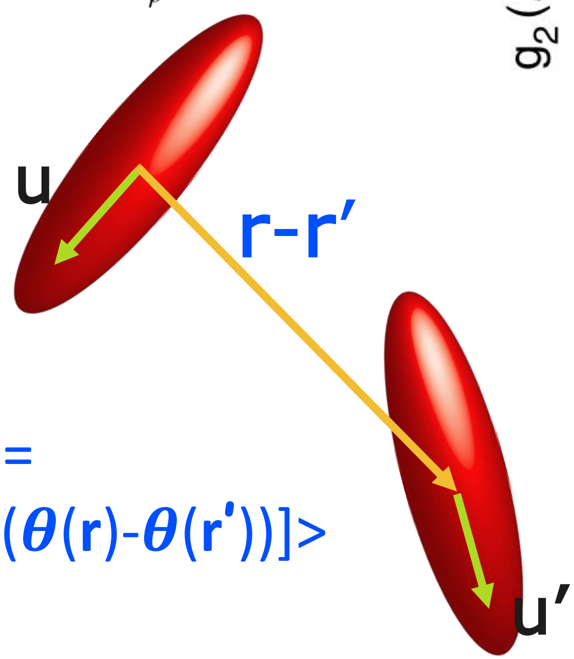


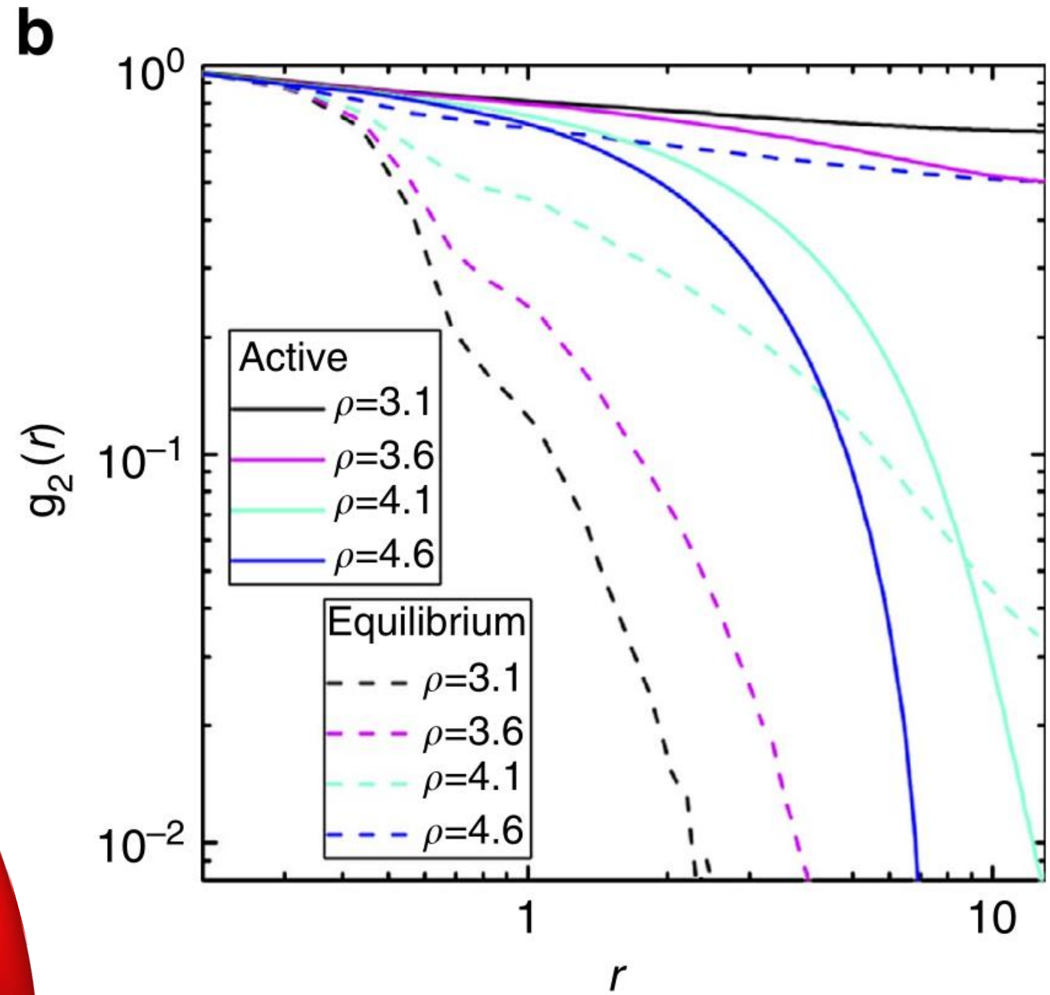
Shi & Ma, Nat. Commun. 2013, 4:3013



$g_2(\mathbf{r})$ correlation





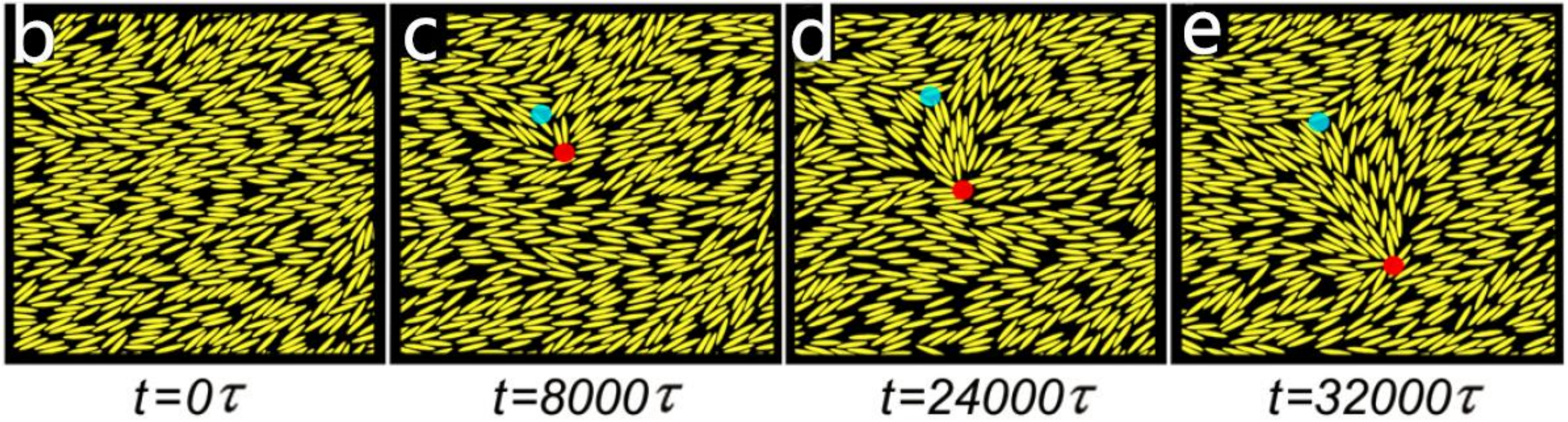


$$g_2(\mathbf{r}-\mathbf{r}')= \langle\cos[2(\boldsymbol{\theta}(\mathbf{r})-\boldsymbol{\theta}(\mathbf{r}'))]\rangle$$

