Mechanical Force Inference in Epithelial Cell Monolayers

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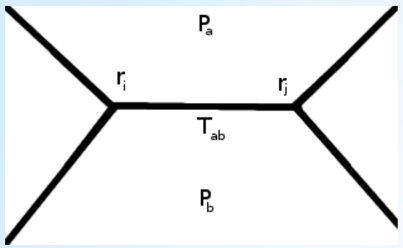
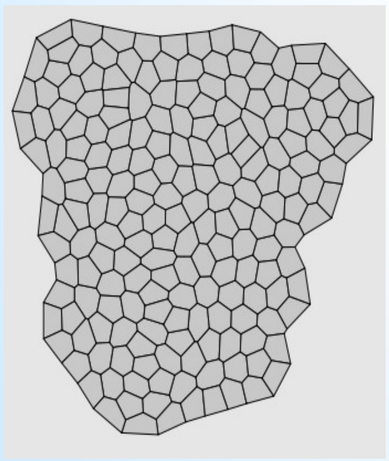
Generalized Vertex Model



$$dH[L_{ab}, A_a] = \sum_{a,b \in cells} T^{ab} dL_{ab} + \sum_{a \in cells} P^a dA_a$$

$$F^{\mu}_{i,j} = T_{ab} \overbrace{\frac{(r_j - r_i)^{\mu}}{|r_j - r_i|}}^{Tension\ Term} + \overbrace{\frac{1}{2}(P_a - P_b)(r_j - r_i)_{\nu} \epsilon^{\mu\nu}}^{Pressure\ Term}$$

$$\frac{\partial H}{\partial L_{ab}} = T_{ab} \quad , \quad \frac{\partial H}{\partial A_a} = P_a$$



$$\frac{\partial H}{\partial r_i} = \sum_{a \in cells} P^a \frac{\partial A_a}{\partial r_i} + \sum_{a,b \in cells} T^{ab} \frac{\partial L_{ab}}{\partial r_i}$$

Mechanical Equilibrium Condition:

$$0 = \sum_{a \in cells} P^a \frac{\partial A_a}{\partial r_i} + \sum_{a,b \in cells} T^{ab} \frac{\partial L_{ab}}{\partial r_i}$$

$$\vec{0} = M \vec{\Psi} \quad , \quad \vec{\Psi} = \begin{pmatrix} T_{ab} \\ \vdots \\ P_a \end{pmatrix}$$

Inference of Parameters

Consider a cell array with b bonds, v vertices, and c cells with B boundary cells:

$$Euler\ characteristic : v + c - b = 1$$
$$Vertex / Bond\ counting : 3v = 2b$$
$$Unknowns : 2v - B - 1$$
$$Constraints : 2(v - B)$$
$$Deg.\ of\ freedom : B - 1$$

Set the overall scale, and we can infer parameters up to B-1 degrees of freedom:

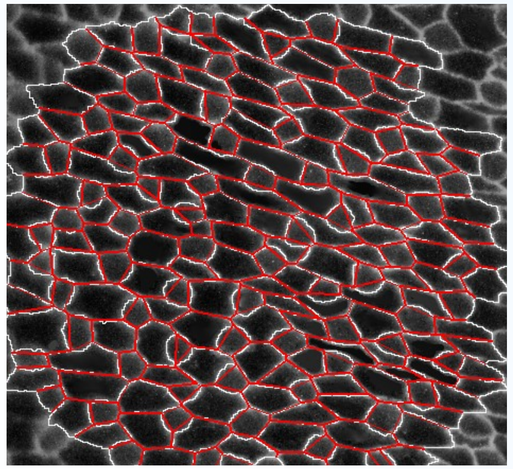
$$M \vec{\Psi} = \vec{f}$$

$$\vec{\Psi} = \tilde{M}^{-1} \vec{f} + \sum_{\nu=1}^{B-1} A_{\nu} \vec{\phi}_{\nu}$$

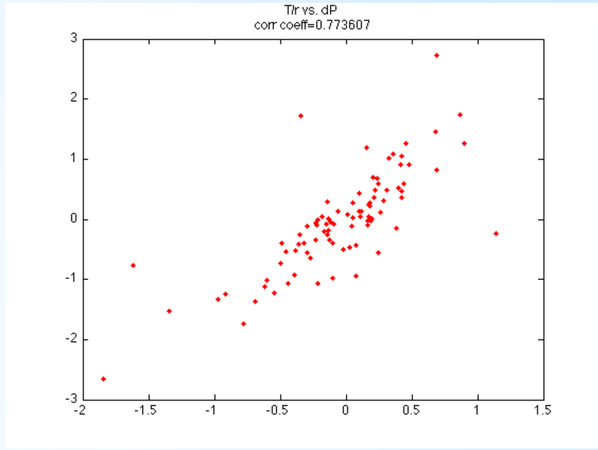
$$M \vec{\phi}_{\nu} = \vec{0} \quad , \quad \nu = \{1, \ldots, B-1\}$$

Avian Cochleal Epithelium

- Precursor hair cells exhibit different morphology than support cells during development
- UseYoung-Laplace equation to calculate pressure difference expected, and correlate with inferred pressure values.
- Assume one tension per cell to sufficiently constrain inversion.
- Results show high correlation between Y-L and inferred values.

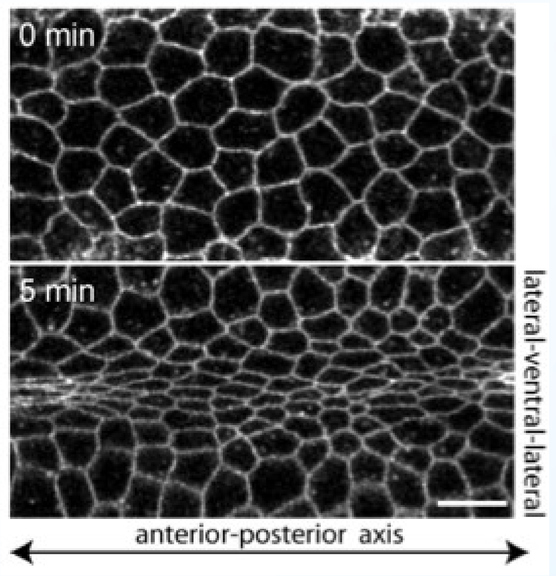


Goodyear, R; Richardson, G, 1997



Drosophila Ventral Furrow Formation

- Time lapse data allows us to test for changes in parameters at different times.
- Examine differences in distribution of tensions above and below some critical angle.
- Assume mechanics is dominated by cortical tension effects to sufficiently constrain inversion.
- Results indicate increased tension anisotropy as time progresses.



Martin, A. C . et al 2010