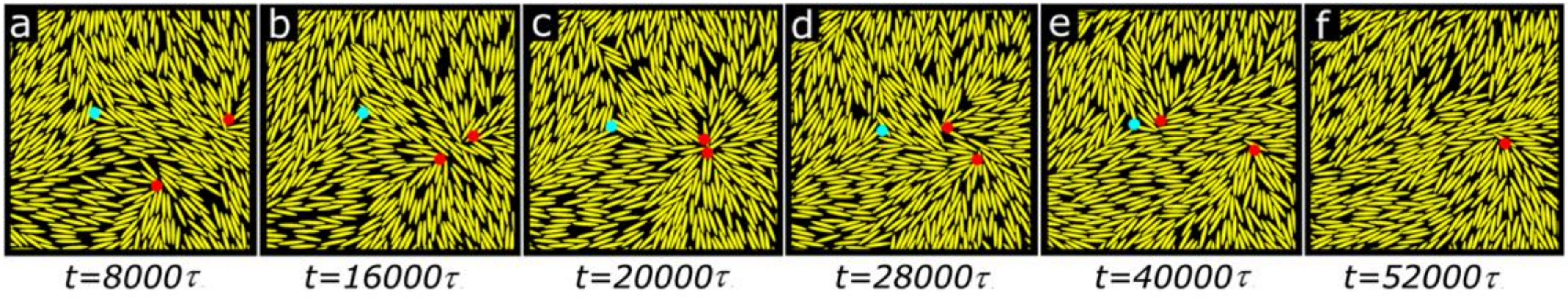
Shi & Ma, Nat. Commun. 2013, 4:3013

Dynamics of topological defects

- Active unbinding of topological defects pair



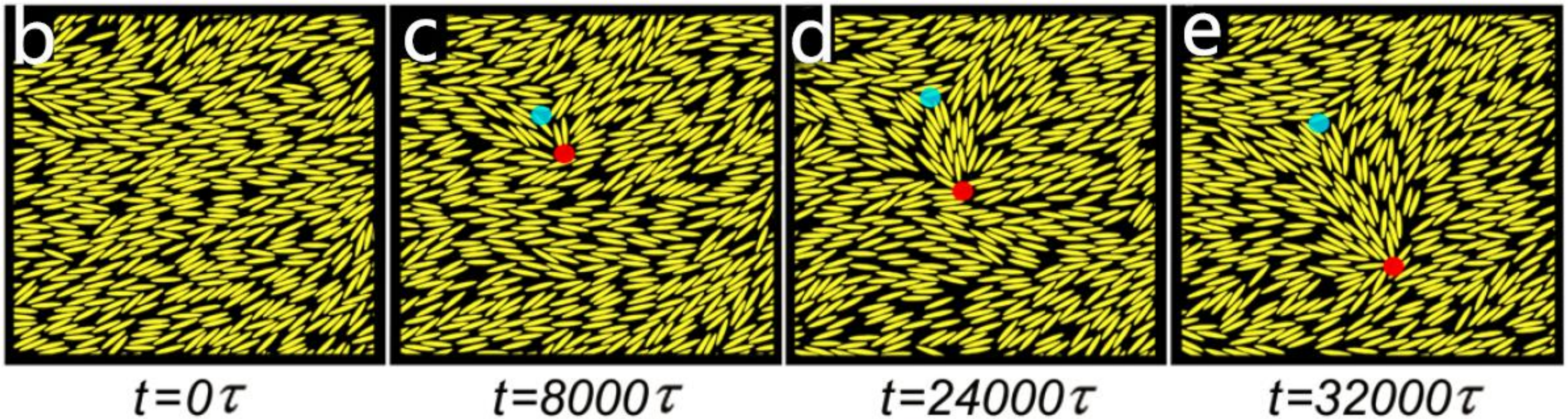
- Collision and annihilation of defects



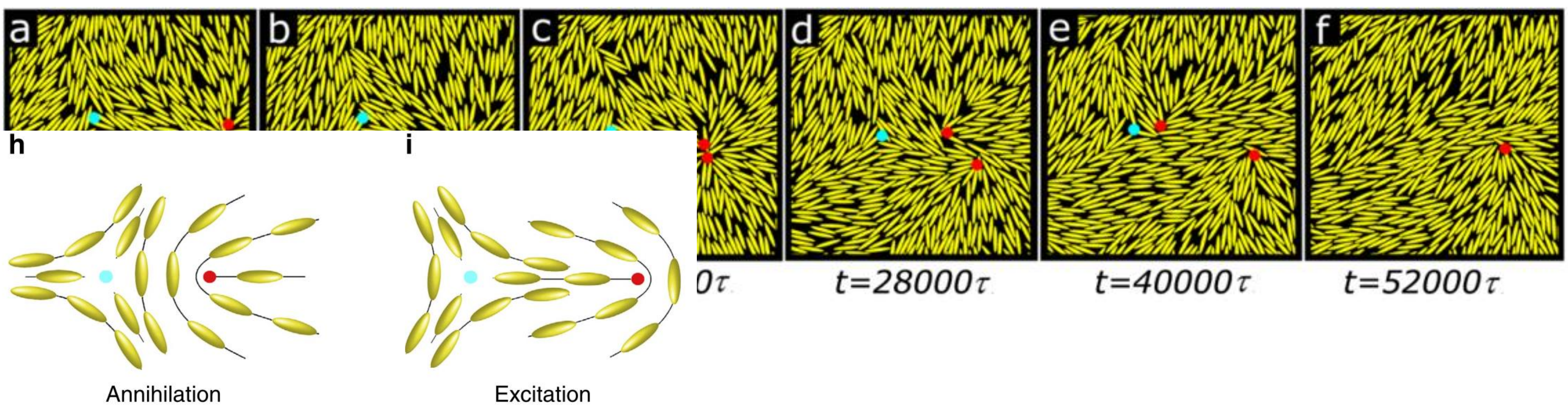
Shi & Ma, Nat. Commun. 2013, 4:3013

Dynamics of topological defects

- Active unbinding of topological defects pair



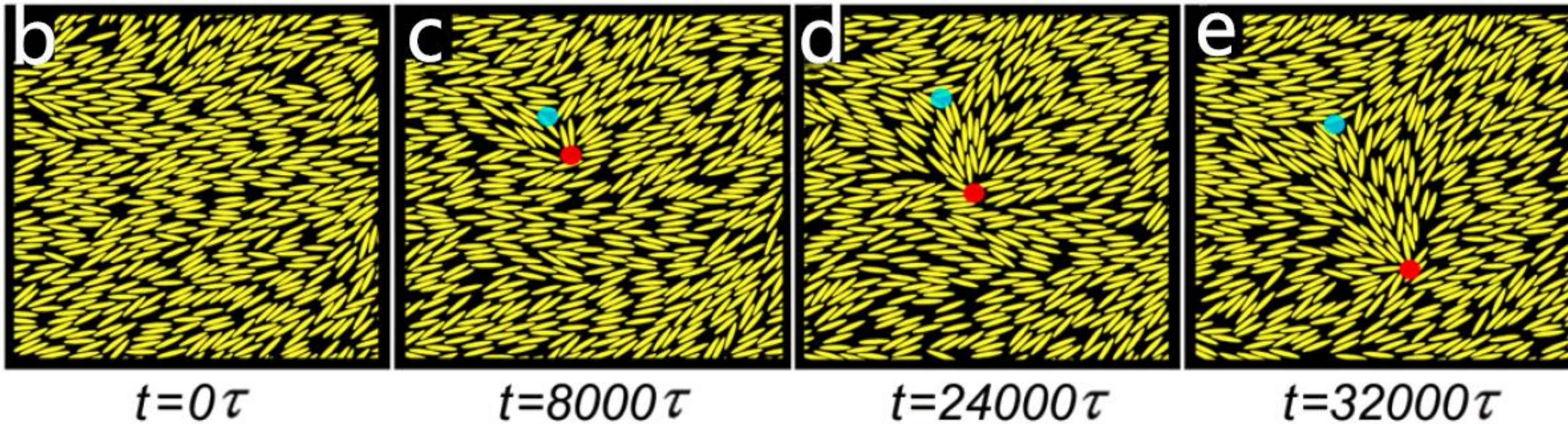
- Collision and annihilation of defects



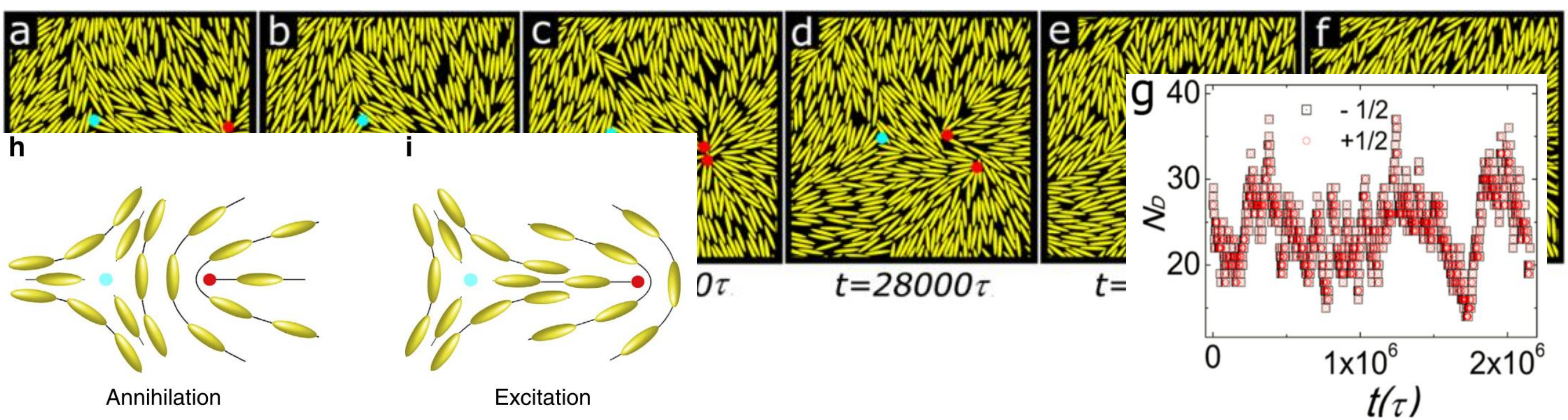
Shi & Ma, Nat. Commun. 2013, 4:3013

Dynamics of topological defects

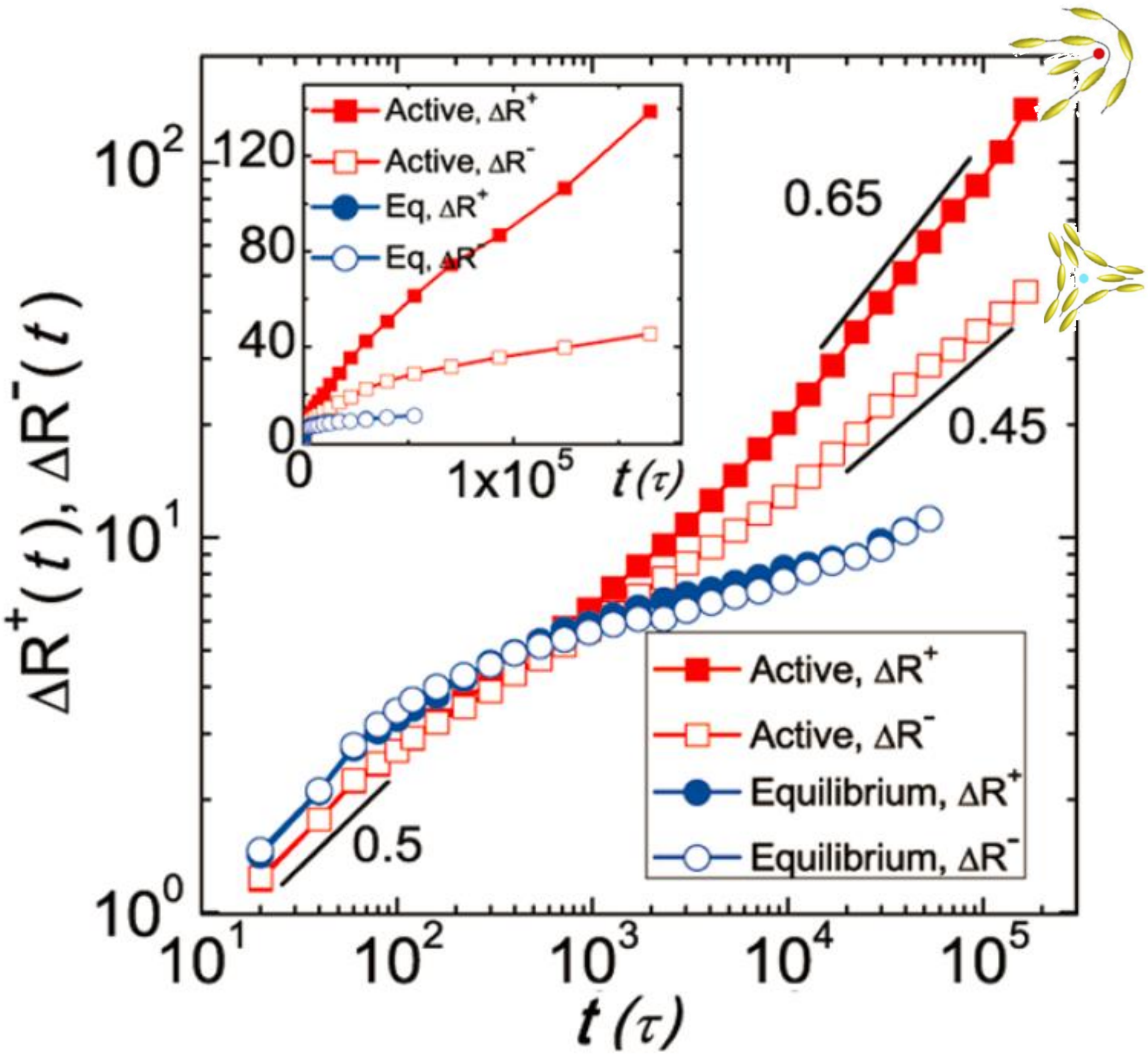
- Active unbinding of topological defects pair



- Collision and annihilation of defects

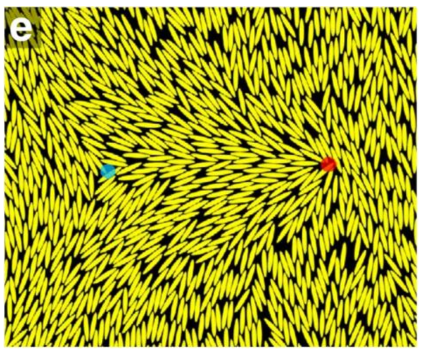


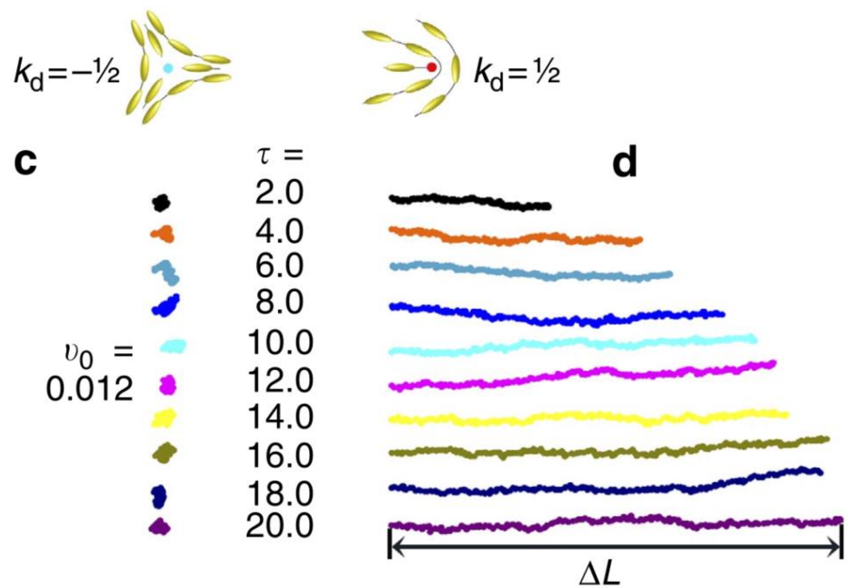
Super-diffusivity



Shi & Ma, Nat. Commun. 2013, 4:3013

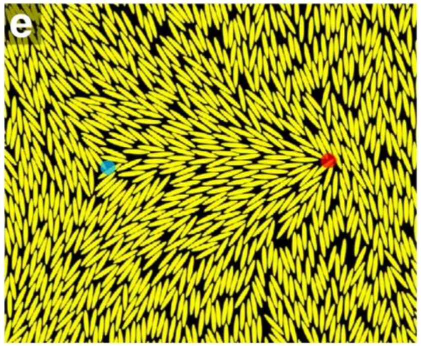
Racing of defects

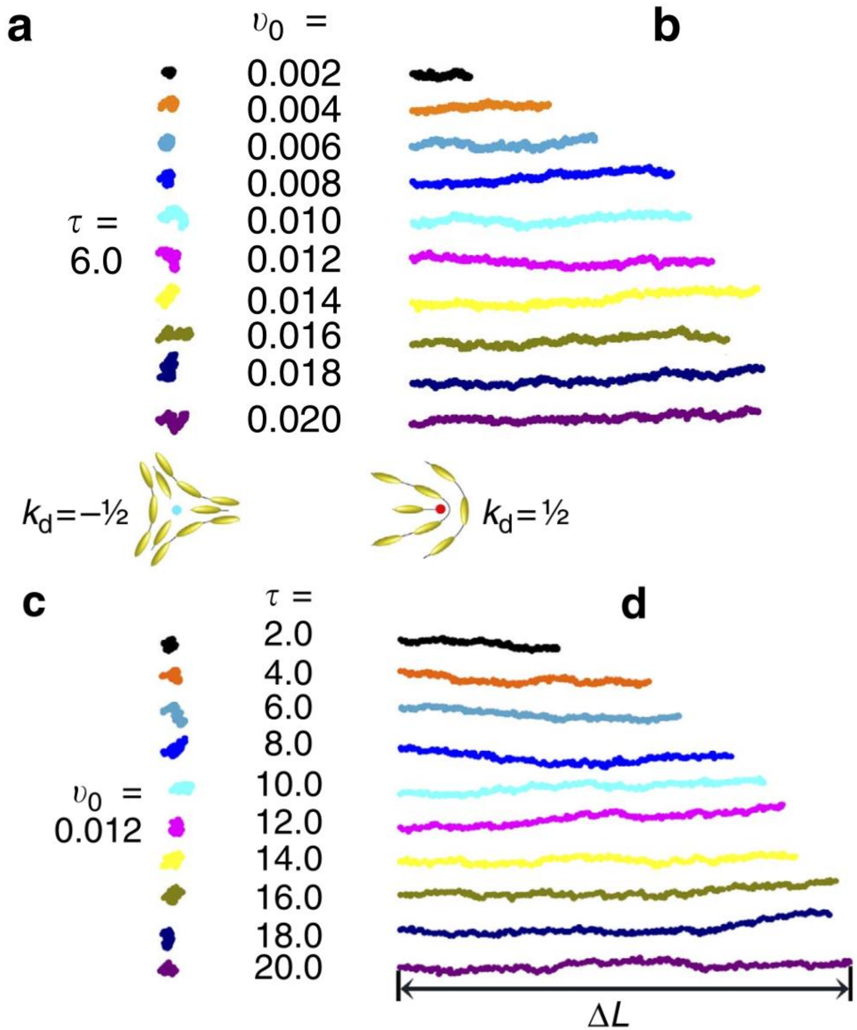




Shi & Ma, Nat. Commun. 2013, 4:3013

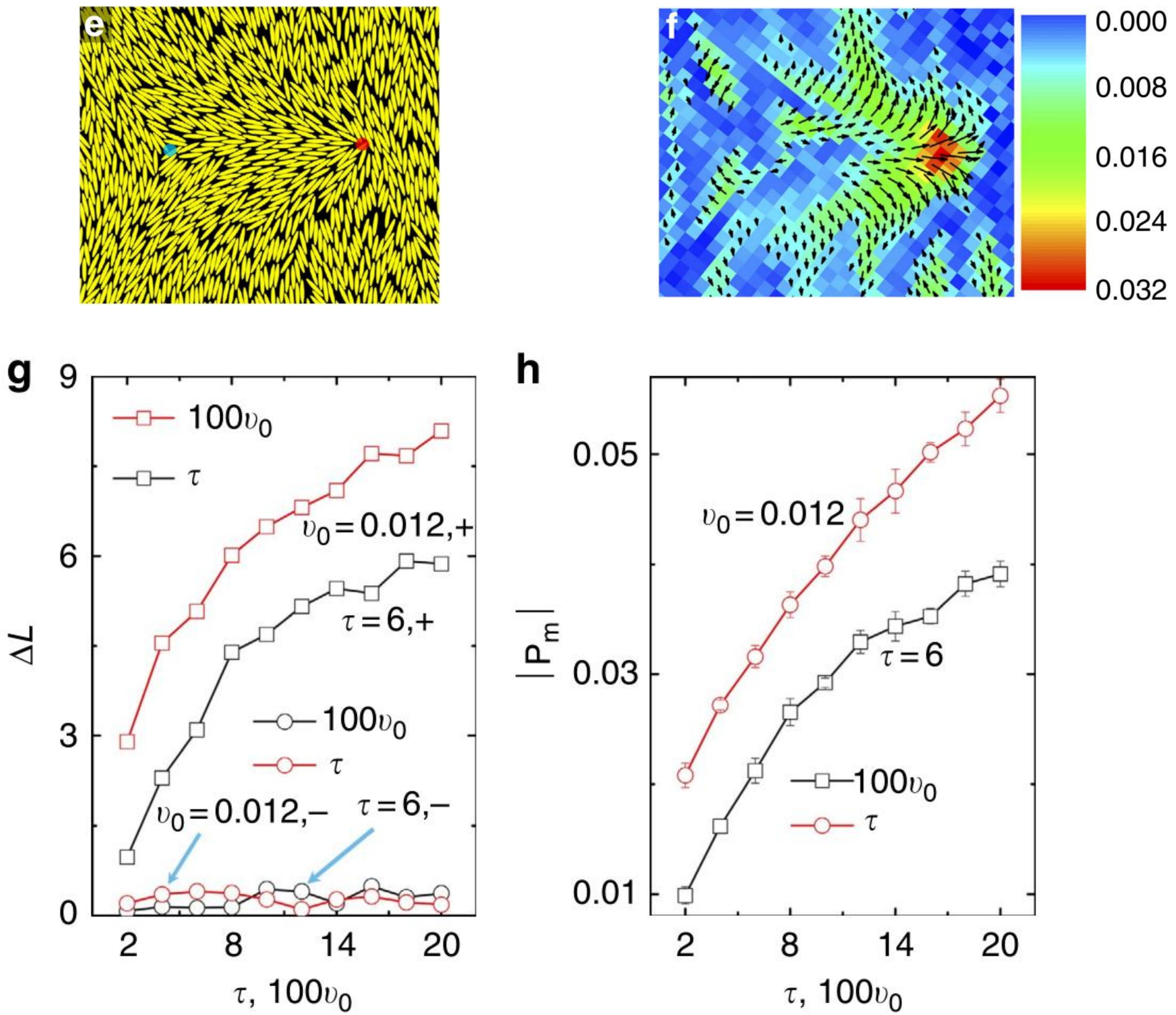
Racing of defects



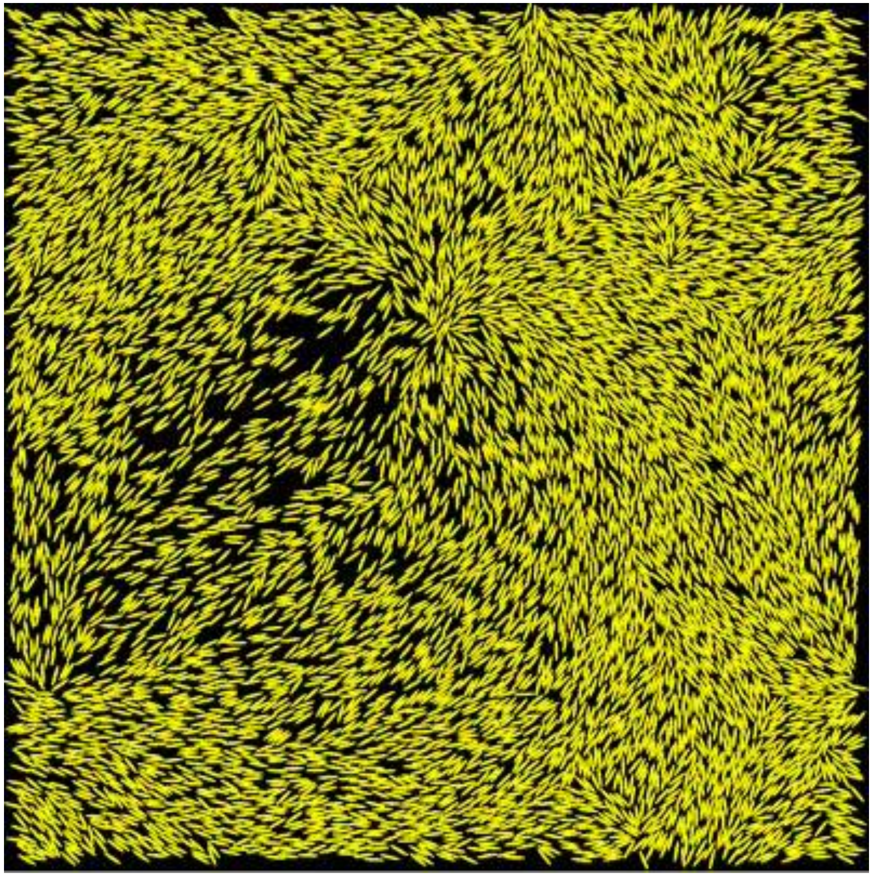


Shi & Ma, Nat. Commun. 2013, 4:3013

Polarity and flows



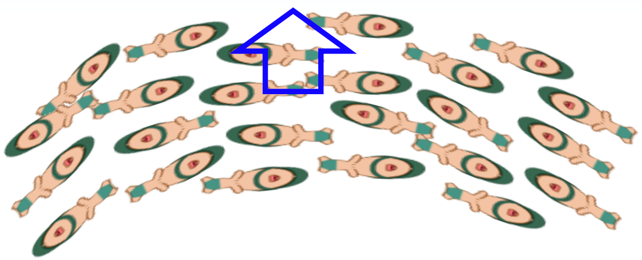
Collective motion in active nematics

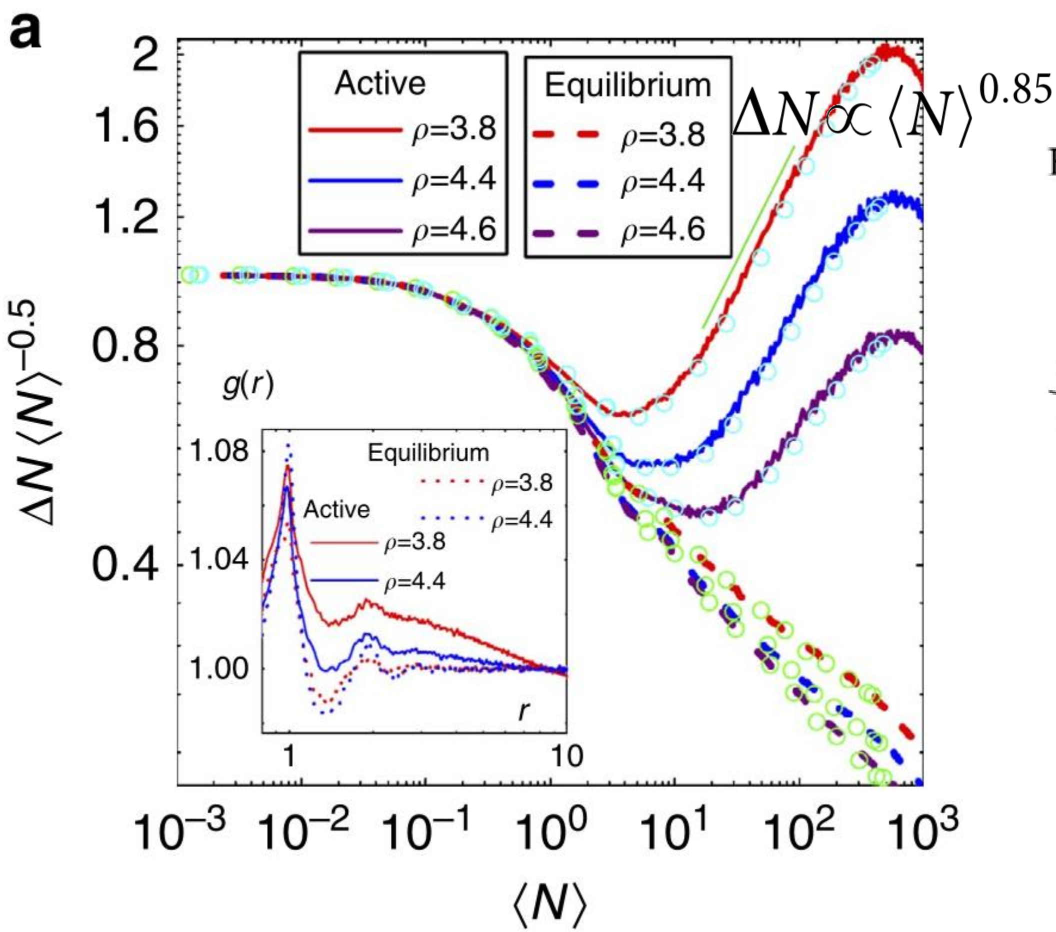


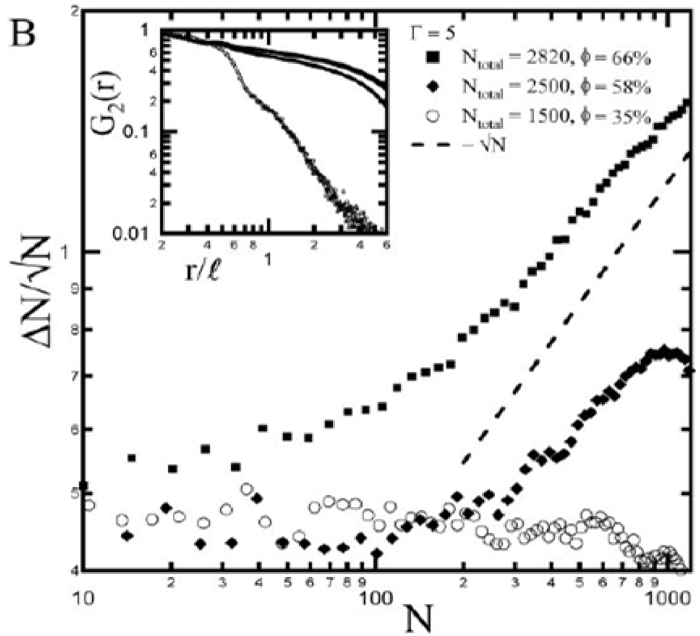
Shi & Ma, Nat. Commun. 2013, 4:3013

Giant number fluctuations

$$\Delta N = \sqrt{\langle N \rangle + \rho_0^2 \int_A \mathbf{dr}_1 \int_A \mathbf{dr}_2 [g(|\mathbf{r}_1 - \mathbf{r}_2|) - 1]},$$





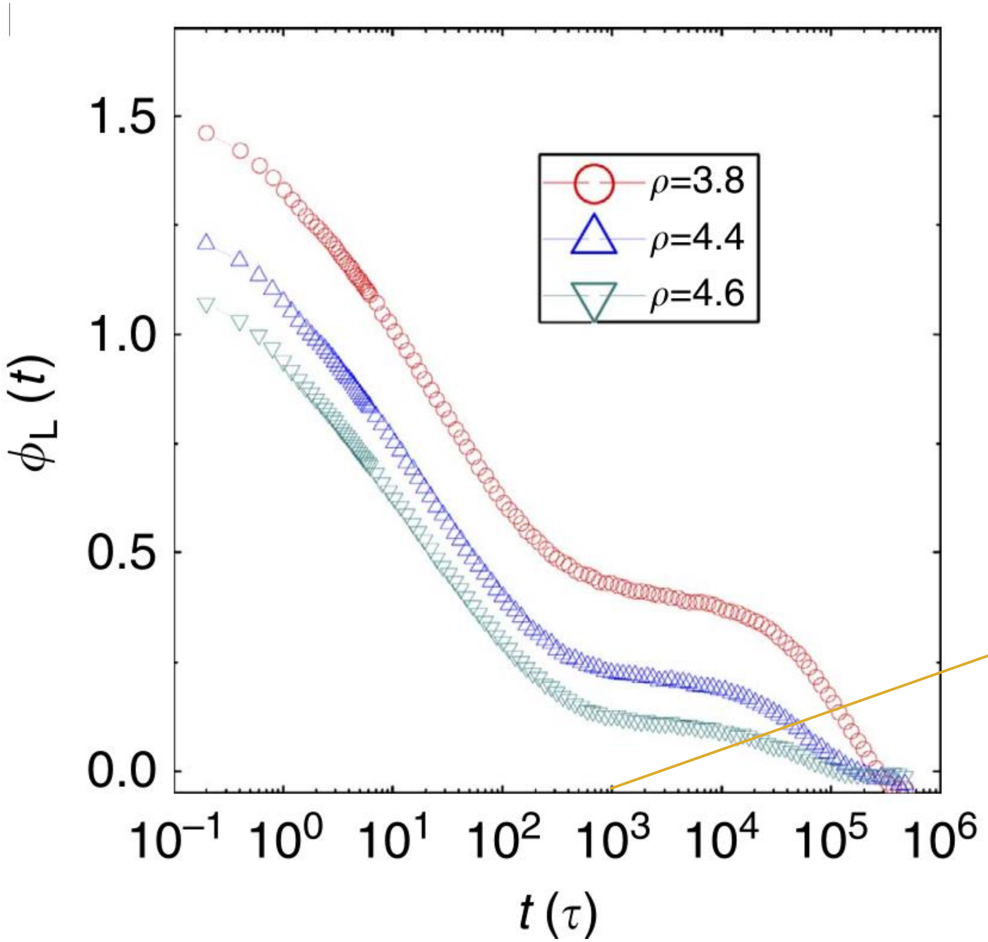


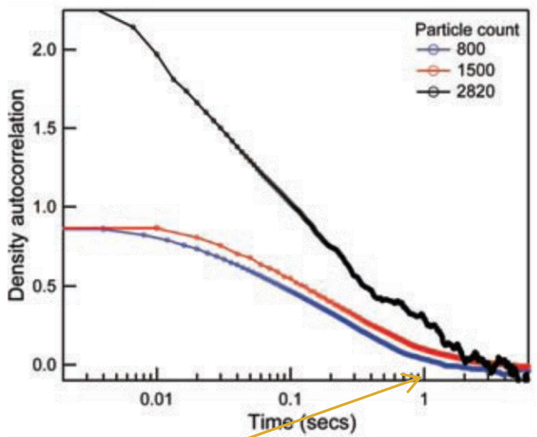
V. Narayan et al. Science (2007).

Shi & Ma, Nat. Commun. 2013, 4:3013

Density Relaxation

Density auto-correlations $\phi(t) = \langle (n_L(t) - \langle n_L(t) \rangle)(n_L(0) - \langle n_L(0) \rangle) \rangle / L^4$

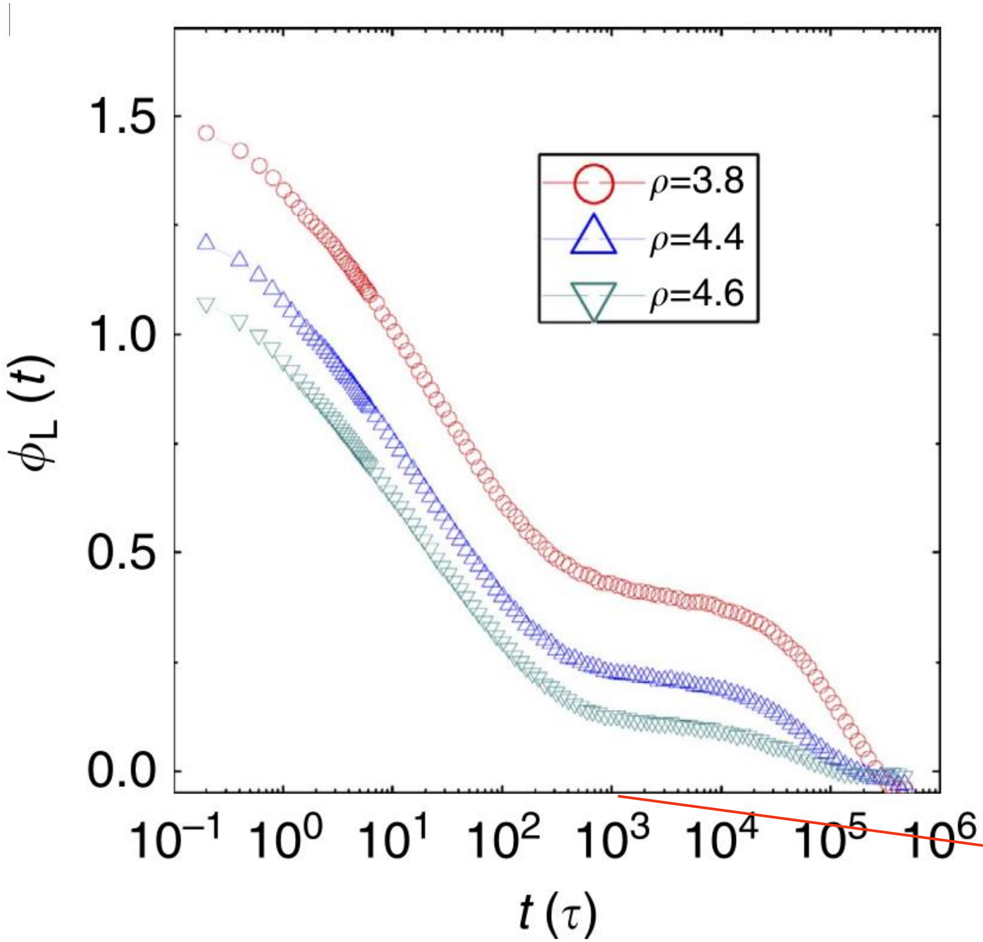


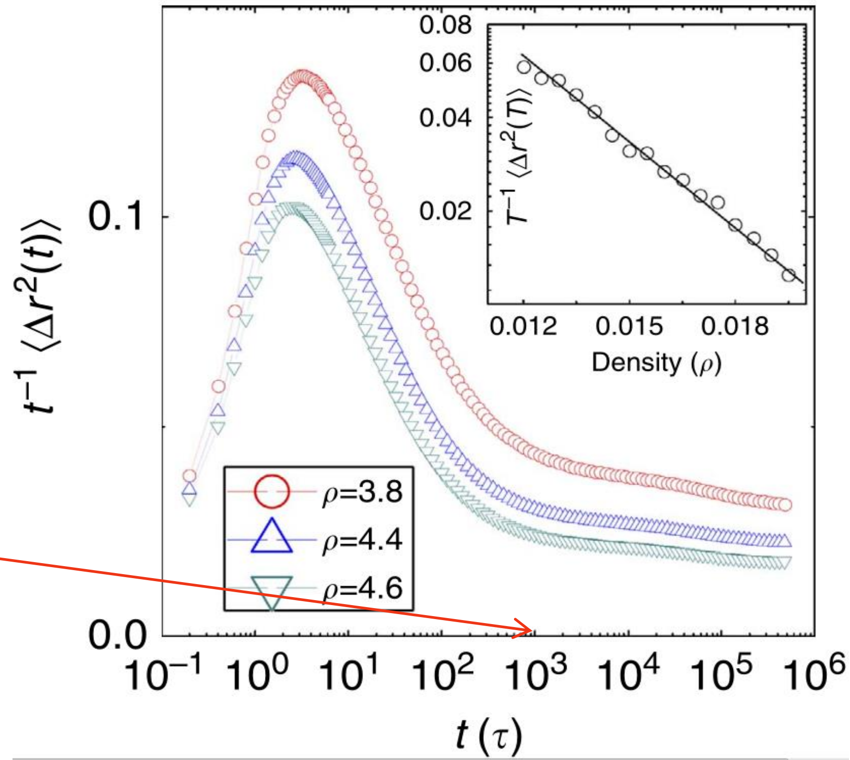


Shi & Ma, Nat. Commun. 2013, 4:3013

Density Relaxation

Density auto-correlations $\phi(t) = \langle (n_L(t) - \langle n_L(t) \rangle)(n_L(0) - \langle n_L(0) \rangle) \rangle / L^4$

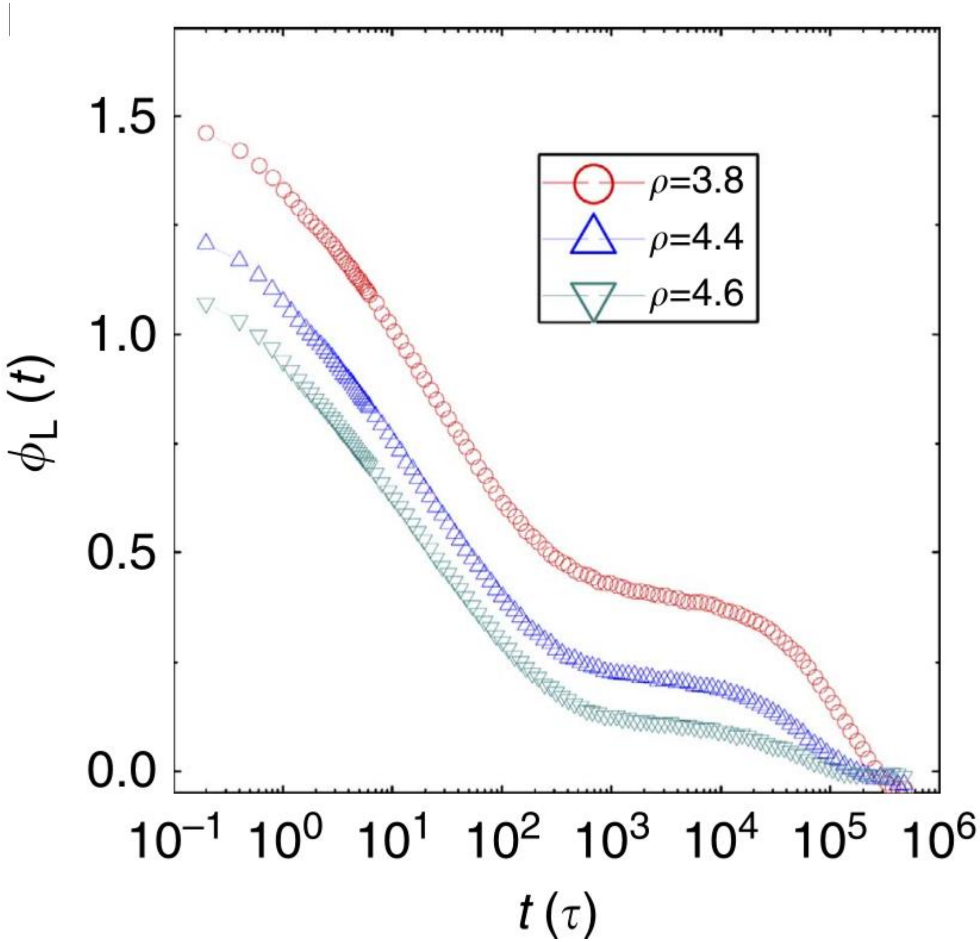


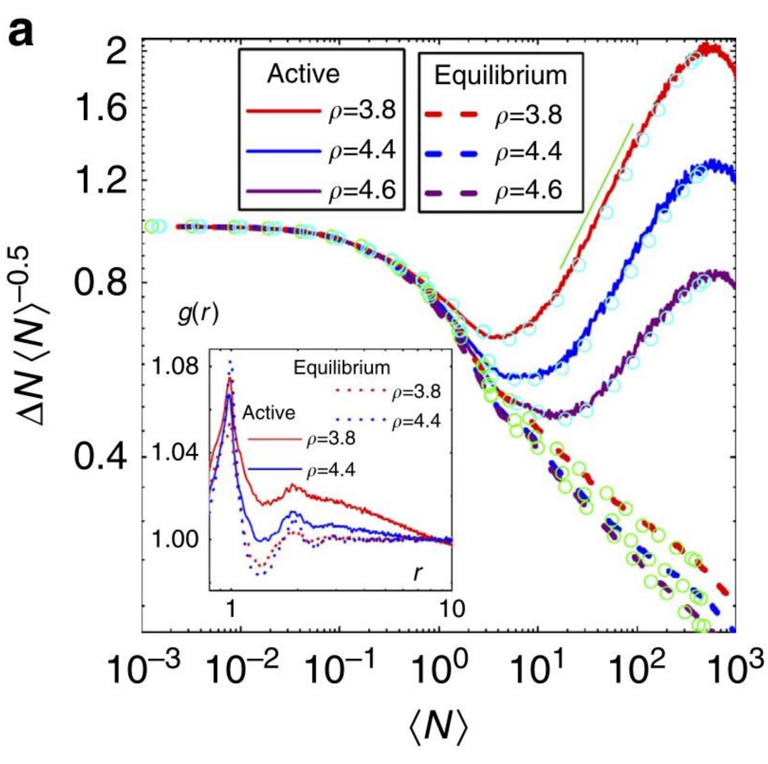


Shi & Ma, Nat. Commun. 2013, 4:3013

Density Relaxation

Density auto-correlations $\phi(t) = \langle (n_L(t) - \langle n_L(t) \rangle)(n_L(0) - \langle n_L(0) \rangle) \rangle / L^4$

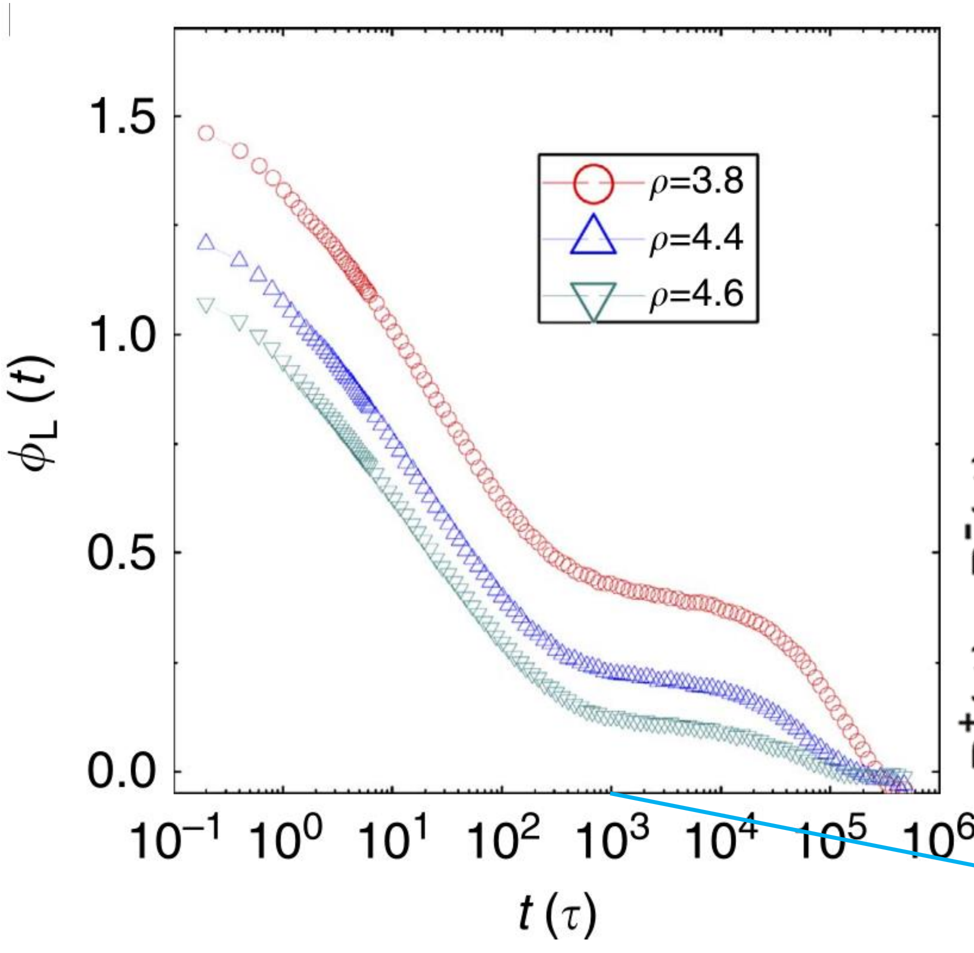


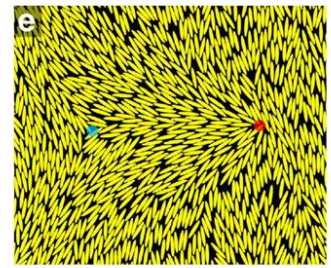


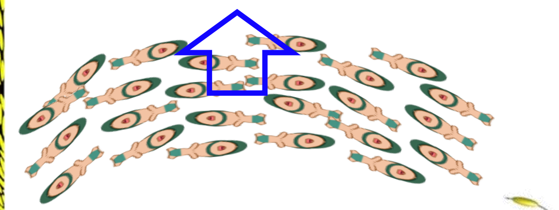
Shi & Ma, Nat. Commun. 2013, 4:3013

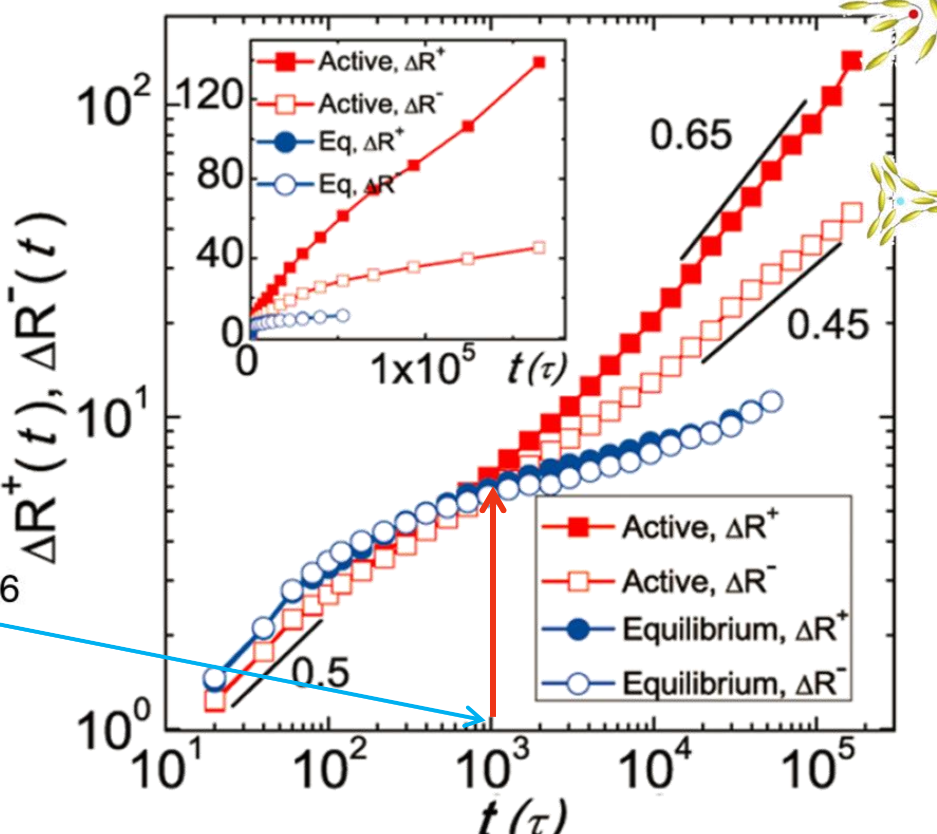
Density Relaxation

Density auto-correlations $\phi(t) = \langle (n_L(t) - \langle n_L(t) \rangle)(n_L(0) - \langle n_L(0) \rangle) \rangle / L^4$



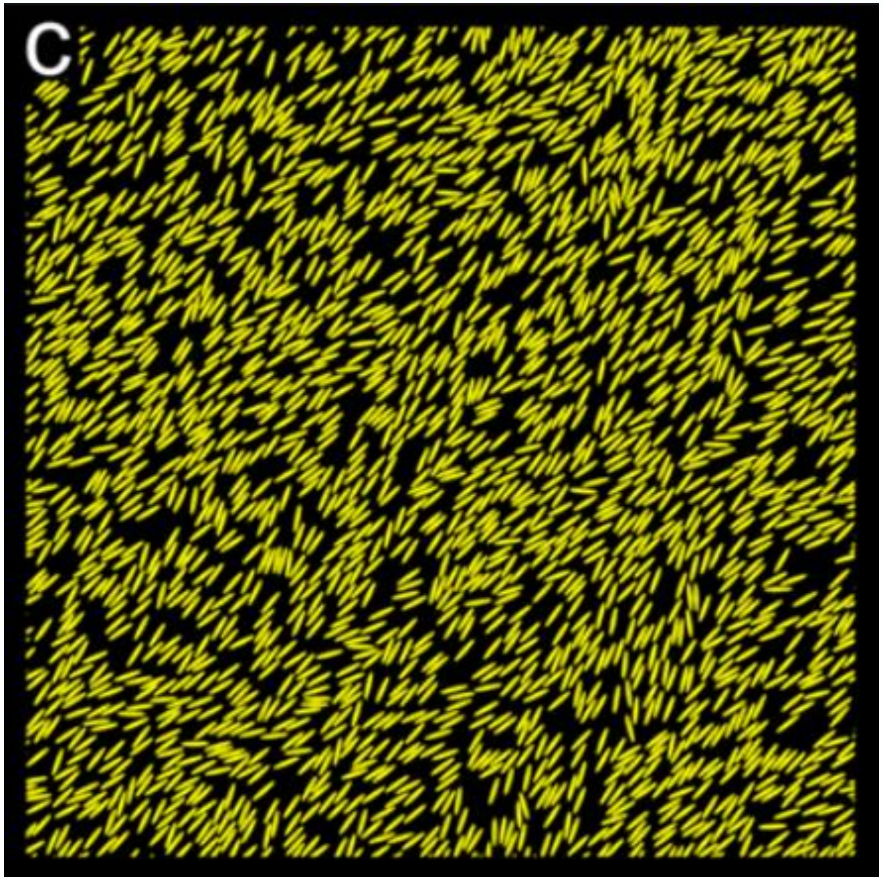


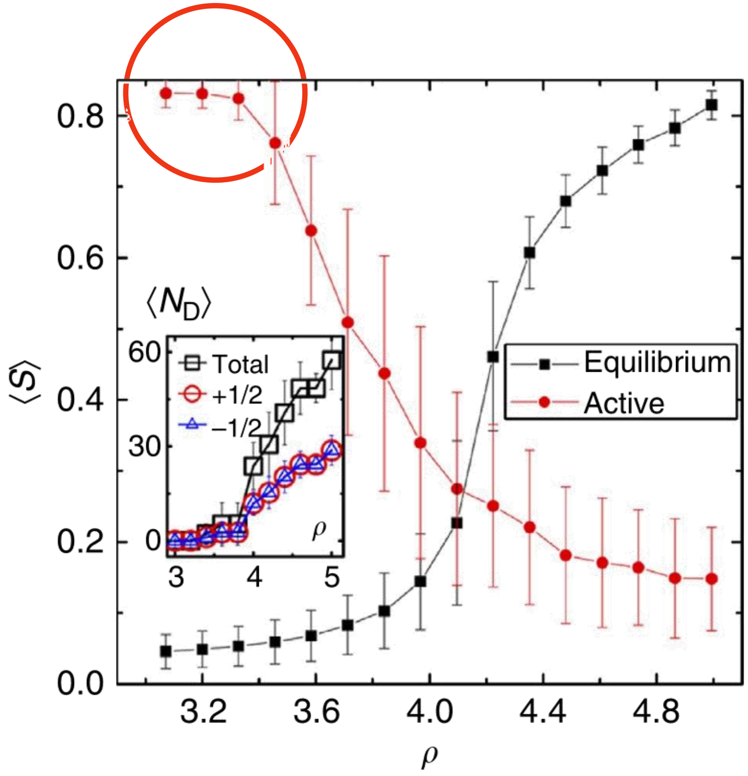




Shi & Ma, Nat. Commun. 2013, 4:3013

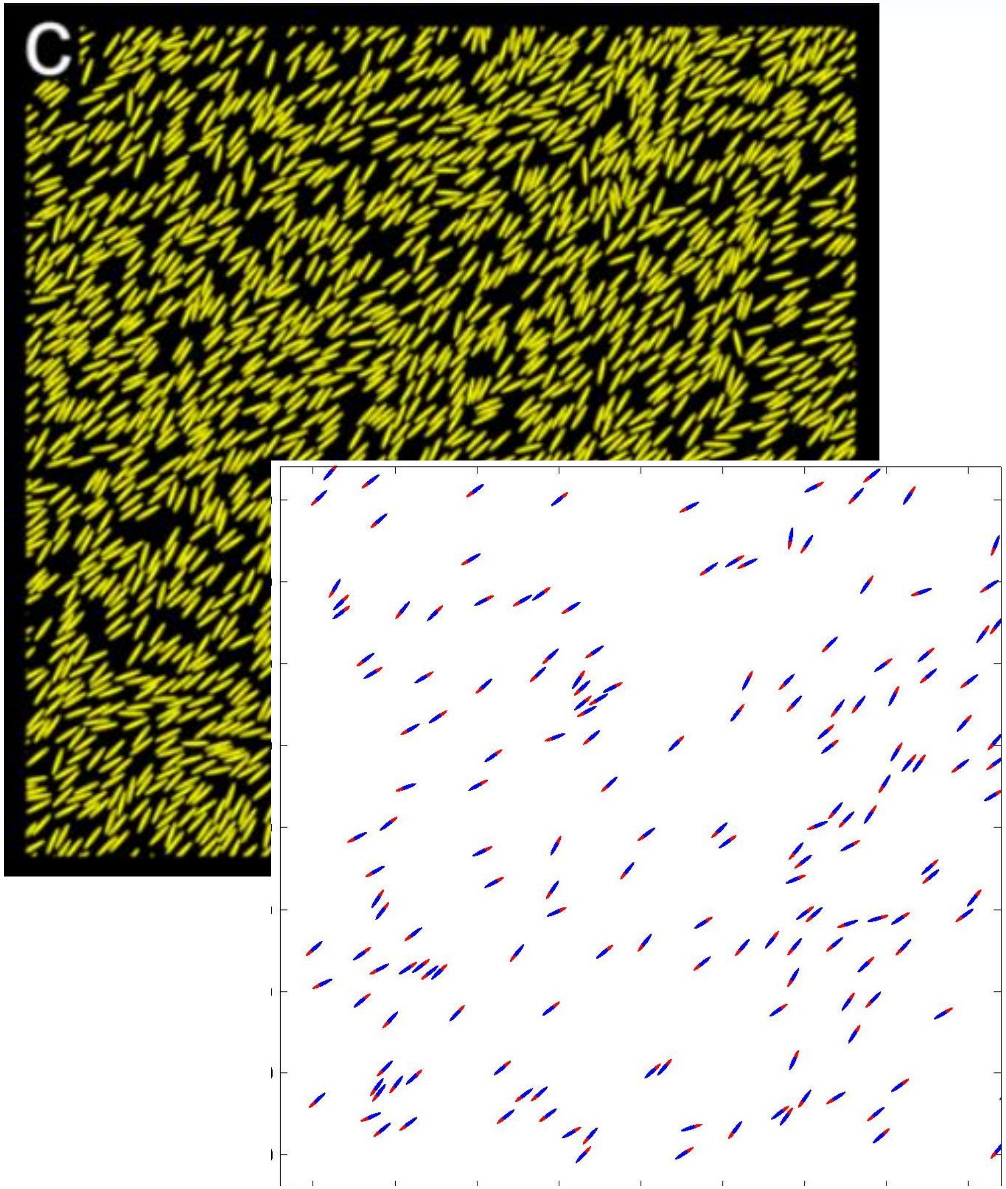
Enhanced ordering effects

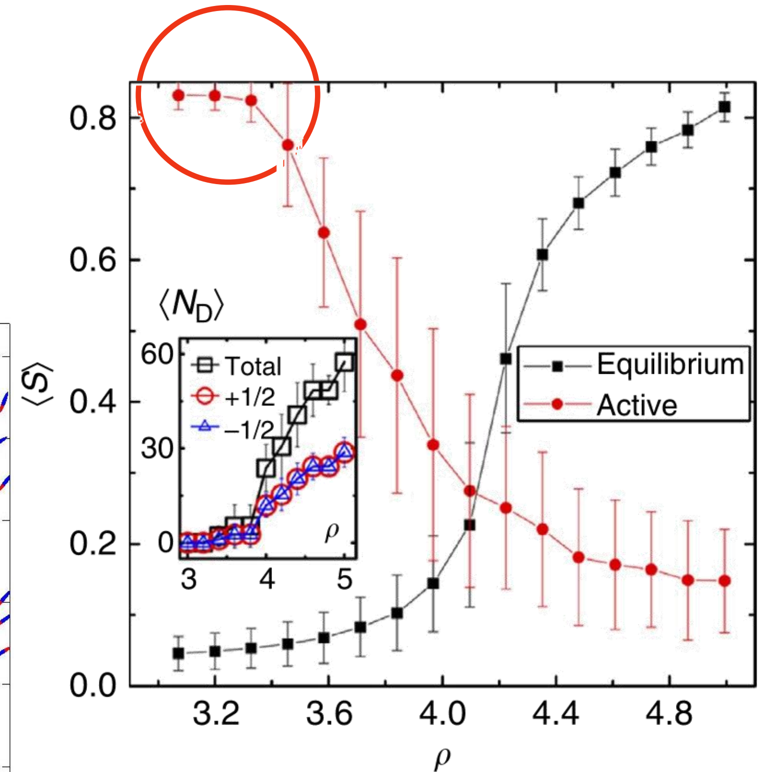




Shi & Ma, Nat. Commun. 2013, 4:3013

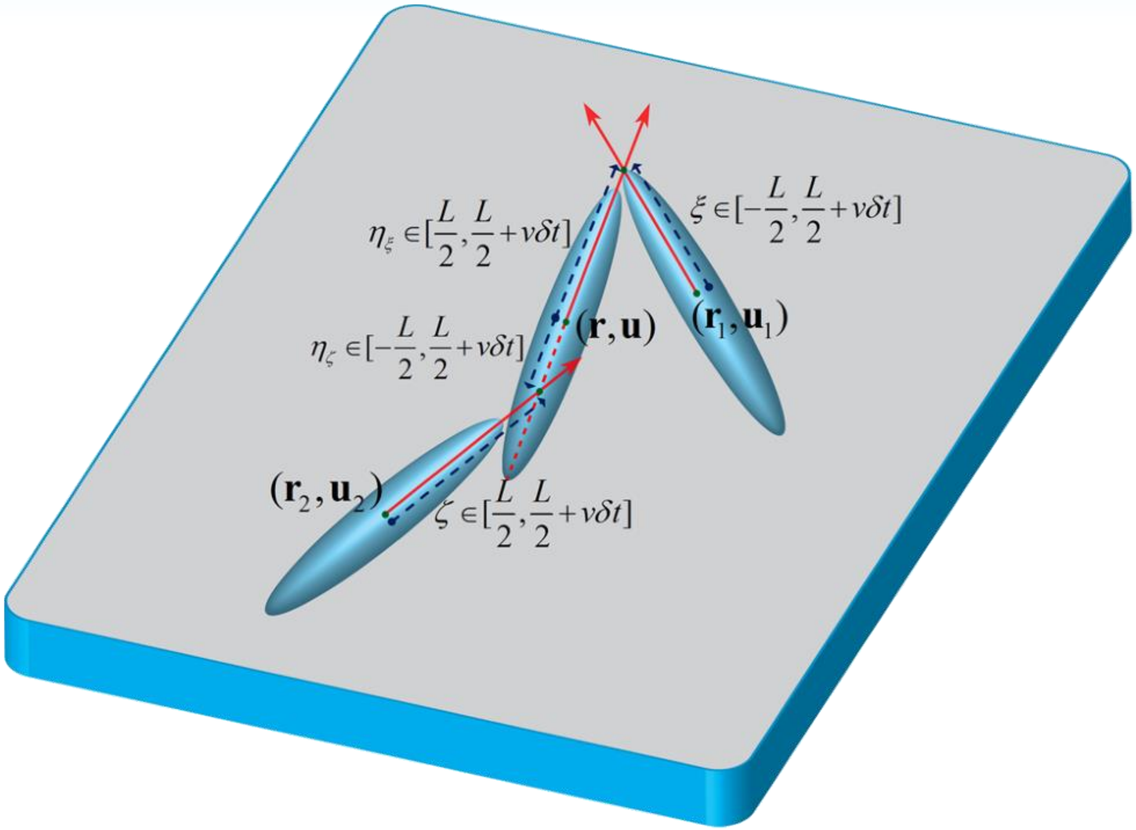
Enhanced ordering effects

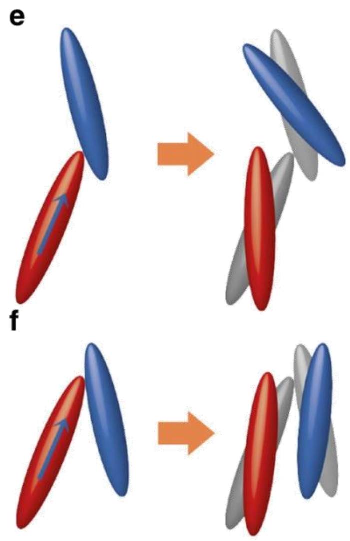




Shi & Ma, Nat. Commun. 2013, 4:3013

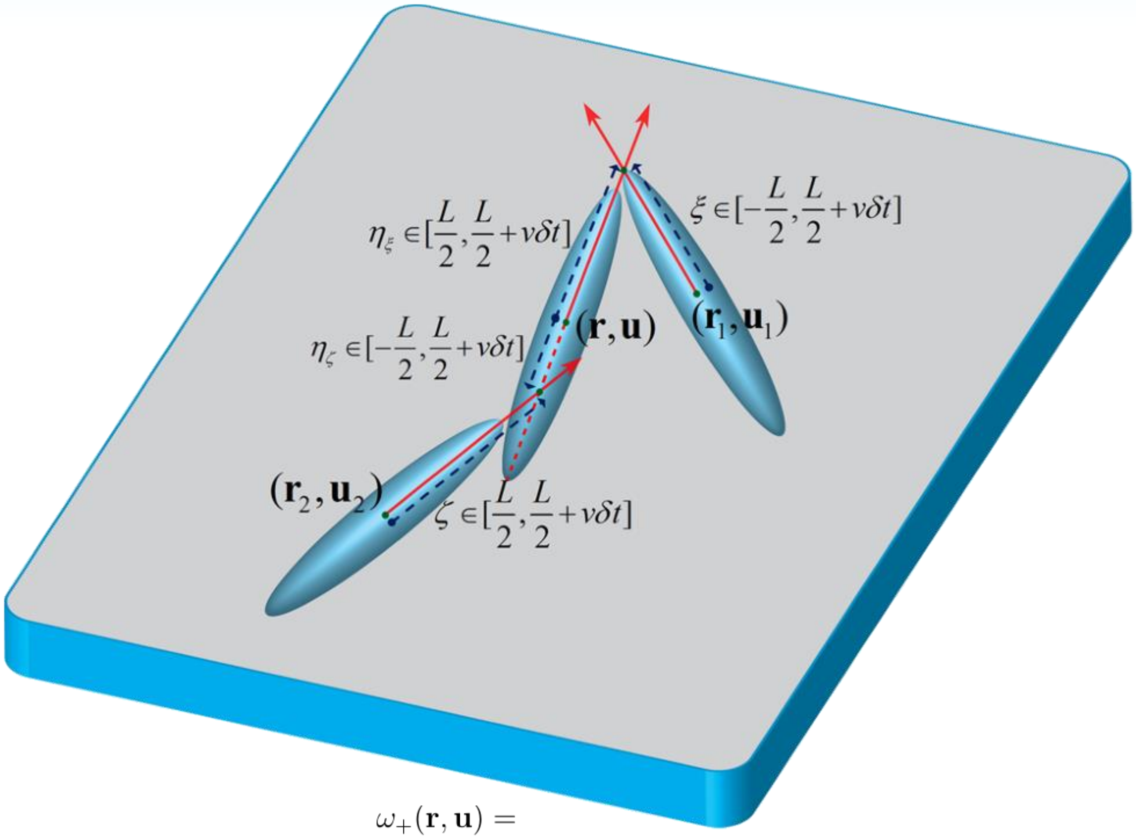
Collision induced rotations

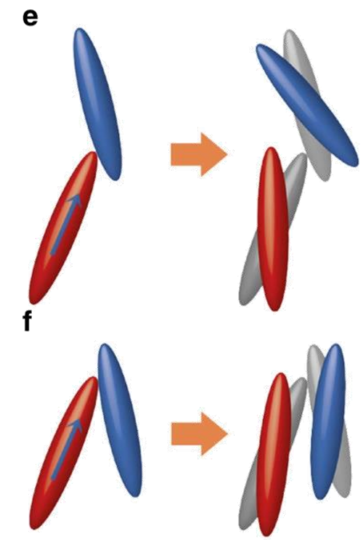




Shi & Ma, Nat. Comms. 2013, 4:3013

Collision induced rotations





$$
\begin{aligned}
&\omega_+(\mathbf{r},\mathbf{u}) = \\
&\sigma_{\mathrm{r}} v \int \mathrm{d}\mathbf{u}' |\mathbf{u}\times\mathbf{u}'| \frac{[\mathbf{u}-(\mathbf{u}\cdot\mathbf{u}')]\times\mathbf{u}}{|[\mathbf{u}-(\mathbf{u}\cdot\mathbf{u}')]\times\mathbf{u}|} \int_{-\frac{L}{2}}^{\frac{L}{2}} \mathrm{d}\xi [f_+(\mathbf{r}+\frac{L}{2}\mathbf{u}-\xi\mathbf{u}',\mathbf{u}') + f_-(\mathbf{r}+\frac{L}{2}\mathbf{u}+\xi\mathbf{u}',\mathbf{u}')] \\
&-\sigma_{\mathrm{r}} v \int \mathrm{d}\mathbf{u}' |\mathbf{u}\times\mathbf{u}'| \frac{[\mathbf{u}'-(\mathbf{u}\cdot\mathbf{u}')]\times\mathbf{u}}{|[\mathbf{u}'-(\mathbf{u}\cdot\mathbf{u}')]\times\mathbf{u}|} \int_{0}^{\frac{L}{2}} \mathrm{d}\eta [f_+(\mathbf{r}+\eta\mathbf{u}-\frac{L}{2}\mathbf{u}',\mathbf{u}') - f_-(\mathbf{r}+\eta\mathbf{u}+\frac{L}{2}\mathbf{u}',\mathbf{u}')] \\
&+\sigma_{\mathrm{r}} v \int \mathrm{d}\mathbf{u}' |\mathbf{u}\times\mathbf{u}'| \frac{[\mathbf{u}'-(\mathbf{u}\cdot\mathbf{u}')]\times\mathbf{u}}{|[\mathbf{u}'-(\mathbf{u}\cdot\mathbf{u}')]\times\mathbf{u}|} \int_{-\frac{L}{2}}^{0} \mathrm{d}\eta [f_+(\mathbf{r}+\eta\mathbf{u}-\frac{L}{2}\mathbf{u}',\mathbf{u}') - f_-(\mathbf{r}+\eta\mathbf{u}+\frac{L}{2}\mathbf{u}',\mathbf{u}')],
\end{aligned}
$$

Shi & Ma, Nat. Commun. 2013, 4:3013

Kinetic equations

$$\partial_t f_\pm + k(f_\pm - f_\mp) \pm \nabla \cdot (\mathbf{v} f_\pm) = D_\mathrm{r}\mathscr{R}^2 f_\pm - \mathscr{R}(\omega_\pm f_\pm)$$

Shi & Ma, Nat. Commun. 2013, 4:3013

Kinetic equations

$$\partial_t f_\pm + k(f_\pm - f_\mp) \pm \nabla \cdot (\mathbf{v} f_\pm) = D_\mathrm{r}\mathscr{R}^2 f_\pm - \mathscr{R}(\omega_\pm f_\pm)$$

- Total number distribution: $f(\mathbf{r},\mathbf{u}) = f_+(\mathbf{r},\mathbf{u}) + f_-(\mathbf{r},\mathbf{u})$

$$\partial_t f(\mathbf{r},\mathbf{u}) + \nabla \cdot [\mathbf{v} f_\mathrm{m}(\mathbf{r},\mathbf{u})] + \mathscr{R}[\omega_+(\mathbf{r},\mathbf{u}) f_+(\mathbf{r},\mathbf{u}) + \omega_-(\mathbf{r},\mathbf{u}) f_-(\mathbf{r},\mathbf{u})] = D_\mathrm{r}\mathscr{R}^2 f(\mathbf{r},\mathbf{u})$$

$$\partial_t f_\mathrm{m}(\mathbf{r},\mathbf{u}) + \nabla \cdot [\mathbf{v} f(\mathbf{r},\mathbf{u})] + \mathscr{R}[\omega_+(\mathbf{r},\mathbf{u}) f_+(\mathbf{r},\mathbf{u}) - \omega_-(\mathbf{r},\mathbf{u}) f_-(\mathbf{r},\mathbf{u})] = - 2k f_\mathrm{m}(\mathbf{r},\mathbf{u}) + D_\mathrm{r}\mathscr{R}^2 f_\mathrm{m}(\mathbf{r},\mathbf{u}).$$

Shi & Ma, Nat. Commun. 2013, 4:3013

Kinetic equations

$$\partial_t f_\pm + k(f_\pm - f_\mp) \pm \nabla \cdot (\mathbf{v} f_\pm) = D_\mathrm{r}\mathscr{R}^2 f_\pm - \mathscr{R}(\omega_\pm f_\pm)$$

- Total number distribution: $f(\mathbf{r},\mathbf{u}) = f_+(\mathbf{r},\mathbf{u}) + f_-(\mathbf{r},\mathbf{u})$

$$\partial_t f(\mathbf{r},\mathbf{u}) + \nabla \cdot [\mathbf{v} f_\mathrm{m}(\mathbf{r},\mathbf{u})] + \mathscr{R}[\omega_+(\mathbf{r},\mathbf{u}) f_+(\mathbf{r},\mathbf{u}) + \omega_-(\mathbf{r},\mathbf{u}) f_-(\mathbf{r},\mathbf{u})] = D_\mathrm{r}\mathscr{R}^2 f(\mathbf{r},\mathbf{u})$$

$$\partial_t f_\mathrm{m}(\mathbf{r},\mathbf{u}) + \nabla \cdot [\mathbf{v} f(\mathbf{r},\mathbf{u})] + \mathscr{R}[\omega_+(\mathbf{r},\mathbf{u}) f_+(\mathbf{r},\mathbf{u}) - \omega_-(\mathbf{r},\mathbf{u}) f_-(\mathbf{r},\mathbf{u})] = - 2k f_\mathrm{m}(\mathbf{r},\mathbf{u}) + D_\mathrm{r}\mathscr{R}^2 f_\mathrm{m}(\mathbf{r},\mathbf{u}).$$

$$f_\mathrm{m}(\mathbf{r},\mathbf{u}) = f_+(\mathbf{r},\mathbf{u}) - f_-(\mathbf{r},\mathbf{u})$$

- Homogeneous condition: $\partial_t f(\mathbf{u}) = -\sigma_\mathrm{r} v \mathscr{R}[f(\mathbf{u})\mathscr{R}(W(\mathbf{u}))] + D_\mathrm{r}\mathscr{R}^2 f(\mathbf{u}),$

$$W(\mathbf{u}) = \int \mathrm{d}\mathbf{u}' |\mathbf{u}\cdot\mathbf{u}'| f(\mathbf{u}')$$

Shi & Ma, Nat. Commun. 2013, 4:3013

Kinetic equations

$$\partial_t f_{\pm} + k(f_{\pm} - f_{\mp}) \pm \nabla \cdot (\mathbf{v} f_{\pm}) = D_{\rm r}\mathscr{R}^2 f_{\pm} - \mathscr{R}(\omega_{\pm} f_{\pm})$$

- Total number distribution: $f(\mathbf{r},\mathbf{u}) = f_+(\mathbf{r},\mathbf{u}) + f_-(\mathbf{r},\mathbf{u})$

$$\partial_t f(\mathbf{r},\mathbf{u}) + \nabla \cdot [\mathbf{v} f_{\rm m}(\mathbf{r},\mathbf{u})] + \mathscr{R}[\omega_+(\mathbf{r},\mathbf{u})f_+(\mathbf{r},\mathbf{u}) + \omega_-(\mathbf{r},\mathbf{u})f_-(\mathbf{r},\mathbf{u})] = D_{\rm r}\mathscr{R}^2 f(\mathbf{r},\mathbf{u})$$

$$\partial_t f_{\rm m}(\mathbf{r},\mathbf{u}) + \nabla \cdot [\mathbf{v} f(\mathbf{r},\mathbf{u})] + \mathscr{R}[\omega_+(\mathbf{r},\mathbf{u})f_+(\mathbf{r},\mathbf{u}) - \omega_-(\mathbf{r},\mathbf{u})f_-(\mathbf{r},\mathbf{u})] = - 2k f_{\rm m}(\mathbf{r},\mathbf{u}) + D_{\rm r}\mathscr{R}^2 f_{\rm m}(\mathbf{r},\mathbf{u}).$$

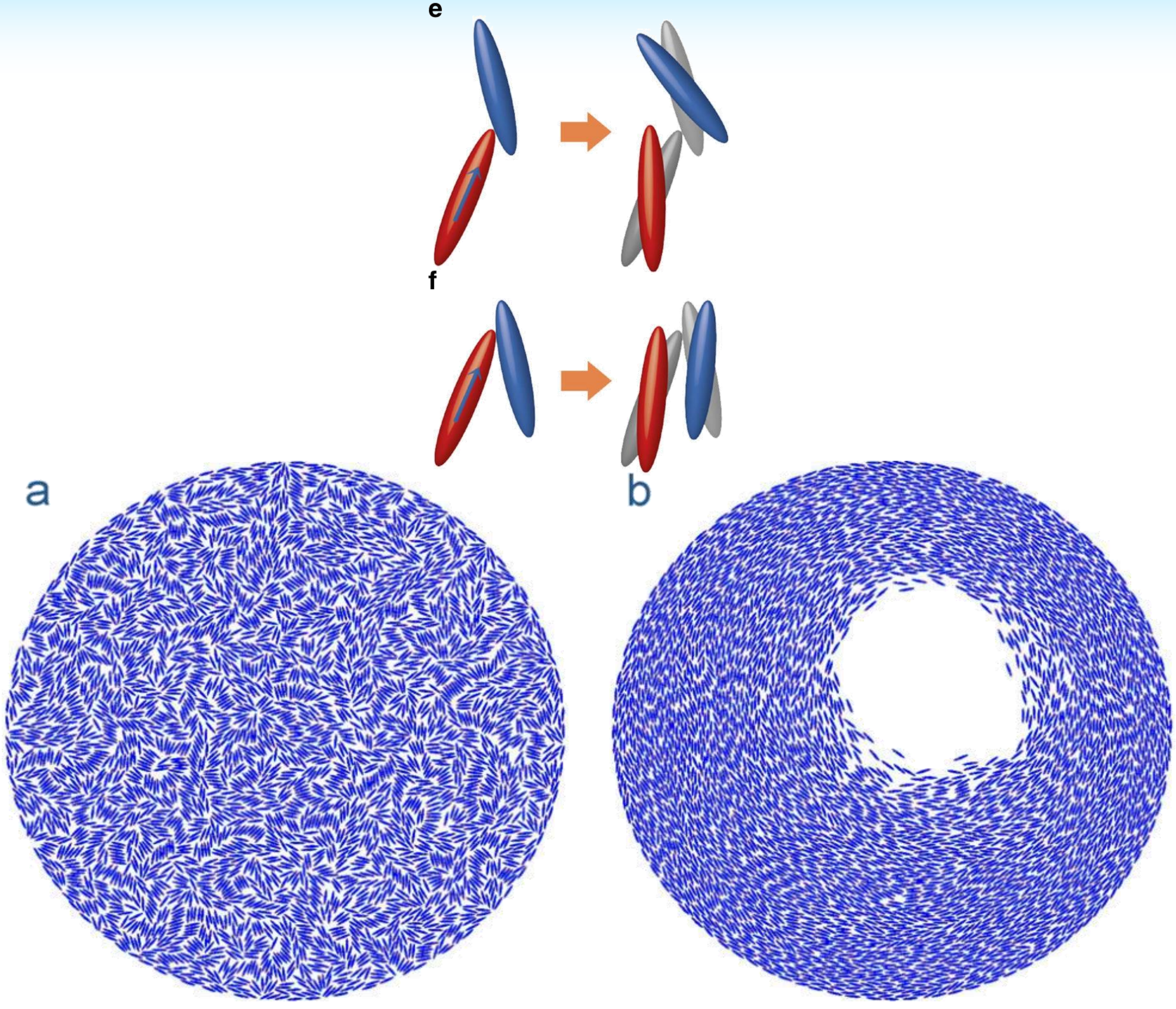
$$f_{\rm m}(\mathbf{r},\mathbf{u}) = f_+(\mathbf{r},\mathbf{u}) - f_-(\mathbf{r},\mathbf{u})$$

- Homogeneous condition: $\partial_t f(\mathbf{u}) = -\sigma_{\rm r} v \mathscr{R}[f(\mathbf{u})\mathscr{R}(W(\mathbf{u}))] + D_{\rm r}\mathscr{R}^2 f(\mathbf{u}),$

  $W(\mathbf{u}) = \int \mathrm{d}\mathbf{u}' |\mathbf{u}\cdot\mathbf{u}'| f(\mathbf{u}')$

- Linear instability for isotropic state: $\partial_t S = (-4D_{\rm r} + \frac{8\sigma_{\rm r} v}{3\pi}\rho)S.$
- Critical density $\rho > D_{\rm r}\frac{3\pi}{2\sigma_{\rm r} v}$

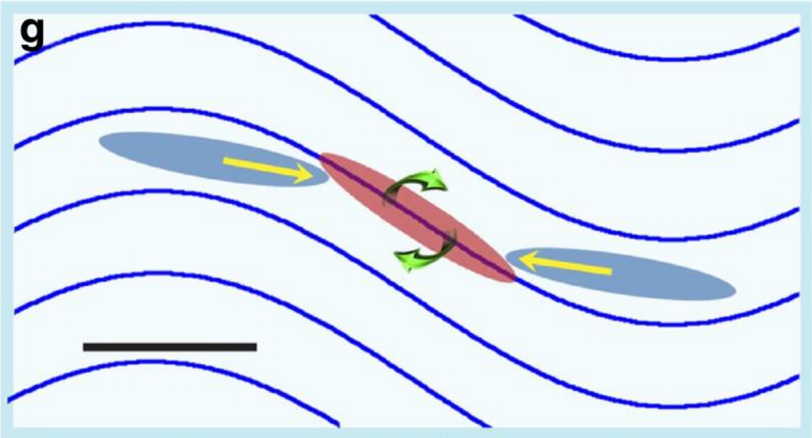
Shi & Ma, Nat. Commun. 2013, 4:3013

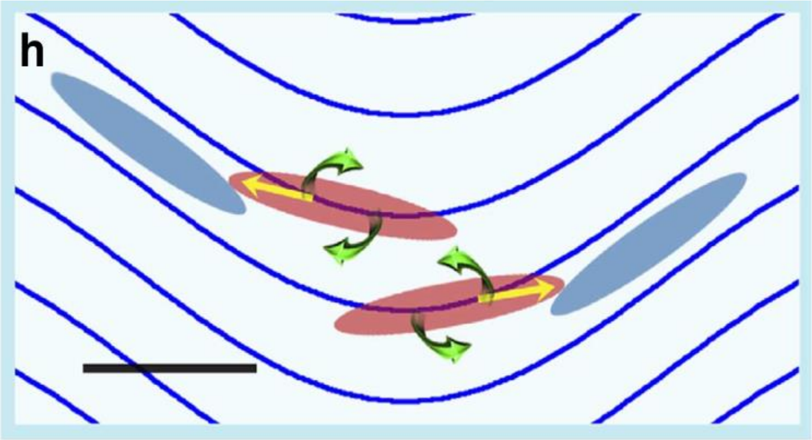


e
f
a
b

$$\partial_t \delta n_y = \left(\frac{v^2\rho}{4k} + \frac{\sigma_r v \rho^2}{3\pi} + \frac{4\sigma_r v \rho^2}{45\pi} - \frac{\sigma_r v \rho^2 (v+k)}{8k} - \frac{4\sigma_r v^2 \rho^2}{3\pi k}\right)\partial_x^2 \delta n_y + \left(\frac{v^2\rho}{4k} + \frac{2\sigma_r v \rho^2}{15\pi}\right)\partial_y^2 \delta n_y$$

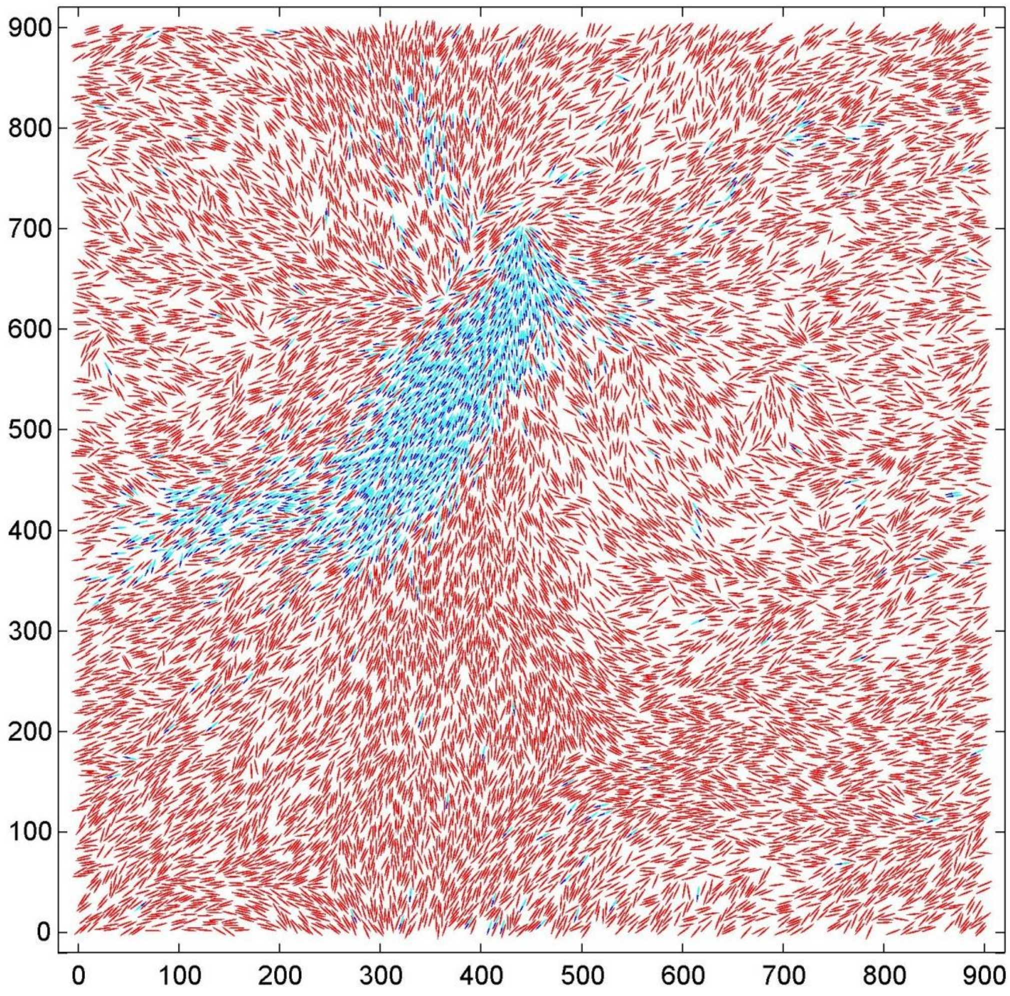
- Linear instability $\boxed{\rho > \rho^\star = \frac{90\pi v}{\sigma_r[(45\pi-152)k+(45\pi+240)v]}}$ $\boxed{v/k > \frac{152/15-3\pi}{16+3\pi}}$



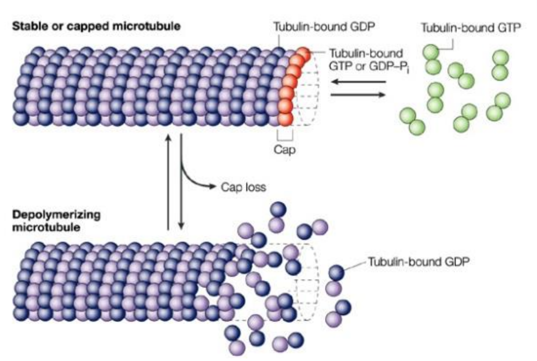


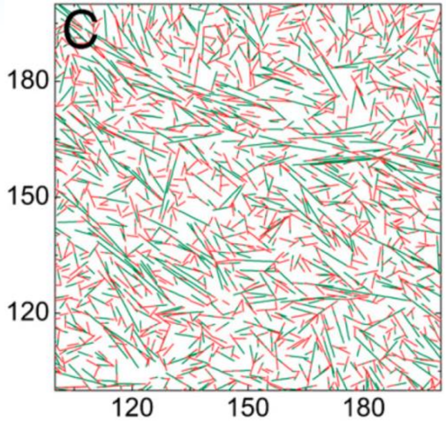
Shi & Ma, Nat. Commun. 2013, 4:3013

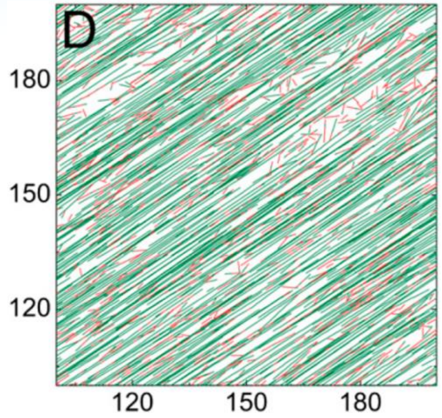
Regulation of collective motion through topological defects

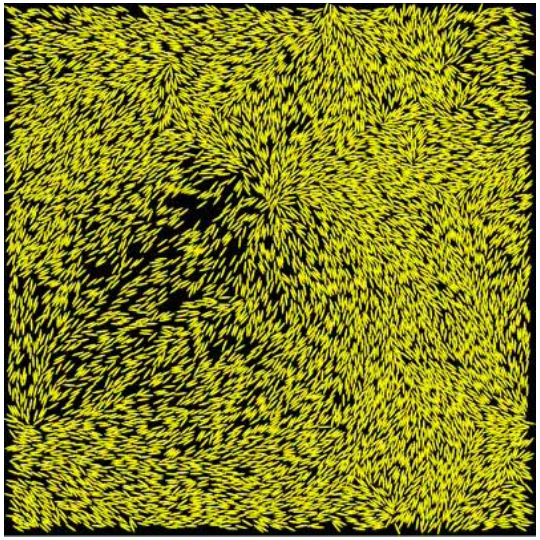


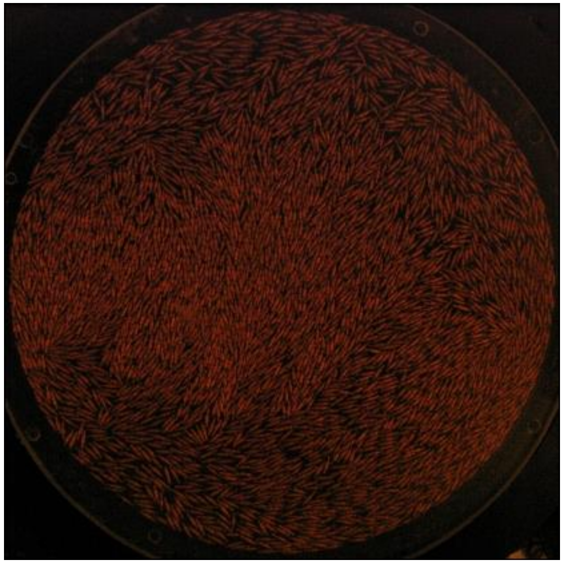
Conclusions











Thank you for your attention!